MATH 127 Calculus for the Sciences

Lecture 14

Today's lecture

Last time

Modelling with Trig functions

1. Given some data from science, model it using a trig function
2. Given a trig function modelling something, retrieve data from it
3. Some models will involve waves, so you use sin or cos. Some will involve triangles, so you use appropriate trig functions.

This time

Course note coverage Section 2.5.3

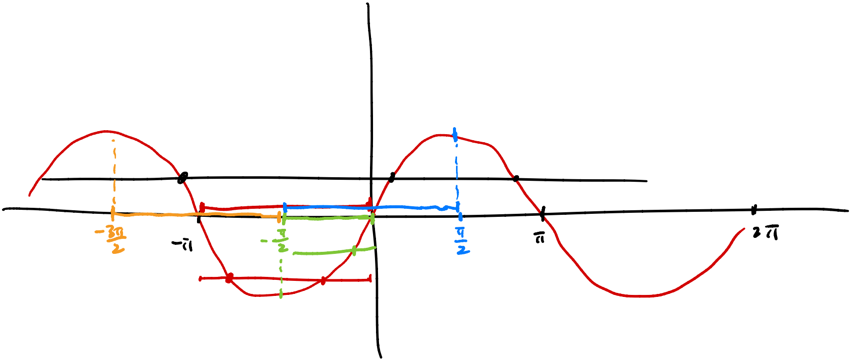
Inverse of trig functions

1. Do they even exist?
2. Definition of $\arcsin, \arccos, \arctan$
3. How to compute them

Practice midterms available on learn.

Inverse of sin

Question What is the inverse function of $\sin(x)$?



Inverse of sin

Question So there is no inverse on all of the real numbers (by horizontal line test), but we can restrict the interval so that $\sin(x)$ does pass the horizontal line test.
For which of the following intervals does $\sin(x)$ pass the horizontal line test?

(A) $[-\pi, 0]$ ✗

(B) $\left[-\frac{\pi}{2}, 0\right]$ ✓

(C) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ✓

(D) $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$ ✓

Inverse of sin

There are many choices of domains such that $\sin(x)$ admits inverse. But we want a nice choice to work with.

Definition The function

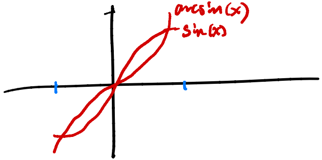
$$\boxed{\arcsin(x)}$$

is the inverse of $\sin(x)$ with respect to the "special domain" $\boxed{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$.
Sometimes we also write

$$\arcsin(x) = \sin^{-1}(x)$$

Remark The notation $\sin^{-1}(x)$ is **not to be confused** with

$$\frac{1}{\sin(x)} = \boxed{\csc(x)}.$$



Inverse of sin

	sin	arcsin
Domain	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[-1,1]$
Range	$[-1,1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$

Example Evaluate the following

- $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \theta = \frac{\pi}{3}$

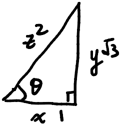
This is the same as asking, for which angle θ, is sin of it equal to $\boxed{\frac{\sqrt{3}}{2}}$.

This angle must lie in the range of arcsin, which is the "special domain" of sin, which is

$$\boxed{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$

Remember how to find sin of an angle? Let's work backwards.

$\sin(\theta) = \frac{\sqrt{3}}{2}$ and θ inside $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$\sin\theta = \frac{y}{z} = \frac{\sqrt{3}}{2}$

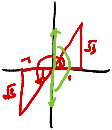
So we can take

$y = \sqrt{3}$

$z = 2$

$x^2 + y^2 = z^2$

$x = \pm 1$



so $x = 1$

$\theta = \frac{\pi}{3}$

Inverse of sin

Example Evaluate the following

(1). $\sin(\arcsin(0.12345))$

(2). $\sin(\arcsin(\boxed{1.23456}))$

	sin	arcsin
Domain	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[-1, 1]$
Range	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$

(1) Use cancellation rule

$$\sin(\arcsin(0.12345)) = 0.12345.$$

(2) Does not exist because 1.23456 is not in the domain of arcsin.

