class 50 min | home 1 hour | home read notes | class

theory examples | examples | theory | theory examples

MATH 127 Calculus for the Sciences

Lecture 5

September 12, 2025

Today's lecture

Office hours/Question session
Mon: 1430-1520 (before class, in my office MC5331)
Wed: after class in this room (but make sure you get to your quiz in time)
Fri: after class in this room
Tutoring center
In person MC 3022 0900 - 2000
Online on Teams

Last time Integration, fundamental theorem of calculus

This time
Course note coverage Section 1.1.4, 1.1.5
An example of how to use FTC
Anti-derivatives
Indefinite integrals

Quiz 2 is next Wednesday!

Coverage:

1. Basic derivatives, e.g. deriative of $\sin(x)$ is...?
2. Tangent line, e.g. find point-slope form of tangent of $y = \ldots$ at $x = \ldots$.
3. Tangent approximation, e.g. if tangent at $x = 3$ is ..., approximate the value at $x = 3.01$
4. Differentials, e.g. what is dy is $y = \ldots$?
5. Differential equaiton, e.g. is ... a differential equation? What's its order? Is ... a solution to it?
6. Definite integrals, e.g. evaluate $\boxed{\int_0^3 x dx}$ (we will do this today).
7. Net change problem, e.g. If $F'(x) = \ldots$, what is $\boxed{\Delta F}$ as x goes from 3 to 5?

Example

Recall from last time.

Theorem (The Fundamental Theorem of Calculus (FTC)) *If $\boxed{F(x)}$ is a differentiable function with $\boxed{F'(x)} = \boxed{f(x)}$, then*

$$\boxed{F(x)}\Big|_a^b = \int_a^b \boxed{f(x)}\,dx.$$

Question If an object has speed $S(t) = 2t$ at time t, what is the distance it travelled from time 0 to time 3?

Denote the distance function by $D(t)$. We want to find

$$D(t)\Big|_0^3 = \underbrace{D(3) - D(0)},$$

but we do not know what $D(t)$ is.

Example

Theorem (The Fundamental Theorem of Calculus (FTC)) *If $F(x)$ is a differentiable function with $\boxed{F'(x)} = \boxed{f(x)}$, then*

$$\boxed{F(x)}\Big|_a^b = \int_a^b f(x)dx.$$

Want:

$$\boxed{D(t)}\Big|_0^3 = D(3) - D(0) = \boxed{???}$$

Know: The derivative of $D(t)$ is exactly

$$\boxed{D'(t) = S(t).}$$

By FTC, we have

$$\underbrace{\boxed{D(t)\Big|_0^3}} = \underbrace{\int_0^3 \boxed{S(t)}dt}.$$

So now we **want:**

$$\boxed{\int_0^3 2t\, dt}$$

Example

Theorem (The Fundamental Theorem of Calculus (FTC)) *If $F(x)$ is a differentiable function with $F'(x) = f(x)$, then*

$$F(x)\Big|_a^b = \boxed{\int_a^b f(x)dx}.$$

Now we want:

$$\int_0^3 2t\,dt = \boxed{???}$$

Know: If we take $F(t) = \boxed{t^2}$, then

$$\boxed{F'(t)} = 2t.$$

By FTC, we have

$$\boxed{\int_0^3 2t\,dt} = \boxed{F(t)\Big|_0^3}. = F(3) - F(0)$$

And the right-hand side is equal to:

$$F(3) - F(0) = 3^2 - 0^2 = 9$$

Example

Theorem (The Fundamental Theorem of Calculus (FTC)) *If $F(x)$ is a differentiable function with $F'(x) = f(x)$, then*

$$F(x)\Big|_a^b = \int_a^b f(x)dx.$$

Written more concisely, we can say

$$\begin{aligned} D(t)\Big|_0^3 &= \int_0^3 S(t)dt \qquad \text{(by FTC)} \\ &= \int_0^3 2t\,dt \quad \leftarrow \\ &= t^2\Big|_0^3 \qquad \text{(by FTC again)} \\ &= 3^2 - 0^2 = 9. \end{aligned}$$

Question If we know $D'(t) = 2t$, and $(t^2)' = 2t$, then can we just say

$$D(t) = t^2?$$

No. because maybe $D(t) = t^2 + 1$

Anti-derivatives

Definition Suppose $\boxed{F'(x)} = \boxed{f(x)}$, then we say $F(x)$ is an **anti-derivative** of $f(x)$.

