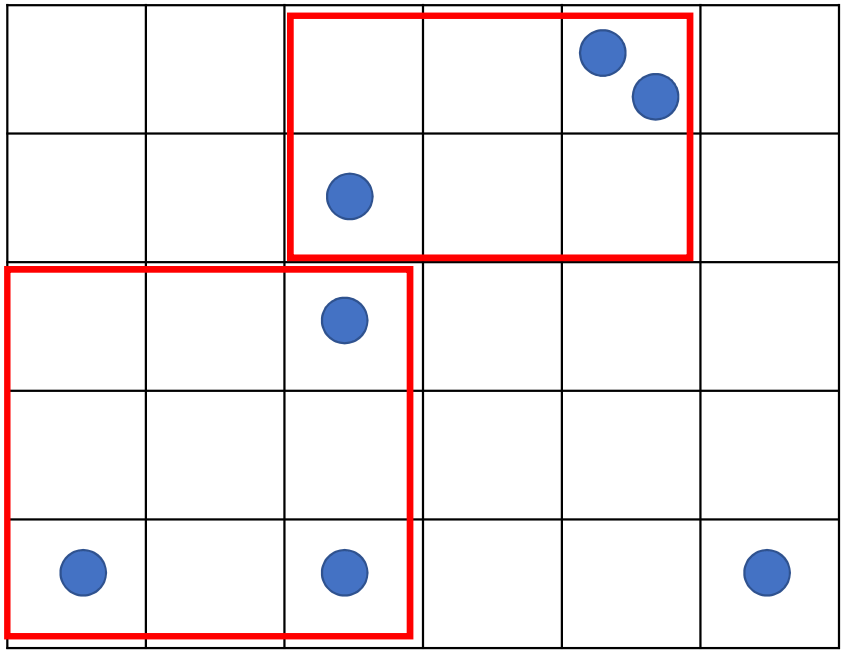
IOI'05 Garden

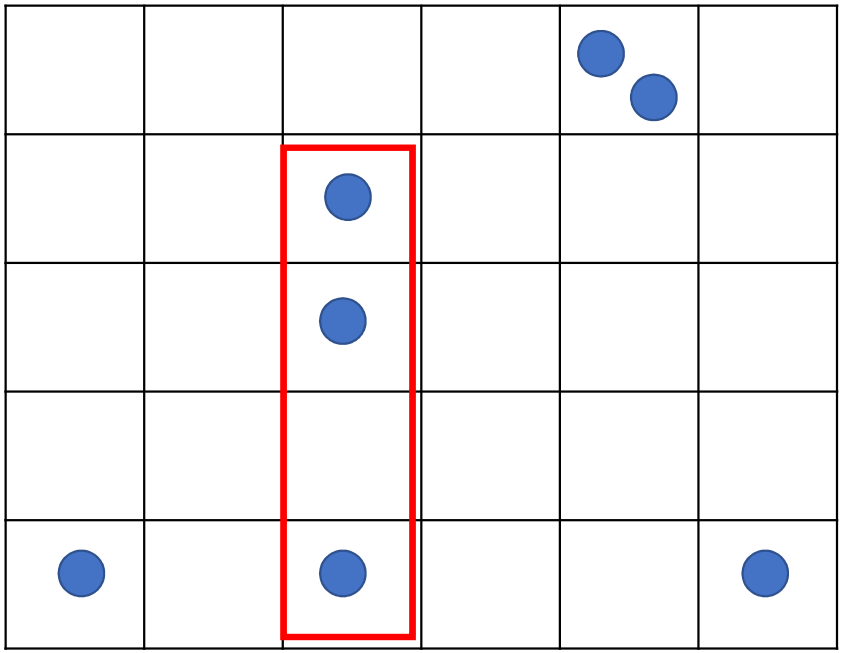
By Fatih Gelgi

Problem



Variation

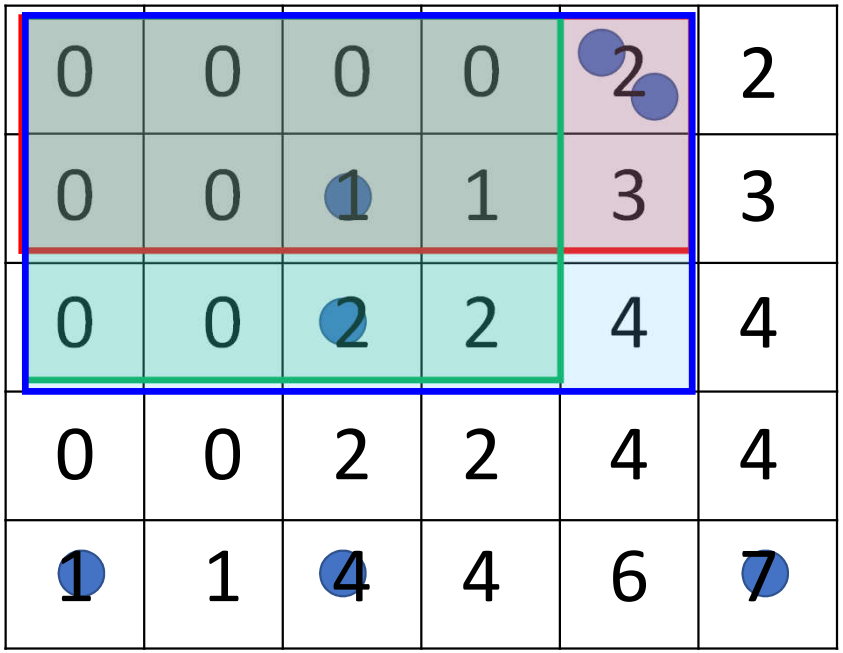
- Min rectangle with K roses?



Basics: Prefix sums

