MATH 127 Calculus for the Sciences

Lecture 3

September 8, 2025

Today's lecture

Last time
Tangent line approximation, differential approximation.

This time
Course note coverage Section 1.1.4, 1.1.5
Differentials recap
Differential equations: definition, order, solution

Differentials recap

Recall from last time: we have the approximation

$$\Delta y \approx f'(a)\Delta x$$

which is more and more accurate when Δx becomes smaller, but is not perfect unless Δx is exactly 0.

To capture this infinitesimal behaviour, we say the **differentials** dy, dx satisfy

$$\boxed{dy = f'(x)dx.}$$

Example

Example Suppose $u = sin(m)$. Find du.

Example

Question For what function $f(x)$ do we have

$$f'(x) = f(x)?$$

Differential equations

A **differential equation** is an equation involving an unknown function, its variable, and its derivatives.

Example The equation

$$f'(x) = f(x)$$

is a differential equation, because it involves the unknown function $f(x)$, the variable x, and the derivative $f'(x)$.

Example The equation

$$\frac{d^2y}{dx^2} = x$$

is a differential equation, because it involves the unknown function $\square$, the variable $\square$, and the (second) derivative of $\square$.

Order of a differential equation

Remark The equation

$$\frac{g^{(2)}(x)}{x} = f'(x) + g(3)^2 + 1$$

has two functions, g and f.
So to say that it is a differential equation, we should specify: regarding to which unknown function and (sometimes) which variable.

Definition Given a differential equation with respect to an unknown function $f(x)$ in the variable x, the **order** of the differential equation is the order of the highest derivative of $f(x)$ involved in the equation.

Example The equation

$$f''(x) + f'(x) = f(x)$$

has order 2, because the highest derivative it involves is the *second* derivative $f''(x)$.

Examples

Example The equation

$$k + f''(x) = 2$$

viewed as a differential equation in the unknown function $f(x)$ and variable x, has degree 2.

Example The equation

$$f'''(3) + f''(x) = 2$$

has order $\square$, because

Example The equation

$$\frac{d^2y}{dx^2} = \frac{d^3z}{dx^3}$$

has order $\square$ when viewed as a differential equation in the unknown function y, and has order $\square$ when viewed as a differential equation in the unknown function z,

Solution of a differential equation

A function that satisfies a differential equation is called a **solution** of that differential equation. As we have seen before,

$$f(x) = e^x, f(x) = 2e^x$$

are both solutions to the differential equation

$$f'(x) = f(x).$$

Example

$$g(x) = \frac{dy}{dx} + C$$

is a solution for any constant value C to the differential equation

$$g'(x) = \frac{d^2y}{dx^2}.$$

when the equation is viewed in regards to the unknown function $\square$ and the variable $\square$.

Family of solutions

Consider the differential equation

$$f'(x) = 1.$$

A solution of this is $f(x) = x$. Another solution is $f(x) = x + 1$.
More generally, any solution of the above can be written in the form

$$f(x) = x + C$$

for some constant value C. We say C is a **parameter** of the **family of solutions** $f(x) = x + C$.

Example $f(x) = x^2 + C_1x + C_2$ is a family of solutions with parameters C_1, C_2 of the differential equation $f''(x) = 2$.

Exercise

Question The function $y = u^2 + f(x) + C$ is a family of solutions of which of the following differential equations (considered in regards to the unknown function y and the variable u)?

$$y = u^2 + f(x) + 1; \quad \frac{dy}{du} = 2u; \quad \frac{dy}{du} = 2u + f'(x); \quad \frac{d^2y}{du^2} = 2.$$