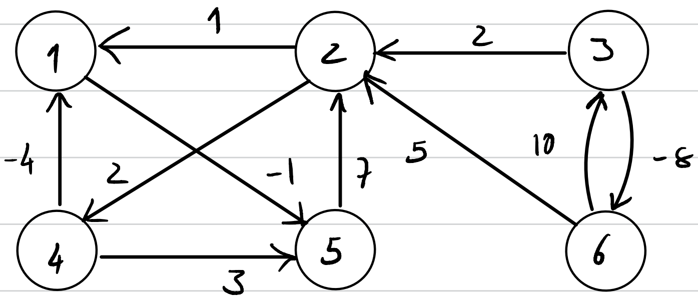
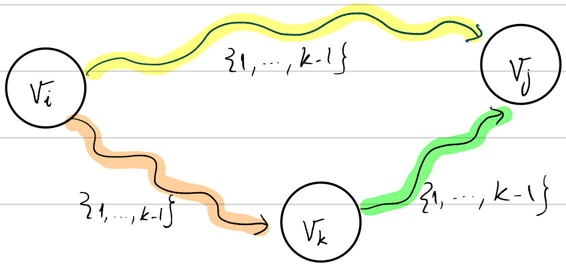
Prática 5

· Caminhos mais curtos entre todos os pares

Ex 25.2-1



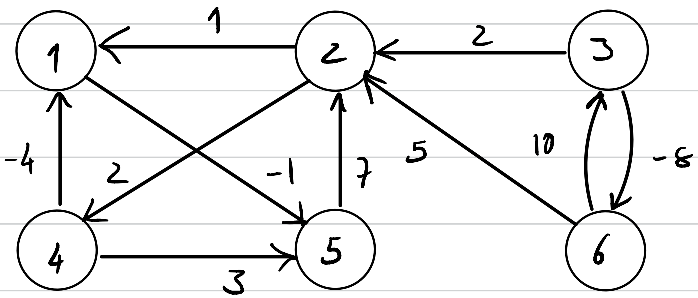


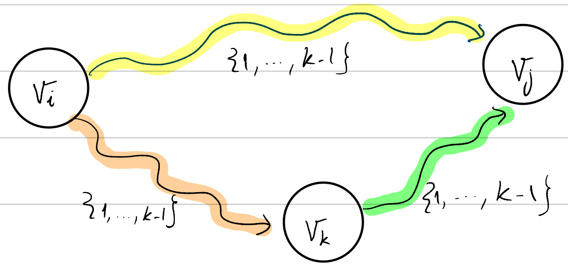
$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)} \right)$$

$$D^{(0)} = \begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \left[\begin{array}{cccccc} \ & \ & \ & \ & \ & \ \\ \ & & & & & \\ \ & & & & & \\ \ & & & & & \\ \ & & & & & \\ \ & & & & & \end{array} \right] \end{array}$$

$$D^{(1)} = \begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} & \left[\begin{array}{cccccc} \ & \ & \ & \ & \ & \ \\ \ & & & & & \\ \ & & & & & \\ \ & & & & & \\ \ & & & & & \\ \ & & & & & \end{array} \right] \end{array}$$

Ex 25.2-1



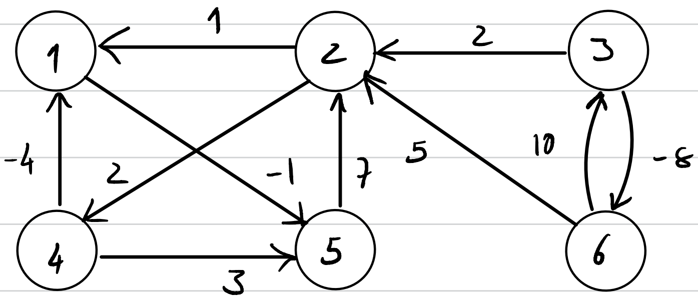


$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

$$D^{(0)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & \infty & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & 3 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$D^{(1)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

Ex 25.2-1

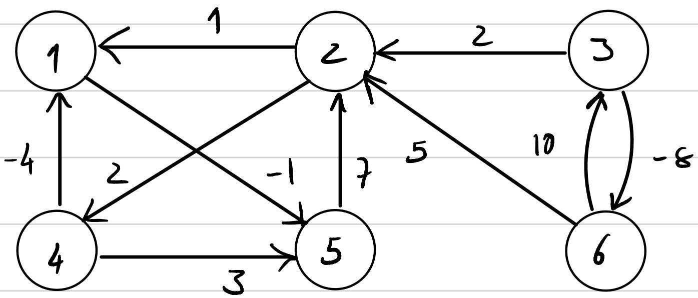


$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)} ,\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)} \right)$$

$$D^{(1)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$D^{(2)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \end{array}$$

Ex 25.2-1

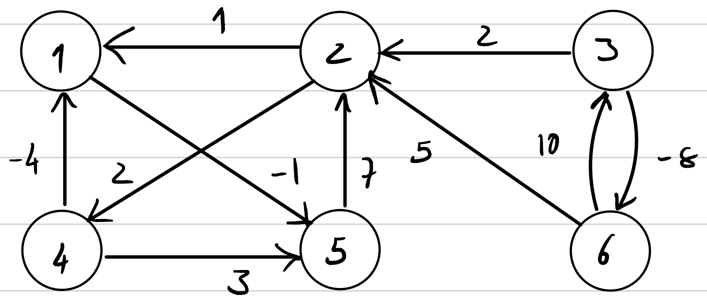


$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)} \right)$$

$$D^{(2)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \end{array}$$

$$D^{(3)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \end{array}$$

Ex 25.2-1

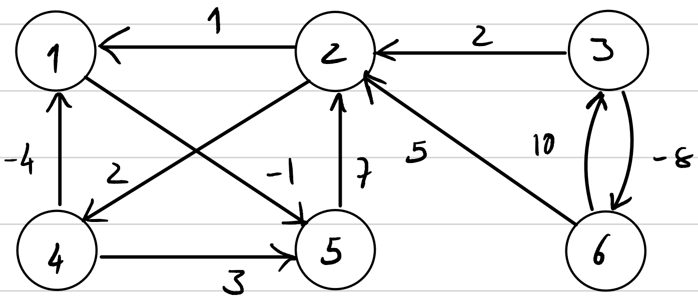


$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

$$D^{(3)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \end{array}$$

$$D^{(4)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \end{array}$$

Ex 25.2-1

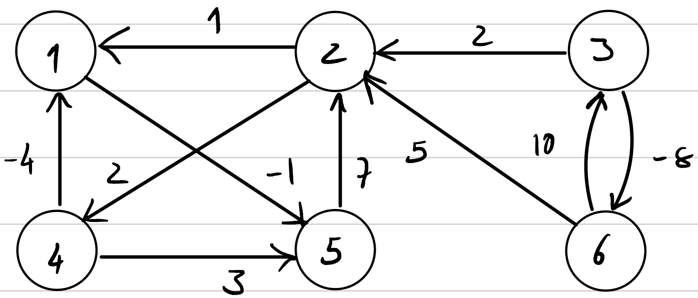


$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)} \right)$$

$$D^{(4)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \end{array}$$

$$D^{(5)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \end{array}$$

Ex 25.2-1

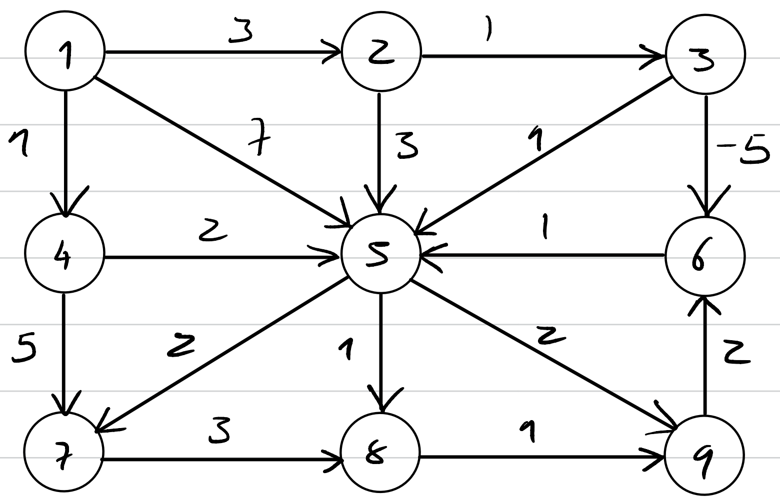


$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)} \right)$$

$$D^{(5)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \end{array}$$

$$D^{(6)} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \end{array}$$

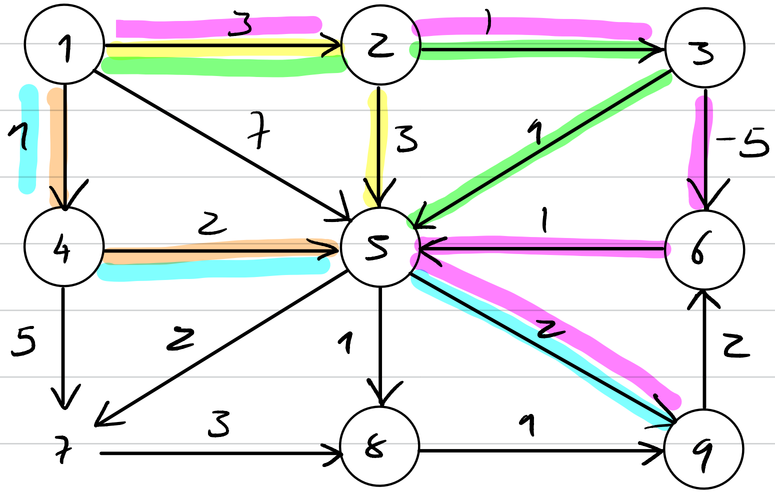
TI 16/17 I.d



$$D_{ij}^{(k)} = \min \left(D_{ij}^{(k-1)}, D_{ik}^{(k-1)} + D_{kj}^{(k-1)} \right)$$

