

Práctica 10



Q1 (21.8-1)

for  $i = 1$  to  $16$

    MakeSet( $x_i$ )

for  $i = 1$  to  $15$  by  $2$

    Union( $x_i, x_{i+1}$ )

for  $i = 1$  to  $13$  by  $4$

    Union( $x_i, x_{i+2}$ )

    Union( $x_1, x_5$ )

    Union( $x_{11}, x_{13}$ )

    Union( $x_9, x_{10}$ )

    FindSet( $x_2$ )

    FindSet( $x_9$ )

] I

I



...



II



...



III

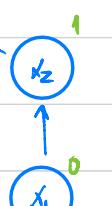
III

Union( $x_1, x_3$ )

Union( $x_5, x_7$ )

:

Union( $x_{13}, x_{15}$ )

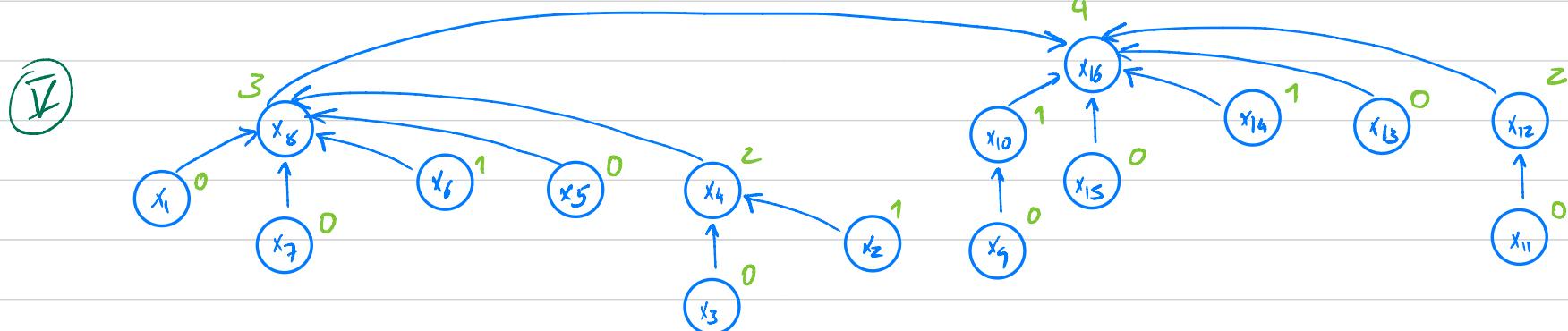
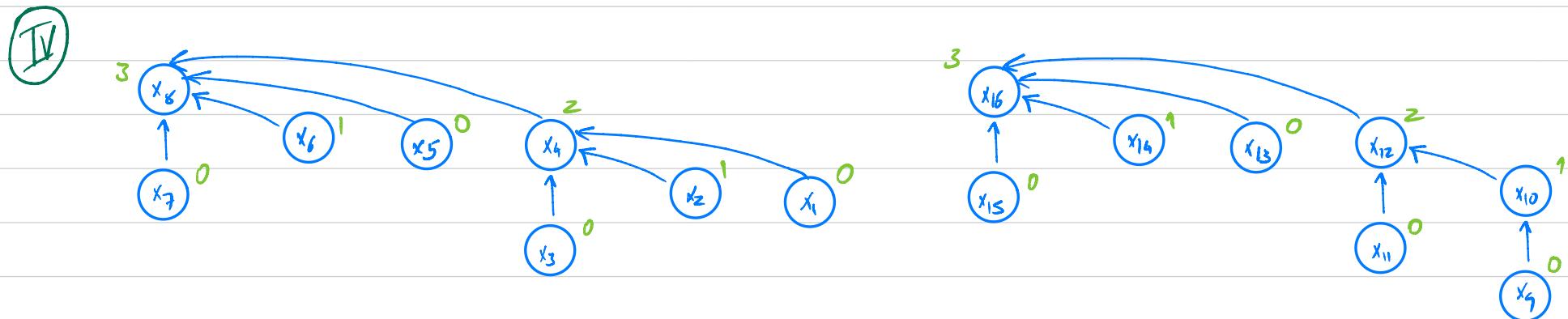
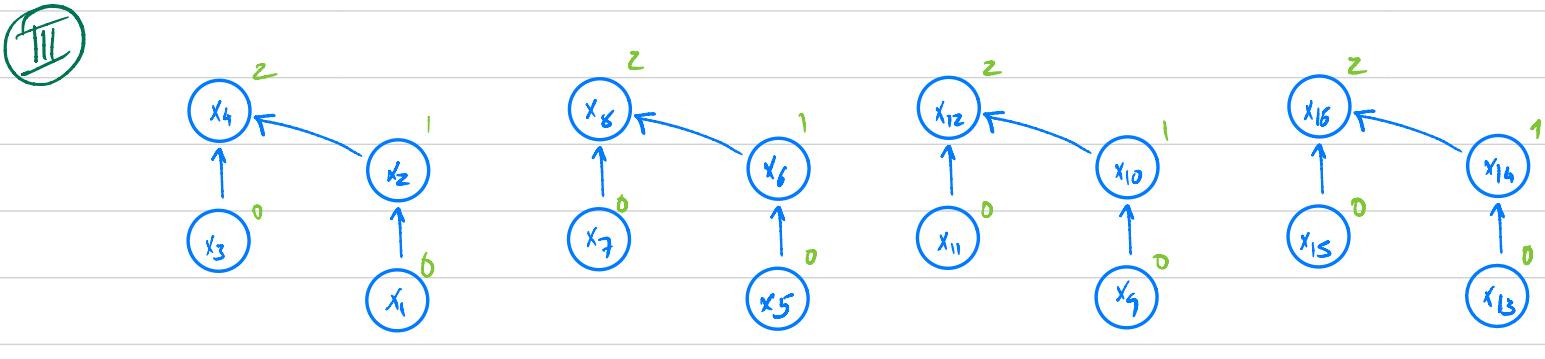


IV



Q1 (21.3-1)

