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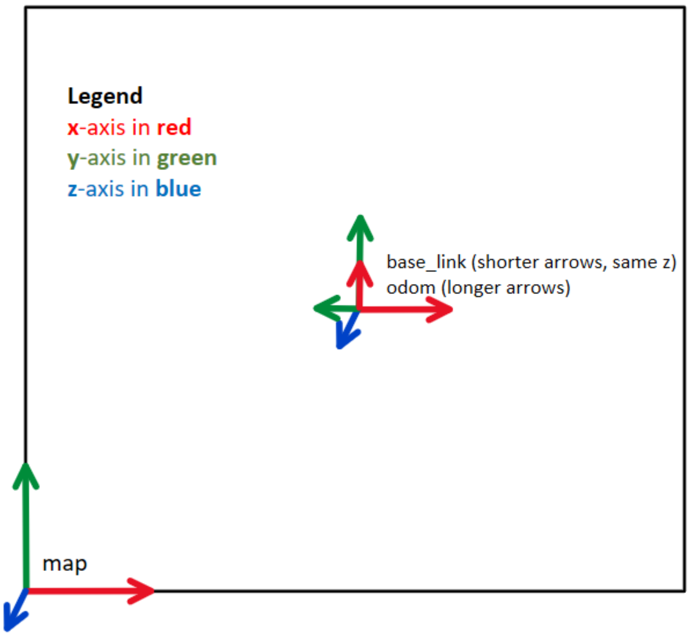
COSC 181

Homework 2 - Kinematics & Transformations

Question 1



Conventions Followed

base_link:

- x forward (north @ start), y left (west @ start), z up.
- Origin is centered between wheels on the plane where wheels touch the ground

odom:

- x east, y north, z up
- Origin is colocated at robot initial base_link (middle of map)

map:

- x east, y north, z up
- Origin: bottom left corner (as specified)

Transformation matrices

0 deg rotation, (5, 5, 0) translation

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

${}^{map}T_{odom}$

90 deg yaw rotation, no translation → $\cos(90°) = 0$, $\sin(90°) = 1$

$$\left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

${}^{odom}T_{base\_link}$

Simply negate the translation

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

${}^{odom}T_{map}$

Transpose of rotational component

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

${}^{base\_link}T_{odom}$

Question 2

From class notes

$R = \frac{\ell}{2} \cdot \frac{v_r + v_\ell}{v_r - v_\ell}$

$\omega = \frac{v_r - v_\ell}{\ell}$

when $v_r > v_\ell$ (turning left)

From HW #1

ℓ = width = 235 mm = 0.235 m

1. Calculate ICC.

$R = \frac{0.235\,m}{2} \cdot \frac{\cancel{\pi}(1\,m/s)}{\cancel{\pi}(0.1\,m/s)} = 5 \cdot 0.235\,m = 1.175\,m$

