Práctica 12

Ex 29.3-5

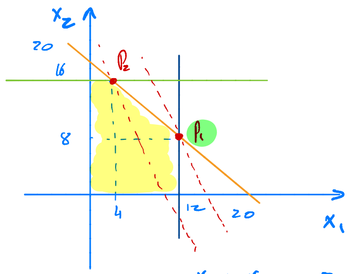
$\max \; 18 x_1 + 12.5 x_2$

$x_1 + x_2 \leq 20$ (I)

$x_1 \leq 12$ (II)

$x_2 \leq 16$ (III)

$x_1, x_2 \geq 0$



$x_1 + x_2 = 20$

$x_2 = 20 - x_1$

$P_1: \; 18 \times 12 + 12.5 \times 8 = 316$

$P_2: \; 18 \times 4 + 12.5 \times 16 = 272$

$\nabla f = (18, \; 12.5)$

• Declive da recta perpendicular ao gradiente: $m = -\frac{18}{12.5} \approx -1.44$

(0)

$$\begin{aligned} z &= 18 x_1 + 12.5 x_2 \\ s_1 &= 20 - x_1 - x_2 \\ s_2 &= 12 - x_1 \\ s_3 &= 16 - x_2 \end{aligned}$$

$\times 18$

$20/1 = 20$

$12/1 = 12$

(1)

$$\begin{aligned} z &= 216 + 12.5 x_2 - 18 s_2 \\ s_1 &= 8 - x_2 + s_2 \\ x_1 &= 12 + 0 - s_2 \\ s_3 &= 16 - x_2 + 0 \end{aligned}$$

12.5

$8/1 = 8$

$16/1 = 16$

(2)

$$\begin{aligned} z &= 316 - 5.5 \cdot s_2 - 12.5 \cdot s_1 \\ x_2 &= 8 + s_2 - s_1 \\ x_1 &= 12 - s_2 + 0 \\ s_3 &= 8 - s_2 + s_1 \end{aligned}$$

Ex 29.4-1

$\max \; c^T x$, $Ax \leq b$, $x \geq 0$ $\rightsquigarrow$ $\min \; b^T y$, $y^T A \geq c^T$, $y \geq 0$

$$\max \begin{bmatrix} 18 & 12.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 20 \\ 12 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\min \begin{bmatrix} 20 & 12 & 16 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \leq \begin{bmatrix} 18 & 12.5 \end{bmatrix}$$

$y_1, y_2, y_3 \geq 0$

$\Downarrow$

$$\boxed{\begin{aligned} &\min \; 20 y_1 + 12 y_2 + 16 y_3 \\ &y_1 + y_2 \leq 18 \\ &y_1 + y_3 \leq 12.5 \\ &y_1, y_2, y_3 \geq 0 \end{aligned}}$$

TZ 08/09 I.2 :

$$\max \quad x_1 - x_2 + x_3$$
$$2x_1 - x_2 + 2x_3 \leq 6$$
$$2x_1 - 2x_2 + x_3 \leq -2$$
$$-x_1 + 2x_2 - 2x_3 \leq -1$$
$$x_1, x_2, x_3 \geq 0$$

$$\max \quad -x_0$$
$$2x_1 - x_2 + 2x_3 - x_0 \leq 6$$
$$2x_1 - 2x_2 + x_3 - x_0 \leq -2$$
$$-x_1 + 2x_2 - 2x_3 - x_0 \leq -1$$
$$x_1, x_2, x_3 \geq 0$$

(1)

$$z = -x_0$$
$$s_1 = 6 - 2x_1 + x_2 - 2x_3 + x_0$$
$$s_2 = -2 - 2x_1 + 2x_2 - x_3 + x_0$$
$$s_3 = -1 + x_1 - 2x_2 + 2x_3 + x_0$$

(2)

$$z = -2 - 2x_1 + 2x_2 - x_3 - s_2$$
$$s_1 = 8 \quad 0 \quad - x_2 \quad - x_3 \quad + s_2 \qquad 8/1 = 8$$
$$x_0 = 2 \quad 2x_1 \quad -2x_2 \quad x_3 \quad + s_2 \qquad 2/2 = 1$$
$$s_3 = 1 \quad 3x_1 \quad -4x_2 \quad +3x_3 \quad + s_2 \qquad 1/4 = 1/4$$

$\times \frac{1}{2}$

(3)

$$z = -3/2 \quad -1/2\, x_1 \quad +1/2\, x_3 \quad -1/2\, s_2 \quad -1/2\, s_3$$
$$s_1 = 31/4 \quad -3/4\, x_1 \quad -7/4\, x_3 \quad 3/4\, s_2 \quad +1/4\, s_3 \qquad -\frac{31}{4} \times \frac{4}{7} = -\frac{31}{7} > 4$$
$$x_0 = 3/2 \quad 1/2\, x_1 \quad -1/2\, x_3 \quad 1/2\, s_2 \quad +1/2\, s_3 \qquad \cancel{3/2} \times \cancel{2} = 3$$
$$x_2 = 1/4 \quad 3/4\, x_1 \quad 5/4\, x_3 \quad 1/4\, s_2 \quad -1/4\, s_3$$

