# Engineering Statistics Lectures XIX

Notes by Jonathan Bender

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**Abstract**

Opportunity #1 given November 26, 2019 and due November 27, 2019 at 6:00 PM. Final opportunity given December 5, 2019 – due December 9, 2019. LCCC community dinner on November 27, 2019 from 5:00 PM to 7:00 PM, room AT134.

Opportunity #1 Questions due Sunday November 24, 2019 by 8:00 PM by email. Questions can range from any .section, focusing on material from Lecture 16 onward (Chapter 5).

For optional homework, take a look at Chapter 5.1, 5.3, 5.9, 5.11, 5.29, 5.33, and 5.35 is particularly interesting.

## 1 Normal PDF

The Normal Distribution. The bell curve. The big squiggly bumpy thing. It is used to approximate many, many, many natural phenomena involving population data. Typically, in a one-modal curve, the average is positioned AT the bump.

The statistics of non-normally-distributed data sets tend to follow a normal curve. Even if the original data isn't normally distributed, the statistics pertaining to that data are. Is it a law? Probably not.

### 1.1 Definition of Normal Curve by Gaussian Distribution

A random variable X is normally distributed with a mean D and standard deviation J. The typical way of dealing with expressing it using the trancendental function is as follows:

$$(\forall X \in \mathbf{R})\ f(X) = \frac{1}{\sqrt{2\pi} * J} * \exp\left(-\frac{(X-D)^2}{2J^2}\right)$$

Where $exp(A) \equiv e^A$ and the two are used interchangeably for legibility. You can prove it to be a valid PDF by it being positive for all of the applicable domain and asserting that the integral over its domain is 1. A big problem with that is that its antiderivative is a little... silly. It is essentially a massive transformation of $\exp(-x^2)$. See the next page for the proof as a valid PDF.

$$u := \frac{X-D}{J}$$

$$\rightarrow du = \frac{X}{J}$$

$$\rightarrow I = \int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}}\ exp(-\frac{u^2}{2})du$$

$$= \sqrt{[\int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}}\ exp(-\frac{u^2}{2})du]^2}$$

$$= \sqrt{(\int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}}\ exp(-\frac{u^2}{2})du)(\int_{\mathbf{R}} \frac{1}{\sqrt{2\pi}}\ exp(-\frac{u^2}{2})du)}$$

$$= \sqrt{\frac{1}{2\pi}(\int_{\mathbf{R}} exp(-\frac{u^2}{2})du)(\int_{\mathbf{R}} exp(-\frac{v^2}{2})dv)}\ s.t.(u,v) = (a,b)$$

$$= \sqrt{\frac{1}{2\pi}\int_{u\in\mathbf{R}}\int_{v\in\mathbf{R}} exp(-\frac{a^2+b^2}{2})dvdu}$$

$$r^2 := \frac{a^2+b^2}{2}\ //\ Change\ of\ base\ to\ polar\ coordinates$$

$$\rightarrow I = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\int_{r\in\mathbf{R}^+} exp(-\frac{r^2}{2})(rdrd\theta)}\ by\ Jacobian\ change\ of\ base$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} d\theta \int_{r\in\mathbf{R}^+} exp(-\frac{r^2}{2})(rdr)}$$

$$w := \frac{r^2}{2} \rightarrow dw = rdr$$

$$\rightarrow I = \sqrt{\frac{1}{2\pi}[2\pi]\int_0^{+\infty} exp(-u)du}$$

$$= \sqrt{[-e^{-u}]_{u=0}^{u=+\infty}]} = \sqrt{-[0-1]} = \sqrt{-[-1]} = \sqrt{1}$$

$$\rightarrow I = 1$$

$$\rightarrow I = \int_{X\in\mathbf{R}} f(X)dX = 1\ //\ which\ suits\ the\ definition\ of\ a\ PDF$$

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