
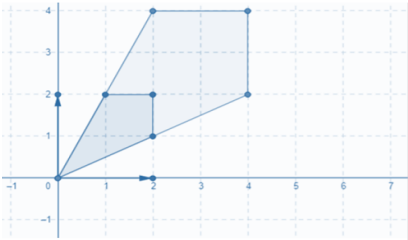
	<p style="text-align: center;">PES University, Bengaluru (Established under Karnataka Act No. 16 of 2013)</p>	<p style="text-align: center;">UE20CS904</p>
<div style="display: flex; align-items: center;">  <div> <p style="text-align: center;">AUG 22 : END SEMESTER ASSESSMENT (ESA) M TECH DATA SCIENCE AND MACHINE LEARNING_ SEMESTER I</p> <p style="text-align: center;">UE20CS904 - Mathematical Foundation</p> </div> </div>		
<p>Time: 3 Hrs</p>	<p style="text-align: center;">Answer All Questions</p>	<p style="text-align: right;">Max Marks: 80</p>
<p style="text-align: center;">Instructions</p> <ul style="list-style-type: none"> • All answers should be handwritten in the answer script. • Graph, if any, has to be plotted in the graph sheet page of the answer script. • Marks will be allotted only when all the steps to arrive at the answer are shown. 		
<p style="text-align: center;">Section A (20 marks)</p>		

1	a)	<p>Calculate the angle between two given vectors. The two vectors are, $a = \vec{i} + 2\vec{j}$ and $b = 9\vec{i} + 3\vec{j}$</p>	2
	b)	<p>Find the vector projection of the vector $a = [3,4]$ on $b = [5, -12]$</p>	2
	c)	<p>In the plot shown below the dark shaded portion represents the original coordinates of an object and the same after transformation is represented by the lightly shaded object. Write the coordinates, the transformation matrix and the coordinates after transformation.</p> 	2
	d)	<p>Write the transformation matrix rotation of a 2d image.</p>	2
	e)	<p>State true or false: (i) A singular value decomposition exists only for square matrix. (ii) $A^T = A^{-1}$ if A is orthogonal</p>	2
2	a)	<p>$A = [1 \ 1 \ 1 \ 1]$ $B = [0 \ 1 \ -1 \ 1]$ $C = [-2 \ 1 \ 0 \ 0]$ Check if the following are true or false: (BA + A) = (B + I)A, where I is the identity matrix. (A + B)C = AC + BC</p>	2
	b)	<p>What will happen when eigenvalues are roughly equal?</p> <p>A. PCA will perform outstandingly B. PCA will perform badly C. Can't Say D. None of above</p>	2

