MATH 127 Calculus for the Sciences

Lecture 15

Today's lecture

Last time
Inverse of trig functions

1. Do they even exist?
2. Definition of $\arcsin, \arccos, \arctan$
3. How to compute them

This time
Course note coverage Section 2.5.4
Calculus of trig functions

1. Derivatives of $\sin, \cos, \tan, \csc$ and all that;
2. Derivatives of $\arcsin$ and whatever;
3. Anti-derivatives of these

Derivatives of trigs

For the first three, just memorize them. For the last three, you could get them using quotient rule, but it's faster to just memorize them as well.

Function	Derivative
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$
$\cot(x)$	$-\csc(x)^2$

easy

memorize

memorize or quotient rule.

$$(\sec(x))' = \left(\frac{1}{\cos(x)}\right)' = \frac{1}{\cos^2(x)} \cdot \sin x$$

$$= \boxed{\frac{\sin x}{\cos^2(x)}}$$

Example

Example Find the derivative of

$$f(x) = \frac{\sin(e^x)}{\csc^2(\ln x)}$$

Challenge Find the domain of this function.

$$f'(x) = \left(\frac{1}{\csc^2(\ln x)} \cdot \sin(e^x)\right)'$$

$$= \left(\frac{1}{\csc^2(\ln x)}\right)' \cdot \sin(e^x) + \frac{1}{\csc^2(\ln x)} \cdot (\sin(e^x))'$$

$$= (\sin^2(\ln x))' \sin(e^x) + \sin^2(\ln x) \cdot (\sin(e^x))'$$

$$= 2\sin(\ln x)(\sin(\ln x))' \sin(e^x) + \sin^2(\ln x) \cos(e^x) \cdot (e^x)'$$

$$= 2\sin(\ln x)\cos(\ln x)\frac{1}{x} \cdot \sin(e^x) + \sin^2(\ln x)\cos(e^x) \cdot e^x.$$

Example

Example Find the derivative of

$$f(x) = |\cot(x)|^x$$

Challenge Find the domain of this function.

$$f(x) = e^{x \cdot \ln|\cot(x)|}$$

$$f'(x) = \left(e^{x\ln|\cot(x)|}\right)'$$

$$= e^{x\ln|\cot(x)|} \cdot \left(x\ln|\cot(x)|\right)'$$

$$= e^{x\ln|\cot(x)|} \cdot \left(1 \cdot \ln|\cot(x)| + x\left(\ln|\cot(x)|\right)'\right)$$

$$= e^{x\ln|\cot(x)|} \cdot \left(\ln|\cot(x)| + x \cdot \frac{1}{\cot(x)} \cdot (\cot x)'\right)$$

$$= e^{x\ln|\cot(x)|}\left(\ln|\cot(x)| - x\frac{\csc^2(x)}{\cot(x)}\right)$$

Example

Example Find the derivative of

$$f(x) = \sqrt{\tan(x) + \sin(\sqrt{x+1})}$$

Challenge Find the domain of this function.

$$f'(x) = \left(\left(\tan(x) + \sin\sqrt{x+1}\right)^{\frac{1}{2}}\right)'$$

$$= \frac{1}{2}\left(\tan(x) + \sin\sqrt{x+1}\right)^{-\frac{1}{2}}\left(\tan(x) + \sin\sqrt{x+1}\right)'$$

$$= \frac{1}{2}\left(\tan(x) + \sin\sqrt{x+1}\right)^{-\frac{1}{2}}\left(\sec^2(x) + \cos\left(\sqrt{x+1}\right)\frac{1}{2}(x+1)^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2}\left(\tan(x) + \sin\sqrt{x+1}\right)^{-\frac{1}{2}}\left(\sec^2(x) + \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot \cos\sqrt{x+1}\right).$$

Derivatives of arcwhatever

For the these three, just memorize them. Sorry for all the memorizing. If you want I can prove them to you after class.

Function	Derivative
$\arcsin(x)$	$\dfrac{1}{\sqrt{1-x^2}}$
$\arccos(x)$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\arctan(x)$	$\dfrac{1}{1+x^2}$

→ memorise.

Example

Example Find the derivative of

$$f(x) = \boxed{x}\,\boxed{\underbrace{\arctan(\underbrace{\ln(x)}_{x>0})}_{\text{no restriction}}}$$

What is the domain?

$$f'(x) = \arctan(\ln x) + x\frac{1}{1+(\ln(x))^2}\cdot\frac{1}{x}$$
$$= \arctan(\ln x) + \frac{1}{1+(\ln x)^2}$$

Domain of $\ln x$: $x > 0$

Domain of arctan = Range of tan

$= (-\infty, \infty)$

Final domain : $x > 0$

Anti-derivatives of whatever

Hopefully they give you cheat sheet for this. There's no way I'm remembering this. They are used in real life, like when you want to find the area of some kind trig wavy thing.

Integral	Result
$\int \tan(x)dx$	$\ln\lvert\sec(x)\rvert + C$
$\int \sec(x)dx$	$\ln\lvert\sec(x)+\tan(x)\rvert + C$
$\int \csc(x)dx$	$-\ln\lvert\csc(x)+\cot(x)\rvert + C$

memorise.

Example

Example Solve the initial value problem

$$y' = \sec(x)\tan(x), \underline{y(0) = 12345.}$$

$$\int \sec(x)\tan(x)\,dx = \sec(x) + C$$

Say $y(x) = \sec(x) + C$

$$y(0) = \sec(0) + C$$

$$= \frac{1}{\cos(0)} + C$$

$$= 1 + C$$

So $1 + C = 12345$

$$C = 12344$$