Example If $f(x) = 2x$, an anti-derivative of $f(x)$ is

$$\boxed{F(x) = x^2};$$

another anti-derivative of $f(x)$ is

$$\boxed{F(x) = x^2 + 1};$$

a third anti-derivative of $f(x)$ is

$$\boxed{F(x) = x^2 - 100}.$$

because $F'(x) = 2x$

If we know $F(x)$ is the anti-derivative of $f(x)$, then all other anti-derivatives are of form

$$\boxed{F(x) + C \quad \text{for some constant } C}.$$

Indefinite integral

Definition Suppose $F(x)$ is the anti-derivative of $f(x)$, then the family of anti-derivatives of $f(x)$ is denoted by

$$\int f(x)dx = F(x) + C$$

where C is called the **constant of integration**.

Given any $F(x)$ and C with $F'(x) = f(x)$, we have

$$\int_a^b f(x)dx = \boxed{\left(F(x)+C\right)\Big|_a^b = \left(F(b)+C\right) - \left(F(a)+C\right) = F(b)-F(a)}\ .$$

So no matter which C we choose, it will always cancel out and we always get the same answer.

$$\int x\,dx = \frac{x^2}{2} + C$$

$$\int \left(\frac{x^2}{2}+C\right)dx = \frac{x^3}{6} + Cx + D$$

Common anti-derivatives

Keep the following in mind.

1. $\int 0dx = C$;
2. $\int x^n dx = \boxed{\frac{x^{n+1}}{n+1}} + C$;
3. $\int e^x dx = \boxed{e^x + C}$;
4. $\int \cos x\, dx = \boxed{\sin x + C}$;
5. $\int \sin x\, dx = \boxed{-\cos x + C}$;

$$\left(\boxed{\frac{x^{n+1}}{n+1}}\right)' = \frac{(n+1)x^n}{n+1} = x^n$$

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If k is a number and f, g are two functions with anti-derivatives F, G respectively, then

$$\int \Big(k \cdot f(x) + g(x)\Big) dx = k \cdot F(x) + G(x) + C$$

because the derivative of $kF(x) + G(x)$ +C is $\boxed{k \cdot f(x) + g(x)}$.

Example

Question If my speed is $(6x^2 + 2e^x)$ m/s, what could my distance function be at time x?

My distance function must be an anti-derivative of my speed.

An anti-derivative of $6x^2 + 2e^x$ is

$(2x^3)' = 2 \cdot 3x^2 = 6x^2$

$$\boxed{2x^3 + 2e^x};$$

$(2e^x)' = 2e^x$

so we can say

$$\int (6x^2 + 2e^x)dx = \boxed{2x^3 + 2e^x + C}$$

Another anti-derivative of $6x^2 + 2e^x$ is

$$\boxed{2x^3 + 2e^x + 100};$$

so we can say

$$\int (6x^2 + 2e^x)dx = \boxed{2x^3 + 2e^x + 100 + C}.$$

Conclusion: my distance function is... $2x^3 + 2e^x + C$ for some constant C

$2x^3 + 2e^x$ + constant.

Example

Question Which of the following sentences is correct?

Suppose $f'(x) = 3x^2$, but we are not sure what $f(x)$ is exactly....

1. $f(x)$ is an anti-derivative of $3x^2$. ✓
2. x^3 is an anti-derivative of $3x^2$. ✓
3. $f(x) = x^3$. ✗
4. $f(x) = x^3 + C$. ✗
5. $f(x) = x^3 + C$ for some number C. ✓
6. Any anti-derivative of $3x^2$ is of form $f(x) + C$. for some C. ✓
7. Any anti-derivative of $3x^2$ is of form $x^3 + C$. for some C ✓
8. Any anti-derivative of $3x^2$ is of form $x^3 + 1 + C$. for some C ✓

Survey

Give some feedback for your experience so far.