Engineering Statistics Lecture XV

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Abstract

Bell Labs – Subsidiary of AT&T, company invented many things ranging from the BJT to the general transistor to the telephone (Namesake – Alexander Graham Bell) to the IC. Basically died at the late 70's AT&T split.

1 Reprisal of Joint Probability

For three variables,

$$P(X = x, Y = y) = \frac{J_x(x)J_y(y)J_z(x, y)}{\#\ of\ possible\ cases}$$

Where $J_x(x)$ refers to a joint probability element which concerns x. and $J_z(x, y)$ refers to a joint probability element which concerns z, a dependent variable upon x and y. Generally, these are ${}_nC_r$ functions such that the sum of all n and the sum of all k in the numerator match that of the denominator.

2 Marginal probabilities

What if we want the chance that, regardless of y, x is fixed? Well, good! We have a way of dealing with it!

$$P(x) = \sum_{j \in Y} P(x, j)$$

Where x is some fixed value and j iterates over the set of possible values for Y. For three independent variables,

$$P(x) = \sum_{j \in Y} \sum_{k \in Z} P(x, j, k)$$

Where, as before, j and k iterate over Y and Z. Think of P(x) to be a lesser-dimensional slice of the relevant space (plane corresponds to line, space corresponds to plane, hyperspace corresponds to space, etc.).

3 Means in multidimensional sets

$$\mu_X = \frac{P(x)}{\|X\|} = \frac{\sum_{j \in Y} P(x,j)}{\#\ of\ terms\ in\ X} = avg(P(x))$$

Where P(x) is the marginal probability of x occurring.

4 Marathon I guess?

Spoz that the fraction of guys that complete a marathon is X, and the fraction of women is Y. Historically-speaking, $f(X = x, Y = y) = Axy\ s.t.\ 0 \leq y \leq x \leq 1$.

To get the probability between two values of x, we take the integral of the PDF f(x) between those two values.

To get the probability within two ranges of x and y respectively, we take the integral of a PDF f(x,y) within those ranges; the area under the surface.

4.1 A. find A.

$$\int\int Axy\ dxdy = 1\ \forall(x,y)\ s.t.\ 0 \leq y \leq x \leq 1$$
$$\rightarrow \int_{y=0}^{x=1}\int_{x=y}^{x=1} Axy\ dxdy = 1$$
$$\rightarrow \int_{x=0}^{x=1}\int_{y=0}^{y=x} Axy\ dydx = 1$$
$$\rightarrow \int_{x=0}^{x=1} \frac{A}{2}xy^2|_{y=0}^{y=x}\ dy = 1$$
$$\rightarrow \int_{x=0}^{x=1} \frac{A}{2}x^3 dx = 1$$
$$\rightarrow \frac{A}{8}x^4|_{x=0}^{x=1} = 1$$
$$\rightarrow \frac{A}{8} = 1$$
$$\rightarrow A = 8$$

$$\rightarrow f(x,y) = 8xy$$

4.2 B. Find g(x), h(y)

Find g(x):

$$
\begin{aligned}
g(x) &= \int_{y=0}^{y=x} f(x,y)dy \\
\rightarrow g(x) &= 8\int_{y=0}^{y=x} xy\ dy \\
&= 4xy^2|_{y=0}^{y=x} \\
\\
\rightarrow g(x) &= 4x^3
\end{aligned}
$$

Find h(y):

$$
\begin{aligned}
h(y) &= \int_{x=y}^{x=1} [f(x,y)]dx \\
\rightarrow h(y) &= \int_{x=y}^{x=1} [8xy]dx \\
&= 4x^2y|_{x=y}^{x=1} \\
&= 4y(1-y^2) \\
\\
\rightarrow h(y) &= 4y(1+y)(1-y)
\end{aligned}
$$

