

Constructing Robust Graphs for Community Detection

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Abstract

We present a framework called *Locally Boosted Graph Aggregation* for aggregating multiple noisy networks into a single network so as to improve the quality of community detection algorithms on the result. LBGA addresses the problem of finding a single network that faithfully and robustly represents multiple noisy, complementary underlying data sources. We define a new random graph model to model such scenarios in community detection called the *local stochastic block model* (LSBM), and we exhibit the utility of LBGA on this synthetic model as well as real data sets. LBGA outperforms existing network aggregation algorithms when ground truth is available, and it produces high-quality representations of real networks. We argue that LBGA is generic and can be adapted to other application domains.

1 Introduction

Community detection methods in machine learning literature generally assume a single truthful input graph. However, in practical scenarios the data that comprise a network come from multiple sources which may be noisy and may disagree [17]. For example, in a social network one may communicate with friends via Instagram and family via Facebook. The best way to aggregate this information is unclear, and the choice of representation heavily impacts the performance of subsequent data mining algorithms [13; 6; 21]. Though the impact of graph representation on subsequent analysis has been studied, few techniques exist for learning conducive graph representations. Aggregation is often ad-hoc in practice, making it difficult to compare algorithms within the same domain using different data sources.

In this paper, we study the problem of constructing a single graph representation that accurately reflects the underlying network structure and allows for better detection of communities scattered across different data sources. We present an aggregation framework called *Locally Boosted Graph Aggregation* (LBGA) which simulates an iterative reward system inspired by boosting and bandit learning. LBGA evaluates the quality of edges locally, so that it can choose aggregations

which most accurately represent the local structure of communities observed in real networks [1; 17]. LBGA relies on the pair of a simple clustering algorithm and a local heuristic quality measure as a proxy for evaluating the quality of intermediate results. We empirically show that our algorithm constructs robust aggregated graph representations for community detection by testing it on synthetic and real-world data sets and comparing it to existing aggregation algorithms.

We emphasize that while this paper specifically addresses community detection, the edge reward mechanism and the graph aggregation steps of LBGA are application agnostic. LBGA can be repurposed for graph aggregation with respect to other applications, a direction we leave for future work. The paper is organized as follows. In Section 2 we review related literature. In Section 3 we discuss in detail the LBGA framework. In Section 4 we present the experimental analysis and results, and Section 5 we discuss future work.

2 Related work

2.1 Graph representation learning and clustering

Our work generally falls under representation learning for graphs, which includes modeling decisions about the nodes and edges of the graph. Rossi et al. [28] taxonomize this field, and show that transformations to heterogeneous graphs can improve the quality of a learning algorithm. Within their taxonomy our work falls under link re-weighting, which includes the work of [35; 14]. Our setting deviates from these works by allowing different edge types between the same pair of vertices. Also, our approach is stochastic, which we find necessary for learning a robust representation.

[33] develops a cross-diffusion based fusion framework called SNF. Both SNF and LBGA emphasize local similarities versus global ones. SNF’s framework of iterative re-weighting of edges based on message-passing is similar to LBGA. The LBGA framework is different because the re-weighting is based on techniques from boosting, and unlike SNF our algorithm does not rely on consistency across the input graphs. In Section 4.5 we demonstrate empirically how LBGA outperforms SNF on all of our synthetic data sets.

Clustering in multilayer networks [25; 32; 31; 22; 3; 16; 30] also has close connections to our work. However, the literature does not address scenarios where the information provided by the different sources is complementary or the

overlap is scarce. Our approach iteratively selects those edge sources that lead to better clustering quality, independently of disagreement across the different features. Also, these approaches differ fundamentally from LBGA in that they do not produce a graph, which could be used for other purposes. As an example of multi-edge clustering algorithms, we consider the GraphFuse algorithm [25] that falls under the category of tensor-based clustering. GraphFuse computes the clustering based on the CP decomposition of the tensor formed by appending the adjacency matrices of the different graph sources. In Section 4.5 we demonstrate that LBGA outperforms GraphFuse in terms of recovering the ground truth clustering across different datasets considered.

