

# Open Problem: Learning Quantum Circuits with Queries

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## Abstract

We pose an open problem on the complexity of learning the behavior of a quantum circuit with value injection queries. We define the learning model for quantum circuits and give preliminary results. Using the test-path lemma of [Angluin et al. \(2009a\)](#), we show that new ideas are likely needed to tackle value injection queries for the quantum setting.

## 1. Introduction

Learning classical circuits with value injection queries (VIQ) was defined by [Angluin et al. \(2009b\)](#). In the VIQ model, one assigns values to subsets of the circuit wires and observes the output. This model has been extended to large alphabet, analog, and probabilistic circuits ([Angluin et al., 2008, 2009a](#)). There has also been work on learning networks with similar kinds of queries ([Angluin et al., 2010; Kempe et al., 2003](#)). Here, we ask how to learn a *quantum* circuit using VIQs. There is much work in physics on inferring the structure of specific quantum systems, but little on their generic learnability. Progress on this problem will bring together rich ideas from both the quantum and learning communities.

## 2. Model

A size  $k$  quantum circuit  $C$  on  $n$  qubits is a list of gates  $G_1, \dots, G_k$ , where each  $G_i$  consists of an  $8 \times 8$  unitary matrix  $A_i : \mathbb{C}^8 \rightarrow \mathbb{C}^8$  and a list  $(i_1, i_2, i_3) \in \{0, 1, \dots, n\}^3$  of the indices of the qubits on which  $A_i$  operates. A *qubit*  $v$  is a unit vector in  $\mathbb{C}^2$ . Denote the basis vectors of  $\mathbb{C}^2$  as  $e_0, e_1$ . The input to a quantum circuit is a unit vector in  $(\mathbb{C}^2)^{\otimes n} = \mathbb{C}^{2^n}$ , the  $n$ -fold tensor product of  $\mathbb{C}^2$  with itself, where each copy of  $\mathbb{C}^2$  corresponds to a qubit. We denote the basis vectors of  $\mathbb{C}^{2^n}$  by  $e_i$  where  $0 \leq i \leq 2^n - 1$  is written as a binary string  $i = b_1, \dots, b_n$  and  $e_i = \bigotimes_{t=1}^n e_{b_t}$  is the tensor product of basis states from the copies of  $\mathbb{C}^2$  (called a *pure state*). In our model all inputs to quantum circuits will be pure states.

The computation of an input vector  $v = v_1 \in \mathbb{C}^{2^n}$  is as follows. In step  $i$  the following is applied to  $v_i$ : swap columns s.t.  $i_1, i_2, i_3$  are the first 3 columns, apply the unitary map  $A_i \otimes I_{2^{n-3}}$ , and then reverse the column swap. Note that operations that only “depend” on three qubits can affect the entire state vector  $v_i$ . After all  $k$  gates are computed,  $v_{k+1}$  is measured, returning index  $i$  w.p.  $|v_i|^2$ , and  $v_{k+1}$  “collapses” to the basis vector  $e_i$ .

An *instance* of our model is a pair of integers  $n, k$ , an unknown quantum circuit  $C$  on  $n$  qubits of size  $k$ , and an unknown permutation  $\sigma$  on  $\{1, \dots, k\}$  masking the order of the gates. A *function injection query* (FIQ) is a tuple  $(x, S, (B_i : i \in S))$  where  $x \in \{0, 1\}^n$ ,  $S \subset \{1, \dots, k\}$ , and  $B_i$  are  $8 \times 8$  unitary matrices. The response to a query is a string  $y$  which is the final measured state of the quantum circuit formed by replacing the matrix  $A_i$

	with measurements	observing full state vector
VIQ	<b>open</b> (fails test-path lemma)	<b>open</b>
FIQ	poly	poly

Table 1: A table summarizing our results, with open problems in bold.

of gate  $G_i$  with  $B_{\sigma(i)}$  for each  $i \in S$  and run on the starting state vector  $e_x$ . We define an *unmeasured* variant where the query response is the entire state vector before measurement.

Given  $v \in \{0, 1\}^3$ , a *value injection* of  $v$  into a gate  $G_j$  of a circuit  $C$  with matrices  $A_j$  operating on bits  $(i_1, i_2, i_3)$  is an augmented matrix  $C'$  with three additional qubits fixed to the states  $e_{v_1}, e_{v_2}, e_{v_3}$  and an additional gate  $H_j$  following  $G_j$  defined by swapping the new qubits with  $i_1, i_2, i_3$ , respectively. One can similarly inject values into a subset of gate outputs. A *value injection query* (VIQ) is a tuple  $(x, S, (v_i \in \{0, 1\}^3 : i \in S))$  where  $x \in \{0, 1\}^n$  and  $S \subset \{1, \dots, k\}$ . The response to a query is a string  $y$  which is the measurement of the first  $n$  qubits of the final state of the quantum circuit formed by injecting each value  $v_i$  into gate  $\sigma(i)$  of  $C$  and running the resulting augmented circuit on input  $e_x$ . We define the analogous unmeasured variant as well.

An algorithm *learns to  $\varepsilon$ -behavioral equivalence* if for any circuit  $C$  it outputs  $C'$  s.t. the distributions over responses to VIQs (FIQs) for  $C$  and  $C'$  differ by  $\leq \varepsilon$  in  $l_1$  norm.

