ISyE 3232 Final Project: Panda QPL[EX]press

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December 12, 2024

1 Data Cleaning

1.1 Notes

While cleaning the 2nd observation data, I found the number in line went to -1 twice, and the number being served never went below 0, so I adjusted the number in the line at the start from 5 to 6.

1.2 Pre-formatting

	enter_laps	serve_laps	exit_laps
0	NaN	NaN	8.00
1	NaN	NaN	41.00
2	NaN	NaN	23.00
3	NaN	13.00	14.00
4	NaN	24.00	22.00
5	NaN	14.00	10.00
6	8.00	59.00	55.00
7	19.00	22.00	45.00
8	34.00	19.00	49.00
9	8.00	22.00	22.00

Table 1: The first 10 rows of the pre-formatted observation 2 data

The first step was converting the data in all 3 tables to match with the 3 columns above. These are just the inter-event times for the 3 events measured. I moved each column down so that each row represents a single person. For example, in the data for observation 2, 6 people were in line so they did not have enter_laps and 3 people were being served so they did not have serve_laps when data collection began. This gives us the 6 and 3 NaN in the first rows of those columns.

1.3 Creating Time Columns

	enter_laps	serve_laps	exit_laps	enter_time	serve_time	exit_time
0	NaN	NaN	8.00	NaN	NaN	8.00
1	NaN	NaN	41.00	NaN	NaN	49.00
2	NaN	NaN	23.00	NaN	NaN	72.00
3	NaN	13.00	14.00	NaN	13.00	86.00
4	NaN	24.00	22.00	NaN	37.00	108.00
5	NaN	14.00	10.00	NaN	51.00	118.00
6	8.00	59.00	55.00	8.00	110.00	173.00
7	19.00	22.00	45.00	27.00	132.00	218.00
8	34.00	19.00	49.00	61.00	151.00	267.00
9	8.00	22.00	22.00	69.00	173.00	289.00

Table 2: The first 10 rows of the observation 2 data

All 3 time columns are just cumulative sums of the laps columns, representing the time the person at that index experienced that event.

