

## MATH/COSC 303

### Assignment 2

**Due:** Feb 4th, assignments are due at the end of lab.

#### Hand Written Questions:

1. Show that the linear system

$$\begin{array}{rcrcrcrcl} 4w - 2x + 3y - z & = & 6 \\ 0x + y + 4z & = & -2 \\ y - 2z & = & 0 \\ 6z & = & 12 \end{array}$$

has no solutions.

2. Consider the following Augmented System

$$\left[ \begin{array}{cccc|c} 1 & 2 & 0 & 0 & 5 \\ 0 & 0 & 2 & -4 & 12 \\ 2 & 3 & -1 & 0 & 9 \\ 0 & 4 & 2 & 3 & 10 \end{array} \right]$$

Reduce the system to upper triangular form by using Gaussian Elimination with

- a) Trivial Pivoting,
  - b) Partial Pivoting, and
  - c) Full Pivoting.
3. a) Determine the complexity of Gaussian Elimination with Back Substitution.  
b) An alternative algorithm for solving linear systems is Gauss-Jordan Elimination. The complexity of Gauss-Jordan Elimination is  $3n^3 + 5n^2 + 2n$ . Suppose  $n = 100$ , roughly how much faster is Gaussian Elimination with Back Substitution?

#### Computer Assisted Questions:

4. Lindfield and Penny, questions 1.1, 1.2, and 1.14.
5. Let

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad y = \begin{bmatrix} -4 \\ -3 \\ -2 \\ -1 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 6 & -2 & -6 \\ 0 & 6 & 0 & -6 \\ 2 & 0 & -2 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -3 & -3 & -3 \\ 4 & 4 & 4 \end{bmatrix}.$$

Use MATLAB to compute the following.

- a)  $63x - 17y$
- b)  $x \cdot (63x - 17y)$
- c)  $8(A^\top - 5B)$
- d)  $B * 8(A^\top - 5B)^\top$
- e)  $[B * 8(A^\top - 5B)^\top][63x - 17y]$

6. Write a MATLAB script to create an upper triangular matrix based on your student number as follows

$$SID = [a_1, a_2, \dots, a_8]$$
$$A = \begin{bmatrix} a_1 & a_2 & \dots & a_8 \\ 0 & a_2 & \dots & a_8 \\ 0 & 0 & \dots & a_8 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_8 \end{bmatrix}$$

Write a MATLAB script to solve  $Ax = \mathbf{ones}(8, 1)$  via backwards substitution.

7. Write a MATLAB script to perform an LU decomposition on a  $n \times n$  matrix.
- Use the `tic` and `toc` commands in MATLAB to time the script decomposing matrices of the form `rand(1,1)`, `rand(2,2)`, `rand(3,3)`, ..., `rand(10,10)`. Decompose 10 matrices of each form.
  - Use a scatter plot (MATLAB command `scatter`) to display dimension versus time using the data found in part (a).