Assignment 3: Functional Error using Adjoint-Weighted Residual

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Abstract

1 Code summary

1.1 Development

For this assignment, the following functionalities have been implemented:

- refactored the code:
  - replaced the dense Jacobian matrix with `SparseMatrixCSC` (implemented in last assignment);
  - moved some code in `shock_example.jl` into function `setup_for_implicit_solve` which returns all necessary data used for the implicit solve, including `solver`, `q`, `area` and `Jac`;
  - moved both gas property and discretization parameters into a file `parameters.jl`;
  - replaced function `calcStateJacobian` which returns a dense matrix with the implementation from last assignment which returns a `SparseMatrixCSC`;
- implemented the adjoint-weighted residual (AWR) method using p enrichment;
- calculated the elementwise localized error;
- applied the AWR to both the subsonic and transonic flows.

1.2 How to run the code

For the subsonic flow, change variable `area_star` to 0.8, and run

```
julia awr.jl,
```

and the results are under directory *results/subsonic*. For transonic flow, change variable `area_star` to 1.0 and run

```
julia awr.jl,
```

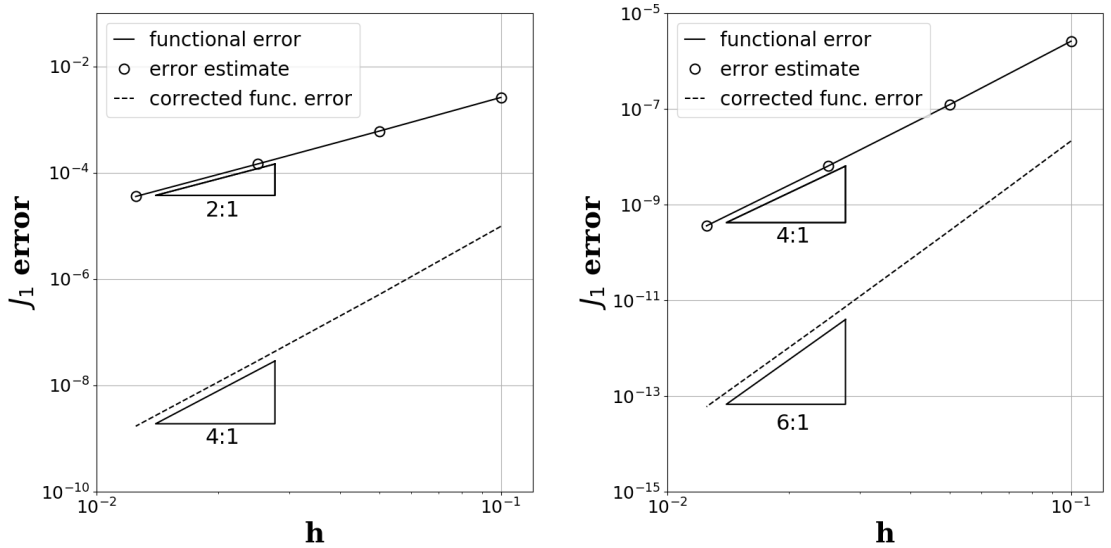
and results are under directory *results/transonic*.

2 Subsonic flow

2.1 Grid convergence study of functional error estimate

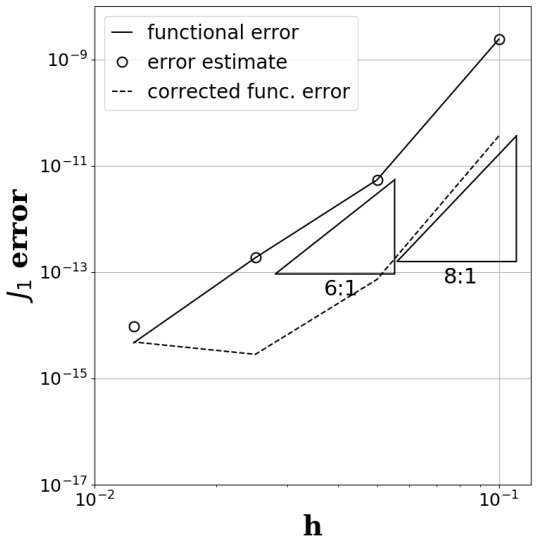
The grid convergence study of the functional error estimate for both J_1 and J_2 is carried out. The results are shown in Figures 1 and 2. As can be seen, in all cases the functional error without the AWR correction

exhibits an accuracy of $p+1$ while the corrected functional error is $2(p+1)$ accurate before approaching machine zero. An exception occurs for the functional J_2 when $p=3$, in which the both the functional error and corrected functional error are much more accurate than expected.