- Union ( $x_1, x_5$ ) { IV }
- Union ( $x_{11}, x_3$ ) { III }
- Union ( $x_9, x_{10}$ ) { I }
- findSet ( $x_2$ ) { VI }
- findSet ( $x_9$ ) { VII }



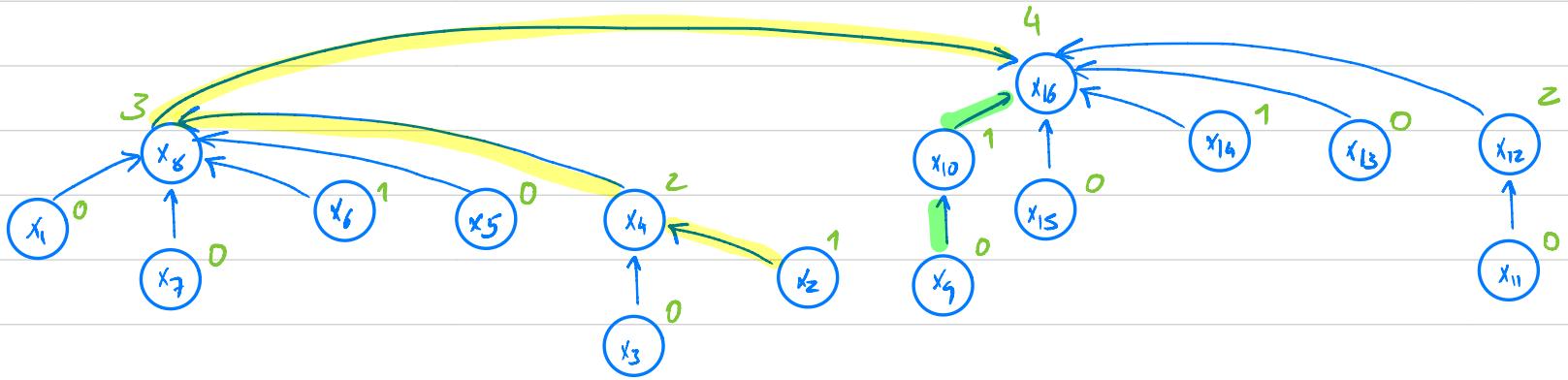
Q1 (21.8 - 1)

findSet( $x_2$ )

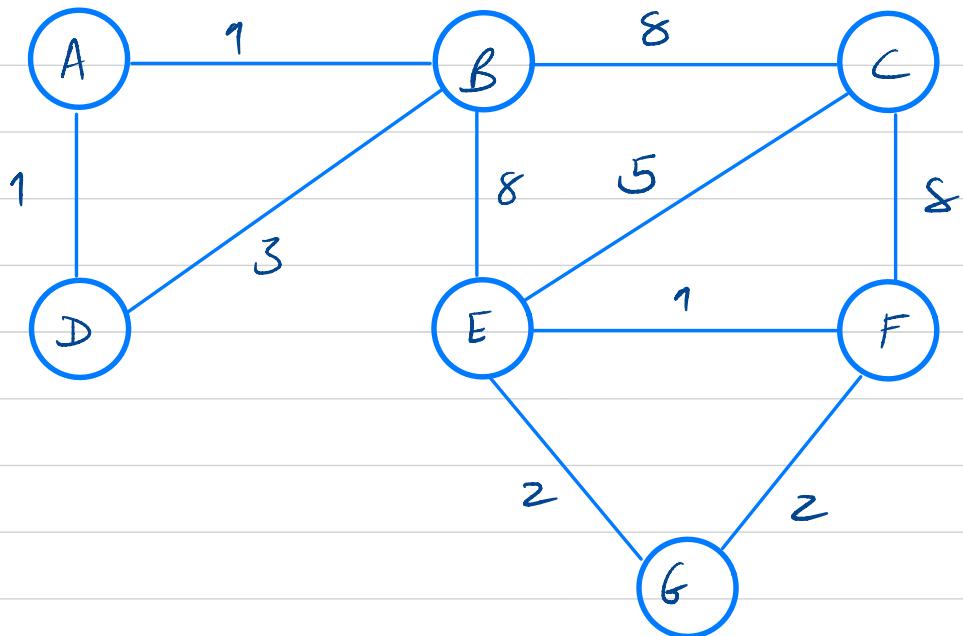
(VI)

findSet( $x_9$ )

(VII)

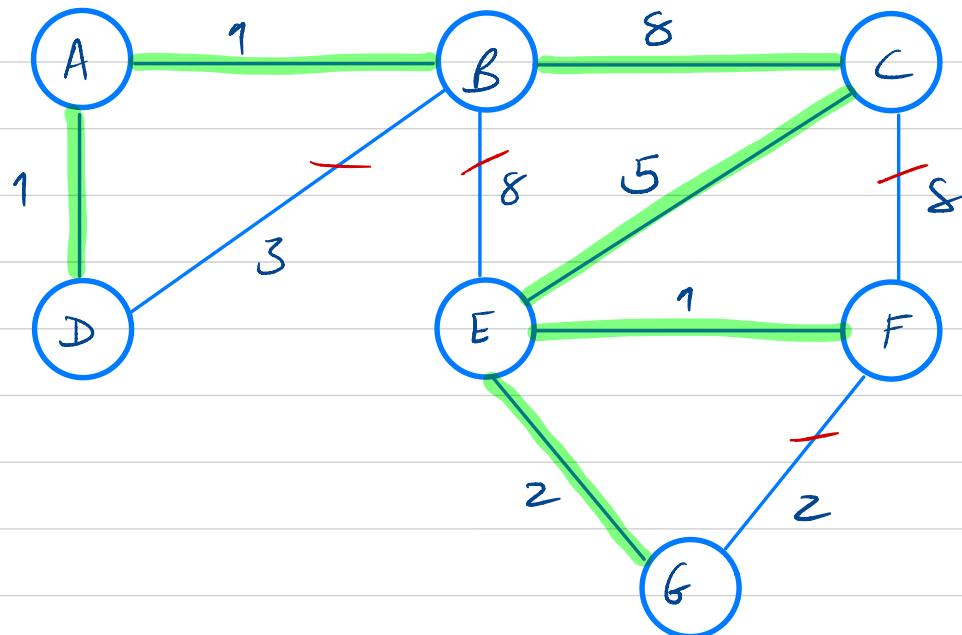


Q2 (T1\_06/07 I.3)



