

Engineering Statistics Lecture XI

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Abstract

HW #2 is due October 15, 2019:

- Section 2.3 #23-37 odd
- Section 2.4 #49-65 odd
- Section 2.5 #73-93 odd

NO CLASS THURSDAY, OCTOBER 10, 2019

1 Random Variables

For a given sample space S , a random variable is any rule that associates a number with each outcome in S , given that the domain is S and the range $R \subseteq S$.

1.1 Example: Roll a die.

X can be a random variable means that a roll of the die can be any value such that $x \in \{1, 2, 3, 4, 5, 6\}$. This works and X is a random variable.

1.2 Example: Flip a coin.

X can be a random variable means that a flip of the coin can yield $\{H, T\}$; Because the set can be enumerated (mapped to a countable set of numbers), X is a random variable.

1.3 Example: Eye color

Let S be $\{\text{brown, blue, hazel, green}\}$: X can be some $\{100 \text{ if brown, } 5 \text{ if blue, } 2 \text{ if hazel, } 50 \text{ if green}\}$. You do not have to have an enumerated set from 1 to however many items are in the set for it to be a "random variable", as frustrating as it may be to those that you are working with.

2 Probability Density Function (PDF)

A probability density function defines how the total probability is allocated (distributed) over all possible values of X .

Roll a die: X is the number result. Let $f(X)$ be the PDF.

$$f(X) \in \{\frac{1}{6}\} \iff X \in \{1 \rightarrow 6\}$$

The sum of this PDF $f(x)$ is as follows:

$$\int_{x \in X} f(x) dx = 1$$

2.1 Concrete stuff

Brendan drives a cement mixer. He charges $\$100 + \$\frac{80}{yd^3}$.

$f(x) = \{A \text{ for all } x \text{ between } 0 \text{ and } 10, 0 \text{ otherwise}\}$.

Find a:

$$\int_0^{10} A dx = 1 \rightarrow A(10 - 0) = 1 \rightarrow A = \frac{1}{10}.$$

Find b by the average value theorem:

$$\sum_{all \ X} xf(x)$$

for discrete values,

$$\int_{all \ X} xf(x) dx$$

for continuous values.

$$B = \frac{1}{10} \int_0^{10} x dx = \frac{1}{20} x^2 \Big|_0^{10} = 5$$
$$\rightarrow B = 5$$