Orbit Mechanics Tutorial

Exercise 1: Keplerian Orbits in Space-Fixed, Earth-fixed and Topocentric Systems

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3 tutorials in the semester:

1. Keplerian Orbits in Space-Fixed, Earth-fixed and Topocentric Systems
2. Numerical Integration of Satellite Orbits
3. Integration of Satellite Orbits with Different Force Models

Wednesdays in room 2601 from 13:15 to 14:45.
Doubts and code issues.

Satellite to analyse: Sentinel 3, consider 3 orbits and compute the undisturbed and disturbed case.

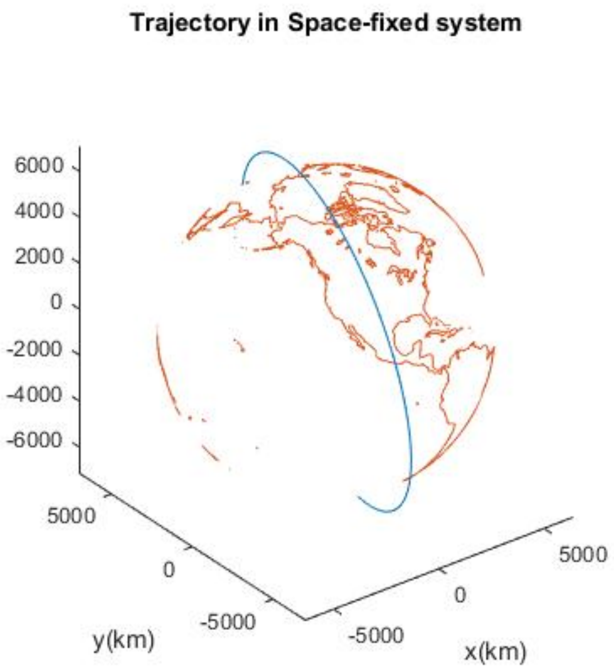
Undisturbed case:
-Ideal situation, where the orbit is a perfect ellipse and only the gravity is considered.

Disturbed case:
-Numerical integration and equations for motion are used.

Keplerian elements:

Satellite	a[km]	e	i[°]	Ω[°]	ω[°]	T_p[sec]
Sentinel-3	7192	0.004	98.3	257.7	144.2	00:00

Task 1: Plot 3 revolutions for the undisturbed case



Functions were already created for the exercise 1!

Load again the Earth_coast() function.

Task 2: Write program yprime

There are many methods to solve numerical integration such as Euler, Runge-Kutta, etc. All of them get errors after some iterations, but some have improvements to minimize this difference.

The most basic case is the Euler's method. Formula to use this time:

$$\ddot{r} = \underbrace{-\frac{GM}{R^3} r}_{\text{Central force of the gravity field}} + \underbrace{F_s}_{\text{Disturbing forces}}$$

Task 2: Write program yprime

There are many methods to solve numerical integration such as Euler, Runge-Kutta, etc. All

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ v_{x_0} \\ v_{y_0} \\ v_{z_0} \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \\ \dot{v}_{x_0} \\ \dot{v}_{y_0} \\ \dot{v}_{z_0} \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ v_{x_1} \\ v_{y_1} \\ v_{z_1} \end{bmatrix} \Rightarrow [\ldots]$$

Initial position and velocity (data from kep2cart!!)	Apply formula from previous slide	New values computed	Repeat the process. Use different time steps. Results are here for 5 and 50 secs

