MATH 127 Calculus for the Sciences

Lecture 1

September 3, 2025

Information

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Learn Use the website called Learn to access course material, including

- Course outline; course schedule; course notes
- Exercises, practice exams.

Odyssey Use the website called Odyssey to find quiz/midterm/exam seating.

Piazza Use the forum called Piazza to ask questions, answer other people's questions, and discuss.

Quiz 1 is on Sept 10. The first quiz includes questions about high school algebra and calculus which we will not review in lecture.

Today's lecture

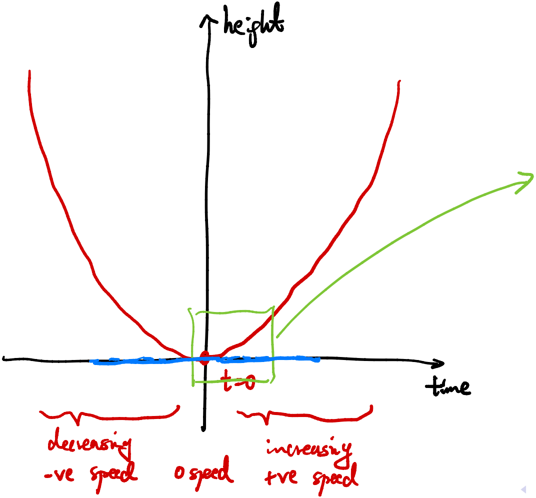
Course note coverage Section 1.1.1 - 1.1.2

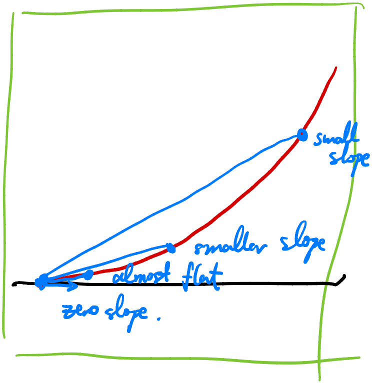
Definition of derivative
Differentiability

Computation of derivative
Common derivatives
Rules of differentiation

Motivation

Question Assume an object has height t^2 at time t. What is the speed of the object at the moment $t = 0$?





Derivative

Definition Let $y = f(x)$ be a function. The $\boxed{\textbf{derivative of } y}$ is the function

$$f'(x) = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon}$$

Given a specific number a, the $\boxed{\textbf{derivative of } y \textbf{ at } x = a}$ is the value

$$f'(a)$$

Remark
The domain of the derivative is where the limit on the right-hand side exists.
If the limit exists everywhere, then we say the function is $\boxed{\textbf{differentiable}}$.
The derivative of a function at a point is a **number**.
If the limit exists at a specific point $x = a$ exists, then we say the function is $\boxed{\textbf{differentiable at } a}$.

Notations for derivatives

Since the derivative of a function is a function, we can take the derivative of a derivative. Doing this n times, we get the n-th derivative of $y = f(x)$.

Notation

The n-th derivative of $f(x)$ is $\boxed{f^{(n)}(x)}$.

The n-th derivative at a is $\boxed{f^{(n)}(a)}$.

Example The derivative of x^2 is the function denoted $(x^2)' = 2x$.

Notation

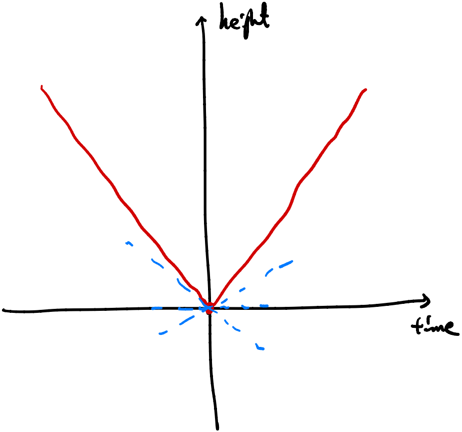
The n-th derivative of $y = f(x)$ is $\boxed{\frac{d^n y}{dx^n}}$.

The n-th derivative at a is $\boxed{\left.\frac{d^n y}{dx^n}\right|_{x=a}}$.

Example The second derivative of $y = x^2$ is the function $\frac{d^2y}{dx^2} = 2$.

