HW_2.R

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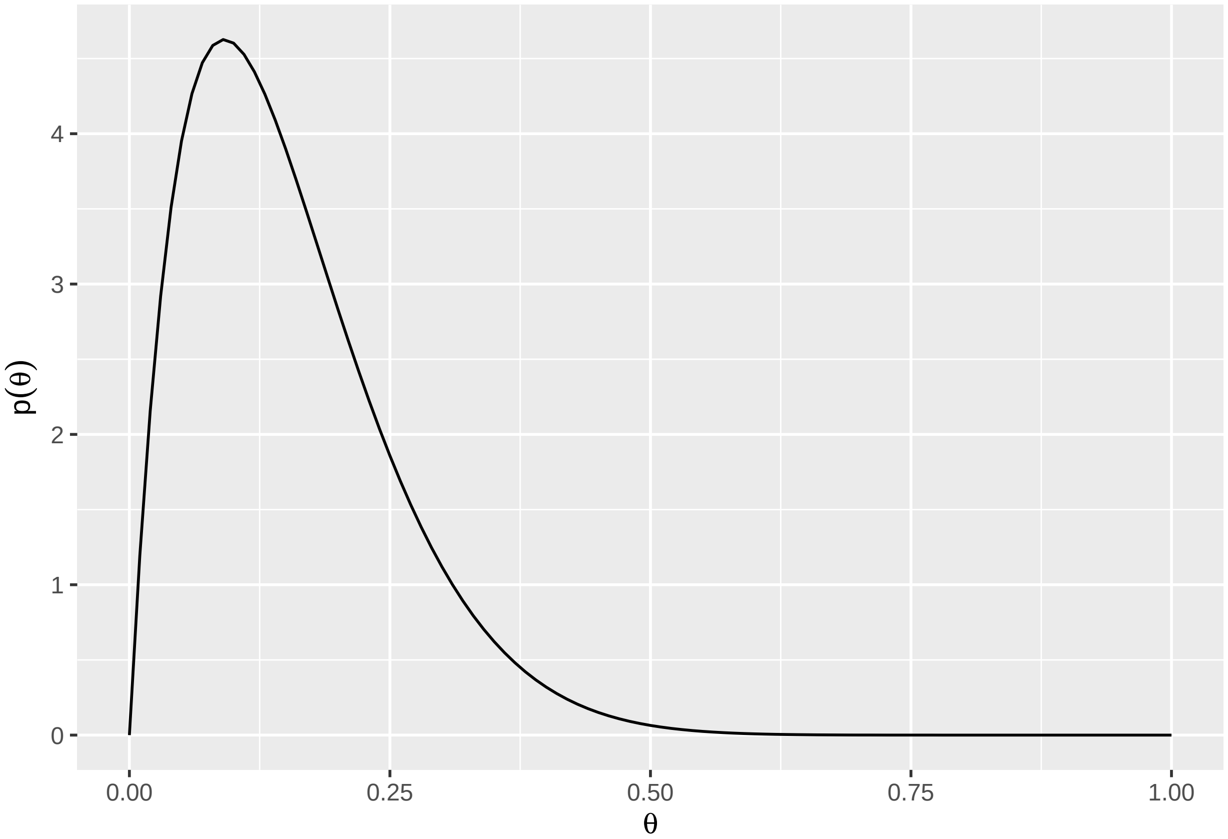
```r
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.5
## v forcats   1.0.0     v stringr   1.5.1
## v ggplot2   3.5.0     v tibble    3.2.1
## v lubridate 1.9.3     v tidyr     1.3.1
## v purrr     1.0.