Task 2: Write program yprime

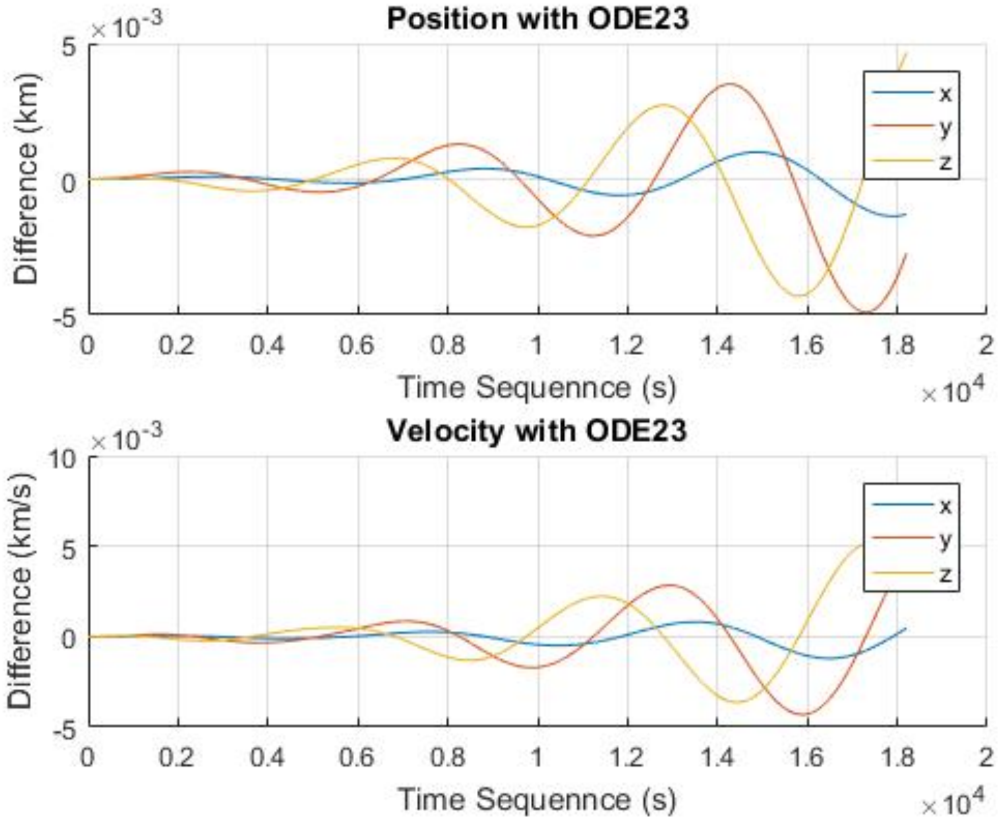
Use also two different integrators from MATLAB like ode23, ode45, ode113 (check MATLAB help for more details).

```
y0=[x0,y0,z0,vx0,vy0,vz0];                      Give initial values
options=odeset('InitialStep',5,'MaxStep',5);    Time steps
[t y]=ode23('yprime',times,y0,options);
```

```
function yp =yprime(t,y)
% Input: position and velocity [r v] (3 components per vector)
% Output: derivative for both vectors [ṙ, v̇]
...
```