**Open Problem** *Determine the query complexity of learning quantum circuits with VIQs.*

Now we give some preliminary results on learning quantum circuits with VIQs and FIQs.

### 3. Function injection queries are tractable

**Proposition 1** *There is a  $O(k \log k + kn)$ -query algorithm for learning a quantum circuit with FIQs (without measurement) to 0-behavioral equivalence.*

**Proof** The algorithm learns the gates individually, and then their order. For the first part, for each gate  $1 \leq j \leq k$  and  $0 \leq i \leq 7$ , query  $Q_{j,i} = (i0^{n-3}, [k] \setminus \{j\}, (I_8)_{l \in S})$ . With  $O(n)$  overhead (see below), assume gate  $A_j$  acts on the first 3 qubits. Fixing  $j$  and looking at all  $Q_{j,i}$ , each output vector consists of the  $i$ th column of  $A_j$ . Repeat on all gates makes  $O(nk)$  FIQs. To find the qubits  $A_j$  acts on: inject the identity everywhere except  $j$  and a 3-cycle permutation at  $j$ , and query each  $e_{1_l}$  where  $1_l$  is the string of 0s with a 1 at position  $l$ .

In step 2, we sort the gates in  $O(k \log k)$  time and compare the order of two gates  $A_j, A_{j'}$  as follows. Fix two permutations  $\sigma, \sigma' \in S_8$  that do not commute. Form  $B_\sigma$  by permuting the columns of the identity matrix according to  $\sigma$ . Inject the identity into every gate except  $j, j'$ ,  $B_\sigma$  into gate  $j$ , and  $B_{\sigma'}$  into gate  $j'$ . Choose  $x$  such that  $\sigma\sigma'(x) \neq \sigma'\sigma(x)$  and input  $e_x$ . If the order of the gates matters, the response to the query will differ, and it is trivial to determine the correct order. Note the comparisons do not require the full state vector. The algorithm has query complexity  $O(nk + k \log k)$ . ■

Measurement adds  $\text{poly}(k/\varepsilon)$  FIQs to the above algorithm to make it  $\varepsilon$ -approximate. Consider the case with two gates  $A_j, A_{j'} \in \mathbb{C}^{8 \times 8}$ , operating on the same qubits. Decompose  $A_j$  according to a basis of the vector space  $SU(8)$ . Inject the identity everywhere but  $j, j'$

and inject all pairwise sums of basis operations to  $A_{j'}$ , query all 8 basis vectors, and measure the outputs. Using the basis coefficients of  $A_j$  as variables, this gives a system of polynomial equalities in which all parameters are  $O(1)$ . It can be solved to accuracy  $\varepsilon/k$ . It is easy to see that the errors across gates grow linearly, and the general case is similar.

#### 4. Value injections are as hard as large-alphabet probabilistic circuits

Angluin et al. (2009a) use the idea of “test paths” (from Angluin et al. (2009b)), to bound the query complexity of learning boolean probabilistic circuits. Their analysis also shows test paths fail for probabilistic circuits with an alphabet size greater than two. We show quantum circuits have the same barrier by constructing a gadget analogous to that of Angluin et al. (2009a) (Lemma 8). Define  $B$  as a matrix as in Figure 1. Further define the standard quantum XOR gate using an extra scratchwork qubit by the mapping  $e_{ijk} \mapsto e_{ij(k \oplus i \oplus j)}$ .

**Lemma 2** *There is a circuit on which every (measured) VIQ leaving a path free makes the last output qubit uniformly random, yet with no VIQ the last output qubit is deterministic.*

**Proof** Define  $C$  on three qubits as in Figure 1. The circuit normally maps  $e_{000} \mapsto \frac{1}{\sqrt{2}}(e_{000} + e_{110})$ ,  $e_{010} \mapsto \frac{1}{\sqrt{2}}(e_{000} - e_{110})$ ,  $e_{100} \mapsto \frac{1}{\sqrt{2}}(e_{111} + e_{101})$ , and  $e_{110} \mapsto \frac{1}{\sqrt{2}}(e_{111} - e_{101})$ , i.e., the last qubit is 1 iff the input’s first qubit is 1. Likewise, when the scratch-work qubit is 1, the output bit is flipped. When there is a VIQ, say, of 1 at the identity gate acting on the 2nd qubit, the mapping becomes  $e_{000} \mapsto \frac{1}{\sqrt{2}}(e_{0110} + e_{1101})$ ,  $e_{010} \mapsto \frac{1}{\sqrt{2}}(e_{0110} - e_{1101})$ ,  $e_{100} \mapsto \frac{1}{\sqrt{2}}(e_{0111} + e_{1100})$ , and  $e_{110} \mapsto \frac{1}{\sqrt{2}}(e_{0111} - e_{1100})$ . The extra qubit introduced by the VIQ is the last index, and the qubit of interest is the third qubit, which is uniformly random. ■

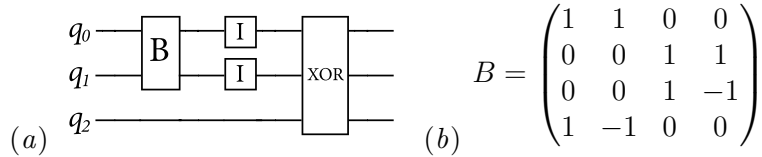


Figure 1: Left: the circuit for Lemma 2. Right: the columns of  $B$  form the Bell basis.

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