[27; 7] present approaches for identifying the right graph aggregation, given a complete ground truth clustering or a portion of it. Our framework requires no such knowledge, but we do use ground truth to validate our experiments on synthetic data (Section 4.3).

2.2 Boosting and bandits

Our framework departs from previous work primarily through its algorithmic inspirations, namely boosting [29] and bandit learning [4]. In boosting, one assumes the existence of a *weak classifier* whose performance is slightly better than random. In a landmark paper [29], Schapire showed how to combine weak classifiers into a PAC-learner by a majority voting scheme. One can consider different graph data sources as weak learners, and ask whether one can “boost” them to a good graph. Unfortunately, our problem setting does not allow pure boosting because boosting requires access to ground truth labels. Even with reliable input, the community detection has no single accepted measure of quality.

In bandit learning an algorithm receives rewards as it explores a set of actions, and the goal is to minimize a notion of regret. The basic model has many variants, but two central ones are expert advice and adversaries. Experts are functions suggesting what action to take in each round, and regret is measured with respect to the best expert. The adversarial setting involves an omniscient adversary who sets the experts and rewards so as to maximize regret. We can imagine graphs as adversarial experts, and adapt bandit learning techniques to compensate. Indeed, LBGA is a reward system based on the given application and uses update techniques from bandit learning to learn a graph representation. In our setting we only care if the aggregate graph is good at the end, while bandit learning often seeks to maximize cumulative rewards during learning. There are bandit settings that only care about the final result [5], but to the best of our knowledge they do not apply to our problem.

The primary technique we use is the Multiplicative Weights Update Algorithm (MWUA). See [2] for an overview and an extensive list of successful applications. The algorithm maintains a weight for each element x_j of a finite set X . In rounds, an element x_i is chosen by sampling proportionally to the weights, a reward $q_{t,i}$ is received, and the weight for x_i is multiplied or divided by $(1 + \varepsilon q_{t,i})$, for some parameter $\varepsilon > 0$. After many rounds, the elements with the highest weight are deemed the best and used for whatever purpose needed. Next, we describe how this algorithm is adapted to

graph aggregation.

3 The Locally Boosted Graph Aggregation framework

LBGA can succinctly be described as running multiplicative weights for each edge, forming a candidate graph representation G_t in each round by sampling edges, and computing local rewards on G_t to update the weights for the next round. When G_t converges we produce it as output. The remainder of this section expands the details of this sketch and our specific algorithm implementing it.

3.1 Framework details

Let H_1, \dots, H_m be a set of unweighted, undirected graphs defined on the same vertex set V . We think of each H_i as “expert advice” suggesting for a pair of vertices $u, v \in V$ whether to include edge (u, v) or not. Our algorithm combines the H_i into a single graph G^* suitable for the proposed application. We present LBGA in the context of community detection, noting generalizations. Each round has four parts: producing the aggregate candidate graph G_t , computing a clustering $A(G_t)$ for use in measuring the quality of G_t , computing the local quality of each edge, and updating the weights for the edges. After T rounds we output $G^* = G_T$.

Aggregated Candidate Graph G_t : In each round construct G_t as follows. Maintain a weight $w_{u,v,i}$ for each graph H_i and each edge (u, v) in $H_1 \cup \dots \cup H_m$. Normalize the set of all weights for an edge $w_{u,v}$ to a probability distribution over the H_i . For each edge u, v , sample an H_i according to this distribution and include the edge in G_t if it is present in the drawn H_i .

Event $A(G_t)$: After the graph G_t is produced, run a clustering algorithm A on it to produce a clustering $A(G_t)$. In this paper we fix A to be the Walktrap algorithm [26], though we have observed the effectiveness of other clustering algorithms as well. In general A can be any event, and in this case we tie it to the application by making it a simple clustering algorithm.

Local quality measure: Define a *local quality measure* $q(G, e, c)$ to be a $[0, 1]$ -valued function of a graph G , an edge e of G , and a clustering c of the vertices of G . The quality of (u, v) in G_t is the “reward” for that edge, and it is used to update the weights of each input graph H_i . More precisely, the reward for (u, v) in round t is $q(G_t, (u, v), A(G_t))$.

