Aalto University, Institute of Industrial Engineering and Management
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Assignment on hedging S& P 100 call options

The aim of this assignment is that you get experience in working with real data, that you acquire an understanding in hedging, and that you get an idea on how to do financial engineering with Matlab. You can work either alone or in groups of 2-3 persons. A report made by a group should be more complete than a report made alone, so that the work load is the same for everyone. You can have a group of 4 IF one of the members have never used Matlab or programmed anything.

The assignment consists of two parts:

1) a Matlab exercise on hedging call options and

2) a report on the results.

The report should be returned no later than Wednesday, December 4, 2019.

The task is to dynamically hedge call options using different hedging strategies (delta hedging, delta-vega hedging) and different rehedging frequencies, and to compare the corresponding hedging performances. The call option is hedged with a replicating portfolio consisting of a certain amount of the underlying and possibly of cash. As the price of the call changes, the amount of stock in the portfolio should be readjusted. Basically, you should try delta hedging and delta-vega hedging. If you are working in a group, you might also try delta-gamma hedging.

The option data is given in and Excel file, with name isx2010C. (you can also try isx2008C; the prices in 2008 were a bit more volatile). This file consist of 12 worksheets. Each worksheet contains daily information on call options maturing between 65 and 1 days, with several strike prices. Each worksheet is named by its maturity date. The Matlab algorithm to get information out of the Excel file is given in Exercise 5, problem on implied volatilities. The data on the worksheet is explained on the last pages of Lecture 6.

You can first concentrate on one worksheet and delta hedging. First, you can choose one at-the-money (ATM) call with maturity approximate 45 days. These options are the most stable and easiest to start with. Then, you should choose in the replicating portfolio a delta amount $\Delta = \partial C/\partial S$ of the underlying SP 100 index S_t (ISX). The value of the index in the replicating portfolio changes as a function of time, and so does the value of the call. After a couple of days, you can first

1. compute the difference A between **the change in the value of the option and the change in the value the replicating portfolio**, and then

2. adjust the replicating portfolio with a new amount delta of the SP 100 index.

You can repeat **??**. and **??**. n times and compute the total mean squared error,

$$E = \frac{1}{n}\sum_{i=1}^{n} A^2.$$

This error described the average accuracy of the hedge. Why should you compute the sum of A^2 and not A? You can test what is the role of the rehedging frequency in the error E. You can try, for example, 1 day, 2 days or one week. Then, you can try hedging using different strikes (OTM, ATM, ITM) and different maturities. Instead of a single option, you can hedge a portfolio of options, for example a butterfly. Note that you don't need to hedge all the options in one worksheet.

When you feel that delta hedging is easy, you can try delta-vega hedging. Note that this time, you should take the replicating call from another sheet (the next one). You can compare the hedging accuracy of both strategies. Previous years, the delta-vega hedging has turned out to be much more difficult than single delta hedging.

To get at least some statistical idea on how different strategies and rehedging frequencies affect the hedging performance, you should repeat the hedging for all the 12 worksheets. Based on data obtained from all the sheets, you can compute averages and standard deviations of the accuracy of different hedging strategies.

Please note that the strike prices may be different for options with different maturity dates (in different worksheets). It is possible that when comparing the hedging of options from different worksheets, you should use the moneyness $E/S_t e^{r(T-t)}$ instead of strike prices E.

Basically, the assignment is made using option data from 2010. You can also try the very turbulent months in fall 2008. In 2010, the markets were quite calm and rational. It is possible that you will find only small errors between different hedging strategies.

Above, only one possible way to work with the hedging problem has been described. However, you are very free to approach the problem in some other way. There is no one correct way of doing the assignment. There is only one important thing: that you understand the idea behind hedging and how different parameters and hedging strategies affect the hedging performance.

Feel free to ask Ruth if there are unclear things or if you need help with Matlab!

Hedging strategies - delta hedging and delta-vega hedging

The delta, denoted by Δ, is given by

$$\Delta = \frac{\partial C^{\mathrm{BS}}}{\partial S} = \mathcal{N}(d_1)$$

and the vega, denoted byκ, is given by

$$\kappa = \frac{\partial C^{\mathrm{BS}}}{\partial \sigma} = \frac{S_t e^{-d_1^2/2}\sqrt{T-t}}{\sqrt{2\pi}} = S_t\sqrt{T-t}\mathcal{N}'(d_1).$$

Hedging strategies in incomplete markets depend on some dynamic risk-measure that has to be minimized. Common strategies include *delta hedging* and *delta-vega hedging*.

Consider a portfolio P^{D} consisting of a short call $C^{\mathrm{BS}} = C^{\mathrm{BS}}(t, S_t; E, T; \sigma)$ and of a certain amount of the underlying stock S_t. Changes in the value of the underlying affect the value of the option. If, for example, the stock price rises, the value of a call option usually increases as well while the value of a put option decreases. When the stock price volatility is constant and trading is continuous, it is theoretically possible to construct a portfolio P^{D} immune to changes in the value of the underlying by holding an instantaneous delta amount, $\Delta = \partial C^{\mathrm{BS}}/\partial S$, of the underlying. Then, following this *delta strategy*, it is possible to eliminate all risk of loss if the option is executed. In practice, complete elimination is not possible as trading is done discretely.

