
Pratica 13



Q1 (Rz 08/09 I.Z)

$$\begin{array}{ll} \max & -2x_1 - x_2 - 2x_3 \\ x_1 + 5x_2 + x_3 & \leq 100 \\ x_1 + 2x_2 + x_3 & \geq 50 \\ 2x_1 + 4x_2 + x_3 & \geq 80 \\ x_1, x_2, x_3 & \geq 0 \end{array} \rightsquigarrow \begin{array}{ll} \max & -2x_1 - x_2 - 2x_3 \\ x_1 + 5x_2 + x_3 & \leq 100 \\ -x_1 - 2x_2 - x_3 & \leq -50 \\ -2x_1 - 4x_2 - x_3 & \leq -80 \\ x_1, x_2, x_3 & \geq 0 \end{array}$$

$$\begin{array}{ll} \min & -x_0 \\ x_1 + 5x_2 + x_3 - x_0 & \leq 100 \\ -x_1 - 2x_2 - x_3 - x_0 & \leq -50 \\ -2x_1 - 4x_2 - x_3 - x_0 & \leq -80 \\ x_1, x_2, x_3, x_0 & \geq 0 \end{array}$$

$$\begin{aligned} z &= -x_0 \\ s_1 &= 100 - x_1 - 5x_2 - x_3 + x_0 \\ s_2 &= -50 + x_1 + 2x_2 + x_3 + x_0 \\ s_3 &= -80 + 2x_1 + 4x_2 + x_3 + x_0 \end{aligned}$$

$$\begin{aligned} z &= -80 + 2x_1 + 4x_2 + x_3 - s_3 \\ s_1 &= 180 - 3x_1 - 1x_2 - 2x_3 + s_3 & 180/1 = 20 \\ s_2 &= 30 - x_1 - \cancel{-2x_2} - 0 + s_3 & 30/2 = 15 \\ x_0 &= 80 - 2x_1 - 4x_2 - x_3 + s_3 & 80/4 = 20 \end{aligned}$$

Q1 (R2 08/09 I.Z)

$$\max -2x_1 - x_2 - 2x_3$$

$$x_1 + 5x_2 + x_3 \leq 100$$

$$x_1 + 2x_2 + x_3 \geq 50$$

$$2x_1 + 4x_2 + x_3 \geq 80$$

$$x_1, x_2, x_3 \geq 0$$

$$z = -x_0$$

$$s_1 = 100 - x_1 - 5x_2 - x_3 + x_0$$

$$s_2 = -50 + x_1 + 2x_2 + x_3 + x_0$$

$$s_3 = -80 + 2x_1 + 4x_2 + x_3 + x_0$$

$$z = -80 + 2x_1 + 4x_2 + x_3 - s_3$$

$$s_1 = 180 - 3x_1 - 1x_2 - 2x_3 + s_3 \quad 180/1 = 20$$

$$s_2 = 30 - x_1 - 2x_2 \quad 0 \quad + s_3 \quad 30/2 = 15$$

$$x_0 = 80 - 2x_1 - 4x_2 - x_3 + s_3 \quad 80/4 = 20$$

$$z = -20 \quad 0 \quad + x_3 \quad + s_3 \quad - 2s_2$$

$$s_1 = 45 \quad 3/2 x_1 \quad -2x_3 \quad -7/2 s_3 \quad + 9/2 s_2 \quad 45/2 = 22.5$$

$$x_2 = 15 \quad -1/2 x_1 \quad 0 \quad 1/2 s_3 \quad -1/2 s_2$$

$$x_0 = 20 \quad 0 \quad (-x_3) \quad -s_3 \quad + 2s_2 \quad 20/1 = 20$$

$$x_1 \quad s_3 \quad s_2 \quad x_0$$

$$z = 0 \quad 0 \quad 0 \quad 0 \quad -x_0$$

$$s_1 = 5 \quad 3/2 x_1 \quad -3/2 s_3 \quad + 1/2 s_2 \quad + 2x_0$$

$$x_2 = 15 \quad -1/2 x_1 \quad 1/2 s_3 \quad -1/2 s_2 \quad 0$$

$$x_3 = 20 \quad 0 \quad -s_3 \quad + 2s_2 \quad -x_0$$

Solução equivalente inicial:

$$(x_1^0, x_2^0, x_3^0) = (0, 15, 20)$$

Q1 (R2 08/09 I.Z)