(a) p=1

(b) p=2

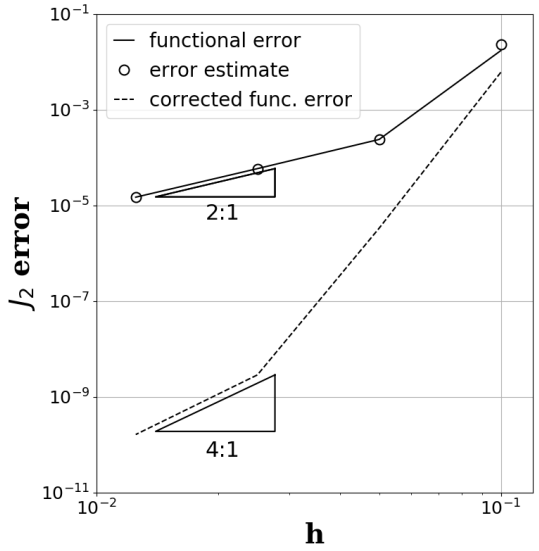


(c) p=3

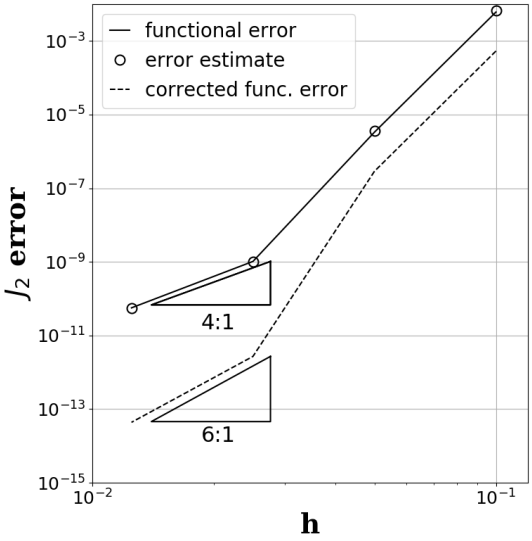
Figure 1: Functional error estimate of J_1

2.2 Elementwize localized error

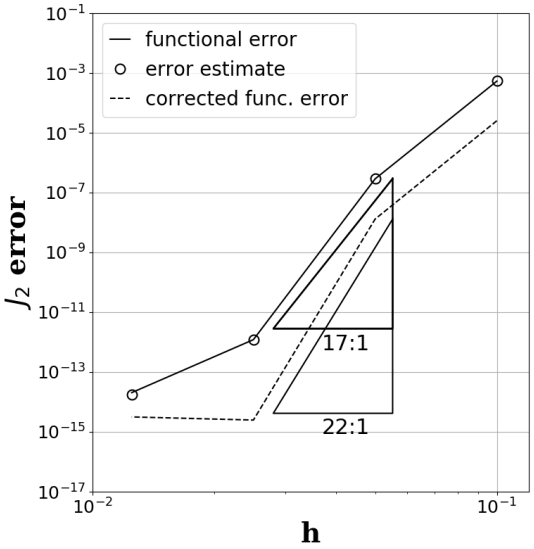
The elementwise localized error versus x using `numelem=80` is plotted in Figures 3 and 4. We can see that with difference degrees of discretization, the element functional error shows both different distribution pattern



