LA CHEAT SHEET NOV WEEK 3

QSNACK (PIECE OF CAKE) EDITION

<u>Definition</u>. *Let* $\beta = \{u_1, u_2, \cdots, u_n\}$ *be a basis for FDVS* V. *For* $x \in V$, *let* $a_1, a_2, \cdots, a_n$ *be the unique scalars* $\ni$

$$x = \sum_{i=1}^{n} a_i u_i$$

the coordinate vector of x *relative to* β, *denoted* $[x]_\beta$, *by*

$$[x]_\beta = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

<u>Definition</u>. V *and* W *are FDVS with basis* $\beta = \{v_1, v_2, \cdots, v_n\}$ *and* $\gamma = \{w_1, w_2, \cdots, w_m\}$, *respectively. Let* $T : V \to W$ *be linear. Then for each* $j, 1 \leq j \leq n$, *there exist unique scalars* $a_{ij} \in F, 1 \leq i \leq m, \ni$

$$T(v_j) = \sum_{i=1}^{m} a_{ij} w_i \textit{for } 1 \leq j \leq n$$

Let $A_{ij} = a_{ij}$, *the matrix* $A = [T]_\beta^\gamma$ *is call the matrix representation of* T *in the ordered bases* β *and* γ. *If* $V = W$ *and* $\beta = \gamma$, *then we write* $A = [T]_\beta$

Theorem. *Let* V *and* W *be FDVS with basis* β *and* γ, *respectively, and let* $T, U : V \to W$ *be linear transformations. Then*

(a) $[T + U]_\beta^\gamma = [T]_\beta^\gamma + [U]_\beta^\gamma$
(b) $[aT]_\beta^\gamma = a[T]_\beta^\gamma$ *for all scalars* a.