$\times -\frac{7}{2}$

$\times 3/2$

(4)

$$\begin{array}{lllllll} & & & x_1 & s_2 & s_3 & \\ z & = & 0 & 0 & 0 & 0 & -x_0 \\ s_1 & = & 5/2 & -5/2\, x_1 & -s_2 & -3/2\, s_3 & +7/2\, x_0 \\ x_3 & = & 3 & x_1 & s_2 & s_3 & -2x_0 \\ x_2 & = & 5/2 & 3/2\, x_1 & s_2 & 1/2\, s_3 & -3/2\, x_0 \end{array}$$

$$(x_1^0, x_2^0, x_3^0) = (0, 5/2, 3)$$

$$z = x_1 - x_2 + x_3$$
$$= x_1 - (5/2 + 3/2\, x_1 + \cancel{s_2} + 1/2\, s_3) + (3 + x_1 + \cancel{s_2} + s_3)$$
$$= 1/2 + 1/2\, x_1 + 1/2\, s_3$$

$$z = 1/2 \quad +1/2\, x_1 \quad +1/2\, s_3$$
$$s_1 = 5/2 \quad -5/2\, x_1 \quad -s_2 \quad -3/2\, s_3$$
$$x_3 = 3 \quad x_1 \quad s_2 \quad s_3$$
$$x_2 = 5/2 \quad 3/2\, x_1 \quad s_2 \quad 1/2\, s_3$$

A solução não é óptima

Rz 16/17 I.c

$$\max \; -4x_1 + 5x_2 + 3x_3$$
$$-x_1 + x_2 + x_3 \leq -2$$
$$-3x_1 + 4x_2 + 4x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

$\wr$

$$\max \; -x_0$$
$$-x_1 + x_2 + x_3 - x_0 \leq -2$$
$$-3x_1 + 4x_2 + 4x_3 - x_0 \leq 1$$
$$x_1, x_2, x_3, x_0 \geq 0$$

$\rightsquigarrow$

(I)

$$z = 0 - x_0$$
$$s_1 = -2 + x_1 - x_2 - x_3 + x_0$$
$$s_2 = 1 + 3x_1 - 4x_2 - 4x_3 + x_0$$

$\wr$

(II)

$$z = -2 + x_1 - x_2 - x_3 - s_1$$
$$x_0 = 2 - x_1 + x_2 + x_3 + s_1$$
$$x_2 \; \{ \; s_2 = 3 + 2x_1 - 3x_2 - 3x_3 + s_1$$

(III)

			x_2	x_3	s_1	x_0
z	=	0	0	0	0	$-x_0$
x_1	=	2	$+x_2$	$+x_3$	$+s_1$	$-x_0$
s_2	=	7	$-x_2$	$-x_3$	$3s_1$	$-2x_0$

$$f = -4x_1 + 5x_2 + 3x_3 = -4(2 + x_2 + x_3 + s_1) + 5x_2 + 3x_3$$
$$= -8 + x_2 - x_3 - 4s_1$$

$$(x_1^0, x_2^0, x_3^0) = (2, 0, 0) \qquad f^0 = -8$$

(I)

$$z = -8 + x_2 - x_3 - 4s_1$$
$$x_1 = 2 + x_2 + x_3 + s_1$$
$$s_2 = 7 - x_2 - x_3 \; 3s_1$$

$$(x_1^*, x_2^*, x_3^*) = (9, 7, 0)$$

(II)

$$z = -1 - 2x_3 - s_1 - s_2$$
$$x_1 = 9 \quad 0 \quad 4s_1 \quad -s_2$$
$$x_2 = 7 \quad -x_3 \quad 3s_1 \quad -s_2$$

$$f^* = -1$$

- <u>Problema dual:</u>

$$\max \; c^T x \quad \rightsquigarrow \quad \min \; b^T y$$

$$Ax \leq b \qquad\qquad y^T A \geq c$$

$$x \geq 0 \qquad\qquad y \geq 0$$

$$\max \; -4x_1 + 5x_2 + 3x_3$$

$$-x_1 + x_2 + x_3 \leq -2$$

$$-3x_1 + 4x_2 + 4x_3 \leq 1$$

$$x_1, x_2, x_3 \geq 0$$

$$\Downarrow$$

$$\min \; -2y_1 + y_2$$

$$-y_1 - 3y_2 \geq -4$$

$$y_1 + 4y_2 \geq 5$$

$$y_1 + 4y_2 \geq 3$$

$$y_1, y_2 \geq 0$$

- Do Teorema da Dualidade Forte concluímos q̄ a solução do problema dual é -1.