2   Arrival Analysis

I decided to use seconds as my time period for number of arrivals as it maintains the integrity of the arrival data while keeping the granularity of the service time data when inputted in QPLEX. In order to calculate it, I made 1 second bins from 0 to the highest time recorded, and saw how many appeared per 1 second bin. This was done using pd.cut()
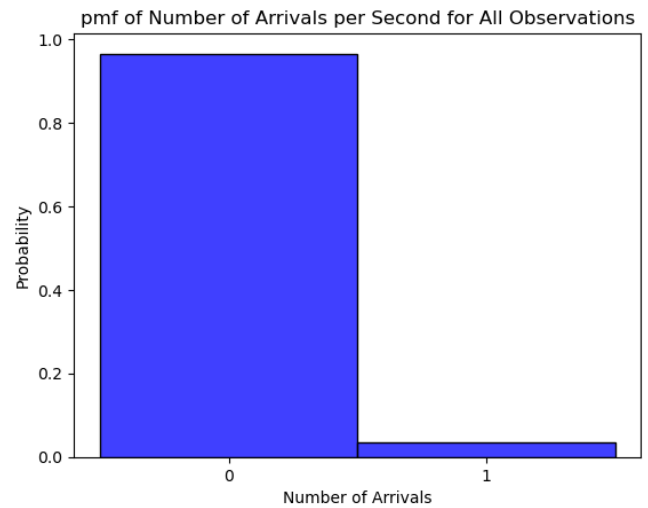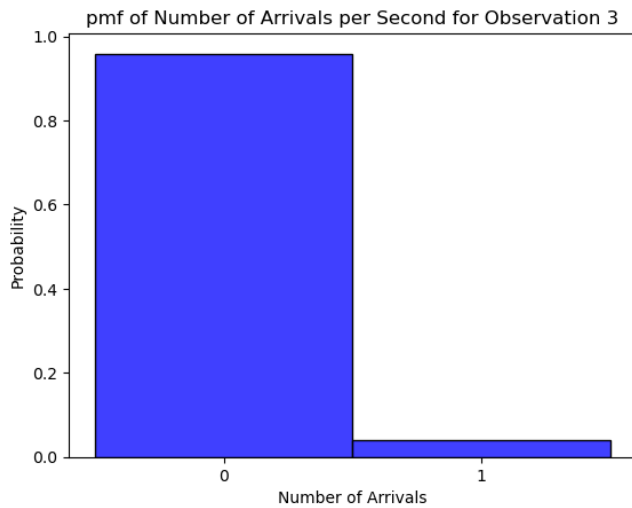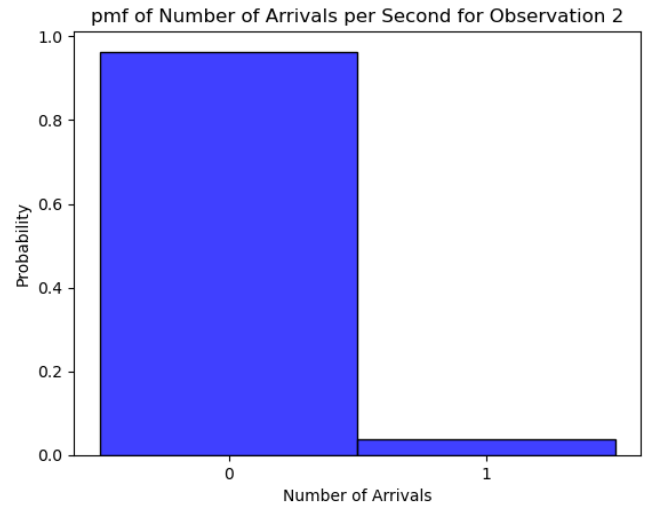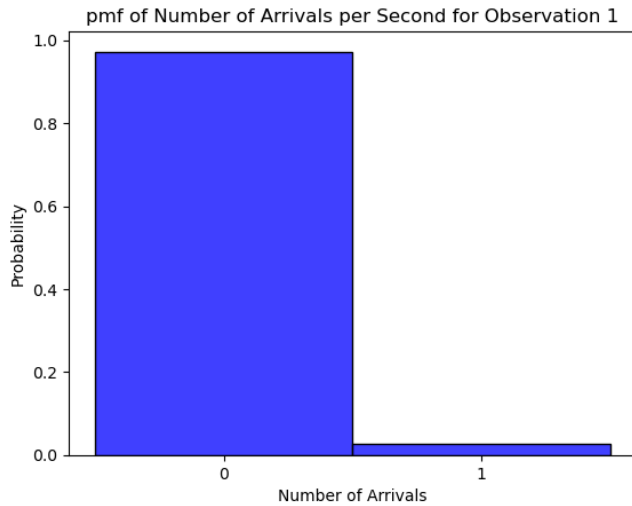
| Number of Arrivals	0	1
Observation 1	0.972	0.028
Observation 2	0.964	0.036
Observation 3	0.970	0.030
Aggregate	0.968	0.032

3   Analysis 1

For the first analysis, I modeled the system as a single-server queue, where the system begins at the checkout, and ends when the customer leaves checkout. If the system is defined as when the customer begins being served (Analysis 2), then it is either a multi-server or network queue, as multiple customers are being served simultaneously.

3.1   Service Times

These service times are calculated as follows:

$$\text{service\_time} = \text{exit\_laps} \textbf{ IF } (\text{exit\_time} - \text{serve\_time}) > \text{exit\_laps} \textbf{ ELSE } \text{NaN}$$

