Scheme 7

Computer Science 61A

October 27, 2016

1 Introduction

In the next part of the course, we will be working with the **Scheme** programming language. In addition to learning how to write Scheme programs, we will eventually write a Scheme interpreter in Project 4!

Scheme is a dialect of the **Lisp** programming language, a language dating back to 1958. The popularity of Scheme within the programming language community stems from its simplicity – in fact, previous versions of CS 61A were taught in the Scheme language.

2 Primitives

Scheme has a set of *atomic* primitive expressions. Atomic means that these expressions cannot be divided up.

```
scm> 123
123
scm> 123.123
123.123
scm> #t
True
scm> #f
False
```

Unlike in Python, the only primitive in Scheme that is a false value is `#f` and its equivalents, `false` and `False`. The `define` special form defines variables and procedures by binding a value to a variable, just like the assignment statement in Python. When a variable is defined, the `define` special form returns a symbol of its name. A procedure is what we call a function in Scheme!

The syntax to define a variable and procedure are:

- `(define <variable name> <value>)`
- `(define (<function name> <parameters>)<function body>)`

```
scm> (define a 3)                  ; a = 3
a
scm> a
3
scm> (define (foo x) x)            ; procedure named foo
foo
scm> (foo a)
3
```

2.1 Questions

1. What would Scheme print?

```
scm> (define a 1)
```

Solution:
`a`

```
scm> a
```

Solution:
`1`

```
scm> (define b a)
```

Solution:
`b`

```
scm> b
```

Solution:
`1`

```
scm> (define c 'a)
```

Solution:
```
c
```

```
scm> c
```

Solution:
```
a
```

3 Call Expressions

Scheme call expressions follow prefix notation, where an operator is followed by zero or more operand subexpressions. Operators may be symbols, such as `+` and `*` or more complex expressions, as long as they evaluate to procedure values.

```
scm> (- 1 1)                ; 1 - 1
0
scm> (/ 8 4 2)              ; 8 / 4 / 2
1
scm> (* (+ 1 2) (+ 1 2))    ; (1 + 2) * (1 + 2)
9
```

To call a function in Scheme, you first need a set of parentheses. Inside the parentheses, you specify a function, then the arguments (remember the spaces!).

Evaluating a Scheme function call works just like Python:

1. Evaluate the operator (the first expression after the `(`), then evaluate each of the operands.
2. Apply the operator to those evaluated operands.

When you evaluate `(+ 1 2)`, you evaluate the `+` symbol, which is bound to a built-in addition function. Then, you evaluate `1` and `2`, which are primitives. Finally, you apply the addition function to `1` and `2`.

Some important built-in functions you'll want to know are:

- `+`, `-`, `*`, `/`
- `equal?`, `=`, `>`, `>=`, `<`, `<=`

3.1 Questions

1. What would Scheme print?
   ```
   scm> (+ 1)
   ```

   Solution:
   ```
   1
   ```

   ```
   scm> (* 3)
   ```

   Solution:
   ```
   3
   ```

```
scm> (+ (* 3 3) (* 4 4))
```

Solution:
25

```
scm> (define a (define b 3))
```

Solution:
a

```
scm> a
```

Solution:
b

```
scm> b
```

Solution:
3

4 Special Forms

There are certain expressions that look like function calls, but *don't* follow the rule for order of evaluation. These are called *special forms*. You've already seen one — `define`, where the first argument, the variable name, doesn't actually get evaluated to a value.

4.1 If Statements

Another common special form is the **`if`** form. An **`if`** expression looks like:

```
(if <condition> <then> <else>)
```

where `<condition>`, `<then>` and `<else>` are expressions. First, `<condition>` is evaluated. If it evaluates to `#t`, then `<then>` is evaluated. Otherwise, `<else>` is evaluated. Remember that only `False` and `#f` are false-y values; everything else is truth-y.

```
scm> (if (< 4 5) 1 2)
1
scm> (if #f (/ 1 0) 42)
42
```

4.2 Boolean Operators

Scheme also has boolean operators **and**, **or**, and **not** like in Python! In addition, **and** and **or** are also special forms because they are short-circuiting operators.

```
scm> (and 1 2 3)
3
scm> (or 1 2 3)
1
scm> (or True (/ 1 0))
True
scm> (and False (/1 0))
False
scm> (not 3)
False
scm> (not True)
False
```

