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Author(s): Hendrik Bessembinder and Paul J. Seguin

Source: *The Journal of Financial and Quantitative Analysis*, Mar., 1993, Vol. 28, No. 1 (Mar., 1993), pp. 21-39

Published by: Cambridge University Press on behalf of the University of Washington School of Business Administration

Stable URL: https://www.jstor.org/stable/2331149

Price Volatility, Trading Volume, and Market Depth: Evidence from Futures Markets

Hendrik Bessembinder and Paul J. Seguin*

Abstract

The relations between volume, volatility, and market depth in eight physical and financial futures markets are examined. Evidence suggests that linking volatility to total volume does not extract all information. When volume is partitioned into expected and unexpected components, the paper finds that unexpected volume shocks have a larger effect on volatility. Further, the relation is asymmetric; the impact of positive unexpected volume shocks on volatility is larger than the impact of negative shocks. Finally, consistent with theories of market depth, the study shows large open interest mitigates volatility.

I. Introduction

A widely documented empirical regularity is the positive contemporaneous correlation between trading volume and price volatility. Karpoff (1987) cites 18 separate studies that document this relation in a variety of financial markets including equities, futures, currencies, and Treasury bills. A subset of these studies also documents an asymmetry in the relation; positive price shocks are associated with larger volumes than are negative price shocks.

This study provides additional empirical evidence on the relations between price volatility and trading activity for a selection of futures contracts. The work extends previous research along a number of dimensions. First, this analysis examines the contribution of market depth, which Kyle (1985) defines as the order flow required to move prices by one unit. It is argued that depth varies with recent trading activity, which is proxied by endogenously determined open interest. Further, when this proxy is large, it is expected that observed volatility, conditional on contemporaneous volume, would be lower.

Second, the study investigates whether the effect of volume on volatility is homogeneous by separating volume into its expected and unexpected components and allowing each component to have a separable effect on observed price volatility. The paper also examines whether the volatility of prices responds asymmetrically

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*School of Business, Arizona State University, Tempe, AZ 85287–3906, and School of Business Administration, University of Michigan, Ann Arbor, MI 48109–1234, respectively. The authors thank Eric Amel, Kalok Chan, Margaret Monroe, Jon Paul, Doug Skinner, and an anonymous **JFQA** referee for valuable comments. They also acknowledge Jay Coughenour's laudable research assistance.

to volume shocks depending on whether volume is above or below its expected value.

Finally, empirical methods are employed that explicitly accommodate persistence in volumes and volatilities. Given the economic and econometric importance attached to these persistencies (e.g., Lamoureux and Lastrapes (1990), Schwert (1990), Bessembinder and Seguin (1992)), this study represents a methodological improvement over previous studies of the volume-volatility relation.

For a sample composed of eight agricultural, metal, currency, and financial futures markets, the paper finds that futures price volatility is positively related to both the expected and unexpected components of volume. However, unexpected shocks have, on average, seven times the effect on price volatility as changes in expected volume. Further, the paper finds the relation between unexpected volume shocks and contemporaneous volatility to be asymmetric, with positive volume shocks having an economically and statistically larger effect on volatility.

The relation between price volatility and open interest, which is used as a proxy for market depth, is also examined. Open interest is partitioned into expected and unexpected components, and it is documented that volatility is negatively related to the expected level of open interest in all eight markets. Further, the change in open interest during the day has explanatory power, even when volume series are included in the specification.

II. Volume, Volatility, and Liquidity

Several theories predict a positive contemporaneous relation between price volatility and trading volume. The "mixture of distributions" hypothesis (Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), and Harris (1986)) assumes that the variance per transaction is monotonically related to the volume of that transaction. Consequently, price changes are sampled from a mixture of normal distributions with either the volume per transaction, number of transactions, or number of information arrivals per observation unit acting as the mixing variable.

Copeland (1976), (1977), Morse (1981), Jennings, Starks, and Fellingham (1981), and Jennings and Barry (1983) develop and extend "sequential arrival of information" models where new information is disseminated sequentially to traders, and traders not yet informed cannot perfectly infer the presence of informed trading. Consequently, the sequential arrival of new information to the market generates both trading volume and price movements, with both increasing during periods characterized by numerous information shocks.

A third explanation is found in Admati and Pfleiderer (1988), who show that traders with trade timing discretion choose to trade when recent volume is large. Therefore, transactions and price movements are bunched in time, and the effect of volume on price movements depends on recent volume levels (Admati and Pfleiderer, Hypothesis 3). Kyle (1985) defines depth as the volume of (unanticipated) order flows required to move prices by one unit. His model implies that larger volumes support more informed traders, and that depth varies with the level of noninformational trading activity.

III. Empirical Methods and Data

Schwert (1990) introduces a procedure for computing unbiased estimates of conditional daily return standard deviations while accommodating known empirical regularities including the persistence of volatility. Following Schwert and Seguin (1990) and Davidian and Carroll (1987), the paper iterates between a conditional mean and a conditional volatility equation of the form,

$$(1) \qquad R_t = \alpha + \sum_{j=1}^{n} \gamma_j R_{t-j} + \sum_{i=1}^{4} \rho_i d_i + \sum_{j=1}^{n} \pi_j \hat{\sigma}_{t-j} + U_t,$$

$$(2) \qquad \hat{\sigma}_t = \delta + \sum_{j=1}^{n} \omega_j \hat{U}_{t-j} + \sum_{i=1}^{4} \eta_i d_i + \sum_{k=1}^{m} \mu_j A_k + \sum_{j=1}^{n} \beta_j \hat{\sigma}_{t-j} + e_t,$$

where R_t is the observed percent change in the futures price on day t, and the four indicator variables (d_i) capture differences in means and variances by day of the week (French (1980), French and Roll (1986)). Fitted values from Equation (1) represent expected returns conditional on the day of the week and on recent levels of volatility. Residuals from Equation (1), denoted U_t, represent unexpected returns. Following Schwert and Seguin (1990), the study employs estimates of daily standard deviations obtained using the transformation,[1]

$$(3) \qquad \hat{\sigma}_t = |\hat{U}_t|\sqrt{\pi/2}.$$

In Equation (2), conditional standard deviations are estimated by regressing volatility estimates against day of the week indicators, lagged unexpected returns, and lags of the estimated standard deviation series. Lags of the estimated standard deviation series are included to measure and accommodate any persistence in futures price volatility.

