

Chap.6 Inner Product Space

§6.1 Inner product and norms

Example

Let $A, B \in M_{m \times n}(F)$,

$$\langle A, B \rangle = \text{Tr}(B^* A)$$

Determine the $\langle \cdot, \cdot \rangle$ is an inner product.

solution:.

- Claim 1. $\langle A, A \rangle \geq 0$
- Claim 2. $\langle A, B \rangle = \overline{\langle B, A \rangle}$
- Claim 3. $\langle kA + B, C \rangle = k\langle A, C \rangle + \langle B, C \rangle$



Notice : A vector space V over F endowed with a specific inner product is called a inner product space. If $F = \mathbb{C}$ we called it "Complex inner product space", whereas if $F = \mathbb{R}$ we called it "real inner product space".

§6.2 Gram-Schmidt Orthogonalization Process