

Engineering Statistics Lectures XX

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Abstract

Final opportunity given December 5, 2019 – due December 9, 2019.

1 More normal curve stuff!

The normal curve is defined as follows:

$$f(x, \mu, \sigma) = \frac{1}{(\sqrt{2\pi})\sigma} \exp\left(\frac{-(x - \mu)^2}{2\sigma^2}\right)$$

Well, the standard curve ($\sigma = 1, \mu = 0$) is:

$$\text{std}(z) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-1}{2} z^2\right) \text{ s.t. } z = \frac{x - \mu}{\sigma}$$

So,

$$P(a < X < b) = P\left(\frac{a - \mu}{\sigma} < Z < \frac{b - \mu}{\sigma}\right)$$

There are tables in the book that get cumulative distribution values $F(z)$ situated based on standard deviation distance from the mean. $\text{std}(z)$ is just a genericization of x in terms of standard deviations away from the mean – the standard deviation is scaled to 1, and the mean is transformed out.

2 Example: Speeds on the highway!

Suppose that the speed of a randomly-selected vehicle from 6:00 PM to 8:00 PM over a highway is a standard r.v. X such that $(\mu = 74, \sigma = 3.5)$: What's the probability that a car is going between 68 and 79 miles per hour?

$$\begin{aligned} P(68 < X < 79) &\equiv \frac{X - 74}{3.5} \\ \rightarrow P(68 < X < 79) &= P\left(\frac{68 - 74}{3.5} < Z < \frac{79 - 74}{3.5}\right) \\ &= P\left(\frac{-6}{3.5} < Z < \frac{5}{3.5}\right) \\ &= P\left(\frac{-12}{7} < Z < \frac{10}{7}\right) \\ &= P(-1.71 < Z < 1.43) \\ &= F(1.43) - F(-1.71) \end{aligned}$$

Well, Deputy Donut (Professor's name choice, not mine!) decides that he will cite the top 13.5% of the drivers. What speed is he going to use as his boundary? Well, there is a value Z_0 in the CDF such that $F(Z_0) = 86.5\%$. This value is the number of standard deviations ahead of the mean that will be our speed value. In the specific case here, 0.8650 is somewhere between $F(1.10) = 0.8643$ and $F(1.11) = 0.8665$. The rate of change is approximately $\frac{0.0022}{0.01}$; Our starting and ending values are (1.100, 0.0000) and (1.101, 0.0022); where is the second value 0.0007?

$$\begin{aligned} 0.0007 &= \frac{0.0022}{0.01}(Z_0 - 1.100) \\ \rightarrow Z_0 &= \frac{0.01}{0.0022}0.0007 + 1.100 \\ &\rightarrow Z_0 \approx 1.103181; \\ Z_0 &= \frac{X_0 - 74}{3.5} \\ \rightarrow X_0 &= 3.5 * (1.103181) + 74 \end{aligned}$$

$$\rightarrow X_0 = 77.86mph$$

Using the point slope formula where the left side is the difference from the lower standard deviation value. This method is known as "linear interpolation".

So, Deputy Donut is going to keep to people who are going about 77.86 miles per hour or higher.

Between now and Thursday, 12/5/2019, Take a look at Chapter 6's problems concerning the normal distribution.