# Engineering Statistics Lecture XIII

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**Abstract**

HW #3 is due October 31, 2019:

- Chapter 4 #1-31 odd

HW #4 is due October 20, 2019 at 8:00 PM:
Submit 2 opportunity questions with, preferably, correct answers.

- Multiple guess questions
- Extended response questions

Opportunity #0 Given October 22, 2019;
Opportunity #0 Due October 24, 2019.

## 1 Brendan's concrete something or another

f(x) is a line from [0,2] over [0,1], then from [2,0] over [1,2]

### 1.1 Find A: A = 1

### 1.2 Find $\mu_w$ : $\mu_w = 1$

## 1.3 Find $\sigma_w^2$

Variance:

$$\sigma_w^2 = \frac{\sum_{i=1}^{n}(f(x_i) - \mu)^2}{n-1} = \int_{all\ x}(f(x) - \mu)dx$$

$$\rightarrow \sigma_w^2 = E[x^2] - E[x]^2$$

Big brain time: $\mu = E[x] = \int x f(x)dx$.

$$\begin{aligned} E[w^2] = \int w^2 f(w)dw &= \int_0^1 w^2 * wdw + \int_1^2 w^2(2-w)dw \\ &= \frac{w^4}{4}|_0^1 + \frac{2}{3}w^3|_1^2 - \frac{w^4}{4}|_1^2 \\ &= \frac{1-0}{4} + \frac{2}{3}(8-1) - \frac{1}{4}(16-1) \\ &= \frac{7}{6} \end{aligned}$$

## 1.4 Margin: 10w - 20

## 1.5 Find E[Margin]

$$\begin{aligned} E[Margin] &= \int (10W - 20)f(w)dW \\ &= 10\int (W-2)f(W)dW \end{aligned}$$

For a linear function f(x) of random variable X, E[f(x)] $= A\mu_x + b$

$$\rightarrow E[f(x)] = AE[x] + B$$

$$\rightarrow E[10W - 20] = 10(1) - 20 = -10$$

(See 1.1)

$$\rightarrow \sigma^2(f(x)) = A^2\sigma_x^2$$

### 1.6 Find cumulative distribution function (CDF)

CDF - Shows how probability mass accumulates as x goes from $-\infty$ to X.

$$
\begin{aligned}
f(X)CDF &= \int_{-\infty}^{X} f(\tau)d\tau \\
&= \int_{0}^{X} \tau d\tau \iff x \leq 1 \\
OR \;\; &= \int_{0}^{1} \tau d\tau + \int_{1}^{X} (2-\tau)d\tau \iff X \in [1,2] \\
&= \frac{1}{2}\tau^2|_0^1 + [2\tau - \frac{1}{2}\tau^2]_1^X \\
&= 2x - \frac{1}{2}x^2 - 1 \iff X \in [1,2]
\end{aligned}
$$

### 1.7 Find $\mathbf{p}(\frac{2}{5} \leq W \leq \frac{4}{3})$

$$
\begin{aligned}
&= CDF(\frac{4}{3}) - CDF(\frac{2}{5}) \\
&= -\frac{1}{2}(\frac{4}{25}) + \frac{1}{2} + 2\frac{4}{3} - \frac{1}{2}\frac{16}{9} - 2 + \frac{1}{2}
\end{aligned}
$$