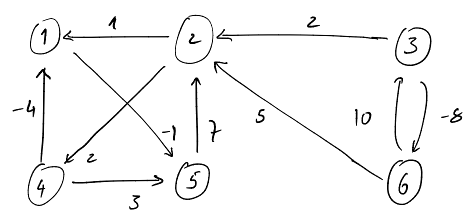
25.2-1



$$D_{ij}^{(k)} = \min\left(D_{ij}^{(k-1)},\ D_{ik}^{(k-1)} + D_{kj}^{(k-1)} \right)$$

$$D^{(0)} = \begin{bmatrix} 0 & 0 & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & \infty & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & 3 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix}$$

$$D^{(1)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix}$$

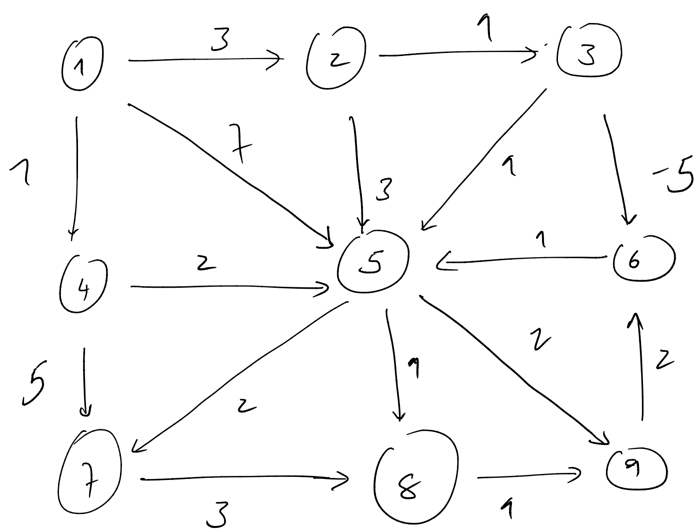
$$D^{(3)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix}$$

$$D^{(4)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

$$D^{(5)} = \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

$$D^{(6)} = \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix}$$

TI 16/17 I.d



- $D^{(2)}(1,5) = \min\left(D^{(1)}(1,5),\ D^{(1)}(1,2) + D^{(1)}(2,5) \right)$

  $= \min(7,\ 3+3) = 6$

- $D^{(2)}(1,9) = \infty$

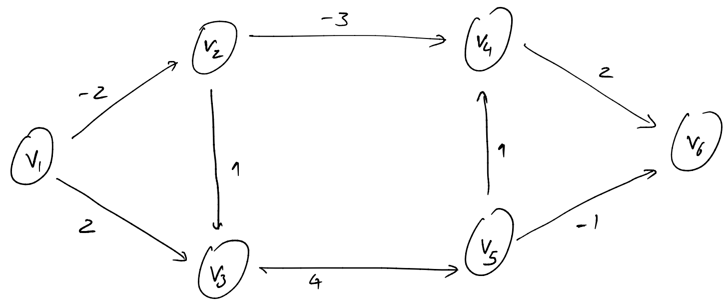
- $D^{(3)}(1,5) = \min\left(D^{(2)}(1,5),\ D^{(2)}(1,3) + D^{(2)}(3,5) \right)$
  $= \min(6,\ 4+1)$
  $= 5$

- $D^{(4)}(1,5) = 3$

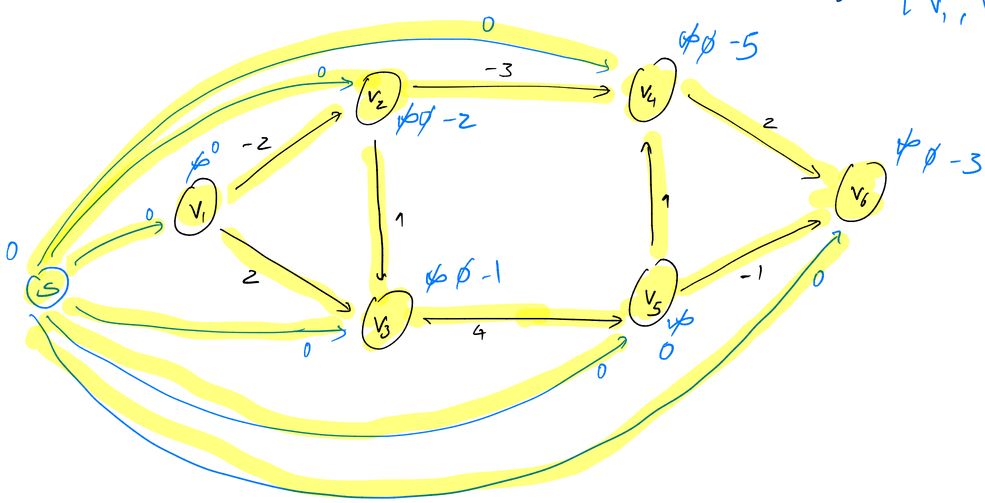
- $D^{(5)}(1,9) = \min\left(D^{(4)}(1,9),\ D^{(4)}(1,5) + D^{(4)}(5,9) \right)$
  $= 5$

