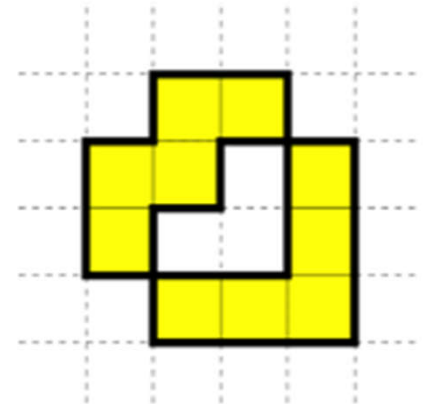
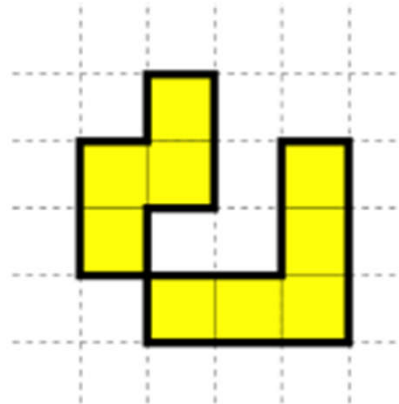
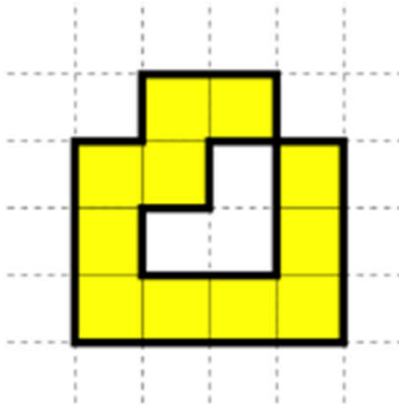
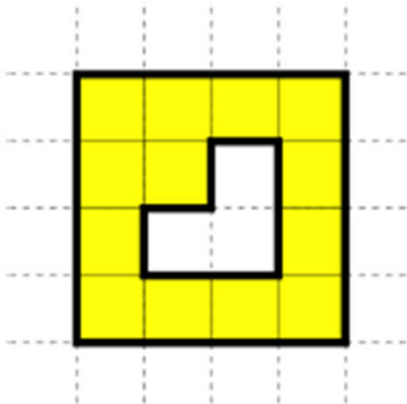


IOI'12 Ideal City

By Fatih Gelgi

Problem

- A path between any two *empty* cells
- A path between any two non-*empty* cells



Problem

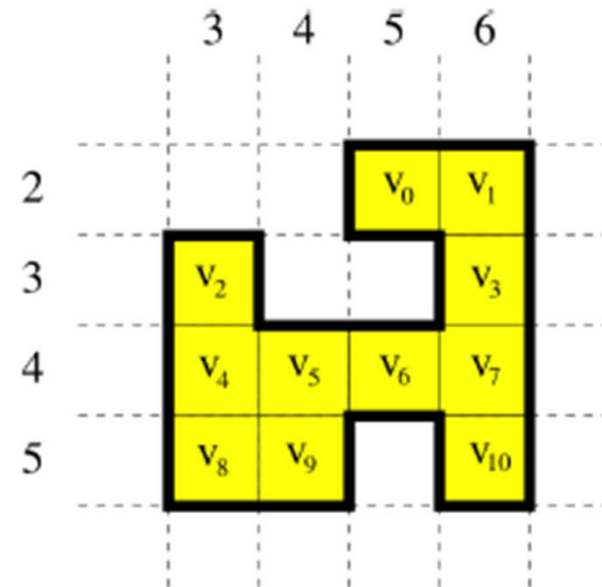
Distance

- $v_0 = (2, 5)$, $v_1 = (2, 6)$, $v_2 = (3, 3) \dots$
- $d(v_1, v_3) = 1$, $d(v_1, v_8) = 6$

Objective

- Find the sum of the total distances of all pairs:

$$\sum d(v_i, v_j), \text{ where } 0 \leq i < j \leq N - 1$$



Problem

Tasks

- Subtask 1 [10%]: $N \leq 200$.
- Subtask 2 [20%]: $N \leq 2000$.
- Subtask 3 [30%]: $N \leq 100,000$, any two non-empty cells i and j such that $X[i] = X[j]$, every cell between them is non-empty too; any two non-empty cells i and j such that $Y[i] = Y[j]$, every cell between them is non-empty too.
- Subtask 4 [40%]: $N \leq 100\,000$.

Solution: Floyd-Warshall

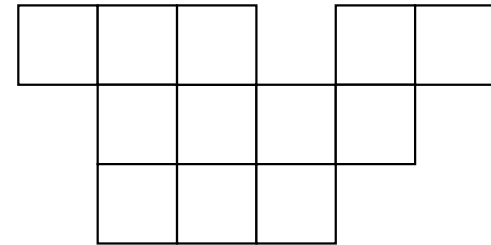
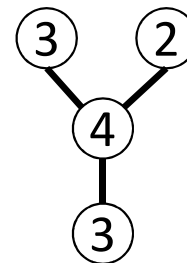
- Straightforward Idea: calculate all pairwise distances
- $O(N^3)$
- Subtask 1: OK

Solution: Breadth First Search

- Iterative Breadth First Search
- Find distances from a vertex v and update the distance sum $\Rightarrow O(N)$
- Do it for all vertices $\Rightarrow O(N^2)$
- Subtask 1, 2: OK

Solution: Horizontal / Vertical Node Groups

Horizontal
Node group



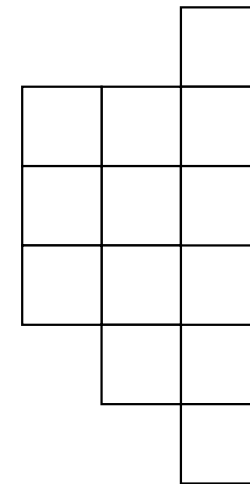
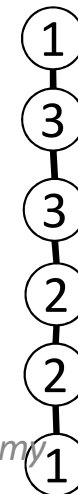
Answer = $\sum \{w(x) * w(y) * d(x,y)\}$ for all (x,y)

Generate trees $\Rightarrow O(N \log N)$

Total $\Rightarrow O(N^2)$

- Subtask 1, 2, 3: OK

Vertical
Node group



Solution: Optimization

Observation:

$$\begin{aligned} & \text{sum } \{w(x) * w(y) * d(x,y)\} \text{ for all } (x,y) \\ &= S(e) * S'(e) \text{ for all edges } e \end{aligned}$$

$O(N \log N)$

- Subtask 1, 2, 3, 4: OK