$\hookrightarrow ICC = (5m - 1.175m,\ 5m,\ 0m) = (3.825m,\ 5m,\ 0m)$

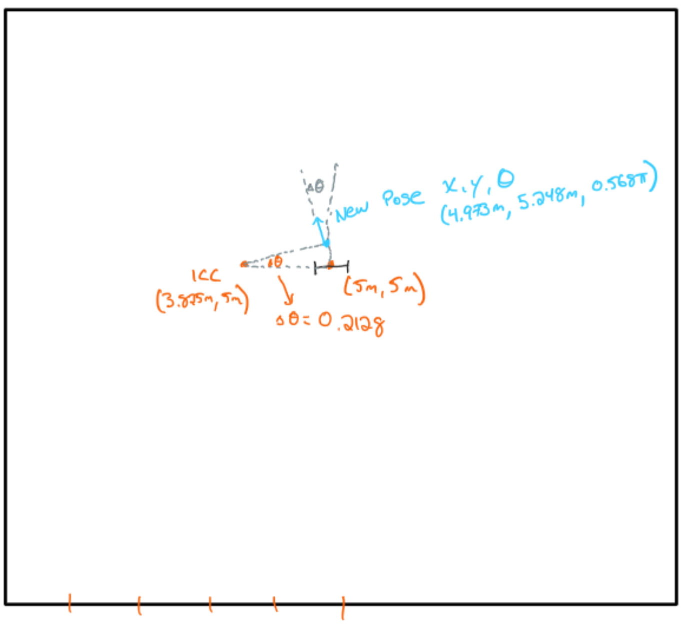
2. Calculate ω

$\omega = \frac{v_r - v_\ell}{\ell} = \frac{0.1\,m/s}{0.235\,m} = 0.4255\ /sec$

I forgot to include π here. For the rest of the hw, I used the results of this calculation. See the correct answer for #2 in red below.

3. Calculate $\Delta\theta$

$\Delta\theta = \omega \cdot \Delta t = 0.4255/sec \cdot 0.5\,sec = 0.2128$



4. Calculate new pose.

$x' = x_{ICC} + R \cdot \cos(\theta)$
$= 3.825m + 1.175m \cdot \cos(0.2128)$
$= 4.973\,m$
$\hookrightarrow -0.027m$ relative to odom

$y' = y_{ICC} + R \cdot \sin(\theta)$
$= 5m + 1.175m \cdot \sin(0.2128)$
$= 5.248m \rightarrow 0.248m$ relative to odom

$\theta' = \theta + \Delta\theta = \frac{\pi}{2} + 0.2128 \approx 0.568\pi$

(Note: θ is relative to odom reference frame x-axis (which is due east))

Updated matrix ${}^{odom}T_{base\_link}$

$$\begin{bmatrix} \cos(.568\pi) & -\sin(.568\pi) & 0 & -0.027 \\ \sin(.568\pi) & \cos(.568\pi) & 0 & 0.248 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Downarrow$

$$\begin{bmatrix} -0.212 & -0.977 & 0 & -0.027 \\ 0.977 & -0.212 & 0 & 0.248 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Corrected Answer for #2.

$\omega = \frac{v_r - v_\ell}{\ell} = \frac{0.1 \cdot \pi}{0.235} = 1.337 \frac{rad}{sec.}$

$\Delta\theta = 1.337 \frac{rad}{sec} \cdot 0.5\,s = 0.668$ radians.

$x' = 3.825m + 1.175m \cdot \cos(0.668) = 4.75\,m$

$y' = 5m + 1.175m \cdot \sin(0.668) = 5.73\,m$

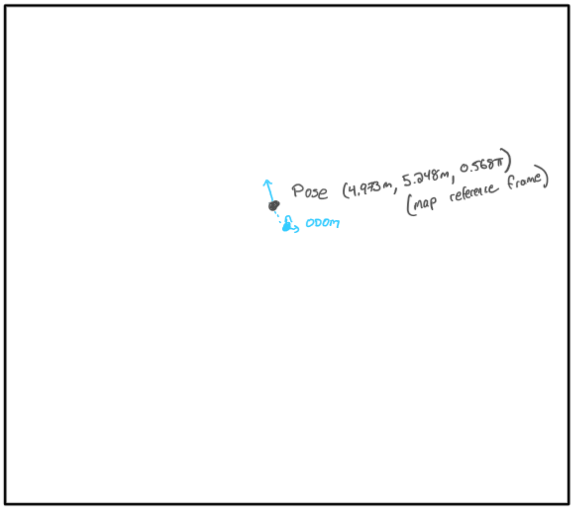
Position (relative to odom) = (-0.25, 0.73 m)

$\theta' = \frac{\pi}{2} + 0.668 = 2.24$ radians

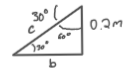
almost $\frac{3\pi}{4}$

$$\begin{bmatrix} -0.62 & -0.78 & 0 & -0.25 \\ 0.78 & -0.62 & 0 & 0.73 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{odom}T_{base\_link}$$

