MATH 4803 Project 1

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March 20, 2025

1 Introduction

In this project we aim to describe change in temperature over time ($\frac{dT}{dt}$) in an apartment. We chose to model this using an ordinary differential equation which takes into account the temperature outside, temperature the thermostat is set to, amount of energy provided by the thermostat as well as various heat sources (shower, appliances). More specifically, we calculate the area of surfaces that transmit heat and consider diffusion to be instantaneous thus simplifying our model. We then use a matrix working as a binary variable to encode these areas in our differential equation. We then run a this simulation multiple times, with slightly adjusted variables each iteration to account for approximated measured values.

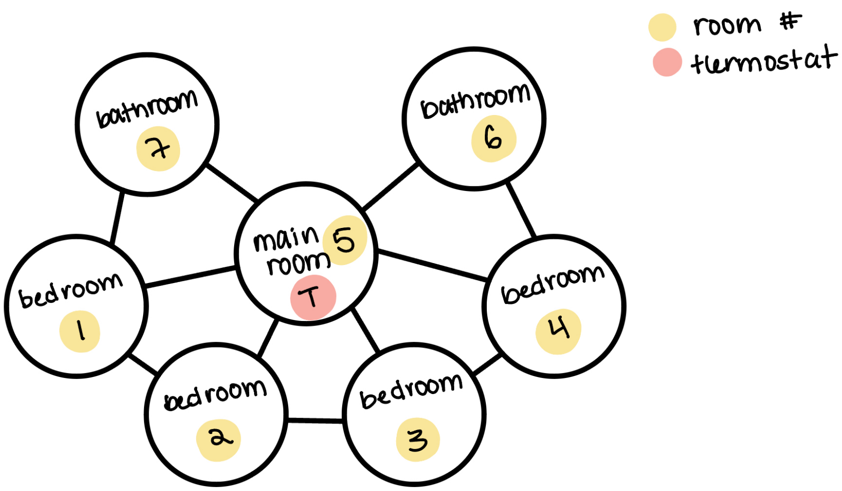


Figure 1: Diagram of James Jones' Apartment

This diagram is a visual aid of how the heat is spread throughout the apartment. Each edge represents where heat is moved from one room to another (considering the appropriate heat transfer coefficient). Note windows are a junction where heat can enter/exit which is not displayed in the figure. This diagram also demonstrates the numbering system of our apartment. Thus the main room is known as room 5, and each 5th column/row in the matrices will correspond to room 5, a.k.a. the main room. For example, we can find the volume of the main room by looking 5th row in V, which gives us $106m^3$.

2 Parameters

2.1 Values found online:

(7) The amount each heat source contributes to each room when on (Watts)

$$\tag{1}$$

$$h_{window} = 3.7\frac{W}{m^2K} = \text{U-Value window}\left(\frac{W}{m^2K}\right) \tag{2}$$

$$h_{wall} = 0.18\frac{W}{m^2K} = \text{U-Value wall} \tag{3}$$

$$h_{door} = 3\frac{W}{m^2K} = \text{U-Value door} \tag{4}$$

$$c = 1210\frac{J}{K^\circ m^3} = \text{the specific heat of air} \tag{5}$$

$$\tag{6}$$

$$\kappa = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2000 & 50 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix} \tag{7}$$

2.2 Roommate Data:

$$g_{low,i} = \text{lowest temperature roommate } i \text{ will tolerate} \tag{8}$$

$$g_{high,i} = \text{highest temperature roommate } i \text{ will tolerate} \tag{9}$$

$$\Omega_i \sim \begin{cases} Normal(\mu = 43200, \sigma = 60) & i = 1 \\ Normal(\mu = 86400, \sigma = 3600) & i = 2 \\ Normal(\mu = 345600, \sigma = 86400) & i = 3 \\ Normal(\mu = 86400, \sigma = 1800) & i = 4 \end{cases} = \text{the inter-arrival time for roommate } i\text{'s showers}$$

$$\tag{10}$$

$$\Phi \sim \begin{cases} Exponential(\frac{1}{\lambda} = 43200) & i = 1 \\ Exponential(\frac{1}{\lambda} = 86400) & i = 2 \\ Exponential(\frac{1}{\lambda} = 28800) & i = 3 \\ Exponential(\frac{1}{\lambda} = 172800) & i = 4 \end{cases} = \text{the inter-arrival time for roommate } i \text{ cooking}$$

$$\tag{11}$$

2.3 Measured Values:

(14) the area of doors between each room (m^2). Thus $A_{i,j}$ corresponds to the area of the door between room i and room j.

