MATH/COSC 303

Assignment 4

Due: Apr 1 in LAB, assignments are due at the END of lab.

Hand Written Questions:

1. Consider the IVP $y' = 2y + ty$, $y(0) = 1$.
   a) Show that the IVP has a unique solution.
   b) Show that $y(t) = e^{2t+\frac{1}{2}t^2}$ solves the IVP.
2. Consider the IVP $y' = \frac{3}{2}y^{1/3}$, $y(0) = 0$, $t \geq 0$.
   a) Show that $y(t) = 0$ solves the IVP.
   b) Show that $y(t) = t^{3/2}$ solves the IVP.
   c) Why does this not contradict the "IVP Unique Solution Theorem".
3. Show that when Heun's method is used to solve the IVP $y' = f(t)$ over the interval $[a, b]$ with $y(a) = y(b) = 0$, the result is the Trapezoidal Rule to approximate the integral $\int_a^b f(t)dt$.
4. Consider the IVP $y' = t^2 - y$, $y(0) = 1$.
   a) Create a second order Taylor polynomial to approximate $y(t)$ near 0.
   b) Create a third order Taylor polynomial to approximate $y(t)$ near 0.

BONUS Consider the IVP $y' = y$, $y(0) = 1$. Use Euler's method to prove that

$$\lim_{h \to 0} (1 + h)^{1/h} = e.$$

Computer Assisted Questions:

5. Consider the IVP $y' = 3y + 3t$, $y(0) = 1$ (solution $y(t) = \frac{4}{3}e^{3t} - t - 1/3$).
   a) Approximate $y(1)$ using Euler's Method with $h = 1, 0.1$, and 0.01. Compute the relative error for each approximation.
   b) Approximate $y(1)$ using Heun's Method with $h = 1, 0.1$, and 0.01. Compute the relative error for each approximation.
   c) Approximate $y(1)$ using Runga-Kutta's 4-step Method with $h = 1, 0.1$, and 0.01. Compute the relative error for each approximation.
6. Consider the IVP $y' = -ty$, $y(0) = 1$ (solution $y(t) = e^{-t^2/2}$).
   a) Approximate $y(1)$ using Euler's Method with $h = 1, 0.1$, and 0.01. Compute the relative error for each approximation.
   b) Approximate $y(1)$ using Heun's Method with $h = 1, 0.1$, and 0.01. Compute the relative error for each approximation.
   c) Approximate $y(1)$ using Runga-Kutta's 4-step Method with $h = 1, 0.1$, and 0.01. Compute the relative error for each approximation.
7. Consider the IVP $y' = t^2 - y$, $y(0) = 1$ (solution $y(t) = -e^{-t} + t^2 - 2t + 2$) – see question 4.
   a) Approximate $y(1)$ using a 2nd and 3rd order Taylor Series Method with $h = 1$. Compute the relative error for each approximation.