- Kruskal
- Prim

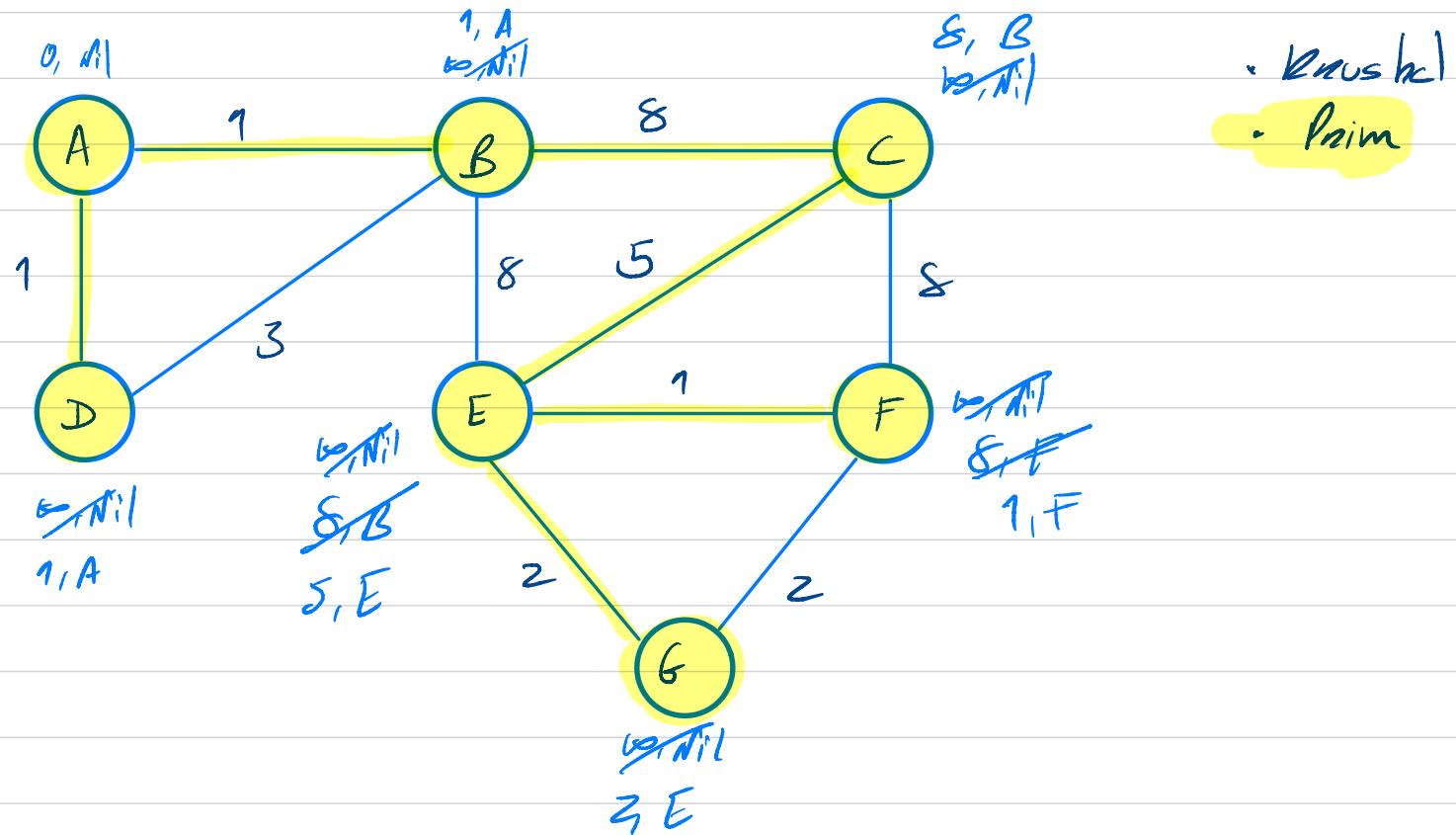
Q2 (T1 06/07 I.3)



