Math Logic and Life

Midterm

(1) Find the truth set of the propositional function $(x^2+1)(x-3)(x^2-2)(2x-3)=0$, when it is given that the set of meanings of this propositional function is each of the following
(a)Z(b)Q(c)R(d)C

(2) Both of the following statements have the set of positive real numbers as their set of meanings. Which statement is true?
a) $(\exists y)(\forall x)(x<2y)$ b)$(\forall x)(\exists y)(x<2y)$

(3) Write the converse and contrapositive of the following expression: "If $\sqrt{2}<\sqrt{5}$, then $2<5$."

(4) Suppose statement P : $(\forall x)(\exists y)(x<2y)$. Write statement $/simP$

(5) Determine which of the following expressions are tautologies.
(a) $P \to P$
(b) $P \cap (Q \cup R) \leftrightarrow (P \cap Q) \cup (P \cap R)$
(c) $P \cup Q \leftrightarrow (\sim P \to Q)$

(6) Assume that "Tim is a boy" is a true statement and that "Tim is ten years old" is a true statement. Which of the following are true?
a) Tim is not a boy and Tim is ten years old
b) If Tim is not a boy then Tim is ten years old
c) Tim is ten years old or Tim is a boy
d) If Tim is not ten years old then Tim is not a boy

(7) Show that $(P \to Q) \cap (P \cap (\sim Q))$ is a contradiction.

(8) Which of the following are propositions?

(9) Determine which of the following statements are true
(a) If $A \supseteq B$, then $A = B$
(b) Since $\emptyset$ is a set that is a member of every set, there is a set that is a number of every set
(c) If $A = B$, then $A \supset B$
(d) If sets $A = \{1,3,5\}$ and $B = \{1,3,7\}$ then $A \cap B = \{1,3\}$
(e) $\{\{1,2,3\}\} \in N \cup P(N)$
(f) $\{\{1,2,3\}\} \subseteq N \cup P(N)$
(g) The empty set is a subset of every set.

(h) If sets $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{3, 4, 6\}$ then $A - B = \{1, 2\}$

(i) $\{1, 2, 3\} \in N \cup P(N)$

(j) $\{1, 2, 3\} \subseteq P(N)$

(10) Write a useful negation of each of the following propositions.
a) If $1 \neq 3$, then $f(1) \neq f(3)$ b) If $a > b$, then $a^2 > b^2$

(11) Give are sets $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 4, 5, 6, 7\}$

(12) Finish the following truth table:

P	Q	∼P	∼Q	∼P∨Q	P∧ ∼Q	∼Q→∼P
T	T					
T	F					
F	T					
F	F					

(13) Prove that if an integer n^2 is even then n is even

(14) If r is real number such that $r^2 = 2$, then r is irrational.

(15) Prove or disprove the following statement. For any sets P, Q, and $R, (P \cap Q) \cup R = P \cap (Q \cup R)$

(16) Prove that for any natural number n, either n is a prime or a perfect square, or n divides $(n - 1)!$.