# MATH 127 Calculus for the Sciences

Lecture 15

## Today's lecture

#### Last time

Inverse of trig functions

- 1. Do they even exist?
- 2. Definition of arcsin, arccos, arctan
- 3. How to compute them

#### This time

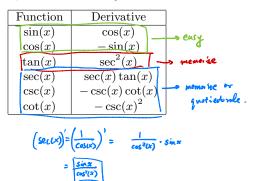
Course note coverage Section 2.5.4

Calculus of trig functions

- 1. Derivatives of sin, cos, tan, csc and all that;
- 2. Derivatives of arcsin and whatever;
- 3. Anti-derivatives of these

#### Derivatives of trigs

For the first three, just memorize them. For the last three, you could get them using quotient rule, but it's faster to just memorize them as well.



Example Find the derivative of

$$f(x) = \frac{\sin(e^x)}{\csc^2(\ln x)}$$

Challenge Find the domain of this function.

$$f'(x) = \left(\frac{1}{\csc^2(l_{nx})} \cdot \sin(l_n^x)\right)'$$

$$= \left(\frac{1}{\csc^2(l_{nx})} \cdot \sin(l_n^x) + \frac{1}{\csc^2(l_{nx})} \cdot (\sin(l_n^x))'\right)'$$

$$= \left(\sin^2(l_{nx}) \cdot \sin(l_n^x) + \sin^2(l_{nx}) \cdot (\sin(l_n^x))'\right)'$$

$$= 2\sin(l_{nx}) \cdot (\sin(l_{nx}) \cdot \sin(l_n^x) + \sin^2(l_{nx}) \cdot \cos(l_n^x) \cdot (e^x)'$$

$$= 2\sin(l_{nx}) \cdot \cos(l_{nx}) \cdot \sin(l_n^x) + \sin^2(l_{nx}) \cdot \cos(l_n^x) \cdot e^x$$

**Example** Find the derivative of

$$f(x) = |\cot(x)|^x$$

Challenge Find the domain of this function.

$$f(x) = e^{x \cdot \ln|\cot(x)|}$$

$$f'(x) = (e^{x \cdot \ln|\cot(x)|})'$$

$$= e^{x \cdot \ln|\cot(x)|} \cdot (x \cdot \ln|\cot(x)|)'$$

$$= e^{x \cdot \ln|\cot(x)|} \cdot (1 \cdot \ln|\cot(x)| + x \cdot \ln|\cot(x)|)'$$

$$= e^{x \cdot \ln|\cot(x)|} \cdot (\ln|\cot(x)| + x \cdot \frac{1}{\cot(x)} \cdot (\cot(x)')$$

$$= e^{x \cdot \ln|\cot(x)|} \cdot (\ln|\cot(x)| - \pi \frac{\csc^2(x)}{\cot(x)})$$

Example Find the derivative of

$$\int (x) = \sqrt{\tan(x) + \sin(\sqrt{x+1})}$$

Challenge Find the domain of this function.

$$\int_{-1}^{1} (x) = \left( \left( \int_{-1}^{1} (x) + S \int_{-1}^{1} (x + 1)^{-\frac{1}{2}} \right)^{2} \right)^{2}$$

$$= \frac{1}{2} \left( \int_{-1}^{1} (x) + S \int_{-1}^{1} (x + 1)^{-\frac{1}{2}} \left( \int_{-1}^{1} (x + 1)^{-\frac{1}{2}} (x + 1)^{-\frac{1}{2}} \right)^{2}$$

$$= \frac{1}{2} \left( \int_{-1}^{1} (x + 1)^{-\frac{1}{2}} (x + 1)^{-\frac{1}{2}} \left( \int_{-1}^{1} (x + 1)^{-\frac{1}{2}} (x + 1)^{-\frac{1}{2}} \right)^{2}$$

$$= \frac{1}{2} \left( \int_{-1}^{1} (x + 1)^{-\frac{1}{2}} \left( \int_{-1}^{1} (x + 1)^{-\frac{1}{2}} (x + 1)^{-\frac{1}{2}} (x + 1)^{-\frac{1}{2}} \right)^{2}$$

#### **Derivatives of** arcwhatever

For the these three, just memorize them. Sorry for all the memorizing. If you want I can prove them to you after class.

Function	Derivative	
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$	-> memor!Le.
$\arccos(x)$	$-\frac{1}{\sqrt{1-x^2}}$	
$\arctan(x)$	$\left(\frac{1}{1+x^2}\right)$	

Example Find the derivative of

$$f(x) = x \arctan(\ln(x))$$

What is the domain?

$$f'(x) = \operatorname{archan}(\ln x) + x \frac{1}{1 + (\ln x)^2} \cdot \frac{1}{x}$$

$$= \operatorname{archan}(\ln x) + \frac{1}{1 + (\ln x)^2}$$

Final domain: x > 0

#### Anti-derivatives of whatever

Hopefully they give you cheat sheet for this. There's no way I'm remembering this. They are used in real life, like when you want to find the area of some kind trig wavy thing.

Integral	Result	
$\int \tan(x)dx$	$\ln \sec(x)  + C$	
$\int \sec(x)dx$	$\ln \sec(x) + \tan(x)  + C$	
$\int \csc(x)dx$	$-\ln \csc(x) + \cot(x)  + C$	
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**Example** Solve the initial value problem

$$y' = \sec(x)\tan(x), \underline{y}(0) = 12345.$$

$$\int Sel(x) fan(x) dx = Sel(x) + C$$

$$Suy \quad y(x) = Sel(x) + C$$

$$y(x) = Sel(x) + C$$

$$= \frac{1}{cos(0)} + C$$

$$= 1 + C$$

$$So \quad |+C = |2345$$

$$C = 12344$$