$$\max -2x_1 - x_2 - 2x_3$$

$$x_1 + 5x_2 + x_3 \leq 100$$

$$x_1 + 2x_2 + x_3 \geq 50$$

$$2x_1 + 4x_2 + x_3 \geq 80$$

$$x_1, x_2, x_3 \geq 0$$

$$x_1 \quad s_3 \quad s_2 \quad x_0$$

$$Z = 0 \quad 0 \quad 0 \quad 0 \quad -x_0$$

$$s_1 = S \quad \frac{3}{2}x_1 \quad -\frac{3}{2}s_3 \quad +\frac{1}{2}s_2 \quad +2x_0$$

$$x_2 = 1S \quad -\frac{1}{2}x_1 \quad \frac{1}{2}s_3 \quad -\frac{1}{2}s_2 \quad 0$$

$$x_3 = 20 \quad 0 \quad -s_3 \quad +2s_2 \quad -x_0$$

Solución equivalente inicial:

$$(x_1^0, x_2^0, x_3^0) = (0, 15, 20)$$

$$\begin{aligned} f &= -2x_1 - x_2 - 2x_3 \\ &= -2x_1 - (15 - \frac{1}{2}x_1 + \frac{1}{2}s_3 - \frac{1}{2}s_2) \\ &\quad - 2(20 - s_3 + 2s_2) \\ &= -5S - \frac{3}{2}x_1 + \frac{3}{2}s_3 - \frac{7}{2}s_2 \end{aligned}$$

$$Z = -5S - \frac{3}{2}x_1 + \frac{3}{2}s_3 - \frac{7}{2}s_2$$

$$s_1 = S \quad \frac{3}{2}x_1 \quad -\frac{3}{2}s_3 \quad +\frac{1}{2}s_2$$

$$S \times \frac{2}{3} = 10/3$$

$$\begin{array}{rcl} \frac{x_1}{3} \downarrow & x_2 = 1S & -\frac{1}{2}x_1 \quad \frac{1}{2}s_3 \quad -\frac{1}{2}s_2 \\ \xrightarrow{\frac{-2}{3}} & x_3 = 20 & 0 \quad -s_3 \quad +2s_2 \end{array}$$

$$20/1 = 20$$

$$Z = -50 \quad 0 \quad -3s_2 \quad -s_1$$

$$s_3 = 10/3 \quad x_1 \quad \frac{1}{3}s_2 \quad -\frac{2}{3}s_1$$

$$x_2 = 50/3 \quad 0 \quad -\frac{1}{3}s_2 \quad -\frac{1}{3}s_1$$

$$x_3 = 50/3 \quad -x_1 \quad \frac{5}{3}s_2 \quad +\frac{2}{3}s_1$$

$$f^* = -50 \quad (x_1^*, x_2^*, x_3^*) = (0, 50/3, 50/3)$$

Q2 (R2 16/17 I.C)

$$\max -4x_1 + 5x_2 + 3x_3$$

$$-x_1 + x_2 + x_3 \leq -2$$

$$-3x_1 + 4x_2 + 4x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$

?

$$\max -x_0$$

$$-x_1 + x_2 + x_3 - x_0 \leq -2$$

$$-3x_1 + 4x_2 + 4x_3 - x_0 \leq 1$$

$$x_1, x_2, x_3, x_0 \geq 0$$

↔

$$\textcircled{I} \quad z = 0 - x_0$$

$$s_1 = -z + x_1 - x_2 - x_3 + x_0$$

$$s_2 = 1 + 3x_1 - 4x_2 - 4x_3 + x_0$$

?

$$\textcircled{II} \quad z = -z + x_1 - x_2 - x_3 - s_1$$

$$x_0 = z - x_1 + x_2 + x_3 + s_1$$

$$x_2 \\ s_2 = 3 + 2x_1 - 3x_2 - 3x_3 + s_1$$

$$x_2 \quad x_3 \quad s_1 \quad x_0$$

$$\textcircled{II} \quad z = 0 \quad 0 \quad 0 \quad 0 \quad -x_0$$

$$x_1 = z + x_2 + x_3 + s_1 - x_0$$

$$s_2 = -x_2 - x_3 - s_1 - 2x_0$$

$$f = -4x_1 + 5x_2 + 3x_3 = -4(z + x_2 + x_3 + s_1) + 5x_2 + 3x_3 \\ = -8 + x_2 - x_3 - 4s_1$$

