The Guide to AR, MA, ARIMA, SARIMA Forecasting Method

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ARIMA models provide another approach to time series forecasting. Exponential smoothing and ARIMA models are the two most widely used approaches to time series forecasting, and provide complementary approaches to the problem. While exponential smoothing models (ESM) are based on a description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

Before we introduce ARIMA models, Please go through the discuss concept of stationarity and the technique of differencing time series here.

Autoregressive (AR) Models

In an autoregression model, we forecast the variable of interest using a linear combination of *past values of the variable*. The term *auto*regression indicates that it is a regression of the variable against itself.

Thus, an autoregressive model of order p can be written as:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t,$$

where εt is white noise. It is **AR(p) model** of order p.

For an AR(1) model, i.e. *p =1, AR model of order 1.*

$$Y_t = c + \phi_1 * y_{t-1} + \varepsilon_t$$

*when **ϕ1=0, yt** is equivalent to **white noise;***

*when **ϕ1=1 and c=0, yt** is equivalent to a **random walk;***

*when **ϕ1=1 and c≠0, yt** is equivalent to a **random walk with drift;***

*when **ϕ1<0 ,yt** tends to **oscillate around the mean.***

Moving average (MA) models

Rather than using past values of the forecast variable in a regression, a moving average model uses past forecast errors in a regression-like model.

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q},$$

where εt is white noise. It is an **MA(q) model** of order q. Of course, we do not *observe* the values of εt, so it is not really a regression in the usual sense. Notice that each value of yt can be thought of as a weighted moving average of the past few forecast errors.

It is possible to write any stationary AR(p) model as an MA(∞) model. For example, using repeated substitution, we can demonstrate this for an AR(1) model:

$$\begin{aligned} y_t &= \phi_1 y_{t-1} + \varepsilon_t \\ &= \phi_1(\phi_1 y_{t-2} + \varepsilon_{t-1}) + \varepsilon_t \\ &= \phi_1^2 y_{t-2} + \phi_1 \varepsilon_{t-1} + \varepsilon_t \\ &= \phi_1^3 y_{t-3} + \phi_1^2 \varepsilon_{t-2} + \phi_1 \varepsilon_{t-1} + \varepsilon_t \end{aligned}$$

Provided *−1 <ϕ1 < 1*, the value of *ϕk1* will get smaller as *k* gets larger. So eventually we obtain:

$$y_t = \varepsilon_t + \phi_1 \varepsilon_{t-1} + \phi_1^2 \varepsilon_{t-2} + \phi_1^3 \varepsilon_{t-3} + \cdots,$$

Non-seasonal ARIMA models

If we combine differencing with autoregression and a moving average model, we obtain a non-seasonal ARIMA model. ARIMA is an acronym for AutoRegressive Integrated Moving Average.

$$y'_t = c + \phi_1 y'_{t-1} + \cdots + \phi_p y'_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t,$$