This takes into account when somebody has spent additional time in the queue. A very imperfect model, but the model is limited by the data.
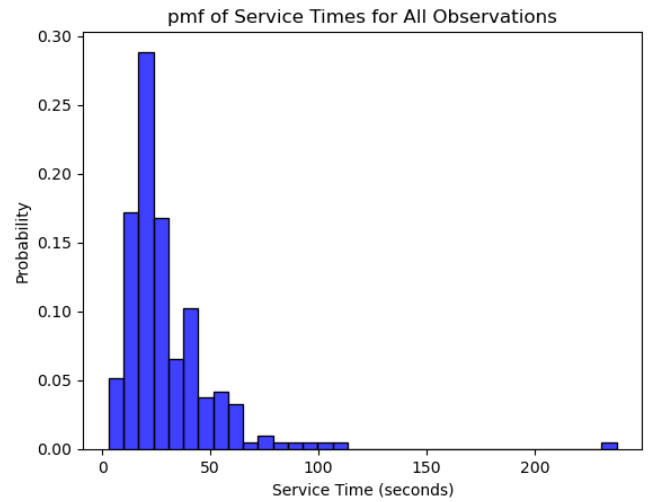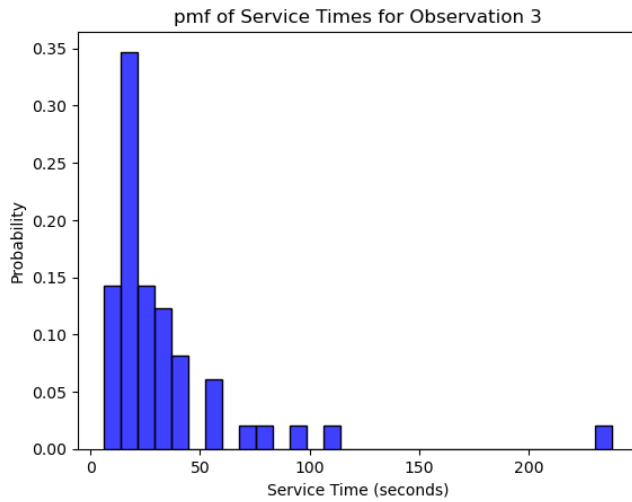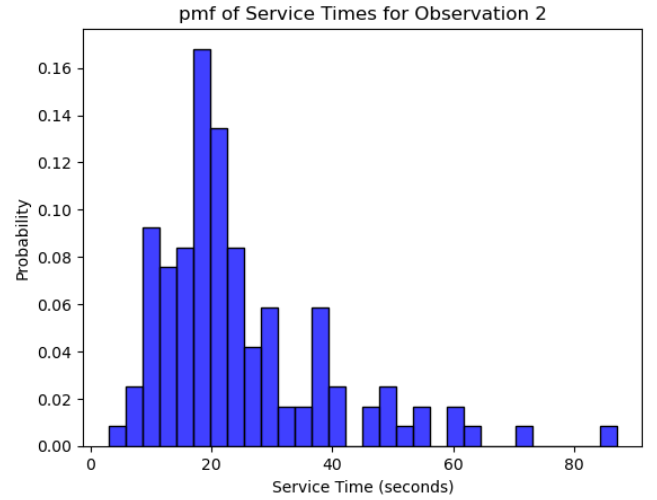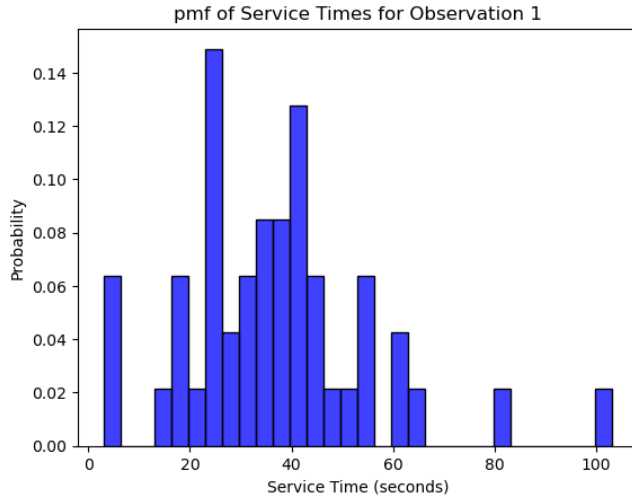
The service times pmfs are visualized above, but not put into tables as they are too long. See the appendix for the full tables.