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

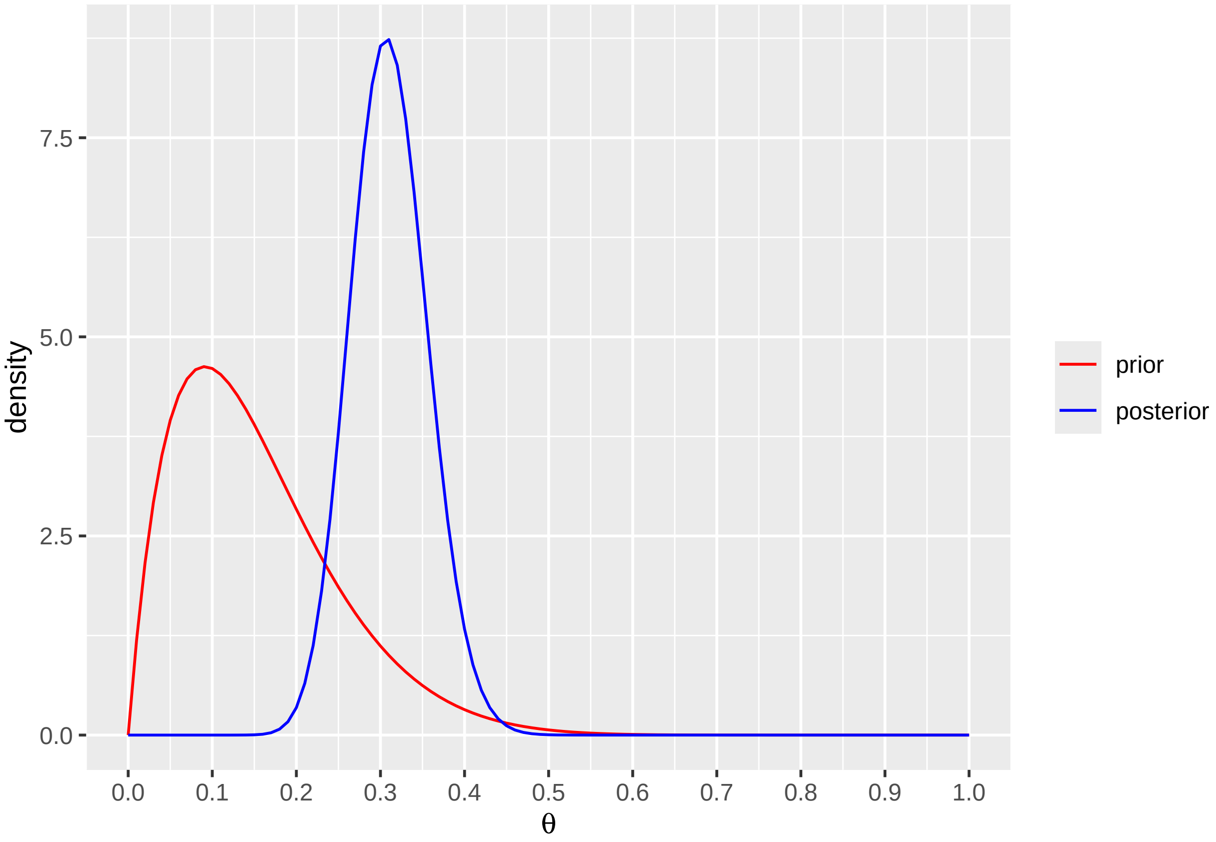
```r
# 5. Beta-Binomial Model
# (a)
# prior parameters
a <- 2
b <- 11
# plot the Beta prior density
xrange <- c(0, 1)
df <- data.frame(xrange)
ggplot(data = df, aes(x = xrange)) +
  geom_function(fun = function(x) dbeta(x, a, b)) +
  labs(x = expression(theta), y = expression(p(theta)))
```