Past (signed) unexpected returns in the specification are included for three reasons. First, many past studies of equity spot market volatilities (including Schwert (1990)), find that these lags have explanatory power. Second, past studies, including 12 cited by Karpoff (1987), have also documented a correlation between volume and contemporaneous signed price changes. With the exception of Rogalski (1978), who examines stocks and associated warrants, none of these studies considers recent (rather than simultaneous) price changes.

Finally, including both lagged signed forecast errors and the lagged $\hat{\sigma}_t$s, which are proportional to unsigned forecast errors, allows the relation between

[1]From Patel and Read ((1981), p. 34, Theorem 2.6.1.2), if $x \sim N(0, \sigma^2)$ then $E(|x|) = \sqrt{(2/\pi)}\sigma$. Since x in this case is a vector of OLS residuals, the assumption that the mean of the distribution is zero is not a problem. The distributional assumption of conditional normality must be maintained. The presence of skewness or kurtosis could impart a bias in mean absolute deviation-based estimates of volatility. However, evidence in Seguin (1991) indicates that the effect of changes in higher moments on inferences made using this class of volatility estimate are negligible for equity returns.

unexpected return and volatility to vary depending on the sign of the unexpected return.[2] More specifically, (2) and (3) imply

$$\frac{\partial \hat{\sigma}_t}{\partial \hat{U}_{t-j}} = \begin{cases} \omega_j + \sqrt{2/\pi}\beta_j & \text{if } \hat{U}_{t-j} > 0 \\ \omega_j - \sqrt{2/\pi}\beta_j & \text{if } \hat{U}_{t-j} < 0. \end{cases} \tag{4}$$

If, for example, $\beta_j < 0$, negative return shocks lead to larger revisions in subsequent conditional volatilities. Though the theoretic basis underpinning this asymmetry is not necessarily pertinent to futures markets,[3] disallowing asymmetry may reduce explanatory power or create a statistical misspecification.

As recommended by Davidian and Carroll (1987), Equations (1) and (2) are estimated sequentially. First, Equation (1) is estimated without lagged volatility estimates. The transformation in Equation (3) is applied to the residuals and Equation (2) is estimated. Fitted values from Equation (2) are then employed as regressors in reestimating (1). Finally, Equation (2) is reestimated using residuals from the consistent estimation of Equation (1).

A. The Selection of Trading Activity Variables

Following Bessembinder and Seguin (1992) and Moser (1992), the effects of trading activity on conditional volatilities are evaluated by including trading activity variables, A_k in Equation 2, where A_k are m trading activity variables. Existing theoretical analyses of volume and volatility do not distinguish between anticipated and unanticipated components of volume. However, it has been frequently documented that trading volumes are highly serially correlated, indicating that volume is highly forecastable. Given the emphasis on differentiating the effects of expected versus unexpected components of variables across a wide spectrum of economic inquiry, the paper empirically examines whether surprises in trading volume convey more information and, thus, have a larger effect on prices than forecastable volumes. Accordingly, multivariate forecasting methods (detailed below) are employed to partition futures volume into expected and unexpected components.

A distinguishing feature of futures markets is that the number of contracts in existence is endogenously determined at each point in time. Thus, open interest data provide an additional measure of trading activity. Surprisingly little research has exploited the existence of open interest data. One exception is Figlewski (1981) who regresses monthly spot volatility on, among other factors, average monthly futures volume and open interest in his study of GNMA futures. He reports positive coefficients for both variables.

Open interest is partitioned into expected and unexpected components, again using multivariate forecasting methods. This allows analysis of open interest along

[2]There are numerous ARCH-GARCH specifications that allow for asymmetries in the return-volatility relation, including the asymmetric GARCH (Engle (1990)), the nonlinear asymmetric GARCH (Glosten, Jaganathan, and Runkle (1989)) and the exponential GARCH (Nelson (1991)). See Pagan and Schwert (1990) for a review.

[3]For equities, a negative correlation between past returns and volatility is predicted due to the existence of leverage (Christie (1982)). Since there are no debt claims against the assets underlying futures positions, it is difficult to conceive of personal or financial leverage effects for futures.

two dimensions. First, the expected portion reflects open interest as of the beginning of the trading day. The second dimension, unexpected open interest, captures unanticipated changes in net contract formation. The marginal explanatory power of variables other than lagged open interest is small. Consequently, expected open interest is approximately equal to yesterday's level, while unexpected open interest is approximately equal to the change in open interest during the day.

Open interest measures are pertinent for at least two reasons. First, since many speculators are "day traders" who do not hold open positions overnight, open interest as of the close of trading likely reflects primarily hedging activity and, thus, proxies for the amount of uninformed trading.[4] Using open interest in conjunction with volume data may provide insights into the price effects of market activity generated by informed versus uninformed traders or hedgers versus speculators.

Second, market depth depends on the willingness and ability of traders to risk capital and take positions in response to a perceived deviation of price from intrinsic value. Willingness is in part determined by traders' risk aversion, while ability is partly determined by existing wealth constraints. If these and other underlying determinants of depth do not change quickly, then a variable constructed from lagged open interest should contain information on current depth. Also, the unexpected change in open interest during the session is, almost tautologically, a close proxy for the current willingness of futures traders, in aggregate, to risk capital.

Unexpected open interest and unexpected volume are not predetermined variables, and their inclusion in Equation (2) does not imply that volume shocks and changes in positions necessarily induce or cause (in either an economic or statistical sense) changes in prices. This work concurs with Clark (1973), Tauchen and Pitts (1983), and Epps and Epps (1976), who argue that volume and volatility are jointly endogenous variables that covary in response to external order or information shocks. The primary econometric objective here is to document partial relations between price revisions and shocks to volume and open interest while conditioning on levels of recent activity. The specification (2) allows for this investigation.

B. Data

Daily settlement prices, trading volumes, and open interests for all outstanding maturities on eight futures markets were obtained over the interval May 1982 to March 1990, from the Columbia Business School Futures Center and from Data Resources Inc., supplemented with data from the Dow Jones News Retrieval Service and the Wall Street Journal. To provide a comprehensive cross section, two currencies (Deutsche marks and Japanese yen), two metals (gold and silver), two agricultural commodities (cotton and wheat), and two financial contracts (Treasury bonds and Treasury bills) were chosen.