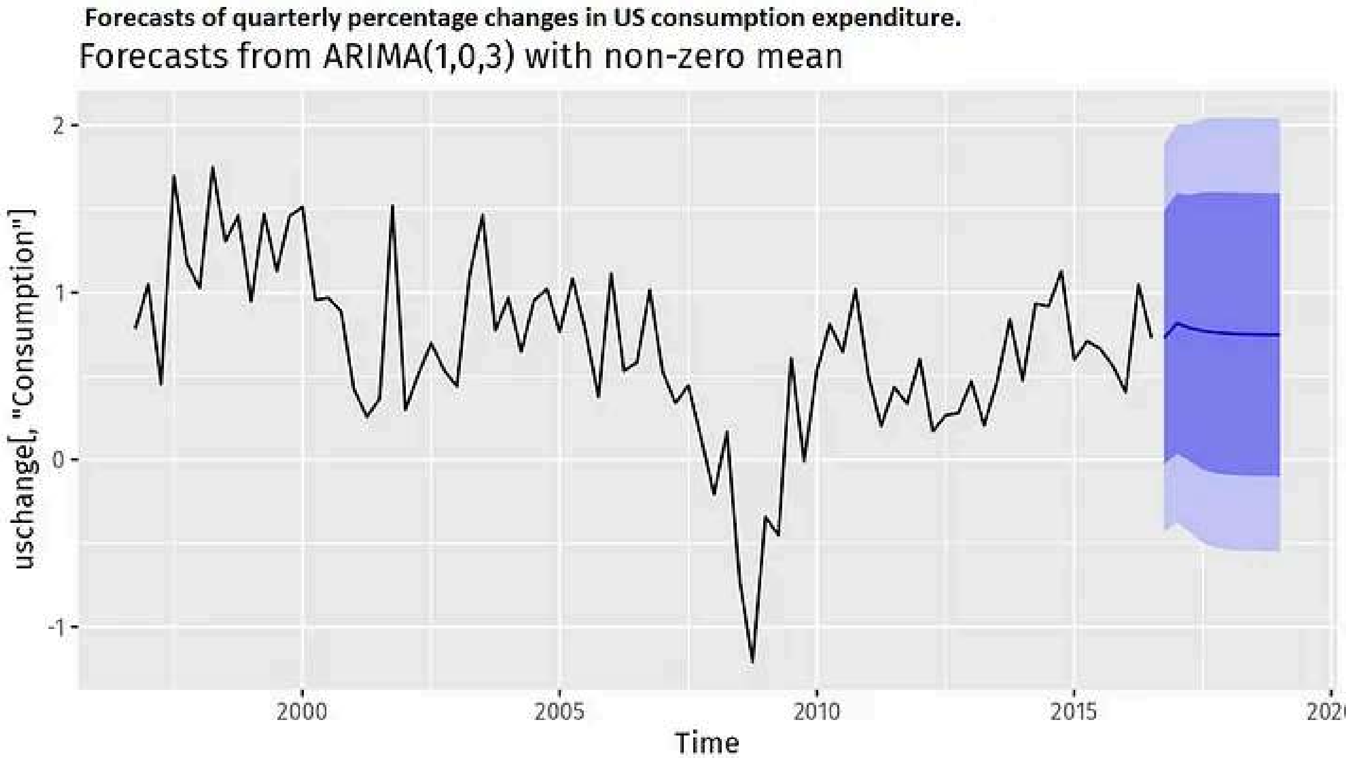
where *y't is the differenced series* (can be differenced more than once). The "predictors" on the right hand side include both lagged values of *yt and lagged errors*.

We call this an *ARIMA(p,d,q)* model, where
p= order of the autoregressive part
d= degree of first differencing involved
q= order of the moving average part

The same stationarity and invertibility conditions that are used for autoregressive and moving average models also apply to an ARIMA model.

Special cases of ARIMA models.

White noise	ARIMA(0,0,0)
Random walk	ARIMA(0,1,0) with no constant
Random walk with drift	ARIMA(0,1,0) with a constant
Autoregression	ARIMA(p,0,0)
Moving average	ARIMA(0,0,q)



Understanding ARIMA models

The `auto.arima()` function is useful, but anything automated can be a little dangerous, and it is worth understanding something of the behaviour of the models.

The constant c has an important effect on the long-term forecasts obtained from these models:

If c=0 and d=0, the long-term forecasts will go to zero.

If c=0 and d=1, the long-term forecasts will go to a non-zero constant.

If c=0 and d=2, the long-term forecasts will follow a straight line.

If c≠0 and d=0, the long-term forecasts will go to the mean of the data.

If c≠0 and d=1, the long-term forecasts will follow a straight line.

If c≠0 and d=2, the long-term forecasts will follow a quadratic trend.

Estimation and order selection

Information Criteria

Akaike's Information Criterion (AIC), which was useful in selecting predictors for regression, is also useful for determining the order of an ARIMA model. It can be written as:

$$\text{AIC} = -2\log(L) + 2(p + q + k + 1),$$

where L is the likelihood of the data, k = 1 if c ≠ 0 and k = 0 if c = 0.

For ARIMA models, the corrected AIC can be written as

$$\text{AICc} = \text{AIC} + \frac{2(p+q+k+1)(p+q+k+2)}{T-p-q-k-2},$$

and the Bayesian Information Criterion can be written as

$$\text{BIC} = \text{AIC} + [\log(T) - 2](p+q+k+1).$$

Good models are obtained by ***minimising the AIC, AICc or BIC***. Our preference is to use the AICc.

AIC should not to be used to select order of differencing(d) of a model, but only for selecting the values of p and q. Because the differencing changes the data on which the likelihood is computed, So we need to use some other approach to choose d, and then we can use the AICc to select p and q.

