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ABSTRACT
We study the price impact of order book events—limit orders, market orders, and cancellations—using the NYSE Trades and Quotes data for fifty U.S. stocks. We show that, over short time intervals, price changes are mainly driven by the *order flow imbalance* (OFI), defined as the imbalance between supply and demand at the best bid and ask prices. Our study reveals a linear relation between OFI and price changes, with a slope inversely proportional to the market depth. These results are shown to be robust to intraday seasonality effects, and stable across time scales and across stocks. This linear price impact model, together with a scaling argument, implies the empirically observed "square-root" relation between the magnitude of price moves and trading volume, but this relation is found to be noisy and less robust than the one based on OFI. We discuss a potential application of OFI as a measure of adverse selection in limit order executions and discuss the implications for intraday volatility dynamics. (*JEL*: G12, C58)

KEYWORDS: high frequency data, liquidity, limit order book, market microstructure, price impact

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The availability 1 of high-frequency records of trades and quotes has stimulated an extensive empirical and theoretical literature on the relation between order flow, liquidity, and price movements in order-driven markets Cont (2011). A particularly important issue for applications is the impact of orders on prices: the optimal liquidation of a large block of shares, given a fixed time horizon, crucially involves assumptions on price impact (Bertsimas and Lo (1998), Almgren and Chriss (2000), Obizhaeva and Wang (2013)). Understanding price impact is also important from a theoretical perspective, since it is a fundamental mechanism of price formation.

Various aspects of price impact have been studied in the literature but there is little agreement on how to model it Bouchaud (2010), and the only consensus seems to be the intuitive notion that imbalance between supply and demand moves prices. Theoretical studies have drawn a distinction between instantaneous price impact of orders and its decay through time and shown that the form of instantaneous impact has important implications. Huberman and Stanzl (2004) show that there are arbitrage opportunities if the instantaneous effect of trades on prices is non-linear and permanent. Gatheral (2010) extends this analysis by showing that if the instantaneous price impact function is non-linear, impact needs to decay in a particular way to exclude arbitrage and if it is linear, it needs to decay exponentially in time. Bouchaud *et al.* (2004) associated the decay of price impact of trades with limit orders, arguing that there is a "delicate interplay between two opposite tendencies: strongly correlated market orders that lead to super-diffusion (or persistence), and mean reverting limit orders that lead to sub-diffusion (or anti-persistence)". This insight implies that looking solely at trades, without including the effect of limit orders amounts to ignoring an important part of the price formation mechanism.

However, the empirical literature on price impact has primarily focused on trades. One approach is to study the impact of "parent orders" gradually executed over time using proprietary data (Engle, Ferstenberg, and Russel (2006), Almgren *et al.* (2005)). Alternatively, empirical studies on public data Evans and Lyons (2002); Gabaix *et al.* (2003); Hasbrouck (1991); Keim and Madhavan (1996); Kempf and Korn (1999); Torre and Ferrari (1997); Plerou *et al.* (2002); Potters and Bouchaud (2003); Cont (2011) have analyzed the relation between the direction and sizes of trades and price changes and typically conclude that the instantaneous price impact of trades is an increasing, nonlinear function of their size. This focus on trades leaves out the information in quotes, which provide a more detailed picture of price formation Engle and Lunde (2003); Cont (2011) and raises a natural question: is volume of trades truly the best explanatory variable for price movements in markets where many quote events can happen between two trades?

We argue that a price impact model that encompasses limit orders, market orders, and cancellations and relates their impact to the concurrent market liquidity would provide a more detailed description of price formation. Obtaining such model is also desirable from the practical point of view because modern order execution algorithms increasingly use limit orders and incorporate market state variables in their decisions. There is also ample empirical evidence that limit orders play an important role in determining price dynamics. Arriving limit orders significantly reduce the impact of trades Weber and Rosenow (2005) and the concave shape of the price impact function changes depending on the contemporaneous limit order arrivals Stephens, Waelbroeck, and Mendoza (2009). The outstanding limit orders (also known as market depth) significantly affect the impact of an individual trade (Knez and Ready (1996)), low depth is associated with large price changes Weber and Rosenow (2006); Farmer *et al.* (2004), and depth influences the relation between trade sizes and returns Hasbrouck and Seppi (2001). The emphasis in the aforementioned studies remains, however, on trades and there are few empirical studies that focus on limit orders from the outset. Notable exceptions are Engle and Lunde (2003), Gomber, Schweickert, and Theissen (2006), Hautsch and Huang (2009) who perform an impulse-response analysis of limit and market orders, Hopman (2007) who analyzes the impact of different order categories over 30 min intervals and Eisler, Bouchaud, and Kockelkoren (2009) who examine the impact of market orders, limit orders, and cancellations at the level of individual events.

Summary

We conduct an empirical investigation of the instantaneous impact of order book events—market orders, limit orders, and cancellations—on equity prices. Although previous studies give a relatively complex description of their impact, we show that their instantaneous effect on prices may be modeled parsimoniously through a *single* variable, the *order flow imbalance* (OFI). This variable represents the net order flow at the best bid and ask and tracks changes in the size of the bid and ask queues by

- increasing every time the bid size increases, the ask size decreases or the bid/ask prices increase,
- decreases every time the bid size decreases, the ask size increases or the bid/ask prices decrease.

Interestingly, this variable treats a market sell and a cancel buy of the same size as equivalent, since they have the same effect on the size of the best bid queue. This aggregate variable explains mid-price changes over short time scales in a linear fashion, for a large sample of stocks, with an average R^2 of 65%. In contrast, order flows deeper in the order book do not substantially contribute to price changes. Our model based on OFI relates prices, trades, limit orders, and cancellations in a simple way: it is *linear*, requires the estimation of a single *price impact coefficient* and it is robust across stocks and across timescales.

Most of variability in the instantaneous price impact, both across time and across stocks is explained by variations in market depth. In fact, we establish an exact inverse relation between the two variables. The coefficient of proportionality in that relation depends dramatically on the depth definition, showing that arbitrary measures of market depth are biased proxies for price impact and may lead to misleading conclusions on market liquidity.

The price impact coefficient exhibits substantial intraday variability, similar to intraday patterns observed in spreads, market depth, and price volatility Ahn, Bae and Chan (2001); Andersen and Bollerslev (1998); Lee, Mucklow, and Ready (1993); McInish and Wood (1992). We explain the diurnal effects in price volatility using the volatility of OFI and market depth, as opposed to unobservable parameters previously invoked in the literature, such as information asymmetry Madhavan, Richardson, and Roomans (1997) or informativeness of trades Hasbrouck (1991). The strong link between price volatility and standard deviation of OFI suggests that our price impact coefficient is a better estimate of Kyle's λ (a useful metric of liquidity Amihud, Mendelson and Pedersen (2006); Kyle (1985)) than traditional estimates based on trades data. We also show that intraday price volatility is mainly driven by OFI and not by trading volume. The positive correlation between price volatility and volume, widely confirmed by empirical studies Karpoff (1987), can be a statistical artifact due to aggregation of data over time, and we establish how such spurious relation can arise in our model.

OFI exhibits positive autocorrelation over short time scales, which can be exploited to improve the quality of order executions. In particular, we show that a limit order fill is more likely to be followed with a price change in the same direction as the OFI before that fill. For example, a limit sell order is more likely to be adversely selected when OFI is positive. Monitoring OFI can therefore help reduce adverse selection in limit order fills.

Outline

The article is structured as follows. In Section 1, we specify a parsimonious model that links stock price changes, OFI and market depth and motivate it by a stylized example of the order book. Section 2 describes our data and presents estimation results for our model. Section 3 discusses potential applications of our results: in subsection 3.1 we use OFI as a measure of adverse selection in limit order executions, in subsection 3.2 we demonstrate how diurnal effects in depth and OFI generate intraday patterns in price impact and price volatility, and in subsection 3.3 we show how a spurious relation between volume and the magnitude of price moves emerges as a statistical artifact from our simple model. Section 4 presents our conclusions.

1 PRICE IMPACT MODEL

1.1 A Stylized Model of the Limit Order Book

To motivate our approach we first consider a stylized version limit order book where the instantaneous effect of order book events can be explicitly computed.

Consider an order book in which the number of shares (depth) at each price level beyond the best bid and ask is equal to D. Order arrivals and cancellations occur only at the best bid and ask. Moreover, when bid (or ask) size reaches D, the next passive order arrives 1 tick above (or below) the best quote, initializing a new best level. Consider a time interval $[t_{k-1}, t_k]$ and denote by L_k^b, C_k^b, respectively, the total size of buy orders that arrived to and canceled from current best bid during that time interval. Also denote by M_k^b the total size of marketable buy orders that arrived to current best ask, and by P_k^b the bid price at time t_k. The quantities L_k^s, C_k^s, M_k^s for sell orders are defined analogously and P_k^s is the ask price.

In this simple order book model there exists a linear relation between order flows $L_k^{b,s}, C_k^{b,s}, M_k^{b,s}$ and price changes $\Delta P_k^{b,s} = (P_k^{b,s} - P_{k-1}^{b,s})$ (also illustrated on Figures 1–3):

$$\Delta P_k^b = \delta \left\lceil \frac{L_k^b - C_k^b - M_k^s}{D} \right\rceil \tag{1}$$

$$\Delta P_k^s = -\delta \left\lceil \frac{L_k^s - C_k^s - M_k^b}{D} \right\rceil, \tag{2}$$

where δ is the tick size[1]. These relations are remarkably simple—they involve no parameters, the impact of all order book events is *additive* and depends only on their net imbalance. Although all of the subsequent analysis can be carried out separately for bid and ask prices, for simplicity we consider mid-price changes normalized by tick size $P_k = \frac{P_k^b + P_k^s}{2\delta}$:

$$\Delta P_k = \frac{\mathrm{OFI}_k}{2D} + \epsilon_k, \tag{3}$$

$$\mathrm{OFI}_k = L_k^b - C_k^b - M_k^s - L_k^s + C_k^s + M_k^b, \tag{4}$$

where OFI_k is the order flow imbalance (or net order flow) and ϵ is the truncation error. We can also rewrite (3) as

$$\Delta P_k = \frac{\mathrm{TI}_k}{2D} + \eta_k, \tag{5}$$

$$\mathrm{TI}_k = M_k^b - M_k^s, \tag{6}$$

where TI_k is the trade imbalance and $\eta_k = \frac{L_k^b - C_k^b - L_k^s + C_k^s}{2D} + \epsilon_k$. When limit order activity dominates, that is absolute values of terms $|L_k^{b,s}|, |C_k^{b,s}|$ are much larger than $|M_k^{b,s}|$, the correlation of price changes with TI_k is weaker than with OFI_k, because limit order submissions and cancellations manifest as noise in (5).

[1]This is easily proven by induction over the number of price changes in $[t_{k-1}, t_k]$. The statement is clearly true when there are no price changes or a single price change of $\pm\delta$. Since any price change of $\pm k\delta$ consists of jumps of size 1, we simply need to sum the OFIs across these jumps on the right side of the equation.

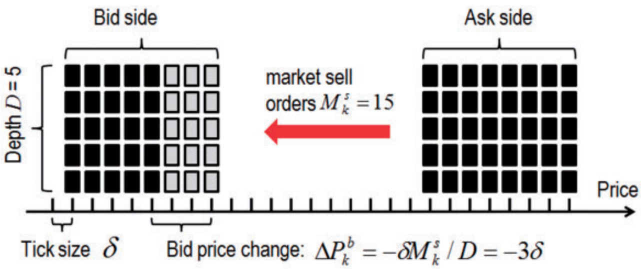

Figure 1 Market sell orders remove M^s shares from the bid (gray squares represent net change in the order book).

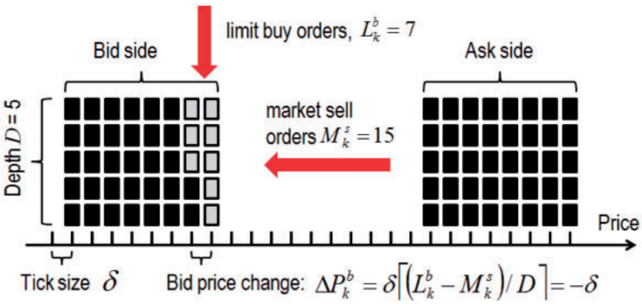

Figure 2 Market sell orders remove M^s shares from the bid, while limit buy orders add L^b shares to the bid.

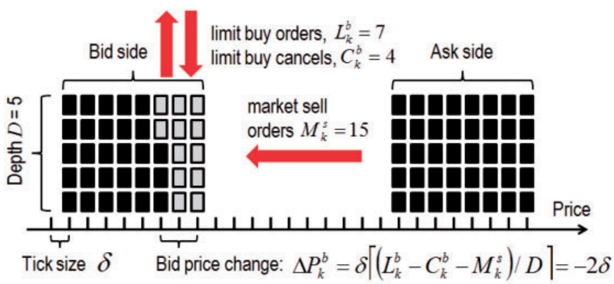

Figure 3 Market sell orders and limit buy cancels remove M^s+C^b shares from the bid, while limit buy orders add L^b shares to the bid.

1.2 Model Specification

Actual order books have complex dynamics: arrivals and cancellations occur at all price levels, the depth distribution across levels has non-trivial features Potters and Bouchaud (2003); Rosu (2009); Zovko and Farmer (2002), and hidden orders together with data-reporting issues create additional errors Avellaneda, Stoikov, and Reed (2011); Hasbrouck (2010). Motivated by the stylized order book example we assume a noisy relation between price changes and OFI, which holds locally for short intervals of time $[t_{k-1,i}, t_{k,i}] \subset [T_{i-1}, T_i]$, where $[T_{i-1}, T_i]$ are longer intervals.

$$\Delta P_{k,i} = \beta_i \mathrm{OFI}_{k,i} + \epsilon_{k,i}. \tag{7}$$

In this model β_i is a *price impact coefficient* for an i-th time interval and $\epsilon_{k,i}$ is a noise term summarizing influences of other factors (e.g., deeper levels of the order book). We allow β_i and the distribution of $\epsilon_{k,i}$ to change with index i, because of well-known intraday seasonality effects. Our discussion from the previous section allows us to interpret $\frac{1}{2\beta_i}$ as an *implied* order book depth. The stylized order book model suggests that price impact coefficient is inversely related to market depth, and we consider the following model:

$$\beta_i = \frac{c}{D_i^{\lambda}} + \nu_i, \tag{8}$$

where c, λ are constants and ν_i is a noise term. The stylized order book model corresponds to $c = \frac{1}{2}, \lambda = 1$. We also consider a relation between price changes and trades:

$$\Delta P_{k,i} = \beta_i^T \mathrm{TI}_{k,i} + \eta_{k,i}, \tag{9}$$

but expect it to be much noisier than (7).