4.3 Questions

1. What does Scheme print?

```
scm> (if (or #t (/ 1 0)) 1 (/ 1 0))
```

Solution:
```
1
```

```
scm> (if (> 4 3) (+ 1 2 3 4) (+ 3 4 (* 3 2)))
```

Solution:
```
10
```

```
scm> ((if (< 4 3) + -) 4 100)
```

Solution:
```
-96
```

```
scm> (if 0 1 2)
```

Solution:
```
1
```

4.4 Lambdas and Defining Functions

Scheme has lambdas too! The syntax is

```
(lambda (<PARAMETERS>)<EXPR>)
```

Like in Python, lambdas are function values. Also like in Python, when a lambda expression is called in Scheme, a new frame is created where the parameters are bound to the arguments passed in. Then, `<EXPR>` is evaluated in this new frame. Note that `<EXPR>` is not evaluated until the lambda function is called.

```
scm> (define x 3)
x
scm> (define y 4)
y
scm> ((lambda (x y) (+ x y)) 6 7)
13
```

Like in Python, lambda functions are also values! So you can do this to define functions:

```
scm> (define square (lambda (x) (* x x)))
```

```
square
scm> (square 4)
16
```

When you do `(define (<FUNCTION NAME> <PARAMETERS>) <EXPR>)`, Scheme will automatically transform it to `(define <FUNCTION NAME> (lambda (<PARAMETERS>) <EXPR>)`. In this way, lambdas are more central to Scheme than they are to Python.

4.5 Let

There is also a special form based around `lambda`: `let`. The structure of `let` is as follows:

```
(let ( (<SYMBOL1> <EXPR1>)
       ...
       (<SYMBOLN> <EXPRN>) )
      <BODY> )
```

This special form is really just equivalent to:

```
( (lambda (<SYMBOL1> ... <SYMBOLN>) <BODY>) <EXPR1> ... <EXPRN>)
```

`let` effectively binds symbols to expressions, then runs the body of the let form. This can be useful if you need to reuse a value multiple times, or if you want to make your code more readable.

For example, we can use the approximation $\sin(x) \approx x$ (which is true for small x) and the trigonometric identity $\sin(x) = 3\sin(x/3) - 4\sin^3(x/3)$ to approximate $\sin(x)$ for any x.

```
(define (sin x)
    (if (< x 0.000001)
        x
        (let ( (recursive-step (sin (/ x 3))) )
            (- (* 3 recursive-step)
               (* 4 (expt recursive-step 3))))))
```

4.6 Questions

1. Write a function that calculates factorial. (Note we have not seen any iteration yet.)

```
(define (factorial x)
```

Solution:

```
    (if (< x 2)
        1
        (* x (factorial (- x 1))))
```

```
                                                  )
```

2. Write a function that calculates the n^{th} Fibonacci number.

```
(define (fib n)
    (if (< n 2)
        1
```

Solution:

```
        (+ (fib (- n 1)) (fib (- n 2))))
```

```
                                                                                    )
```

5 Pairs and Lists

To construct a (linked) list in Scheme, you can use the constructor `cons` (which takes two arguments). `nil` represents the empty list. If you have a linked list in Scheme, you can use selector `car` to get the first element and selector `cdr` to get the rest of the list. (`car` and `cdr` don't stand for anything anymore, but if you want the history go to `http://en.wikipedia.org/wiki/CAR_and_CDR`.)

```
scm> nil
()
scm> (null? nil)
#t
scm> (cons 2 nil)
(2)
scm> (cons 3 (cons 2 nil))
(3 2)
scm> (define a (cons 3 (cons 2 nil)))
a
scm> (car a)
3
scm> (cdr a)
(2)
scm> (car (cdr a))
2
scm> (define (len a)
       (if (null? a)
         0
         (+ 1 (len (cdr a)))))
len
scm> (len a)
2
```

If a list is a ”good looking” list, like the ones above where the second element is always a linked list, we call it a **well-formed list**. Interestingly, in Scheme, the second element does not have to be a linked list. You can give something else instead, but `cons` always takes exactly 2 arguments. These lists are called **malformed list**. The difference is a dot:

```
scm> (cons 2 3)
(2 . 3)
scm> (cons 2 (cons 3 nil))
(2 3)
scm> (cdr (cons 2 3))
3
scm> (cdr (cons 2 (cons 3 nil)))
(3)
```

In general, the rule for displaying a pair is as follows: use the dot to separate the `car` and `cdr` fields of a pair, but if the dot is immediately followed by an open parenthesis, then remove the dot and the parenthesis pair. Thus, `(0 . (1 . 2))` becomes `(0 1 . 2)`

