

Prática 11

(1)

T2 08/09 I.3

| tipo | Assembagem ( $A_i$ ) | Polimento ( $P_i$ ) | Empacotamento ( $E_i$ ) | Lucro ( $L_i$ ) |
|------|----------------------|---------------------|-------------------------|-----------------|
| 1    | 2                    | 3                   | 2                       | 1.5             |
| 2    | 4                    | 2                   | 3                       | 2.5             |
| 3    | 3                    | 3                   | 2                       | 3.0             |
| 4    | 7                    | 4                   | 5                       | 4.0             |

Tempo total: 10.000 mins  
 assembagem  
 50.000 mins  
 polimento  
 60.000 mins  
 empacotamento

$x_1, x_2, x_3, x_4$   
 $x_i \rightarrow$  quantidade produzida do produto i

$$\max \sum_{i=1}^4 L_i \cdot x_i$$

$$\text{st } \sum_{i=1}^4 A_i \cdot x_i \leq 10.000$$

$$\sum_{i=1}^4 P_i \cdot x_i \leq 50.000$$

$$\sum_{i=1}^4 E_i \cdot x_i \leq 60.000$$

R2 08/09 I.3

Quantidade mínima por nutriente por kg

| Nutriente | A  | B  | C  | D |
|-----------|----|----|----|---|
| qt        | 90 | 50 | 20 | 2 |

| Ingrediente | A   | B   | C  | D  | Custo |
|-------------|-----|-----|----|----|-------|
| I1 (g/kg)   | 100 | 80  | 40 | 10 | 40    |
| I2 (g/kg)   | 200 | 150 | 20 | 0  | 60    |
| E           | 0   | 0   | 0  | 0  | 0     |

$$\min 40x_1 + 60x_2$$

$$100x_1 + 200x_2 \geq 90$$

$$80x_1 + 150x_2 \geq 50$$

$$40x_1 + 20x_2 \geq 20$$

$$10x_1 \geq 2$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Variáveis do problema:

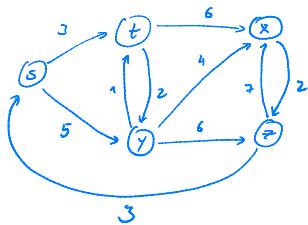
$x_1$  - quantidade de I1 em kg

$x_2$  - " " I2 em kg

$x_3$  - " " E em kg

(2)

Ex 29.2-2

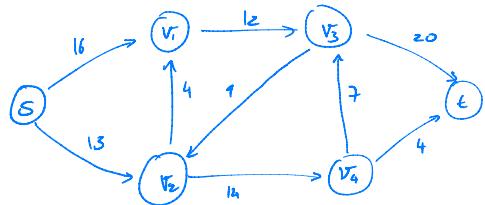
• Caminho mais curto entre  $s \circ g$ 

$$\begin{array}{ll} \min & d_g \\ \end{array}$$

$$\begin{array}{l} d_s = 0 \\ d_t \leq d_s + 3 \\ d_y \leq d_s + 5 \\ d_t \leq d_y + 1 \\ d_y \leq d_t + 2 \\ d_x \leq d_t + 6 \\ d_x \leq d_y + 4 \end{array}$$

$$\begin{array}{l} d_x \leq d_z + 7 \\ d_z \leq d_x + 2 \\ d_z \leq d_y + 6 \\ d_s \leq d_z + 3 \\ d_x, d_s, d_t, d_y, d_z \geq 0 \end{array}$$

Ex 29.2-4



$$\max f_{s1} + f_{s2}$$

$$\begin{array}{ll} 0 \leq f_{s1} \leq 16 & 0 \leq f_{z4} \leq 14 \\ 0 \leq f_{s2} \leq 13 & 0 \leq f_{s2} \leq 9 \\ 0 \leq f_{13} \leq 12 & 0 \leq f_{43} \leq 7 \\ 0 \leq f_{21} \leq 4 & 0 \leq f_{3t} \leq 20 \\ & 0 \leq f_{46} \leq 4 \end{array}$$

$$\begin{array}{l} f_{s1} + f_{z1} = f_{13} \\ f_{s2} + f_{z2} = f_{21} + f_{43} \\ f_{13} + f_{43} = f_{3t} + f_{z2} \\ f_{z4} = f_{43} + f_{4t} \end{array}$$

Ex 29.1-4

$$\begin{array}{ll} \min & 2x_1 + 7x_2 + x_3 \\ \text{st} & x_1 - x_3 = 7 \\ & 3x_1 + x_2 \geq 24 \\ & x_2 \geq 0 \\ & x_3 \leq 0 \end{array}$$

$$\begin{array}{l} x_1 = x_1^+ - x_1^- \\ x_3 = -x_3^+ \end{array}$$

$$\begin{array}{ll} \min & z(x_1^+ - x_1^-) + 7x_2 - x_3^+ \\ & x_1^+ - x_1^- + x_3^+ = 7 \\ & 3(x_1^+ - x_1^-) + x_2 \geq 24 \\ & x_2, x_1^+, x_1^-, x_3^+ \geq 0 \end{array}$$