Seasonal ARIMA models (SARIMA)

A seasonal ARIMA model is formed by including additional seasonal terms in the ARIMA models we have seen so far. It is written as follows:

$$\text{ARIMA} \quad \underbrace{(p, d, q)}_{\substack{\uparrow \\ \text{Non-seasonal part} \\ \text{of the model}}} \quad \underbrace{(P, D, Q)_m}_{\substack{\uparrow \\ \text{Seasonal part of} \\ \text{of the model}}}$$

where m = number of observations per year

ACF/PACF for Seasonal ARIMA

The seasonal part of an AR or MA model will be seen in the seasonal lags of the PACF and ACF.

For example, an ARIMA(0,0,0)(0,0,1)12 model will show:

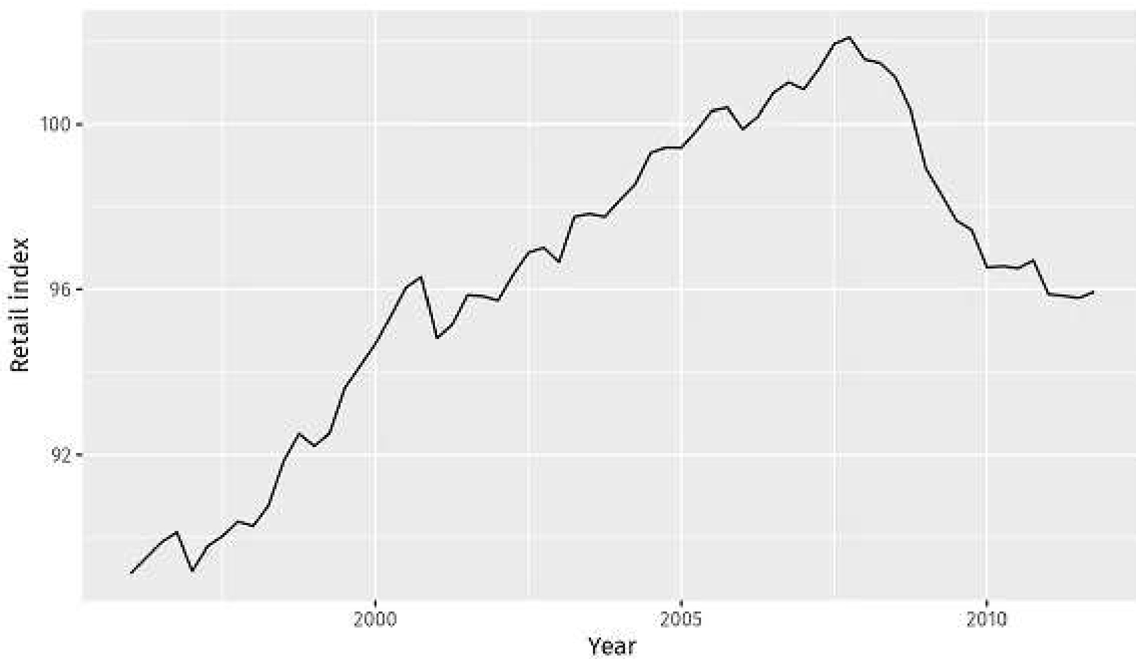
1) a spike at lag 12 in the ACF but no other significant spikes;
2) exponential decay in the seasonal lags of the PACF (i.e., at lags 12, 24, 36, …)

Similarly, an ARIMA(0,0,0)(1,0,0)12 model will show:

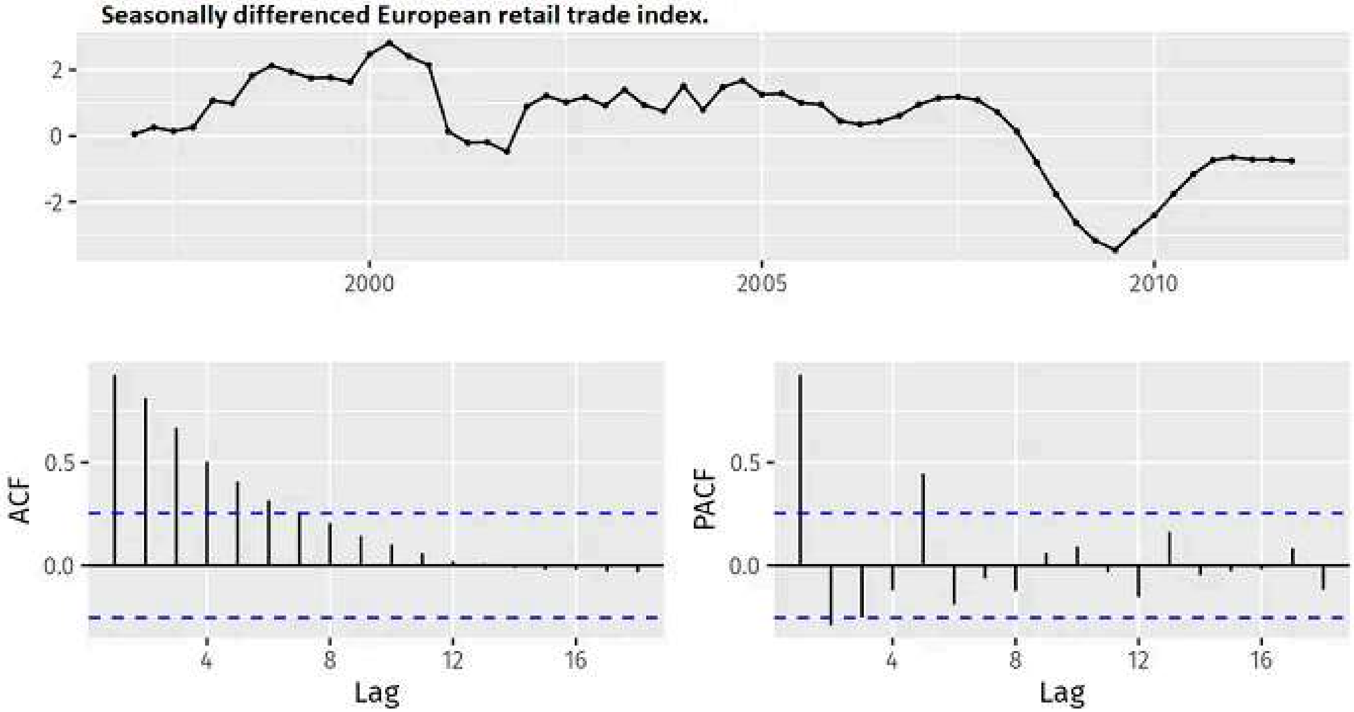
1) exponential decay in the seasonal lags of the ACF;
2) a single significant spike at lag 12 in the PACF.

Example: European quarterly retail trade

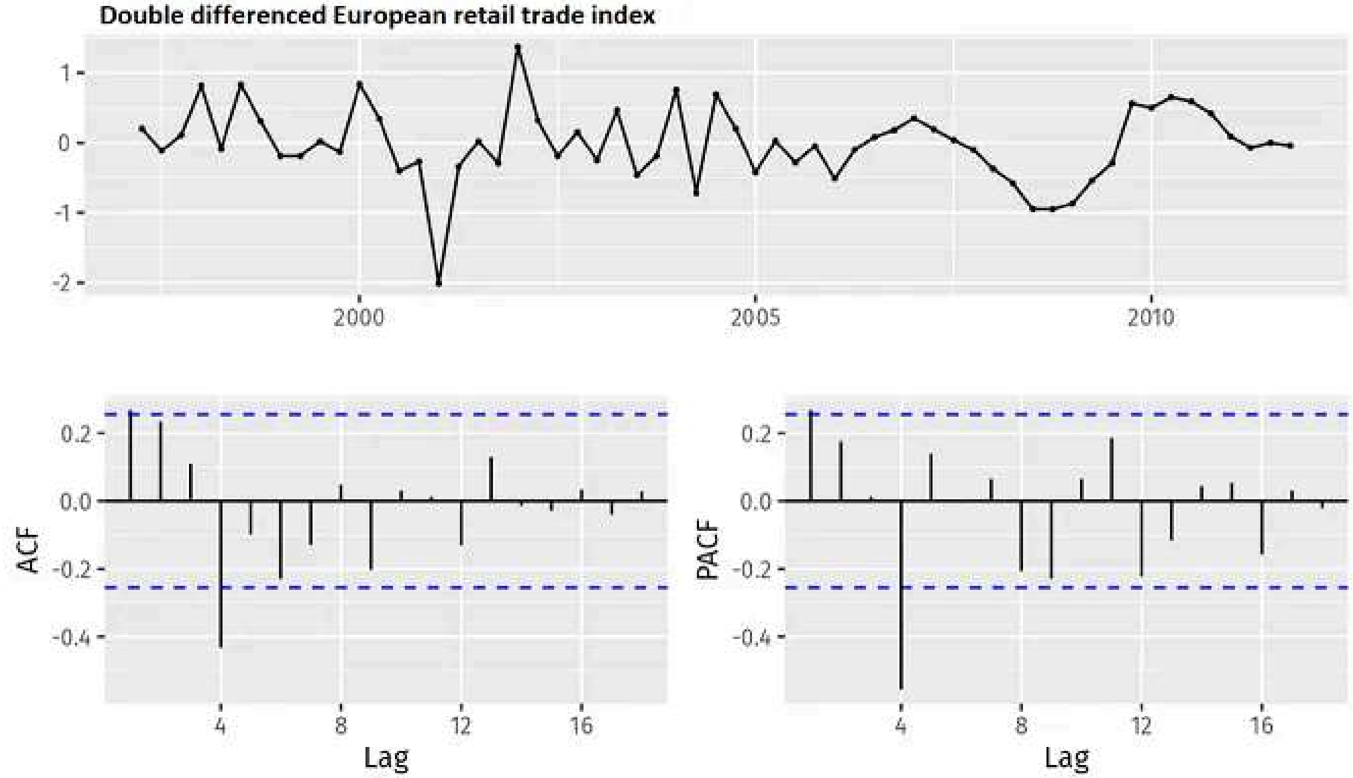
The data are clearly non-stationary, with some seasonality.