Question 3



1. Solve this triangle:



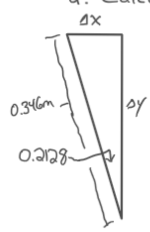
$c = \frac{0.2m}{\sin(30°)} = \frac{0.2m}{1/2} = 0.4m$

$b = \frac{0.2m}{\tan(30°)} = 0.2m \cdot \sqrt{3} = 0.346m$

The expected reading would be 0.4m

- Note that the sonar is located at front of robot, which is 200mm long.

2. Calculate point. Solve this triangle



Forward distance = reading + offset = 0.346m + 0.1m = 0.446m

$\Delta x = 0.446m \cdot \sin(0.2128) = 0.094\,m$

$\Delta y = 0.446m \cdot \cos(0.2128) = 0.436m$

Laser hit point: $4.973m - 0.094m = x' = 4.879$

$5.248m + 0.436m = y' = 5.684$

$z = 0$

The sonar would hit at (4.879, 5.684, 0) in the map frame.

Question 4

$p_{base\_link} = {}^{base\_link}T_{odom} \times {}^{odom}T_{world} \times p_{dest}$

$$-\begin{bmatrix} -.212 & 0.977 \\ -0.977 & -0.212 \end{bmatrix} \begin{bmatrix} -0.027 \\ 0.248 \end{bmatrix} = -\begin{bmatrix} 0.248 \\ -0.026 \end{bmatrix}$$

1. Calculate ${}^{base\_link}T_{odom}$ by finding inverse of ${}^{odom}T_{base\_link}$

$$\begin{bmatrix} -0.212 & -0.977 & 0 & -0.027 \\ 0.977 & -0.212 & 0 & 0.248 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \left[\begin{array}{ccc|c} -0.212 & 0.977 & 0 & -0.248 \\ -0.977 & -0.212 & 0 & 0.026 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

2. Calculate p_{base_link}

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} -0.212 & 0.977 & 0 & -0.248 \\ -0.977 & -0.212 & 0 & 0.026 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} 4.8m \\ 4.8m \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -.401 \\ 0.238 \\ 0 \\ 1 \end{bmatrix}$$

Question 5

Part 1: Rotation.

$\theta = 0$

$\theta' = 1.182\pi$

$\hookrightarrow$ Faster $\Rightarrow \theta' = -0.818\pi$

Turning right, so $v_\ell > v_R$.

$v_{max} = 1.25\ m/s$.

Take $v_\ell = 1.25\ m/s$,

$v_r = -1.25\ m/s$,

$$\omega = \frac{v_\ell - v_r}{\ell} = \frac{2.5\ m/s}{0.235\ m} = \frac{10.64\ rad}{sec.}$$

$$\theta' = \omega \Delta t = 0.818\pi = 10.64 \cdot \Delta t.$$

$$\Delta t = \frac{0.818 \cdot \pi}{10.64} = 0.242\ s$$

$\rightarrow \Delta t = 0.242\ s$

Part 2: Translation

Distance $= \sqrt{(.401)^2 + (.238)^2} = 0.466\ m$.

Take $\boxed{v_\ell = v_R = v_{max} = 1.25\ m/s.}$

$1.25\ m/s \cdot \Delta t = 0.466\ m$

$$\boxed{\Delta t = \frac{0.466\ m}{1.25\ m/s} = 0.373\ s}$$

Part 3: Rotation

$\theta = -0.818\pi$

$\theta' = -0.250\pi$

$\Delta\theta = 0.568\pi$

Left turn, so $v_r > v_\ell$. take

$\boxed{v_r = 1.25\ m/s,\ v_\ell = -1.25\ m/s}$, $\omega = \frac{10.64\ rad}{sec.}$