To obtain an aggregate measure of activity in each market, volumes and open interest are summed across all outstanding maturities. To obtain a representative

[4]The authors thank Barry Schachter who suggested this interpretation.

"return" series, the percentage change in the settlement price[5] of the contract closest to expiration is used, except within the delivery month, when the change in the second nearest contract is used. Each return is computed using successive prices on a contract with a given expiration date. The resulting returns series have the desirable properties that i) they could be realized by a trader following the simple strategy of rolling into the second nearest contract at the beginning of the delivery month, and ii) they are computed using prices from the most actively traded contracts and, thus, are most likely to be representative of "firm" prices at which reasonable quantities could be transacted.

At the suggestion of the referee, the robustness of the paper's results to the use of returns computed from a "composite" contract constructed in the manner of Clark (1973) was investigated. The "near" and "Clark" returns are very highly correlated (e.g., 0.997 for Deutsche mark returns). Empirical results are robust to use of returns constructed in this manner.

Panel A of Table 1 presents some details of the contracts examined here, including contract specifications and futures exchange. The average dollar value of a contract and the average dollar value of contracts traded per day are also presented.

Panel B of Table 1 reports means, standard deviations, and partial autocorrelations for percent price changes and their absolute values, contract volumes, and open interests. The most volatile market is silver with a return standard deviation exceeding 2 percent per day. In contrast, the daily standard deviation for Treasury bills is 0.028 percent. There is little evidence of return predictability; with two exceptions, partial autocorrelations of returns at all lags are statistically insignificant.

TABLE 1

Panel A Characteristics of Futures Markets in Sample

Deliverable Good	Futures Exchange	Contract Size	Mean Contract Size ($ 000)[a]	Mean Daily Dollar Volume ($ Millions[b])
Yen	Chicago Mercantile Exchange	12,500,000 yen	71	1,224
Deutsche Mark	Chicago Mercantile Exchange	125,000 DM	57	1,290
Treasury Bond	Chicago Board of Trade	$100,000 face value, 8% coupon	83	15,871
Treasury Bill	Chicago Mercantile Exchange	$1,000,000 face value	982	10,295
Wheat	Chicago Board of Trade	5000 bushels	17	192
Cotton	New York Cotton Exchange	50,000 lb	33	167
Gold	Commodity Exchange, Inc	100 troy oz	39	1,499
Silver	Commodity Exchange, Inc	5000 troy oz	31	602

[a]Contract size times mean nearby futures price (5/82 to 3/90)
[b]Contract size times mean nearby futures price times mean volume (5/82 to 3/90)

(continued on next page)

[5]A quoted T-bill futures price differs from a delivery price. The quoted price, Q, obligates the short to deliver 90-day T-bills at contract maturity for a price such that the *annualized* return to the buyer over the 90 days is $(100 - Q)$ percent. Quoted futures prices are converted to implied delivery prices, $D = (75 + (Q/4))$ and returns are computed based on these delivery prices.

TABLE 1 (contınued)

Panel B Summary Statistics on Futures Returns, Risk, and Trading Volumes

(Daıly Observations, 5/82 to 3/90)

	Mean	Standard Deviation	Partial Autocorrelation at Lag· 1	2	3	4	5	Unıt Root Test Statıstıc[1]
Yen Futures								
Return (%)	0 008	0 672	0 0035	0 0240	0 0376	0 0126	0 0143	
Absolute Return (%)	0 474	0 476	0 1150*	0 0528	0 1259*	0 0422	0 0735	
Volume	17491	11213	0 7585*	0 2333*	0 2131*	0 1487*	0 1234*	−6 24*
Open Interest	37574	15848	0 9826*	−0 0700	0 0169	−0 0393	−0 0031	−4 12*
Deutsche Mark Futures								
Return (%)	0 004	0 740	−0 0161	0 0192	0 0332	−0 0284	0 0042	
Absolute Return (%)	0 546	0 499	0 0520	0 0499	0 0916*	0 0790*	0 0844*	
Volume	22395	11506	0 7358*	0 2073*	0 2091*	0 1796*	0 1176*	−6 47*
Open Interest	44395	18254	0 9834*	−0 0160	−0 0258	−0 0230	0 0139	−4 14*
T-Bond Futures								
Return (%)	0 034	0 796	0 0391	0 0251	0.0039	−0 0238	−0 0401	
Absolute Return (%)	0 593	0 531	0 0775*	0 0731	0 1203*	0 1333*	0 1258*	
Volume	191494	114531	0 6703*	0 3645*	0 2106*	0 2048*	0 2344*	−5 78*
Open Interest	258909	86020	0 9974*	0 0200	−0 0580	−0 0083	−0 0184	−1 79
T-Bıll Futures								
Return (%)	0 001	0 028	0 0693	0 0476	−0 0127	−0 0352	−0 0086	
Absolute Return (%)	0 018	0 021	0 2350*	0 1399*	0 1828*	0 1944*	0 1719*	
Volume	10485	7353	0 8177*	0 2533*	0 2327*	0 1605*	0 1158*	−5 36*
Open Interest	37025	10713	0 9968*	−0 1356*	−0 0823*	−0 0134	−0 0234	−2 57
Wheat Futures								
Return (%)	−0 005	1 267	0 0059	−0 1045*	0 0303	−0 0148	−0 0425	
Absolute Return (%)	0 934	0 856	0 2423*	0 1447*	0 1223*	0 0576	0 1524*	
Volume	11524	6119	0 6083*	0 1509*	0 1620*	0 0867*	0 1340*	−9 35*
Open Interest	46008	14220	0 8591*	0 2804*	0 2646*	0 1729*	0 1392*	−4 20*
Cotton Futures								
Return (%)	0 051	1 301	0 0423	−0 0754*	−0 0353	−0 0485	−0 0118	
Absolute Return (%)	0 948	0 897	0 2212*	0 1102*	0 1276*	0 1089*	0 1544*	
Volume	5059	2604	0 5834*	0 0945*	0 1775*	0 1290*	0 0720	−9 34*
Open Interest	29224	7777	0 9974*	−0 1324*	−0 0450	−0 0492	−0 0120	−1 87
Gold Futures								
Return (%)	−0 021	1 264	−0 0457	0 0423	0 0372	−0 0641	−0 0086	
Absolute Return (%)	0 847	0 939	0 2288*	0 1354*	0 1371*	0 1261*	0 1067*	
Volume	38307	16483	0 5127*	0 1813*	0 1107*	0 0960*	0 0487	−11 31*
Open Interest	139219	16710	0 9894*	−0 0857*	−0 0052	−0 0131	−0 0067	−1 42
Sılver Futures								
Return (%)	−0 021	2 057	0 0167	0 0350	0 0512	−0 0498	−0 0023	
Absolute Return (%)	1 403	1 504	0 2415*	0 1023*	0 1025*	0 1524*	0 1415*	
Volume	19526	10244	0 5927*	0 2007*	0 1222*	0 1335*	0 0600	−10 05*
Open Interest	72807	19823	0 9953*	−0 0106	−0 0277	−0 0151	−0 0123	−2 37