(a) p=1



(b) p=2

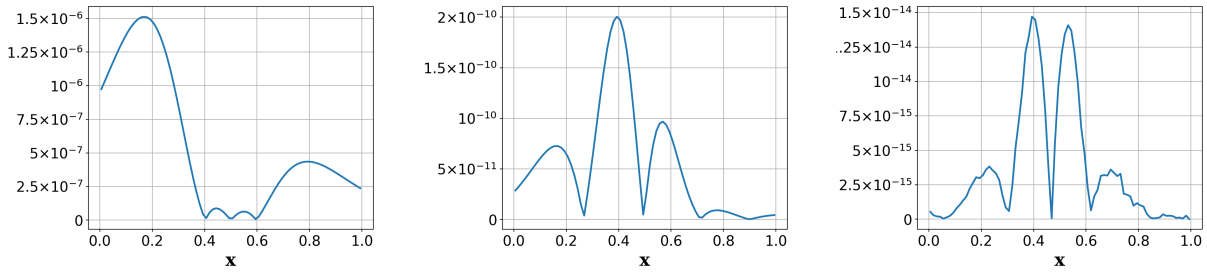


(c) p=3

Figure 2: Functional error estimate of J_2

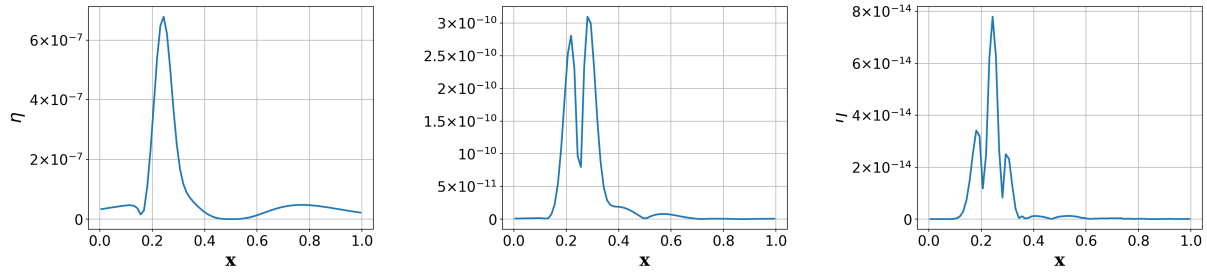
and magnitude. For J_1, when $p = 1$, the mesh should be refined in region $[0, 0.3]$ and coarsened in region $[0.4, 0.5]$; when $p = 2$, the mesh should be refined in region $[0.3, 0.45]$ and coarsened in region $[0.7, 1]$; when $p = 3$ all the element error is already small enough so that no refinement is needed.

The coarsening/refinement strategy for J_2 is simpler: for both $p = 1$ and $p = 2$, the refinement and coarsening region should be $[0, 0.2] \cup [0.4, 1]$ and $[0.2, 0.3]$, respectively. As with J_1, when $p = 3$ all the element error is already small enough so that no refinement is needed.



(a) p=1 (b) p=2 (c) p=3

Figure 3: Error indicator of J_1



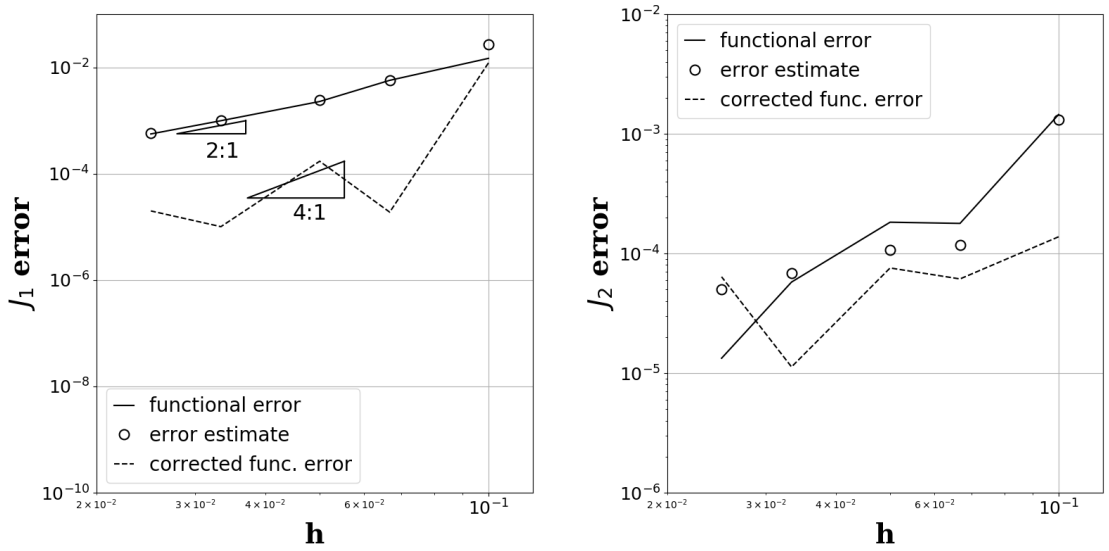
(a) p=1 (b) p=2 (c) p=3

Figure 4: Error indicator of J_2

3 Transonic flow

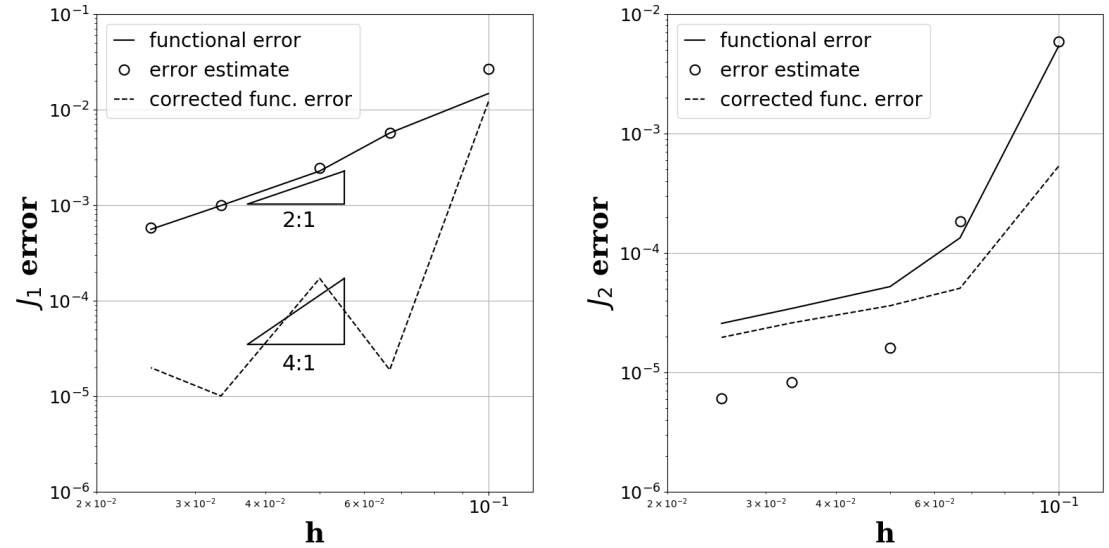
3.1 Grid convergence study of functional error estimate

