**Midterm Answer**

(1) (a)(b)(d)

(2) proof: ($\Rightarrow$) let $n = 2k+1, k \in \mathbb{Z}$

$n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4(k^2+k) + 1$ is odd

($\Leftarrow$) We have that $n^2$ is odd

Claim: $n$ is odd

Suppose not, $n = 2k, k \in \mathbb{Z}$ is even

$n^2 = 4k^2 \rightarrow\leftarrow$ to $n^2$ is odd

Hence, $n$ is odd.

(3) (b) $\because \forall x \exists y = x+1$ such that $x < 2y$

(4) $(\exists x)(\forall y)(x \geq 2y)$

(5) (a) $1 \neq 3$ and $f(1) = f(3)$

(6)

P: Tom finishes his work

Q: Tom can go home tonight

| $P$ | $Q$ | $\sim P$ | $\sim Q$ | $\sim P \rightarrow \sim Q$ |
|---|---|---|---|---|
| T | T | F | F | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

(7) (b)(d)

(8) (1)$\{2,3,4,5\}$ (2)$\{1,2,3,4,5,6,7\}$ (3)$\{1\}$

(9) (c)(d)(f)(g)(i)(j)

(10) (a) $\{3\}$ (b) $\{\frac{3}{2}, 3\}$ (c) $\{\pm\sqrt{2}, \frac{3}{2}, 3\}$ (d) $\{\pm i, \pm\sqrt{2}, \frac{3}{2}, 3\}$

(11) (b)

(12) ($\subseteq$) Given $x \in (A \cup B) - (A \cap B)$

$\Rightarrow x \in A$ or $B$ but $x \notin A \cap B$

$\Rightarrow$ If $x \in A$, but $x \notin A \cap B \Rightarrow x \in A$ but $x \notin B \Rightarrow x \in A - B$

If $x \in B$, but $x \notin A \cap B \Rightarrow x \in B$ but $x \notin A \Rightarrow x \in B - A$

$\Rightarrow x \in (A - B) \cup (B - A)$

($\supseteq$) Given $x \in (A - B) \cup (B - A)$

$\Rightarrow$ If $x \in A$, but $x \notin B$

$x \in A \Rightarrow x \in A \cup B$ and $x \notin B \Rightarrow x \notin A \cap B$

$x \in (A \cup B) - (A \cap B)$

If $x \in B$ but $x \notin A$

$x \in B \Rightarrow x \in A \cup B$ and $x \notin A \Rightarrow x \notin A \cap B$

$x \in (A \cup B) - (A \cap B)$

(13) Converse: If $2 < 5$, then $\sqrt{2} < \sqrt{5}$

Contrapositive: If $2 \geq 5$, then $\sqrt{2} \geq \sqrt{5}$

(14) (a)(b)(c)

(15) True, Given $x \in A \cap C \Rightarrow x \in A$ and $x \in C$,

then $x \in A \subseteq B \Rightarrow x \in B$ and $x \in C \subseteq D \Rightarrow x \in D$

$\therefore x \in B \cap D$ Hence $A \cap C \subseteq B \cap D$