

# MATH 127 Calculus for the Sciences

## Lecture 5

September 12, 2025

## Today's lecture

**Office hours/Question session**

Mon: 1430-1520 (before class, in my office MC5331)

Wed: after class in this room (but make sure you get to your quiz in time)

Fri: after class in this room

**Tutoring center**

In person MC 3022

Online on Teams

**Last time** Integration, fundamental theorem of calculus

**This time**

**Course note coverage** Section 1.1.4, 1.1.5

An example of how to use FTC

Anti-derivatives

Indefinite integrals

## Quiz 2 is next Wednesday!

Coverage:

1. Basic derivatives, e.g. derivative of  $\sin(x)$  is...?
2. Tangent line, e.g. find point-slope form of tangent of  $y = \dots$  at  $x = \dots$
3. Tangent approximation, e.g. if tangent at  $x = 3$  is ..., approximate the value at  $x = 3.01$ .
4. Differentials, e.g. what is  $dy$  is  $y = \dots$ ?
5. Differential equation, e.g. is ... a differential equation? What's its order? Is ... a solution to it?
6. Definite integrals, e.g. evaluate  $\int_0^3 x dx$  (we will do this today).
7. Net change problem, e.g. If  $F'(x) = \dots$ , what is  $\Delta F$  as  $x$  goes from 3 to 5?

## Example

Recall from last time.

**Theorem (The Fundamental Theorem of Calculus (FTC))** If  $F(x)$  is a differentiable function with  $F'(x) = f(x)$ , then

$$F(x) \Big|_a^b = \int_a^b f(x) dx.$$

**Question** If an object has speed  $S(t) = 2t$  at time  $t$ , what is the distance it travelled from time 0 to time 3?

Denote the distance function by  $D(t)$ . We want to find

$$D(t) \Big|_0^3 = D(3) - D(0),$$

but we do not know what  $D(t)$  is.

## Example

**Theorem (The Fundamental Theorem of Calculus (FTC))** *If  $F(x)$  is a differentiable function with  $F'(x) = f(x)$ , then*

$$F(x) \Big|_a^b = \int_a^b f(x) dx.$$

**Want:**

$$D(t) \Big|_0^3 = D(3) - D(0) = \boxed{???}$$

**Know:** The derivative of  $D(t)$  is exactly

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By FTC, we have

$$\boxed{\phantom{000}} = \int_1^3 \boxed{\phantom{000}} dt.$$

So now we want:

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## Example

**Theorem (The Fundamental Theorem of Calculus (FTC))** If  $F(x)$  is a differentiable function with  $F'(x) = f(x)$ , then

$$F(x) \Big|_a^b = \int_a^b f(x) dx.$$

Now we want:

$$\int_1^3 2t dt = \boxed{???}$$

**Know:** If we take  $F(t) = \boxed{\phantom{000}}$ , then

$$F'(t) = 2t.$$

By FTC, we have

$$\int_1^3 2t dt = \boxed{\phantom{000}}.$$

And the right-hand side is equal to:

## Example

**Theorem (The Fundamental Theorem of Calculus (FTC))** If  $F(x)$  is a differentiable function with  $F'(x) = f(x)$ , then

$$F(x) \Big|_a^b = \int_a^b f(x) dx.$$

Written more concisely, we can say

$$\begin{aligned} D(t) \Big|_0^3 &= \int_0^3 S(t) dt && \text{(by FTC)} \\ &= \int_0^3 2t dt \\ &= (t^2) \Big|_0^3 && \text{(by FTC again)} \\ &= 3^2 - 0^2 = 9, \end{aligned}$$

**Question** If we know  $D'(t) = 2t$ , and  $(t^2)' = 2t$ , then can we just say

$$D(t) = t^2?$$

## Anti-derivatives

**Definition** Suppose  $F'(x) = f(x)$ , then we say  $F(x)$  is an **anti-derivative** of  $f(x)$ .

**Example** If  $f(x) = 2x$ , an anti-derivative of  $f(x)$  is

;

another anti-derivative of  $f(x)$  is

;

a third anti-derivative of  $f(x)$  is

.

If we know  $F(x)$  is the anti-derivative of  $f(x)$ , then all other anti-derivatives are of form



## Indefinite integral

**Definition** Suppose  $F(x)$  is the anti-derivative of  $f(x)$ , then the family of anti-derivatives of  $f(x)$  is denoted by

$$\int f(x)dx = F(x) + C$$

where  $C$  is called the **constant of integration**.

Given any  $F(x)$  and  $C$  with  $F'(x) = f(x)$ , we have

$$\int_a^b f(x)dx = \boxed{\phantom{F(b) - F(a)}}.$$

So no matter which  $C$  we choose, it will always cancel out and we always get the same answer.

$$1 \quad \int \rho d\alpha = C;$$

5.  $\int \sin x =$  ;

$$\int (k \cdot f(x) + g(x)) dx = k \cdot F(x) + G(x) + C$$

## Example

**Question** If my speed is  $(6x^2 + 2e^x)$  m/s, what could my distance function be at time  $x$ ?

My distance function must be an anti-derivative of my speed.

An anti-derivative of  $6x^2 + 2e^x$  is

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so we can say

$$\int (6x + 2e^x)dx = \boxed{\phantom{0000}}.$$

Another anti-derivative of  $6x^2 + 2e^x$  is

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so we can say

$$\int (6x + 2e^x)dx = \boxed{\phantom{00000000}}.$$

**Conclusion:** my distance function is...

## Example

**Question** Which of the following sentences is correct?

Suppose  $f'(x) = 3x^2$ , but we are not sure what  $f(x)$  is exactly....

1.  $f(x)$  is an anti-derivative of  $3x^2$ .
2.  $x^3$  is an anti-derivative of  $3x^2$ .
3.  $f(x) = x^3$ .
4.  $f(x) = x^3 + C$ .
5.  $f(x) = x^3 + C$  for some number  $C$ .
6. Any anti-derivative of  $3x^2$  is of form  $f(x) + C$ .
7. Any anti-derivative of  $3x^2$  is of form  $x^3 + C$ .
8. Any anti-derivative of  $3x^2$  is of form  $x^3 + 1 + C$ .

## Survey

Give some feedback for your experience so far.