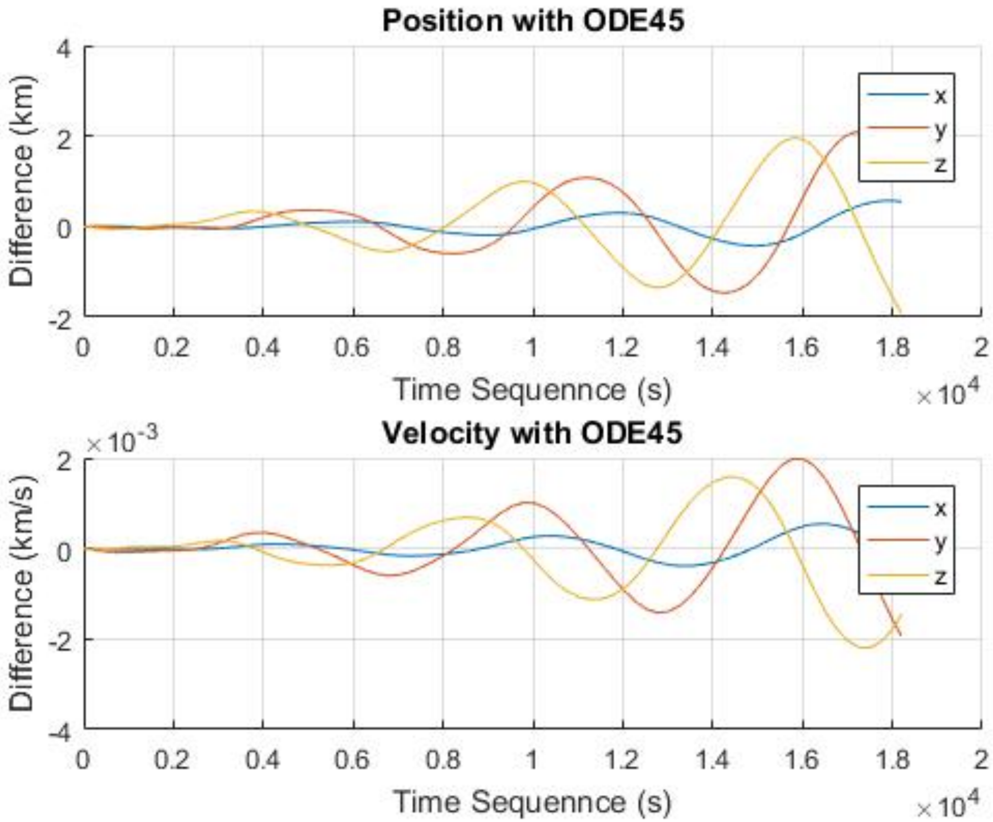
Task 3: Plot the results

Here for 5 seconds



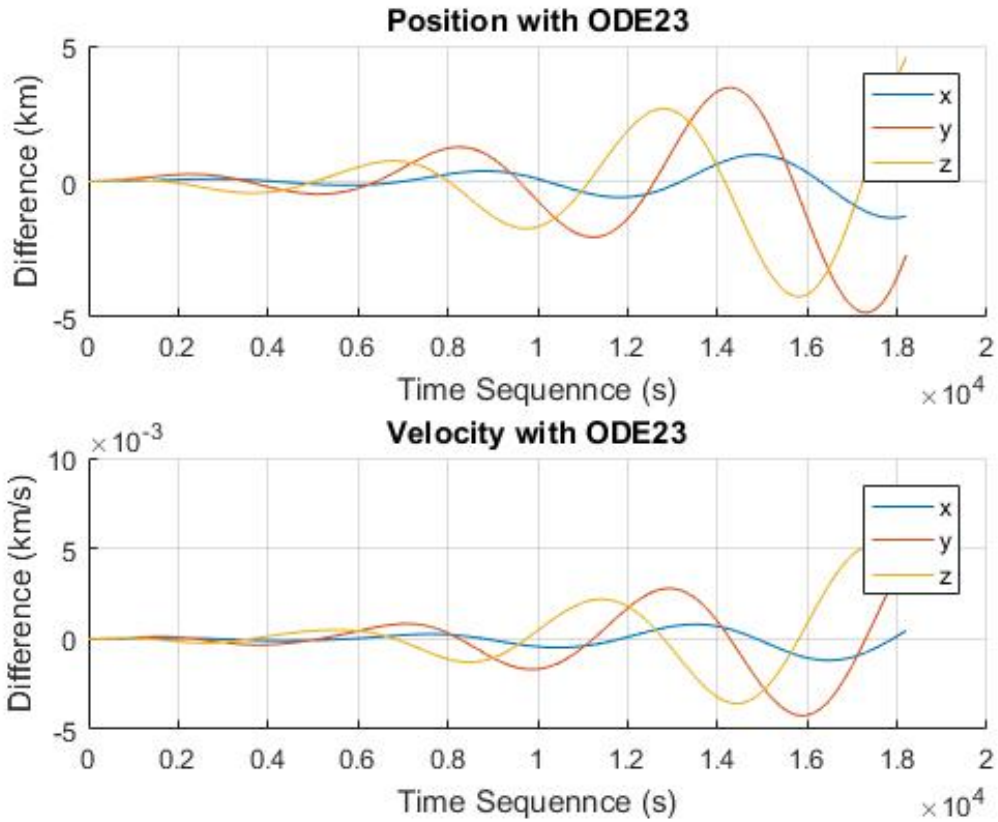
Task 3: Plot the results

Using ode45 for 5 sec.



