MATH 127 Calculus for the Sciences

Lecture 13

Today's lecture

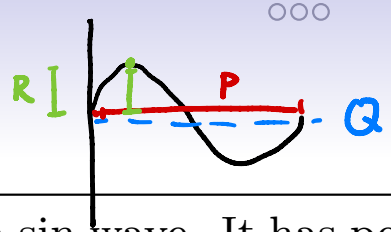
Last time
Trigonometric functions

1. Radian vs degree
2. sin and cos of special angles
3. Compute sin and cos given tan
4. Trig identities
5. Graphing trig functions

This time
Course note coverage Section 2.5.2
Modelling with Trig functions

1. Given some data from science, model it using a trig function
2. Given a trig function modelling something, retrieve data from it
3. Some models will involve waves, so you use sin or cos. Some will involve triangles, so you use appropriate trig functions.

Find function



Example Suppose an orange is moving according to a sin wave. It has period P. An average height of Q, an amplitude of R.
Find a function $f(x)$ that describes the position of this thing at time x.

Step 1. Since the orange satisfies a sin wave, the function we are looking for is of form

$$f(x) = a \sin(bx) + c$$

for some numbers a, b, c.

Step 2. Since the average is Q while the average of sin is always 0, we should shift sin up by Q, so

$$c = Q$$

Step 3. Since the amplitude is R while the amplitude of sin is always 1, we should scale sin by R, so

$$a = R$$

Step 4. Since the period is P while the period of sin is always 2π, we want to speed sin up so that

$$b \cdot P = 2\pi$$

$$P = \frac{2\pi}{b} \qquad b = \frac{2\pi}{P}$$

Find function

Example Suppose an orange is moving according to a sin wave. It has period P. An average height of Q, an amplitude of R.
Find a function $f(x)$ that describes the position of this thing at time x.

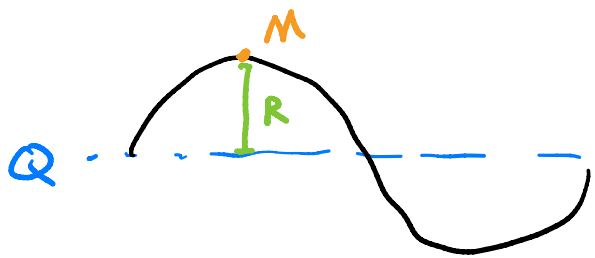
Step 5. In conclusion, the function we are looking for is

$$f(x) = R\sin\left(\frac{2\pi}{P}x\right) + Q$$

Example (1) What if I replace sin with cos in the above example. Which step do you need to modify?
(2) What if instead of telling you the amplitude, I tell you the maximum value? M

(1) $f(x) = R\cos\left(\frac{2\pi}{P}x\right) + Q$

(2) $R = M - Q$



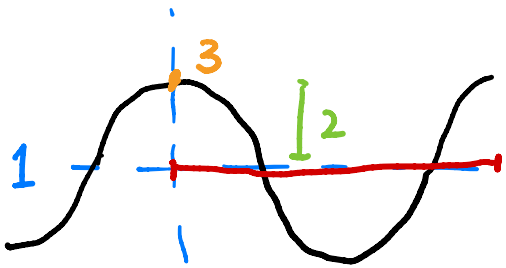
Example

Example Suppose a wavy thing is moving according to a cos wave. It has an average height of 1. Its maximum is 3 and the distance between a two maximums is 123.
Find a function $f(x)$ that describes this wavy thing.

Step 1. Since the thing satisfies a cos wave, we write

$$f(x) = a\cos(bx) + c$$

for some numbers a, b, c.



Step 2. Since the average is 1 we have so

$$c = 1$$

Step 3. Next we want the amplitude, but we only know the average 1 and maximum value 3. The amplitude is

$$a = 3 - 1 = 2$$

Step 4. Next we want the amplitude, but we only know the distance between two maxima... but wait, that is also the period! So

$$b \cdot 123 = 2\pi \quad \text{and} \quad b = \frac{2\pi}{123}$$

Conclusion:

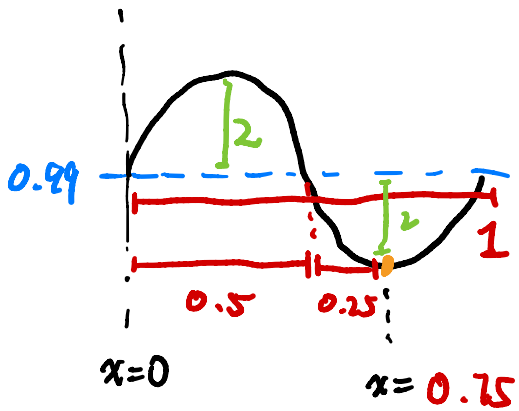
$$f(x) = 2\cos\left(\frac{2\pi}{123}x\right) + 1$$

Example

Example Suppose a wavy UFO satisfies

$$f(x) = 2\sin(2\pi x) + 0.99$$

1. What is the average height?
2. What is the minimum height?
3. Assuming $x > 0$, when does it reach minimum for the first time?