- $D^{(6)}(1,9) = \min\left(D^{(5)}(1,9),\ D^{(5)}(1,6) + D^{(5)}(6,9) \right)$
  $= \min(5,\ -1+3)$
  $= 2$

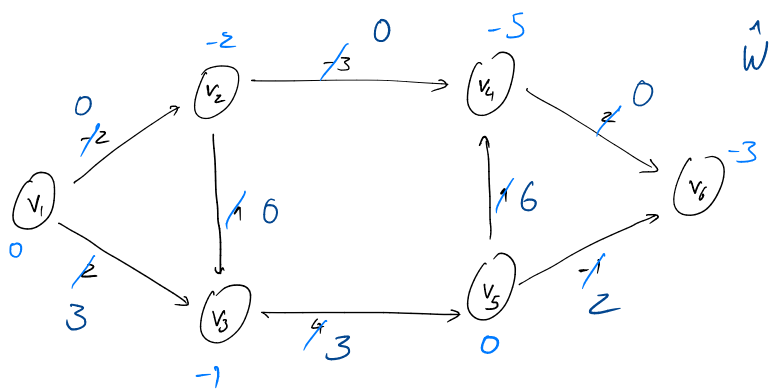
RI 08/09 II.3



Topological Sort:
[v_1, v_2, v_3, v_5, v_4, v_6]



	v_1	v_2	v_3	v_4	v_5	v_6
$h(v)$	0	-2	-1	-5	0	-3



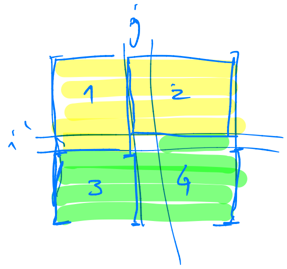
$$\hat{w}(u,v) = w(u,v) + h(u) - h(v)$$

25.2.4

Com os subscripts estamos a calcular:

$D_{ij}^{(k)} = \min(D_{ij}^{(k-1)}, D_{ik}^{(k-1)} + D_{kj}^{(k-1)})$

Sem os subscripts podemos estar a calcular:



(1) $D_{ij}^{(k)} = \min(D_{ij}^{(k-1)}, D_{ik}^{(k)} + D_{kj}^{(k)})$, $k \leq i$ && $k < j$

(2) $D_{ij}^{(k)} = \min(D_{ij}^{(k-1)}, D_{ik}^{(k-1)} + D_{kj}^{(k)})$, $k < i$ && $k \geq j$

(3) $D_{ij}^{(k)} = \min(D_{ij}^{(k-1)}, D_{ik}^{(k)} + D_{kj}^{(k-1)})$, $k > i$ && $k < j$

(4) $D_{ij}^{(k)} = \min(D_{ij}^{(k-1)}, D_{ik}^{(k-1)} + D_{kj}^{(k-1)})$, $k \geq i$ && $k \geq j$

- $D_{ik}^{(k-1)} = D_{ik}^{(k)}$
- $D_{kj}^{(k-1)} = D_{kj}^{(k-1)}$

25.3-3

Não mudam.