3.2   Queue Length

	Mean	95th Percentile
Observation 1	24.095	27
Observation 2	7.929	15
Observation 3	34.542	43
Aggregate	22.189	41

3.3 Intensity Analysis

The intensity is calculated using:

$$\text{Arrival Rate} = \lambda = E[\text{number of arrivals}]$$
$$\text{Service Rate} = \mu = \frac{1}{\text{mean service time}}$$
$$\text{Intensity} = \rho = \frac{\lambda}{\mu}$$

Which gives us:

	Arrival Rate	Service Rate	Intensity
Observation 1	0.02750	0.02704	1.01706
Observation 2	0.03626	0.04023	0.90127
Observation 3	0.02980	0.02939	1.01380
Aggregate	0.03229	0.03379	0.95570

Units for arrival rate and service rate are in $\frac{1}{\text{second}}$
Because observation 2 has an intensity < 0.95, we can run QPLEX analysis on it.

3.4 QPLEX Validation

	Mean	95th Percentile
Observations	7.929	15
QPLEX	2.919	9

In this case, the model underestimated significantly.

3.5 QPLEX Generated Trade-off Curve
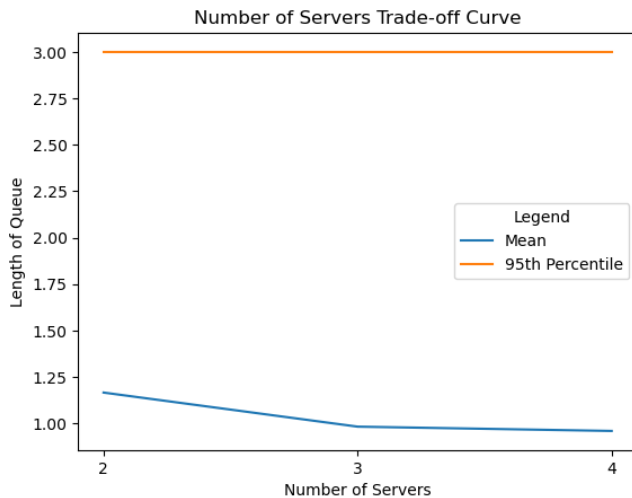
Number of Servers	Mean	95th Percentile
2	1.166	3
3	0.983	3
4	0.959	3

4 Analysis 2

For the second analysis, I modeled the queue as a multi-server queue, where the service begins at the first step in the assembly line, and ends when the customer leaves checkout. The underlying assumptions that allow us to model this as a multi-server queue and not a network are:

- Having each server move with the customer is no different to having each server pass the customer off to the next server

- Customer's service times are independent

Both of these are false, which is why we will explore it further in Analysis 3. Unfortunately, the data collected is designed around a model such as this one and not a network model.

4.1 Service Times

These service times are calculated as follows:

$$\text{service\_time}_i = \text{exit\_time}_i - \text{serve\_time}_i$$

This gives us the full time from when the customer begins getting served to when they leave the checkout.

The service times pmfs are visualized above, but not put into tables as they are too long. See the appendix for the full tables.