so, we take first seasonal difference, with lag =4, quarterly.



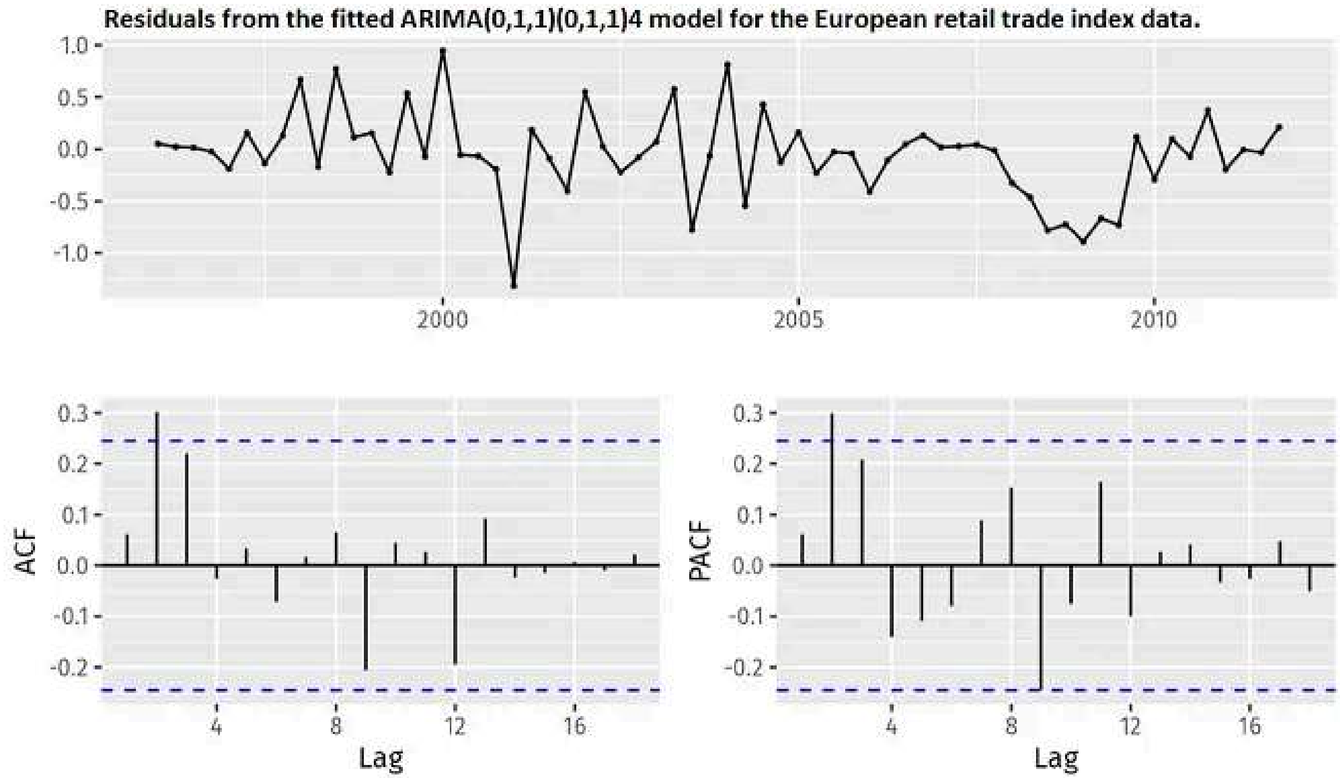
After 1st Seasonal difference (lag=4), the series appear to be non-stationary in above fig, so we take an additional first difference.