$$\star (x_1^0, x_2^0, x_3^0) = (z, 0, 0) \quad f^0 = -8$$

Q2 (R2 16/17 I.C)

$$f = -4x_1 + 5x_2 + 3x_3 = -4(z + x_2 + x_3 + s_1) + 5x_2 + 3x_3 \\ = -8 + x_2 - x_3 - 4s_1$$

$$\star (x_1^0, x_2^0, x_3^0) = (z, 0, 0) \quad f^0 = -8$$

$$\textcircled{I} \quad \begin{matrix} & x_2 & x_3 & s_1 & x_0 \end{matrix}$$

$$z = 0 \quad 0 \quad 0 \quad 0 \quad -x_0$$

$$x_1 = z + x_2 + x_3 + s_1 - x_0$$

$$s_2 = +x_2 - x_3 - 3s_1 - 2x_0$$



\textcircled{I}

$$z = -8 + x_2 - x_3 - 4s_1$$

$$x_1 = z + x_2 + x_3 + s_1$$

$$s_2 = +x_2 - x_3 - 3s_1$$

\textcircled{II}

$$z = -1 - 2x_3 - s_1 - s_2$$

$$x_1 = 1 \quad 0 \quad 4s_1 - s_2$$

$$x_2 = -x_3 - 3s_1 - s_2$$

$$(x_1^*, x_2^*, x_3^*) = (1, 7, 0)$$

$$f^* = -1$$

Q3 (To 08/09 J.z)

$$\max x_1 - x_2 + x_3$$

$$2x_1 - x_2 + 2x_3 \leq 6$$

$$2x_1 - 2x_2 + x_3 \leq -2$$

$$-x_1 + 2x_2 - 2x_3 \leq -1$$

$$x_1, x_2, x_3 \geq 0$$

}

$$\max -x_0$$

$$2x_1 - x_2 + 2x_3 - x_0 \leq 6$$

$$2x_1 - 2x_2 + x_3 - x_0 \leq -2$$

$$-x_1 + 2x_2 - 2x_3 - x_0 \leq -1$$

$$x_1, x_2, x_3 \geq 0$$

① $Z = -x_0$

$$S_1 = 6 - 2x_1 + x_2 - 2x_3 + x_0$$

$$S_2 = -2 - 2x_1 + 2x_2 - x_3 + x_0$$

$$S_3 = -1 + x_1 - 2x_2 + 2x_3 + x_0$$

② $Z = -z - 2x_1 + 2x_2 - x_3 - S_2$

$$S_1 = 8 \quad 0 \quad -x_2 - x_3 + S_2 \quad 8/1 = 8$$

$$x_0 = z \quad 2x_1 \quad -2x_2 \quad x_3 + S_2 \quad z/2 = 1$$

$$S_3 = 1 \quad 3x_1 \quad -4x_2 + 3x_3 + S_2 \quad 1/4 = 1/4$$

Q3 (To 08/09 I.2)

$$\max x_1 - x_2 + x_3$$

$$2x_1 - x_2 + 2x_3 \leq 6$$

$$2x_1 - 2x_2 + x_3 \leq -2$$

$$-x_1 + 2x_2 - 2x_3 \leq -1$$

$$x_1, x_2, x_3 \geq 0$$

}

$$\max -x_0$$

$$2x_1 - x_2 + 2x_3 - x_0 \leq 6$$

$$2x_1 - 2x_2 + x_3 - x_0 \leq -2$$

$$-x_1 + 2x_2 - 2x_3 - x_0 \leq -1$$

$$x_1, x_2, x_3 \geq 0$$

$$(3) z = -\frac{1}{2}x_1 - \frac{1}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}s_2 - \frac{1}{2}s_3$$

$$s_1 = \frac{3}{4}x_1 - \frac{3}{4}x_2 - \frac{7}{4}x_3 + \frac{3}{4}s_2 + \frac{1}{4}s_3 - \frac{3}{4} \times \frac{6}{7} = -\frac{31}{7} > 1$$

$$x_0 = \frac{3}{2} - \frac{1}{2}x_1 - \frac{1}{2}x_3 - \frac{1}{2}s_2 + \frac{1}{2}s_3 \quad \frac{3}{2} \times 2 = 3$$