Update Rule: Update the weights using MWUA as follows. Define two learning rate parameters $\varepsilon > 0, \nu > 0$, with the former being used to update edges from G_t that are present in H_i and the latter for edges not in H_i . In particular, suppose $q_{u,v}$ is the quality of the edge (u, v) in G_t . Then, the update rule is defined as follows:

$$w_{u,v,i} = \begin{cases} w_{u,v,i}(1 + \varepsilon q_{u,v}), & \text{if } (u, v) \in H_i \\ w_{u,v,i}(1 - \nu q_{u,v}), & \text{if } (u, v) \notin H_i. \end{cases}$$

3.2 Quality measures for community detection

We presently describe the quality measure we use for community detection. First we define *edge consistency*, which

measures whether an edge has endpoints in the same cluster or across clusters:

$$EC_{u,v} = \begin{cases} 1, & \text{if } c(u) = c(v) \\ -1, & \text{if } c(u) \neq c(v). \end{cases}$$

We also define *neighborhood overlap* (NO), which asserts that vertices sharing many neighbors are likely to be in the same community. NO declares the quality of (u, v) to be the (normalized) cardinality of the intersection of the neighborhoods of u and v , namely $NO_{u,v} = \frac{|N(u) \cap N(v)|}{|N(u) \cap N(v)| + \log(|V|)}$, where $N(x)$ is the neighborhood of x . Our quality metric, *consistentNO*, combines edge consistency with neighborhood overlap by multiplying the two functions. We have also run experiments using more conventional neighborhood metrics, such as the Dice and Jaccard indices [9]. ConsistentNO outperforms them by at least 10% in our experiments and for brevity we omit the results.

3.3 LBGA implementation

We give pseudocode for our implementation of LBGA in Algorithm 1. The runtime of LBGA is $O(T(|E|Q(n) + A(n)))$, where $|E|$ is the number of edges, $Q(n)$ is the runtime of evaluating the quality function, $A(n)$ is the runtime of evaluating the event A , and T is the number of rounds. Algorithm 1 improves this by fixing edges whose weights have grown $> 1 - \delta$ or $< \delta$ for a new parameter δ . As LBGA learns, the sampling procedure becomes substantially sublinear in the number of edges. Penalizing non-edges ($\nu > 0$) also improves runtime, and LBGA is stable to minor variations in ε and δ . Moreover, our algorithm empirically scales linearly with the size of the input.

4 Experimental analysis

We describe the datasets used for analysis and provide quantitative results for the performance of Algorithm 1. In all of our experiments LBGA was run with parameters $\varepsilon = \nu = 0.2$, $\delta = 0.05$.

4.1 Synthetic datasets

Our primary synthetic data model is a generalization of the stochastic block model of [34]. We construct a probability distribution $G(n_i, p_i, r_i)$ over graphs as follows. Given a number n of vertices and a list of cluster (block) sizes $\mathbf{n} = \{n_1, \dots, n_k\}$ with $n = \sum_i n_i$, partition the n vertices into k blocks $\{b_1, \dots, b_k\}$ with $|b_i| = n_i$. Define k graphs G_1, \dots, G_k and set the probability of an edge occurring in G_i with both endpoints in block b_i to p_i , all others occurring with probability r_i . We call this model the *local stochastic block model* (LSBM). To contrast, we define the *global stochastic block model* (GSBM) by setting the probability of an edge occurring in G_i with endpoints in the same block (any block, not just block b_i) to be p_i , all others with probability r_i . Finally, we include *Erdős-Rényi random graphs* [11] alongside LSBM (e.g., LSBM-3) to capture a range of structure and noise combinations.