(15) the area of walls between each room (m^2). Thus $A_{i,j}$ corresponds to the total area of wall(s) between room i and room j.

(16) the area of windows in each room (m^2).

$$(12)$$

$$V = \begin{bmatrix} 22.812780 \\ 22.812780 \\ 22.812780 \\ 22.812780 \\ 106.531944 \\ 7.534944 \\ 7.534944 \end{bmatrix} = \text{the volume of each room}(m^3) \tag{13}$$

$$A_{doors} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1.7286 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.7286 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.7286 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.7286 & 0 & 0 \\ 1.7286 & 1.7286 & 1.7286 & 1.7286 & 0 & 1.8382 & 1.8382 \\ 0 & 0 & 0 & 0 & 1.8382 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.8382 & 0 & 0 \end{bmatrix} \tag{14}$$

$$A_{walls} = \begin{bmatrix} 0 & 8.418 & 8.418 & 8.418 & 8.649 & 3.3048 & 0 \\ 8.418 & 0 & 8.418 & 8.418 & 14.169 & 0 & 0 \\ 8.418 & 8.418 & 0 & 8.418 & 14.169 & 0 & 0 \\ 8.418 & 8.418 & 8.418 & 0 & 8.649 & 0 & 3.3048 \\ 8.649 & 14.169 & 14.169 & 8.649 & 0 & 3.0866 & 3.0866 \\ 3.3048 & 0 & 0 & 0 & 3.0866 & 0 & 0 \\ 0 & 0 & 0 & 3.3048 & 3.0866 & 0 & 0 \end{bmatrix} \tag{15}$$

$$A_{windows} = \begin{bmatrix} 1.1524 & 1.1524 & 1.1524 & 1.1524 & 4.04 & 0 & 0 \end{bmatrix} \tag{16}$$

$$\gamma^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 4 & 1 & 1 \end{bmatrix} = \text{the number of vents in each room} \tag{17}$$

2.4 Calculated Values:

$C = cV =$ the heat capacity of each room($\frac{J}{K}$)

$P = h_{door}A_{doors} + h_{wall}A_{walls} =$ the conductivity between each room($\frac{W}{K}$)

$\pi =$ the temperature outside averaged over 5 years

2.5 Tested Value:

$\Gamma = 200W =$ The amount of energy thermostat can move per vent per time (W)

3 Random Variables

$\tau =$ the temperature the thermostat is set to

$$\tau' = \text{the amount of energy provided by the thermostat} = \begin{cases} \Gamma & \tau > T_5 \\ -\Gamma & \tau < T_5 \\ 0 & o/w \end{cases}$$

$$\delta = \begin{bmatrix} \delta_{stove} \\ 1 \\ \delta_{shower} \\ \delta_{shower} \end{bmatrix} = \text{whether each heat source is on}$$

4 Assumptions

- Perfect Insulation: Heat/Cooling enters room through vents, windows, and doors but not through walls

- Assume temperature in rooms mixes instantaneously after some source or sink of heat is introduced

- Assume window type is double-glazed energy efficient

- Assume the shower time follows a normal distribution

- We use the meteostat module to sample hourly temperature in Atlanta over time which uses averages over many years. We assume that there is little difference in the results between using the average and the actual measurement for a specific date when modeling temperature over time.

- We assume that roommates will only be in their own room, main room, or outside. This is because (we hope) no one is spending more than an hour in the bathroom, and when you are in the shower, the thermostat temperature is irrelevant to you.

- We note that U-values vary by source and we chose one source listed in references for consistency however this may not be an actual average.