Derivatives might not exist

Question Assume an object has height $|t|$ at time t. What is the speed of the object at the moment $t = 0$?



$$\lim_{\varepsilon \to 0^-} \frac{f(\varepsilon) - f(0)}{\varepsilon} = -1$$

$$\lim_{\varepsilon \to 0^+} \frac{f(\varepsilon) - f(0)}{\varepsilon} = 1$$

limit does not exist.

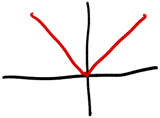
not differentiable at $t=0$

speed is not well-defined at $t=0$.

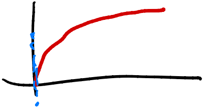
Examples of non-differentiable functions

The following are some (not all) of the cases where a function is not differentiable.

Example If f has a **sharp corner** at $x = a$, e.g. $f(x) = |x|$.



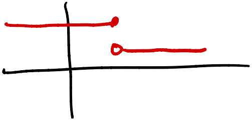
Example If f has a **vertical tangent** at $x = a$, e.g. $f(x) = x^{1/3}$.



$f'(0) = \infty$

but ∞ is not a real number.

Example If f is **not continuous** at $x = a$, e.g. $f(x) = \begin{cases} 1, & \text{if } x \leq 1, \\ 0.5, & \text{if } x > 1. \end{cases}$



$$\lim_{\varepsilon \to 0^-} \frac{f(\varepsilon) - f(0)}{\varepsilon} = 0$$

$$\lim_{\varepsilon \to 0^+} \frac{f(\varepsilon) - f(0)}{\varepsilon} = \infty.$$

Common derivatives

You should know the derivatives of the following functions from high school.

1. If $f(x) = c$ for c a constant, then $f'(x) = 0$.
2. If $f(x) = x^n$ for an integer $n \neq 0$, then $f'(x) = nx^{n-1}$.
3. If $f(x) = e^x$, then $f'(x) = e^x$.
4. If $f(x) = \sin(x)$, then $f'(x) = \cos(x)$.
5. If $f(x) = \cos(x)$, then $f'(x) = -\sin(x)$.

$y=1$, $y'=0$.
$y=x^2$, $y'=2x$
$y=e^x$, $y'=e^x$
$y=\sin x$, $y'=\cos x$
$y=\cos x$, $y'=-\sin x$.

Derivatives rules

You should know the following rules from high school.
Suppose c is a number, and f, g, h are functions.

1. If $h(x) = cf(x)$, then $h'(x) = cf'(x)$. (Constant rule)
2. If $h(x) = f(x) + g(x)$, then $h'(x) = f'(x) + g'(x)$. (Sum rule)
3. If $h(x) = f(x) \cdot g(x)$, then $h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$. (Product rule)
4. If $h(x) = f \circ g(x) = f(g(x))$, then $h'(x) = f'(g(x))g'(x)$. (Chain rule)

The quotient rule

Question (Quotient rule)

If $h(x) = \dfrac{f(x)}{g(x)}$, how to compute $h'(x)$ in terms of f and g?

$$h'(x) = \left(\frac{f(x)}{g(x)}\right)'$$

$$= \left(f(x) \cdot \frac{1}{g(x)}\right)'$$

$$= f'(x) \cdot \frac{1}{g(x)} + f(x) \cdot \left(\frac{1}{g(x)}\right)' \quad \text{product rule.}$$

Let $F(u) = \frac{1}{u}$, then $\frac{1}{g(x)} = F(g(x))$. We know $F'(u) = -\frac{1}{u^2}$

so $\left(\frac{1}{g(x)}\right)' = \frac{-1}{g(x)^2} \cdot g'(x)$ chain rule.

$$\Longrightarrow h'(x) = \frac{f'(x)}{g(x)} - \frac{f(x) \cdot g'(x)}{g(x)^2}$$

Example

Question Assume the mass M (in kg) of an object of radius r (in cm) is

$$M(r) = \frac{e^r + 3r}{r^3}$$

Find the rate of change of the mass with respect to the radius.

$$M'(r) = \left(\frac{e^r+3r}{r^3}\right)'$$

$$= \frac{e^r+3}{r^3} + \frac{(e^r+3r)\cdot(-3)}{r^4}$$

$$= \frac{re^r+3r-3e^r-9r}{r^4}$$

$$= \frac{-2e^r-6r}{r^4}.$$

unit is : $\frac{kg}{cm}$.