```r
pbeta(0.25, a, b, lower.tail = FALSE) # upper tail probability
```

```
## [1] 0.1583818
```

```r
# (b)
y <- 30
n <- 90

# posterior parameters and inference
a_post <- a + y
b_post <- b + n - y
# plot prior and posterior densities
ggplot(data = df, aes(x = xrange)) +
  geom_function(fun = function(x) dbeta(x, a, b), aes(color = "prior")) + # prior
  geom_function(fun = function(x) dbeta(x, a_post, b_post), aes(color = "posterior")) + # posterior
  scale_x_continuous(breaks = c(0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1)) +
  scale_colour_manual(breaks = c("prior", "posterior"),
                      values = c("red", "blue")) +
  labs(x = expression(theta), y = "density") +
  theme(legend.title = element_blank())
```



```
# (c)
# posterior summaries
a_post/(a_post + b_post) # posterior mean
```

```
## [1] 0.3106796
```

```
(a_post-1)/(a_post + b_post - 2) # posterior mode
```

```
## [1] 0.3069307
```

```
qbeta(0.5, a_post, b_post) # posterior median
```

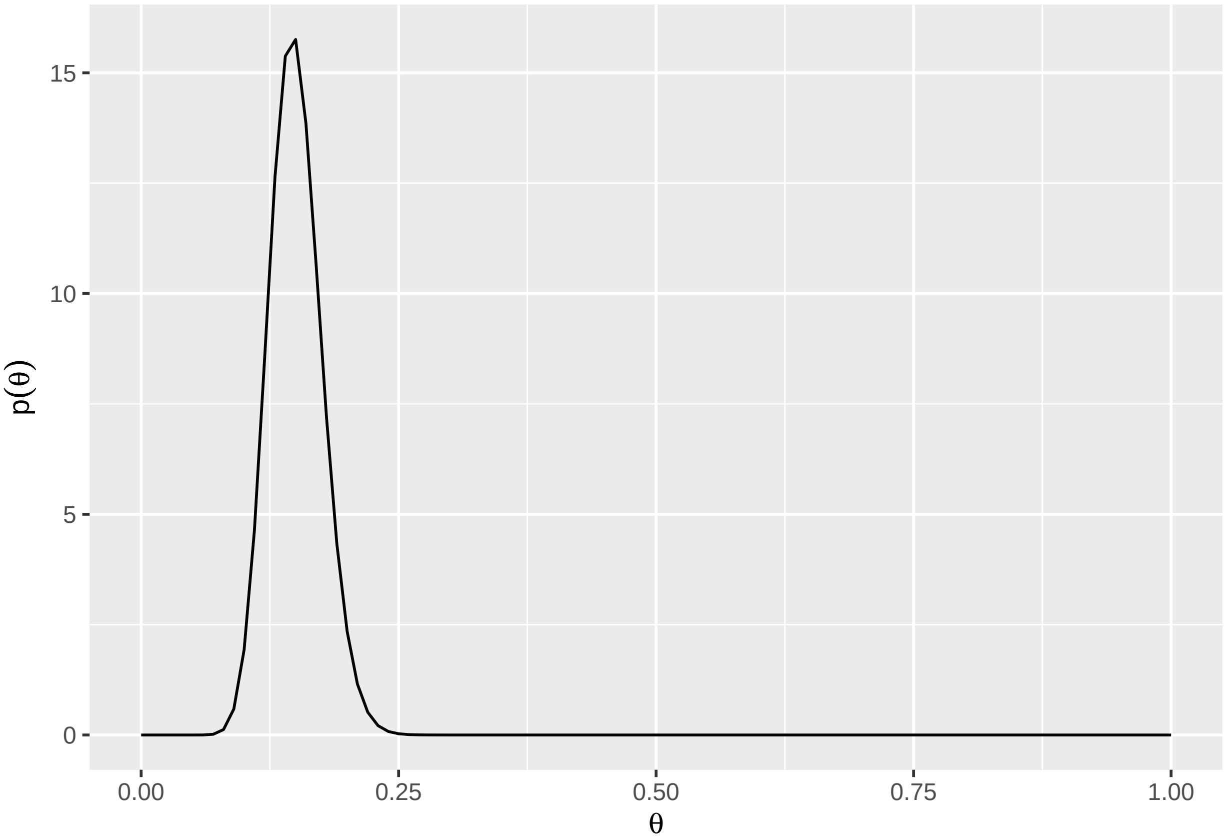
```
## [1] 0.30945
```