5 Equations

$$\frac{dT_i}{dt} = \frac{1}{C_i}\left(A_{windows,i}h_{window}(\pi - T_i) + \kappa_i\delta + \tau_i'\gamma_i + \sum_j P_{i,j}(T_j - T_i)\right), \forall i$$

6 Explanations

τ : τ is calculated the following way. At each time point, the surrounding temperature of each roommate is taken, i.e. if roommate 1 is in the main room, their surrounding temp is the temp of the main room, if roommate 4 is in their own bedroom, their surrounding temp is the temp of their bedroom. This surrounding temp is then compared to compared to g_{low}, g_{high}. If the roommate is outside or showering, then this comparison is discarded. Otherwise, if the surrounding temp falls outside the roommate's preferred temperature bounds, they are added to the list of all the roommates who also want to change the temperature. A person is randomly selected from this list. If the temperature is too high and the thermostat is set above g_{high}, the roommate will set it to g_{high}. If the temperature is too high but the thermostat is less than or equal to g_{high}, the roommate will decrease the thermostat temperature by 1 degree. It works similarly when the temperature is too low, except now you use g_{low} instead of g_{high}, and you increment temperature instead of decrementing.

κ : κ represents the amount of watts that each appliance releases as heat when it is on. The row of the matrix represents which room the appliance is in, thus the 6th row has a 10,000 watt appliance. Each column represents a unique appliance, the first column is the stove, the second column is the fridge, third column is the shower is bathroom 1/room 6, the fourth column is the shower in bathroom 2/room 7.

δ : δ is a vector of binary variables that represent when appliances are on. Each δ inside is 1 when the corresponding appliance is on, 0 when the appliance is off. It is assumed that the fridge will always be on, so the second item, which corresponds to the fridge, it equivalent to 1.

τ' : τ' represents the amount of watts released by the thermostat, or more accurately, by the central heating and cooling. The physical thermostat is located in the main room, as shown in the diagram. If the temperature there is below τ(not τ'), the thermostat will release Γ Watts, split evenly throughout the vents. For the opposite, it will release -Γ.

7 Results

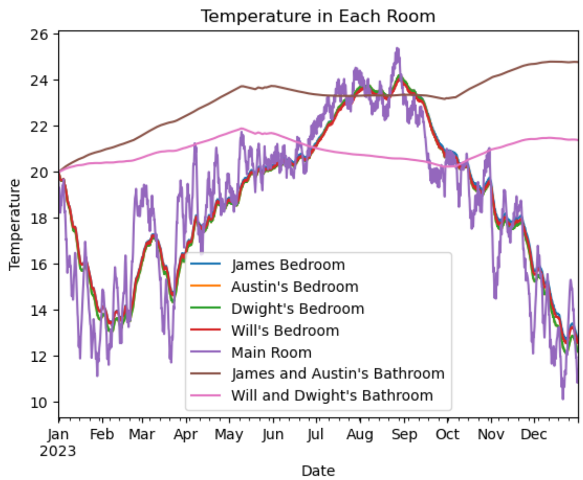
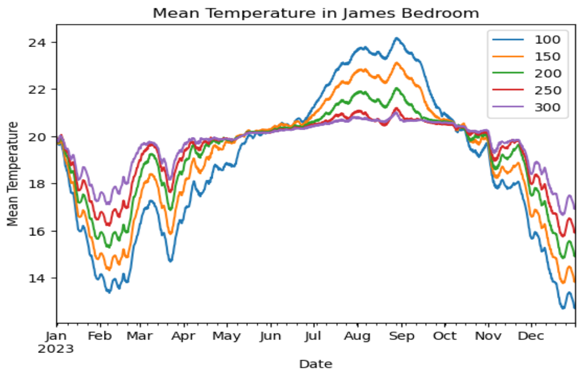


