1. .

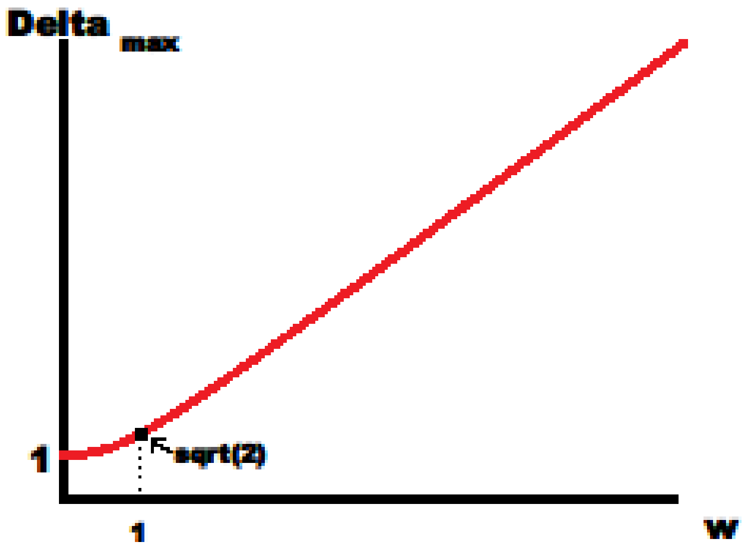
a) .

$$r \equiv 0$$
$$z = -Ks^{-1}(w + z)$$
$$\dot{z} = -Kz - Kw$$
$$y = z$$
$$A_M = -K, \qquad B_M = -K, \qquad C_M = 1$$
$$M(s) = -(s + K)^{-1}K = -\frac{K}{s + K}$$

b) If $\Delta = 0$, then we need roots of M(s) in the left half plane, thus K>0 is stable.

c) $\bar{\sigma}(\Delta) < \frac{1}{\|M(j\omega)\|_2} = \|1 + j\omega\|_2 = \sqrt{1 + \omega^2}$

For large w, $\bar{\sigma} \approx |\omega|$, for small w, $\bar{\sigma} \approx 1$



d) The system is more robust to uncertainties at high frequencies than at low frequencies.

2. .

$$\dot{\hat{x}} = A\hat{x} + Bu + K_o(y - \hat{y}) + Mr$$
$$\hat{y} = C\hat{x} + Nr$$

a) $\hat{x} = x - e$

$$\hat{y} = C(x - e) + Nr = Cx - Ce + Nr \rightarrow y - \hat{y} = Ce - Nr$$

$$(\dot{x} - \dot{e}) = A(x - e) + Bu + K_o(Ce - Nr) + Mr$$
$$\dot{e} = Ae - K_o(Ce - Nr) - Mr = (A - K_oC)e + (N - M)r$$

To get e independent from r, $\dot{e}$ must also be independent of r, thus N=M

b) .

$$\dot{\hat{x}} = A\hat{x} + B(-K_x\hat{x}) + K_o(y - \hat{y}) + Mr$$
$$\hat{y} = C\hat{x}$$
$$\dot{\hat{x}} = (A - C - BK_x)\hat{x} + K_oy + Mr$$

If we set $M = -K_o$

$$\dot{\hat{x}} = (A - C - BK_x)\hat{x} + K_o(y - r)$$

3. $J = \frac{1}{2}\int_0^\infty (qx^TQx + \rho u^TRu)d\tau = \int_0^\infty \left(x^T\left(\frac{q}{2}Q\right)x + u^T\left(\frac{r}{2}R\right)u\right)d\tau\,,\ K = R^{-1}B^TP,$

$$H = \begin{bmatrix} A & -B\left(\frac{r}{2}R\right)^{-1}B^T \\ -\left(\frac{q}{2}Q\right) & -A^T \end{bmatrix}$$

a. For $q \approx 0,\ H \approx \begin{bmatrix} A & -B\left(\frac{r}{2}R\right)^{-1}B^T \\ 0 & -A^T \end{bmatrix} \rightarrow \phi_{cl}(s)\phi_{cl}(-s) = \det[sI - H] =$
$(sI - A)(sI + A^T) \rightarrow \phi_{cl}(s) = (sI - A)$
For small q, the system is uncontrolled, and the eigenvalues are those of A.

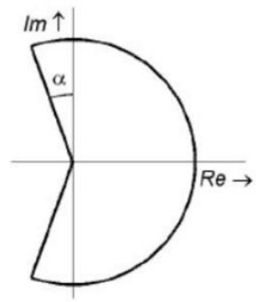
b. For $r \approx 0,\ H \approx \begin{bmatrix} A & -\frac{2}{r}BR^{-1}B^T \\ -\left(\frac{q}{2}Q\right) & -A^T \end{bmatrix} \rightarrow \phi_{cl}(s)\phi_{cl}(-s) = \det[sI - H] \approx$
$\det\left[s + \left(\frac{q}{2}Q\right)\right]\det\left[s + \frac{2}{r}BR^{-1}B^T\right]$
For small r, we have infinite gains in the feedback loop and the system is unstable. This is because if we have no penalty associated with the input, we will get the input going towards infinity.

4. To have all poles and modes decay with $re\{\lambda\} \leq -5$ and with $\zeta \geq \frac{1}{\sqrt{2}}$ $\left(\lambda = -\omega_n\zeta \pm i\sqrt{1-\zeta^2}\right) \rightarrow |im\{\lambda\}| \leq 1/\sqrt{2}$

$$re\{\lambda + 5\} > 0$$

$$\zeta = \cos(\alpha) \geq \frac{1}{\sqrt{2}} \rightarrow \alpha = 45°$$

We simply check the Nyquist plot of A_{cl} according to the modified D-contour that uses $\alpha = 45°$. The system A_{cl} has poles and modes that meet the ζ requirement and only if the number of CCW encirclements of -1 found by this modified Nyquist contour equals the number of RHP open loop poles.



Modified contour

We must also test whether the real part of the poles are less than -5, and to do so we check $(A_{cl} + 5I)$ with a normal D-Contour. Passing the Nyquist test here verifies that the poles decay faster than e^{-5t}. If both of these conditions are met, then we pass both performance requirements.