Data: Unweighted graphs H_1, \dots, H_m on the same vertex set V , a clustering algorithm A , a local quality metric q , three parameters $0 < \varepsilon, \nu, \delta < 1/2$

Result: A graph G

Initialize a vector $\mathbf{w}_{u,v} = \mathbf{1}$ for all $u \neq v \in V$

Let U be the edge set of $H_1 \cup \dots \cup H_m$

Let $G_{\text{learned}} = (V, \emptyset)$

while $|U| > 0$ **do**

 Let G be a copy of G_{learned}

for $(u, v) \in U$ **do**

 Let $p_{u,v} = \frac{\sum_i w_{u,v,i} 1_{\{(u,v) \in H_i\}}}{\sum_i w_{u,v,i}}$

 Flip a coin with bias $p_{u,v}$

 If heads, include (u, v) in G .

end

 Cluster G using A

for $(u, v) \in U$ **do**

 Set $p = q(G, A(G), (u, v))$

for $i = 1, \dots, m$ **do**

if $(u, v) \in H_i$ **then**

 Set $w_{u,v,i} = w_{u,v,i}(1 + \varepsilon p)$

else

 Set $w_{u,v,i} = w_{u,v,i}(1 - \nu p)$

end

end

 Let $p_{u,v} = \frac{\sum_i w_{u,v,i} 1_{\{(u,v) \in H_i\}}}{\sum_i w_{u,v,i}}$

if $p_{u,v} > 1 - \delta$ **then**

 Add (u, v) to G_{learned} , remove it from U

end

if $p_{u,v} < \delta$ **then**

 Remove (u, v) from U

end

end

end

Output G

Algorithm 1: Optimized implementation of LBGA. Note that 1_E denotes the characteristic function of the event E .

4.2 Real datasets

We use a subset of *DBLP* [18], an online database of research in computer science. We take data from two conferences: the Symposium on the Theory of Computing (STOC), and the Symposium on Foundations of Computer Science (FOCS). Each of the 3153 nodes represents an author, and we use two graphs on this vertex set: the *co-authorship* graph and the *title similarity* graph. For the latter, we add an edge between two author vertices if any of their paper titles contain at least three words in common (excluding stop words), and the weight of this edge is the number of such pairs.

RealityMining [10] was a 9-month experiment in 2004 which tracked a group of 90 individuals at MIT via their cell phones. The dataset includes voice calls, bluetooth scan events, cell tower usage, and self-reported friendship and proximity data. We used data between 2004-09-23 19:00:00 and 2005-01-07 18:00:00 (UTC-05:00). Nodes are individuals in the study, and weighted edges correspond to the total duration of voice calls, the total amount of time two individuals used the same cell tower, the total number of bluetooth events, and the results of the friendship/proximity surveys for

Dataset	Parameters
LSBM-1	$m = k = 4, n_i = 125, p_i = 0.2, r_i = 0.05$
LSBM-2	$m = k = 4, n_i = 125, p_i = 0.3, r_i = 0.05$
LSBM-3	$m = 5, k = 4, n_i = 125, p_i = 0.3, r_i = 0.05,$ $i = 1, \dots, m, p_5 = r_5 = 0.01$
GSBM-4	$m = k = 4, n_i = 125, p_1 = 0.1625,$ $p_2 = 0.125, p_3 = 0.125, p_4 = 0.0875, r_i = 0.05$
GSBM-5	$m = k = 4, n_i = 125, p_1 = 0.15, p_2 = 0.1,$ $p_3 = p_4 = 0.05, r_i = 0.05, i = 1, \dots, m$
ER only	$m = 4, p_i = r_i = 0.01$
DBLP	$n = 3153, m = 2$
RMining	$n = 90, m = 6$
Enron	$n = 145, m = 2, \alpha = 0.9$

Table 1: Description of datasets analyzed. Total number of vertices in each synthetic source graph is $n = 500$. The number of graph sources is m . The number of clusters is k . The number of vertices in cluster i is n_i . The within- and across-cluster edge probabilities for graph source i are p_i and r_i , respectively.

a total of 6 graphs.

Finally, the *Enron* email dataset [15] is a corpus of over 600,000 emails sent between 145 employees of the Enron Corporation in the early 2000’s. We produced two graphs from the Enron data, one for peer-to-peer email communication and one for topic similarity in the email content. The vertices are individuals. For email, edges are weighted by the number of emails sent. We used Mallet [20] to generate the LDA topic model for the content of Enron email data, aggregating into one document all the email content sent by each employee. Each document is represented by 60 topics. We measure cosine distance of the topic vectors, and consider an edge as present if the cosine distance was above a specified threshold value α discussed in Section 4.4. Table 1 contains a summary of all the datasets used for the experimental analysis and their parameters.