When the stock price volatility is also stochastic, perfect hedging is not possible even theoretically. There are at least two sources of randomness: one from the stock price process, another from the volatility process. The randomness from the volatility is then hedged by holding an instantaneous amount of replicating options, in addition to the short option and the underlying stocks, in the portfolio. We call this kind of hedging *delta-vega hedging*.

Let us have a practical look at the delta-vega hedging strategy. Now the portfolio P^{DV} consists of three main components: a short option C^{H} with maturity T, an instantaneous α amount of the underlying stock with stochastic volatility, and an instantaneous amount η of a replicating option $C^{\mathrm{Rep}} = C^{\mathrm{Rep}}(t, S_t; E, T_2; \sigma)$ on the same underlying and with the same strike prices than the hedged option, but a longer maturity, $T_2 > T$.

The amount α of stocks to hold is

$$\alpha(\sigma) = -\frac{\partial C^{\mathrm{BS}}}{\partial S} + \frac{\partial C^{\mathrm{BS}}/\partial \sigma}{\partial C^{\mathrm{Rep}}/\partial \sigma}\frac{\partial C^{\mathrm{Rep}}}{\partial S} \tag{1}$$

and the amount η of the replicating options to hold is

$$\eta(\sigma) = -\frac{\partial C^{\mathrm{BS}}/\partial \sigma}{\partial C^{\mathrm{Rep}}/\partial \sigma}. \tag{2}$$

The hedging ratios α and η can be written in terms of the greeks delta $\Delta(\sigma)$ and vega $\kappa(\sigma)$ as

$$\alpha(\sigma) = -\Delta^{\mathrm{BS}}(\sigma) + \frac{\kappa^{\mathrm{BS}}(\sigma)}{\kappa^{\mathrm{Rep}}(\sigma)}\Delta^{\mathrm{Rep}}(\sigma) \tag{3}$$

and

$$\eta(\sigma) = -\frac{\kappa^{\mathrm{BS}}(\sigma)}{\kappa^{\mathrm{Rep}}(\sigma)}, \tag{4}$$

where Δ^{BS} and κ^{BS} refer to the delta and vega of the hedged option C^{BS}, and Δ^{Rep} and κ^{Rep} refer to those of the replicating option C^{Rep}.

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To start with:

From the Excell sheet 1, we choose an initial day, for example n=45 days to maturity. We denote this moment of time by t_0. We then choose a call option with price C_0 on that date. We compute its implied volatility I_0 to compute delta Δ_0. Then, we make two portfolios: we take a long position in portfolio OP containing one call and a short position in portfolio RE containing an amount delta of the underlying asset S. The initial values are $OP_0 = C_0$ and $RE_0 = \Delta_0 S_0$. Ideally, changes in the value of the option portfolio OP are neutralized by opposite changes in the replicating portfolio RE. Notice that the values of the portfolios are not the same; only changes in their values should be the same (for a short moment of time). We decide to rehedge every second day.

Then, the next day t_1, we compute how much the value of each portfolio has changed: $dOP_{01} = OP_1 - OP_0 = C_1 - C_0$ and $dRE_{01} = RE_1 - RE_0 = \Delta_0(S_1 - S_0)$. We denote $A_0 = dOP_{01} - dRE_{01}$. We don't want to rehedge yet.

The third day t_2, we compute how much the value of each portfolio has changed: $dOP_{12} = OP_2 - OP_1 = C_2 - C_1$ and $dRE_{21} = RE_2 - RE_1 = \Delta_0(S_2 - S_1)$. We denote $A_1 = dOP_{21} - dRE_{21}$. Then, we rehedge. We compute the implied volatility I_2 and the corresponding delta Δ_2. Then, we adjust the replicating portfolio so that it contains an amount Δ_2 of the underlying asset with value S_2. Now, $OP_2 = C_2$ and $RE_2 = \Delta_2 S_2$. (here, you don't need to think wherefrom the money comes to buy the underlying assets. When later you feel comfortable with this assignment, you can make a variation were you keep track also of the money used, if you want to.)

The fourth day t_3, we compute how much the value of each portfolio has changed: $dOP_{32} = OP_3 - OP_2 = C_3 - C_2$ and $dRE_{32} = RE_3 - RE_2 = \Delta_2(S_3 - S_2)$. We denote $A_2 = dOP_{32} - dRE_{32}$. This time, we don't rehedge.

You can proceed by computing A_i and rehedging every second day until the maturity, day t_n. Then, you can compute the squared sum of A_i, $0 \leq i \leq n-1$. Alone, this squared sum does not tell you a lot. You can change the frequency of rehedging, the strike prices used, the maturities used, the excell sheets used, and compare the squared sums $\sum_{i=0}^{n-1} A_i^2$ obtained.