	c)	Find out if the following is a concave function or convex function for the interval $(-5, -2)$ $f(x) = -x^2 - 7x$	2
	d)	Find the critical points of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$	2
	e)	Find the Jacobian Matrix of $f(x, y) = (x^2 + y^2 - 1, x^2y + \sin(xy^2))$	2
Section B (30 marks)			
3	a)	Check whether the vectors $a = [3 \ 1 \ 2]$, $b = [-1 \ 3 \ 4]$, $c = [5 \ 0 \ 1]$ are linearly independent and find the rank of the matrix. (3 marks) (ii) Check if this matrix is orthogonal. (2 marks)	5
	b)	Find Eigen Values of A, A^2 and A^{-1} for $A = \begin{bmatrix} 4 & 2 & 1 & 3 \end{bmatrix}$	5
	c)	Mr. Johns sells Mango, Apple and Peach. The price of a kg of Mango, 3 kgs of Apple, and a kg of Peach is Rs 145. The price of 3 kgs of Mango, 4 kgs of Apple, and a kg of Peach is Rs 280. The price of 2 kgs of Apple, and a kg of Peach is Rs 65. Find out the price of a kg of each fruit.	5
	d)	If $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ Calculate $f'(x), f(0), f(\infty), f(-\infty)$	5
	e)	Mr X is an investor. His portfolio primarily tracks the performance of the Nifty index and he wants to add the stock of company 'A'. Before adding the stock to his portfolio, he wants to assess if there exists a relationship between Nifty and Stock A. <div> <div>Year</div> <div>Nifty</div> <div>Stock A</div> </div> <div> <div>2015</div> <div>1692</div> <div>682016</div> <div>1978</div> <div>102</div> </div> <div> <div>2017</div> <div>1884</div> <div>110</div> </div> <div> <div>2018</div> <div>2151</div> <div>112</div> </div> <div> <div>2019</div> <div>2519</div> <div>154</div> </div> Help Mr X to assess the same.	5
	f)	Obtain an orthonormal basis for R^3 by applying Gram-Schmidt to the linearly independent set $\{v_1 = (1,1,0), v_2 = (1,1,1), v_3 = (3,1,1)\}$	5
SECTION C – (3 * 10 = 30 MARKS)			
4	a)	<p>(i) For the image matrix on the left, use the kernel on the right and perform a convolution operation so as to replace the no '18' with appropriate value. Write your observation if this operation will increase or decrease the intensity values. (2 marks)</p> <div> <div> <div>1</div><div>2</div><div>3</div><div>4</div><div>5</div><div>6</div><div>7</div> </div> <div> <div>8</div><div>8</div><div>10</div><div>11</div><div>12</div><div>13</div><div>14</div> </div> <div> <div>15</div><div>16</div><div>17</div><div>18</div><div>19</div><div>20</div><div>21</div> </div> <div> <div>22</div><div>23</div><div>24</div><div>25</div><div>26</div><div>27</div><div>28</div> </div> <div> <div>29</div><div>30</div><div>31</div><div>32</div><div>33</div><div>34</div><div>35</div> </div> </div> <div>★</div> <div> <div>0.1</div><div>0.2</div><div>0.3</div> </div> <div> <div>0.4</div><div>0.5</div><div>0.6</div> </div> <div> <div>0.7</div><div>0.8</div><div>0.9</div> </div>	10
		(ii) Find the Single value decomposition of the following matrix	

		$A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$ The Σ or the D matrix is given as $\begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{10} \end{bmatrix}$ (5 Marks)																									
		(iii) Calculate Euclidean, Manhattan and Chebyshev distance between $A = (2,3,4)$ and $B = (1,0,-1)$ (3 Marks)																									
4	b)	<p>Consider a firm operating two plants in two different locations. They both produce the same output (say, 10 units) using the same type of inputs. Although the amounts of inputs vary between the plants the output level is the same.</p> <ol style="list-style-type: none">1. The firm management suspects that the production cost in Plant 2 is higher than in Plant 1. Verify?2. The manager of the Plant 2 claims that the reason of the cost differences is the higher input prices in her region than in the other.3. Is the available information supports her claim? <p>The following information was collected from the managers of these plants.</p> <table><caption>PLANT 1</caption><tr><th>Input</th><th>Price</th><th>Amount used</th></tr><tr><td>Input 1</td><td>3</td><td>9</td></tr><tr><td>Input 2</td><td>5</td><td>10</td></tr><tr><td>Input 3</td><td>7</td><td>8</td></tr></table> <table><caption>PLANT 2</caption><tr><th>Input</th><th>Price</th><th>Amount used</th></tr><tr><td>Input 1</td><td>4</td><td>8</td></tr><tr><td>Input 2</td><td>7</td><td>12</td></tr><tr><td>Input 3</td><td>3</td><td>9</td></tr></table>	Input	Price	Amount used	Input 1	3	9	Input 2	5	10	Input 3	7	8	Input	Price	Amount used	Input 1	4	8	Input 2	7	12	Input 3	3	9	10
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4	c)	<p>Consider the data given below and fit a linear regression line $y = ax + b$ using gradient descent.</p> <table><tr><td>X</td><td>0</td><td>0.4</td><td>0.6</td><td>1</td></tr><tr><td>Y</td><td>0</td><td>1</td><td>0.48</td><td>0.95</td></tr></table> <p>Initialize the weights a and b to 0.8, 0.2 respectively. Update the weights such that the error is minimum using gradient descent. Use the function sum of squared errors $\sum (y - \hat{y})^2$ where \hat{y} is the y-predicted value and y is the actual given y. Plot the linear regression line after updating the values of a and b in two iterations.</p>	X	0	0.4	0.6	1	Y	0	1	0.48	0.95	5 + 5														
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Y	0	1	0.48	0.95																							