↓

$$\begin{array}{l} \max -2x_1^+ + 2x_1^- - 7x_2 + x_3^+ \\ x_1^+ - x_1^- + x_3^+ \leq 7 \\ -x_1^+ + x_1^- - x_3^+ \leq -7 \\ -3x_1^+ + 3x_1^- - x_2 \leq -24 \\ x_1^+, x_1^-, x_2, x_3^+ \geq 0 \end{array}$$

$$\left\{ \begin{array}{l} \min z x_1^+ - 2x_1^- + 7x_2 - x_3^+ \\ x_1^+ - x_1^- + x_3^+ \leq 7 \\ -x_1^+ + x_1^- - x_3^+ \leq -7 \\ -3x_1^+ + 3x_1^- - x_2 \leq -24 \\ x_1^+, x_1^-, x_2, x_3^+ \geq 0 \end{array} \right.$$

(3)

T2 08/09 I.1

$$\begin{array}{ll}
 \min & -x_1 - 2x_2 + x_3 \\
 \text{s.t.} & 2x_1 + x_2 + x_3 \leq 14 \\
 & -4x_1 - 2x_2 - 3x_3 \geq -28 \\
 & 2x_1 + 5x_2 + 5x_3 \leq 30 \\
 & x_1, x_2, x_3 \geq 0
 \end{array}
 \quad
 \begin{array}{ll}
 \max & x_1 + 2x_2 - x_3 \\
 \text{s.t.} & 2x_1 + x_2 + x_3 \leq 14 \\
 & 4x_1 + 2x_2 + 3x_3 \leq 28 \\
 & 2x_1 + 5x_2 + 5x_3 \leq 30 \\
 & x_1, x_2, x_3 \geq 0
 \end{array}$$

$$z = x_1 + \cancel{2x_2} - x_3$$

$$s_1 = 14 - 2x_1 - x_2 - x_3 \quad | :1 = 14$$

$$s_2 = 28 - 4x_1 - 2x_2 - 3x_3 \quad | :2 = 14$$

$$\textcircled{s}_3 = 30 - 2x_1 - 5x_2 - 5x_3 \quad | :5 = 6$$

 $\downarrow$ 

$$z = 12 + \cancel{1/5 \cdot x_1} - 3x_3 - 2/5 \cdot s_3$$

$$\textcircled{s}_1 = 8 - 8/5 \cdot x_1 + 1/5 \cdot s_3 \quad \rightarrow 8 \times \frac{5}{8} = 5$$

$$s_2 = 16 - 16/5 \cdot x_1 - x_3 + 2/5 \cdot s_3 \quad \rightarrow 16 \times \frac{5}{16} = 5$$

$$x_2 = 6 - 2/5 \cdot x_1 - x_3 - 1/5 \cdot s_3 \quad \rightarrow 6 \times \frac{5}{2} = 3 \times 5 = 15$$

 $\downarrow$ 

$$z = 13 - 3x_3 - 15/40 \cdot s_3 - 1/8 \cdot s_1$$

$$x_1 = \textcircled{5} \quad 0 + 1/8 \cdot s_3 - 5/8 \cdot s_1$$

$$s_2 = 0 - x_3 + 0 + 2s_1$$

$$x_2 = \textcircled{4} - x_3 - 1/4 \cdot s_3 + 1/4 \cdot s_1 \quad \text{Optimal: } -13$$

$$\text{Sol: } (5, 4, 0)$$

RZ 08/09 I.1

$$\min 100 + x_1 - 5x_2 + 2x_3$$

$$\text{s.t. } x_1 + 3x_2 \leq 50$$

$$x_1 + 2x_2 + x_3 \leq 50$$

$$x_1 + x_3 \leq 72$$

$$x_1, x_2, x_3 \geq 0$$

$$\max -x_1 + 5x_2 - 2x_3$$

$$x_1 + 3x_2 \leq 50$$

$$x_1 + 2x_2 + x_3 \leq 50$$

$$x_1 + x_3 \leq 72$$

$$x_1, x_2, x_3 \geq 0$$

$$\begin{aligned} z &= -x_1 + 5x_2 - 2x_3 \\ \frac{5}{3}s_1 &= 50 - x_1 - 3x_2 \quad 50/3 < 17 \\ -\frac{2}{3}s_3 &= 50 - x_1 - 2x_2 - x_3 \quad 50/2 = 25 \\ s_3 &= 72 - x_1 - x_3 \end{aligned}$$

$$z = 250/3 - 8/3 x_1 - 2x_3 - 5/3 s_1$$

$$x_2 = 50/3 - 1/3 x_1 - 1/3 s_1$$

$$s_2 = 50/3 - 1/3 x_1 - x_3 + 2/3 s_1$$

$$s_3 = 72 - x_1 - x_3$$

$$\bullet \quad 100 - 250/3 = \frac{300 - 250}{3} = \frac{50}{3}$$

$$(x_1^*, x_2^*, x_3^*) = (0, 50/3, 0)$$