There are many useful operations and shorthands on lists. One of them is `list` special form `list` takes zero or more arguments and returns a list of its arguments. Each argument is in the car field of each list element. It behaves the same as quoting a list, which also creates the list.

```
scm> (list 1 2 3)
(1 2 3)
scm> '(1 2 3)
(1 2 3)
scm> (car '(1 2 3))
1
scm> (equal? '(1 2 3) (list 1 2 3))
#t
scm> '(1 . (2 3))
(1 2 3)
scm> '(define (square x) (* x x))
(define (square x) (* x x))
```

1. Define a function that takes 2 lists and concatenates them together. Notice that simply calling `(cons a b)` would not work because it will create a deep list. Instead, think recursively!

```
(define (concat a b)
```

Solution:

```
(if (null? a)
  b
  (cons (car a) (concat (cdr a) b)))
```

```
                                                  )

scm> (concat '(1 2 3) '(2 3 4))
(1 2 3 2 3 4)
```

2. Define `replicate`, which takes an element `x` and a non-negative integer `n`, and returns a list with `x` repeated `n` times.

```
(define (replicate x n)
```

Solution:

```
(if (= n 0)
    nil
    (cons x (replicate x (- n 1))))
```

```
                                             )

scm> (replicate 5 3)
(5 5 5)
```

3. A **run-length encoding** is a method of compressing a sequence of letters. The list `(a a a b a a a a)` can be compressed to `((a 3) (b 1) (a 4))`, where the compressed version of the sequence keeps track of how many letters appear consecutively.

   Write a Scheme function that takes a compressed sequence and expands it into the original sequence. *Hint:* try to use functions you defined earlier in this worksheet.

```
(define (uncompress s)
```

Solution:

```
  (if (null? s)
    s
    (concat (replicate (car (car s)) (car (cdr (car s)))) (
       uncompress (cdr s))))
```

```
                                                      )
scm> (uncompress '((a 1) (b 2) (c 3)))
(a b b c c c)
```

4. Define `deep-apply`, which takes a nested list and applies a given procedure to every element. `deep-apply` should return a nested list with the same structure as the input list, but with each element replaced by the result of applying the given procedure to that element. Use the built-in `list?` procedure to detect whether a value is a list. The procedure `map` has been defined for you.

```
(define (map fn lst)
  (if (null? lst)
    nil
    (cons (fn (car lst)) (map fn (cdr lst)))))
(define (deep-apply fn nested-list)
```

Solution:

```
(if (list? nested-list)
    (map (lambda (x) (deep-apply fn x)) nested-list)
    (fn nested-list))
```

```
                                                              )
```

```
scm> (deep-apply (lambda (x) (* x x)) '(1 2 3))
(1 4 9)
scm> (deep-apply (lambda (x) (* x x)) '(1 ((4) 5) 9))
(1 ((16) 25) 81)
scm> (deep-apply (lambda (x) (* x x)) 2)
4
```

6 Extra Questions

1. Fill in the following to complete an abstract tree data type:

```
(define (make-tree root branches) (cons root branches))

(define (root tree)                                        )

(define (branches tree)                                     )
```

Solution:
```
(define (root tree) (car tree))
(define (branches tree) (cdr tree))
```

2. Using the abstract data type above, write a function that sums up the entries of a tree, assuming that the entries are all numbers. Hint: you may want to use the `map` function you defined above, as well as an additional helper function.

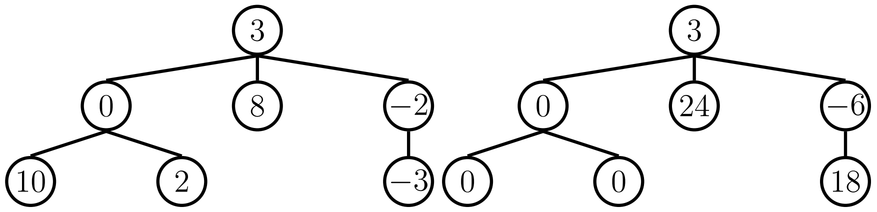
```
(define (tree-sum tree)
```

Solution:
```
    (+ (root tree) (sum (map tree-sum (branches tree))))
```

```
                                                   )
```

Solution:
```
(define (sum lst)
    (if (null? lst) 0 (+ (car lst) (sum (cdr lst)))))
```

3. Using the abstract data type above, write a Scheme function that creates a new tree where the entries are the product of the entries along the path to the root in the original tree. Hint: you may want to write helper functions.



```
(define (path-product-tree t)
```

Solution:

```
(define (path-product t product)
    (let ((prod (* product (root t))))
        (make-tree prod
            (map (lambda (t) (path-product t prod))
                 (branches tree)))))
(path-product t 1))
```