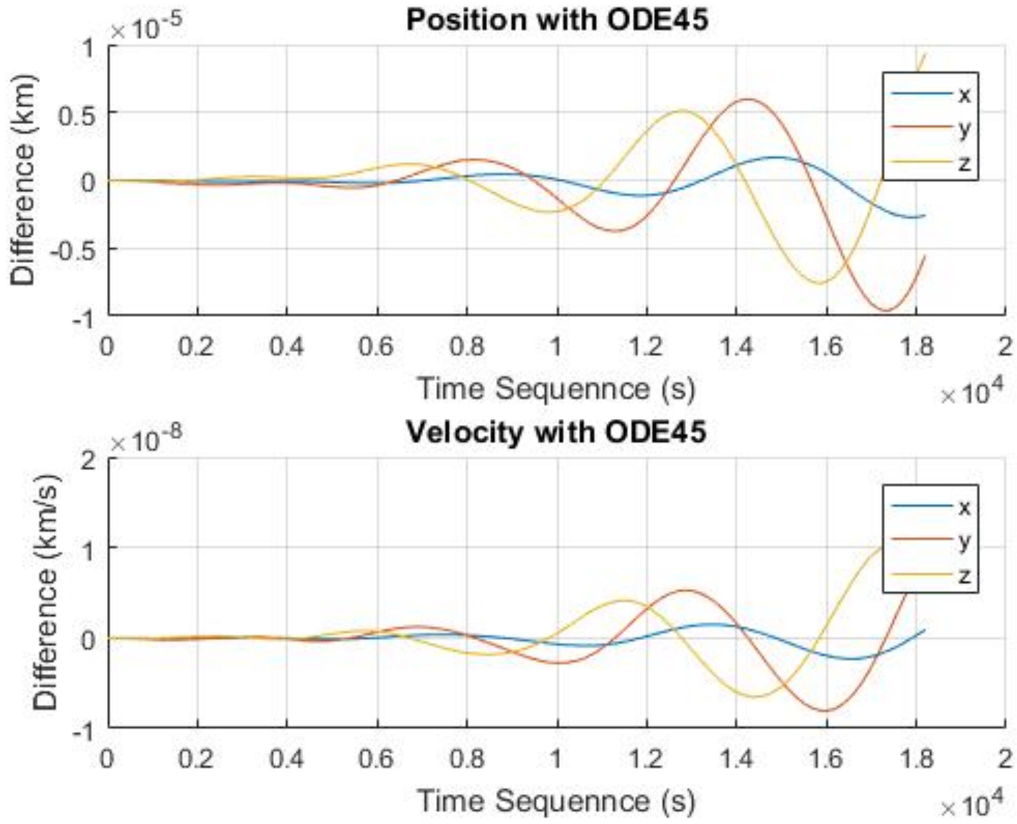
Task 3: Plot the results

Ode23 for 50 seconds



Task 3: Plot the results

Ode45 for 50 seconds



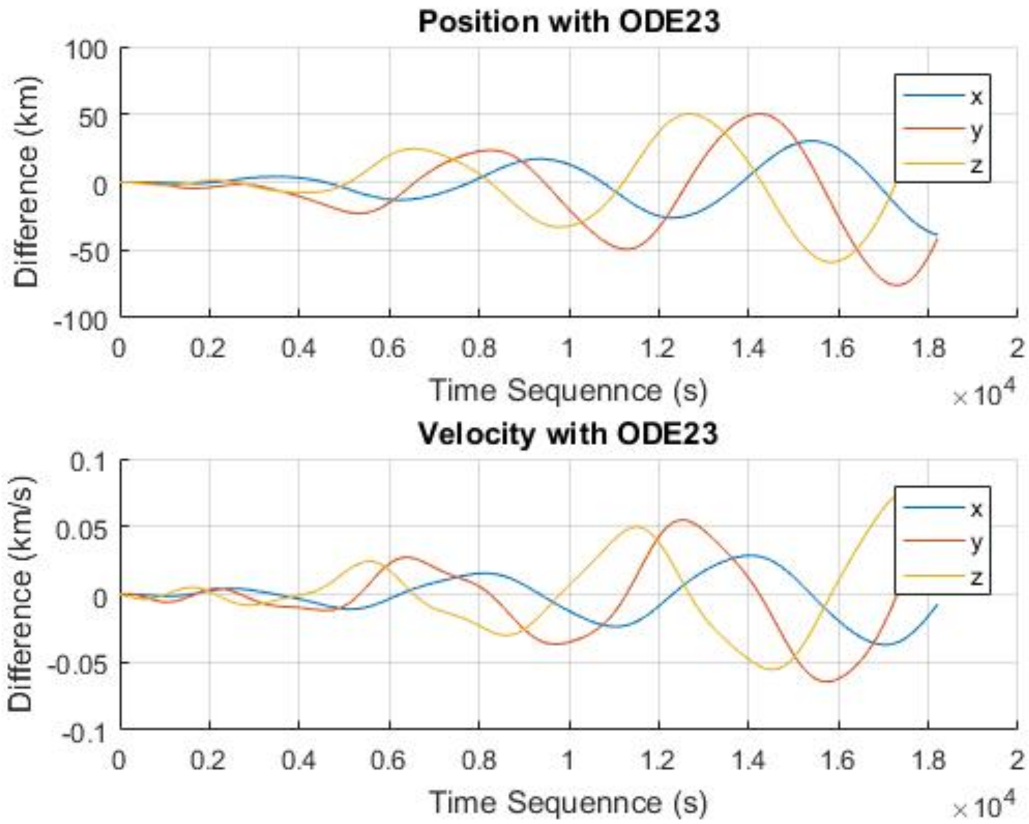
Task 4: Consider the disturbed case

Use the formula given in the lecture:

