Institute for Astronomical and Physical Geodesy
Technische Universität München

Winter Semester 2017/2018

ESPACE: Orbit Mechanics, Exercise 1

Keplerian Orbits in Space-fixed, Earth-fixed and Topocentric systems

Orbits of several satellites are given in an inertial, geocentric reference system (space-fixed) by the Keplerian orbital elements: semi major axis a, eccentricity e, inclination i, right ascension of the ascending node Ω, argument of the perigee ω, and perigee passing time T_0 on Nov. 13, 2017.

Satellite	**a [km]**	**e**	**i [deg]**	**Ω [deg]**	**ω [deg]**	**T_0 [h]**
GOCE	6629	0.004	96.6	210	144.2	02:00
GPS	26560	0.01	55	30	30	11:00
MOLNIYA	26554	0.7	63	200	270	07:30
GEO	geostationary	0	0	0	50	00:00
MICHIBIKI	geosynchronous	0.075	41	200	270	04:10

For the following computations precession, nutation, polar motion and variations in the length of day are neglected. The Earth fixed reference system then rotates with an angular rate of $\omega_{Earth} = 2\pi/86164s$ about the $\mathbf{e}_3$-axis of the inertial space-fixed reference system. At the time t_0 = Nov. 13, 2017, 00:00 the sidereal angle is 03h 29m.

1) Create a MATLAB-function kep2orb.m that computes polar coordinates r (radius) and ν (true anomaly) based on input orbital elements. Formulate your program in a way that the time t can be used as input parameter.

2) Plot the orbit for the 5 satellites in the orbital plane for one orbital revolution.

3) Plot the mean anomaly M, the eccentric anomaly E, and the true anomaly ν as well as the difference $\nu - M$ for one orbital revolution for the GPS satellite and the Molniya satellite.

4) Create a MATLAB-function kep2cart.m that uses kep2orb.m, which transforms Keplerian elements to position and velocity in an inertial (space-fixed) system.

5) Compute position and velocity vectors of the 5 satellites for a period of one day. Visualize your results. Plot the trajectory in 3D and 2D (projection to x-y, x-z and y-z planes) as well as a time series of the magnitude of velocity.

6) Create a MATLAB-function cart2efix.m that transforms position and velocity in a spacefixed system into position and velocity in an Earth-fixed system.

7) Plot the trajectory of the satellites in 3D for the first two orbital revolutions.

8) Calculate and draw the satellite ground-tracks on the Earth surface.

9) Create a MATLAB-function efix2topo.m that transforms position and velocity in an Earthfixed system into position and velocity in a topocentric system centered at the station Wettzell which position vector in an Earth-fixed system is given by: $\mathbf{r}_W$ = (4075.53022, 931.78130, 4801.61819)^T km.

10) Plot the trajectory of the satellites as observed by Wettzell using the MATLAB-function skyplot.m.

11) Calculate visibility (time intervals) for the satellites at the station Wettzell and visualize them graphically.

Use the following values for your computations.
Geocentric gravitational constant $GM = 398.6005 \cdot 10^{12}$ m^3/s^2
Earth's radius $R_E = 6371 \cdot 10^3$ m

Prepare a written report with a short description of the way how to perform the computations and comment your results. Include the MATLAB-functions kep2orb.m, kep2cart.m, cart2efix.m and efix2topo.m.

Due date for delivery of written report: 04. December 2017

Please send your written report (as .pdf) to:
Mario Fuentes (`mario.fuentes@tum.de`)
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