MATH 127 Calculus for the Sciences

Lecture 11

Today's lecture

Last time
Exponential and logarithmic functions

- Real life application of exp.
- Simplify things using exp and log rules.
- Derivative of exp and log function.

This time
Course note coverage

- Integrate $\frac{1}{x}$. (Section 2.4.4)
- Riemann sum approximation of ln. (part of Section 2.4.3: this could be on Quiz 4)
- Logarithmic scale (real life application of log). (section 2.4.6)

Quiz 4 is next Wednesday!

Coverage:

1. Topics that were on the coverage of Quiz 2-3 might show up in Quiz 4.
2. Integral functions.
3. Inverse function.
4. Exp and log functions.
5. More on log functions (will cover today).

Derivatives of exp/log functions

Example What is the derivative of $\log_{2x} 3^x$? What is its domain?

Easier examples

Example What is the derivative of $\ln(x)$? What is its domain?

Example What is the derivative of $\ln|x|$?

Approximating $\ln x$

Now that we know $\ln|x|$ is the anti-derivative of $\frac{1}{x}$. In particular, for $x > 0$,

$$\int_1^x \frac{1}{t}dt = \ln|t|\Big|_1^x = \ln x - \ln 1 = \ln x.$$

That is

$$\boxed{\ln x = \int_1^x \frac{1}{t}dt.}$$

In real life, this formula allows us to estimated $\ln x$. Because knowing that $\ln x$ is an integral, we can use

$$\boxed{}$$

to approximate it!

Approximating $\ln x$

Example Approximate $\ln 4$ using right Riemann approximation by dividing a interval into 6 pieces.

Logarithmic scale

Logarithmic scale would be a good motivation for why we need logarithmic functions.

- Earthquake scale is measured on a logarithmic scale:
  an earthquake of level 6 is 10 times stronger than level 5.
- pH value is measured on a logarithmic scale:
  a pH value of 3 is 100 times less acidic than a pH value of 5.
- Volume is a bit different:
  a sound that is 130 dB is 1000 times louder than 100 dB.

Logarithmic scale

Definition We say a function is a **logarithmic scale of base** a if

adding b to the output means the input has to be multiplied by a^b.

By how is this description in any sense logarithmic? Here's why:

If $f(x)$ is a logarithmic scale of base a, then we know

$$f(x) = \log_a\left(\frac{x}{A}\right)$$

for some number A.

For the above, we see a needs to satisfy the condition

$$\boxed{}.$$

Logarithmic scale

Definition We say a function is a **logarithmic scale of base** a if

adding b to the output means the input has to be multiplied by a^b.

Example The earthquake scale is on a base-10 logarithmic scale. The input is intensity and the output is the magnitude.

1. If you add 3 to the magnitude, then the intensity needs to be multiplied by a factor of

$$\boxed{}$$

2. If one earthquake has magnitude 4 and another has magnitude 6.5, then the intensity of the second one is

$$\boxed{\text{times stronger}}$$

than the first one.

Logarithmic scale

Definition We say a function is a **logarithmic scale of base** a if

adding b to the output means the input has to be multiplied by a^b.

Example Some aliens measure earthquake on a base-9 logarithmic scale. The input is badness and the output is the level.

1. If one earthquake is level 3 and another has level 6.5, then the badness of the second one is

   $\boxed{\quad\quad\text{times more bad}}$

   than the first one.

2. We can compute this value using exp rules

$$\boxed{\quad} = \boxed{\quad\quad} = \boxed{\quad\quad} = \boxed{\quad\quad}$$

Logarithmic scale

Definition We say a function is a **logarithmic scale of base** a if

adding b to the output means the input has to be multiplied by a^b.

Example The volume of sound is a bit weird. The output is volume/decible and input is intensity. It satisfies

$$D(I) = 10 \log_{10} \left(\frac{I}{10^{-12}} \right)$$

And this is not a logarithmic scale. But if we divide the output by 10, then we do get a logarithmic scale of base-10.

1. If sound is 25 dB higher in volume than another sound, then its intensity is

$$\boxed{\quad\quad \text{times stronger}}$$

2. Let us try to estimate this value

$$\boxed{\quad\quad} = \boxed{\quad\quad\quad\quad} \approx \boxed{\quad\quad\quad} = \boxed{\quad\quad}$$

3 questions on your opinions

I went to many teaching seminars where old people talk about how I should teach. Now I want to know what you think.