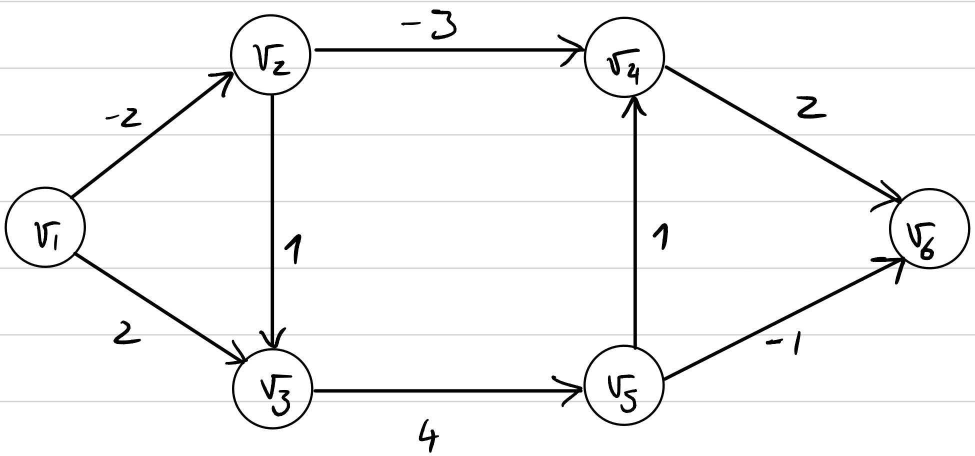
- $D^{(2)}(1,5) =$
- $D^{(2)}(1,9) =$
- $D^{(3)}(1,5) =$
- $D^{(4)}(1,5) =$
- $D^{(5)}(1,9) =$
- $D^{(6)}(1,9) =$

TI 16/17 I.d

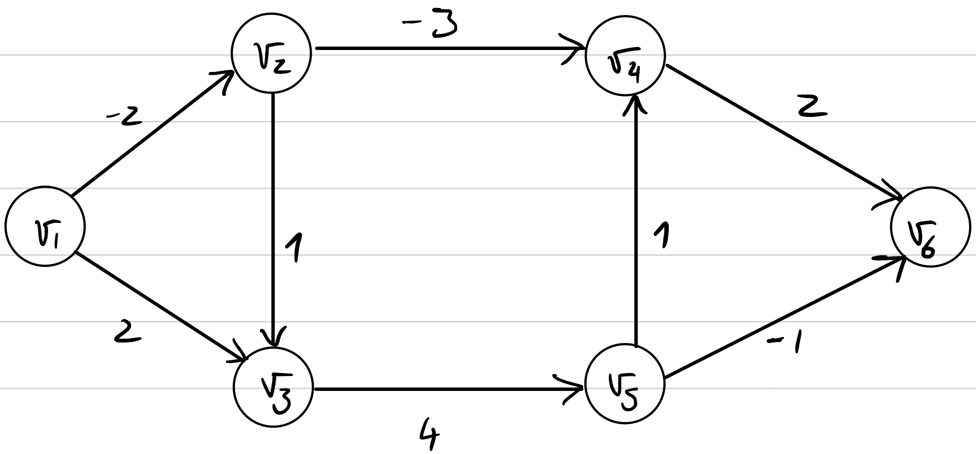


- $D^{(2)}(1,5) = 6$
- $D^{(2)}(1,9) = \infty$
- $D^{(3)}(1,5) = 5$
- $D^{(4)}(1,5) = 3$
- $D^{(5)}(1,9) = \min\Big(\underbrace{D^{(4)}(1,9)}_{\infty},\ \underbrace{D^{(4)}(1,5) + D^{(4)}(5,9)}_{2+2}\Big) = 4$
- $D^{(6)}(1,9) = \min\Big(\underbrace{D^{(5)}(1,9)}_{4},\ \underbrace{D^{(5)}(1,6)}_{-1} + \underbrace{D^{(5)}(6,9)}_{3}\Big) = 2$