4.3 Validation procedure

We now state how we evaluate the quality of LBGA’s output.

Recovery of Inherent Clusters. Since the output of LBGA is a graph, we use the walktrap clustering algorithm to extract communities for analysis. When ground truth communities are available we compare them with LBGA’s communities using Normalized Mutual Information [8]. Otherwise we relate our clusters to known features of the dataset.

Quality of Graph Representation. In addition to producing a good clustering, an ideal graph representation also removes cross-community edges and produces a sparser representation. We use the standard Newman modularity measure [24] and conductance [17] to measure this. Note that *higher* modularity scores and *lower* conductance scores signify stronger community structure. We note two extreme graph representation cases, the empty graph which is perfectly modular and the union graph which is a trivial aggregation. To signal these cases in our results, we display the *sparsity* of the produced graph G^* , i.e. the fraction of edges in G^* out of the total set of edges in all input graphs.

Recovery of Graph Source Contribution. In our experiments some input graphs contribute more to uncovering the

underlying community structure, and should have higher edge weights on average. As such we display the average weights of the input graphs, e.g. in Figure 1.

4.4 Experimental results

Table 2 states the numerical results of our experiments. As a baseline, we computed the modularity and conductance values of the union of the input graphs with respect to the ground truth (for synthetic) or the Walktrap clusterings (for the real world). For synthetic examples, we compare our results with GraphFuse [25] and SNF [33] algorithms. Overall, LBGA converges to graphs with high modularity and low conductance. LBGA produces graph representations that induce correct clusterings in almost all cases where ground truth is known, the challenging case being when the noise rate is close to known detectability thresholds.

Synthetic

Figure 1 depicts a run of Algorithm 1 with consistentNO on the dataset LSM-3. LBGA converges quickly to a graph with a perfect clustering and high modularity. We plot the number of edges in G_t over time and the average vertex-pair weight for each input graph. LBGA produces a graph using with 40% sparsity and weights edges from the Erdős-Rényi source appropriately. Our algorithm hence achieves a high quality graph while preserving and highlighting the underlying community structure.

Figures 1 and 2 also demonstrate that LBGA does not falsely boost noise to report community structure where none is present. Figure 2 depicts the behavior of LBGA on a dataset of only Erdős-Rényi random graphs. LBGA produces aggregate graphs whose modularity values are low and conductance values are high. We see a clear phase transition in performance around $p = 0.3$ corresponding to the Erdős-Rényi graphs becoming triangle-dense and therefore less distinguishable from graphs that are a single community. Tolerance to such dense levels of noise is unavoidable.

DBLP

Table 2 shows LBGA producing an aggregate DBLP graph of modularity exceeding the baseline and using significantly fewer edges. Our algorithm selects title similarity as having more influence in recovering communities for the STOC/FOCS conferences. We also manually inspected the resulting clusters, they incorporate both membership and coauthorship. For example, Mikko Koivisto, Thore Husfeldt, Petteri Kaski, and Andreas Björklund have coauthored over 15 papers in combinatorial optimization. They naturally fall within a small coauthorship cluster. However, title similarity splits these researchers across two clusters due to differences in their non-coauthored work. They fall in the same cluster in the aggregate graph, which includes researchers who are either coauthors with one of the four or have done much work in the same field.

RealityMining & Enron

For RealityMining, LBGA’s output contains two dense clusters corresponding exactly to the MIT Media Lab and the Sloan Business School, with only three edges crossing the cut. In addition, this graph uses only 63.5% of the total edges