- Prefix sums in 2D: r[i][j] = roses in the rectangle (1,1)-(i,j)

  r[i][j] = r[i-1][j] + r[i][j-1] - r[i-1][j-1] + grid[i][j]

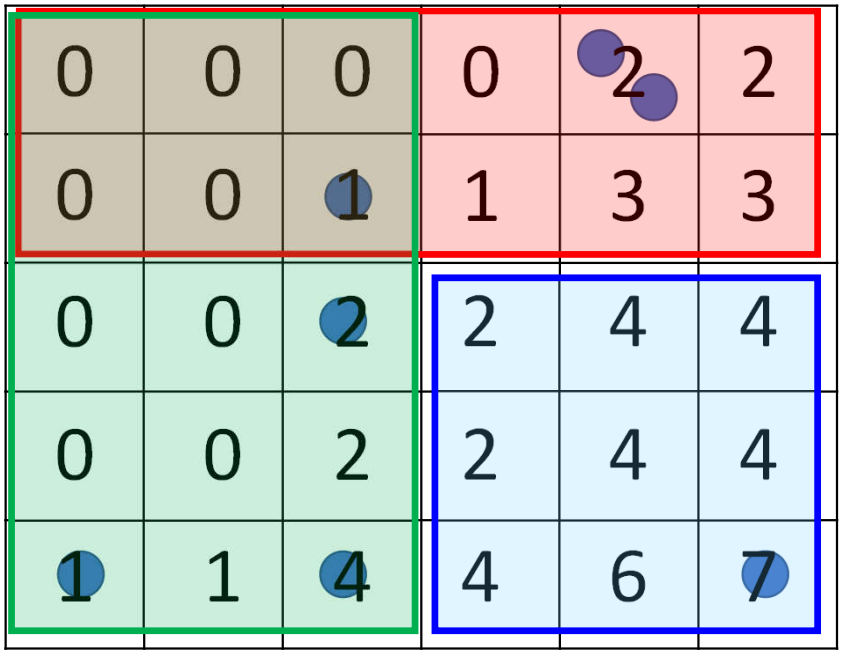


time: O(WL)

Basics: Prefix sums

- Roses in Rectangle (y1,x1) (y2,x2) = ?

