Prática 11

T2 08/09 I.3

Tipo	Assemblagem (A_i)	Polimento (P_i)	Empacotamento (E_i)	Lucro (L_i)
1	2	3	2	1.5
2	4	2	3	2.5
3	3	3	2	3.0
4	7	4	5	4.0

Tempo kkl: 10.000 mins assemblagem
50.000 mins polimento
60.000 mins empacotamento

x_1, x_2, x_3, x_4
$x_i \Rightarrow$ quantidade produzida do produto i

$$\max \sum_{i=1}^{4} L_i \cdot x_i$$

st

$$\sum_{i=1}^{4} A_i \cdot x_i \leq 10.000$$

$$\sum_{i=1}^{4} P_i \cdot x_i \leq 50.000$$

$$\sum_{i=1}^{4} E_i \cdot x_i \leq 60.000$$

R2 08/09 I.3

Quantidade mínima por nutriente por kg

Nutriente	A	B	C	D
Qt	90	50	20	2

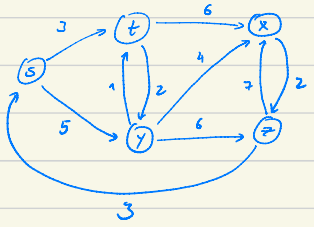
Ingrediente	A	B	C	D	Custo
I1 (g/kg)	100	80	40	10	40
I2 (g/kg)	200	150	20	0	60
E	0	0	0	0	0

Variáveis do problema:
x_1 - quantidade de I1 por kg
x_2 - " " I2 por kg
x_3 - " " E por kg

$$\min \quad 40 \times x_1 + 60 \times x_2$$
$$100 \times x_1 + 200 \times x_2 \geq 90$$
$$80 \times x_1 + 150 \times x_2 \geq 50$$
$$40 \times x_1 + 20 \times x_2 \geq 20$$
$$10 \times x_1 \geq 2$$
$$x_1 + x_2 + x_3 = 1$$
$$x_1, x_2, x_3 \geq 0$$

Ex 29.2-2

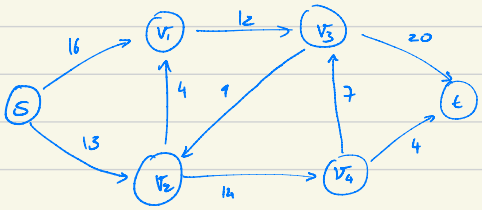
- Caminho mais curto entre s e y



$\max\ d_y$

$d_s = 0$
$d_t \le d_s + 3$
$d_y \le d_s + 5$
$d_t \le d_y + 1$
$d_y \le d_t + 2$
$d_x \le d_t + 6$
$d_x \le d_y + 4$
$d_x \le d_z + 7$
$d_z \le d_x + 2$
$d_z \le d_y + 6$
$d_s \le d_z + 3$
$d_x, d_s, d_t, d_y, d_z \ge 0$

Ex 29.2-4



$\max\ f_{s1} + f_{s2}$

$0 \le f_{s1} \le 16$
$0 \le f_{s2} \le 13$
$0 \le f_{13} \le 12$
$0 \le f_{21} \le 4$
$0 \le f_{24} \le 14$
$0 \le f_{32} \le 9$
$0 \le f_{43} \le 7$
$0 \le f_{3t} \le 20$
$0 \le f_{4t} \le 4$

$f_{s1} + f_{21} = f_{13}$
$f_{s2} + f_{32} = f_{21} + f_{24}$
$f_{13} + f_{45} = f_{3t} + f_{32}$
$f_{24} = f_{43} + f_{4t}$

Ex 29.1-4

$\min\ 2x_1 + 7x_2 + x_3$
s.t. $x_1 - x_3 = 7$
$3x_1 + x_2 \ge 24$
$x_2 \ge 0$
$x_3 \le 0$

$x_1 = x_1^+ - x_1^-$
$x_3 = -x_3^+$

$\min\ 2(x_1^+ - x_1^-) + 7x_2 - x_3^+$
$x_1^+ - x_1^- + x_3^+ = 7$
$3(x_1^+ - x_1^-) + x_2 \ge 24$
$x_2, x_1^+, x_1^-, x_3^+ \ge 0$