4.2 Queue Length

The queue is modeled here as the line before service begins.
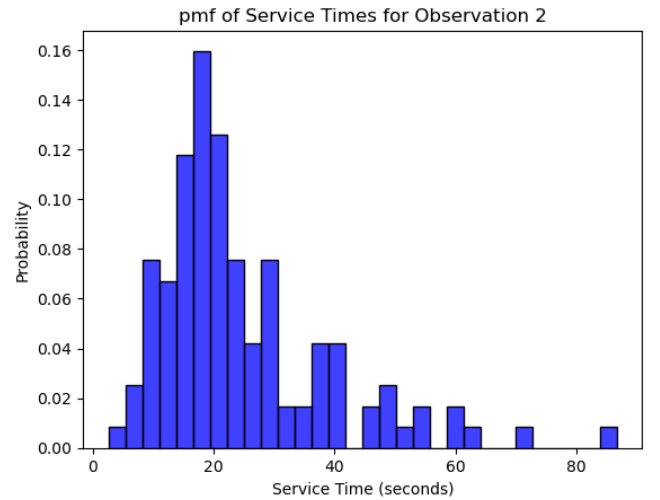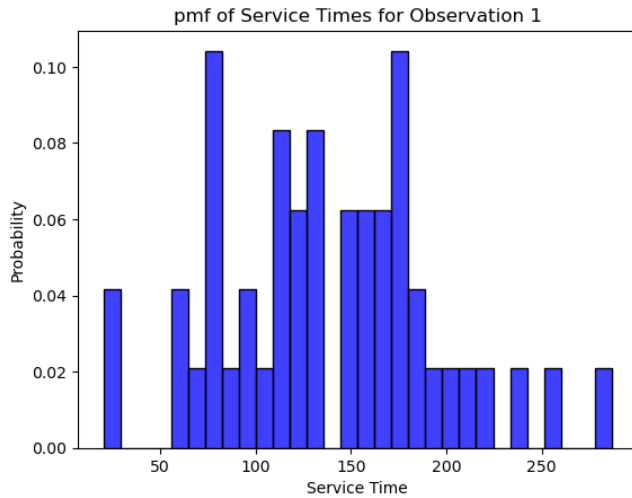
	Mean	95th Percentile
Observation 1	20.385	23
Observation 2	4.463	9
Observation 3	16.526	21
Aggregate	13.791	22

Table 3: Queue Length Statistics

4.3 Number Being Served
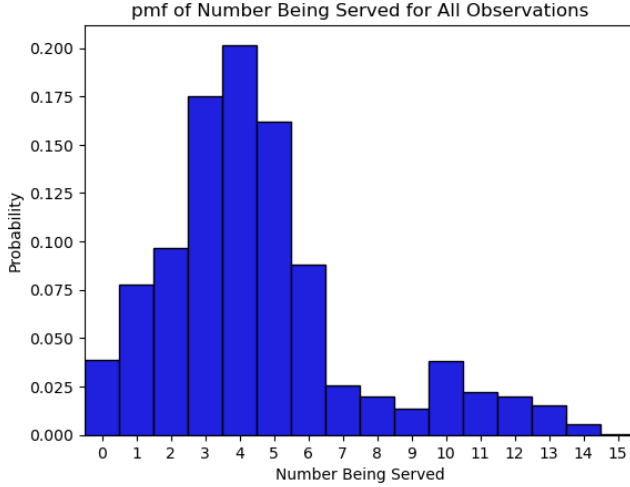
	Average Number of Servers
Observation 1	3.677
Observation 2	3.323
Observation 3	6.655
Aggregate	4.546

I remember there being 4 servers during my observation so the data makes sense. Notably, Observation 3 had a far higher average number, which makes sense given it's higher intensity in the last analysis.

4.4 Intensity Analysis

Below are the intensities of each observation, with the number of servers set to the rounded average.

|	Arrival Rate	Service Rate	Number of Servers	Intensity
Observation 1	0.02750	0.00738	4	0.93227
Observation 2	0.03626	0.01052	3	1.14925
Observation 3	0.02980	0.00375	7	1.13631
Aggregate	0.03229	0.00700	5	0.92200

Since the intensity of Observation 1 is less than 0.95, we can run validation with QPLEX.

4.5 QPLEX Validation

	Mean	95th Percentile
Observations	20.385	23
QPLEX	23.001	42

Table 4: QPLEX Validation for Observation 1

It can be seen that the high intensity causes QPLEX to overestimate the number in the line. QPLEX was run while the mean was below the observed maximum.

