

Advance Calculus Exercise

Question: Let \mathcal{T}_1 and \mathcal{T}_2 be topologies on a set X . Prove or disprove that $\mathcal{T}_1 \cup \mathcal{T}_2$ and $\mathcal{T}_1 \cap \mathcal{T}_2$ are topologies on X .

Solution.

■

Question: In any metric space, $\overline{B(x, r)} \subseteq \overline{B(x, r)} \forall r \geq 0$

Solution.

■

Question: Let X be a metric space, $S \subseteq X$. Then S is closed $\Leftrightarrow \partial S \subseteq S$

Solution.

■

Question: (De-Morgan's Law)

Show that:

$$(a) \left(\bigcup_{\alpha \in I} E_{\alpha} \right)^c = \bigcap_{\alpha \in I} (E_{\alpha})^c$$

$$(b) \left(\bigcap_{\alpha \in I} E_{\alpha} \right)^c = \bigcup_{\alpha \in I} (E_{\alpha})^c$$

Solution.

■

Question: (Inverse Image)

$$f^{-1}(B) = \{x \in X \mid f(x) \in B\}$$

is called the inverse image of a set B with the function $f : X \rightarrow Y$

Show that:

$$(a) f^{-1}(f(B)) \supseteq B$$

$$(b) f(f^{-1}(B)) \subseteq B$$

Solution.

■