```
(((a_post)*(b_post)/(a_post + b_post))^2)*(a_post + b_post + 1)^0.5 # posterior standard deviation
```

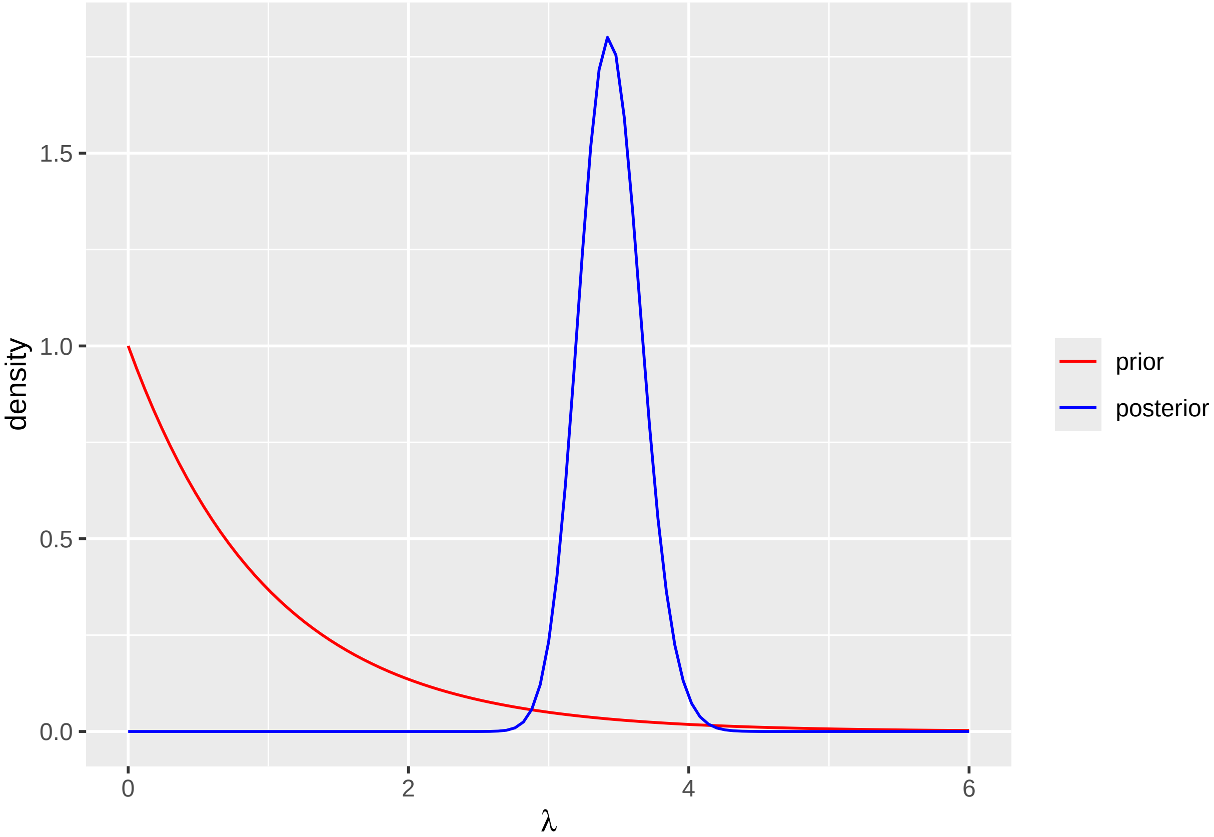
```
## [1] 4962.024
```

```
# 8. Gamma Model

# (a)
# write your R code here
# prior parameters
a <- 30
b <- 170
# plot the Beta prior density
xrange <- c(0, 1)
df <- data.frame(xrange)

ggplot(data = df, aes(x = xrange)) +
```

```r
  geom_function(fun = function(x) dbeta(x, a, b)) +
  labs(x = expression(theta), y = expression(p(theta)))
```



```r
# prior parameters
a <- 1
b <- 1

#===============================================================================

# given data
y <- c(3, 2, 2, 3, 5, 4, 3, 4, 5, 5, 5, 3, 6, 4, 6, 4, 5, 5, 3, 2, 2, 3, 5, 1, 1, 4, 4, 5, 5, 3, 3, 2,

#===============================================================================

# posterior parameters and inference
a_post <- a + sum(y)  #gamma posterior distribution formula
b_post <- b + length(y) #gamma posterior distribution formula
posterior_dist <- function(lambda) dgamma(lambda, shape = a_post, rate = b_post)

xrange <- c(min(y), max(y))
df <- data.frame(xrange)

# plot prior and posterior densities
ggplot(data = df, aes(x = xrange)) +
  geom_function(fun = function(lambda) dgamma(lambda, shape = a, rate = b), aes(color = "prior")) +   #
  geom_function(fun = posterior_dist, aes(color = "posterior")) +   # posterior
  scale_colour_manual(breaks = c("prior", "posterior"),
                      values = c("red", "blue")) +
  labs(x = expression(lambda), y = "density") +
  theme(legend.title = element_blank())
```