The specification (7)–(8) may be regarded as a model of instantaneous price impact of order book events, arriving within time interval $[t_{k-1}, t_k]$. An order submitted or canceled at time $\tau \in [t_{k-1}, t_k]$ contributes a signed quantity e_τ to supply/demand. In any given time interval, these contributions are likely be unbalanced, leading to an order flow imbalance OFI_k, which affects supply/demand and leads to a corresponding price adjustment. If an individual order goes in the same direction as the majority of orders ($\mathrm{sgn}(e_\tau) = \mathrm{sgn}(\mathrm{OFI}_k)$), it reinforces the concurrent OFI and can affect the price. If the order goes against the concurrent OFI ($\mathrm{sgn}(e_\tau) = -\mathrm{sgn}(\mathrm{OFI}_k)$), it is compensated by other orders and has an instantaneous impact of zero. In our model all events (including trades) have a linear price impact, on average equal to β_i during the i-th interval. Their realized impact however depends on the concurrent orders.

The idea that the concurrent limit order activity can make a difference in terms of trades' impact was demonstrated in Stephens, Waelbroeck, and Mendoza (2009), where authors show that the shape of the price impact function essentially depends on the contemporaneous limit order activity. Our approach can also be related to the model proposed in Eisler, Bouchaud, and Kockelkoren (2009), where order book events have a linear impact on prices, which depends on their signs and types[2]. The major difference of our model lies in the aggregation across time and events.

[2] Note that in our case all order book events have the same average impact, equal to β_i, regardless of their type. As shown in Eisler, Bouchaud, and Kockelkoren (2009), average impacts of different event types are empirically very similar, allowing to reasonably approximate them with a single number.

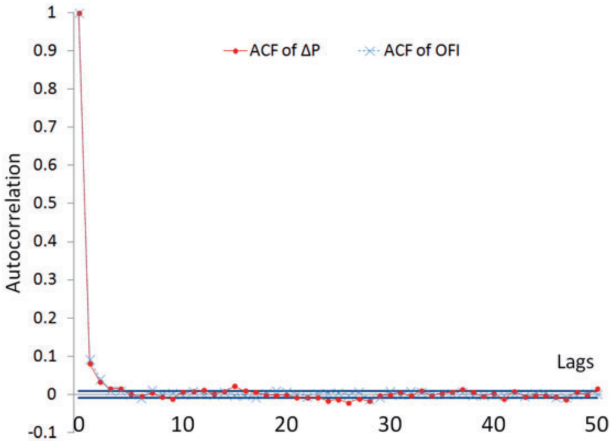

Figure 4 ACF of the mid-price changes $\Delta P_{k,i}$, the Order flow imbalance $\mathrm{OFI}_{k,i}$ and the 5% significance bounds for the SLB.

As shown in Eisler, Bouchaud, and Kockelkoren (2009), time series of individual order book events have complicated auto- and cross-correlation structures, which typically vanish after 10 s. In our data the autocorrelations at a timescale of 10 s are small and quickly vanish as well (ACF plots for a representative stock are shown on Figure 4). Finally, the model used in Hasbrouck and Seppi (2001) for explaining the price impact of trades is similar to (9). Although the focus there is on trades, authors allow the price impact coefficient to depend on contemporaneous liquidity factors and change through time.

At the same time, the linear relation (7) is different from many earlier models that consider only the effect of transactions Gabaix *et al.* (2003); Hasbrouck (1991); Kempf and Korn (1999); Torre and Ferrari (1997); Plerou *et al.* (2002); Potters and Bouchaud (2003). Instead of modeling price impact of trades as a (nonlinear) function of trade size, we show that the instantaneous price impact of a series of events (including trades) is a linear function of their size after these events are aggregated into a single imbalance variable. We will show that, first, the effect of trades on prices is adequately captured by the OFI and, second, that if one leaves out all events except trades, the relation (7) leads to an apparent concave relation between the magnitude of price changes and trading volume.

The next section provides an overview of the estimation results for our model.

2 ESTIMATION AND RESULTS

2.1 Data

Our main data set consists of one calendar month (April, 2010) of trades and quotes data for fifty stocks. The stocks were selected by a random number generator from S&P 500 constituents, which were obtained from Compustat. The data for individual stocks were obtained from the TAQ consolidated quotes and TAQ consolidated trades databases[3].

Consolidated quotes contain best bid/ask price changes and round-lot changes in best bid/ask sizes. Quote data entries consist of a stock ticker, a timestamp (rounded to the nearest second), bid price and size, ask price and size and various flags including exchange flag. Consolidated trade entries consist of timestamps, prices, sizes, and various flags. These two data sets are often referred to as Level 1 data, as opposed to Level 2 data, which includes quote updates deeper in the book, or information on individual orders.

At the same time TAQ data have important limitations—the timestamps are rounded to the nearest second, and it may omit odd-lot trades and quotes. To perform several detailed robustness checks we also use an auxiliary data set consisting of NASDAQ ITCH 4.0 messages for the same calendar month (April, 2010) for one representative stock from our main data set (Schlumberger). This data are accessible through LOBSTER website[4] which also provides NASDAQ order book history for the selected stock. We used LOBSTER data for the top five order book levels without any additional pre-processing.

The TAQ data were used to compute the National Best Bid and Offer sizes and prices (NBBO) at each quote update. We find that the ratio between the number of NBBO quote updates and the number of trades is roughly 40–1 in our data. Many empirical studies have focused exclusively on trades rather than quotes, but the sheer difference in sizes of these data sets suggests that more information may be conveyed by quotes than by trades. Using the exchange flag, we also considered one exchange at a time and obtained similar empirical results.

2.2 Variables

Every observation of the bid and the ask consists of the bid price P^b, the bid queue size q^b (in number of shares), the ask price P^s and size q^s. We enumerate them by n and compute differences between consecutive observations $(P^b_n, q^b_n, P^s_n, q^s_n)$ as follows:

$$e_n = q^b_n \mathbb{1}_{\{P^b_n \geq P^b_{n-1}\}} - q^b_{n-1} \mathbb{1}_{\{P^b_n \leq P^b_{n-1}\}} - q^s_n \mathbb{1}_{\{P^s_n \leq P^s_{n-1}\}} + q^s_{n-1} \mathbb{1}_{\{P^s_n \geq P^s_{n-1}\}}. \tag{10}$$

The variables e_n are signed contributions of order book events to supply/demand. When a passive buy order arrives, q^b increases but P^b remains the same, leading to $e_n = q^b_n - q^b_{n-1}$ which is the size of that order. If q^b decreases, we have $e_n = q^b_n - q^b_{n-1}$, representing the size of a marketable sell order or buy order cancellation. If P^b

[3] The TAQ data were obtained through Wharton Research Data Services (WRDS).

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changes, then $e_n = q_n^b$ or $e_n = -q_{n-1}^b$, representing, respectively, the size of a price-improving order or the last order in the queue that that was removed. Symmetric computations are done for the ask side.

We use two uniform time grids $\{T_0, \ldots, T_I\}$ and $\{t_{0,0}, \ldots, t_{I,K}\}$ with time steps $T_i - T_{i-1} = 30$ min and $t_{k,i} - t_{k-1,i} = \Delta t = 10$ s[5]. Within each long time interval $[T_{i-1}, T_i]$ we compute 180 price changes and OFIs indexed by k:

$$\Delta P_{k,i} = \frac{P^b_{N(t_{k,i})} + P^s_{N(t_{k,i})}}{2\delta} - \frac{P^b_{N(t_{k-1,i})} + P^s_{N(t_{k-1,i})}}{2\delta} \tag{11}$$

$$\mathrm{OFI}_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} e_n, \tag{12}$$

where $N(t_{k-1,i}) + 1$ and $N(t_{k,i})$ are the index of the first and the last order book event in the interval $[t_{k-1,i}, t_{k,i}]$. The tick size δ is equal to 1 cent in our data. Note that in our empirical study OFI is computed from fluctuations in best bid/ask prices and their sizes according to (12), because data on individual orders are not available in our main data set. If that data are available, OFI can be computed according to (4). We believe that a computation based on (4) can lead to better empirical results because aggressive order terms M^b, M^s will capture information on hidden orders and unreported odd-lot sized orders within the spread, to the extent that aggressive orders interact with hidden orders. Since TAQ data report only round-lot sized quote changes, we note that units of OFI are round lots (100 shares), and assume in (12) that both sides of the market are equally affected by missing quote updates[6].

We define trade imbalance during a time interval $[t_{k-1,i}, t_{k,i}]$ as the difference between volumes of buyer- and seller-initiated trades during that interval, and also define trading volume within that time interval:

$$\mathrm{TI}_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} b_n - s_n \qquad \mathrm{VOL}_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} b_n + s_n, \tag{13}$$

where b_n, s_n are sizes of buyer- and seller-initiated trades (in round lots) that occurred at the n-th quote (equal to zero if no trade occurred at that quote). In contrast with TI, the OFI measure computed using (12) does not hinge on trade classification, which is known to be problematic for TAQ data (see Appendix for more details on matching trades with quotes and trade classification).

[5]Results for other timescales are reported in the Appendix

[6]As we demonstrate in the Appendix, neither missing odd-lot sized observations nor potential mis-sequencing of quote updates across different exchanges during NBBO computation change our qualitative findings.

Whereas previous studies Chordia, Roll, and Subrahmanyam (2008); Hasbrouck (1991); Hasbrouck and Seppi (2001); Kempf and Korn (1999); Plerou *et al.* (2002); Torre and Ferrari (1997) focused on trade imbalance[7], the OFI is a more general measure. It encompasses effects of all order book events, including trades.

For each interval $[T_{i-1}, T_i]$ we also estimate depth by averaging the bid/ask queue sizes right before or right after a price change, consistently with the definition of depth in the stylized order book model:

$$D_i = \frac{1}{2}\left[\frac{\sum_{n=N(T_{i-1})+1}^{N(T_i)}\left(q_n^b \mathbb{1}_{\{P_n^b < P_{n-1}^b\}} + q_{n-1}^b \mathbb{1}_{\{P_n^b > P_{n-1}^b\}}\right)}{\sum_{n=N(T_{i-1})+1}^{N(T_i)} \mathbb{1}_{\{P_n^b \neq P_{n-1}^b\}}} + \frac{\sum_{n=N(T_{i-1})+1}^{N(T_i)}\left(q_n^s \mathbb{1}_{\{P_n^s > P_{n-1}^s\}} + q_{n-1}^s \mathbb{1}_{\{P_n^s < P_{n-1}^s\}}\right)}{\sum_{n=N(T_{i-1})+1}^{N(T_i)} \mathbb{1}_{\{P_n^s \neq P_{n-1}^s\}}}\right].$$

2.3 Empirical Findings

This section reports detailed results for a representative stock, Schlumberger (SLB) and some average results across stocks. Detailed results for other stocks in our sample are presented in the Appendix. During the sample period the average price of Schlumberger stock was 67.94 dollars and the average daily volume was 947.6 million shares. The daily average number of NBBO quote updates is about 440 thousands, and the average daily number of trades is around 10 thousands. The average spread is 1 cent, its 95-th percentile is 2 cents and the average best NBBO quote size is 39 round lots (3900 shares).

The model (7) is estimated by an ordinary least squares regression:

$$\Delta P_{k,i} = \hat{\alpha}_i + \hat{\beta}_i \mathrm{OFI}_{k,i} + \hat{\epsilon}_{k,i} \tag{14}$$

with separate half-hour subsamples indexed by i. Figure 5 presents a scatter plot of $\Delta P_{k,i}$ against $\mathrm{OFI}_{k,i}$ for one of such subsamples.

In general we find that $\hat{\beta}_i$ is statistically significant[8] in 98% of samples, and $\hat{\alpha}_i$ is significant in 10% of samples, which is close to the Type-I error rate. The average t-statistics for $\hat{\alpha}_i, \hat{\beta}_i$ are, respectively -0.21 and 16.27 for SLB (cross-sectional averages are -0.02 and 12.08). To check for higher order/nonlinear dependence we estimate an augmented regression:

$$\Delta P_{k,i} = \hat{\alpha}_i^Q + \hat{\gamma}_i \mathrm{OFI}_{k,i} + \hat{\gamma}_i^Q \mathrm{OFI}_{k,i}|\mathrm{OFI}_{k,i}| + \hat{\epsilon}_{k,i}^Q. \tag{15}$$

[7] Hopman (2007) computes the supply/demand imbalance based on limit orders and trades, but not cancellations.

[8] Given a relatively large number of observations we use the z-test with a 95% significance level. Since regression residuals demonstrate heteroscedasticity and autocorrelation, Newey–West standard errors are used to compute t-statistics.

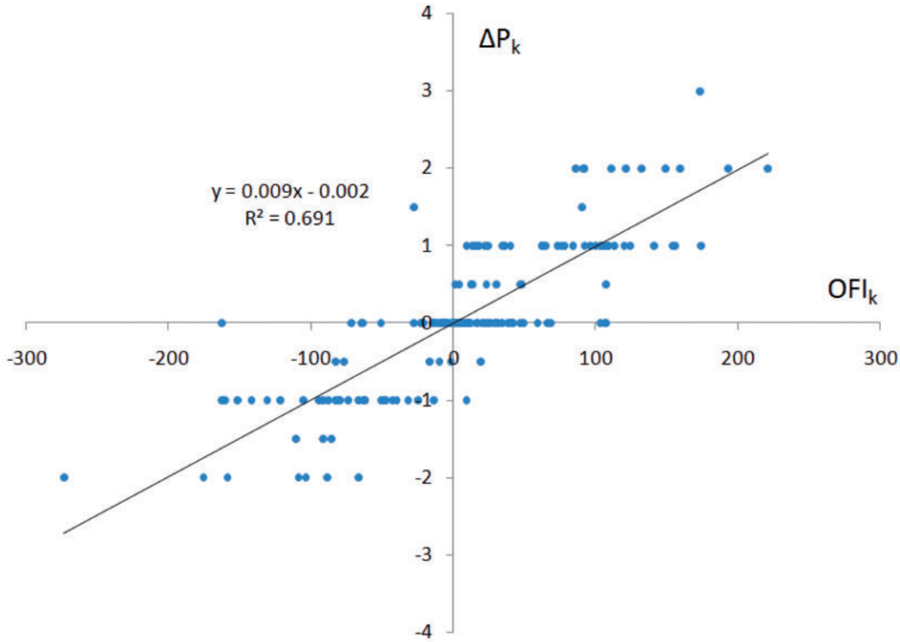

Figure 5 Scatter plot of $\Delta P_{k,i}$ against $\mathrm{OFI}_{k,i}$ for the SLB, April 1, 2010 11:30–12:00 pm.