*Denotes statistical significance (p-value $<$ 0 05)

[1]This column reports modified Dickey-Fuller unit root test statistics For each volume and open interest series, the regression, $\Delta Y_t = \mu + \beta_1 Y_{t-1} + \Sigma_{j=1}^{5} \gamma_j \Delta Y_{t-j}$ ıs estimated, where $\Delta Y_{t-j} = Y_{t-j} - Y_{t-j-1}$ The test statıstics reported are associated with β_1 Here a * denotes rejection of the null of a unit root, since the test statistic is less than −2 87, the critical (α = 0 05) test statistic reported in Fuller (1976)

(continued on next page)

There is, however, evidence of persistence in volatility. Bollerslev (1988) recommends analyzing autocorrelations of the absolute value of a series to determine the time series properties of the variance of the original series. Here, the first order partial autocorrelations for the absolute value of returns are uniformly positive and are statistically significant for five contracts.[6] Further, there is at

[6]Since the largest first order partial autocorrelation is only 0.24, it is assumed that the variance series are stationary. Finding stationarity is econometrically fortunate since this avoids a variety of econometric complications. See Engle and Bollerslev (1986) and Plosser and Schwert (1978).

TABLE 1 (continued)

Panel C Correlations between Futures Returns, Absolute Returns, and Trading Volume
(Daily observations, 5/82 to 3/90)

	Correlation With	
	Absolute Returns	Trading Volume
Yen Futures		
Returns	0 139*	0 002
Absolute Returns		0 334*
Deutsch Mark Futures		
Returns	0 128*	0 084*
Absolute Returns		0 371*
T-Bond Futures		
Returns	0 095*	−0 028
Absolute Returns		0 176*
T-Bill Futures		
Returns	0 245*	0 154*
Absolute Returns		0 536*
Wheat Futures		
Returns	0 049*	−0 024
Absolute Returns		0 373*
Cotton Futures		
Returns	0 121*	−0 021
Absolute Returns		0 351*
Gold Futures		
Returns	−0 011	0 027
Absolute Returns		0 550*
Silver Futures		
Returns	−0 008	−0 019
Absolute Returns		0 408*

*Denotes statistical significance (p-value < 0 05)

least one positive and significant partial autocorrelation at higher lags for every contract. These results reinforce the need to accommodate volatility persistence in subsequent tests.

The most active market, measured by raw or dollar value-weighted volumes or open interest, is Treasury bonds, while the least active is cotton, with typical daily volumes roughly 40 times smaller, and dollar volumes roughly 100 times smaller. Volumes and open interests are, not surprisingly, highly autocorrelated. First order autocorrelation coefficients for volume vary from 0.513 (gold) to 0.818 (Treasury bills), while those for open interest uniformly exceed 0.85. Partial autocorrelation coefficients at higher lags are significant for every volume series and half the open interest series.

The final column in Panel B reports modified (5 lag) Dickey-Fuller test statistics for the presence of unit roots in volume and open interest series. Determining whether a series contains a unit root represents an important preliminary step in partitioning these series into expected and unexpected components. The existence of a unit root is rejected for all volume series and three open interest series. However, for the remaining five open interest series, tests suggest the existence of a unit root. The existence of a unit root is rejected for all 16 series after first differencing. Though not reported, the smallest (in absolute value) t-statistic was -15.3 after differencing.

Panel C reports contemporaneous correlations between signed returns, absolute returns, and trading volumes for each market. There is no evidence of skewness in the two metal contract return series. However, the remaining six contracts have statistically significant positive correlations between signed and unsigned returns, indicating positive return skewness. Consistent with prior studies, absolute returns are highly correlated with contemporaneous trading volumes in each of the eight markets. However, as discussed below, analyzing relations between volatility and total volume does not extract the maximum amount of information from the volume series.

There is less evidence of significant correlations between volumes and signed returns. Karpoff (1987) cites 12 studies that document a positive relation between the two in asset markets. Karpoff (1988) argues that this relation may be the manifestation of asymmetric costs for short versus long positions in asset markets and presents empirical results demonstrating that the relation is absent in futures markets, where such cost asymmetries do not exist. The results here are consistent with Karpoff's in six markets, but significant positive correlations between returns and volumes for the D-mark and T-bill contracts are found.

C. Expected and Unexpected Activity Variables

Results of the above tests indicate the presence of a unit root in five of the open interest series. Consequently, estimation involving these series is performed using first differences. Initially, univariate Box-Jenkins methods are employed to partition volume and open interest into expected and unexpected components. Following Schwert (1990), this study does not attempt to find an ideal model for each series, but instead chooses an arbitrarily long set of autoregressive coefficients. Thus, the eight volume and three stationary open interest series are partitioned using an AR(10), while an ARIMA(10,1,0) is used for the five nonstationary open interest series. This step yields 16 series of one-step-ahead forecast errors,

$$\text{(5)} \qquad \hat{\varepsilon}_{it} = \text{Activity}_{it} - E\{\text{Activity}_{it} | \text{Activity}_{i,t-\tau},\ \tau = 1, \ldots 10\} \quad \text{for } i = 1, \ldots, 16.$$

Numerous studies (including Schwert (1990) and Gallant, Rossi, and Tauchen (1990)) provide evidence that past volatilities have predictive power for forecasting volumes. To capture this power, the one-step-ahead forecast errors are regressed from the univariate specifications against lags of absolute return (scaled by $\sqrt{\pi/2}$), volume, (differenced) open interest, and days until the next contract expiration,

$$\text{(6)} \qquad \hat{\varepsilon}_{it} = \psi + \sum_{j=1}^{10} \rho_{ij} \hat{\sigma}_{it-j} + \sum_{k=1}^{10} \lambda_{ik} \text{Vol}_{it-k} + \sum_{m=1}^{10} \mu_m \, \text{OpIn}_{it-m} + \phi_i \, \text{DTE}_i + v_{it},$$

where Vol_i, OpIn_i, and DTE_i are volumes, open interests, and days-to-expiration corresponding to series $i = 1, \ldots, 16$.