RI 08/09 - II.3

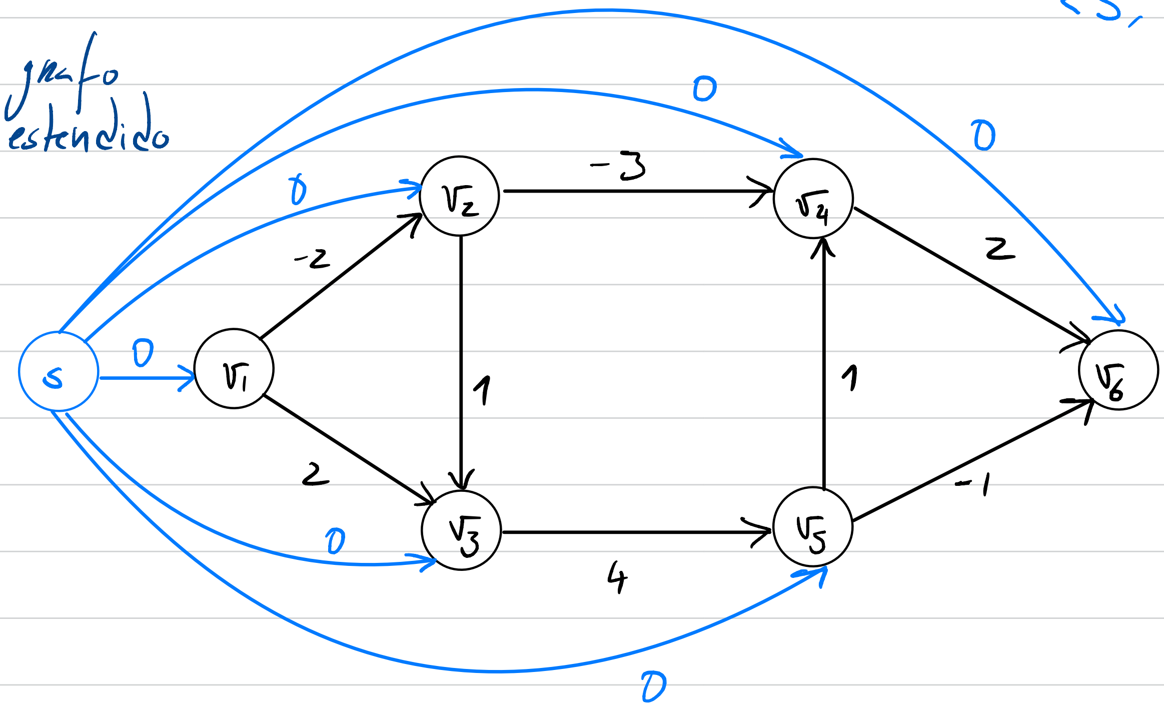


RI 08/09 - II.3

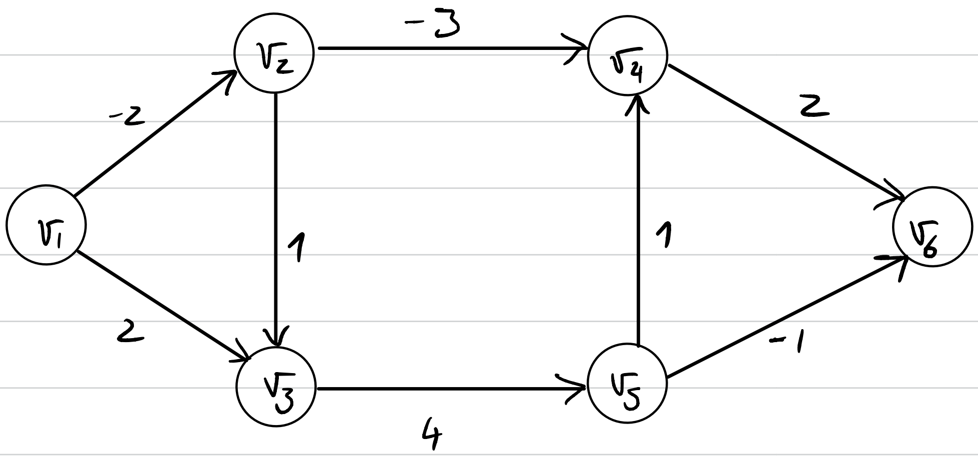


Topological Order:
$\langle s, v_1, v_2, v_3, v_5, v_4, v_6 \rangle$

$\bar{G}$: grafo estendido

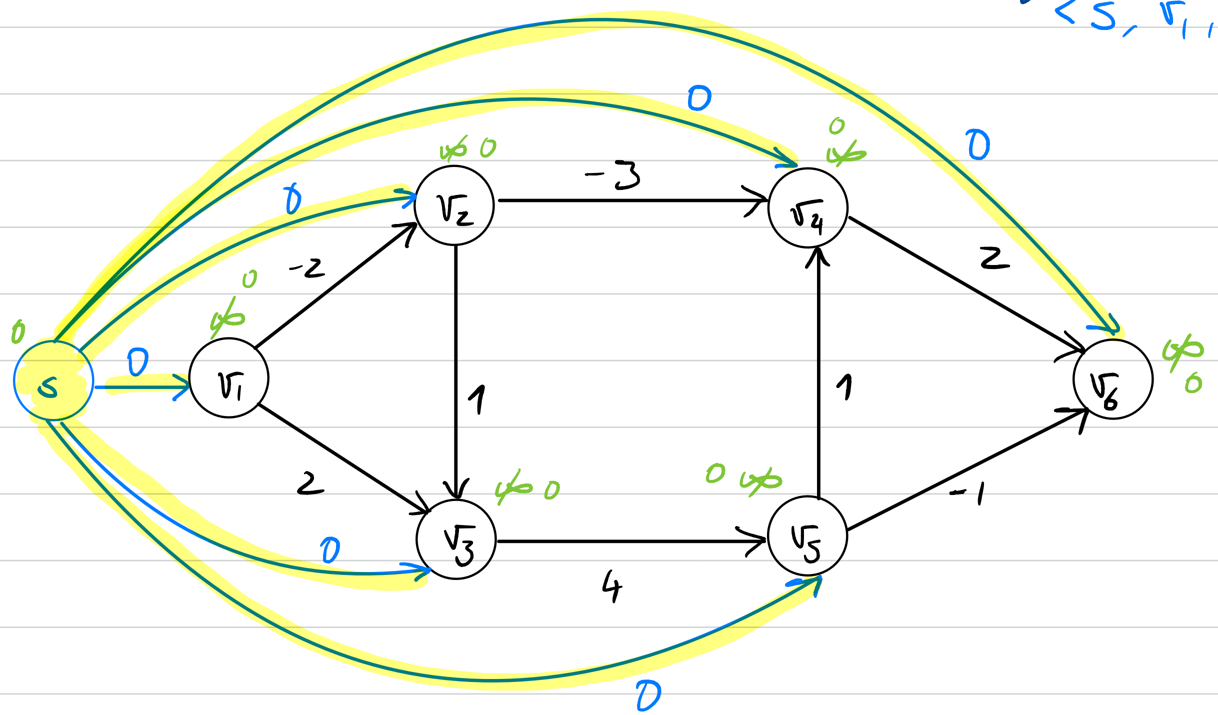


RI 08/09 - II.3

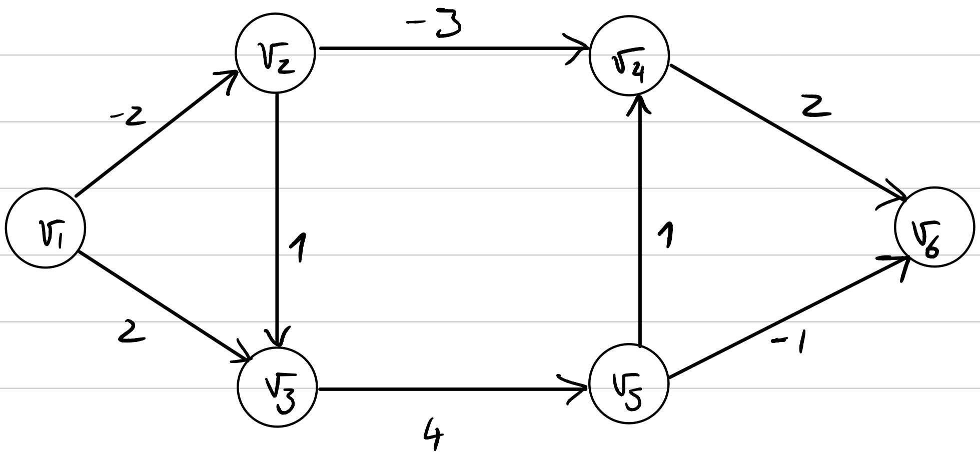


Topological Order:
$\langle s, v_1, v_2, v_3, v_5, v_4, v_6 \rangle$

$\bar{G}$:

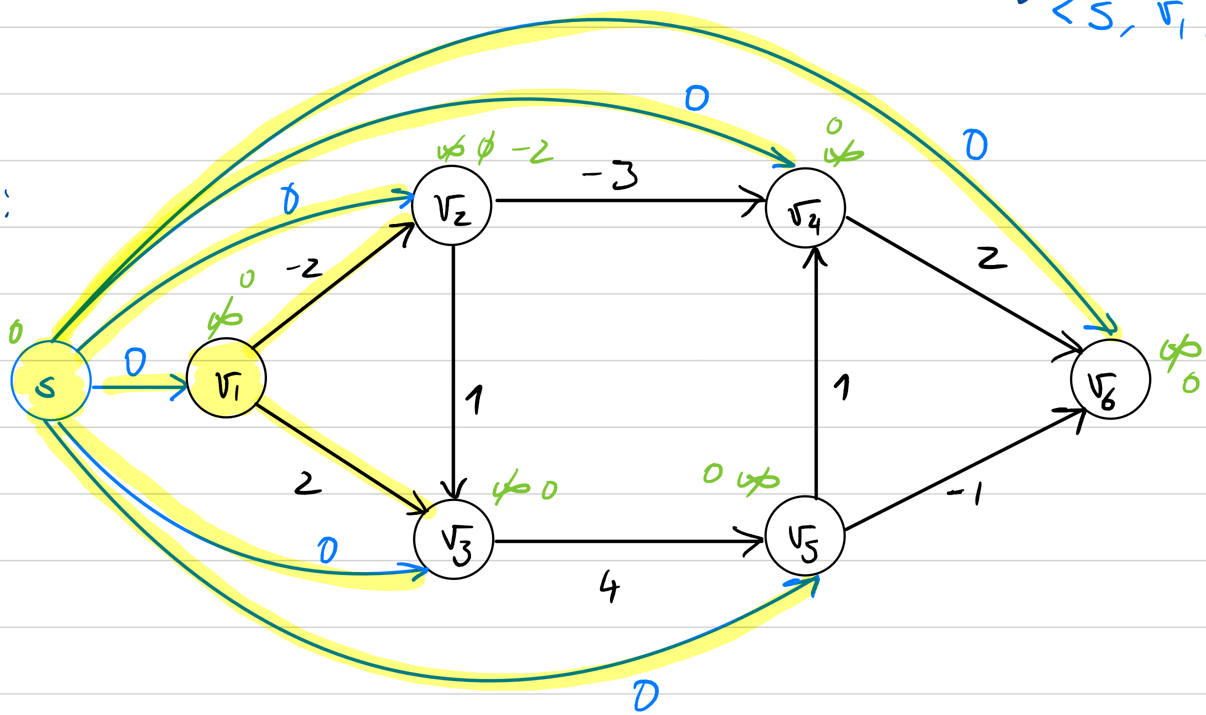


RI 08/09 - II.3

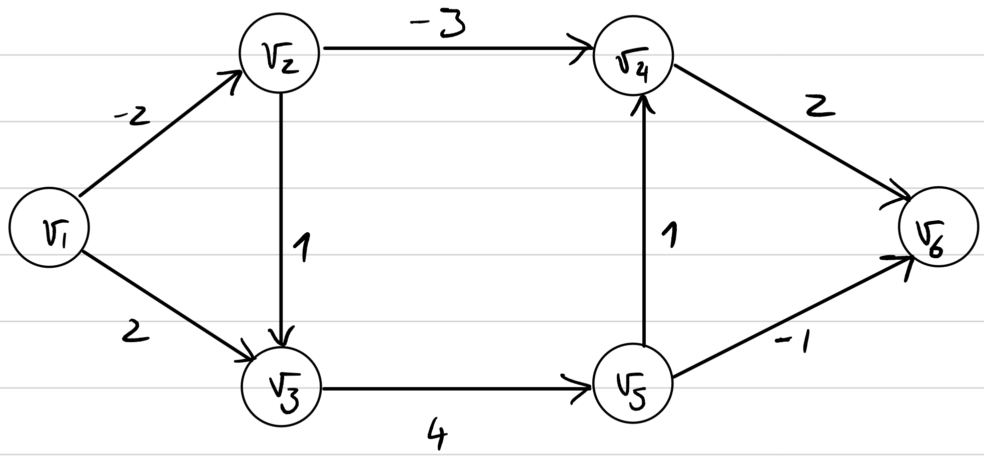


Topological Order:
$\langle s, v_1, v_2, v_3, v_5, v_4, v_6 \rangle$

$\overline{G}$:

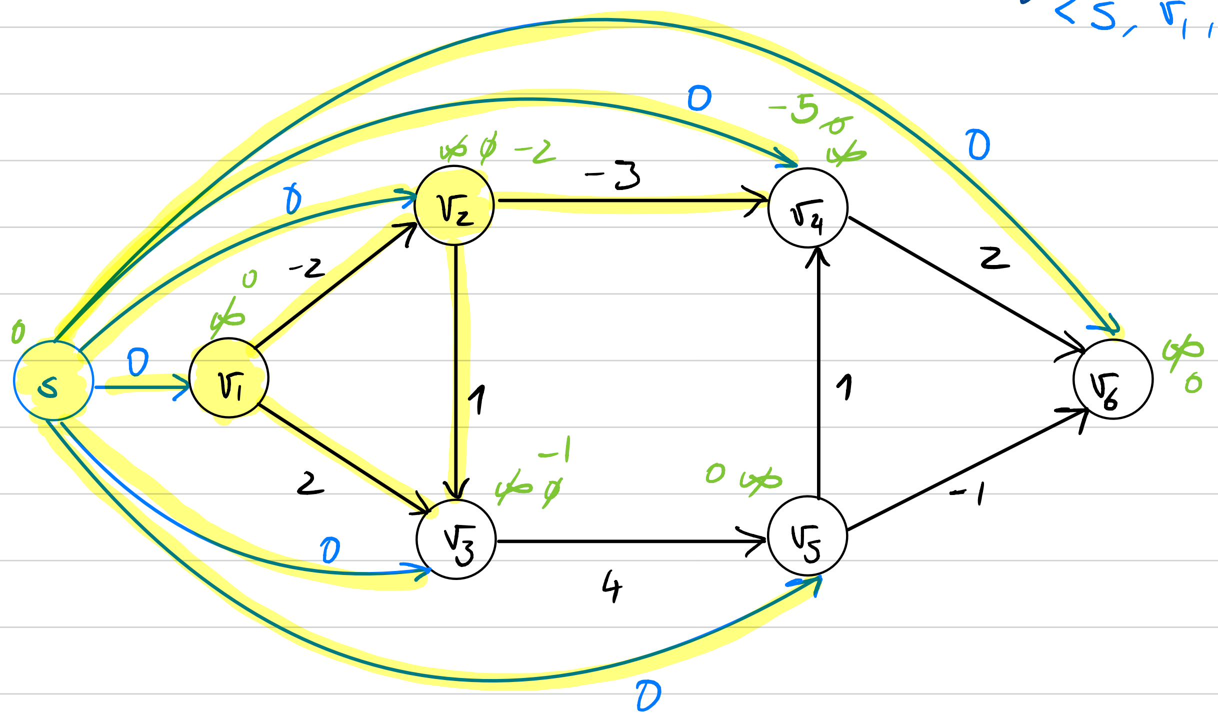


RI 08/09 - II.3

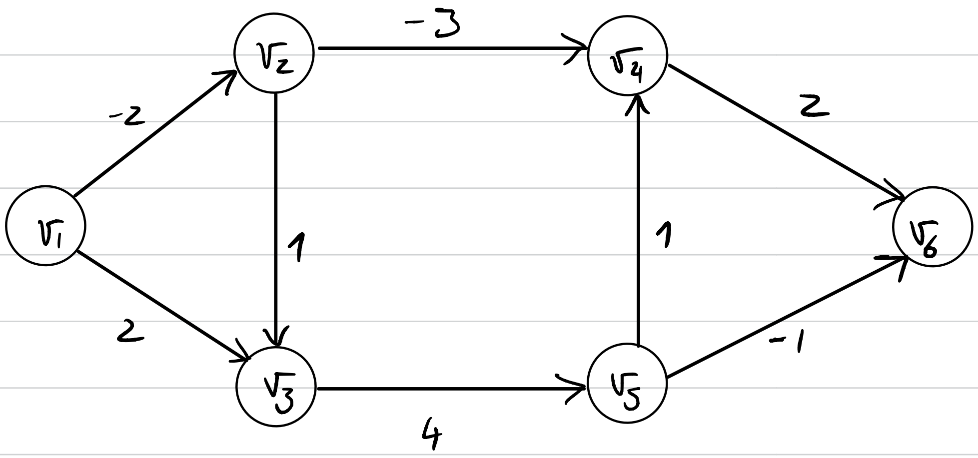


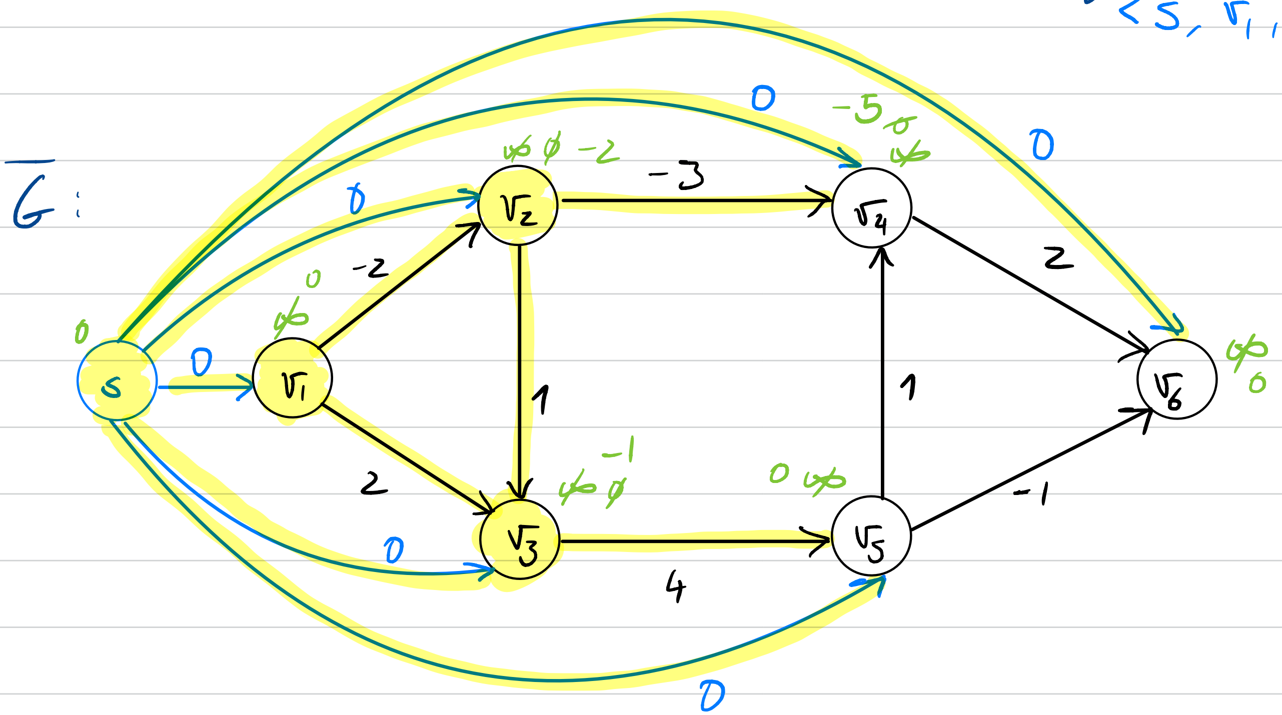
Topological Order:
< s, v_1, v_2, v_3, ~~v~~ v_5, v_4, v_6 >

$\bar{G}$:



RI 08/09 - II.3

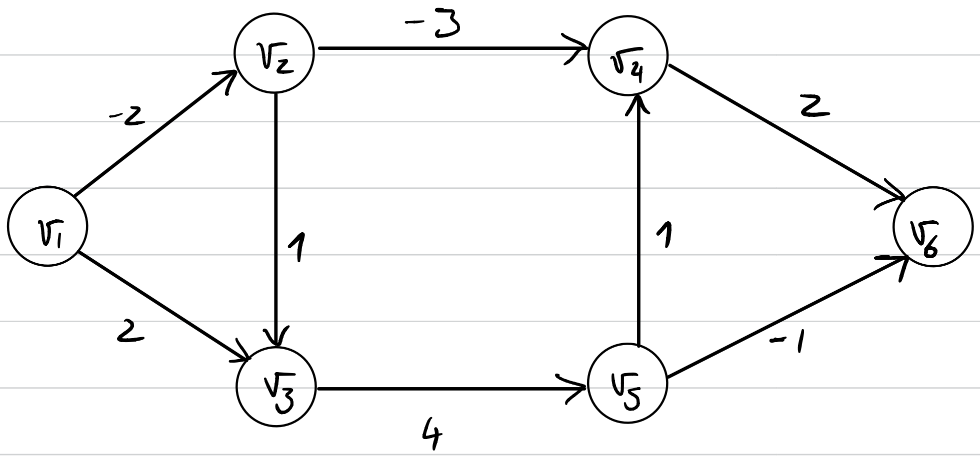




Topological Order:

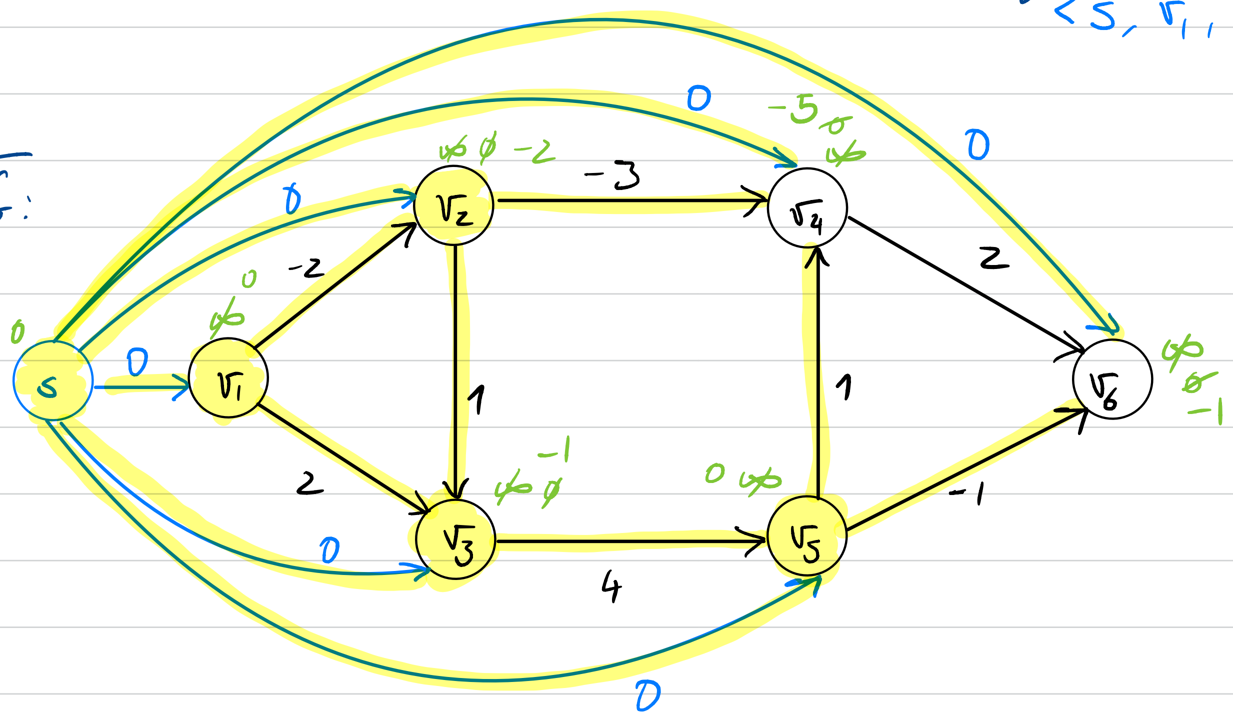
$\langle s, v_1, v_2, v_3, v_5, v_4, v_6 \rangle$

RI 08/09 - II.3

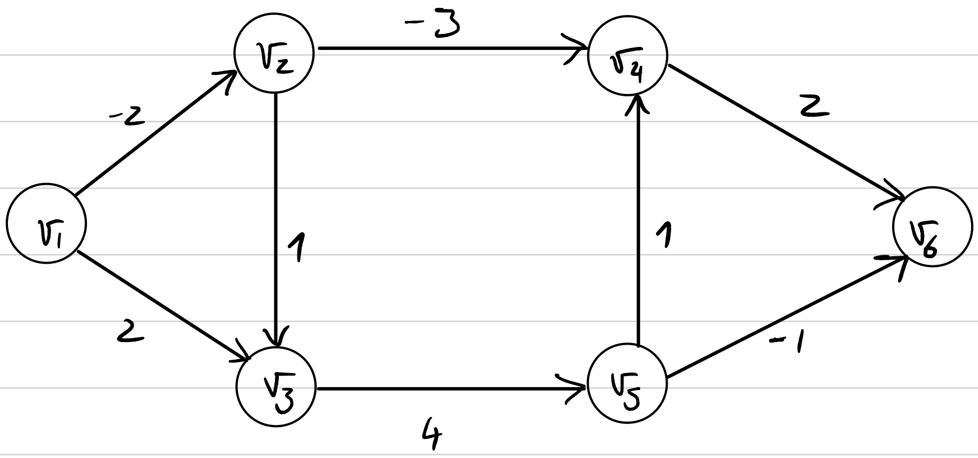


Topological Order:
$\langle s, v_1, v_2, v_3, v_5, v_4, v_6 \rangle$

$\bar{G}$:

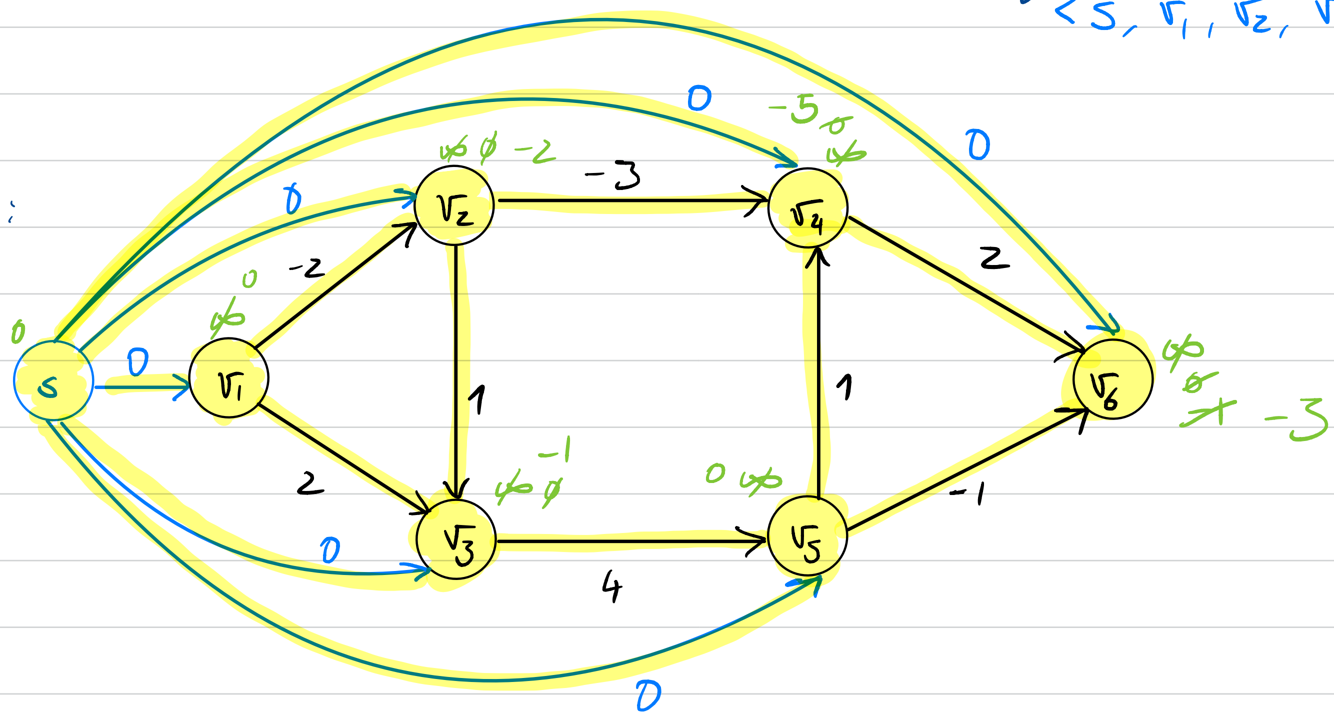


RI 08/09 - II.3



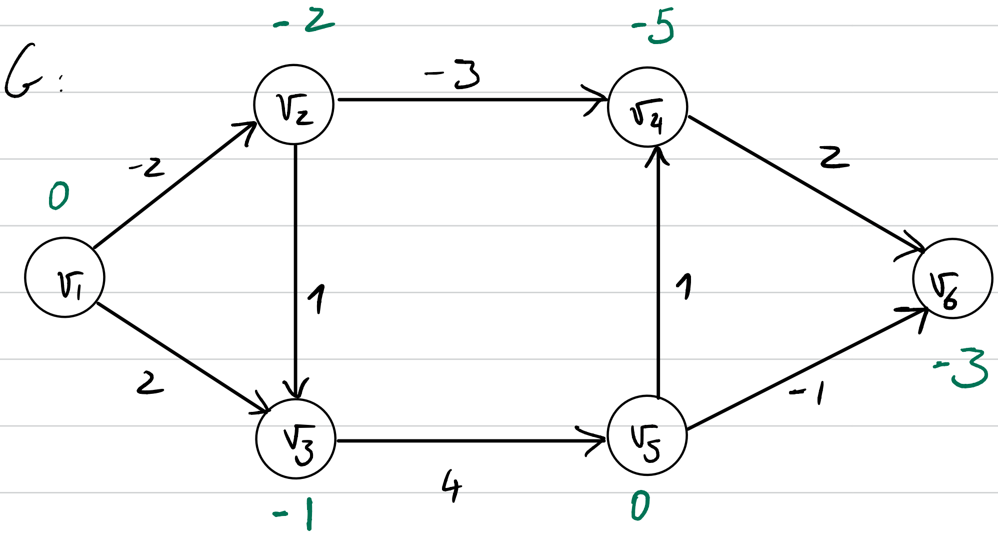
Topological Order:
< s, v_1, v_2, v_3, v_5, v_4, v_6 >

G:



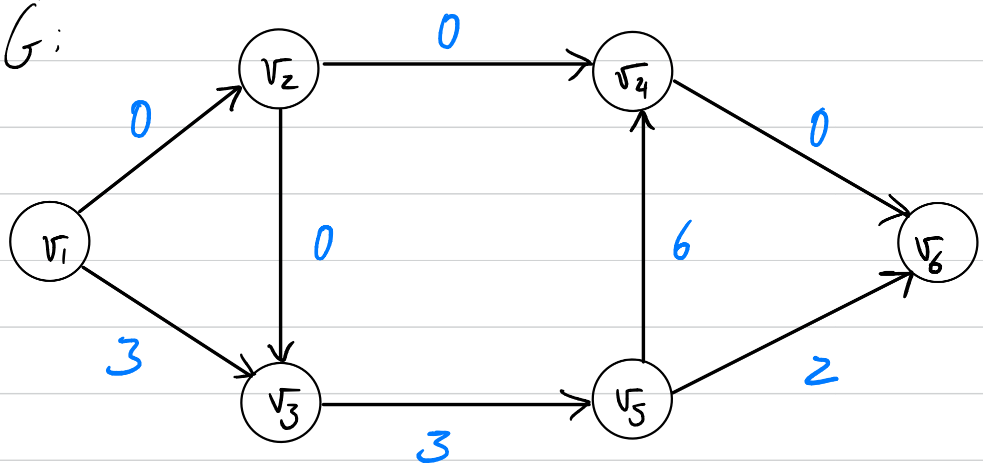
RI 08/09 - II.3

G:



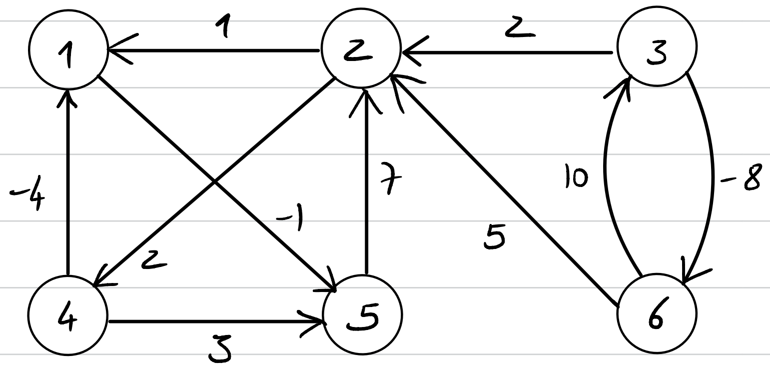
• Alturas de Johnson apresentadas em cima de cada nó.

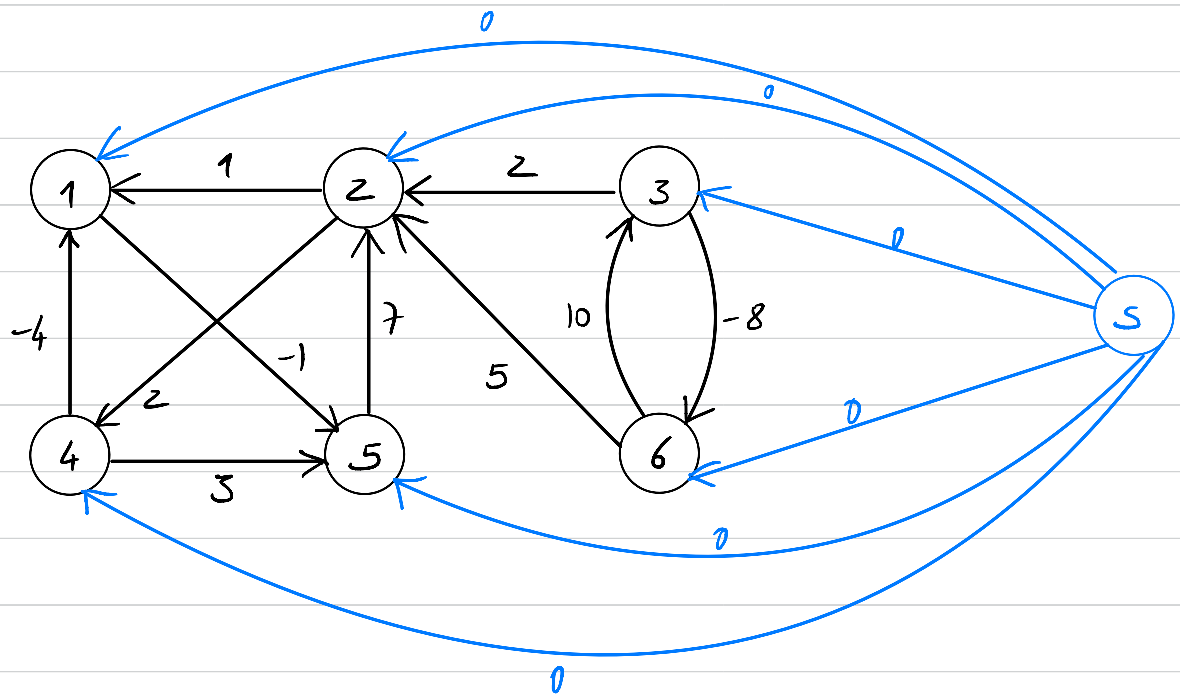
$\hat{G}$:



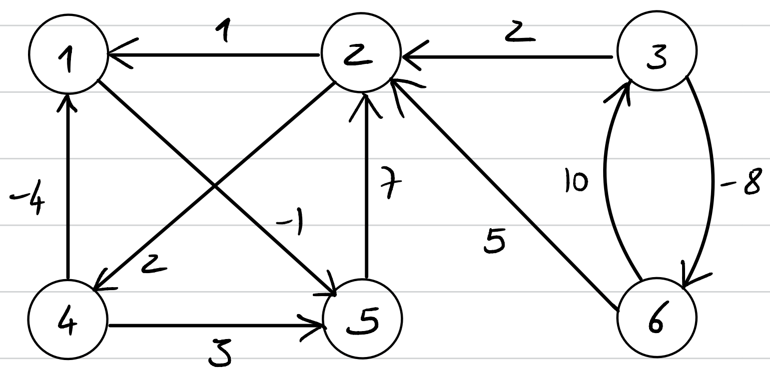
$$\hat{w}(u,v) = w(u,v) + h(u) - h(v)$$

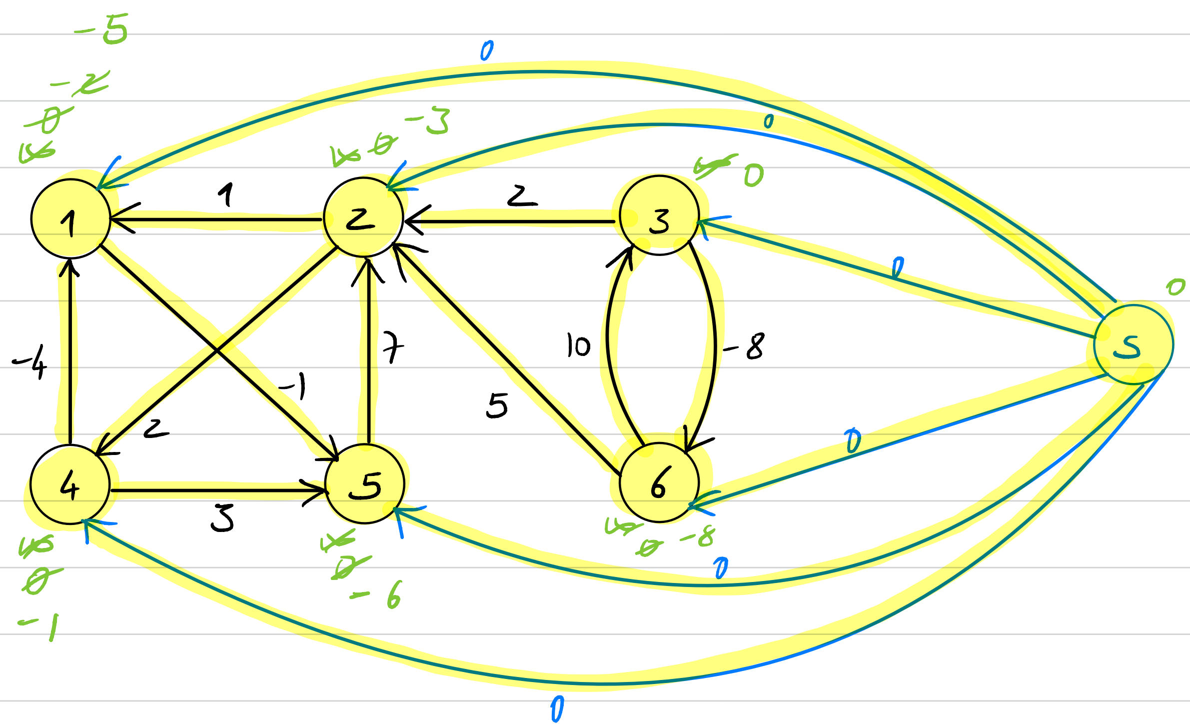
25.3-1



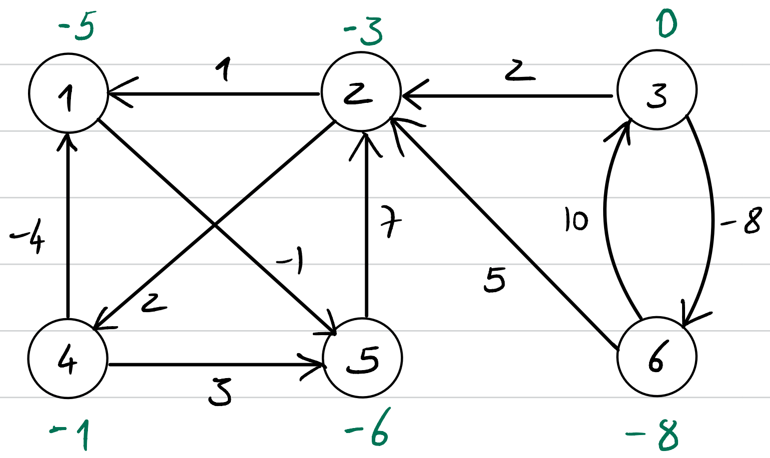


25.3-1

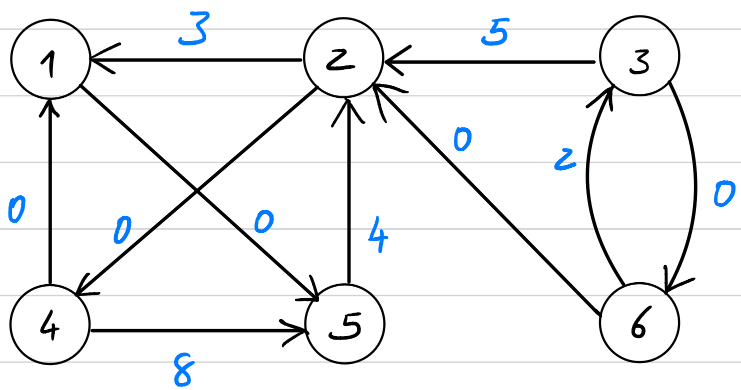




25.3-1



$\hat{G}$:



$$\hat{w}(u,v) = w(u,v) + h(u) - h(v)$$

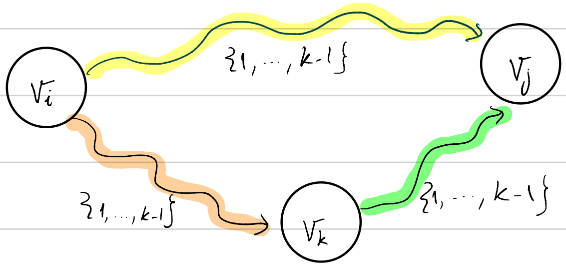
T1 08/09 II.2 Considere os algoritmos para o cálculo de caminhos mais curtos entre todos os pares de vértices. Indique se cada uma das seguintes afirmações é verdadeira (V) ou falsa (F).

1. É possível implementar o algoritmo de Floyd-Warshall por forma a que a memória necessária à sua execução seja $O(V^2)$.
2. No algoritmo de Floyd-Warshall, a matriz $D^{(k)}$ contém os custos dos caminhos mais curtos, entre todos os pares de vértices, que contenham no máximo $k - 1$ arcos.
3. No algoritmo de Johnson, o valor de $h(u)$ é o mínimo entre 0 e o custo do caminho mais curto que termina no vértice u.
4. O tempo de execução do algoritmo de Johnson é $O(V^5)$.
5. Devido ao procedimento de repesagem utilizado no algoritmo de Johnson, um caminho mais curto entre dois vértices pode deixar de o ser.
6. O tempo de execução do algoritmo de Floyd-Warshall é $O(V^3)$.

T1 08/09 II.2 Considere os algoritmos para o cálculo de caminhos mais curtos entre todos os pares de vértices. Indique se cada uma das seguintes afirmações é verdadeira (V) ou falsa (F).

1. É possível implementar o algoritmo de Floyd-Warshall por forma a que a memória necessária à sua execução seja $O(V^2)$. V
2. No algoritmo de Floyd-Warshall, a matriz $D^{(k)}$ contém os custos dos caminhos mais curtos, entre todos os pares de vértices, que contenham no máximo $k-1$ arcos. F
3. No algoritmo de Johnson, o valor de $h(u)$ é o mínimo entre 0 e o custo do caminho mais curto que termina no vértice u. V
4. O tempo de execução do algoritmo de Johnson é $O(V^5)$. F
5. Devido ao procedimento de repesagem utilizado no algoritmo de Johnson, um caminho mais curto entre dois vértices pode deixar de o ser. F
6. O tempo de execução do algoritmo de Floyd-Warshall é $O(V^3)$. ✓

25.2-4



$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

```
Floyd Warshall (W)
    n ← rows (W)
    D^(0) ← W
    for k ← 1 to n
        D^(k) ← new n×n matrix
        for i ← 1 to n
            for j ← 1 to n
                D^(k)[i,j] ← min ( D^(k-1)[i,j], D^(k-1)[i,k] + D^(k-1)[k,j] )
```

$\Downarrow$

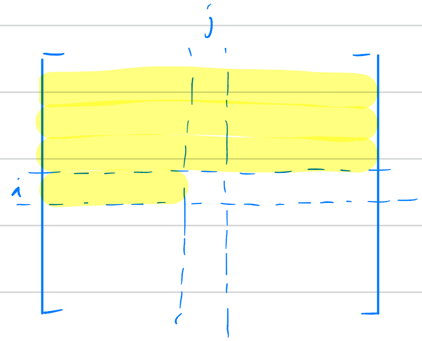
```
Floyd Warshall' (W)
    n ← rows (W)
    D ← W
    for k ← 1 to n
        for i ← 1 to n
            for j ← 1 to n
                D[i,j] ← min ( D[i,j], D[i,k] + D[k,j] )
```

Pergunta: A versão simplificada é correcta?