Figure 2: Temperature over 1 year with timestep 1 hour

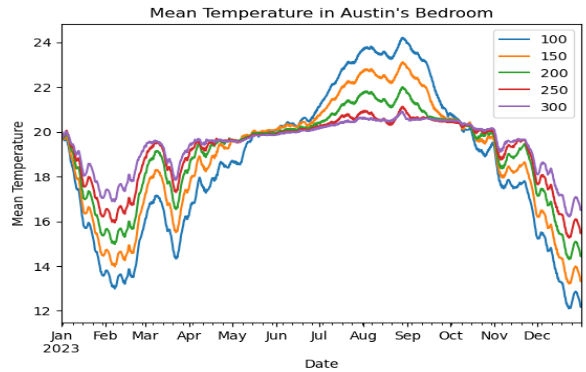
8 Sensitivity Analysis

We performed sensitivity analysis for Γ ranging from 100 to 300 with steps of 50. Figures are below. First, notice how throughout the year, in bedrooms and the main room, a higher Γ produces smaller deviations from the overall mean, indicating the outside temperature has less effect. However, in the bathrooms, where the outside temperature has no direct effects, stronger air conditioning causes much larger variations than in the bedrooms as the outside temperature changes, likely due to the over-correction coming from other rooms. One observation that can be made is that the mean temperature of James' bedroom is slightly higher than the other bedrooms, as the Y-axis is slightly shifted compared to the other roommates' bedrooms. This is reflected in the parameters, as James has the highest g_{high}, meaning he is the fine with higher temperatures compared to his roommates. Another observation is that the mean temperature of bathroom 1 vs bathroom 2 is lower, regardless of gamma. This is also a reflection of the parameters, as Dwight was modeled as showering less, therefore there is less heat coming from the showers in that bathroom.
Another interesting note is that in the summer, the temperature appears to be lower bounded by roommates lower bound, but the temperature can get much higher than their upper bound, and the opposite can be seen in the winter. This makes sense, as the roommates should be using the thermostat purely as a constraint against uncomfortable temperatures (primarily those induced from the outside).

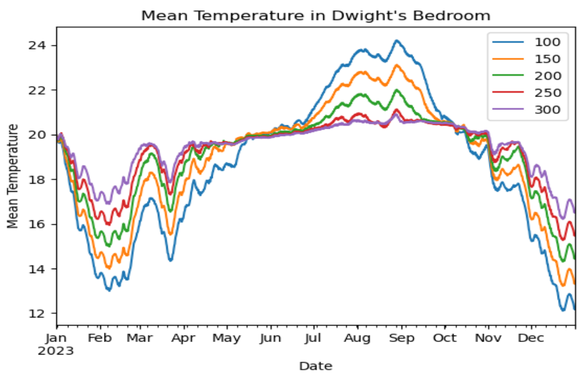
Also notice that the graphs for standard deviation have periodic spikes dropping to 0 in between for the bedrooms which is strange.



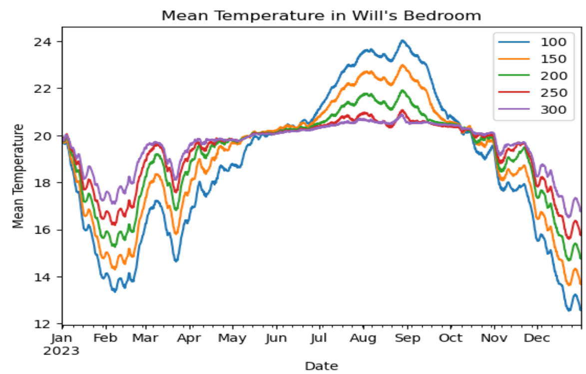
(a) Mean Temperature in James Bedroom given change in Gamma



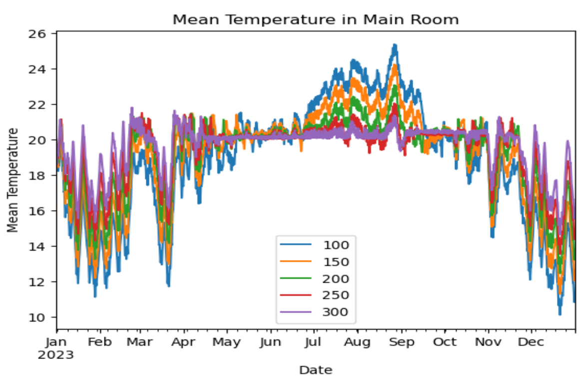
(b) Mean Temperature in Austin Bedroom given change in Gamma



