Engineering Statistics Lectures XVIII

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November 19, 2019

Abstract

Opportunity #1 given November 26, 2019 and due November 27, 2019 at 6:00 PM. Final opportunity given sometime during the week of December 10, 2019. LCCC community dinner on November 27, 2019 from 5:00 PM to 7:00 PM, room AT134.

Opportunity #1 Questions due Sunday November 24, 2019 by 8:00 PM by email. Questions can range from any .section, focusing on material from Lecture 16 onward (Chapter 5).

For optional homework, take a look at Chapter 5.1, 5.3, 5.9, 5.11, 5.29, 5.33, and 5.35 is particularly interesting.

1 Negative Binomial

4 conditions:

1. Independent trials.
2. Trial can be a success or a failure (S or F).
3. P(S) is constant across all trials.
4. Trials continue until k successes are observed.

For all positive integers k. X is the total number of trials before/including the kth success. Spoz k=3:

$$P(X = x) = \binom{x-1}{k-1} * p^{k-1}q^{(x-k)-1} * p$$

Such that the p^{k-1} term refers to the number of successes prior to the kth success and the q^{x-k} term refers to the number of failures prior to the kth success. The "* p" term on the end refers to the fact that for any negative binomial problem of length x, there will ALWAYS be some combination of x-1 successes/failures followed by a single success on the end. SO:

$$P(X = x) = \binom{x-1}{k-1} * p^k q^{x-k}$$

Because the sum of the exponents of p and q must be x due to there having been x trials!

2 BASEBALL

Spoz the Tigers and Indians play a 3-game series. What's the chance that the Tigers win the series (Best of three – at most one loss)? P(S) = 0.6

$$P(X = 2) = \binom{1}{1} p^2 q^0 = 0.36$$

$$P(X = 3) = \binom{2}{1} p^2 q^1 = 0.288$$

Therefore, the probability that the Tigers win the series is P(2) + P(3) = 0.648.

That's no fun, though.

What if the tigers and indians play a 5-game series? What's the new chance of winning? ? Well, that's just the sum of all possible chances from X=3 to X=5. SO:

$$P(X) = \sum_{x=3}^{5} \binom{x-1}{2} * p^3 q^{x-3}$$

So:

$$P(X) = \binom{2}{2} p^3 q^0 + \binom{3}{2} p^3 q^1 + \binom{4}{2} p^3 q^2$$

So:

$$P(X) = 0.2593$$

The arithmetic has been left as an exercise.

3 Average of a Negative Binomial

$\mu = \frac{k}{p}$ for reasons that are left as an investigative proof to the reader.

4 Sample variance of a Negative Binomial

$\sigma^2 = \frac{k(1-p)}{p^2} = \frac{k}{p} q = \mu * q$ For any constant p.