The coefficients $\hat{\gamma}_i^Q$ have an average t-statistic of -0.32 across stocks and are statistically significant only in 17% of our samples. We reject the hypothesis of quadratic (convex or concave) instantaneous price impact, and take this as strong evidence for a linear price impact model (7), because other kinds of non-linear dependence would likely be picked up by this quadratic term.

The goodness of fit is surprising for high-frequency data, with an R^2 of 76% for SLB and 65% on average across stocks[9], suggesting that a one-parameter linear model (7) performs well regardless of stock-specific features, such as average spread, depth, or price level. The definition of R^2 as a percentage of explained variance has an interesting consequence in our case. Since OFI is constructed from order book events taking place only at the best bid/ask, our results show that activity at the top of the order book is the most important factor driving price changes. In the Appendix we confirm this by showing that OFIs from deeper order book levels only marginally contribute to short-term price dynamics. Even though

[9]We note that OFI includes the contributions e_n of price-changing order book events, leading to a possible endogeneity in the regression (14). This problem is inherent to all price impact modeling, because the explanatory variables (events or trades) sometimes mechanically lead to price changes. To test that the high R^2 in our regressions is not due to this endogeneity, we estimated (14) on a subsample of stocks, excluding the price-changing events from OFI. With this change the R^2 declined, but remained high, in the 35%–60% region.

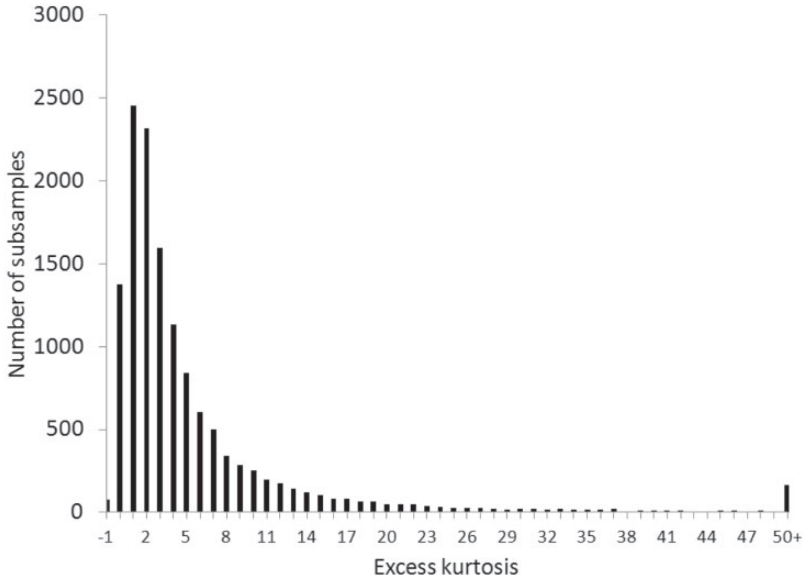

Figure 6 Distribution of excess kurtosis in the residuals $\hat{\epsilon}_{k,i}$ across stocks and time.

large price movements sometimes occur at this timescale, they mostly correspond to large readings of OFI. Figure 6 confirms this by demonstrating a relatively low level of excess kurtosis in regression residuals.

When the amount of passive order submissions and cancellations is much larger than the amount of trades, the stylized order book model predicts that trade imbalance TI explains price changes significantly worse than OFI. To empirically confirm this we estimate following regressions using the same half-hour subsamples[10]:

$$\Delta P_{k,i} = \hat{\alpha}_i^T + \hat{\beta}_i^T \mathrm{TI}_{k,i} + \hat{\eta}_{k,i} \tag{16a}$$

$$\Delta P_{k,i} = \hat{\alpha}_i^D + \hat{\theta}_i^O \mathrm{OFI}_k + \hat{\theta}_i^T \mathrm{TI}_{k,i} + \hat{\epsilon}_{k,i}^D. \tag{16b}$$

When either OFI or TI variable is taken individually, that variable has a statistically significant correlation with price changes. The average t-statistics of slope coefficients in simple regressions (14, 16a) are, correspondingly 16.27 and 5.31 for SLB (cross-sectional averages are 12.08 and 5.08). The average R^2 for the two regressions are 65% and 32%, respectively, confirming the prediction that relation

[10]These regressions contain only linear terms, because we found no evidence of non-linear price impacts in our data (for neither OFI nor TI).

between price changes and trade imbalance is more noisy. When the two variables are used in a multiple regression (16b), the dependence of price changes on trade imbalance becomes much weaker. The average t-statistic of TI coefficient drops to 1.56 for SLB (1.51 across stocks) and it remains statistically significant in only 47% of SLB samples (43% of all stock samples). The dependence on OFI remains strong with an average t-statistic 13.91 for SLB (9.53 across stocks), and the coefficient is statistically significant in almost all samples. We conclude that OFI explains price movements better than trade imbalance, and OFI is a more general measure of supply/demand imbalance because it adequately includes the effect of trade imbalance.

Finally, we use time series of D_i and $\hat{\beta}_i$ for each stock to estimate the relation (8) with the following two regressions:

$$\log\hat{\beta}_i = \alpha\hat{_{L,i}} - \hat{\lambda}\log D_i + \hat{\epsilon}_{L,i} \tag{17}$$

$$\hat{\beta}_i = \alpha\hat{_{M,i}} + \frac{\hat{c}}{D_i^{\hat{\lambda}}} + \hat{\epsilon}_{M,i}. \tag{18}$$

Both regressions are estimated using ordinary least squares[11]. For SLB we find $\hat{c}=0.56$, $\hat{\lambda}=1.08$, and an R^2 of (17) is 92%. The results for all stocks are shown in Table B.4. We observe that depth significantly correlates with price impact coefficients for the vast majority of stocks, confirming our intuition that $\frac{1}{2\beta_i}$ is the implied order book depth. Interestingly, estimates $\hat{c}, \hat{\lambda}$ across stocks are very close to values predicted by the stylized order book model. With the t-statistics[12] in Table B.4 the null hypotheses $\{c=0.5\}$ and $\{\lambda=1\}$ cannot be rejected for most stocks based on conventional significance levels. The restricted model with $\lambda=1$ also demonstrates a good quality of fit, making this a good approximation. Figure 7 illustrates these results with a log–log scatter plot for D_i and $\hat{\beta}_i$. Some stocks (namely APOL, AZO, and CME) have poor fits in regression (17), mainly due to outliers in the dependent variable. After removing these outliers and re-estimating the regression, the estimates $\hat{c}, \hat{\lambda}$ for these stocks fell in line with estimates for other stocks.

To assess the stability of these findings, we re-estimated (17,18) with observations pooled across days but not across intraday time intervals, resulting in thirteen estimates $\hat{c}_i, \hat{\lambda}_i$ for each stock. Although these estimates demonstrate some diurnal variability, they are relatively stable and most of variability in price impact coefficients is explained by variations in depth (Figure 9).

We repeated the analysis with different depth variables, taking D_i to be equal to arithmetic or geometric average of queue sizes over the i-th time interval. Overall,

[11]We note that an estimate $\hat{\lambda}_i$ is used in regression (18). This "plug-in" approach leads to potential errors in explanatory variable, and standard errors for $\hat{c}$ may be underestimated. However, the good quality of fit in regression (17) with an average R^2 of 76% indicates that $\hat{\lambda}_i$ are estimated with good precision. We believe that errors in variable $\frac{1}{D_i^{\hat{\lambda}}}$ are small and do not affect our results.

[12]Since the residuals of these regressions appear to be autocorrelated, the t-statistics are computed with Newey–West standard errors.

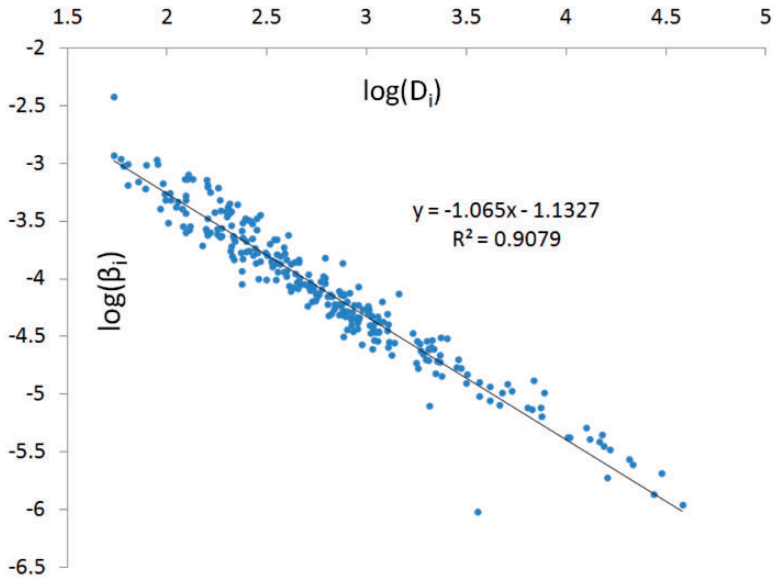

Figure 7 Log–log scatter plot of the price impact coefficient estimate $\hat{\beta}_i$ against average market depth D_i for the SLB.

the results were the same, except for the level of $\hat{c}$ estimates, which were about 40% lower across stocks for the arithmetic average depth, and even lower for the geometric average. The systematic difference in these coefficients implies that taking an arbitrary measure of depth (such as arithmetic average of queue sizes) as a proxy of price impact may lead to significant biases, that is one would dramatically under- or over-estimate price impact in a given stock. Instead of looking at arbitrary depth measures, we suggest computing price impact coefficients β_i and/or implied depth $\frac{1}{2\beta_i}$ to precisely characterize price sensitivity to order flow.

3 APPLICATIONS

3.1 Monitoring Adverse Selection

Time intervals that are involved in modern high-frequency trading applications are usually so short that price changes are relatively infrequent events. Therefore price changes provide a very coarse and limited description of market dynamics. However, OFI tracks best bid and ask queues and fluctuates on a much faster timescale than prices. It incorporates information about build-ups and depletions of order queues and it can be used to interpolate market dynamics between price changes (see, e.g., Figure 8). Our results confirm that such interpolation is in fact valid because OFI closely approximates price changes over short time intervals (e.g.,

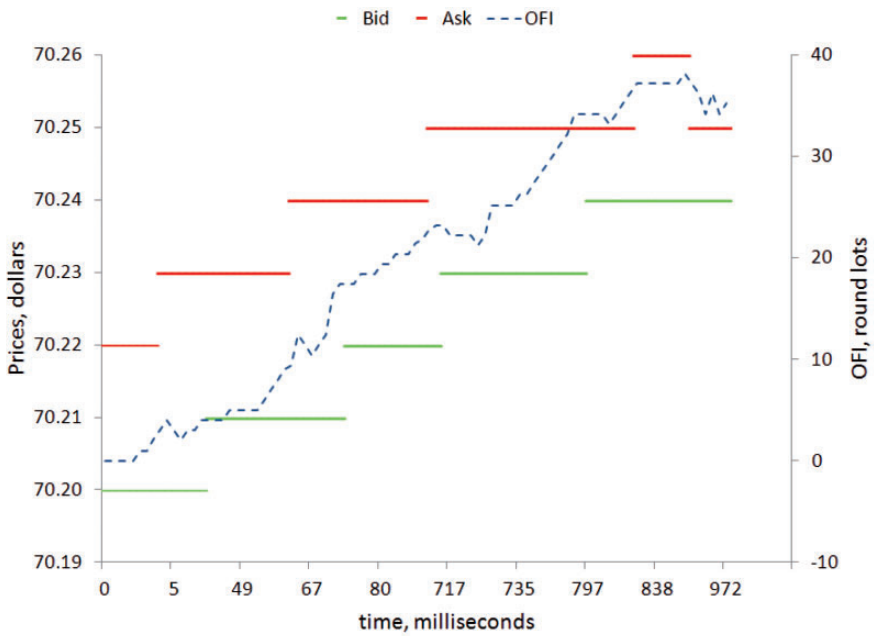

Figure 8 Price dynamics and cumulative OFI on NASDAQ for a 1 s time interval starting at 11:16:39.515 on April 28, 2010, SLB.

results for 50 ms time intervals are shown in the Appendix). To study one possible application of OFI for high-frequency trading we turn to our auxiliary data set, because it contains accurate timestamps up to a millisecond.

Given the strong link between OFI and price changes, and the positive autocorrelation of OFI over short time intervals (Figure 4), we propose to use it as a measure of adverse selection in the order flow. For example, when a limit order is filled, and its execution was preceded by positive OFI, a positive price change is more likely to happen after the limit order execution. This is because the pre-execution positive OFI is likely to persist in the future, and can lead to a post-execution positive price change. For a limit sell order a positive post-execution price change implies that the order was executed at a loss, that is adversely selected.

To test our hypothesis, we consider all limit order executions in our auxiliary data set. For each execution we compute the pre-execution order flow imbalance $\mathrm{OFI}_k^{\mathrm{pre}}$ and the post-execution mid-price change $\Delta P_k^{\mathrm{post}}$. The pre-execution OFI is computed from best bid and ask quote updates with timestamps in $[t_k-200, t_k-1]$ ms, where t_k is the time of the k-th limit order execution. Similarly the post-execution price change is defined as the difference in mid-quote prices between t_k+200 ms and t_k[13]. Then we consider 30 min subsamples of data indexed by i, and

[13]If there are multiple quotes with timestamp t_k+200 or t_k, we take the last one.

estimate the following regression:

$$\Delta P_{k,i}^{\text{post}} = \alpha_i^p + \beta_i^p \text{OFI}_{k,i}^{\text{pre}} + \epsilon_{k,i}^p. \tag{19}$$

The average R^2 of these regressions across a month is 2.93%, the average t-statistic[14] of β_i^p is 2.68 and this coefficient is significant at a 5% level in 63% of subsamples. The average β_i^p is 0.0105. We conclude that pre-execution OFI are positively correlated with post-execution price changes.