4.6 QPLEX Generated Trade-off Curve
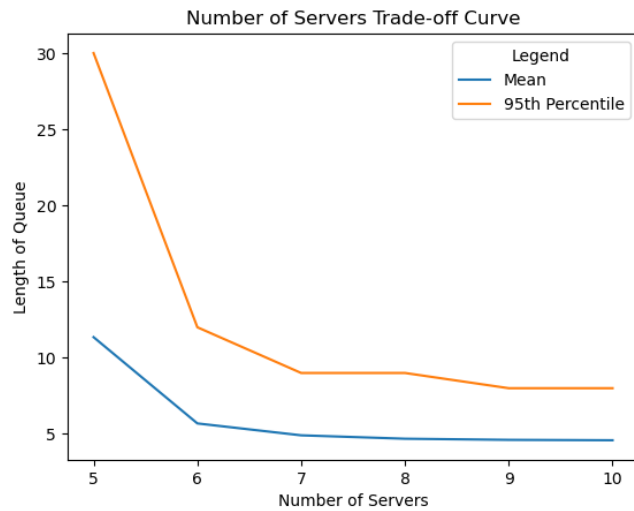
Number of Servers	Mean	95th Percentile
5	11.352	30
6	5.689	12
7	4.908	9
8	4.685	9
9	4.610	8
10	4.585	8

5 Analysis 3

For the third analysis, I modeled the queue as a single-server network queue, where the service begins at the first step in the assembly line, and ends when the customer leaves checkout. The underlying assumption that allows us to model this as a network is that each step in the line has an equal service time pmf that is just the original service time pmf scaled by $\frac{1}{\text{the number of steps}}$. This is false, but unfortunately, the data collected is insufficient to model it correctly. I figured I would still do it because it would be interesting.

Since in Analysis 2 we determined the number of servers was closest to 5, I will model this as such, with 4 steps in the line.

Both queue length and number being served are omitted from this analysis as they are both the same as Analysis 2.

5.1 Service Times
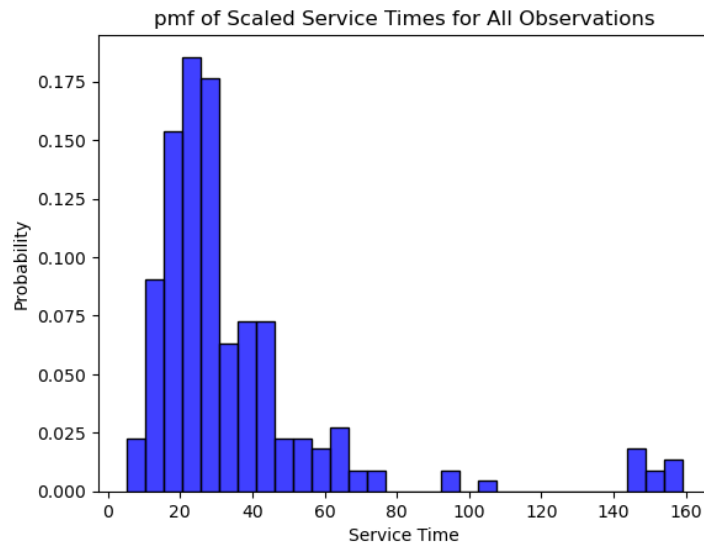
5.2 QPLEX Generated Trade-off Curves

Number of Servers	Mean	95th Percentile
2	4.787	14
3	1.979	5
4	1.735	4
5	1.688	4

Number of Servers Trade-off Curve at Node 2

Number of Servers	Mean	95th Percentile
2	4.787	14
3	1.979	5
4	1.735	4
5	1.688	4

Number of Servers Trade-off Curve at Node 3

Number of Servers	Mean	95th Percentile
2	4.787	14
3	1.979	5
4	1.735	4
5	1.688	4

Number of Servers Trade-off Curve at Node 4
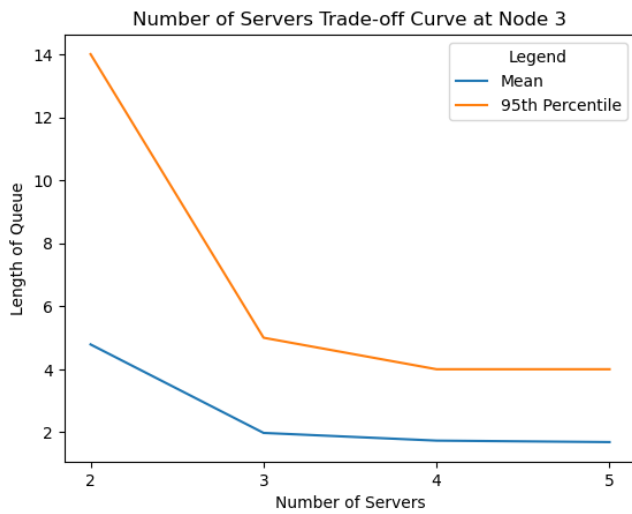
Number of Servers	Mean	95th Percentile
2	4.787	14
3	1.979	14
4	1.735	14
5	1.688	14