$\Downarrow$

$\min\ 2x_1^+ - 2x_1^- + 7x_2 - x_3^+$
$x_1^+ - x_1^- + x_3^+ \le 7$
$-x_1^+ + x_1^- - x_3^+ \le -7$
$-3x_1^+ + 3x_1^- - x_2 \le -24$
$x_1^+, x_1^-, x_2, x_3^+ \ge 0$

$\Leftarrow$

$\max\ -2x_1^+ + 2x_1^- - 7x_2 + x_3^+$
$x_1^+ - x_1^- + x_3^+ \le 7$
$-x_1^+ + x_1^- - x_3^+ \le -7$
$-3x_1^+ + 3x_1^- - x_2 \le -24$
$x_1^+, x_1^-, x_2, x_3^+ \ge 0$

T2 08/09 I.1

$$\begin{aligned} \min\ & -x_1 - 2x_2 + x_3 \\ \text{s.t.}\ & 2x_1 + x_2 + x_3 \le 14 \\ & -4x_1 - 2x_2 - 3x_3 \ge -28 \\ & 2x_1 + 5x_2 + 5x_3 \le 30 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

$$\begin{aligned} \max\ & x_1 + 2x_2 - x_3 \\ \text{s.t.}\ & 2x_1 + x_2 + x_3 \le 14 \\ & 4x_1 + 2x_2 + 3x_3 \le 28 \\ & 2x_1 + 5x_2 + 5x_3 \le 30 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

$z = x_1 + 2x_2 - x_3$

$s_1 = 14 - 2x_1 - x_2 - x_3$ $\quad 14/1 = 14$

$s_2 = 28 - 4x_1 - 2x_2 - 3x_3$ $\quad 28/2 = 14$

$s_3 = 30 - 2x_1 - 5x_2 - 5x_3$ $\quad 30/5 = 6$

$\frac{2}{5} \times 30 = \frac{2}{5} \times 5 \times 6 = 12$

↓

$z = 12 + 1/5 \cdot x_1 - 3x_3 - 2/5 \cdot s_3$

$s_1 = 8 - 8/5 \cdot x_1 + 1/5 \cdot s_3$ $\quad \rightarrow 8 \times \frac{5}{8} = 5$

$s_2 = 16 - 16/5 \cdot x_1 - x_3 + 2/5 \cdot s_3$ $\quad \rightarrow 16 \times \frac{5}{16} = 5$

$x_2 = 6 - 2/5 x_1 - x_3 - 1/5 \cdot s_3$ $\quad \rightarrow 6 \times \frac{5}{2} = 3 \times 5 = 15$

↓

$z = 13 - 3x_3 - 15/40 \cdot s_3 - 1/8 s_1$

$x_1 = 5 \quad 0 + 1/8 \cdot s_3 - 5/8 s_1$

$s_2 = 0 - x_3 + 0 + 2 s_1$

$x_2 = 4 - x_3 - 1/4 \cdot s_3 + 1/4 \cdot s_1$

Óptimo: -13

Sol: $(5, 4, 0)$

R2 08/09 I.1

$$\min\ 100 + x_1 - 5x_2 + 2x_3$$

$$\text{s.t.}\quad x_1 + 3x_2 \le 50$$

$$x_1 + 2x_2 + x_3 \le 50$$

$$x_1 + x_3 \le 17$$

$$x_1, x_2, x_3 \ge 0$$

$$\max\ -x_1 + 5x_2 - 2x_3$$

$$x_1 + 3x_2 \le 50$$

$$x_1 + 2x_2 + x_3 \le 50$$

$$x_1 + x_3 \le 17$$

$$x_1, x_2, x_3 \ge 0$$

$z = -x_1 + 5x_2 - 2x_3$

$s_1 = 50 - x_1 - 3x_2$ (5/3) $\quad 50/3 < 17$

$s_2 = 50 - x_1 - 2x_2 - x_3$ (-2/3) $\quad 50/2 = 25$

$s_3 = 17 - x_1 - x_3$

$$z = 250/3 - 8/3\,x_1 - 2x_3 - 5/3\,s_1$$

$$x_2 = 50/3 - 1/3\,x_1 - 1/3\,s_1$$

$$s_2 = 50/3 - 1/3\,x_1 - x_3 + 2/3\,s_1$$

$$s_3 = 17 - x_1 - x_3$$

- $100 - 250/3 = \frac{300 - 250}{3} = \frac{50}{3}$

$$(x_1^*, x_2^*, x_3^*) = (0,\ 50/3,\ 0)$$