T1 08/09 II.2

1 - V 3 - V 5 - F
2 - F 4 - F 6 - V

25.2-6

- Existe um ciclo negativo sse $D_{ii} < 0$ para algum índice i

$\boxed{\Leftarrow}$ ✓

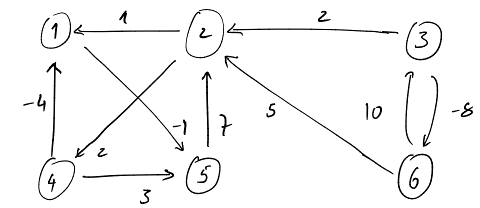
$\boxed{\Rightarrow}$ Seja p o caminho circular com o menor número de vértices.

- $|p| = 1$
  - $p = \langle i \rangle$ para algum i $\Rightarrow D_{ii}^{(0)} = W_{ii} < 0 \Rightarrow$ Valores de D nunca são aumentados

- $|p| > 1$
  - $\langle i, \ldots, k, \ldots, i \rangle$ onde k é o maior índice em p

$D^{(k)}(i,i) = \min(\underbrace{D^{(k-1)}(i,i)}_{\geq 0}, \underbrace{D^{(k-1)}(i,k) + D^{(k-1)}(k,i)}_{< 0})$

25.1-1 -

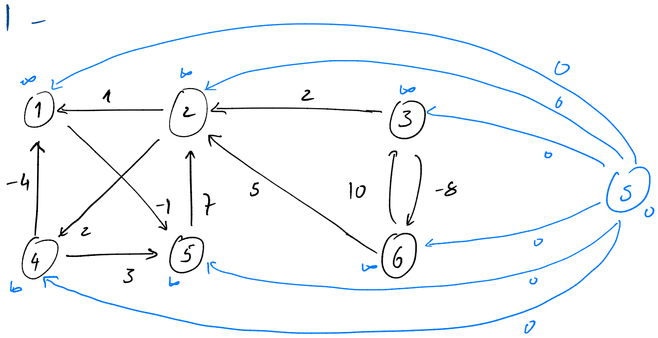


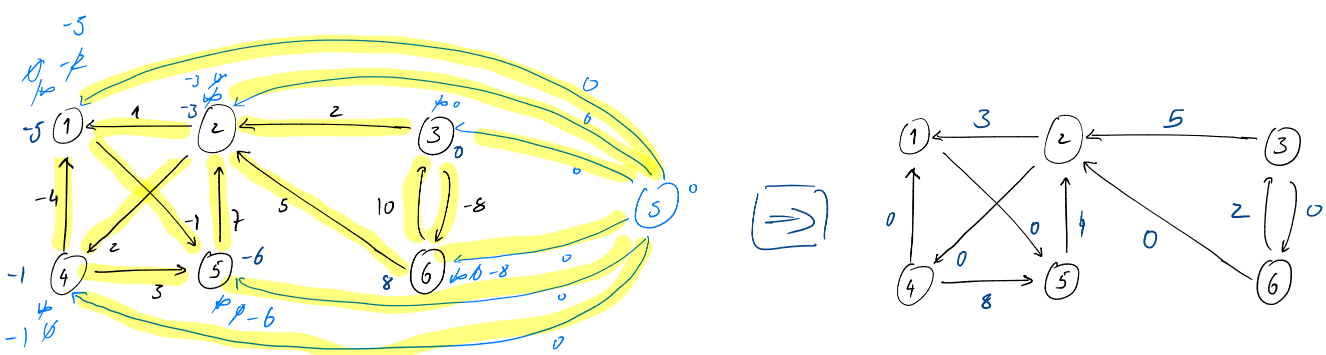
(Left as an exercise)

Slow: $D_{ij}^{(n)} = \min\limits_{1 \le k \le N} \{ D_{ik}^{(n-1)} + V_{kj} \}$

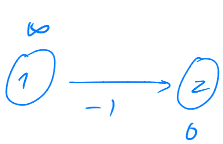
Faster: $D_{ij}^{(2n)} = \min\limits_{1 \le k \le N} \{ D_{ik}^{(n)} + D_{kj}^{(n)} \}$

25.3-1 -





25.3-4



$2 = s$

$\delta(s,s) = 0$

$\delta(s,1) = +\infty$

$\hat{w}(1,2) = -1 + h(1) - h(2)$
$= -1 - 0 + \infty$

Num grafo com um único componente as alturas estão bem definidas porque qualquer que seja o s escolhido temos que: $\forall v \in V.\ s \leadsto v$, de onde segue que:

$\delta(s,v) < \infty$