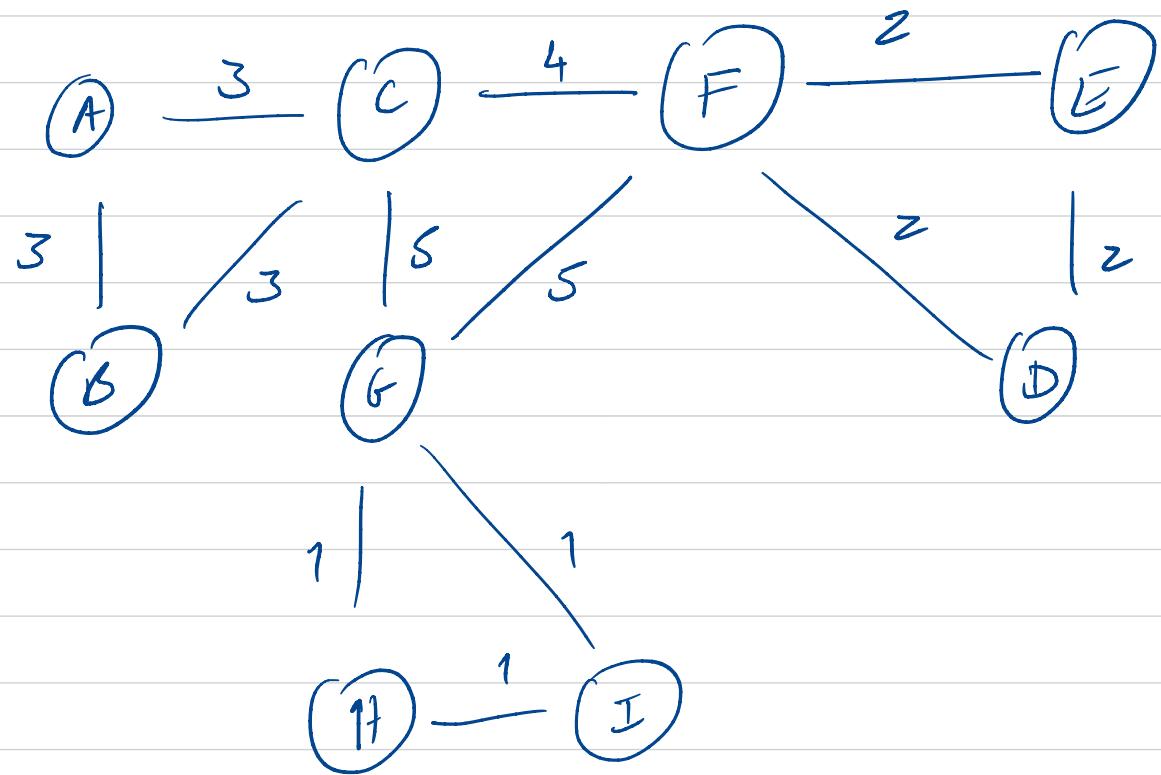
- Kruskal
- Prim

$$\begin{aligned}W(T) &= 1 + 1 + 1 + 2 + 5 + 8 \\&= 18\end{aligned}$$

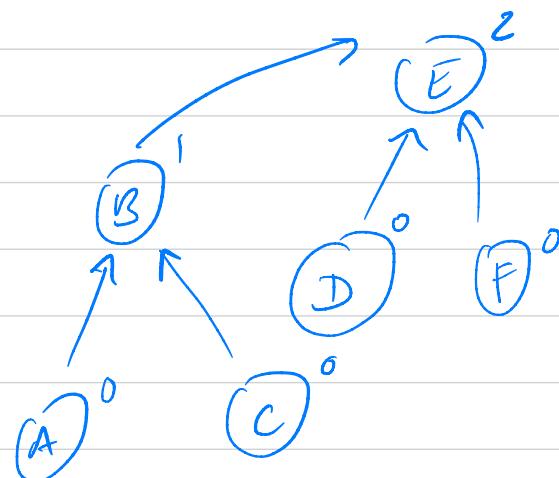
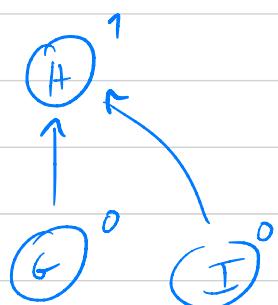
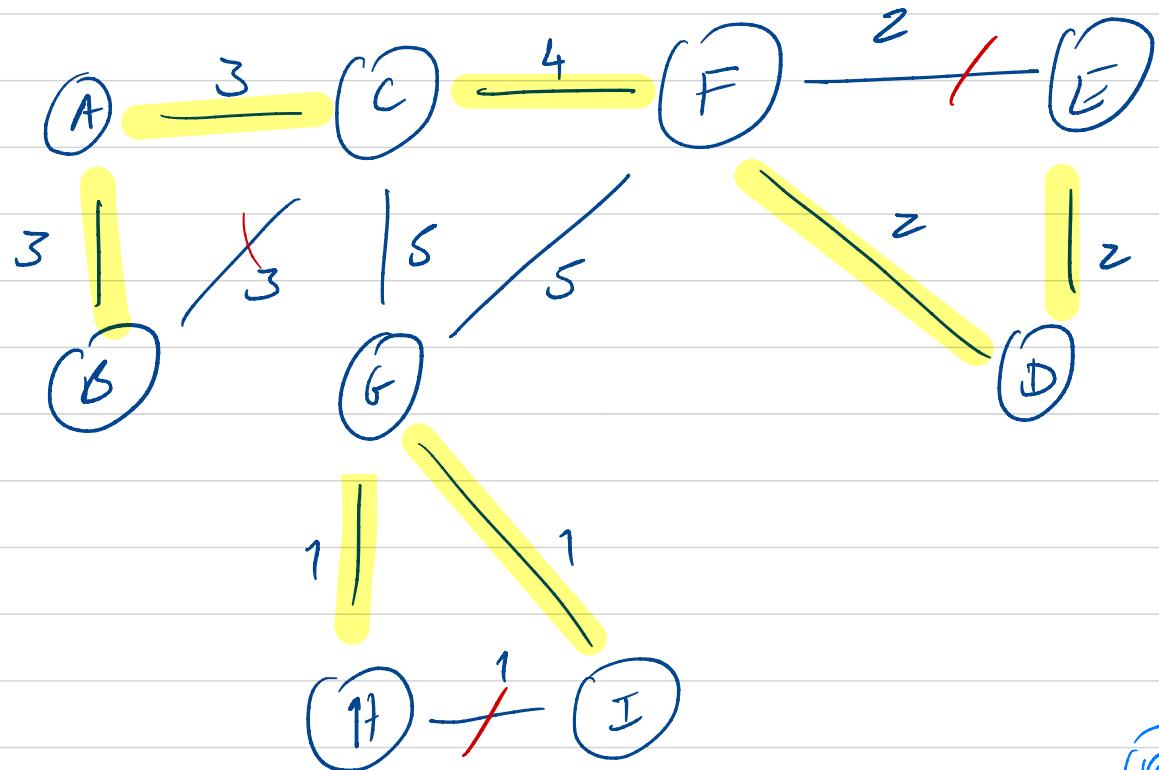
Q2 (T1 06/07 I.3)



