Study of Rigid Body Models in the PySPH Framework

Mid-Semester Review

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March 14, 2016

Picking up from Pre-Project

- Work done during Pre-Project:
  - Establish proficiency Python programming
  - Identify Potential Topics
  - Obtain (beginner's) understanding of the pySPH framework

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  - Implement physically consistent models
  - Implement Solid-Fluid coupling using IISPH technique

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- It is a Meshfree Particle Method where the domain discretization is performed as:
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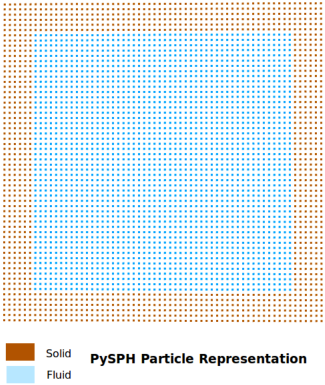
- The SPH methodology uses the Lagrangian Formulation of the governing equations (i.e. all the governing equations are expressed in terms of the Substantial Derivative)



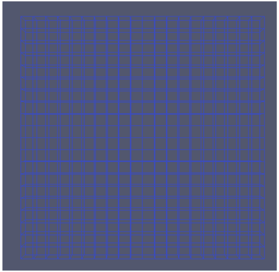
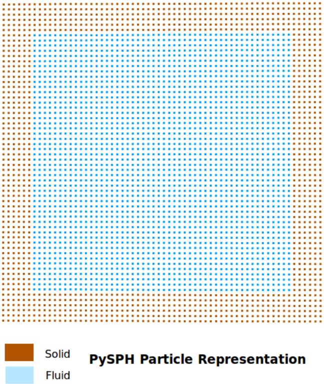
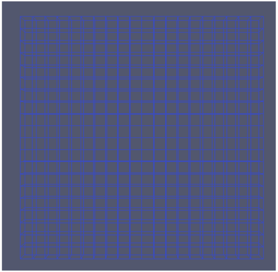


Figure : Discretization in Meshfree and Grid Based Methods

The SPH Method: Spatial Discretization





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- Particle Distribution can be completely arbitrary
- No explicit interconnectivity between particles

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- The Kernel is a Real function and has non-zero values on a compact/infinite sized support
- The support depends on the **Smoothing Length**, h.

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  - It's integral over the domain should be 1
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$$\therefore [A]_d = \sum_b A(\mathbf{r_b}, t)\ w_h(|\mathbf{r_{ab}}|)\ V_b,$$

$$\forall a \in \Omega, \mathbf{r}_{ab} = \mathbf{r}_a - \mathbf{r}_b$$

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  - FEM constructs these basis functions from the grid and SPH constructs them based on the kernel
  - The FEM basis functions are stationary, while those of SPH move along with the particles.
    i.e. if **'B'** are the SPH Basis functions then $\frac{D\mathbf{B}}{Dt} = 0$

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- **Soft Contact** Method
  - The bodies are allowed to "overlap"
  - The amount and rate of the overlap gives incremental contact forces
  - These contact forces are applied to Newton's laws to obtain new out of balance forces and moments
  - From the forces and moments, the velocities and displacements are evaluated

- DEM terminology:
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- Since the bodies are represented as collections of particles in pySPH, explicit Boxing and Contact Search is not needed
- the NNPS keeps track of all those elements which are in contact and the smoothing length effectively computes the amount of overlap

- Models the contacts as a Spring and Dashpot system

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- Current PoC model implementation

  Repulsive Force, $f_{i,s} = -k(d - |\mathbf{r_{ij}}|)\frac{\mathbf{r_{ij}}}{|\mathbf{r_{ij}}|}$

  Damping Force, $f_{i,d} = \eta\ \mathbf{v_{ij}}$

  Shear Force, $f_{i,t} = k_t\ \mathbf{v}_{\mathbf{ij},t}$

  where,

  k = Spring Coefficient,

  k_t = Coefficient of Shear,

  η = Damping Coefficient,

  d = diameter of elements comprising the body

  $\mathbf{r_{ij}} = \mathbf{r}_i - \mathbf{r}_j$,

  $\mathbf{v_{ij}} = \mathbf{v}_i - \mathbf{v}_j$

  Relative Tangential Velocity,

$$\mathbf{v}_{\mathbf{ij},t} = \mathbf{v_{ij}} - \left(\mathbf{v_{ij}} \cdot \frac{\mathbf{r_{ij}}}{|\mathbf{r_{ij}}|}\right)\frac{\mathbf{r_{ij}}}{|\mathbf{r_{ij}}|}$$

