PES UNIVERSITY	**PES University, Bengaluru** (Established under Karnataka Act No. 16 of 2013)	**UE20CS904**

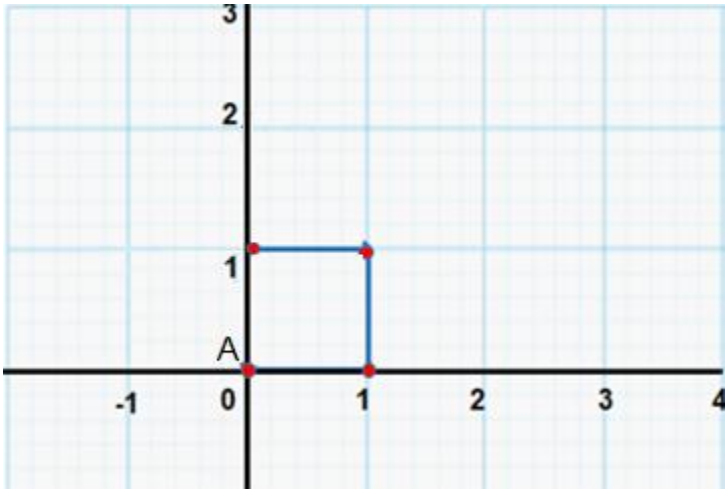
October 2021 : END SEMESTER ASSESSMENT (ESA)
M TECH DATA SCIENCE AND MACHINE LEARNING_ SEMESTER I

UE20CS904 - Mathematical Foundation

Time: 3 Hrs	Answer All Questions	Max Marks: 80

Section A (20 marks)

1	a)	A = $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} \\ -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$ Find out if the following condition holds for the above matrix. $A^T = A^{-1}$	2
	b)	The revenue generation function of an IT company is $3000x - 20x^2 + 200$ rupees where x is the number of employees. Find out the marginal revenue generation when 10 employees are hired.	2
	c)	Calculate the angle between two given vectors. The two vectors are, $a = 2\vec{i} - 4\vec{j}$ and $b = 11\vec{i} + 2\vec{j}$	2
	d)	Find the Jacobian matrix of : F(x) F(x) = $\begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2 \end{pmatrix} = \begin{pmatrix} {x_1}^2 + 5x_2 - 5 \\ \sin\frac{x_1}{x_2} + {x_2}^2 x_1 \end{pmatrix}$	2
	e)	Find out whether the function is increasing or decreasing $f(x) = -8x^2 + 15$	2
2	a)	Find out the minima of the following function for the interval (-5, -2) $f(x) = x^3 + 2x$	2
	b)	Find out whether the function is concave or convex $f(x) = -8x^2 + 15$	2
	c)	Statement : For any two matrices A & B , $A^T B^T = (BA)^T$ Check whether the statement is True for the following matrices A = $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$	2
	d)	Write the transformation matrix for reflection of a 2d image along the y axis.	2
	e)	Find the minimum value of $f(x)$ when $x < 5$. Where , $f(x) = 3x^6 + 5x^4 + 1$	2

		Section B (30 marks)	
3	a)	Find out the inverse of the following matrix. $A = \begin{bmatrix} 2 & 1 & 2 \\ -3 & 4 & 5 \\ 6 & 1 & 0 \end{bmatrix}$	5
	b)	The following box was rotated at an angle of 60° counter Clockwise around the origin (or point A). Find out the distance between the coordinate point of the box which passes through the y axis (excluding the origin or the point A) after the transformation and the point A.  	5
	c)	Find X & Y , if $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$	5
	d)	Mr. Murgan sells 3 different products. He sells products X, Y & Z. If he sells one unit of X , 5 units of Y and a unit of Z he makes a profit of 1080 Rs. If he sells one of Y and a unit of Z he makes a profit of 540 Rs If he sells 2 units of X and buys two units of Y and a unit of Z from another seller same as his selling price he incurs a loss of 180 rupees. Find out the price of product X , Y , & Z?	5
	e)	Find the covariance for the following set of vectors. $A = \begin{bmatrix} 1 & 3 & 5 & 7 & 8 \\ 2 & 2 & 0 & 1 & 4 \end{bmatrix}$	5
	f)	Transform the following basis into orthogonal basis using Gram-Schmidt Process. U1 = (1,-1,1) U2 = (1,0,1) U3 = (1,1,2)	5
		Section C (30 marks)	
4	a)	Find out the Eigen values and the Eigen vector for the following matrix. $A = \begin{bmatrix} 5 & -10 & -5 \\ 2 & 14 & 2 \\ -4 & -8 & 6 \end{bmatrix}$	10

	b)	Find the Singular value decomposition of the following matrix $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$	10
	c)	We have recorded the weekly average price of a stock over 6 consecutive days. Y shows the weekly average price of the stock and x shows the number of the days. Try to fit the best possible function 'f' to stablish the relationship between the number of the day and conversion rate.(Applying Gradient descent) where f(x) = y = a + b * x . X = Day | Y = Price of the stock 1 | 10 2 | 14 3 | 18 4 | 22 5 | 25 6 | 33 The initial values of a & b are, a= 4.9 & b=4.401.The learning rate is mentioned as .05. The error rate of a & b should be less than .01. Plot the predicted and actual data in a graph.	10