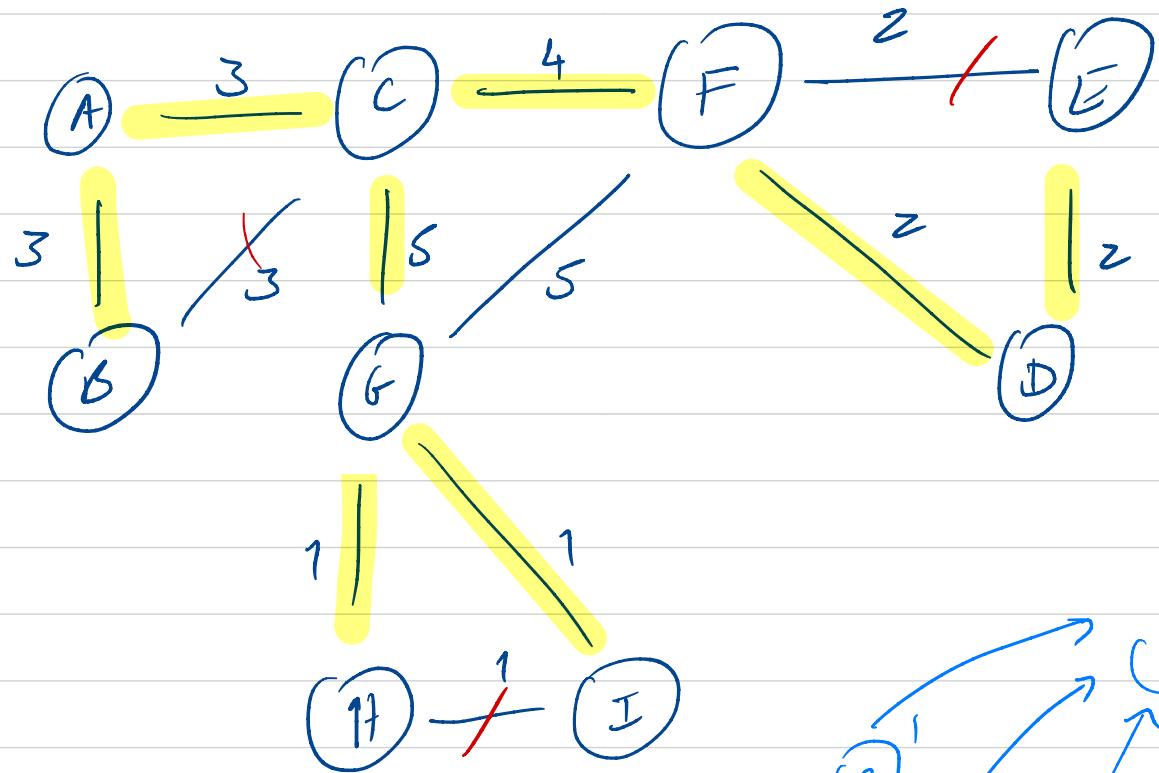
03 (EE 20/21 I.b)



03 (EE 20/21 I.b)



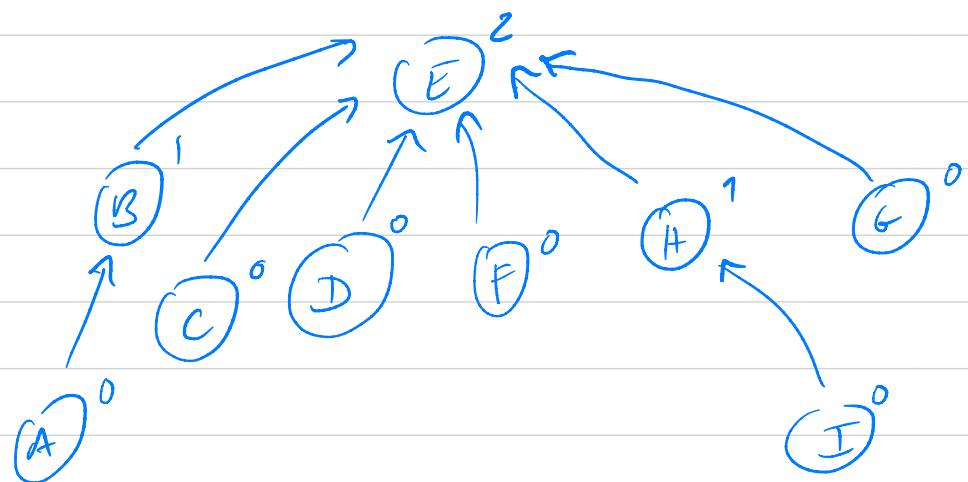
Q3 (EE 20/21 Ib)



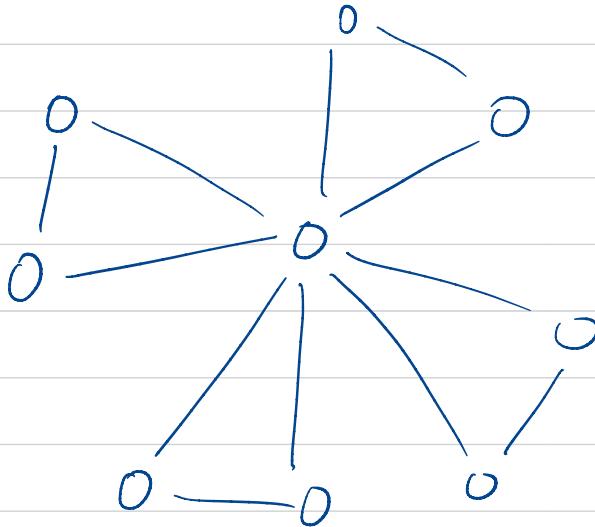
Nº de errores:

$$3^3 \times 2 = 54$$

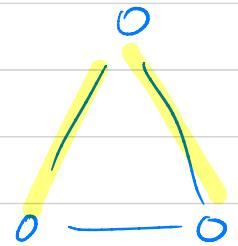
$$\omega(T) = \underline{\underline{21}}$$



Q4 (T1 08/09 II.I)