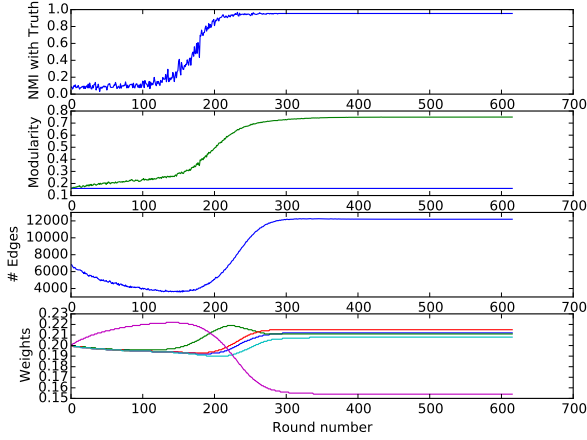


Figure 1: Graph representation learning for LSMB-3. The LBGA parameters are $\varepsilon = \nu = 0.2$, $\delta = 0.05$. Plots in order top to bottom: 1. NMI of $A(G_t)$ with the ground truth clustering, 2. modularity of G_t w.r.t. $A(G_t)$, with the horizontal line showing the modularity of the union of the input graphs w.r.t. ground truth, 3. the number of edges in G_t , 4. the average probability weight of vertex pairs for H_i . The Erdős-Rényi graph converges to low weight by round 300, even though it is initially favored. Hence LBGA can recover from bad luck and does not boost noise.

available. For Enron, LBGA achieves similar results. In addition, Figure 3 depicts the output of LBGA, showing a clear community structure. The small clusters are lower-level employees, and the big clusters are managers. Moreover, there was a known fantasy sports club within the network [19], and these individuals all fall in a single cluster of G^* .

The modularity of RealityMining and Enron are smaller with LBGA than the baseline. We argue that this is caused by small clusters produced by LBGA, modularity is known to be inaccurate for small clusters [12]. Indeed, LBGA outperforms the baseline with respect to conductance.

4.5 Comparison with GraphFuse and SNF

We compare LBGA with GraphFuse [25], a multi-graph clustering algorithm and SNF [33], a graph fusion algorithm. We use NMI with the ground truth as the performance measure. For the comparison analysis we have only considered the synthetic datasets where the notion of ground truth is known. Table 2 contains the comparison results.

LBGA outperforms SNF in all cases, and by a particularly large margin on the LSMB model. LBGA also outperforms GraphFuse on both the global block models and the lower-noise local block models LSMB-2 and LSMB-3. LBGA also produces very sparse representations that may be useful for future analysis, while GraphFuse and SNF produce only a clustering.

4.6 Sensitivity analysis

We analyze the sensitivity of LBGA to noise. In Figure 4 we display NMI for the LSMB model and varying intra-cluster edge probability p_i and varying signal-to-noise ratios. As expected, NMI falls as the noise rate r_i increases. LBGA

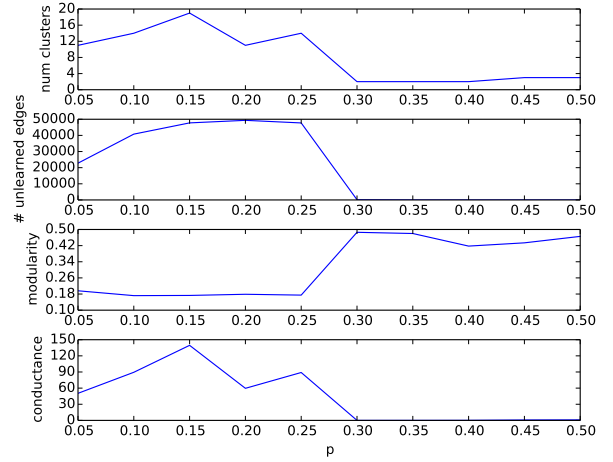


Figure 2: Statistics about the aggregate graph produced by LBGA after 500 rounds on a suite of 4 Erdős-Rényi random graphs on 500 nodes and varying edge probability p .

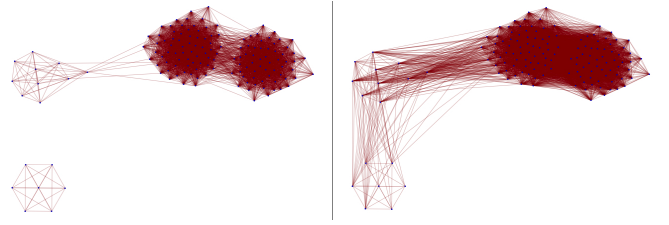


Figure 3: Left: the results of LBGA on the Enron dataset. Right: the input graph of topics.

reaches higher quality and maintains the quality longer for denser graphs, which is also consistent with our expectations. At a signal to noise ratio of 2 or less, the NMI drops to non-useful levels regardless of p_i . The sharp drop in quality is related to well-known phase transitions for community detectability [23].