We also estimated regression (19) with 50 ms and 100 ms time intervals for pre- and post-execution variables, and obtained similar results, with stronger correlations for smaller time intervals[15]. When we split $\text{OFI}_{k,i}^{\text{pre}}$ into multiple OFI variables over non-overlapping subintervals of $[t_k - 200, t_k - 1]$, we find that only the variable closest to t_k—the execution time—is statistically significant and positively correlated with post-execution price change. These results suggest that limit order traders need to actively monitor order flows and react to emerging OFIs as quickly as possible to avoid being adversely selected.

3.2 Intraday Volatility Dynamics

The link between price impact and market depth established here has important implications for intraday volatility. Market depth is known to follow a predictable diurnal pattern (Ahn, Bae and Chan (2001), Lee, Mucklow, and Ready (1993)), and Equation (8) implies that instantaneous price impact must also have a *predictable* intraday pattern. To demonstrate it, we averaged $\hat{\beta}_i$ for each stock and each intraday half-hour interval across days, resulting in diurnal effects for that stock, normalized these effects by a grand average $\hat{\beta}_i$ for that stock and averaged normalized diurnal effects across stocks. The same procedure was repeated for depths D_i. We also re-estimated (17,18) with observations pooled across days but not across intraday time intervals, resulting in thirteen estimates $\hat{\lambda}_i, \hat{c}_i$ for each stock. The overall average diurnal effects for these quantities are shown on Figure 9.

We found that between 9:30 am and 10 am the depth is two times lower than on average, indicating that the market is relatively shallow. In a shallow market, incoming orders can easily affect mid-prices and price impact coefficients between 9:30 am and 10 am are in fact two times higher than on average. Moreover, price impact coefficients between 9:30 am and 10 am are five times higher than between 3:30 pm and 4 pm.

The intraday pattern in price impact can be used to explain intraday patterns in price volatility, observed by many studies (Ahn, Bae and Chan (2001), Andersen and Bollerslev (1998), Hasbrouck (1991),

[14] Here we also use Newey–West standard errors because residuals demonstrate significant autocorrelation.

[15] For instance, with 50 ms time intervals the average t-statistic of β_i^p is 3.41 and this coefficient is significant in 75% of samples. The average R^2 becomes 3.32%.

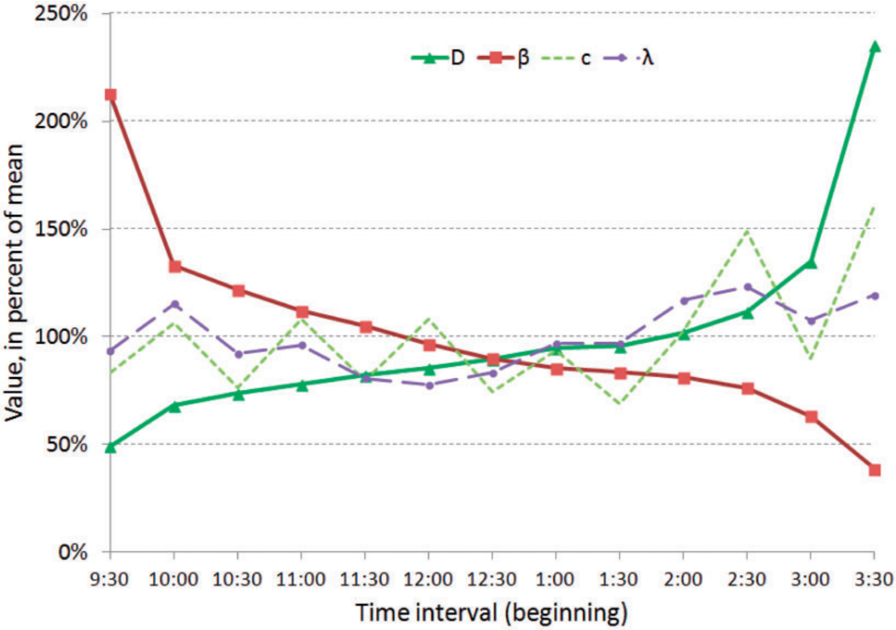

Figure 9 Diurnal effects in the price impact coefficient $\hat{\beta}_i$, the average depth D_i and the parameters $\hat{c}_i, \hat{\lambda}_i$. Most of the intraday variation in price impact coefficients comes from variations in depth, while parameters $\hat{c}_i, \hat{\lambda}_i$ are relatively more stable.

Madhavan, Richardson, and Roomans (1997)). Similarly to the price impact coefficient and the market depth, we computed the intraday patterns in variances of $\Delta P_{k,i}$ and $\mathrm{OFI}_{k,i}$, using our half-hour subsamples. Taking the variance on both sides in Equation (7), we obtain a link between $\mathrm{var}[\Delta P_{k,i}]$, $\mathrm{var}[\mathrm{OFI}_{k,i}]$, and β_i:

$$\mathrm{var}[\Delta P_{k,i}] = \beta_i^2 \mathrm{var}[\mathrm{OFI}_{k,i}] + \mathrm{var}[\epsilon_{k,i}]. \tag{20}$$

The average variance patterns are plotted on Figure 10. Notice that price volatility has a sharp peak near the market open, while volatility of OFI peaks near the market close. The latter peak is offset by low price impact, which gradually declined throughout the day. For the i-th half-hour interval, Equation (20) implies that $\mathrm{var}[\Delta P_{k,i}] \approx \hat{\beta}_i^2 \mathrm{var}[\mathrm{OFI}_{k,i}]$ because $\mathrm{var}[\epsilon_{k,i}]$ is relatively small, which is also demonstrated[16] on Figure 10. Since the R^2 in regression (14) is high, the ratio $\frac{\mathrm{var}[\epsilon_{k,i}]}{\mathrm{var}[\mathrm{OFI}_{k,i}]}$ is small, and we can rewrite (20) as $\beta_i \approx \frac{\sigma_{P,i}}{\sigma_{O,i}}$, where $\sigma_{P,i} = \sqrt{\mathrm{var}[\Delta P_{k,i}]}$ and $\sigma_{O,i} = \sqrt{\mathrm{var}[\mathrm{OFI}_{k,i}]}$. This bears strong resemblance to the definition of Kyle's λ (Kyle (1985))—a metric that is used in the asset pricing literature to gauge liquidity risk (see Amihud, Mendelson and Pedersen (2006) and references therein). This

[16] $\hat{\beta}_i^2 \mathrm{var}[\mathrm{OFI}_{k,i}]$ was computed from the average patterns of $\hat{\beta}_i$ and $\mathrm{var}[\mathrm{OFI}_{k,i}]$.

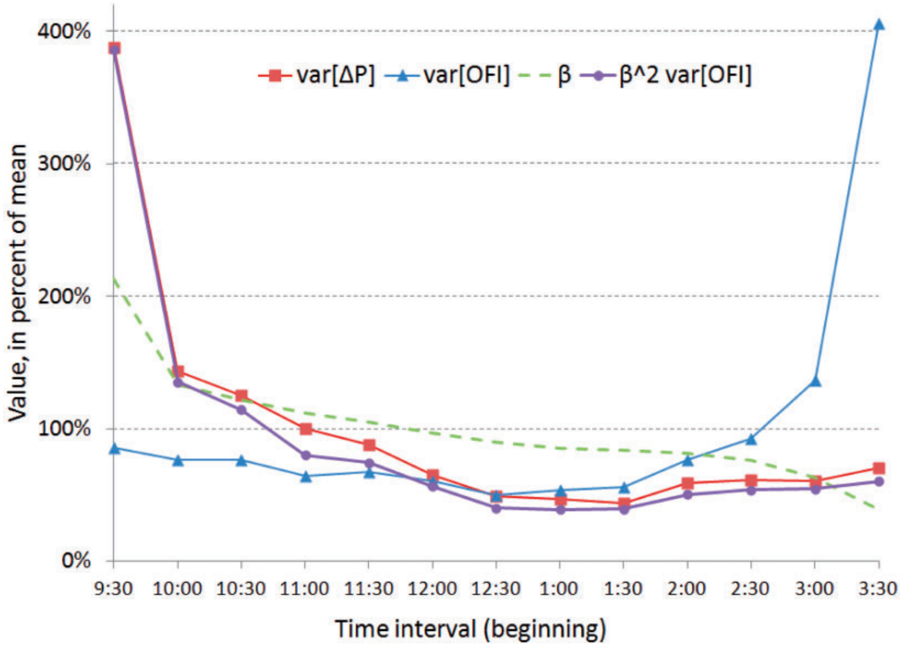

Figure 10 Diurnal variability in variances var$[\Delta P_{k,i}]$, var$[\mathrm{OFI}_{k,i}]$, the price impact coefficient $\hat{\beta}_i$ and the expression $\beta_i^2\mathrm{var}[\mathrm{OFI}_k]_i$.

metric is traditionally estimated as a slope β_i^L in regression (16a), but our analysis suggests that β_i is a better estimate. Although one could also write $\beta_i^L \approx \frac{\sigma_{P,i}}{\sigma_{T,i}}$, where $\sigma_{T,i} = \sqrt{\mathrm{var}[\mathrm{TI}_{k,i}]}$, this would be a poorer approximation because $\frac{\mathrm{var}[\eta_{k,i}]}{\mathrm{var}[TI_{k,i}]} > \frac{\mathrm{var}[\epsilon_{k,i}]}{\mathrm{var}[\mathrm{OFI}_{k,i}]}$ as shown by R^2 values in Table B.3.

The intraday pattern in price variance was explained in an earlier study Madhavan, Richardson, and Roomans (1997) using a structural model. The authors argued that price volatility is higher in the morning because of a higher inflow of public and private information. In another study Hasbrouck (1991) the morning peak of price volatility is explained mostly by higher intensity of public information. Both studies agree that the impact of trades is larger in the morning. Our model contributes to this discussion by explaining the peak of price volatility using tangible quantities, rather than unobservable information variables. Our findings also suggest that price impact and information asymmetry may be, in fact, two sides of the same coin. If there is more private information in the morning than in the evening and if limit order traders are aware of this information asymmetry, their participation will likely diminish in the morning, leading to lower depth near market open. At the same time, low depth implies a higher price impact in our model, making the information advantages harder to realize at the market open.

3.3 Volume and Volatility

The positive correlation between magnitudes of price changes and trading volume is empirically confirmed by many authors (see Karpoff (1987) for a review). Recently, trading volume became an important metric for order execution algorithms—these algorithms often attempt to match a certain percentage of the total traded volume to reduce the price impact. However, it remains unclear whether trading volume truly determines the magnitude of price moves and whether it is a good metric for price impact. Casting doubt on this assertion, it was found in Jones, Kaul, and Lipson (1994) that the relation between *daily* volatility and *daily* volume is essentially due to the number of trades and not the volume per se (also see Chan and Fong (2000) for a following discussion).

We provide further evidence that volume is not a driver of price volatility, now on *intraday* timescales. First, we prove that even when prices are purely driven by OFI and not by volume, a concave relation between magnitude of price changes and transaction volume emerges as an artifact due to aggregation of data across time. Second, we confirm that such relation exists in the data, but it becomes statistically insignificant after accounting for magnitude of OFI.

Comparing the definitions of VOL and OFI we note that both quantities are sums of random variables. As the aggregation time window $[t_{k-1}, t_k]$ becomes progressively larger, the behavior of these sums (under certain assumptions) will be governed by the Law of Large Numbers and the Central Limit Theorem. We consider a general time interval $[0, T]$ and denote by $N(T)$ the number of order book events during that time interval. We also denote by OFI(T) and VOL(T), respectively, the OFI and the traded volume during $[0, T]$. The following proposition shows a link between OFI(T) and VOL(T) as T grows.

Proposition 3.1: *Assume that*

1. $\frac{N(T)}{T} \to \Lambda$, *as* $T \to \infty$, *where* Λ *is the average arrival rate of order book events.*
2. $\{e_i\}_{i \geq 1}$ *form a covariance-stationary sequence and have a linear-process representation* $e_i = \sum_{j=0}^{\infty} a_j Y_{i-j}$, *where* Y_i *is a two-sided sequence of i.i.d random variables with* $E[Y_i] = 0$ *and* $E[Y_i^2] = 1$, *and* a_j *is a sequence of constants with* $\sum_{j=0}^{\infty} a_j^2 = \sigma^2 < \infty$. *Moreover,* $\mathrm{cov}(e_1, e_{1+n}) \sim cn^{2(H-1)}$ *as* $n \to \infty$, *where* $0 < H < 1$ *is a constant that governs the decay of the autocorrelation function.*
3. $\{w_i\}_{i \geq 1}, w_i = b_i + s_i$ *are random variables with a finite mean* $\mu\pi$, *where* π *is the proportion of order book events that correspond to trades and* μ *is the mean trade*

size. $E|w_i|^p < \infty$ *for some* $p > 1$ *and* $\sum_{N \geq 1} \frac{1}{N}(E|\frac{1}{N}\sum_{i \leq N} w_i|^q)^{r/q} < \infty$ *for some* r, q *such that* $0 < r \leq q \leq \infty$ *and* $r/q \leq 1 - 1/p$.

$$\text{Then} \qquad \frac{(\mu\pi)^H}{\sigma}\frac{\text{OFI}(T)}{\text{VOL}^H(T)} \overset{T\to\infty}{\Rightarrow} \xi \sim N(0,1)$$

where $\Rightarrow$ *denotes convergence in distribution.*

The proof of this proposition is given in the Appendix. If the time interval $[0, T]$ includes a large enough number of order book events, Proposition 3.1 implies that

$$\text{OFI}(T) \sim \xi \frac{\sigma}{(\mu\pi)^H}\text{VOL}^H(T) \simeq N\left(0, \frac{\sigma^2}{(\mu\pi)^{2H}}\text{VOL}^{2H}(T)\right). \tag{21}$$

If the time intervals $[t_{k-1,i}, t_{k,i}]$ are large enough to support this approximation then substituting (21) in (7) yields

$$\Delta P_{k,i} \sim N\left(0, \frac{\sigma^2\beta_i^2}{(\mu\pi)^{2H}}\text{VOL}_{k,i}^{2H} + \sigma_i\right)$$

where $\sigma_i = \text{var}[\epsilon_{k,i}]$. Note that even if $\sigma_i = 0$, that is even if volume cannot affect price volatility through the residual variance, Proposition 3.1 predicts a spurious relation between price volatility and volume.