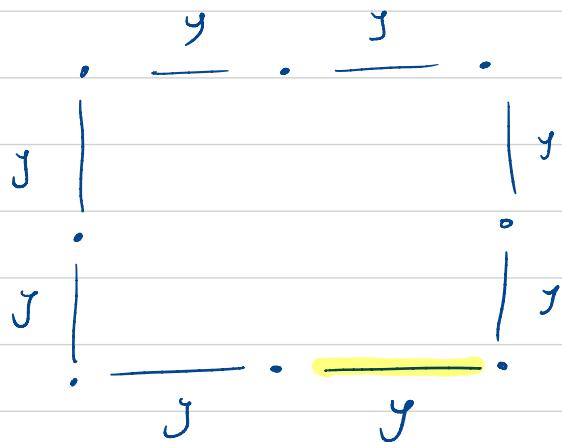
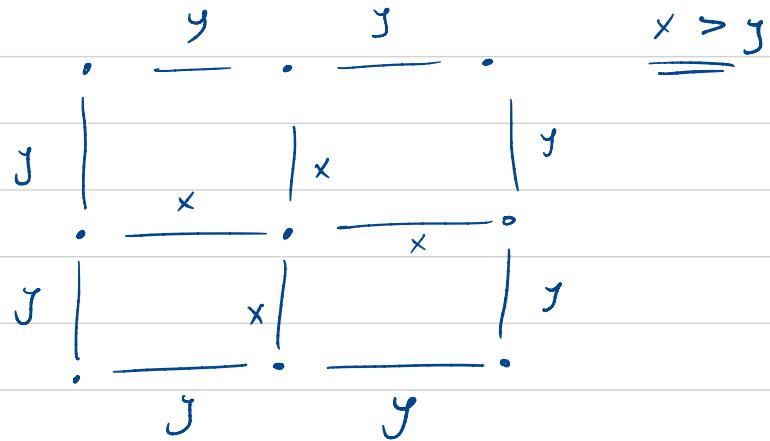
Quantos MSTs?  
3



- Todos os ancos pesam o mesmo

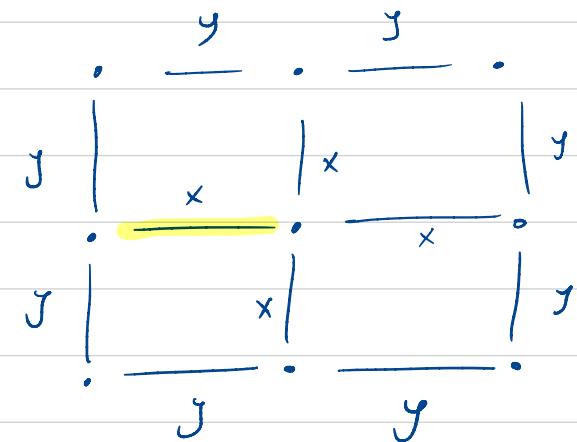
$$\text{Total: } \underline{s^4 = 81}$$

Q5 (R1 08/09 II.1)



8 MTS passíveis

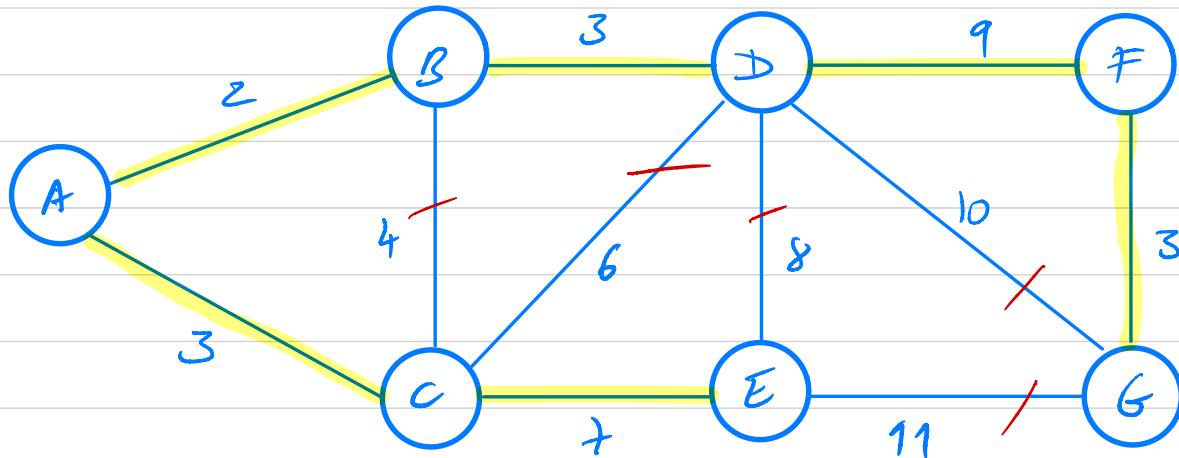
Qual o min  $\tilde{g}$  pode ser mantido?



Nº total de MTS:  $8 \times 4 = 32$

↳ Qual o arco interior  $\tilde{g}$  vai ser mantido?

Q6 (TI\_07/08 - II.1)



- Vma MST
- Peso do MST
- n° de MSTs  $\Rightarrow 1$

$$\begin{aligned}W(T) &= 2 + 3 + 3 + 3 + 9 + 7 \\&= 2 + 9 + 9 + 7 \\&= \underline{\underline{27}}\end{aligned}$$

Q7 (CLRS Ex 21.3-4)

MakeSet(x)

$$x.p = x$$

$$x.rank = 0$$

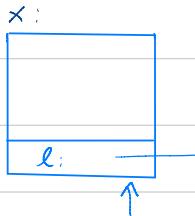


MakeSet(x)

$$x.p = x$$

$$x.rank = 0$$

x.l = x



Union(x, y)

let  $R_x = \text{FindSet}(x)$

let  $R_y = \text{FindSet}(y)$

if ( $R_x == R_y$ ) return

if ( $R_y.rank > R_x.rank$ )

$R_y.p := R_x$

else if ( $R_y.rank > R_x.rank$ )

$R_x.p := R_y$

else

$R_y.p := R_x$

$R_y.rank := R_x.rank + 1$



Union(x, y)

let  $R_x = \text{FindSet}(x)$

let  $R_y = \text{FindSet}(y)$

if ( $R_x == R_y$ ) return

if ( $R_y.rank > R_x.rank$ )

$R_y.p := R_x$

ExtendList( $R_x, R_y$ )

else if ( $R_y.rank > R_x.rank$ )

