

Sensitivity Analysis

- Simple sensitivity analysis: $E[T] = \beta_0 + \beta_1 X$. If you're missing some X , how would you fill them in to get really different estimates of β than you get from a CCA?

You can impute values of X that are missing to see how different your estimates would be.

- Stochastic: you implicitly posit some model for missing values of X_M , impute them according to that model, get parameter estimates from the completed datasets and plot the distribution of those.

Note that this implicitly sets some model for missing X , so the “sensitivity” really has to do with the model you specify.

- Optimized: If you want to get a sense of how bad things could be, you would want to find values of X_M such that $\hat{\beta}^*$ on the completed data would be really different from your $\hat{\beta}$.

Essentially, we have this problem: Let $\hat{\beta}$ be the obtained estimate from the analysis. We want to find X_M such that $\mathbf{c}'\hat{\beta}^* - \mathbf{c}'\hat{\beta}$ is large, where c is some contrast vector. Thus, we have

$$\begin{aligned} \min_{X_M} & -(\mathbf{c}'\hat{\beta}^* - \mathbf{c}'\hat{\beta})^2 \\ \text{subject to } & X_M \in \mathcal{X} \end{aligned}$$

It will be difficult to solve this analytically, but we could try gradient descent. This may be sensitive to starting point, so we may want to try several random initializations. Finally, we should keep track of imputed X_M and the squared difference at each step, so that we can plot observed vs. imputed X and how different the imputed parameter estimates are.

Note, if the math proves unwieldy to update all missing X_i at once, we could do a series of stochastic updates, where we take a subset of the i for which there is missingness, update those X_i and then turn around and update another set of X_i . It would make sense to sort by missingness pattern to do this.

- Paper would propose the idea of sensitivity analysis. Describe the two algorithms (stochastic search vs. gradient descent). Demonstrate them on a data set or two. Close with recommendations.