6 Appendix

6.1 Service Times Tables (4.1)

Service Time	Observation 1	Observation 2	Observation 3	Aggregate
3	0.021	0.008	0	0.009
6	0.043	0	0.020	0.014
7	0	0.008	0	0.005
8	0	0.017	0	0.009
9	0	0.017	0.020	0.014
10	0	0.042	0.041	0.033
11	0	0.034	0.020	0.023
12	0	0.017	0.020	0.014
13	0	0.017	0.020	0.014
14	0	0.042	0.041	0.033
15	0	0.050	0.020	0.033
16	0.021	0.034	0	0.023
17	0	0.059	0	0.033
18	0.043	0.067	0.102	0.070
19	0.021	0.042	0.041	0.037
20	0	0.042	0.102	0.047
21	0	0.050	0.041	0.037
22	0.021	0.042	0.061	0.042
23	0.021	0.034	0	0.023
24	0.021	0.008	0.020	0.014
25	0.064	0.042	0	0.037
26	0.043	0	0.020	0.014
27	0.021	0.025	0	0.019
28	0.021	0.017	0.020	0.019
29	0	0.017	0.020	0.014
30	0.043	0.042	0.082	0.051
31	0.021	0.017	0.020	0.019
32	0	0	0.020	0.005
33	0.021	0	0	0.005
35	0.021	0.008	0	0.009
36	0.043	0.008	0	0.014
37	0.021	0.008	0.020	0.014
38	0.021	0.034	0	0.023
39	0.043	0.017	0	0.019
40	0.043	0	0.020	0.014
41	0	0.017	0	0.009
42	0.085	0.008	0.020	0.028
44	0.021	0	0.020	0.009
45	0.021	0.008	0	0.009
46	0.021	0	0	0.005
47	0	0.008	0	0.005
48	0	0.008	0	0.005
49	0.021	0.008	0	0.009
50	0	0.008	0	0.005
52	0.021	0.008	0	0.009
53	0.021	0	0	0.005
55	0.021	0.017	0.020	0.019
56	0.021	0	0	0.005
57	0	0	0.020	0.005
60	0.021	0.008	0.020	0.014
61	0	0.008	0	0.005
62	0.021	0.008	0	0.009
63	0.021	0	0	0.005

	Observation 1	Observation 2	Observation 3	Aggregate
71	0	0.008	0	0.005
74	0	0	0.020	0.005
77	0	0	0.020	0.005
81	0.021	0	0	0.005
87	0	0.008	0	0.005
93	0	0	0.020	0.005
103	0.021	0	0	0.005
110	0	0	0.020	0.005
238	0	0	0.020	0.005

6.2   Service Times Tables (5.1)

Service Time	Observation 1	Observation 2	Observation 3	Aggregate
21	0.021	0	0	0.005
23	0.021	0	0	0.005
27	0	0.008	0	0.005
38	0	0.016	0	0.009
43	0	0.016	0	0.009
46	0	0.016	0	0.009
47	0	0.008	0	0.005
48	0	0.008	0	0.005
52	0	0.008	0	0.005
53	0	0.008	0	0.005
55	0	0.008	0	0.005
56	0	0.016	0	0.009
57	0	0.016	0	0.009
58	0	0.008	0	0.005
59	0.021	0.016	0	0.014
60	0.021	0.016	0	0.014
62	0	0.008	0	0.005
63	0	0.016	0	0.009
64	0	0.016	0	0.009
67	0	0.016	0	0.009
69	0	0.041	0	0.023
71	0	0.033	0	0.018
72	0.021	0	0	0.005
73	0	0.016	0	0.009
75	0.042	0.016	0	0.018
77	0	0.008	0	0.005
78	0	0.008	0	0.005
79	0.021	0.008	0	0.009
80	0	0.024	0	0.014
81	0	0.016	0	0.009
82	0.042	0	0	0.009
83	0.021	0.008	0	0.009
84	0	0.008	0	0.005
85	0	0.033	0	0.018
86	0	0.016	0	0.009
87	0	0.016	0	0.009
88	0	0	0.020	0.005
90	0	0.008	0	0.005
91	0	0.016	0	0.009
92	0	0.008	0	0.005
93	0	0.008	0	0.005
94	0	0.016	0	0.009
95	0.021	0.024	0	0.018
96	0.021	0	0	0.005