$R_x.p := R_y$

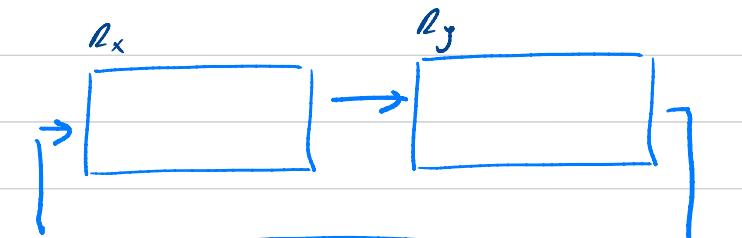
ExtendList( $R_y, R_x$ )

else

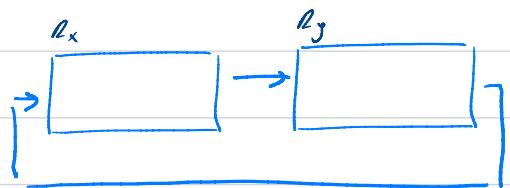
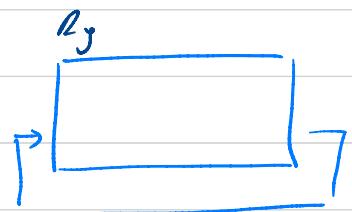
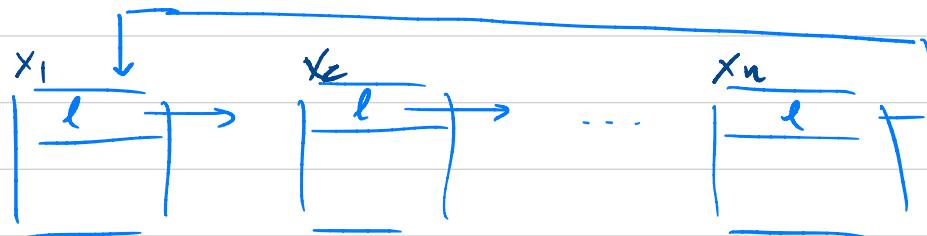
$R_y.p := R_x$

$R_y.rank := R_x.rank + 1$

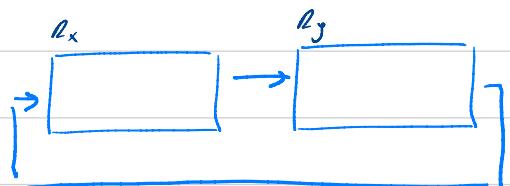
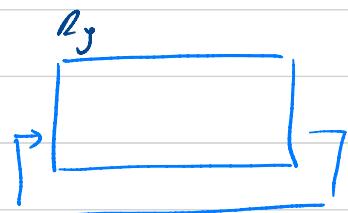
ExtendList( $R_x, R_y$ )



Joining Circular lists:



Joining Circular Lists:



PrintSet( $x$ )

let head =  $x$

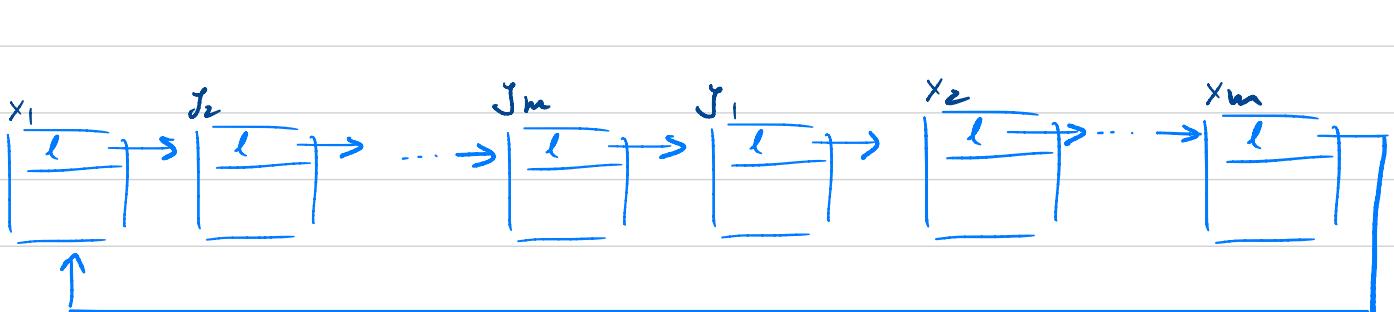
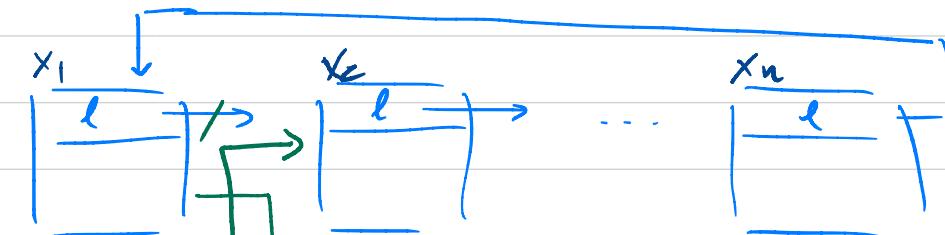
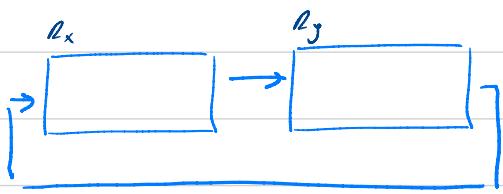
do {

print( $x$ )

$x = x \cdot \ell$

} while ( $x \neq \text{head}$ )

Joining Circular Lists:



`ExtendList(x, y)`

$tmp := x.\ell$

$x.\ell := y.\ell$

$y.\ell := tmp$

Q8 (CLAS 23.2-2)