Interestingly, the recent theory of market microstructure invariants (Kyle and Obizhaeva (2011)) also predicts a relation between the volatility of OFI and trading volume. In their analysis, OFI is defined differently based on unobservable "bets", however it is natural to assume positive correlation between OFI and the imbalance of "bets", since the latter reach exchanges in form of actual orders.

We can recast this statement in a testable form for the magnitudes (absolute values) of price changes. Assuming $\epsilon_{k,i} \approx 0$, the scaling argument in Proposition 3.1 together with our linear price impact model imply that

$$|\text{OFI}_{k,i}| \approx \frac{\sigma}{(\mu\pi)^H}\text{VOL}_{k,i}^H|\xi_{k,i}| \tag{22}$$

$$|\Delta P_{k,i}| \approx \frac{\beta_i\sigma}{(\mu\pi)^H}\text{VOL}_{k,i}^H|\xi_{k,i}|. \tag{23}$$

We denote by $\theta_i = \frac{\beta_i\sigma}{(\mu\pi)^H}$ and take logarithms in (23) to obtain

$$\log|\Delta P_{k,i}| = \log\hat{\theta}_i + \hat{H}_i \log \text{VOL}_{k,i} + \log|\hat{\xi}_{k,i}|. \tag{24}$$

Based on Proposition 3.1, we expect this relation to be indirect (i.e., come through $|\text{OFI}_{k,i}|$) and noisy. To confirm this empirically, we estimate three regressions[17]:

$$|\Delta P_{k,i}| = \hat{\alpha}_i^O + \hat{\beta}_i^O |\mathrm{OFI}_{k,i}| + \hat{\epsilon}_{k,i}^O \tag{25a}$$

$$|\Delta P_{k,i}| = \hat{\alpha}_i^V + \hat{\beta}_i^V \mathrm{VOL}_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^V \tag{25b}$$

$$|\Delta P_{k,i}| = \hat{\alpha}_i^W + \hat{\phi}_i^O |\mathrm{OFI}_{k,i}| + \hat{\phi}_i^V \mathrm{VOL}_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^W. \tag{25c}$$

These regressions are estimated for every half-hour subsample with the exponents $\hat{H}_i$ pre-estimated by (24). The averages of $\hat{H}_i$ and their standard deviation for each stock are presented on the left panel in Table B.5. The exponent varies considerably across stocks and time, but is generally below 1/2 in our data. The average results of regressions (25a)–(25c) for each stock are presented on the middle and right panels. We observe that $|\mathrm{OFI}_{k,i}|$ explains the magnitude of price moves better than $\mathrm{VOL}_{k,i}^{\hat{H}_i}$. Although both variables appear to be statistically significant when taken individually, the t-statistics for $\mathrm{VOL}_{k,i}$ drop to marginally significant levels in the multiple regression. Thus, the dependence between absolute values of price moves and traded volume seems to come mostly from correlation between $\mathrm{VOL}_{k,i}$ and $|\mathrm{OFI}_{k,i}|$. Interestingly, the number of trades variable (suggested in Jones, Kaul, and Lipson (1994)) is also statistically significant on a stand-alone basis, but becomes insignificant when added to (25c) as a third variable. Given the recent proliferation of order splitting, the size of most orders is equal to one lot, so $\mathrm{VOL}_{k,i}$ is almost the same as the number of trades variable.

4 CONCLUSION

We have introduced OFI, a variable that cumulates the sizes of order book events, treating the contributions of market, limit and cancel orders equally, and provided empirical and theoretical evidence for a linear relation between high-frequency price changes, and OFI for individual stocks. We have shown that this linear model is robust across stocks and time-scales with a price impact coefficient inversely proportional to market depth. These relations suggest that prices respond to changes in the supply and demand for shares at the best quotes, and that the impact coefficient fluctuates with the amount of liquidity provision, or depth, in the market. Moreover, we have demonstrated that OFI is a more general metric of supply/demand dynamics than trade imbalance, which can be used to analyze intraday changes in volatility and monitor possible adverse selection in limit order

[17] Here we estimate linear regressions rather than log-linear ones to directly test whether the effect of VOL is consumed by |OFI| variable.

executions. Trades seem to carry little to no information about price changes after the simultaneous OFI is taken into account. If trades do not help to explain price changes after controlling for the OFI, it is highly possible that the relation between the magnitude of price changes, or price volatility and traded volume simply captures the noisy scaling relation between these variables. Overall, these findings seem to give an intuitive picture of the price impact of order book events, which is somewhat simpler than the one conveyed by previous studies.

APPENDIX A: PROCESSING OF TRADES AND QUOTES DATA

We considered only quotes with timestamps $\in$ [9:30 am, 4:00 pm], positive bid/ask prices and sizes and quote mode $\notin \{4, 7, 9, 11, 13, 14, 15, 19, 20, 27, 28\}$. Similarly, trades were considered only if they had timestamps $\in$ [9:30 am, 4:00 pm], positive price and size, correction indicator $\leq$ 2 and condition $\notin$ {"*O*", "*Z*", "*B*", "*T*", "*L*", "*G*", "*W*", "*J*", "*K*"}.

From the filtered quotes data we construct the NBBO quotes. This is done by scanning through the filtered quotes data, while maintaining a matrix with the best quotes for every exchange. When a new entry is read, we check the exchange flag of that entry and update the corresponding row in the exchange matrix. Using this matrix, the NBBO prices are computed at each entry as the highest bid and the lowest ask across all exchanges. The NBBO sizes are simply the sums of all sizes at the NBBO bid and ask across all exchanges. For more details on TAQ data set we refer the reader to Hasbrouck (2010), which discusses some particularities of that data, such as possible mis-sequencing of data across exchanges and lack of odd-lot sized orders. With our auxiliary data set we checked that neither of these issues significantly affects our results.

After the NBBO quotes are computed, we applied a simple quote test to the NBBO quotes and the filtered trades data. This test matches trades with NBBO quotes and computes the direction of matched trades. A trade is matched with a quote, if:

1. Trade is not inside the spread, that is
   (a) Trade price $\geq$ NBBO ask: in this case the trade is considered to be a buy trade.
   (b) Trade price $\leq$ NBBO bid: in this case the trade is considered to be a sell trade.
2. Trade date = quote date.
3. Trade timestamp $\in$ [quote timestamp, quote timestamp + 1 s].
4. If the above conditions allow to match a trade with several quotes, it is matched with the earliest quote.

This matching algorithm cannot identify the direction of trades occuring within the bid–ask spread. By comparing the number of matched trades with the overall number of trades in our sample we found that, depending on the stock, 59–95% of trades cannot be matched. Although these percentages appear to be extremely large, the volume percentage of unmatched trades is only 10–39% depending on the stock with an average of 17% across stocks and we believe that omitting these trades does not affect our results. There are other routines to estimate trade direction, including the tick test and the Lee–Ready rule Lee and Ready (1991). Although the latter is used quite frequently, there seems to be no compelling evidence of superiority of either of these heuristics Odders-White (2000); Theissen (2001). To test the robustness of our findings to the choice of a trade direction test, we compared our results on a subsample of stocks, applying alternatively the tick test or our quote test and results were virtually the same.

Finally, we removed observations with high bid–ask spreads to filter out "stub quotes" and data errors. To apply this filter coherently across stocks, we computed the 95-th percentile of bid–ask spread distribution for each stock and removed 5% of that stock's quotes with spreads above that percentile. For the representative stock in our sample (SLB), the removed observations fall mostly on the first minutes after market opening: 15.8% of them occur between 9:30 am and 9:35 am, and 42.1% of them occur between 9:30 am and 10:00 am. The average bid-ask spread of the removed quotes is 3.44 cents with a standard deviation 11.98 cents, the average queue size of these quotes is 11.78 round lots with a standard deviation 12.89 lots. The average time interval between two removed quotes is 1.03 s with a standard deviation 41.64 s. All of the results and tables in this article are generated using the filtered data.

APPENDIX B: ROBUSTNESS CHECKS

B.1 Cross-sectional Evidence

Table B.1 presents the average mid-price, daily transaction volume, daily number of best quote updates, daily number of trades, spread and the depth at the best bid and ask for fifty randomly chosen U.S. stocks. One round lot is equal to hundred shares. All values are calculated from the filtered data, that consists of twenty-one trading day during April, 2010.

Table B.2 presents a cross-section of results (averaged across time) for regressions:

$$\Delta P_{k,i} = \hat{\alpha}_i + \hat{\beta}_i \mathrm{OFI}_{k,i} + \hat{\epsilon}_{k,i},$$
$$\Delta P_{k,i} = \hat{\alpha}_i^Q + \hat{\beta}_i^Q \mathrm{OFI}_{k,i} + \hat{\gamma}_i^Q \mathrm{OFI}_{k,i} |\mathrm{OFI}_{k,i}| + \hat{\epsilon}_{k,i}^Q,$$

where $\Delta P_{k,i}$ are the 10 s mid-price changes in ticks and $\mathrm{OFI}_{k,i}$ are the contemporaneous OFIs. These regressions were estimated using 273 half-hour subsamples (indexed by i) for each stock and their outputs were averaged across subsamples. Each subsample typically contains about 180 observations (indexed by k). The t-statistics were computed using Newey–West standard errors. For brevity, we report

Table B.1 Descriptive statistics

Name	Ticker	Price	Daily volume, shares	Number of best quote updates	Number of trades	Average spread, ticks	Maximum spread, ticks	Best quote size, shares
Advanced Micro Devices	AMD	9.61	20,872,996	417,204	6687	1	1	103,484
Apollo Group	APOL	62.92	1,949,337	172,942	4095	2	5	1525
American Express	AXP	45.21	8,678,723	55,9701	7748	1	24	7918
Autozone	AZO	179.03	243,197	43,682	1081	9	35	750
Bank of America	BAC	18.43	164,550,168	1,529,395	15,008	1	1	320,801
Becton Dickinson	BDX	78.07	1,130,362	61,029	2968	2	5	1530
Bank of New York Mellon	BK	31.77	6,310,701	285,619	5518	1	1	12,199
Boston Scientific	BSX	7.13	25,746,787	309,441	6768	1	1	296,501
Peabody Energy corp	BTU	47.14	5,210,642	298,616	7267	1	3	2949
Caterpillar	CAT	67.20	6,664,891	392,499	8224	1	2	3835
Chubb	CB	52.22	1,951,618	149,010	3601	1	2	4251
Carnival	CCL	40.16	4,275,911	215,427	5503	1	2	5330
Cincinnati Financial	CINF	29.41	688,914	51,373	1528	1	2	4157
CME Group	CME	322.83	418,955	38,504	1412	31	103	541
Coach	COH	41.91	3,126,469	176,795	4458	1	2	4061
ConocoPhillips	COP	56.09	9,644,544	426,614	8621	1	2	8402
Coventry Health Care	CVH	24.16	1,157,022	79,305	2213	1	2	3838
Denbury Resources	DNR	17.88	5,737,740	263,173	4643	1	1	18,622
Devon Energy	DVN	66.98	3,260,982	177,006	5805	2	4	1847
Equifax	EFX	35.34	799,505	62,957	1945	1	3	3925
Eaton	ETN	78.53	1,757,136	67,989	3580	2	6	1254
Fiserv	FISV	52.56	1,038,311	58,304	2208	1	3	2026
Hasbro	HAS	39.48	1,322,037	86,040	2672	1	2	3438
HCP	HCP	32.63	2,872,521	213,045	4357	1	2	4810
Starwood Hotels	HOT	50.59	3,164,807	150,252	5106	2	4	2174

(continued)

Table B.1 Continued

Name	Ticker	Price	Daily volume, shares	Number of best quote updates	Number of trades	Average spread, ticks	Maximum spread, ticks	Best quote size, shares
Kohl's	KSS	56.88	3,064,821	128,196	4936	1	3	2688
L-3 Communications	LLL	94.64	670,937	72,818	2141	2	6	867
Lockheed Martin	LMT	84.14	1,416,072	88,254	3333	2	5	1495
Macy's	M	23.40	8,324,639	491,756	6469	1	1	17,567
Marriott	MAR	34.45	5,014,098	238,190	5499	1	2	6511
McAfee	MFE	40.04	2,469,324	109,073	3561	1	2	4018
McGraw-Hill	MHP	34.90	1,954,576	102,389	3261	1	2	4183
Medco Health Solutions	MHS	63.22	2,798,098	109,382	4680	1	3	2534
Merck	MRK	36.03	13,930,842	448,748	7997	1	1	23,137
Marathon Oil	MRO	32.33	5,035,354	341,408	5522	1	1	14,259
MeadWestvaco	MWV	26.96	1,035,547	92,825	2312	1	3	3741
Newmont Mining	NEM	53.43	5,673,718	435,295	7717	1	2	3847
Omnicom	OMC	41.17	3,357,585	150,800	4359	1	2	6492
MetroPCS Communications	PCS	7.53	4,424,560	107,967	2901	1	1	52,304
Pultegroup	PHM	11.80	6,834,683	262,420	4604	1	1	31,856
PerkinElmer	PKI	23.98	1,268,774	78,114	2127	1	2	7163
Ryder System	R	44.01	631,889	47,422	2085	2	5	1147
Reynolds American	RAI	54.44	773,387	56,236	2076	1	4	2177
Schlumberger	SLB	67.94	9,476,060	440,839	10,286	1	2	3942
Teco Energy	TE	16.52	1,070,815	70,318	1807	1	1	14,816
Time Warner Cable	TWC	53.21	177,0234	88,286	3554	2	3	2174
Whirlpool	WHR	97.73	1,424,264	134,152	3348	4	9	958
Windstream	WIN	11.03	2,508,830	104,887	2937	1	1	79,834
Watson Pharmaceuticals	WPI	42.51	895,967	63,094	2024	1	3	2884
XTO Energy	XTO	48.13	7,219,436	612,804	5040	1	7	22,479
Grand mean		51.75	7,512,376	223,232	4552	2	6	22,665