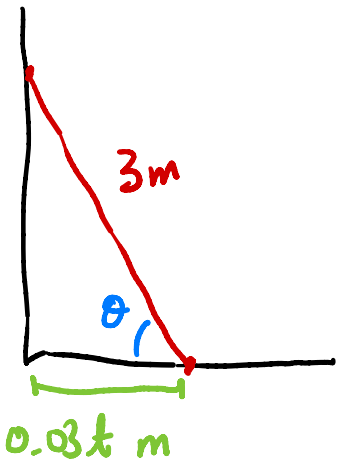
(1) average height = 0.99

(2) minimum height = 0.99 − 2 = −1.01

(3) when $x = 0.75$

Example

Example A 3 meter long ladder is lying against a wall, but it is sliding at the bottom: its feet is $3t$ cm away from the wall at time t. At what time will the angle between the ladder and the floor become $\pi/6$?



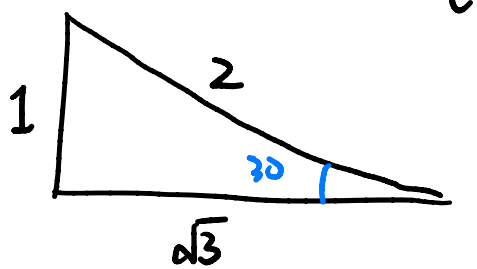
$$\cos\theta = \frac{0.03t}{3} = 0.01t$$

If $\theta = \frac{\pi}{6}$, then

$$\cos\frac{\pi}{6} = 0.01t.$$

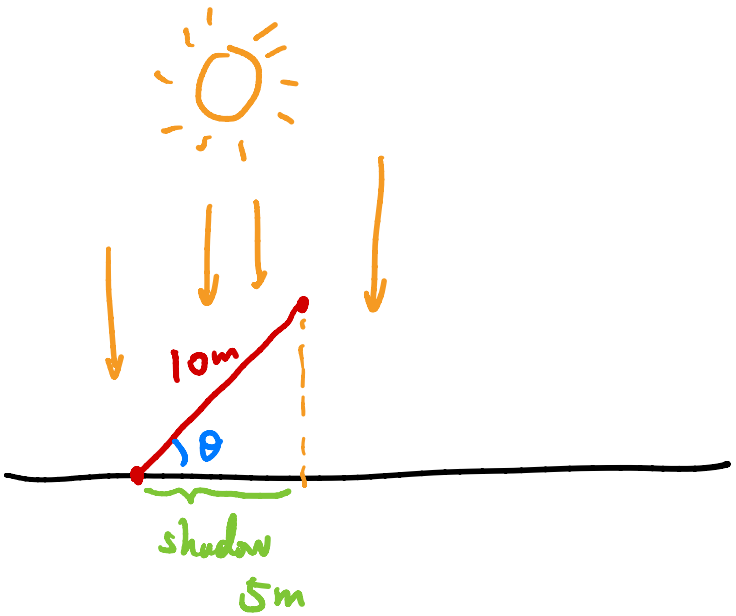
$$\frac{\sqrt{3}}{2} = 0.01t$$

$$t = \frac{100\sqrt{3}}{2} = 50\sqrt{3}$$

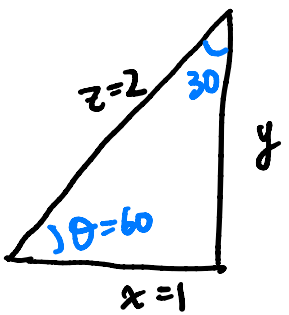


Example

Example Suppose the sun is straight up above us and we have metal rod. If I measure the shadow of the metal rod to be 5 m while the metal rod is in fact 10 m, what is the angle between the metal rod and the ground?



$$\cos\theta = \frac{5}{10} = \frac{1}{2}$$



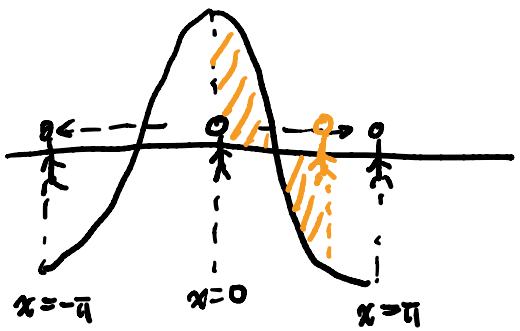
$$\cos\theta = \frac{x}{z}$$

Say $x=1$, $z=2$

So $\theta = 60^\circ$ or $\frac{\pi}{3}$

Example

Example Suppose I stand on the x-axis, and I move smoothly from left to right and right to left, following a sin function. Suppose my average position on the x-axis at time t is $x = 0$, and my movement amplitude is π with a period of 2π. At each time t, what is the area under the curve $\cos(x)$, from the origin to my position, expressed as an integral function? What is the rate of change at time $\frac{\pi}{2}$?



$$\text{Area}(t) = \int_0^{x(t)} \cos(u)\,du$$

$$= \sin(u)\Big|_0^{x(t)}$$

$$= \sin(x(t))$$

$$= \sin(\pi \sin(t))$$

$x(t) = a \sin(bt) + c$

a = amplitude $= \pi$

$b \cdot \text{period} = 2\pi$, $b = 1$

$x(0) = 0$. $x(0) = a \cdot \sin(b \cdot 0) + c$

$= c$

so $c = 0$. $\boxed{x(t) = \pi \sin(t)}$

$\text{Area}'(t) = \cos(\pi \sin(t)) \cdot (\pi \sin(t))'$

$= \cos(\pi \sin t)\, \pi \cos(t)$

when $t = \frac{\pi}{2}$, $\cos(\frac{\pi}{2}) = 0$, so $\text{Area}'(\frac{\pi}{2}) = 0$