Prim ( $G, w, r$ )

for each  $v \in G.V$

$$v.key = \infty; \pi[\cdot] = \text{Nil}$$

$$r.key := 0;$$

let  $Q$  be a min-priority queue with content  $G.V$

while  $Q \neq \emptyset$

let  $u = \text{ExtractMin}(Q)$

for each  $v \in G.\text{Adj}[u]$

$$\text{if } (v.key > w(u, v)) \text{ and } v \in Q$$

$$v.key := w(u, v); \pi[v] := u$$

$\Rightarrow$

Prim ( $w, n, r$ )

let  $key[1..n]$  be a new array

let  $\pi[1..n]$  be a new array

let  $done[1..n]$  be a new array

for  $i=1$  to  $n$

$$key[i] := \infty; \pi[i] := \text{Nil}; done[i] := \text{false}$$

$$key[R] := 0$$

for  $j=1$  to  $n-1$

$$\text{let } v^* = \min \{ key[i] \mid done[i] = \text{false} \}$$

Pick  $u$  s.t.  $key[u] = w^*$

$done[u] := \text{true}$

for  $i=1$  to  $n$

$$\text{if } (w[u, i] < key[i]) \text{ and } done[i] = \text{false}$$

$$key[i] := w[u, i]$$

$$\pi[i] := u$$

matrix de pesos

nº de vértices

vértice de origem

# Q8 (CLRS 23.2-2)

Prim ( $G, w, r$ )

for each  $v \in G.V$

$$v.key = \infty; \pi[v] = \text{Nil}$$

$$r.key := 0;$$

let  $Q$  be a min-priority queue with content  $G.V$

while  $Q \neq \emptyset$

let  $u = \text{ExtractMin}(Q)$

for each  $v \in G.\text{Adj}[u]$

$$\text{if } (v.key > w(u, v)) \text{ and } v \in Q$$

$$v.key := w(u, v); \pi[v] := u$$

$\Rightarrow$

Prim ( $W, n, \pi$ )

let  $key[1..n]$  be a new array

let  $\pi[1..n]$  be a new array

let  $done[1..n]$  be a new array

for  $i=1$  to  $n$

$$key[i] := \infty; \pi[i] := \text{Nil}; done[i] := \text{false}$$

$$key[R] := 0$$

for  $j=1$  to  $n-1$

$$O(n) \left( \begin{array}{l} \text{let } v^* = \min \{ key[i] \mid done[i] = \text{false} \} \\ \text{pick } u \text{ s.t. } key[i] = w^* \end{array} \right)$$

$$done[u] := \text{true}$$

for  $i=1$  to  $n$

$$O(n) \left( \begin{array}{l} \text{if } (w[u, i] < key[i] \text{ and } done[i] = \text{false}) \\ \quad key[i] := w[u, i] \\ \quad \pi[i] := u \end{array} \right)$$

$O(n^2)$

Qq (CLRS 23.1-6)

- Se para qualquer corte num grafo pesado existe existe um único arco leve q̄ cruza o corte então o grafo admite uma única MST.

- Provar a implicação
- contra-exemplo para o sentido inverso.

a) Para todo o corte, existe  
um único arco leve q̄  
cruza o corte  
(+)



O grafo admite uma  
única MST T.

- Sejam  $T_1$  e  $T_2$  duas quaisquer MSTs de  $G$ .  
Vamos provar que  $T_1 = T_2$  dada (+).

- Provar que  $T_1 = T_2$  corresponde a provar que:

$$\forall u, v \in V. (u, v) \in T_1 \Leftrightarrow (u, v) \in T_2$$



Como a prova é simétrica basta provar um sentido  
da implicação.

23.1 - 6 Queremos provar que:  $\forall m, o \in V. (m, o) \in T_1 \Rightarrow (m, o) \in T_2$

- Tome-se qualquer arco  $(m, o)$  em  $T_1$ .

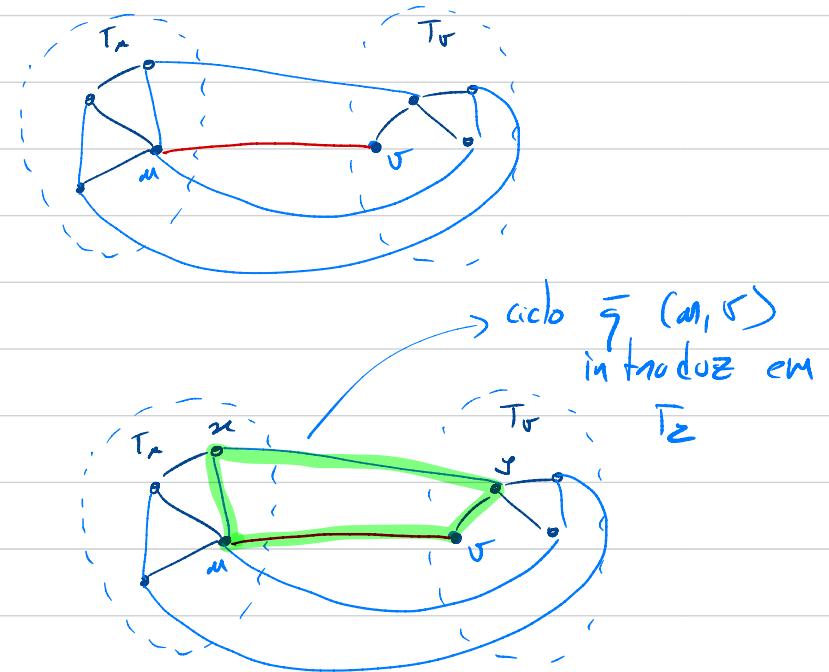
Se  $(m, o) \notin T_2$ , não há nada a provar. Admitimos, portanto, que  $(m, o) \notin T_2$ .

A remoção do arco  $(m, o)$  induz um corte  $(T_L, T_R)$ , onde  $T_L$  contém os vértices atingíveis a partir de  $m$  usando arcos em  $T_1 \setminus \{(m, o)\}$  e  $T_R$  contém os vértices atingíveis a partir de  $o$ .

- Como  $T_1$  é HST concluímos  $\bar{g} (m, o)$  é um arco leve  $\bar{g}$  cruza o corte.

- Como  $(m, o) \notin T_2$ , a sua inclusão em  $T_2$  dá origem a um caminho circular,  $P_C$ , que cruza o corte duas vezes:

- uma vez pelo arco  $(m, o)$  e outra por um arco de  $T_2$ ; seja  $(x, y)$  esse arco



- Como  $(m, o)$  é leve, usamos a hipótese pl para concluir que:  
 $w(m, o) < w(x, y)$ .

$$\text{Segue } \bar{g}: T_2' = (T_2 \setminus \{(x, y)\}) \cup \{(o, y)\}$$

é árvore abrangente e  $w(T_2') < w(T_2)$ . Concluímos  $\bar{g} T_2$  não é HST, contradizendo a contradição.