5 Conclusion

LBGA offers a flexible, local aggregation method for combining different graph sources in order to better represent community structure in networks. We derive LBGA from a solid theoretical foundation in boosting and bandit learning, and demonstrated LBGA as a proof of concept on synthetic and real networks. LBGA also simplifies the task of designing a graph aggregation algorithm into utilizing a principled quality measure q and global event A . Doing so allows us to connect the utility of the graph representation to the application of interest.

There are some natural directions to pursue in further studying LBGA. For community detection, we can improve LBGA in a number of ways. Our consistentNO metric is simple, and a more sophisticated metric comparing a local neighborhood to a given null model is likely to provide improvements. Additionally, we use the walktrap clustering algorithm as a black box in the “event” step of LBGA, and walktrap

Union Graph				GraphFuse	SNF	LBGA: ConsistentNO			
Dataset	Mod.	Cond.	NMI	NMI	NMI	Modularity	Conductance	NMI	Sparsity
GSBM-4	0.178	10.678	1.000	0.716	0.658	0.739	0.084	1.000	0.433
GSBM-5	0.093	15.368	0.636	0.616	0.436	0.727	2.121	0.619	0.235
LSBM-1	0.103	14.725	0.724	0.686	0.099	0.679 ± 0.114	9.962 ± 23.0	0.503 ± 0.05	0.263 ± 0.04
LSBM-2	0.166	11.233	0.992	0.760	0.180	0.740	0.084	0.992	0.420
LSBM-3	0.166	11.216	1.000	0.779	0.209	0.737	0.104	1.000	0.422
ER only	-0.002	24.729	-	-	-	0.193	112.947	-	0.230
DBLP	0.386	1368.859	-	-	-	0.695	159.286	-	0.632
RMining	0.452	70.314	-	-	-	0.246	0	-	0.646
Enron	0.559	134.572	-	-	-	0.444	0.594	-	0.631

Table 2: LBGA performance results, compared to GraphFuse, SMF, and a baseline union aggregation. All datasets in this table were run with *consistentNO* using $\varepsilon = \nu = 0.2, \delta = 0.05$. Union modularity and conductance for real datasets was computed with the walktrap clustering. Values were averaged over 10 trials, and when variances $\sigma^2 > 10^{-4}$ were observed, values are reported with $\pm\sigma$.

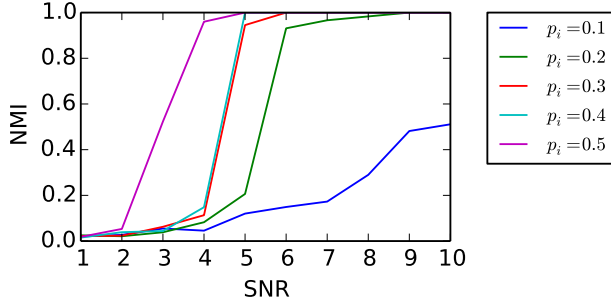


Figure 4: Performance of LBGA (measured by NMI) as a function of SNR for the LSBM model with different probabilities p_i for *consistentNO*.

makes some simplifying decisions to arrive at a final clustering. A direction for future work is to use a modified walktrap event that outputs raw similarity values before constructing a clustering, and incorporating this data into the quality measure. Finally, preliminary results of the authors and others show that LBGA can also be improved by incorporating a consensus technique using LBGA as a black box, and that LBGA can detect hierarchical community structure. Further study of these is needed for a better understanding of LBGA.

Another primary direction is to study the utility of LBGA for other data mining techniques, such as link prediction. Since LBGA is modular, one can adapt LBGA to a new application domain simply by defining an event and quality function.

A final direction is to use proof techniques from boosting and bandit learning to provide strong theoretical guarantees on the performance of LBGA. The authors have some preliminary results in this direction that have not been included in this manuscript for brevity.

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