

# MATH 127 Calculus for the Sciences

## Lecture 13



# Today's lecture

## Last time

Trigonometric functions

1. Radian vs degree
2. sin and cos of special angles
3. Compute sin and cos given tan
4. Trig identities
5. Graphing trig functions

## This time

**Course note coverage** Section 2.5.2

Modelling with Trig functions

1. Given some data from science, model it using a trig function
2. Given a trig function modelling something, retrieve data from it
3. Some models will involve waves, so you use sin or cos. Some will involve triangles, so you use appropriate trig functions.

## Find function

**Example** Suppose an orange is moving according to a sin wave. It has period  $P$ . An average height of  $Q$ , an amplitude of  $R$ .  
Find a function  $f(x)$  that describes the position of this thing at time  $x$ .

**Step 1.** Since the orange satisfies a sin wave, the function we are looking for is of form

for some .

**Step 2.** Since the average is  $Q$  while the average of sin is always , we should shift sin up by , so

$$c =$$

**Step 3.** Since the amplitude is  $R$  while the amplitude of sin is always , we should scale sin by , so

$$a =$$

**Step 4.** Since the period is  $P$  while the period of sin is always , we want to speed sin up so that

$$b \cdot \quad = 2\pi$$

## Find function

**Example** Suppose an orange is moving according to a sin wave. It has period  $P$ . An average height of  $Q$ , an amplitude of  $R$ .  
Find a function  $f(x)$  that describes the position of this thing at time  $x$ .

Step 5. In conclusion, the function we are looking for is

$$f(x) = R \sin\left(\frac{2\pi}{P}x\right) + Q$$

**Example** (1) What if I replace sin with cos in the above example. Which step do you need to modify?  
(2) What if instead of telling you the amplitude, I tell you the maximum value?



## Example

**Example** Suppose a wavy UFO satisfies

$$f(x) = 2 \sin(2\pi x) + 0.99$$

1. What is the average height?
2. What is the minimum height?
3. Assuming  $x > 0$ , when does it reach minimum for the first time?

## Example

**Example** A 3 meter long ladder is lying against a wall, but it is sliding at the bottom: its feet is  $3t$  cm away from the wall at time  $t$ . At what time will the angle between the ladder and the floor become  $\pi/6$ ?

## Example

**Example** Suppose the sun is straight up above us and we have metal rod. If I measure the shadow of the metal rod to be 5 m while the metal rod is in fact 10 m, what is the angle between the metal rod and the ground?



## Example

**Example** Suppose I stand on the  $x$ -axis, and I move smoothly from left to right and right to left, following a sin function. Suppose my average position on the  $x$ -axis at time  $t$  is  $x = 0$ , and my movement amplitude is  $\pi$  with a period of  $2\pi$ . At each time  $t$ , what is the area under the curve  $\cos(x)$ , from the origin to my position, expressed as an integral function? What is the rate of change at time  $\frac{\pi}{2}$ ?