4.3 C. Find $\mu_X,\ \mu_Y$

$$\begin{aligned}\mu_X &= \int_{x=0}^{x=1} xg(x)dx\\ &= \int_{x=0}^{x=1} 4x^4dx\\ &= \frac{4}{5}x^5|_0^1\end{aligned}$$

$$\rightarrow \mu_X = \frac{4}{5}$$

Find μ_Y:

$$\begin{aligned}\mu_Y &= \int_{y=0}^{y=1} yh(y)dy\\ &= \int_0^1 y * 4y(y-y^3)dy\\ &= \int_0^1 4(y^3-y^5)dy\\ &= [y^4 - \frac{2}{3}y^6]_{y=0}^{y=1}\end{aligned}$$

$$\rightarrow \mu_Y = \frac{1}{3}$$

4.4 D. What's P$(0.1 \leq X \leq 0.3,\ 0.2 \leq Y \leq 0.4)$

Find the probability by summing up the contents under the curve. However, Y is strictly bounded by X: $Y \leq X$!! However, the same rule follows: $Y \ngeq X$. So, the bound becomes:

$$0.2 \leq X \leq 0.3, 0.2 \leq Y \leq 0.3$$

So, we take the integral as:

$$\begin{aligned}
P(etc) &= \int_{x=0.2}^{x=0.3} \int_{y=0.2}^{y=0.3} f(x)dydx \\
&= \int_{x=0.2}^{x=0.3} \int_{y=0.2}^{y=0.3} 8xydydx \\
&= \int_{x=0.2}^{x=0.3} [4xy^2]_{y=0.2}^{y=0.3} dx \\
&= \int_{x=0.2}^{x=0.3} 4(\frac{9}{100} - \frac{4}{100})xdx \\
&= \frac{1}{5} * \frac{1}{2}[x^2]_{x=0.2}^{x=0.3} \\
&= \frac{1}{10}[\frac{9}{100} - \frac{4}{100}]
\end{aligned}$$

5 Brian has two pumps?

Spoz that Brian runs a gas station with two pumps.

Let X be the fraction of time that a customer has to wait in line (only has two pumps) for the first pump. Let Y be the fraction of time that a customer has to wait in line for the other pump. Historically-speaking, after much data compiled and taken using python, the PDF f(x,y), given that 10 minutes is the maximum wait time possible:

$$\begin{aligned} f(x,y) = &\{A(2x+3y) && iff\ 0 \le x \le 1\} \\ &\{ && 0 \le y \le 1\} \\ &\{ && \} \\ &\{0 && elsewhere\} \end{aligned}$$

5.1 A. Find A

$$\begin{aligned} A &= \frac{1}{F(x,y)} \\ \rightarrow F(x,y) &= \int_{y=0}^{y=1}\int_{x=0}^{x=1}(2x+3y)dxdy \\ &= \int_{y=0}^{y=1}(x^2+3xy)|_{x=0}^{x=1}dy \\ &= \int_{y=0}^{y=1}(1+3y)dy \\ &= [y+\frac{3}{2}y^2]_{y=0}^{y=1} \\ &= \frac{5}{2} \end{aligned}$$

$$\rightarrow A = \frac{2}{5}$$

5.2 B. Find μ_X, μ_Y

Find μ_X by finding g(x):

$$\begin{aligned} g(x) &= \int_{y=0}^{y=1} \frac{2}{5}(2x+3y)dy \\ &= \frac{2}{5}[2xy + \frac{3}{2}y^2]_{y=0}^{y=1} \\ &= \frac{2}{5}[2x + \frac{3}{2}] \\ &= \frac{4x+3}{5} \end{aligned}$$

$$\begin{aligned} \to \mu_X &= \int_{x=0}^{x=1} \frac{4x^2+3x}{5} dx \\ &= \frac{\frac{4}{3}x^3 + \frac{3}{2}x^2}{5}|_{x=1} \end{aligned}$$

$$\to \mu_X = \frac{17}{30}$$

Find μ_Y by finding h(y):

$$h(y) = \int_{x=0}^{y=1} \frac{2}{5}(2x+3y)dx$$

5.3 Find the probability that the wait time for the first pump is 3-8 minutes and that the time waiting for pump 2 is 3-7 minutes