Our aim now is to find an appropriate ARIMA model based on the ACF and PACF shown in above fig.

The significant spike at lag 1 in the ACF suggests a non-seasonal MA(1) component.
The significant spike at lag 4 in the ACF suggests a seasonal MA(1) component.
Consequently, we begin with an ***ARIMA(0,1,1)(0,1,1)4*** model, indicating a first and seasonal difference, and non-seasonal and seasonal MA(1) components.

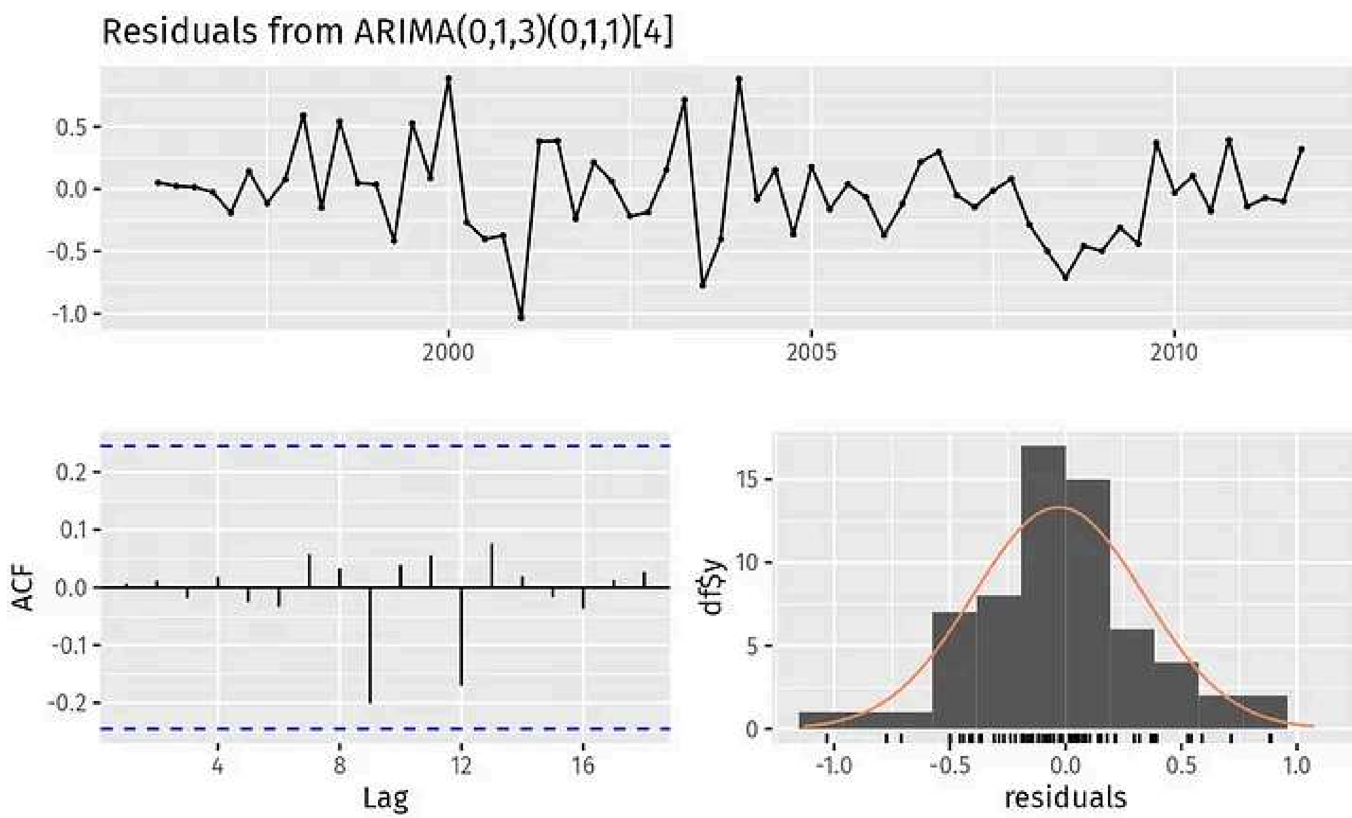
The residuals for the fitted *ARIMA(0,1,1)(0,1,1)4* model are shown below.