Table B.2 Relation between price changes and OFI

Ticker	Average results							Hypothesis testing		
	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}_i$	$t(\hat{\beta}_i)$	$\hat{\gamma}_i^Q$	$t(\hat{\gamma}_i^Q)$	R^2%	$\{\alpha_i \neq 0\}$%	$\{\beta_i \neq 0\}$%	$\{\gamma_i^Q \neq 0\}$%
AMD	−0.0032	−0.24	0.0008	11.10	1.4E-07	0.93	64	0	100	36
APOL	0.0038	0.13	0.0555	10.74	−2.2E-04	−2.42	63	17	96	6
AXP	0.0019	0.11	0.0082	14.12	−3.8E-06	−1.37	69	16	100	8
AZO	0.0101	0.34	0.1619	7.02	−9.3E-04	−1.40	47	25	99	6
BAC	−0.0018	−0.13	0.0002	19.08	1.9E-09	−0.08	79	3	100	14
BDX	−0.0008	−0.07	0.0536	10.77	−1.1E-04	−0.74	63	12	100	12
BK	−0.0078	−0.26	0.0069	15.56	−4.0E-06	−0.89	74	6	100	8
BSX	0.0000	0.01	0.0003	7.55	7.8E-08	3.51	58	3	88	51
BTU	0.0048	0.15	0.0242	14.75	−3.5E-05	−2.05	72	16	100	3
CAT	0.0147	0.30	0.0194	14.80	−1.9E-05	−1.72	71	19	100	5
CB	−0.0086	−0.05	0.0191	12.61	−3.5E-07	−0.04	64	10	100	18
CCL	−0.0067	−0.24	0.0140	14.16	−1.2E-05	−1.03	70	7	100	11
CINF	−0.0030	−0.02	0.0260	11.66	−7.0E-06	0.38	70	4	99	30
CME	0.0506	0.06	0.6262	5.46	−7.2E-03	−1.66	35	18	96	7
COH	−0.0221	−0.54	0.0179	13.13	−1.7E-05	−1.18	69	5	100	7
COP	−0.0008	0.10	0.0084	12.79	−5.8E-06	−1.86	68	13	100	5
CVH	−0.0034	−0.07	0.0217	11.74	7.6E-06	0.37	65	7	99	20
DNR	−0.0008	−0.07	0.0045	13.78	−1.3E-07	0.19	69	5	99	22
DVN	0.0112	0.20	0.0370	12.11	−1.0E-04	−2.72	65	19	100	2
EFX	−0.0032	−0.06	0.0222	9.47	6.4E-05	0.87	56	6	99	32
ETN	−0.0076	0.10	0.0712	11.01	−2.3E-04	−1.81	65	17	100	4
FISV	−0.0002	0.10	0.0397	11.09	−2.3E-05	−0.28	63	10	100	16
HAS	−0.0031	−0.01	0.0222	12.36	4.7E-06	0.28	67	6	100	23
HCP	−0.0078	−0.21	0.0150	13.82	−1.4E-05	−0.63	67	5	100	10
HOT	−0.0012	0.05	0.0345	12.94	−7.2E-05	−2.06	68	14	100	4
KSS	−0.0030	−0.05	0.0317	14.10	−5.4E-05	−1.38	71	13	100	5
LLL	0.0160	0.42	0.1000	12.34	−3.8E-04	−1.56	67	22	98	7
LMT	0.0006	0.00	0.0520	14.14	−1.2E-04	−1.49	72	17	100	4
M	−0.0010	0.07	0.0043	16.61	8.8E-08	0.15	75	6	100	19
MAR	−0.0039	0.02	0.0121	15.10	−4.1E-06	−0.43	71	10	100	10
MFE	0.0087	0.22	0.0205	13.19	−3.8E-05	−0.63	68	11	100	11
MHP	−0.0073	−0.18	0.0211	12.41	5.8E-06	0.18	68	5	99	24
MHS	−0.0055	−0.20	0.0334	11.97	−8.3E-05	−1.64	66	12	100	4
MRK	−0.0065	−0.26	0.0032	13.26	−5.4E-07	−0.61	69	4	100	14
MRO	0.0018	0.12	0.0058	14.16	−3.6E-07	0.32	69	8	100	23
MWV	−0.0011	0.02	0.0205	12.55	−1.7E-05	−0.31	68	9	100	17
NEM	−0.0102	−0.26	0.0170	13.90	−1.9E-05	−2.15	71	12	100	5
OMC	−0.0099	−0.36	0.0144	12.40	−4.5E-06	−0.19	65	4	100	20
PCS	−0.0006	−0.05	0.0015	6.52	1.8E-06	3.79	53	2	86	51
PHM	0.0006	0.02	0.0027	11.27	8.4E-07	1.20	66	3	99	36
PKI	−0.0004	−0.05	0.0102	7.96	4.1E-05	2.15	53	3	96	51
R	0.0006	0.03	0.0667	10.90	3.7E-05	−0.21	63	14	100	16
RAI	−0.0070	−0.10	0.0396	11.39	2.6E-05	−0.03	66	9	100	19
SLB	−0.0077	−0.21	0.0198	16.27	−1.8E-05	−1.67	76	10	100	2
TE	0.0011	0.05	0.0049	7.76	1.4E-05	3.27	54	4	91	55
TWC	−0.0130	−0.15	0.0384	12.24	−5.6E-05	−0.73	64	12	99	9
WHR	0.0628	0.73	0.1278	11.10	−3.3E-04	−1.44	65	25	100	7
WIN	−0.0004	−0.04	0.0009	4.32	1.5E-06	3.98	44	1	72	43
WPI	−0.0090	−0.27	0.0270	11.46	2.9E-05	0.26	66	5	99	23
XTO	−0.0088	−0.25	0.0029	13.26	2.7E-07	0.48	65	3	100	28
Average	0.0002	−0.02	0.0398	12.08	−2.0E-04	−0.32	65	10	98	17

the R^2, the average $\hat{\alpha}_i$, and the average $\hat{\beta}_i$ only for the first regression (with a single $\mathrm{OFI}_{k,i}$ term). There is almost no difference between averages of estimates $\hat{\beta}_i$ and $\hat{\beta}_i^Q$ and the R^2 in two regressions. The last three columns report the percentage of samples where the coefficient(s) passed the z-test at the 5% significance level.

Table B.3 presents the average results of regressions:

$$\Delta P_{k,i} = \hat{\alpha}_i + \hat{\beta}_i \mathrm{OFI}_{k,i} + \hat{\epsilon}_{k,i},$$
$$\Delta P_{k,i} = \hat{\alpha}_i^T + \hat{\beta}_i^T \mathrm{TI}_{k,i} + \hat{\eta}_{k,i},$$
$$\Delta P_{k,i} = \hat{\alpha}_i^D + \hat{\theta}_i^O \mathrm{OFI}_{k,i} + \hat{\theta}_i^T \mathrm{TI}_{k,i} + \hat{\epsilon}_{k,i}^D,$$

where $\Delta P_{k,i}$ are the 10 s mid-price changes, $\mathrm{OFI}_{k,i}$ are the contemporaneous OFIs and $\mathrm{TI}_{k,i}$ are the contemporaneous trade imbalances. These regressions were estimated using 273 half-hour subsamples (indexed by i) for each stock and their outputs were averaged across subsamples. Each subsample typically contains about 180 observations (indexed by k). The t-statistics were computed using Newey–West standard errors. For each of three regressions, Table B.3 reports the average R^2, the average t-statistic of the coefficient(s), the percentage of samples where the coefficient(s) passed the z-test at the 5% significance level and the F-statistic of the regression.

Table B.4 presents the results of regressions:

$$\log \hat{\beta}_i = \alpha\hat{_{L,i}} - \hat{\lambda} \log D_i + \hat{\epsilon}_{L,i},$$
$$\hat{\beta}_i = \alpha\hat{_{M,i}} + \frac{\hat{c}}{D_i^{\hat{\lambda}}} + \hat{\epsilon}_{M,i},$$

where $\hat{\beta}_i$ is the price impact coefficient for the i-th half-hour subsample and D_i is the average market depth for that subsample. These regressions were estimated for each of the 50 stocks, using 273 estimates of $\hat{\beta}_i$ for that stock, obtained from (14). The second regression uses estimates $\hat{\lambda}$ obtained from the first regression. The t-statistics were computed using Newey–West standard errors. The last three columns provide three alternative fit measures—the R^2 of the linear regression (17), the squared correlation between $\hat{\beta}_i$ and fitted values $\hat{\hat{\beta}}_i = \frac{\hat{c}}{D_i^{\hat{\lambda}}}$ and the squared correlation between $\hat{\beta}_i$ and $\hat{\hat{\beta}}_i^* = \frac{\hat{c}}{D_i}$.

Table B.5 presents the average results of regressions:

$$|\Delta P_{k,i}| = \hat{\alpha}_i^O + \hat{\beta}_i^O |\mathrm{OFI}_{k,i}| + \hat{\epsilon}_{k,i}^O,$$
$$|\Delta P_{k,i}| = \hat{\alpha}_i^V + \hat{\beta}_i^V \mathrm{VOL}_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^V,$$
$$|\Delta P_{k,i}| = \hat{\alpha}_i^W + \hat{\phi}_i^O |\mathrm{OFI}_{k,i}| + \hat{\phi}_i^V \mathrm{VOL}_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^W,$$

where $\Delta P_{k,i}$ are the 10 s mid-price changes, $\mathrm{OFI}_{k,i}$ are the contemporaneous OFIs and $\mathrm{VOL}_{k,i}$ are the contemporaneous trade volumes. The exponents $\hat{H}_i$ were estimated in each subsample beforehand using a logarithmic regression: $\log|\Delta P_{k,i}| = \log\hat{\theta}_i + \hat{H}_i \log \mathrm{VOL}_{k,i} + \log|\hat{\xi}_{k,i}|$. These regressions were estimated using 273 half-hour subsamples (indexed by i) for each stock and their outputs were averaged across subsamples. Each subsample typically contains about 180 observations (indexed by k). The t-statistics were computed using Newey–West

Table B.3 Comparison of OFI and trade imbalance

Ticker	Order flow imbalance				Trade imbalance				Both covariates					
	R^2%	$t(\hat{\beta}_i)$	$\{\beta_i \neq 0\}$%	F	R^2%	$t(\hat{\beta}_i^T)$	$\{\beta_i^T \neq 0\}$%	F	R^2%	$t(\hat{\theta}_i^O)$	$t(\hat{\theta}_i^T)$	$\{\theta_i^O \neq 0\}$%	$\{\theta_i^T \neq 0\}$%	F
AMD	64	11.10	100	382	39	5.06	95	140	67	7.64	1.59	99	45	214
APOL	63	10.74	96	396	30	5.04	95	83	66	8.95	1.58	96	44	211
AXP	69	14.12	100	449	34	5.55	92	101	71	11.31	1.90	100	55	241
AZO	47	7.02	99	179	30	4.88	96	87	54	5.78	2.87	98	81	118
BAC	79	19.08	100	774	45	7.03	98	157	80	13.55	0.80	99	25	397
BDX	63	10.77	100	362	28	4.85	92	79	65	8.90	1.53	100	46	195
BK	74	15.56	100	610	36	5.36	93	117	75	11.90	0.80	100	26	313
BSX	58	7.55	88	338	31	3.60	71	106	62	5.74	0.88	82	24	189
BTU	72	14.75	100	527	35	6.03	97	103	74	11.96	1.63	100	44	277
CAT	71	14.80	100	498	33	5.75	94	94	72	12.14	1.55	100	46	262
CB	64	12.61	100	378	33	5.47	95	102	66	9.41	1.57	99	44	202
CCL	70	14.16	100	478	32	5.31	94	93	71	11.44	1.17	100	37	247
CINF	70	11.66	99	552	39	5.35	96	141	72	8.28	1.28	98	40	297
CME	35	5.46	96	112	24	4.31	88	63	44	4.73	2.78	96	71	78
COH	69	13.13	100	457	29	4.75	93	80	70	11.06	1.12	100	31	238
COP	68	12.79	100	450	35	5.69	92	107	70	10.25	1.76	100	49	240
CVH	65	11.74	99	418	35	5.05	93	114	67	8.43	1.35	97	37	222
DNR	69	13.78	99	471	32	4.89	92	101	70	10.43	1.27	99	37	246
DVN	65	12.11	100	414	33	5.57	95	96	68	9.61	2.12	98	60	226
EFX	56	9.47	99	289	31	4.75	89	101	60	7.13	2.26	98	55	167
ETN	65	11.01	100	389	25	4.43	86	69	67	9.85	1.47	99	43	209
FISV	63	11.09	100	380	28	4.82	93	79	65	9.08	1.25	100	38	201
HAS	67	12.36	100	427	32	5.15	95	97	68	9.67	1.17	100	34	223
HCP	67	13.82	100	417	31	5.07	90	91	68	10.92	1.33	100	42	217

(continued)