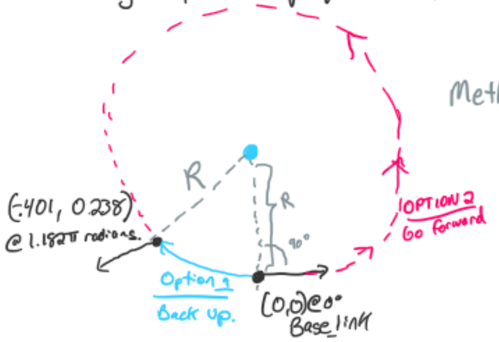
$$\boxed{\Delta t = \frac{0.568\pi\ rad}{10.64\ \frac{rad}{sec}} = 0.168\ s}$$

Transformation Matrix

$$^{0}T_{b1} = \begin{bmatrix} 0.707 & 0.707 & 0 & -0.2 \\ -0.707 & 0.707 & 0 & -0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 5 (using arc)

Diagram, from perspective of base_link.



Method: Constant velocity wheels will make robot follow the arc of a circle with a single fixed ICC. So, we fit a circle that is touching and tangential to the initial position and crosses the target position.

1. Find ICC.

→ will be located at $(0, R)$.

→ circles have constant radius, so we have that dist(ICC, Target) = dist(origin, ICC) = R

$$R = \sqrt{(-.401)^2 + (R - .238)^2}$$

$$R = \sqrt{0.16 + R^2 - 0.476R + 0.566}$$

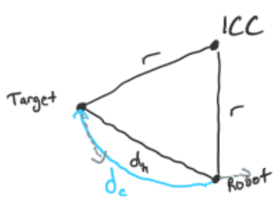
$$\cancel{R^2} = 0.16 + \cancel{R^2} - 0.476 \cdot R + 0.566$$

$$0.476 \cdot R = 0.217$$

$$R = 0.455\,m$$

ICC @ $(0, 0.455)$.

2. Calculate arc angle and arc length



$d_h = 0.466\,m$ (calculated above).

$$\cos(\Theta) = \frac{2r^2 - d_h^2}{2r^2} = 1 - \frac{d_h^2}{2r^2} \quad \text{law of cosines}$$

$$\Theta = \cos^{-1}\left(1 - \frac{(0.466)^2}{2 \cdot (0.455)^2}\right) = 1.075 \text{ radians.}$$

61.61°

$$d_c = r \cdot \Theta = 1.075 \cdot 0.455 = 0.489\,m$$

3. Determine movement parameters.

$|v_r| > |v_l|$ (turning backwards)

$\Delta\Theta = 1.075$ rads

$d_c = -0.489\,m$ (moving backwards)

$v_r = -1.25$ m/s (use maximum for right wheel).

$$R = \frac{l}{2} \cdot \frac{v_r + v_l}{v_r - v_l} \quad \text{(class notes)}$$

$$0.455_m = \frac{0.235_m}{2} \cdot \frac{1.25 + v_l}{1.25 - v_l}$$

→ solve for $v_l = 0.74$ m/s, backwards (-0.74 m/s)

$$\omega = \frac{v_r - v_l}{l} = \frac{1.25 - 0.74}{0.235} = 2.17 \frac{\text{rad}}{\text{sec}}$$

$$\omega \cdot \Delta t = \Delta\Theta \rightarrow \Delta t = \frac{1.075}{2.17} = 0.49\,s$$

For arc, $v_r = -1.25$ m/s

$v_l = -0.74$ m/s

$\Delta t = 0.49\,s$.

4. Use rotation to achieve final orientation.

Current angle: -1.075 rads

Target angle: 1.182π rads $= -0.818\pi$ rads $= -2.570$ rads

$\Delta\Theta = -1.49$ rads

$\omega = 10.64 \frac{\text{rads}}{\text{sec}}$

$\Delta t = \frac{\Delta\Theta}{\omega} = 0.14\,s$

$v_R = -1.25$ m/s

$v_l = 1.25$ m/s