

# Engineering Statistics Lectures XIX

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## Abstract

Opportunity #1 given November 26, 2019 and due November 27, 2019 at 6:00 PM. Final opportunity given December 5, 2019 – due December 9, 2019. LCCC community dinner on November 27, 2019 from 5:00 PM to 7:00 PM, room AT134.

Opportunity #1 Questions due Sunday November 24, 2019 by 8:00 PM by email. Questions can range from any .section, focusing on material from Lecture 16 onward (Chapter 5).

For optional homework, take a look at Chapter 5.1, 5.3, 5.9, 5.11, 5.29, 5.33, and 5.35 is particularly interesting.

## 1 Normal PDF

The Normal Distribution. The bell curve. The big squiggly bumpy thing. It is used to approximate many, many, many natural phenomena involving population data. Typically, in a one-modal curve, the average is positioned AT the bump.

The statistics of non-normally-distributed data sets tend to follow a normal curve. Even if the original data isn't normally distributed, the statistics pertaining to that data are. Is it a law? Probably not.

### 1.1 Definition of Normal Curve by Gaussian Distribution

A random variable  $X$  is normally distributed with a mean  $D$  and standard deviation  $J$ . The typical way of dealing with expressing it using the transcendental function is as follows:

$$(\forall X \in \mathbf{R}) f(X) = \frac{1}{\sqrt{2\pi} * J} * e^{-\frac{(X - D)^2}{2J^2}}$$

You can prove it to be a valid PDF by it being positive for all of the applicable domain and asserting that the integral over its domain is 1.

$$\begin{aligned}
u &:= \frac{X - D}{J} \\
\rightarrow du &= \frac{X}{J} \\
\rightarrow I &= \int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du \\
&= \sqrt{\left[\int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du\right]^2} \\
&= \sqrt{\left(\int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du\right) \left(\int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du\right)} \\
&= \sqrt{\frac{1}{2\pi} \left(\int_{\mathbf{R}} \exp\left(-\frac{u^2}{2}\right) du\right) \left(\int_{\mathbf{R}} \exp\left(-\frac{v^2}{2}\right) dv\right) \text{ s.t. } (u, v) = (a, b)} \\
&= \sqrt{\frac{1}{2\pi} \int_{u \in \mathbf{R}} \int_{v \in \mathbf{R}} \exp\left(-\frac{a^2 + b^2}{2}\right) dv du} \\
r^2 &:= \frac{a^2 + b^2}{2} \\
\rightarrow I &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \int_{r \in \mathbf{R}^+} \exp\left(-\frac{r^2}{2}\right) (r dr d\theta) \text{ by Jacobian change of base}} \\
&= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} d\theta \int_{r \in \mathbf{R}^+} \exp\left(-\frac{r^2}{2}\right) (r dr)} \\
w &:= \frac{r^2}{2} \rightarrow dw = r dr \\
\rightarrow I &= \sqrt{\frac{1}{2\pi} [2\pi] \int_0^{+\infty} \exp(-u) du} \\
&= \sqrt{[-e^{-u}]_{u=0}^{u=+\infty}} = \sqrt{-[0 - 1]} = \sqrt{-[-1]} = \sqrt{1} \\
\rightarrow I &= 1 \\
&\rightarrow \int_{X \in \mathbf{R}} f(X) dX = 1 \\
&\blacksquare
\end{aligned}$$