The unexpected component of each series is defined as $\hat{v}_{it}$, the estimated residual from this second step, while the expected component is defined as the difference between the actual series and the unexpected component, $\text{Activity}_{it} - \hat{v}_{it}$. Thus, the anticipated or expected component of each volume and open interest series is conditioned on past values of volume, open interest, volatility, and stage of the contract life cycle. Though statistically significant, the explanatory power of these second step regressions is typically low, so results are not meaningfully altered when univariate forecasts are used.

IV. Empirical Results

Equations (1) and (2) are estimated for each of the eight contracts. The results of estimating the conditional mean Equation (1) appear in Table 2. Consistent with weak-form efficiency, ex ante known regressors have little predictive power for realized returns, with the largest adjusted R^2 equalling 3 percent. Day-of-the-week dummies are rarely significant, with only yen and cotton displaying significant day-of-the-week effects. Lagged returns have significant explanatory power for only one contract (yen). In contrast, lagged volatilities are positive and jointly significant for six markets.[7] This last result is consistent with the existence of a positive relation between required rates of return and expected volatility (French, Schwert, and Stambaugh (1987) and Tauchen and Hussey (1991)).

Table 3 reports estimates for the conditional volatility Equation (2), including expected and unexpected volumes and open interests. All eight volatility series exhibit significant persistence. The sum of estimated coefficients associated with 10 lagged estimated volatilities ranges from 0.31 for Treasury bills to 0.66 for cotton. Note that these estimates probably underestimate the degree of persistence since i) past volumes are related to past volatilities and ii) the specification includes past volumes (in the expected volume variable). This strong persistence reinforces this paper's beliefs that researchers interested in documenting contemporaneous determinants of volatility must accommodate past volatility shocks.[8] These results contrast with those of Lamoureax and Lastrapes (1989) who find that GARCH effects in equity spot volatility are not significant when volume is included in the specification.

The sum of estimated coefficients on lagged (signed) unexpected return shocks is positive for six of the eight contracts, indicating that positive return shocks have a larger effect on subsequent estimated volatilities (from Equation (4)), though this relation is statistically significant for Treasury bills only. Therefore, volatilities generated in these nonequity markets where issuer leverage is irrelevant show little evidence of the "leverage effect."

[7] It is recognized that standard errors associated with lagged volatilities are biased downward due to the "generated regressor" problem when estimating (1) or (2). Fortunately, inferences central to this analysis are not affected. First, generated regressands do not impart biases on standard errors, so use of a generated dependent variable in the volatility regression does not complicate inference. Further, the important activity variables (volume and open interest) are partitioned in a linear auxiliary regression, so the standard errors associated with these variables are unbiased under the null that the coefficient is zero (Pagan (1984), p. 232).

[8] Aside from the economic importance of controlling for past volatility, this control is also econometrically necessary. In formulations estimated without these lags, Durbin-Watson statistics uniformly fall far below 2.0.

TABLE 2

Autoregressive Model for Daily Futures Returns (5/82 to 3/90)

Futures Market	Yen	D-Mark	T-Bond	T-Bill	Wheat	Cotton	Gold	Silver
Intercept	−0 0407	−0 0552	−0.0759	−0.0043	0 0358	−0 0836	−0 0817	−0.1640
	(−0 789)	(−0 912)	(−1 136)	(−1 974)*	(0 291)	(−0.856)	(−0 865)	(−1.234)
Daily Dummies								
Monday	0.0258	−0 0371	−0 0161	−0 0010	−0.0768	−0.0989	−0 0038	−0.0597
	(0 495)	(−0 665)	(−0 259)	(−0 472)	(−0 840)	(−1.014)	(−0.037)	(--0.365)
Tuesday	0 0265	−0 0429	0 0785	0.0000	−0.0886	−0 2297	−0 0396	−0.0820
	(0 550)	(−0.803)	(1 302)	(−0 004)	(−1.065)	(−2 632)**	(−0.427)	(−0 578)
Wednesday	0.0966	0.0371	−0.0443	0.0004	0 0946	0 0438	−0 0072	0.0695
	(2 114)*	(0 728)	(−0 774)	(0.214)	(1 161)	(0 507)	(−0.084)	(0.513)
Thursday	0 1055	0 0448	0 0049	0 0021	−0.1074	−0.0618	0 0357	0 0206
	(2.200)*	(0.862)	(0 084)	(1.074)	(−1.249)	(−0 678)	(0 401)	(0.148)
Sum of 10 Lagged Volatilities	−0 003	0 087	0 143	0.243	−0.003	0.179	0 064	0.090
	(0 0024)	(1 7310)*	(5 3426)**	(25.3444)**	(0 0037)	(11 4615)**	(1 8428)*	(3.5797)**
Sum of 10 Lagged Unexpected Returns	0 161	0 078	0.013	−0 025	−0 062	−0.083	−0.057	−0.013
	(5 7863)**	(1 2617)	(0 0325)	(0.1293)	(0 6492)	(1 1430)	(0 6008)	(0 0379)
Durbin-Watson	1 999	1 998	1 997	1.999	2 000	1 997	1.997	1 998
Adjusted R^2	0 0056	0 0050	0.0030	0 0305	0.0216	0 0175	0 0181	0.0150
Regression *F*-Statistic	1 47*	1 42*	1 25	3 60**	2.82**	2 46**	2.51**	2.25**