- A solução do problema dual usa as restrições principais do problema. Queremos calcular y_1^* e y_2^* tais que: $-2y_1^* \times y_2^* = -1$
  É fácil confirmar q̄ $(1, 1)$ satisfaz as restrições.

R2 08/09 I.2

$$\max \; -2x_1 - x_2 - 2x_3$$
$$x_1 + 5x_2 + x_3 \le 100$$
$$x_1 + 2x_2 + x_3 \ge 50$$
$$2x_1 + 4x_2 + x_3 \ge 80$$
$$x_1, x_2, x_3 \ge 0$$

$\rightsquigarrow$

$$\max \; -2x_1 - x_2 - 2x_3$$
$$x_1 + 5x_2 + x_3 \le 100$$
$$-x_1 - 2x_2 - x_3 \le -50$$
$$-2x_1 - 4x_2 - x_3 \le -80$$
$$x_1, x_2, x_3 \ge 0$$

$$\min \; -x_0$$
$$x_1 + 5x_2 + x_3 - x_0 \le 100$$
$$-x_1 - 2x_2 - x_3 - x_0 \le -50$$
$$-2x_1 - 4x_2 - x_3 - x_0 \le -80$$
$$x_1, x_2, x_3, x_0 \ge 0$$

$$z = -x_0$$
$$s_1 = 100 - x_1 - 5x_2 - x_3 + x_0$$
$$s_2 = -50 + x_1 + 2x_2 + x_3 + x_0$$
$$s_3 = -80 + 2x_1 + 4x_2 + x_3 + x_0$$

$$z = -80 + 2x_1 + 4x_2 + x_3 - s_3$$
$$s_1 = 180 - 3x_1 - 9x_2 - 2x_3 + s_3 \qquad 180/9 = 20$$
$$s_2 = 30 - x_1 - 2x_2 \quad 0 \quad + s_3 \qquad 30/2 = 15$$
$$x_0 = 80 - 2x_1 - 4x_2 - x_3 + s_3 \qquad 80/4 = 20$$

$$z = -20 \quad 0 \quad + x_3 + s_3 - 2s_2$$
$$s_1 = 45 \quad 3/2\, x_1 - 2x_3 - 7/2\, s_3 + 9/2\, s_2 \qquad 45/2 = 22.5$$
$$x_2 = 15 - 1/2\, x_1 \quad 0 \quad 1/2\, s_3 - 1/2\, s_2$$
$$x_0 = 20 \quad 0 \quad - x_3 - s_3 + 2s_2 \qquad 20/1 = 20$$

		x_1	s_3	s_2	x_0
$z =$	0	0	0	0	$-x_0$
$s_1 =$	5	$3/2\, x_1$	$-3/2\, s_3$	$+1/2\, s_2$	$+2x_0$
$x_2 =$	15	$-1/2\, x_1$	$1/2\, s_3$	$-1/2\, s_2$	0
$x_3 =$	20	0	$-s_3$	$+2s_2$	$-x_0$

• Solução exequível inicial:
$$(x_1^0, x_2^0, x_3^0) = (0, 15, 20)$$

$$f = -2x_1 - x_2 - 2x_3$$
$$= -2x_1 - (15 - 1/2\, x_1 + 1/2\, s_3 - 1/2\, s_2) - 2(20 - s_3 + 2s_2)$$
$$= -55 - 3/2\, x_1 + 3/2\, s_3 - 7/2\, s_2$$

$$
\begin{array}{lclllll}
z & = & -55 & -3/2\, x_1 & +3/2\, s_3 & -7/2\, s_2 & \\
s_1 & = & 5 & 3/2\, x_1 & -3/2\, s_3 & +1/2\, s_2 & 5 \times \frac{2}{3} = 10/3 \\
x_2 & = & 15 & -1/2\, x_1 & 1/2\, s_3 & -1/2\, s_2 & \\
x_3 & = & 20 & 0 & -s_3 & +2\, s_2 & 20/1 = 20
\end{array}
$$

$\times \frac{1}{3}$

$\times -\frac{2}{3}$

$$
\begin{array}{lcllll}
z & = & -50 & 0 & -3 s_2 & -s_1 \\
s_3 & = & 10/3 & x_1 & 1/3\, s_2 & -2/3\, s_1 \\
x_2 & = & 50/3 & 0 & -1/3\, s_2 & -1/3\, s_1 \\
x_3 & = & 50/3 & -x_1 & 5/3\, s_2 & +2/3\, s_1
\end{array}
$$

$$f^* = -50$$

$$(x_1^*, x_2^*, x_3^*) = (0, 50/3, 50/3)$$