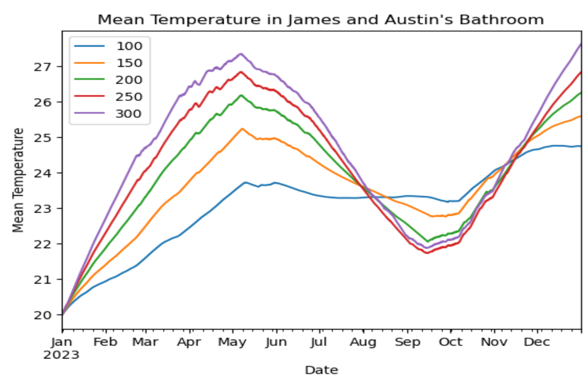
(c) Mean Temperature in Dwight Bedroom given change in Gamma



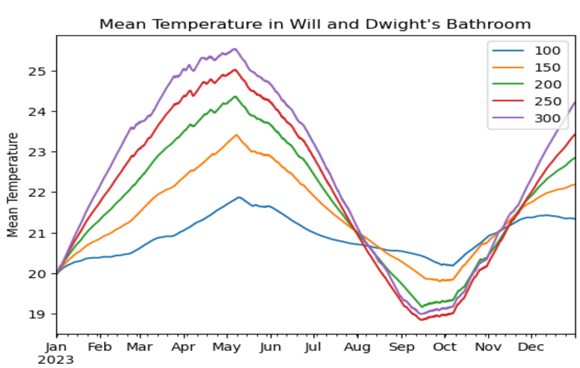
(d) Mean Temperature in Will Bedroom given change in Gamma



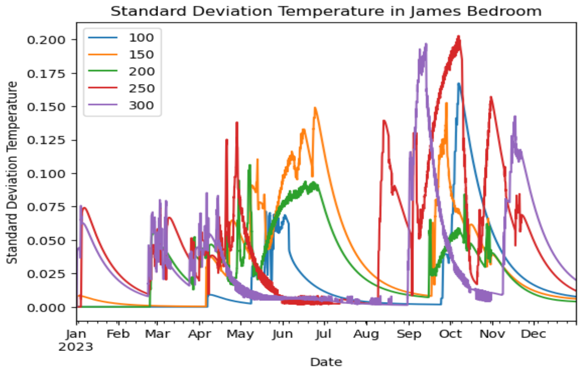
(e) Mean Temperature in Main Room given change in Gamma



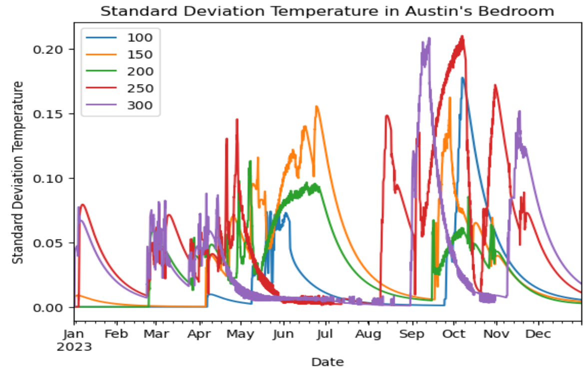
(f) Mean Temperature in James and Austin's bathroom given change in Gamma



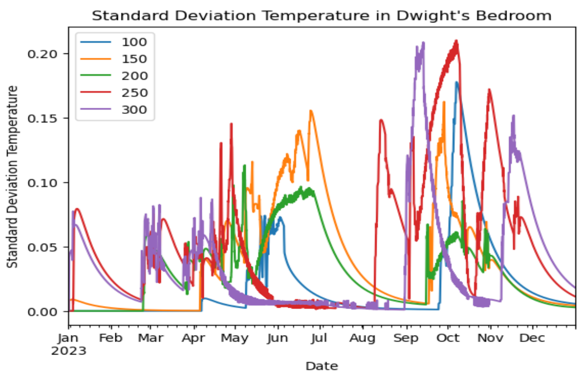
(g) Mean Temperature in Dwight and Will Bedroom given change in Gamma