Table B.3 Continued

Ticker	Order flow imbalance				Trade imbalance				Both covariates					
	R^2%	$t(\hat{\beta}_i)$	$\{\beta_i \neq 0\}$%	F	R^2%	$t(\hat{\beta}_i^T)$	$\{\beta_i^T \neq 0\}$%	F	R^2%	$t(\hat{\theta}_i^O)$	$t(\hat{\theta}_i^T)$	$\{\theta_i^O \neq 0\}$%	$\{\theta_i^T \neq 0\}$%	F
HOT	68	12.94	100	438	27	4.75	88	74	70	11.00	1.48	100	40	231
KSS	71	14.10	100	525	31	5.16	93	91	72	11.86	1.14	100	37	274
LLL	67	12.34	98	485	36	6.00	95	117	70	9.68	2.14	98	57	270
LMT	72	14.14	100	516	35	5.80	96	105	73	11.35	1.83	100	51	277
M	75	16.61	100	640	35	5.10	93	108	76	12.80	1.13	100	38	330
MAR	71	15.10	100	498	34	5.54	95	105	72	11.41	1.18	100	36	258
MFE	68	13.19	100	463	31	4.82	88	93	69	10.27	0.89	100	30	239
MHP	68	12.41	99	489	31	5.09	93	96	70	9.94	1.04	99	33	257
MHS	66	11.97	100	414	28	4.81	89	80	68	10.03	1.50	99	40	218
MRK	69	13.26	100	451	31	4.99	92	93	70	10.41	1.02	100	29	235
MRO	69	14.16	100	465	35	5.38	96	104	70	10.67	1.12	100	35	241
MWV	68	12.55	100	452	34	5.30	96	102	69	9.66	1.01	100	33	237
NEM	71	13.90	100	490	34	5.77	92	100	72	11.38	1.90	100	54	260
OMC	65	12.40	100	411	30	4.90	93	88	67	9.85	1.22	100	39	216
PCS	53	6.52	86	297	35	4.08	74	169	58	4.47	1.43	81	35	195
PHM	66	11.27	99	416	35	4.76	93	115	68	8.40	1.22	98	38	224
PKI	53	7.96	96	263	28	3.98	82	89	57	6.16	1.70	93	47	148
R	63	10.90	100	352	27	4.80	96	71	65	9.02	1.58	100	44	188
RAI	66	11.39	100	422	36	5.60	98	111	68	8.64	1.42	100	43	224
SLB	76	16.27	100	644	32	5.31	89	94	77	13.91	1.56	100	47	336
TE	54	7.76	91	301	37	4.65	82	175	60	5.27	1.96	86	45	200
TWC	64	12.24	99	377	31	5.21	86	93	66	9.67	1.70	99	45	201
WHR	65	11.10	100	394	29	5.03	95	85	67	9.27	1.86	100	52	217
WIN	44	4.32	72	243	41	4.74	75	249	58	2.60	2.55	58	47	206
WPI	66	11.46	99	437	32	4.80	93	100	68	8.95	1.35	99	46	232
XTO	65	13.26	100	399	21	3.78	78	54	66	11.72	1.42	100	40	209
Average	65	12.08	98	429	32	5.08	91	103	67	9.53	1.51	97	43	231

Table B.4 Relation between the price impact coefficient and market depth

Ticker	Parameter estimates						Fit measures		
	$\hat{c}$	$\hat{\lambda}$	$t(\hat{c}=0)$	$t(\hat{c}=0.5)$	$t(\hat{\lambda}=0)$	$t(\hat{\lambda}=1)$	$R^2(\%)$	$\text{corr}[\hat{\beta},\hat{\hat{\beta}}]^2(\%)$	$\text{corr}[\hat{\beta},\hat{\beta}^*]^2(\%)$
AMD	0.53	1.04	31.06	2.0	22.5	1.0	78	86	86
APOL	0.30	0.38	4.59	−3.2	1.1	−1.8	3	34	35
AXP	0.45	1.01	26.27	−3.2	45.8	0.6	90	87	87
AZO	0.45	0.70	5.47	−0.7	5.2	−2.2	14	17	16
BAC	0.87	1.10	31.80	13.6	19.1	1.7	80	89	89
BDX	0.48	1.03	23.91	−1.2	22.2	0.6	74	71	71
BK	0.47	1.04	28.28	−1.9	68.5	2.5	94	94	94
BSX	0.51	1.02	15.23	0.3	24.1	0.4	73	81	81
BTU	0.58	1.10	45.36	6.2	53.8	5.0	93	90	90
CAT	0.48	1.01	35.12	−1.4	20.7	0.2	91	91	90
CB	0.53	1.09	32.41	2.1	63.6	5.5	93	91	91
CCL	0.45	1.04	35.26	−3.6	41.9	1.5	89	86	86
CINF	0.43	1.03	25.48	−4.2	52.5	1.7	93	90	90
CME	1.21	0.35	2.10	1.2	1.4	−2.7	1	2	2
COH	0.61	1.11	15.35	2.9	44.7	4.3	81	83	82
COP	0.32	0.94	13.77	−8.1	22.6	−1.6	82	79	79
CVH	0.54	1.13	26.92	2.2	37.9	4.2	88	90	89
DNR	0.55	1.10	40.77	3.6	44.9	3.9	92	90	90
DVN	0.34	0.91	16.15	−7.8	19.3	−2.0	48	61	61
EFX	0.43	1.05	19.58	−3.0	27.1	1.2	84	80	80
ETN	0.64	1.11	13.55	2.9	20.8	2.1	65	63	63
FISV	0.47	1.04	25.33	−1.7	34.1	1.3	85	80	80
HAS	0.52	1.08	27.80	1.3	49.8	3.8	90	86	85
HCP	0.37	1.00	33.13	−11.3	64.7	0.0	95	94	94
HOT	0.61	1.13	28.19	5.2	37.7	4.3	87	87	87
KSS	0.59	1.09	28.99	4.3	41.7	3.4	90	85	85
LLL	0.57	1.02	15.30	1.9	14.5	0.3	53	65	65

(continued)

Table B.4 Continued

Ticker	Parameter estimates						Fit measures		
	$\hat{c}$	$\hat{\lambda}$	$t(\hat{c}=0)$	$t(\hat{c}=0.5)$	$t(\hat{\lambda}=0)$	$t(\hat{\lambda}=1)$	$R^2(\%)$	$\text{corr}[\hat{\beta},\hat{\hat{\beta}}]^2(\%)$	$\text{corr}[\hat{\beta},\hat{\hat{\beta}}^*]^2(\%)$
LMT	0.72	1.17	7.93	2.4	15.6	2.3	69	63	63
M	0.52	1.06	24.92	1.0	52.1	3.0	94	92	92
MAR	0.50	1.06	22.26	0.0	52.7	3.1	92	89	89
MFE	0.47	1.06	22.12	−1.3	45.3	2.7	92	89	89
MHP	0.45	1.02	20.58	−2.1	38.1	0.6	83	78	78
MHS	0.71	1.16	19.88	5.9	39.5	5.3	88	87	86
MRK	0.31	0.94	21.38	−12.8	36.3	−2.3	87	84	84
MRO	0.55	1.09	28.87	2.4	51.9	4.2	94	94	94
MWV	0.54	1.13	28.16	2.2	39.3	4.6	90	87	87
NEM	0.51	1.07	31.09	0.4	39.3	2.6	89	88	88
OMC	0.52	1.04	36.61	1.1	19.5	0.7	86	90	90
PCS	0.43	1.06	22.79	−3.4	18.6	1.1	53	83	83
PHM	0.62	1.10	39.56	7.7	36.6	3.3	87	92	92
PKI	0.49	1.14	29.01	−0.5	34.7	4.4	80	87	86
R	0.50	1.05	17.43	−0.1	15.8	0.7	58	59	59
RAI	0.51	1.07	26.19	0.4	47.3	3.1	88	79	79
SLB	0.56	1.08	23.39	2.5	47.6	3.6	92	94	93
TE	0.35	1.10	12.12	−5.1	25.1	2.2	70	85	86
TWC	0.55	1.07	22.29	1.9	18.9	1.2	73	85	84
WHR	1.09	1.25	12.66	6.9	13.4	2.7	51	54	53
WIN	17.21	1.80	13.95	13.5	12.2	5.4	35	72	74
WPI	0.39	0.99	19.57	−5.6	32.6	−0.4	79	77	77
XTO	0.97	1.19	27.70	13.46	35.64	5.77	88	91	90
Grand mean	0.88	1.05	23.55	0.59	33.40	1.99	76	79	79

Table B.5 Comparison of traded volume and OFI

Ticker	Avg	Stdev	Order flow imbalance				Traded volume				Both covariates					
	$\hat{H}$	$\hat{H}$	$R^2(\%)$	$t(\hat{\beta}_i^O)$	$\beta_i^O \neq 0(\%)$	F	$R^2(\%)$	$t(\hat{\beta}_i^V)$	$\beta_i^V \neq 0(\%)$	F	$R^2(\%)$	$t(\hat{\phi}_i^O)$	$t(\hat{\phi}_i^V)$	$\phi_i^O \neq 0(\%)$	$\phi_i^V \neq 0(\%)$	F
AMD	0.06	0.08	63	11.7	100	356	14	4.6	87	34	63	10.8	1.2	99	38	182
APOL	0.24	0.08	53	9.1	97	258	25	6.9	100	63	57	7.6	3.3	94	86	144
AXP	0.16	0.08	55	11.3	100	249	20	6.8	100	48	57	9.7	2.9	100	82	133
AZO	0.43	0.22	39	6.3	98	131	32	5.8	100	93	50	5.0	3.9	97	98	98
BAC	0.09	0.08	73	17.6	100	560	24	6.0	89	61	74	15.3	1.3	97	40	285
BDX	0.26	0.10	55	9.4	100	261	27	6.5	100	71	58	7.6	3.1	99	85	147
BK	0.11	0.07	68	14.1	100	437	19	6.7	97	46	68	12.6	2.0	100	58	225
BSX	–0.17	2.41	68	10.3	100	486	14	3.4	97	33	69	10.1	0.0	99	13	246
BTU	0.24	0.07	58	11.4	100	283	23	7.1	99	57	60	9.7	2.6	100	81	151
CAT	0.22	0.07	56	11.0	100	250	19	6.3	99	44	57	9.7	2.3	100	68	131
CB	0.19	0.09	56	11.1	100	261	23	6.5	99	58	58	9.1	2.8	100	76	141
CCL	0.14	0.07	60	12.2	100	309	19	6.7	99	45	62	10.8	2.5	100	77	162
CINF	0.13	0.12	67	12.0	100	505	30	6.2	98	85	69	10.3	2.1	100	58	268
CME	0.49	0.24	28	4.8	98	78	30	5.3	100	83	42	3.9	4.1	94	99	71
COH	0.19	0.07	60	11.3	100	299	22	6.5	99	52	61	9.8	2.4	100	73	157
COP	0.16	0.07	56	10.5	100	277	20	6.1	97	49	58	9.2	2.5	100	74	145
CVH	0.18	0.10	62	11.4	100	352	27	6.1	100	72	64	9.2	2.4	100	73	189
DNR	0.08	0.07	64	13.4	100	376	17	6.4	95	38	65	12.0	1.9	99	57	193
DVN	0.26	0.07	52	9.6	97	236	24	6.9	100	59	55	8.0	3.2	96	85	131
EFX	0.20	0.11	52	9.1	100	241	26	5.6	99	69	56	7.3	2.8	99	77	137
ETN	0.26	0.10	55	9.1	99	252	27	6.6	99	70	58	7.6	3.1	98	85	142
FISV	0.19	0.11	57	10.1	100	284	25	6.0	100	65	59	8.3	2.4	100	70	153
HAS	0.20	0.09	61	11.3	100	328	26	6.3	100	67	63	9.5	2.5	100	76	175
HCP	0.14	0.07	57	11.8	100	268	21	7.1	99	50	59	10.0	2.8	100	80	143
HOT	0.23	0.08	57	10.5	99	263	24	7.2	100	60	60	9.0	3.2	99	88	145

(continued)

Table B.5 Continued

Ticker	Avg	Stdev	Order flow imbalance				Traded volume				Both covariates					
	$\hat{H}$	$\hat{H}$	R^2(%)	$t(\hat{\beta}_i^O)$	$\beta_i^O \neq 0$(%)	F	R^2(%)	$t(\hat{\beta}_i^V)$	$\beta_i^V \neq 0$(%)	F	R^2(%)	$t(\hat{\phi}_i^O)$	$t(\hat{\phi}_i^V)$	$\phi_i^O \neq 0$(%)	$\phi_i^V \neq 0$(%)	F
KSS	0.24	0.08	60	11.6	100	318	25	6.8	99	61	62	9.8	2.6	99	78	169
LLL	0.33	0.12	58	10.3	97	323	34	7.2	100	101	63	7.9	3.4	96	92	188
LMT	0.28	0.09	61	11.6	100	327	31	7.6	100	85	64	9.3	3.1	100	85	182
M	0.11	0.07	69	15.2	100	463	20	6.3	100	46	69	13.5	2.0	100	63	238
MAR	0.15	0.07	61	13.3	100	324	21	7.0	99	50	62	11.5	2.5	100	74	170
MFE	0.16	0.09	60	11.7	100	318	24	7.0	98	62	62	9.7	2.6	100	73	170
MHP	0.20	0.10	62	11.6	100	377	25	6.1	100	62	64	9.7	2.0	100	56	199
MHS	0.23	0.08	56	10.0	100	258	24	6.7	100	58	58	8.4	2.9	100	80	139
MRK	0.10	0.07	62	12.1	100	330	17	5.5	99	40	63	10.8	1.9	100	60	170
MRO	0.09	0.06	61	12.7	100	333	16	6.4	97	36	63	11.5	2.0	100	56	172
MWV	0.18	0.10	62	11.3	100	330	28	6.9	100	75	64	9.2	2.6	100	79	180
NEM	0.20	0.07	56	10.6	100	253	20	6.3	99	47	58	9.3	2.6	100	79	135
OMC	0.15	0.09	57	11.0	100	286	20	6.4	98	48	59	9.4	2.5	100	75	151
PCS	0.11	0.18	62	8.9	100	411	18	3.8	98	54	63	8.4	0.8	100	28	214
PHM	0.07	0.08	64	11.5	100	384	15	5.4	91	34	65	10.7	1.2	100	41	195
PKI	0.11	0.11	55	9.0	99	266	20	4.8	98	47	57	7.8	1.9	98	55	141
R	0.27	0.11	56	9.8	99	259	28	6.3	100	74	59	7.9	3.1	99	87	147
RAI	0.25	0.10	61	10.6	100	334	28	5.9	99	73	63	8.8	2.6	100	75	182
SLB	0.24	0.07	62	12.5	99	330	19	5.8	97	46	63	11.2	1.9	99	56	171
TE	0.09	1.69	60	9.6	100	371	18	4.5	84	48	61	8.7	1.3	99	43	196
TWC	0.25	0.10	55	10.5	100	253	27	6.8	100	73	58	8.4	3.1	99	83	142
WHR	0.34	0.11	56	9.2	99	272	29	6.6	100	78	59	7.5	3.2	98	88	156
WIN	0.06	0.26	48	5.5	86	340	10	2.9	50	34	49	5.3	0.6	85	31	179
WPI	0.22	0.10	61	11.0	100	361	28	5.9	100	75	64	9.0	2.4	99	71	196
XTO	0.08	0.06	53	11.3	100	238	15	6.6	100	32	55	10.0	2.8	100	82	125
Average	0.18	0.18	58	10.9	99	313	23	6.1	97	58	61	9.3	2.4	99	70	168

standard errors. For each of three regressions, Table B.5 reports the average R^2, the average t-statistic of the coefficient(s), the percentage of samples where the coefficient(s) passed the z-sest at the 5% significance level and the F-statistic of the regression.