$$x_2 = \frac{1}{4}x_1 + \frac{3}{4}x_3 + \frac{1}{4}s_2 - \frac{1}{4}s_3$$

$$(4) z = 0 \quad 0 \quad 0 \quad 0 \quad -x_0$$

$$s_1 = \frac{5}{2} - \frac{5}{2}x_1 - s_2 - \frac{3}{2}s_3 + \frac{7}{2}x_0$$

$$x_3 = 3 - x_1 - s_2 - s_3 - 2x_0$$

$$x_2 = \frac{5}{2} - \frac{3}{2}x_1 - s_2 - \frac{1}{2}s_3 - \frac{3}{2}x_0$$

$$(x_1^0, x_2^0, x_3^0) = (0, \frac{5}{2}, 3)$$

$$z = x_1 - x_2 + x_3$$

$$= x_1 - (\frac{5}{2} + \frac{3}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{2}s_3) + (3 + x_1 + \frac{5}{2} + s_3)$$

$$= \frac{1}{2}x_1 + \frac{1}{2}s_3$$

Q3 (Tb 08/09 I.2)

$$\max x_1 - x_2 + x_3$$

$$2x_1 - x_2 + 2x_3 \leq 6$$

$$2x_1 - 2x_2 + x_3 \leq -2$$

$$-x_1 + 2x_2 - 2x_3 \leq -1$$

$$x_1, x_2, x_3 \geq 0$$

}

$$\max -x_0$$

$$2x_1 - x_2 + 2x_3 - x_0 \leq 6$$

$$2x_1 - 2x_2 + x_3 - x_0 \leq -2$$

$$-x_1 + 2x_2 - 2x_3 - x_0 \leq -1$$

$$x_1, x_2, x_3 \geq 0$$

$$(3) \quad z = -\frac{3}{2} - \frac{1}{2}x_1 + \frac{1}{2}x_3 - \frac{1}{2}s_2 - \frac{1}{2}s_3$$

$$\begin{array}{l} s_1 = \frac{3}{4} - \frac{3}{4}x_1 - \frac{7}{4}x_3 \quad \frac{3}{4}s_2 + \frac{1}{4}s_3 \quad -\frac{3}{4} \times \frac{4}{7} = -\frac{3}{7} \\ x_0 = \frac{3}{2} + \frac{1}{2}x_1 - \frac{1}{2}x_3 \quad \frac{1}{2}s_2 + \frac{1}{2}s_3 \\ x_2 = \frac{1}{4} + \frac{3}{4}x_1 + \frac{5}{4}x_3 \quad \frac{1}{4}s_2 - \frac{1}{4}s_3 \quad \frac{7}{4} \times \frac{4}{7} = 3^{\frac{1}{4}} \end{array}$$

$$(4) \quad \begin{array}{ccccc} & x_1 & s_2 & s_3 & \\ z & = & 0 & 0 & 0 & -x_0 \\ s_1 & = & \frac{5}{2} - \frac{5}{2}x_1 & -s_2 & -\frac{3}{2}s_3 & +\frac{7}{2}x_0 \\ x_3 & = & 3 & x_1 & s_2 & s_3 & -2x_0 \\ x_2 & = & \frac{5}{2} & \frac{3}{2}x_1 & s_2 & \frac{1}{2}s_3 & -\frac{3}{2}x_0 \end{array}$$

$$(x_1^0, x_2^0, x_3^0) = (0, \frac{5}{2}, 3)$$

$$z = x_1 - x_2 + x_3$$

$$= x_1 - (\frac{5}{2} + \frac{3}{2}x_1 + \frac{5}{2} + \frac{1}{2}s_3) + (3 + x_1 + \frac{5}{2} + s_3)$$

$$= \frac{1}{2} + \frac{1}{2}x_1 + \frac{1}{2}s_3$$

$$z = \frac{1}{2} + \frac{1}{2}x_1 + \frac{1}{2}s_3$$

$$s_1 = \frac{5}{2} - \frac{5}{2}x_1 - s_2 - \frac{3}{2}s_3$$

$$x_3 = 3 + x_1 + s_2 + s_3$$

$$x_2 = \frac{5}{2} + \frac{3}{2}x_1 + s_2 + \frac{1}{2}s_3$$

A solução não é óptima

Q6 (T2 08/09 I.1)