Test statistics for individual coefficients are *t*-statistics computed using White (1980) standard errors Test statistics for lagged coefficients are *F*-statistics for the hypothesis that the sum of the 10 coefficients is zero

* (**) Denotes statistical significance at 0.10 (0.01) level

TABLE 3

Regressions of Estimated Daily Return Standard Deviations on Expected and Unexpected Trading Activity (5/82 to 3/90)

Futures Market	Yen	D-Mark	T-Bond	T-Bill	Wheat	Cotton	Gold	Silver
Intercept	0 4386 (9 395)**	0 4226 (8 811)**	0 5777 (7 568)**	0 0176 (6 998)**	0 4452 (3.803)**	0 5273 (5 554)**	0 9974 (5 394)**	1 1490 (3.912)**
Expected Volume	0 1213 (4.348)**	0 0762 (2 783)**	0.0031 (1 333)	0.0151 (5.997)**	0 2020 (2 429)*	0 1769 (0 864)	0 0849 (2 872)**	0 0970 (1 389)*
Unexpected Volume	0 4517 (15 875)**	0.4617 (18.997)**	0 0300 (10 231)**	0 0273 (11 790)**	0.9805 (11 573)**	2 2726 (15.649)**	0.4574 (19.525)**	1.0939 (19.296)**
Expected Open Interest	−0.0661 (−4 202)**	−0 0329 (−2.650)**	−0 0069 (−2 736)**	−0.0036 (−4 119)**	−0 0594 (−2.762)**	−0.0728 (−1 889)*	−0 0475 (−3 725)**	−0 0679 (−2 589)**
Unexpected Open Interest	−0.0997 (−2 321)*	−0 0777 (−1 982)*	0.0576 (2 014)*	−0 0140 (−1 098)	−0 1402 (−1 804)*	−0.9484 (−2 037)*	−0 1004 (−0 8805)	−0.2431 (−0 777)
Sum of 10 Lagged Volatilities	0 3963 (59 406)**	0 4403 (71.047)**	0 5314 (120 465)**	0 3051 (29 825)**	0 6281 (243 317)**	0.6611 (281 664)**	0.5055 (124 356)**	0.5734 (183 506)**
Sum of 10 Lagged Unexpected Returns	0 0676 (2 326)	0 0084 (0 043)	−0.0304 (0 511)	0 0789 (3 200)*	0 0396 (0.948)	−0 0361 (0.745)	0.0018 (0 002)	0 0480 (1.267)
Durbin-Watson	2.016	2 020	1 968	1.989	2 019	1 990	2 016	1.991
Adjusted R^2	0 2866	0 3080	0 1982	0 3515	0 2938	0 2802	0.3907	0 3359

The dependent variable is the absolute value of $\sqrt{\pi/2}$ times the unexpected return in the futures market. Volumes are in units of 10,000 contracts. Expected and unexpected series are fitted values and residuals from multivariate forecasting models fitted to the original series Test statistics for individual coefficients are t-statistics computed using White (1980) standard errors Test statistics for lagged coefficients are F-statistics for the hypothesis that the sum of the 10 coefficients is zero

* (**)Denotes statistical significance at 0 10 (0.01) level

A. Effects of Volume and Open Interest

All 16 coefficient estimates for expected and unexpected volume are positive, and 14 are significantly so. However, magnitudes and statistical significance levels indicate that expected and unexpected volumes have heterogeneous effects on volatility. Estimated coefficients associated with volume shocks are uniformly higher than those associated with expected volume, with the ratio of the two varying from 1.8 for Treasury bills to 12.8 for cotton. Typically, a one unit change in unexpected volume has roughly seven times the effect on volatility as a unit change in expected or anticipated volume.

A pervasive, important finding of this study is that all eight coefficients relating expected open interest to volatility are negative and significant. Documenting a significant negative coefficient for the effect of expected open interest on volatility (conditional on trading volume) is consistent with the joint hypothesis outlined above that i) expected open interest is related to the number of traders or amount of capital affiliated with a market, ii) these factors enhance market depth, and iii) there are lower volatility shocks associated with a given volume in deeper markets.

Seven of the eight estimated coefficients relating unexpected changes in open interest to volatility are also negative (the Treasury bond coefficient is the exception). These negative coefficients imply that an increase in open interest during the trading day lessens the impact of a volume shock on volatility. The magnitude of this mitigating effect can be roughly estimated by comparing the coefficient associated with unexpected open interest to the coefficient associated with unexpected volume. For example, for the yen contract, the marginal effect of an unexpected volume of 10,000 contracts on volatility is 0.4517 ± 0.0997 (or $\pm$ 22.1 percent), depending on whether open interest was reduced or increased.

Point estimates imply that a trade that increases both volume during the day and open interest at the end of the day has a smaller effect on volatility than a trade that increases volume but decreases open interest or leaves it unchanged.[9] This finding may represent a useful starting point for researchers interested in the impact of different types of trades on price revision. For example, if end-of-day open interest is dominated by relatively uninformed hedgers, then the change in open interest provides information on the average informativeness of traders during the day. The point estimates suggest that for two sessions with equal levels of both expected and unexpected volumes, the price revision will be smaller for the session with an increase in open interest. This result may reflect the fact that more of the unexpected volume in this session is attributable to comparatively uninformed hedgers.

B. Asymmetries in Volume and Open Interest Shocks

Table 4 reports results obtained when the effects of unexpected changes in volume and open interest on volatility are allowed to vary with the sign of the shock. Indicator variables are defined that equal zero for a negative shock (volumes below

[9]The effect of a trade on open interest depends on the current net positions of the accounts (not the traders) involved. To illustrate, assume A has a long position of two contracts, while B and C are short two. If A and B trade with A taking the long position, open interest increases. If they trade and B takes the long position, open interest declines. If B and C trade, open interest remains unchanged.

their expected levels) and one for a positive shock (volumes above their expected levels). The product of the indicator and the unexpected volume series is also created. Therefore, the coefficient associated with the unexpected series represents the marginal impact of a negative shock on volatility, while the marginal effect of a positive shock can be estimated by adding the coefficients associated with the unexpected series and the product of the series and the indicator variable.