Both the ACF and PACF show significant spikes at lag 2, and almost significant spikes at lag 3, indicating that some additional non-seasonal terms need to be included in the model.

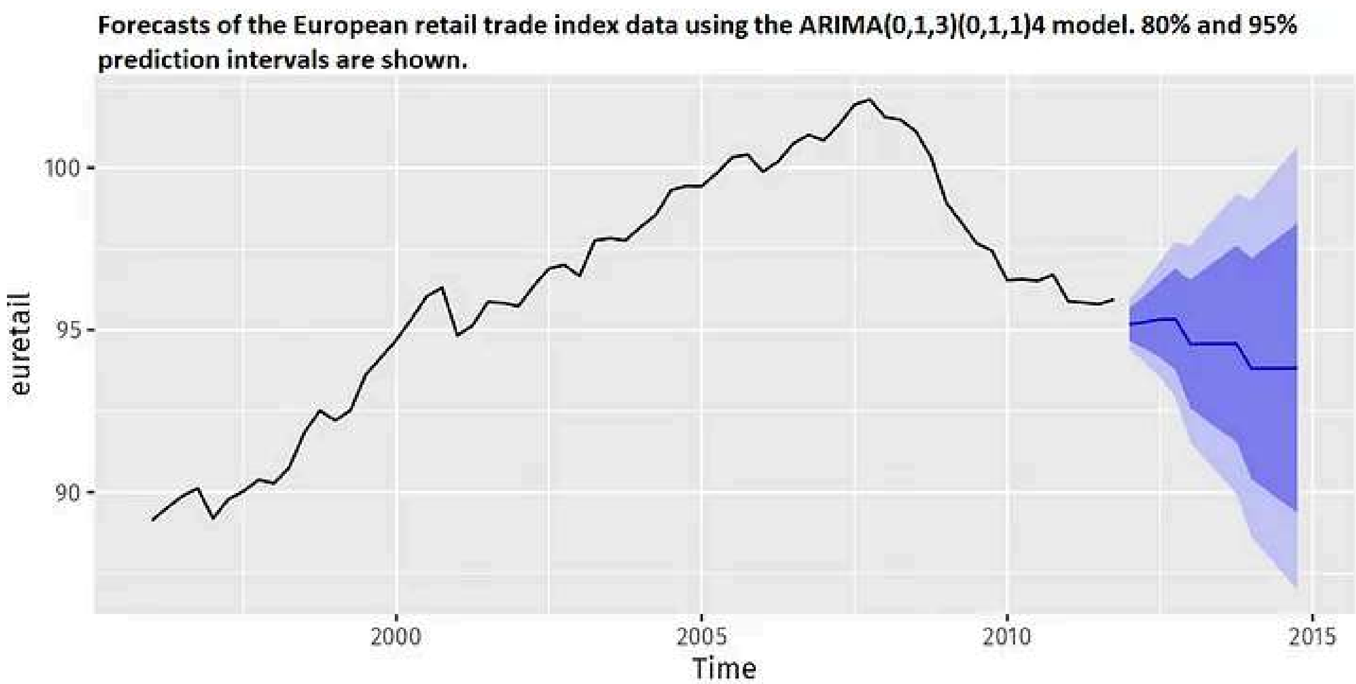
The AICc of the ***ARIMA(0,1,2)(0,1,1)4 model is 74.36,*** while that for the ***ARIMA(0,1,3)(0,1,1)4 model is 68.53.***

The author tried other models with AR terms as well, but none that gave a smaller AICc value. Consequently, we choose the **ARIMA(0,1,3)(0,1,1)4** model and residuals are plotted in below fig.



All the spikes are now within the significance limits, so the residuals appear to be white noise. The Ljung-Box test also shows that the residuals have no remaining autocorrelations.

we now have a seasonal ARIMA model that passes the required checks and is ready for forecasting.



Forecasts from the model for the next three years are shown in above fig. The forecasts follow the recent trend in the data, because of the double differencing. The large and rapidly increasing prediction intervals show that the retail trade index could start increasing or decreasing at any time — while the point forecasts trend downwards.

In conclusion, ARIMA and SARIMA are powerful tools for time series forecasting, especially when dealing with non-stationary data and seasonal patterns. Choosing between ARIMA and SARIMA depends on the nature of the data and whether there are clear seasonal patterns present. These models can be useful for making predictions and understanding trends in various fields, including finance, economics, and environmental science.