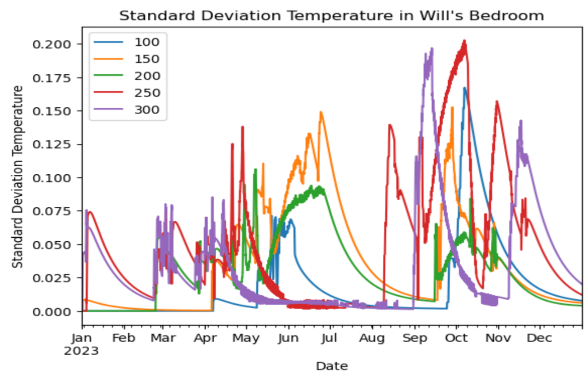
(a) Standard deviation in Temperature in James Bedroom given change in Gamma



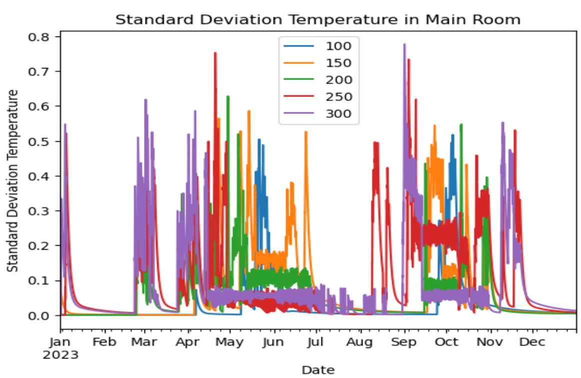
(b) Standard deviation in Temperature in Austin Bedroom given change in Gamma



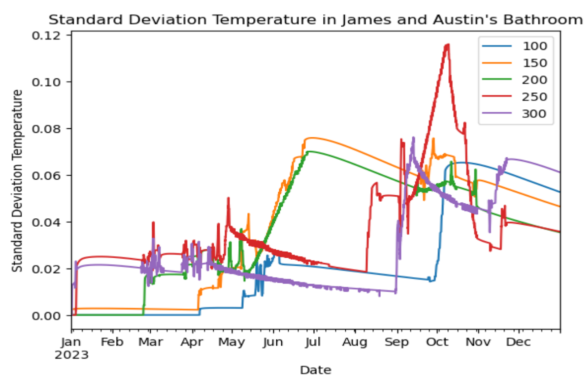
(c) Standard deviation in Temperature in Dwight Bedroom given change in Gamma



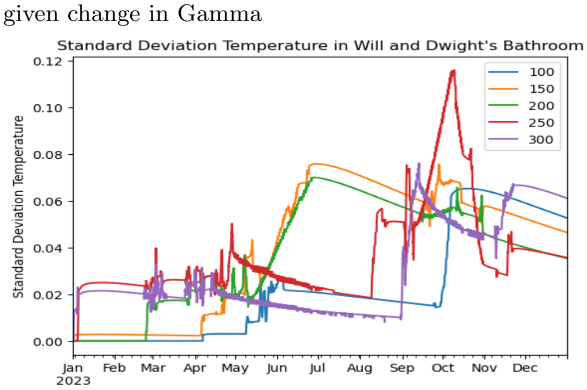
(d) Standard deviation in Temperature in Will Bedroom given change in Gamma



(e) Standard deviation in Temperature in Main Room given change in Gamma



(f) Standard deviation in Temperature in James and Austin's bathroom given change in Gamma



(g) Standard deviation in Temperature in Dwight and Will Bedroom given change in Gamma

9 Future work

Further sensitivity analysis is needed as is futher analysis on the standard deviation. A warm up period would also be a good addition to the simulation model. Analysis on thermostat setting might also be a good addition, but we didn't have time yet.

References

- Heat Transfer Coefficient - Window
- Amps given of By Refrigerator (General Average)
- Refrigerator Model Specifications
- U-Values
- Heat Transfer Coefficient - Walls
- Wattage of Appliances (General List, not used for Fridge)