B.2 Transaction Prices

To reconcile our results with earlier studies that operate in transaction time, we repeated regressions (14), (16a), (16b) with differences between transaction prices $\Delta_L P_k^t = P_k^t - P_{k-L}^t$ for L trades, instead of differences in mid-prices ΔP_k. We picked at random five stocks from our sample (BDX, CB, MHS, PHM, and PKI), and computed $\Delta_L P_k^t$ for $L = 2, 5, 10$ trades (we avoided using $L = 1$ because of possible issues with trade and quote matching). Using the same inter-trade time intervals we computed concurrent OFI and TI variables. To ensure that there is an ample amount of data for each regression, we pooled data across days for each stock and each intraday time interval, resulting in thirteen samples for each stock over a month of data. The results averaged across time and stocks are presented in Table B.6 and closely mirror our results for mid-prices. The variable OFI_k explains price changes better than TI_k on stand-alone basis. Moreover, the effect of trades on prices seems to be captured by the OFI, that is the variable TI_k loses its statistical significance[18]. When used together with OFI_k in the regression. The increase in R^2 from adding TI_k as an extra regressor is almost nill (0.65%, 0.18%, 0.24% for $L = 1, 2, 5$ respectively).

Interestingly, we found that the relation between trade price changes and OFI_k (or TI_k) is sometimes concave. We estimated regressions (14) and (16a) for trade price changes $\Delta_L P_k^t$ with additional quadratic variable $\text{OFI}_k|\text{OFI}_k|$ and found that average t-statistics of its coefficient are, respectively -3.02, -4.10 and -3.85 for $L = 1, 2, 5$ trades. The quadratic term is significant at 5% level in 60%, 74% and 85% of samples for respective values of L, and we did not observe any pattern in these t-statistics, neither across stock nor across time. In the trade imbalance regression the coefficient near quadratic variable $\text{TI}_k|\text{TI}_k|$ is also significant with average t-statistics -3.64, -5.53, -5.48 for respective lag values and it is significant in even a larger fraction of samples.

From these results it appears that price impact is concave when prices are sampled at trade times, but it is linear when they are sampled at regular time intervals. This effect may be a consequence of sampling data at special times (i.e., trade times), which may introduce systematic down biases into the dependent variable. For example, if traders submit large orders when they expect their impact to be minimal, that would lead to a concave (sublinear) price impact. Supporting the idea of a sampling bias, we found that when mid-price changes are sampled at trade times, the price impact of OFI_k is again concave in a substantial fraction

[18] Here we also use Newey–West standard errors because regression residuals have statistically significant autocorrelation.

Table B.6 Comparison of OFI and trade imbalance for transaction prices

Lag (trades)	Order flow imbalance				Trade imbalance				Both covariates					
	R^2(%)	$t(\hat{\beta}_i)$	$\{\beta_i \neq 0\}$(%)	F	R^2(%)	$t(\hat{\beta}_i^T)$	$\{\beta_i^T \neq 0\}$(%)	F	R^2(%)	$t(\hat{\theta}_i^O)$	$t(\hat{\theta}_i^T)$	$\{\theta_i^O \neq 0\}$(%)	$\{\theta_i^T \neq 0\}$(%)	F
$L=2$	14	15.03	100	464	1	2.97	69	26	15	14.19	−2.90	100	71	245
$L=5$	38	16.68	98	753	8	4.79	88	113	39	15.13	−0.14	98	14	379
$L=10$	51	14.85	98	655	13	4.85	88	100	51	13.21	0.70	98	11	329

of our samples. We also regressed changes in last trade prices sampled regularly at a 1 min frequency on OFI_k, and observed concave price impact once again. This may again be attributed to a dependent variable bias—since trades are relatively infrequent, for many time intervals the trade prices are going to be stale and trade price changes are equal to zero, while mid-price changes are not.

B.3 Order Flow at Higher Order Book Levels

The level of detail in our Level 2 auxiliary data set allows us to analyze contributions of order flows at different price levels to price formation and to confirm our claim that price changes are mostly driven by activity at the top of the order book (thus Level 1 data are sufficient to study the impact of limit orders on prices).

For example, consider the bid side of the order book with 10 shares at the top two levels. Absent any activity on the ask side and the second bid level, an OFI of –11 shares will lead to a bid price change of −1 tick. However, if 9 orders at the second bid level cancel before that order flow happens, the same OFI of –11 shares will lead to a price change of −2 ticks. In other words, if order activity up to second (third, fourth, etc) level is important, tracking OFI only at the best prices will give a flawed picture of price dynamics. To test this assertion, we compute variables $\mathrm{OFI}^m, m=2,\ldots,5$ from m-th level queue fluctuations similarly to (12) and relabel $\mathrm{OFI}^1=\mathrm{OFI}$. Then we fit five regressions, similar to (14), where variables $\mathrm{OFI}^m,\ m=2,\ldots,5$ are added one at a time:

$$\Delta P_{k,i}=\hat{\alpha}_i^M+\sum_{m=1}^{M}\hat{\beta}_i^{m,M}\mathrm{OFI}_{k,i}^m+\hat{\epsilon}_{k,i}^M,\ \ M=1,\ldots,5. \tag{B1}$$

The average results across time for a representative stock are shown on Figure B.1. The average increase in explanatory power (measured by R^2) from adding OFI^2 as a regressor is 6.22%, which is quite small compared to the stand-alone R^2 of 70.83% for OFI^1. The effect of $\mathrm{OFI}^3-\mathrm{OFI}^5$ is very small, and their coefficients appear to be only marginally significant, in contrast with those of OFI^1 and OFI^2. The cross-time average of coefficients $\hat{\beta}_i^{1,1}$ in the simple regression[19] with OFI^1 is 0.0597. In the

[19]This coefficient is higher than the one obtained with NBBO data, because NASDAQ best quote depth is smaller than NBBO depth.

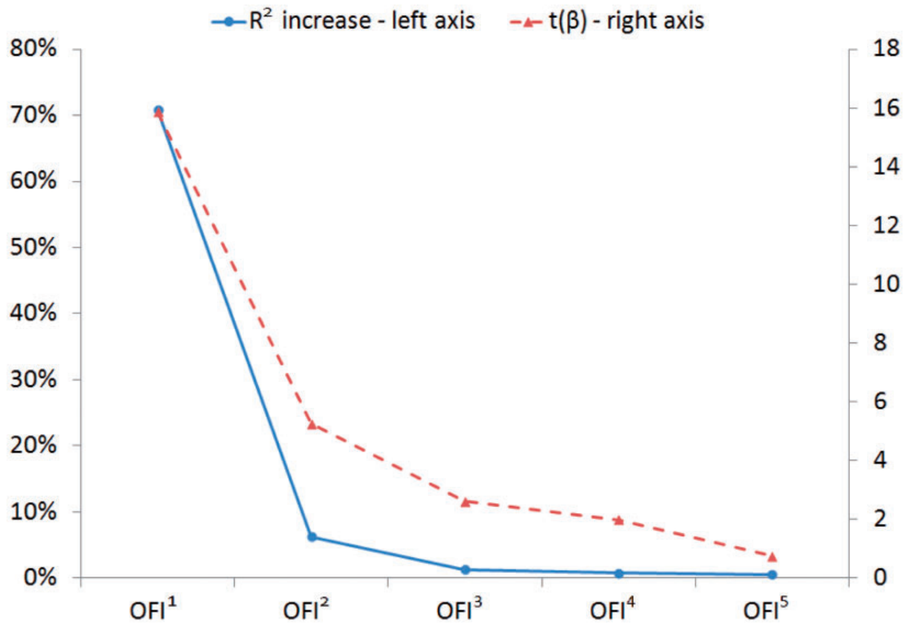

Figure B.1 Cross-time average increase in R^2 from inclusion of variables $OFI^2 - OFI^5$, and cross-time average Newey–West t-statistics of their coefficient in the regression with all five variables, with NASDAQ ITCH data for the SLB.

multiple regression with OFI^1 and OFI^2 the averages of their respective coefficients are 0.0673 and 0.0406. We conclude that second-level activity, as summarized by OFI^2, has only a second-order influence on price changes, which are mainly driven by OFI^1. The effect of $OFI^3 - OFI^5$ is almost nill.

B.4 Choice of Timescale

Using the auxiliary Level 2 data set, we verify that our results are robust to potential issues in TAQ data, namely odd-lot sized orders at the best bid and offer, and mis-sequencing in quote data across exchanges during NBBO construction. We also compare our results across a wide range of timescales. The auxiliary data comes from a single exchange (NASDAQ), has information on orders of all sizes and has timestamps up to a millisecond.

We estimate the regression (14) for a variety of timescales Δt, ranging from 50 ms to 5 min using separate intraday subsamples as before. The size of these samples was different in order to stabilize the number of observations per sample. More precisely, data for the smallest timescales (50, 100, and 500 ms) was separated into 1 min instead of 30 min subsamples to make numerical computations feasible.

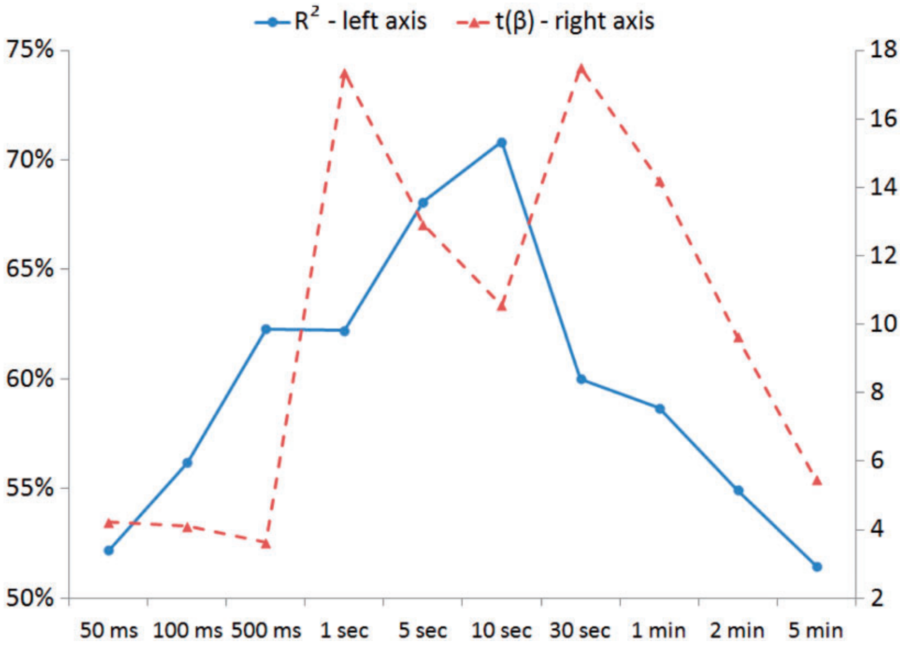

Figure B.2 Average R^2 and Newey–West t-statistics for OFI coefficient across time for different Δt, with NASDAQ ITCH data for the SLB.

Data for the largest timescales (30 s to 5 min) was pooled across days preserving separate 30 min intraday intervals to have a large number of observations per sample. The average R^2 and Newey–West t-statistics for OFI across time for each Δt are presented on Figure B.2.

The goodness of fit is stable across Δt, despite pronounced discreteness of data for very short time intervals. The OFI variable is statistically significant at a 95% level[20] in more than 80% of samples for Δt below 1 s, 100% of samples for Δt between 1 s and 2 min, and 92% of samples for Δt equal 5 min.

Notably there are many large price changes even when we consider Δt equal to 50 ms, but they usually correspond to high values of OFI. This is consistent with findings in Hasbrouck and Saar (2010), where authors describe the sporadic character of order activity in modern markets. When a subset of traders reacts to market updates in a matter of several milliseconds, this creates short intervals of increased activity with possibly large price changes and large OFI, and many time intervals with no activity when both variables are equal to zero. From our findings it appears that the simple model (7) can capture both of these regimes.

[20]Using Newey–West t-statistics.

When a quadratic term $\hat{\gamma}_i^Q \mathrm{OFI}_{k,i}|\mathrm{OFI}_{k,i}|$ is added to the regression, the coefficient $\hat{\gamma}_i^Q$ is significant in a handful of samples (10 out of 871) for Δt bigger or equal to 1 s. For Δt under one second, the quadratic term is significant in about 16% of samples, and its contribution is marginal (about 3% increase in average R^2). We conclude that the relation between price changes and OFI is linear, irrespective of a timescale.

APPENDIX C: PROOF OF PROPOSITION 1

First, we note that Assumption (1) ensures $N(T) \to \infty$ as $T \to \infty$. With this we can use Assumption (3) and apply the law of large numbers for weakly dependent variables (e.g., see Theorem 7 in Lyons (1988)) to the traded volume.

$$\mathbb{P}\left(\frac{\mathrm{VOL}(T)}{N(T)} = \frac{\sum_{i=1}^{N(T)} w_i}{N(T)} \overset{T\to\infty}{\to} \mu\pi\right) = 1. \tag{C1}$$

Second, event contributions e_i have a finite variance σ^2 and, using Assumption (2), we apply a central limit theorem for strongly dependent sequences (see Chapter 4.6 in Whitt (2002)):

$$\frac{\mathrm{OFI}(T)}{\sigma N^H(T)} \equiv \frac{\sum_{i=1}^{N(T)} e_i}{\sigma N^H(T)} \overset{T\to\infty}{\Rightarrow} N(0,1). \tag{C2}$$

Although the denominator $\sigma N^H(T)$ is random, it goes to infinity by Assumption (1) and Anscombe's lemma ensures that we can use such a normalization in the central limit theorem (Embrechts, Kluppelberg, and Mikosch, 1997, Lemma 2.5.8). Since the function $g(x) = x^H, H > 0, x \geq 0$ is continuous, the convergence in (C1) takes place almost surely and the limit in (C1) is deterministic, we can combine (C1) and (C2) in the following way:

$$\frac{(\mu\pi)^H}{\sigma} \frac{\mathrm{OFI}(T)}{\mathrm{VOL}^H(T)} \equiv \frac{\frac{\sum_{i=1}^{N(T)} e_i}{\sigma N^H(T)}}{\left(\frac{\sum_{i=1}^{N(T)} w_i}{\mu\pi(N(T))}\right)^H} \overset{T\to\infty}{\Rightarrow} N(0,1). \tag{C3}$$

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