Inverse of sin

Example Evaluate the following

(1). $\arcsin(\sin(\pi/5))$

(2). $\arcsin(\sin(3\pi/4))$

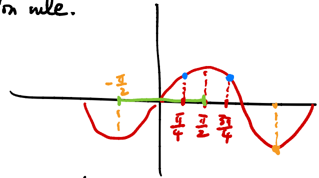
	sin	arcsin
Domain	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[-1,1]$
Range	$[-1,1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\arcsin(\sin \frac{3\pi}{2}) = -\frac{\pi}{2}$

(1) $\frac{\pi}{5}$ is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Therefore we can apply the cancellation rule.

$$\arcsin(\sin(\tfrac{\pi}{5})) = \frac{\pi}{5}.$$



(2) $\arcsin(\sin(3\pi/4)) = \theta$

Then θ is the value in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ such that

$$\sin\theta = \sin(3\pi/4)$$

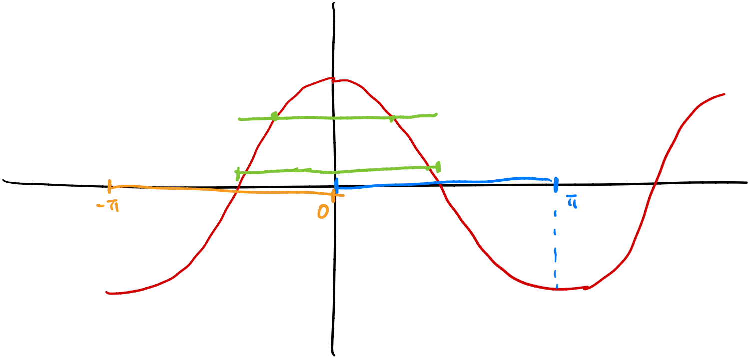
by inspecting the graph

$$\theta = \frac{\pi}{4}.$$

Inverse of cos

The story of the inverse of cos is very similar.

Question What is an interval over which $\cos(x)$ passes the horizontal line test?



Inverse of cos

Definition The function

$$\boxed{\arccos(x)}$$

is the inverse of $\cos(x)$ with respect to the "special domain" $\boxed{[0, \pi]}$. Sometimes we also write

$$\arccos(x) = \cos^{-1}(x)$$

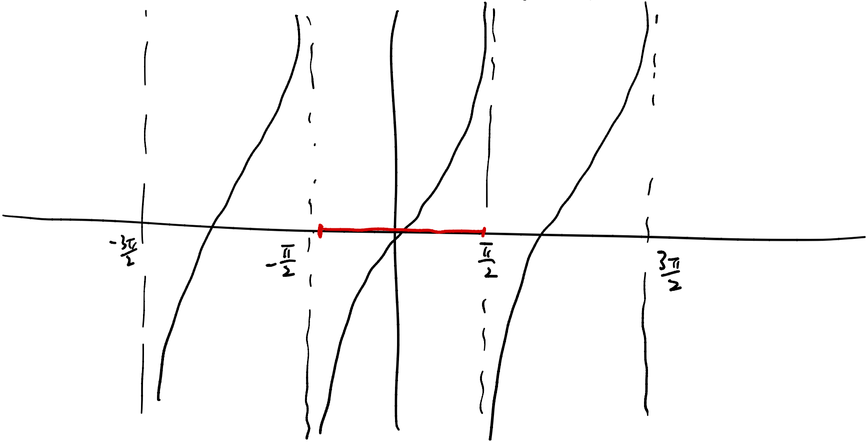
Question What are the domain and range of arccos?

	cos	arc cos
Dom	$[0, \pi]$	$[-1, 1]$
Ran	$[-1, 1]$	$[0, \pi]$

Inverse of tan

Question What is the domain of $\tan(x)$?
What is an interval over which $\tan(x)$ passes the horizontal line test?

Domain $x \neq \frac{\pi}{2} + k\pi$ for any integer k.



Inverse of tan

Definition The function

$$\boxed{\arctan(x)}$$

is the inverse of $\tan(x)$ with respect to the "special domain" $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.
Sometimes we also write

$$\arctan(x) = \tan^{-1}(x)$$

Question What are the domain and range of arctan?

	tan	arctan
Dom	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$(-\infty, \infty)$
Ran	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Inverse of tan

Example You know how to find things like $\tan(\arctan(\frac{3}{4}))$, but can you find

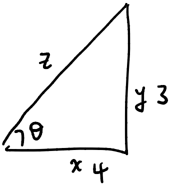
$$\sin\left(\arctan\left(\frac{3}{4}\right)\right)?$$

	tan	arctan
Dom	$(-\frac{\pi}{2}, \frac{\pi}{2})$	$(-\infty, \infty)$
Ran	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

$\arctan(\frac{3}{4}) = \theta$

Then θ is the value in $\boxed{(-\frac{\pi}{2}, \frac{\pi}{2})}$ such that $\tan\theta = \frac{3}{4}$

Want $\sin\theta$

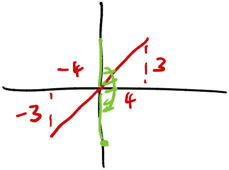


$\tan\theta = \frac{y}{x} = \frac{3}{4}$

So we can take

$y = 3$ or $\cancel{-3}$

$x = 4$ or $\cancel{-4}$



$z = \sqrt{x^2+y^2}$

$= 5$

therefore

$\sin\theta = \frac{y}{z}$

$= \frac{3}{5}$