- Total Contact Force,

$$F_c = \sum_{i \in RigidBody} (f_{i,s} + f_{i,d} + f_{i,t})$$

- Total Torque due to contact,

$$T_C = \sum_{i \in RigidBody} [r_i \times (f_{i,s} + f_{i,d} + f_{i,t})]$$

Newton's Equations for Rigid Body Dynamics

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$$M_I \frac{d\mathbf{V_I}}{dt} = \sum_{k \in I} m_k \frac{d\mathbf{v_k}}{dt}$$

$$I_I \frac{d\Omega_\mathbf{I}}{dt} = \sum_{k \in I} m_k \left(\mathbf{r_k} - \mathbf{R_I}\right) \times \frac{d\mathbf{v_k}}{dt}$$

where,

$M_I, \mathbf{V_I}, \mathbf{R_I} =$ Mass, Velocity and Center of Gravity of "I",
$I_I, \Omega_\mathbf{I} =$Inertial Tensor(MoI), Angular Velocity of "I"
$m_k, \mathbf{v_k}, \mathbf{r_k} =$ Mass, Velocity and Position of k^{th} particle

- Test Cases: Zhang et.al[1] performed experimental studies to validate a coupled Finite Volume Particle (FVP) DEM to simulate interactions between fluids and solid bodies

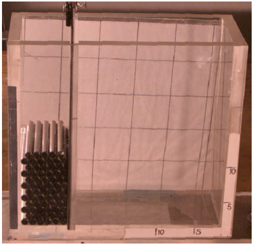

Figure : Experimental Setup

- Tank Dimensions: l = h = 26cm , w = 10cm
- Cylinder Properties: l = 9.9cm ,d = 1cm, $\rho = 2.7 \times 10^3 \frac{kg}{m^3}$

[1] Zhang et.al, Simulation of solid-fluid mixture flow using moving particle methods, J. Comput.Phys. 228 (2009) 25522565, http://dx.doi.org/10.1016/j.jcp.2008.12.005

Validation of RigidBodyCollision model: Expected Profiles

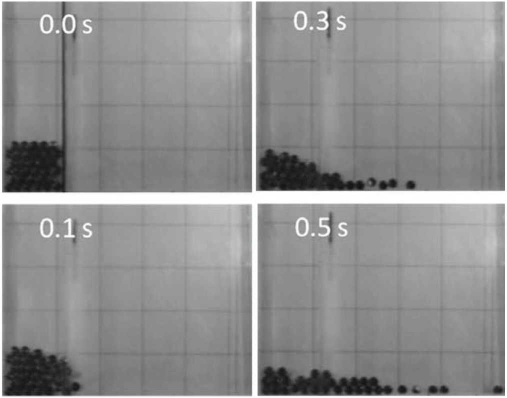

Figure : Evolution of the system at various times (6 Cylinder stack)

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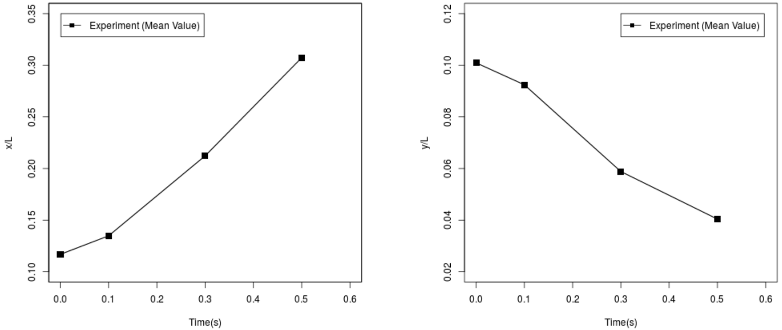

Figure : 6 Cylinder System Center of Mass

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- Conclusion: The current PoC model is not physically consistent

Making RogidBodyCollision model Physically Consistent

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- Canelas,et.al.[2]implemented a Modified, Non-Linear Hertzian model in their recently published work (Feb 2016)

[2]R.B.Canelas, et.al., SPH-DCDEM model for Arbitrary geometries in free surface solid-fluid flows, Computer Physics Communications (2016), http://dx.doi.org/10.1016/j.cpc.2016.01.006

PoA for Remaining Semester

- Implement the Non-Linear Hertzian model in pySPH
- Validate the Model

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- Perform Validation Study for the Solid-Fluid Coupled Model with Fluid equations solved with the IISPH formulation

Thank you!

Questions?