$$\mathbf{r} = -\frac{GM}{r^3}\left[\mathbf{r} - \frac{3}{2}J_2\frac{a_e^2}{r^2}\begin{pmatrix} 5\left(\frac{z}{r}\right)^2 - 1 \\ 5\left(\frac{z}{r}\right)^2 - 1 \\ 5\left(\frac{z}{r}\right)^2 - 3 \end{pmatrix}\right]$$

Task 4: Consider the disturbed case

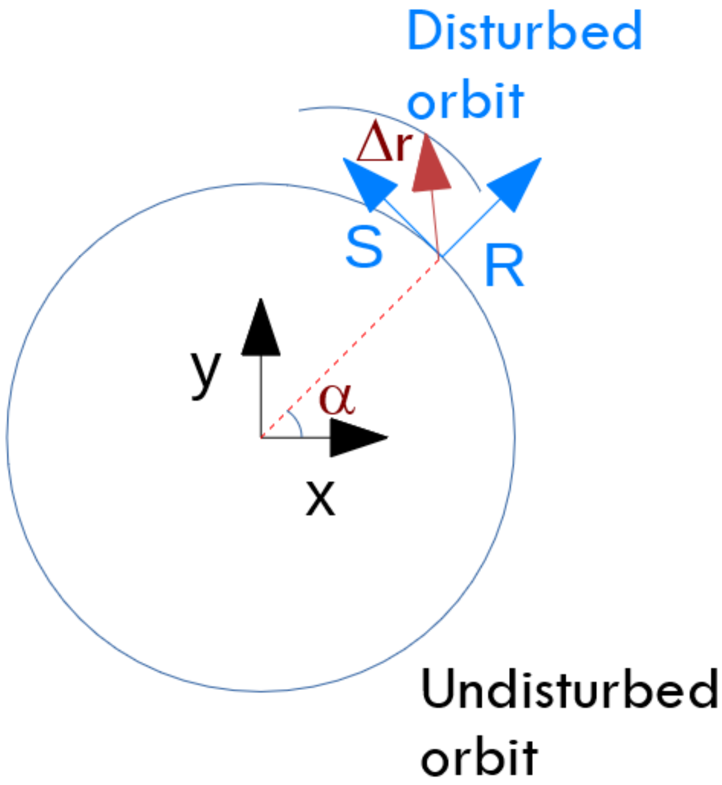
Plot the results. Here ode23 for 5 sec.



