

## Chap.6 Inner Product Space

### §6.1 Inner product and norms

#### Example

Let  $A, B \in M_{m \times n}(F)$ ,

$$\langle A, B \rangle = \text{Tr}(B^* A)$$

Determine the  $\langle \cdot, \cdot \rangle$  is an inner product.

solution:.

- Claim 1.  $\langle A, A \rangle \geq 0$
- Claim 2.  $\langle A, B \rangle = \overline{\langle B, A \rangle}$
- Claim 3.  $\langle kA + B, C \rangle = k\langle A, C \rangle + \langle B, C \rangle$



Notice : A vector space  $V$  over  $F$  endowed with a specific inner product is called a inner product space. If  $F = \mathbb{C}$  we called it "Complex inner product space", whereas if  $F = \mathbb{R}$  we called it "real inner product space".

### §6.2 Gram-Schmidt Orthogonalization Process