25.2-4

$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

$$D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ \underbrace{\overset{?}{D}[i,k] + \overset{?}{D}[k,j]}_{\text{Estes valores já podem ter sido actualizados!}}\right)$$



· 4 possibilidades

(I) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k)}[k,j]\right)$

(II) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k-1)}[k,j]\right)$

(III) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k)}[k,j]\right)$

(IV) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k-1)}[k,j]\right)$

25.2-4

$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

$$D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ \underbrace{\overset{?}{D}[i,k] + \overset{?}{D}[k,j]}_{\text{Estes valores já podem ter sido actualizados!}}\right)$$

- 4 possibilidades

I) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k)}[k,j]\right)$

se $k < i \wedge k < j$

II) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k-1)}[k,j]\right)$

III) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k)}[k,j]\right)$

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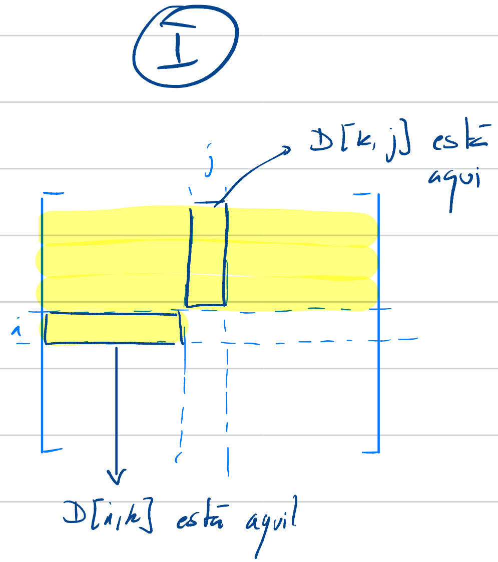


25.2-4

$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

$$D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ \underbrace{\overset{?}{D}[i,k] + \overset{?}{D}[k,j]}_{\text{Estes valores já podem ter sido actualizados!}}\right)$$

· 4 possibilidades

I $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k)}[k,j]\right)$

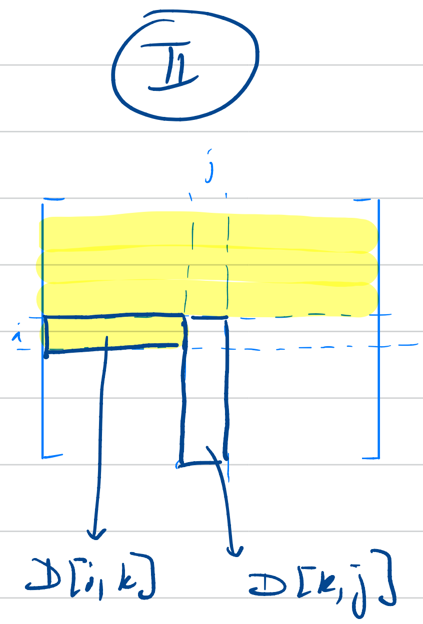
se $k < i \wedge k < j$

II $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k-1)}[k,j]\right)$

se $k \geq i \wedge k < j$

III $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k)}[k,j]\right)$

IV $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k-1)}[k,j]\right)$



25.2-4

$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

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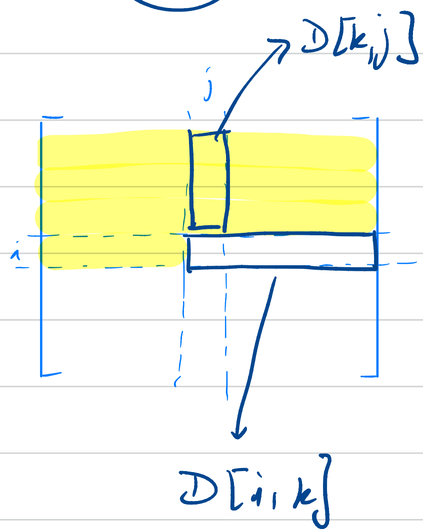
se $k \geq i \land k < j$

III $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k)}[k,j]\right)$
se $k < i \land k \geq j$

IV $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k-1)}[k,j]\right)$

III



25.2-4

$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

$$D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ \underbrace{D^{?}[i,k] + D^{?}[k,j]}_{\text{Estes valores já podem ter sido actualizados!}}\right)$$

· 4 possibilidades

IV

(I) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k)}[k,j]\right)$
se $k < i \wedge k < j$

(II) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k-1)}[k,j]\right)$
se $k \geq i \wedge k < j$

(III) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k)}[k,j]\right)$
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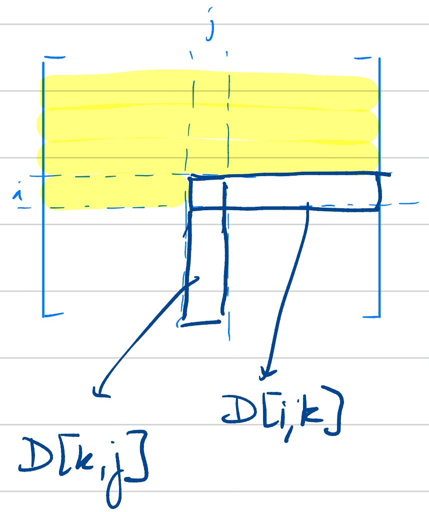
se $k \geq i \wedge k \geq j$



25.2-4

$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

• 4 possibilidades

I $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k)}[k,j]\right)$

se $k < i \wedge k < j$

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se $k < i \wedge k \geq j$

IV $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k-1)}[k,j]\right)$

se $k \geq i \wedge k \geq j$

$$\left.\begin{array}{l} \bullet\ D^{(k-1)}[i,k] = D^{(k)}[i,k] \\ \bullet\ D^{(k-1)}[k,j] = D^{(k)}[k,j] \end{array}\right\} \text{Porquê?}$$

25.2-4

$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\right)$$

· 4 possibilidades

- $D^{(k-1)}[i,k] = D^{(k)}[i,k]$
- $D^{(k-1)}[k,j] = D^{(k)}[k,j]$

O vértice intermédio k não melhora o caminho mais curto entre i e k (mutatis mutandis p/ k e j)

(I) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k)}[k,j]\right)$ se $k < i \land k < j$

(II) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k)}[i,k] + D^{(k-1)}[k,j]\right)$ se $k \geq i \land k < j$

(III) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k)}[k,j]\right)$ se $k < i \land k \geq j$

(IV) $D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k-1)}[k,j]\right)$ se $k \geq i \land k \geq j$

$$D^{(k)}[i,j] = \min\left(D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k-1)}[k,j]\right)$$

25.3-3.

Relação entre $\hat{w}$ e w se $w(u,v) \geq 0 \ \forall (u,v) \in E$

- Se $w(u,v) \geq 0 \ \forall (u,v) \in E$, então concluímos que:

$$\forall v \in V. \ h(v) = 0$$

- Segue que, para todo $(u,v) \in E$:

$$\begin{aligned} \hat{w}(u,v) &= w(u,v) + h(u) - h(v) \\ &= w(u,v) + 0 - 0 \\ &= w(u,v) \end{aligned}$$

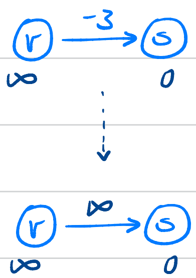
25.3-4

① No cálculo das alturas de Johnson não precisamos de construir o grafo estendido. Basta escolher como source um vértice s em V.

② O método apresentado em ① funciona se o grafo tiver um único componente.

25.3-4

1. No cálculo das alturas de Johnson não precisamos de construir o grafo estendido. Basta escolher como source um vértice s em V.



2. O método apresentado em (1) funciona se o grafo tiver um único componente.

- Para qualquer $s \in V$, temos que:

$$\forall v \in V.\ \delta(s, v) < \infty \implies \text{Alturas bem definidas}$$

25.2-6

- Proponha um algoritmo para identificar ciclos negativos depois da aplicação do algoritmo de Floyd-Warshall.

$\boxed{\Rightarrow}$

- Existe um ciclo negativo sse $D_{ii} < 0$ para algum $1 \le i \le n$.

$\boxed{\Leftarrow}$ ✓

$\boxed{\Rightarrow}$ Existe um caminho negativo $\Rightarrow$ $D_{ii} < 0$ para algum i

- Seja $p = \langle i_1, \dots, i_m = i_1 \rangle$ o ciclo negativo mais curto em G.

- Procedemos por indução no tamanho de $|p|$

  - Base ($|p| = 1$) $\quad p = \langle i_1, i_1 \rangle \quad W(i_1, i_1) < 0$

  - Passo ($|p| > 1$) $\quad p = \langle i_1, \dots, i_m \rangle$ com $m > 1$

    - $k = \min\{i_2, \dots, i_{m-1}\}$
    - $$D^{(k)}(i_1, i_m) = \min\Big(\underbrace{\underbrace{D^{(k-1)}(i_1, i_m)}_{(\dagger)}}_{>0},\ \underbrace{\underbrace{D^{(k-1)}(i_1, i_k)}_{(\ddagger)} + \underbrace{D^{(k-1)}(i_k, i_m)}_{(\dagger\dagger\dagger)}}_{<0}\Big)$$

$(\dagger), (\ddagger), (\dagger\dagger\dagger) \Rightarrow$ todos estes valores estão correctamente calculados porque p é o caminho negativo mais curto