Engineering Statistics Lectures XVI

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Abstract

HW #3 due today. Opportunity #1 given November 21, 2019 and due November 26, 2019. Final opportunity given sometime during the week of December 10, 2019

Chapter 5 begins today!

1 Discrete Probability Functions

1.1 Uniform

Each of the k values of x has equal probability. In example, rolling a die, selecting a 3-digit integer at random, et cetera.

1.2 Mean

For a set of k items indexed within X,

$$\mu_x = \sum_{x_i \in X} \frac{x_i}{k}$$

1.3 Sample variance / population variance

For this same definition as above,

$$\sigma_x^2 = \frac{1}{k-1} \sum_{x_i \in X} (x_i - \mu_x)^2$$

2 Binomial PDF

Four conditions for a binomial PDF exist:

1. Some integer n fixed trials.
2. Each trial is a success or failure (S or F).
3. The trials are independent – the occurrences of one trial do not affect the state of any subsequent trials.
4. P(S) = p = constant; the probability of success is in and of itself constant.

Very short version – A binomial PDF is defined around constant, unchanging conditions by which one trial is completely independent of another. Any subject of change means that the circumstances are NOT a binomial PDF! A random variable X is a binomial random variable if the four conditions are met AND X is the number of successes in some number of n trials.

3 Example: Unfair coin.

Flip an unfair coin 5 times such that S = tails and P(S) = $\frac{3}{4}$. What's P(X=4)?

Define p to be P(S) and q to be P(F) = 1 - P(S).

$$\begin{aligned}
P(X=4) &= \binom{5}{4} * (\frac{3}{4})^4 * (1 - \frac{3}{4}) \\
&= 5 * \frac{81}{256} * \frac{1}{4} \\
&= \frac{405}{1024} \\
P(X=4) &\approx 0.3955
\end{aligned}$$

$$\begin{aligned}
P(X<3) &= \sum_{x=0}^{2} \binom{5}{x} p^x q^{(5-x)} \\
&= \sum_{x=0}^{2} \frac{5!}{x!(5-x)!} (\frac{3}{4})^x (\frac{1}{4})^{(5-x)} \\
&= \frac{53}{512}
\end{aligned}$$

If for any reason you are given a CDF table and you want to find an individual probability, take the value from the CDF table that you want and subtract the previous instance of the CDF.

What if we make it 10 times instead, and P(S) = 0.6?

3.1 A. What is μ?

$\mu = np = 10 * 0.6 = 6$. 6 times on average – we expect 6 successes and 4 failures on average.

3.2 B. What is σ_x^2?

$\sigma^2 = np(1-p) = npq = 10 * 0.6 * 0.4 = 2.4$. So, the population variance is around 2.4.

3.3 C. What is P($X \leq 3$)?

$$\sum_{x=0}^{3} \binom{10}{x} (0.6)^x (0.4)^{10-x}$$
$$=0.05476$$

3.4 D. What is P($X > 5$)?

= 1 - P($X \leq 5$). = 1 - 0.3669

3.5 E. What is P($X \in [3, 7]$)?

P(X=7) - P(X=3) $\approx$ 0.7779

4 Brian has two pumps?

Spoz that Brian runs a gas station with two pumps.

Let X be the fraction of time that a customer has to wait in line (only has two pumps) for the first pump. Let Y be the fraction of time that a customer has to wait in line for the other pump. Historically-speaking, after much data compiled and taken using python, the PDF f(x,y), given that 10 minutes is the maximum wait time possible:

$$\begin{aligned} f(x,y) = &\{A(2x+3y) && iff\ 0 \le x \le 1\} \\ &\{ && 0 \le y \le 1\} \\ &\{ && \} \\ &\{0 && elsewhere\} \end{aligned}$$

4.1 A. Find A

$$\begin{aligned} A &= \frac{1}{F(x,y)} \\ \rightarrow F(x,y) &= \int_{y=0}^{y=1} \int_{x=0}^{x=1} (2x+3y)dxdy \\ &= \int_{y=0}^{y=1} (x^2+3xy)|_{x=0}^{x=1} dy \\ &= \int_{y=0}^{y=1} (1+3y)dy \\ &= [y + \frac{3}{2}y^2]_{y=0}^{y=1} \\ &= \frac{5}{2} \end{aligned}$$

$$\rightarrow A = \frac{2}{5}$$

4.2 B. Find μ_X, μ_Y

Find μ_X by finding g(x):

$$
\begin{aligned}
g(x) &= \int_{y=0}^{y=1} \frac{2}{5}(2x+3y)dy \\
&= \frac{2}{5}[2xy + \frac{3}{2}y^2]_{y=0}^{y=1} \\
&= \frac{2}{5}[2x + \frac{3}{2}] \\
&= \frac{4x+3}{5}
\end{aligned}
$$

$$
\begin{aligned}
\rightarrow \mu_X &= \int_{x=0}^{x=1} \frac{4x^2+3x}{5}dx \\
&= \frac{\frac{4}{3}x^3 + \frac{3}{2}x^2}{5}|_{x=1}
\end{aligned}
$$

$$\rightarrow \mu_X = \frac{17}{30}$$

Find μ_Y by finding h(y):

$$h(y) = \int_{x=0}^{y=1} \frac{2}{5}(2x+3y)dx$$