Task 4: Consider the disturbed case

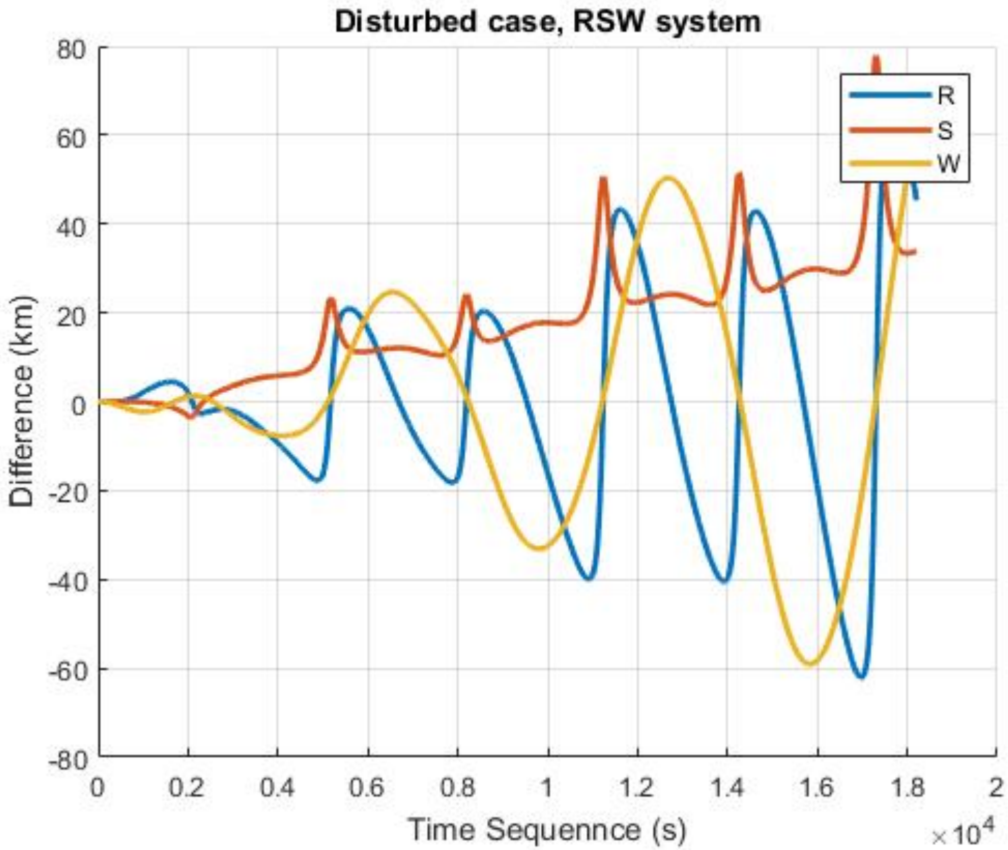
Observe the differences in the RSW system.

1) Compute the base unit vectors of the RSW system based on the unperturbed orbit, e.g. starting from position and velocity vectors
2) Decompose the difference vector between perturbed and unperturbed orbit into this base.



Task 4: Consider the disturbed case

Plot the result. Ode23, step=5.



Task 5: Own integrator for the undisturbed case

Consider both cases: Euler and Runge-Kutta.

Euler

$$y(t_{n+1}) = y(t_n) + \dot{y}(t_n) * \Delta t$$

Runge-Kutta

$$\dot{y}(t_{n+1}) = f(t_n, y(t_n))$$

$$k_1 = f(t_n, y(t_n)) * \Delta t$$

$$k_2 = f(t_n + \frac{h}{2}, y(t_n) + \frac{k_1}{2}) * \Delta t$$

$$k_3 = f(t_n + \frac{h}{2}, y(t_n) + \frac{k_2}{2}) * \Delta t$$

$$k_4 = f(t_n + \frac{h}{2}, y(t_n) + k_3) * \Delta t$$

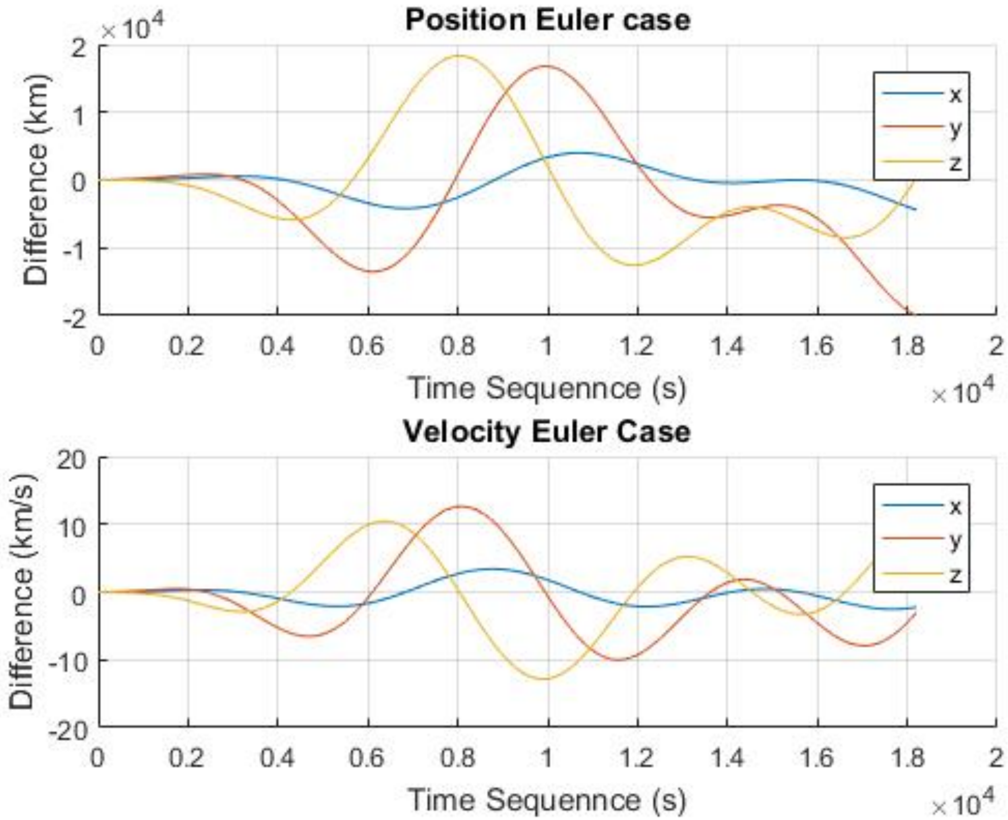
A way to do it in MATLAB:

```
function out=OwnIntegrator('yprime',t,var_0)
```