Significant asymmetries exist for both unexpected trading activity variables, though the natures of the asymmetries differ. The coefficients associated with unexpected volume shocks are uniformly positive and significant: as above, negative volume shocks are associated with lower levels of volatility. All but one of the cross-product terms on unexpected volume are positive, six significantly so. This reinforces the previous finding that positive shocks are associated with higher levels of volatility, and further indicates that positive shocks have a larger effect on volatility than negative shocks.

A rough guide to the magnitude of the asymmetry is the ratio of the estimated coefficient associated with positive shocks (which is the sum of the unexpected volume coefficient plus the cross-product coefficient) to the estimated coefficient associated with negative shocks (which is the unexpected volume coefficient). Across all eight markets, the average of these ratios is 1.76. This ratio indicates little asymmetry for wheat (with a ratio of 0.91). In contrast, positive volume shocks have more than twice the effect on price revisions as negative shocks for yen (with a ratio of 2.37) and gold (2.41).

The asymmetry associated with open interest takes a different form. Estimated coefficients associated with unexpected open interest are negative. This suggests that, ceteris paribus, an unanticipated reduction in open interest is associated with higher volatility. Further, for six contracts, the coefficients associated with the cross-products are positive and larger in magnitude than the unexpected open interest coefficient. Therefore, the sum of the two, which measures the effect of an unanticipated increase in open interest, is positive, indicating that an unanticipated increase in open interest is also associated with higher volatility. Finally, note that for two contracts (yen and Treasury bills), the cross-product coefficient is roughly twice the magnitude of the coefficient on negative shocks. Therefore, the positive shock effect is of the same magnitude but of the opposite sign as the negative shock coefficient. For these series, it may be parsimonious to model volatility as a function of the absolute value of unanticipated open interest.

C. Inter-Market Comparisons

Given the diversity of specifications for the eight contracts examined, it is difficult to use the numbers presented in Tables 3 and 4 to make inter-market comparisons of depth. Though cross-sectional comparisons are not central to this study, the estimates provided in Tables 3 and 4 can be used to construct reasonable—though inexact—measures for determining those markets that are most able to withstand volume shocks. Measures of Kyle's notion of depth as the order flow necessary to move prices by one unit are presented. Table 5 provides dollar trading volumes required to move prices for two different definitions of "a unit."

TABLE 4

Regressions of Estimated Daily Return Standard Deviations on Trading Activity, Allowing for Asymmetries (5/82 to 3/90)

Futures Market	Yen	D-Mark	T-Bond	T-Bill	Wheat	Cotton	Gold	Silver
Intercept	0.3974 (8 739)**	0 3933 (8 183)**	0 5606 (7 178)**	0.0157 (6 635)**	0 4541 (3 970)**	0 4766 (4 937)**	0.8359 (4 525)**	1 1713 (3 979)**
Expected Volume	0 0857 (3 108)**	0.0578 (2 156)*	0 0004 (0 196)	0 0140 (5 979)**	0.2155 (2 481)*	0 0128 (0 059)	0 0531 (1.790)*	0 0095 (0.129)
Unexpected Volume	0 2314 (6.097)**	0 2953 (8 596)**	0 0186 (2 95)**	0.0207 (6.092)**	1 0435 (9 353)**	1.5093 (5 576)**	0 2357 (7 108)**	0 8611 (9 047)**
Unexpected Volume *Voldum	0 3196 (3 998)**	0 2814 (3 940)**	0 0177 (1.985)*	0 0087 (1 597)	−0 0910 (−0 409)	1 1054 (2 361)*	0 3313 (4 639)**	0 3022 (1 814)*
Expected Open Interest	−0.0710 (−4 589)**	−0 0384 (−3 136)**	−0 0080 (−3.307)**	−0.0035 (−4.183)**	−0 0602 (−2 840)**	−0 0745 (−1.943)*	−0 0488 (−3 991)**	−0 0849 (−3 233)**
Unexpected Open Interest	−0 1734 (−2 742)**	−0 0832 (−1 717)*	−0 0138 (−0.297)	−0 0361 (1 347)	−0.1074 (−1 081)	−2.0599 (−2 568)**	−0 5821 (−3 066)**	−1 6303 (−4 314)**
Unexpected Open Interest *Opindum	0 3209 (1 965)*	0 0438 (0 371)	0.1441 (1 621)	0 0438 (1 226)	−0 0430 (−0 313)	2 1029 (1 650)*	0 7534 (2 358)*	3 3068 (3 593)**
Sum of 10 Lagged Volatilities	0.4258 (69 354)**	0 4530 (75 950)**	0 5511 (128 024)**	0 3127 (31 494)**	0 6268 (240 697)**	0 6697 (289 331)**	0 5452 (146 009)**	0 5865 (192 817)**
Sum of 10 Lagged Unexpected Returns	0 0654 (2 209)	0 0050 (0 015)	−0 0350 (0 668)	0 0769 (3 056)*	0 0407 (0 9963)	−0 0335 (0 6438)	−0 0066 (0 0254)	0 04144 (0 9534)
Durbin-Watson	2 038	2 028	1 973	2 010	2 019	1 990	2 016	1.986
Adjusted R^2	0 3004	0 3165	0 2032	0 3558	0 2934	0 2839	0 4058	0 3449

The dependent variable is the absolute value of $\sqrt{\pi/2}$ times the unexpected return in the futures market Volumes are in units of 10,000 contracts Expected and unexpected series are fitted values and residuals from multivariate forecasting models fitted to the original series Voldum and Opindum are equal to 1 if unexpected volume and unexpected open interest are respectively positive, and zero otherwise Test statistics for individual coefficients are *t*-statistics computed using White (1980) standard errors Test statistics for lagged coefficients are *F*-statistics for the hypothesis that the sum of the 10 coefficients is zero

* (**)Denotes statistical significance at 0 10 (0 01) level

The second column displays the approximate dollar trading volumes required to alter the futures price by 1 percent. To illustrate, this dollar amount (e.g., $126.2 million for cotton) divided by the average value of a contract from Table 1 ($33,000 for cotton) yields a number of contracts (3,824 for cotton) that, when multiplied by the estimated coefficient linking volatility to unexpected positive volume shocks (per 10,000 contracts) from Table 5 (1.5093+1.1054 = 2.6147 for cotton), yields 1 percent.