97	0	0.016	0	0.009
99	0	0.016	0	0.009
100	0	0.016	0	0.009
101	0	0.033	0.020	0.023
102	0	0.033	0.020	0.023
103	0.021	0.008	0.020	0.014
104	0	0.008	0	0.005
105	0	0.033	0	0.018
106	0	0	0.020	0.005
107	0	0.008	0	0.005
108	0	0.016	0	0.009
109	0	0.008	0	0.005
110	0	0.016	0	0.009
111	0	0.024	0	0.014
112	0	0.024	0	0.014
113	0.021	0	0	0.005
114	0.021	0.008	0	0.009
115	0	0.008	0.020	0.009
116	0	0.008	0.020	0.009
117	0.042	0.008	0	0.014
118	0	0.016	0	0.009
119	0	0.008	0	0.005
120	0	0.008	0	0.005
121	0.021	0	0.040	0.014
122	0	0.008	0	0.005
123	0	0	0.020	0.005
124	0.021	0	0	0.005
125	0.021	0.008	0	0.009
126	0	0	0.020	0.005
129	0.042	0	0	0.009
130	0.021	0.008	0.020	0.014
132	0.021	0	0.040	0.014
133	0	0	0.020	0.005
139	0	0.008	0	0.005
146	0.021	0	0	0.005
148	0.021	0	0.020	0.009
150	0	0.008	0	0.005
151	0	0.008	0	0.005
152	0.021	0	0	0.005
153	0	0.008	0	0.005
155	0	0.016	0	0.009
156	0.021	0.008	0	0.009
159	0.042	0	0	0.009
161	0	0	0.020	0.005
164	0.021	0.008	0	0.009
166	0.021	0	0	0.005
167	0	0	0.020	0.005
168	0.021	0	0	0.005
171	0	0.008	0	0.005
172	0.021	0	0	0.005
173	0	0	0.020	0.005
175	0.042	0	0	0.009
176	0	0	0.040	0.009
178	0.021	0	0	0.005
179	0	0	0.020	0.005
181	0.042	0	0	0.009
182	0	0.008	0	0.005
183	0.021	0	0	0.005
185	0	0	0.020	0.005
189	0	0	0.020	0.005

191	0	0	0.020	0.005
196	0.021	0	0	0.005
198	0	0	0.020	0.005
206	0.021	0	0	0.005
209	0.021	0	0	0.005
219	0	0.008	0	0.005
221	0.021	0	0	0.005
224	0	0	0.020	0.005
237	0.021	0	0	0.005
238	0	0	0.020	0.005
245	0	0	0.040	0.009
248	0	0	0.020	0.005
251	0	0	0.020	0.005
254	0.021	0	0	0.005
255	0	0	0.020	0.005
263	0	0	0.020	0.005
266	0	0	0.020	0.005
267	0	0	0.020	0.005
287	0.021	0	0	0.005
292	0	0	0.020	0.005
294	0	0	0.020	0.005
377	0	0	0.020	0.005
383	0	0	0.020	0.005
412	0	0	0.020	0.005
576	0	0	0.020	0.005
577	0	0	0.020	0.005
579	0	0	0.020	0.005
586	0	0	0.020	0.005
598	0	0	0.040	0.009
616	0	0	0.020	0.005
628	0	0	0.020	0.005
636	0	0	0.020	0.005