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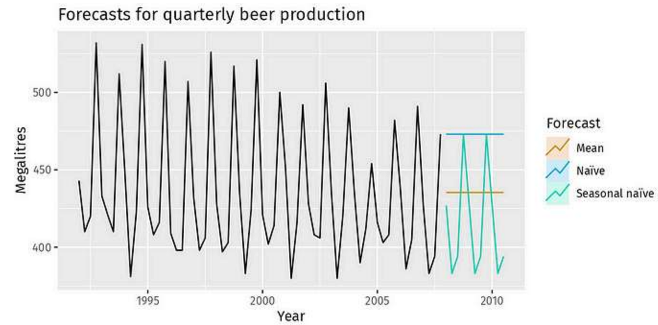

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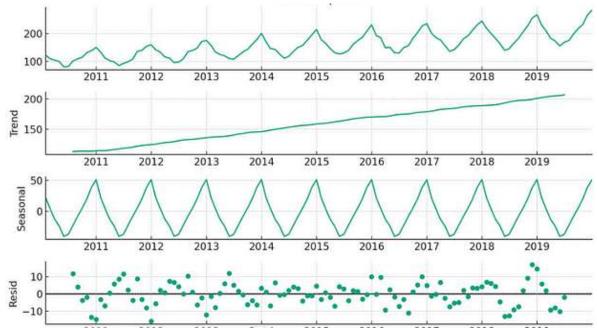

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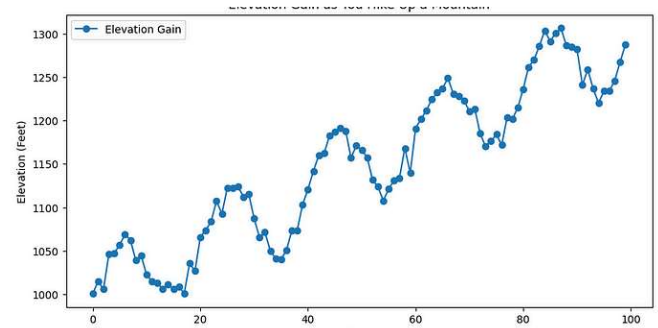

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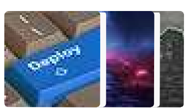

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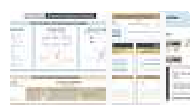

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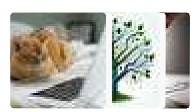

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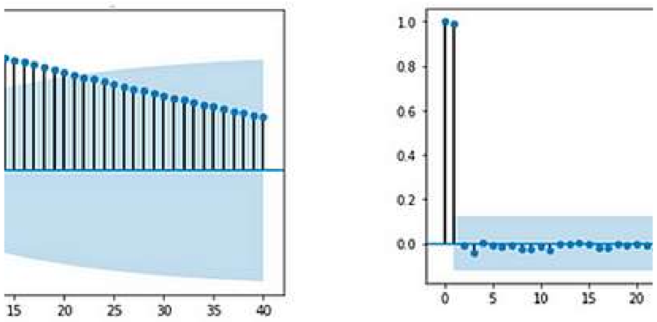

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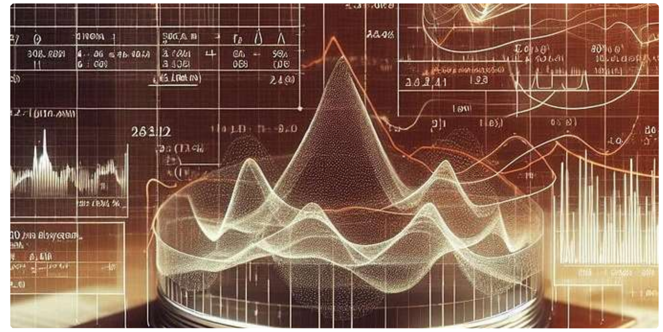

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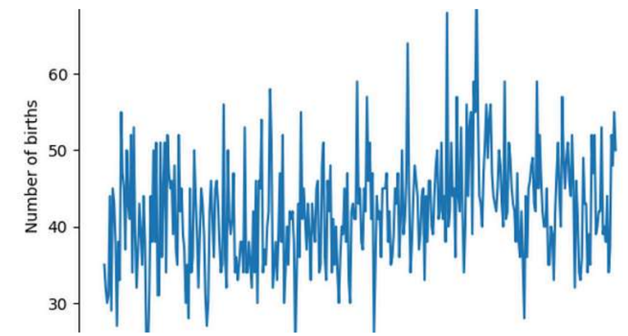

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