  = r[y2][x2] - r[y1-1][x2] - r[y2][x1-1] + r[y1-1][x1-1]



time: O(1)

Solution: Brute force

- Pick any rectangle: $O(W^2 L^2)$ rectangles each O(1) time

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

  Start at x1=x2=1

0	0	0	0	2	2
0	0	1	1	3	3
0	0	2	2	4	4
0	0	2	2	4	4
1	1	4	4	6	7

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

  If less than K roses in the current window, x2=x2+1

0	0	0	0	2	2
0	0	1	1	3	3
0	0	2	2	4	4
0	0	2	2	4	4
1	1	4	4	6	7

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

  If more than K roses in the current window, x1=x1+1

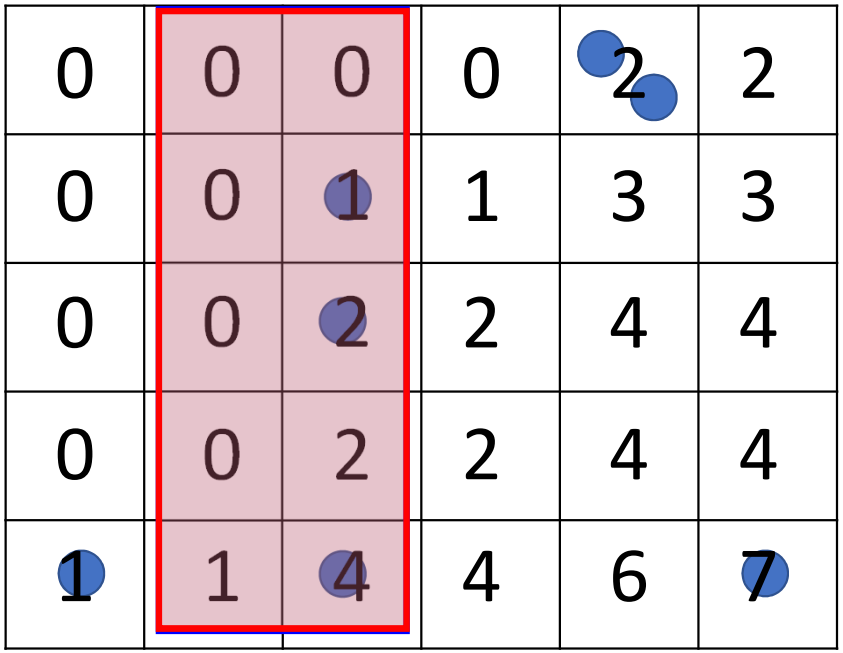
0	0	0	0	2	2
0	0	1	1	3	3
0	0	2	2	4	4
0	0	2	2	4	4
1	1	4	4	6	7

best = INF

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

  If exactly K roses in the current window, update solution and x1=x1+1

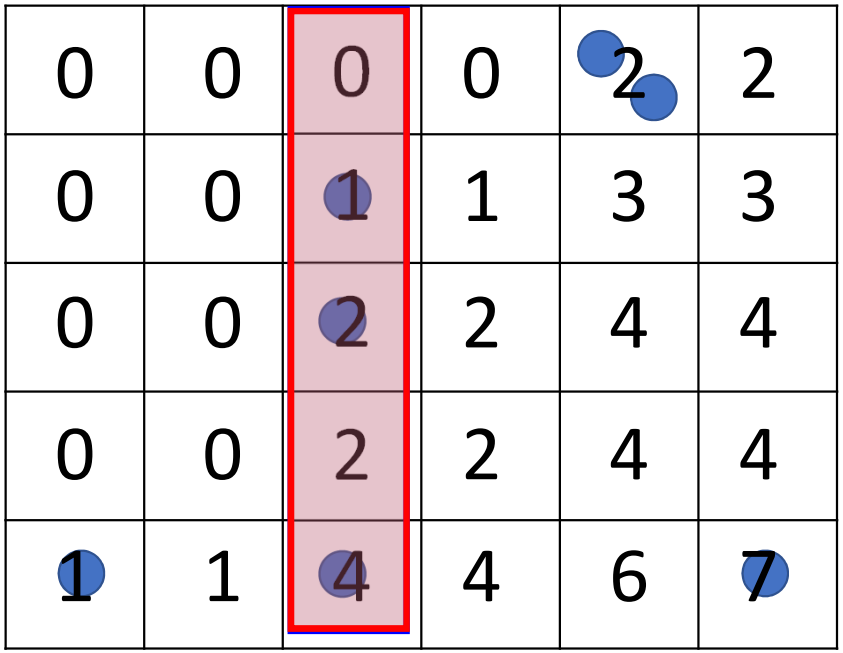


best = 14

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

  If exactly K roses in the current window, update solution and x1=x1+1



best = 12

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

0	0	0	0	2	2
0	0	1	1	3	3
0	0	2	2	4	4
0	0	2	2	4	4
1	1	4	4	6	7

best = 12

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

0	0	0	0	2	2
0	0	1	1	3	3
0	0	2	2	4	4
0	0	2	2	4	4
1	1	4	4	6	7

best = 12

Basics: Sliding window

- s[y] = size of min rectangle with K roses in the area (1,1)-(y,l)

