Advance Calculus Exercise

Question: Let $\mathscr{T}_1$ and T_2 be topologies on a set X. Prove or disprove that $\mathscr{T}_1 \cup \mathscr{T}_2$ and $\mathscr{T}_1 \cap \mathscr{T}_2$ are topologies on X.

Solution.

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Question: In any metric space, $\overline{B(x,r)} \subseteq \overline{B}(x,r) \forall r \geq 0$

Solution.

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Question: Let X be a metric space, $S \subseteq X$. Then S is closed $\Leftrightarrow \partial S \subseteq S$

Solution.

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Question: (De-Morgan's Law)
Show that:
(a) $(\bigcup\limits_{\alpha \in I} E_\alpha)^c = \bigcap\limits_{\alpha \in I} (E_\alpha)^c$
(b) $(\bigcap\limits_{\alpha \in I} E_\alpha)^c = \bigcup\limits_{\alpha \in I} (E_\alpha)^c$

Solution.

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Question: (Inverse Image)

$$f^{-1}(B) = \{x \in X \mid f(x) \in B\}$$

is called the inverse image of a set B with the function $f : X \to Y$
Show that:
(a) $f^{-1}(F(B)) \supseteq B$
(b) $f(f^{-1(B)}) \subseteq B$

Solution.

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