$$\min -x_1 - 2x_2 + x_3$$

$$\text{s.t. } 2x_1 + x_2 + x_3 \leq 14$$

$$-4x_1 - 2x_2 - 3x_3 \geq -28$$

$$2x_1 + 5x_2 + 5x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

$$\max x_1 + 2x_2 - x_3$$

$$\text{s.t. } 2x_1 + x_2 + x_3 \leq 14$$

$$4x_1 + 2x_2 + 3x_3 \leq 28$$

$$2x_1 + 5x_2 + 5x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

$$z = x_1 + 2x_2 - x_3$$

$$s_1 = 14 - 2x_1 - x_2 - x_3 \quad 14/1 = 14$$

$$s_2 = 28 - 4x_1 - 2x_2 - 3x_3 \quad 28/2 = 14$$

$$s_3 = 30 - 2x_1 - 5x_2 - 5x_3 \quad 30/5 = 6$$

$$\frac{z}{5} \times 30 = \frac{z}{8} \times 15 \times 6 = 12$$

↓

$$z = 12 + 1/5 \cdot x_1 - 3x_2 - 2/5 \cdot s_3$$

$$s_1 = 8 - 8/5 \cdot x_1 + 1/5 \cdot s_3 \rightarrow 8 \times \frac{5}{8} = 5$$

$$s_2 = 16 - 16/5 \cdot x_1 - x_2 + 2/5 \cdot s_3 \rightarrow 16 \times \frac{5}{16} = 5$$

$$x_2 = 6 - 2/5 x_1 - x_3 - 1/5 \cdot s_3 \rightarrow 6 \times \frac{5}{2} = 3 \times 5 = 15$$

Q6 (T2 08/09 I.1)

$$z = 12 + 1/5 \cdot x_1 - 3x_3 - 2/5 \cdot s_3$$

$$s_1 = 8 - 8/5 \cdot x_1 + 1/5 \cdot s_3 \rightarrow 8 \times \frac{5}{8} = 5$$

$$s_2 = 16 - 16/5 \cdot x_1 - x_3 + 2/5 \cdot s_3 \rightarrow 16 \times \frac{5}{16} = 5$$

$$x_2 = 6 - 2/5 x_1 - x_3 - 1/5 \cdot s_3 \rightarrow 6 \times \frac{5}{2} = 3 \times 5 = 15$$

↓

$$z = 13 - 3x_3 - 15/40 \cdot s_3 - 1/8 s_1$$

$$x_1 = 5 \quad 0 \quad + 1/8 \cdot s_3 - 5/8 s_1$$

$$s_2 = 0 \quad -x_3 \quad + 0 \quad + 2s_1$$

$$x_2 = 4 \quad -x_3 \quad -1/4 \cdot s_3 \quad + 1/4 \cdot s_1$$

Optimo: -15

Sol: (5, 4, 0)

Q7 (RZ 08/09 I.1)

$$\min 100 + x_1 - 5x_2 + 2x_3$$

$$\text{s.t. } x_1 + 3x_2 \leq 50$$

$$x_1 + 2x_2 + x_3 \leq 50$$

$$x_1 + x_3 \leq 9z$$

$$x_1, x_2, x_3 \geq 0$$

$$\max -x_1 + 5x_2 - 2x_3$$

$$x_1 + 3x_2 \leq 50$$

$$x_1 + 2x_2 + x_3 \leq 50$$

$$x_1 + x_3 \leq 9z$$

$$x_1, x_2, x_3 \geq 0$$

$$\begin{aligned} z &= -x_1 + 5x_2 - 2x_3 \\ \frac{5}{3} s_1 &= 50 - x_1 - 3x_2 \quad 50/3 < 17 \\ -\frac{2}{3} s_2 &= 50 - x_1 - 2x_2 - x_3 \quad 50/2 = 25 \\ s_3 &= 9z - x_1 - x_3 \end{aligned}$$

$$z = 250/3 - 8/3 x_1 - 2x_3 - 5/3 s_1$$

$$x_2 = 50/3 - 1/3 x_1 - 1/3 s_1$$

$$s_2 = 50/3 - 1/3 x_1 - x_3 + 2/3 s_1$$

$$s_3 = 9z - x_1 - x_3$$

$$\bullet 100 - 250/3 = \frac{300 - 250}{3} = \frac{50}{3}$$

$$(x_1^*, x_2^*, x_3^*) = (0, 50/3, 0)$$