The grid convergence study of the functional error estimate for both J_1 and J_2 are carried out. The results are shown in Figures 1 and 2. As can be seen, in all cases, the functional error without AWR correction exhibits an accuracy of $p + 1$ while the corrected function error is $2(p + 1)$ accurate before approaching machine zero. An exception occurs for the functional J_2 when $p = 3$, in which the both the functional error and corrected functional error is much more accurate than expected. For both functionals, the mesh can be coarsened in region $[0, 0.2]$ due to the relatively small localized error.



(a) p=1 (b) p=2

Figure 5: Functional error estimate of J_1 for transonic flow



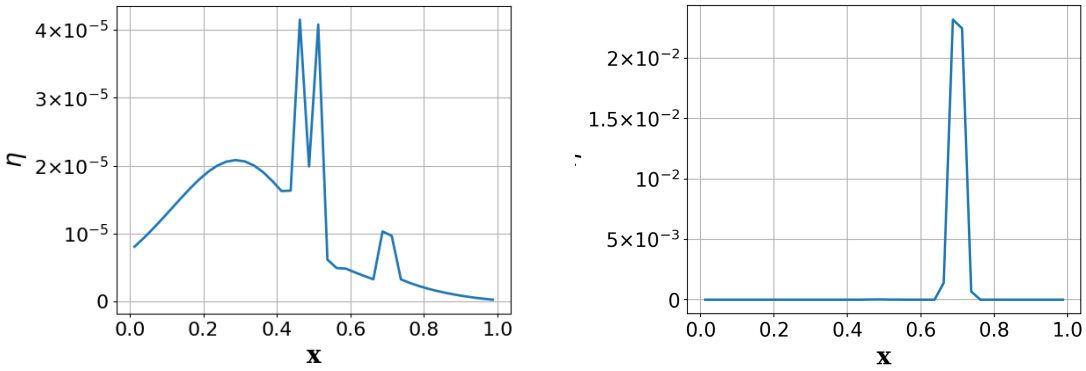
(a) p=1 (b) p=2

Figure 6: Functional error estimate of J_2 for transonic flow

3.2 Elementwize localized error

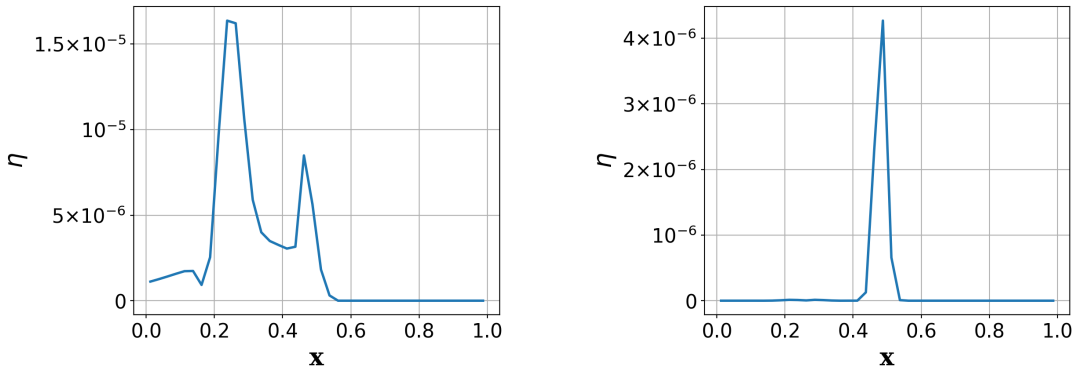
As with subsonic flow, the elementwise localized error versus x using `numelem=40` is plotted in Figures 7 and 8.

For both J_1 and J_2, when $p = 1$, the refinement/coarsening region should be $[0.45, 5]$ and $[0.6, 1]$, respectively; when $p = 2$ the mesh should be refined in region $[0.65, 0.75]$ and coarsened in the rest region.



(a) p=1 (b) p=2

Figure 7: Error indicator of J_1 for transonic flow



(a) p=1 (b) p=2

Figure 8: Error indicator of J_2 for transonic flow