Where:
out -> computed vectors
'yprime' -> handle function
t-> time vector
var_0-> initial values

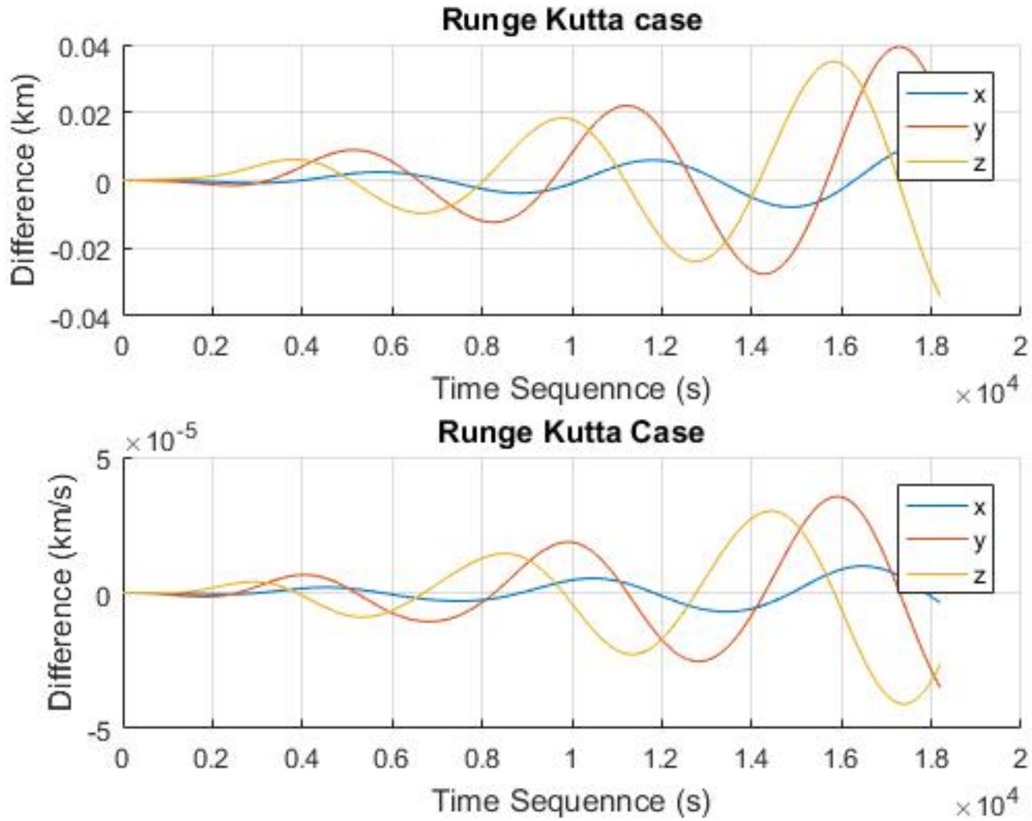
Task 5: Own integrator for the undisturbed case

Result for Euler 50s.



Task 5: Own integrator for the undisturbed case

Result for RK 50s.



To consider

Results will have some variations due to the MATLAB version, internal parameters and the features of your own computer. You will have similar results but don't be afraid if they vary slightly to the ones in the slides or to the ones of your classmates.

Provide more cases in your results as the ones given here, these are just to give a hint of the result, but cases for at least two size steps and two MATLAB methods should be done.