MATH 127 Calculus for the Sciences

Lecture 20

Today's lecture

Last time
Continuity and continuity rules

This time
Course note coverage Section 3.1.3 and 3.1.4
Intermediate value theorem (IVT)
Classifying discontinuities

Classifying discontinuity

Given a function $f(x)$, we can look at three things

(1) the value at a: $f(a)$

(2) the left limit: $\lim_{x\to a^-} f(x)$

(3) the right limit: $\lim_{x\to a^+} f(x)$

There are some possibilities when the function is not continuous

- (1)=(2), but (3) is a number different from them.
- (1)=(3), but (2) is a number different from them.
- (2)=(3), but (1) is a number different from them.
- (1)=(2), but (3) is ∞.
- (1)=(3), but (2) is ∞.
- (2)=(3), but a is not in the domain of f, so (1) does not exist.

Jump discontinuity

(1) the value at a: $f(a)$

(2) the left limit: $\lim_{x\to a^-} f(x)$

(3) the right limit: $\lim_{x\to a^+} f(x)$

There are some possibilities when the function is not continuous

- (1)=(2), but (3) is a number different from them.
- (1)=(3), but (2) is a number different from them.

Removable discontinuity

(1) the value at a: $f(a)$

(2) the left limit: $\lim\limits_{x \to a^-} f(x)$

(3) the right limit: $\lim\limits_{x \to a^+} f(x)$

There are some possibilities when the function is not continuous

- (2)=(3), but (1) is a number different from them.
- (2)=(3), but a is not in the domain of f, so (1) does not exist.

Infinity discontinuity

(1) the value at a: $f(a)$

(2) the left limit: $\lim_{x\to a^-} f(x)$

(3) the right limit: $\lim_{x\to a^+} f(x)$

There are some possibilities when the function is not continuous

- (1)=(2), but (3) is ∞.
- (1)=(3), but (2) is ∞.

Intermediate value theorem

Suppose there is a UFO:

- Moving smoothly in the air, up and down, every day.
- We know for sure that its height is 1000m at 7AM, and 2000m at 7PM.

Then I ask

- Do we know that the UFO has height 500m at some point? If so, when does it happen?
- Do we know that the UFO has height 1000m at some point? If so, when does it happen?
- Do we know that the UFO has height 1500m at some point? If so, when does it happen?
- If I want to set a floating camera to catch the UFO, what heights can I set it to, to be guaranteed to catch the UFO?

Intermediate value theorem

Conclusion:
If a thing has height U at some time, moves continuously, and reaches height V at a later time, then:

for any W between U and V, the thing must reach height W within that period of time.

Intermediate value theorem

Conclusion:

If a thing has height U at some time, moves continuously, and reaches height V at a later time, then:
for any W between U and V, the thing must reach height W within that period of time.

Let us rephrase this:

If $f(a) = U$ and $f(b) = V$, and f is continuous, then:
for any W between U and V, then f must achieve an output of W within the interval $[a, b]$

Intermediate value theorem

Conclusion:

If $f(a) = U$ and $f(b) = V$, and f is continuous, then:
for any W between U and V, then f must achieve an output of W within the interval $[a, b]$

Rephrase again:

If $f(a) = U$ and $f(b) = V$, and f is continuous, then:
for any W such that

$$U < W < V,$$

there exists some c inside $[a, b]$ such that

$$f(c) = W.$$

Intermediate value theorem

Conclusion:

If $f(a) = U$ and $f(b) = V$, and f is continuous, then:
for any W such that
$$U < W < V,$$
there exists some c inside $[a, b]$ such that
$$f(c) = W.$$

Final rephrasing:

Theorem (Intermediate value theorem (IVT))
If f is continuous on the interval $[a, b]$, then for any W such that
$$f(a) < W < f(b),$$
there exists c inside $[a, b]$ such that
$$f(c) = W.$$

Example

Example
Does the polynomial $x^3 + x + 1$ have a root in the interval $[-1, 1]$?

Example This is the same a asking: Does there exist a value c in the interval $[-1, 1]$ such that the function $f(x) = \boxed{}$ satisfies $f(c) = \boxed{}$?

Example

Example

Does the polynomial $x^2 - 1$ have a root in the interval $[-2, 2]$?
Use the intermediate value theorem to show this.