0	0	0	0	2	2
0	0	1	1	3	3
0	0	2	2	4	4
0	0	2	2	4	4
1	1	4	4	6	7

best = 12

time = O(L)

Basics: Sliding window

- Generalize the idea:

s[y1][y2] = size of min rectangle with K roses in the area (y1,1)-(y2,L)

0	0	0	0	2	2
0	0	1	1	3	3
0	0	2	2	4	4
0	0	2	2	4	4
1	1	4	4	6	7

time = O(L)

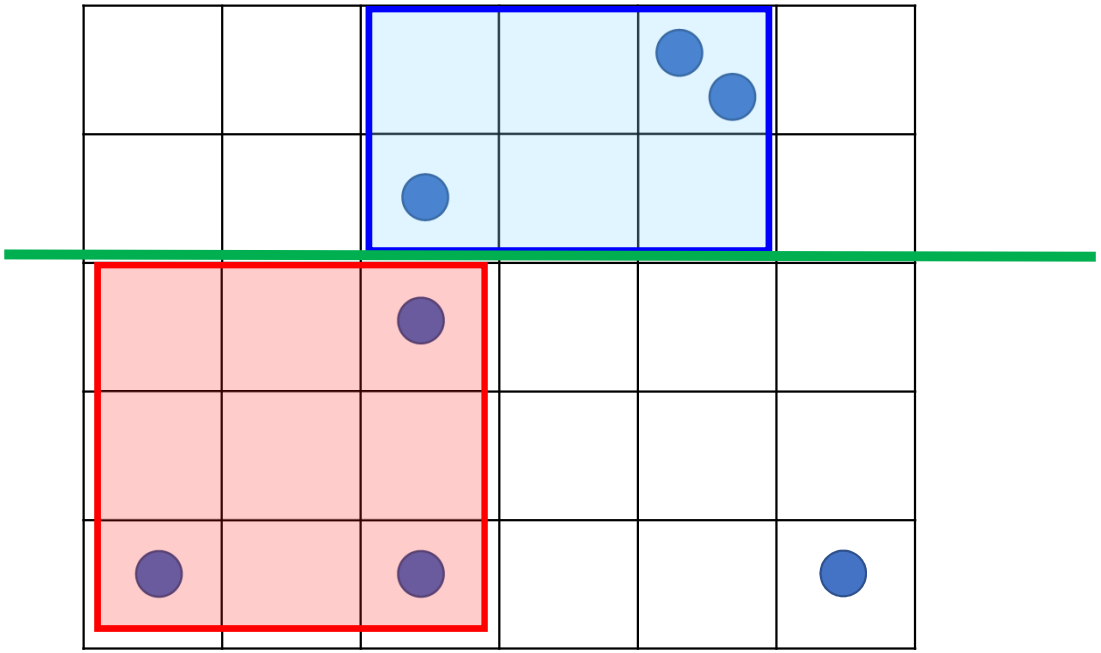
Solution: Sliding window

- Pick any pair (y1,y2): $O(W^2 L)$ – check all horizontal strips or
- Pick any pair (x1,x2): $O(W L^2)$ – check all vertical strips

- Total time $\min\{W^2 L, W L^2\}$

Problem: Two rectangles

- **Observation**: the rectangles can be separated either by a horizontal line or a vertical line



Solution: Sliding window

- Check for all horizontal lines y:

Sy[i][j] = min rectangle with K roses in (i-j) horizontal strip

ans = min {Sy[y1][y2]} + min {Sy[y3][y4]} where y1<=y2<= y <y3<=y4

- Check for all vertical lines x:

Sx[i][j] = min rectangle with K roses in (i-j) vertical strip

ans = min {Sx[x1][x2]} + min {Sx[x3][x4]} where x1<=x2<= x <x3<=x4

Time: $O(W^3 + L^3)$