TABLE 5

Estimates of Market "Depth" by Deliverable Good

Deliverable Good	Capital Required to Move Futures Price by 1 0% ($ million)	Capital Required to Move Futures Price by Average Absolute Return ($ million)
Cotton	126 21	119.65
Wheat	178 48	166.70
Silver	266.48	373 88
Gold	687.83	582.59
Marks	988.38	539 66
Yen	1,288.57	610 78
T-Bonds	22,865 01	13,558 95
T-Bills	333,900.03	6,010 20

For each of the goods listed in the first column, the second column displays the approximate unexpected dollar volume required to alter the futures price by 1% This dollar amount ($126.2 million for cotton) divided by the average value of a contract from Table 1 ($33,000 for cotton) yields a sufficient number of contracts (3,824 contracts for cotton) which, when multiplied by the estimated coefficient linking volatility to unexpected positive volume shocks (per 10,000 contracts) from Table 5 (1.5093 + 1.1054 = 2.6147), yields 1%. The third column displays the approximate unexpected dollar volume required to alter the futures price by the average absolute price change for that contract. This dollar amount ($119.7 million for cotton) divided by the average value of a contract from Table 1 ($33,000 for cotton) yields a sufficient number of contracts (3,626 contracts for cotton) which, when multiplied by the estimated coefficient linking volatility to unexpected positive volume shocks (per 10,000 contracts) from Table 5 (1.5093 + 1.1054 = 2 6147) yields the mean of the absolute return series for that good (0.948% for cotton).

Arguably, defining a "unit" as 1 percent may not yield a fair benchmark. A 1-percent daily movement in the future price of gold is not rare, while a 1-percent movement in the price of a 90-day Treasury bill is virtually unknown. This is reflected in the daily standard deviations or the means of the absolute percent returns displayed in Table 1, Panel B. The mean absolute return for gold is roughly 47 times larger than that for Treasury bills. To accommodate this difference, the experiment was repeated by calculating the approximate dollar volume required to alter the futures price by that contract's average absolute return. This dollar amount (e.g., $119.7 million for cotton) divided by the average value of a contract from Table 1 ($33,000) yields the number of contracts (3,626) that, when multiplied by the estimated coefficient linking volatility to unexpected positive volume shocks (per 10,000 contracts) from Table 5 (1.5093 + 1.1054 = 2.6147 for cotton), yields the mean of the absolute return series for that good (0.948 percent for cotton). The relative rankings and conclusions of this analysis are identical when the positive shock or negative shock coefficient from Table 5, or the unexpected coefficient from Table 3, is employed. Further, the rankings are robust to alternate assumptions made about the change in open interest associated with these hypothetical volume shocks.

Regardless of the measure employed, depth is an order of magnitude greater on the two financial markets. Even after adjusting for the small daily dispersion of Treasury bill prices, market depth for the bill and bond futures contracts is roughly 12 and 25 *times* the median depth of the remaining six contracts. By either measure, depth on the metal and currency markets is three- to five-fold greater than depth on the two agricultural markets.

Though this study focuses primarily on intertemporal changes in depth, volume, and volatility, note that estimated market depth also appears to be related to typical order flows cross-sectionally. The Spearman rank correlation between mean daily dollar volumes (reported in the right column of Table 1, Panel A) and the quantity of capital required to move prices by 1 percent (reported in the center column of Table 5) is 0.881, with an associated Pearson moment correlation of 0.498. This cross-sectional finding confirms the intuition that market depth is closely related to existing or typical trading volumes, and that unexpected order flows are absorbed with smaller price effects in more active markets.

V. Summary and Conclusions

This study examines relations between trading activity and volatility in eight futures markets, employing econometric methods that accommodate volatility persistence, asymmetries in the volume-volatility relation, and interactions of conditional return means and conditional return volatilities. It documents a number of new empirical regularities that consistently appear across eight financial and nonfinancial futures contracts.

Consistent with prior studies, this work not only documents a strong positive relation between contemporaneous volume and volatility, but also finds that the impact of an unanticipated volume shock is between two and 13 times greater than the effect of changes in anticipated or expected volume. Further, the effect of unanticipated volume shocks on contemporaneous volatility is asymmetric, with positive shocks associated with (on average) 76-percent greater volatility.

These findings are consistent with the hypothesis that volatility is affected by existing market depth. There are numerous potential explanations for the regularities documented in this study. Here, the paper outlines its interpretation, which centers on the relation between market depth and capital in place at the beginning of a trading session. Traders begin a session with a predetermined amount of liquid capital. Rational trading strategies, including arbitrage, mitigate the price impacts of order shocks and keep prices close to intrinsic values. Traders' capital in place will depend on anticipated order flows. Thus, market depth decreases (i.e., order flows cause larger price revisions) when the actual order flow differs from the anticipated level. A negative volume shock implies that fewer orders than expected were brought to the market, and some of the market-making capital is underutilized. When more than the expected number of orders arrives, a capital shortage can arise. If a shortage of capital has more deleterious effects on depth than a surplus, market depth during positive volume shocks will be smaller than depth during negative volume shocks.

A second dimension of futures market activity, open interest, is also examined with the finding that expected open interest is negatively related to volatility in all markets. Expected open interest is viewed as a proxy for the number of traders or amount of capital dedicated to a market at the beginning of a trading session. Consequently, finding that higher levels of expected open interest reduce volatility is consistent with the belief that variations in open interest reflect changes in market depth.

The evidence indicates that the effect of volume on volatility depends on whether volume generates changes in open interest. When unexpected changes in open interest are included in the specification, and the coefficients are allowed to vary by sign, the relation between unexpected open interest and volatility is v-shaped. Trades that result in changes in open interest appear to have a larger impact on prices than do trades that leave open interest unaltered. This finding suggests that the volume-volatility relation might also depend on the class of traders involved. For example, activity by "locals" or other short-term speculators could affect volume figures more than end-of-day open interest numbers, while trades by hedgers affect both.

There are a number of potential extensions of this analysis. First, transactions level data might be employed to determine whether the above results are robust to the measurement interval. A second potentially important factor is the type of traders involved in a transaction. Data sets that allow for the identification of volume by trader type may shed light on this question. Finally, the authors of this paper are surprised at the paucity of empirical or theoretic research on the role of open interest, and hope that their evidence that open interest provides information beyond that contained in volume data will stimulate further analysis.

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