Engineering Statistics Lectures XX

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Abstract

Final opportunity given December 5, 2019 – due December 9, 2019.

1 More normal curve stuff!

The normal curve is defined as follows:

$$f(x, \mu, \sigma) = \frac{1}{(\sqrt{2\pi})\sigma} exp(\frac{-(x-\mu)^2}{2\sigma^2})$$

Well, the standard curve ($\sigma = 1, \mu = 0$) is:

$$std(z) = \frac{1}{\sqrt{2\pi}} exp(\frac{-1}{2} z^2) \ s.t. \ z = \frac{x-\mu}{\sigma}$$

So,

$$P(a < X < b) = P(\frac{a-\mu}{\sigma} < Z < \frac{b-\mu}{\sigma})$$

There are tables in the book that get cumulative distribution values F(z) situated based on standard deviation distance from the mean. std(z) is just a genericization of x in terms of standard deviations away from the mean – the standard deviation is scaled to 1, and the mean is transformed out.

2 Example: Speeds on the highway!

Suppose that the speed of a randomly-selected vehicle from 6:00 PM to 8:00 PM over a highway is a standard r.v. X such that ($\mu = 74, \sigma = 3.5$): What's the probability that a car is going between 68 and 79 miles per hour?

$$\begin{aligned}
P(68 < X < 79)?Z &\equiv \frac{X-74}{3.5} \\
\rightarrow P(68 < X < 79) &= P(\frac{68-74}{3.5} < Z < \frac{79-74}{3.5}) \\
&= P(\frac{-6}{3.5} < Z < \frac{5}{3.5}) \\
&= P(\frac{-12}{7} < Z < \frac{10}{7}) \\
&= P(-1.71 < Z < 1.43) \\
&= F(1.43) - F(-1.71)
\end{aligned}$$

Well, Deputy Donut (Professor's name choice, not mine!) decides that he will cite the top 13.5% of the drivers. What speed is he going to use as his boundary? Well, there is a value Z_0 in the CDF such that F(Z_0) = 86.5%. This value is the number of standard deviations ahead of the mean that will be our speed value. In the specific case here, 0.8650 is somewhere between F(1.10) = 0.8643 and F(1.11) = 0.8665. The rate of change is approximately $\frac{0.0022}{0.01}$; Our starting and ending values are (1.100, 0.0000) and (1.101, 0.0022); where is the second value 0.0007?

$$0.0007 = \frac{0.0022}{0.01}(Z_0 - 1.100)$$

$$\rightarrow Z_0 = \frac{0.01}{0.0022}0.0007 + 1.100$$

$$\rightarrow Z_0 \approx 1.103181$$

$$Z_0 = \frac{X_0 - 74}{3.5}$$

$$\rightarrow X_0 = 3.5 * 1.103181 + 74$$

$$\rightarrow X_0 = 77.86mph$$

So, Deputy Donut is going to keep to people who are going about 77.86 miles per hour or higher.

Between now and Thursday, 12/5/2019, Take a look at Chapter 6's problems concerning the normal distribution.