

Broadcom Risk Analysis Notes

12.11.2023

Simple Returns

Let P_t be the closing price of a stock on day t .

Thus, we say the return R_t on a given day t is

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$$

growth factor for e.g. 1.02
↓
we must subtract 1 to get the 2% change in price

and can be thought of the percentage change in price from the day before day t , day $t-1$, to day t .

We can calculate the gross return $R_{t-k,t}$ from day $t-k$ to day t by the following;

$$R_{t-k,t} = (1 + R_{t-k}) \cdot (1 + R_{t-k+1}) \cdot \dots \cdot (1 + R_t) - 1$$

Log Returns

The log return r_t is defined as

$$r_t = \ln \left(\frac{P_t}{P_{t-1}} \right)$$

on a given day t .

Similarly to simple returns, we can define the gross log return from day $t-k$ to day t ;

$$r_{t-k,t} = r_{t-k} + r_{t-k+1} + \dots + r_t$$

Rolling Volatility

We denote the rolling volatility over a k -day period as how much a metric (in this case log returns) fluctuates over k days,

$$\sigma_{k,t} = \text{std}(r_{t-k+1}, r_{t-k+2}, \dots, r_t)$$

$$= \sqrt{\left(\frac{\sum r^2}{k} - \left(\frac{\sum r}{k} \right)^2 \right)} \quad \text{for } r \in (r_{t-k+1}, r_t).$$

We annualise this value to consider the annual risk of the stock;

$$\sigma_{\text{annual}} = \sigma_{\text{daily}} \cdot \sqrt{252}$$

there are 252 trading days in a calendar year

Var annual = Var daily \cdot 252
as variances scales linearly

GARCH Model

We use the GARCH (Generative Autoregressive Conditional Heteroskedasticity) to forecast future

volatility. The idea is that vol. is not const. and tends to cluster, high vol. days tend to be followed by high vol. days and low vol. days tend to be followed by low vol. days.

Let us denote the following:

- Unconditional volatility: standard deviation of all historical returns (assumes constant risk over time)
- Conditional volatility: volatility at a given time, based on past returns - this is what GARCH estimates

The simplest version of the model GARCH (1,1)

suggests

$$\text{Var}_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta \text{Var}_{t-1}$$

long-term avg. variance (points to α_0)
 yesterday's shock matters (points to ϵ_{t-1}^2)
 yesterday's variance persists, if large then forecasted will also be large (points to βVar_{t-1})

or

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

forecasted variance (points to σ_t^2)
 yesterday's variance (points to σ_{t-1}^2)

where α_0 , α_1 and β_1 are model parameters and ϵ_{t-1}^2 is

the "squared shock" from yesterday (how big yesterday's surprise

return was.