```
# posterior summaries
a_post / b_post               # posterior mean
```

```
## [1] 3.442857
```

```
qgamma(0.5, shape = a_post, rate = b_post)    # posterior median
```

```
## [1] 3.438096
```

```
(a_post - 1) / b_post         # posterior mode
```

```
## [1] 3.428571
```

```
a_post / (b_post ^ 2)     # posterior variance
```

```
## [1] 0.04918367
```

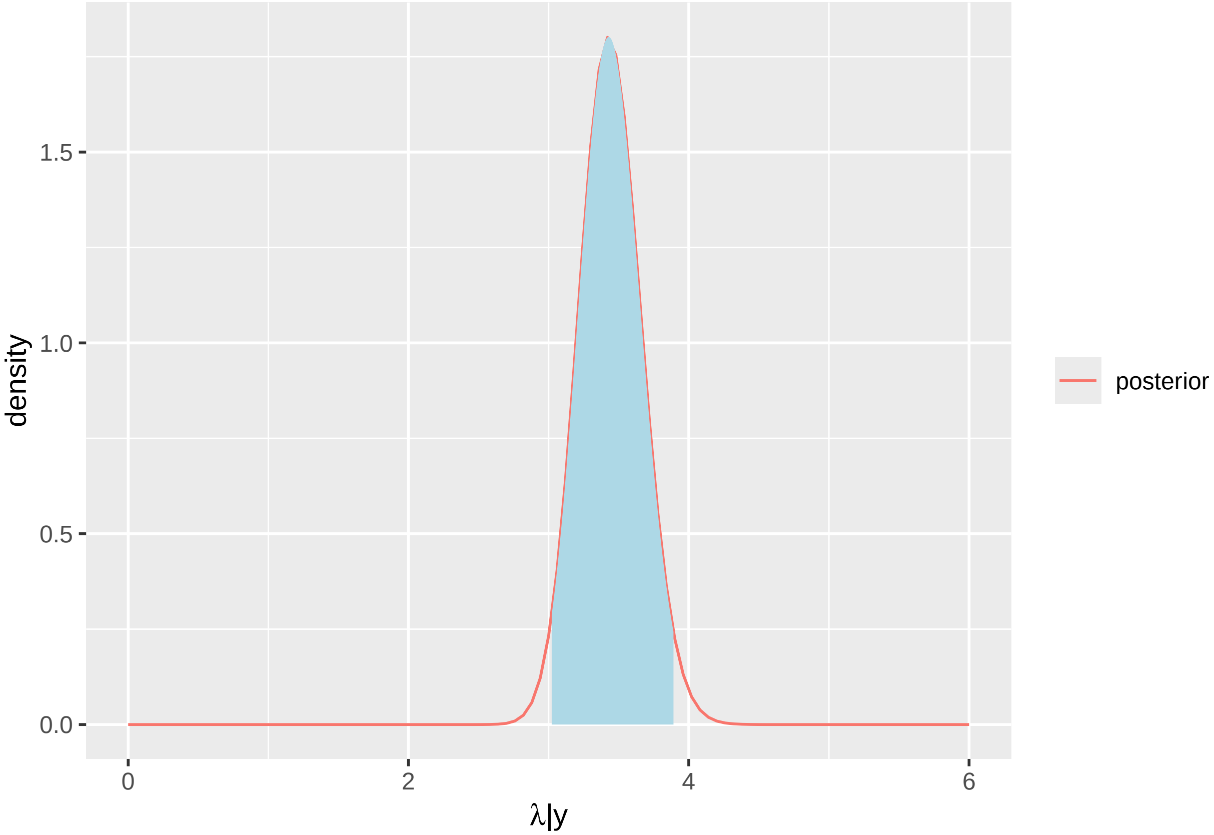
```
# posterior probabilities
1 - pgamma(2, shape = a_post, rate = b_post)    # posterior probability P(lambda|y >= 3)
```

```
## [1] 1
```

```
# 95% equal-tail credible interval
qgamma(c(0.025, 0.975), shape = a_post, rate = b_post)   # lower bound and upper bound
```

```
## [1] 3.021873 3.890895
```

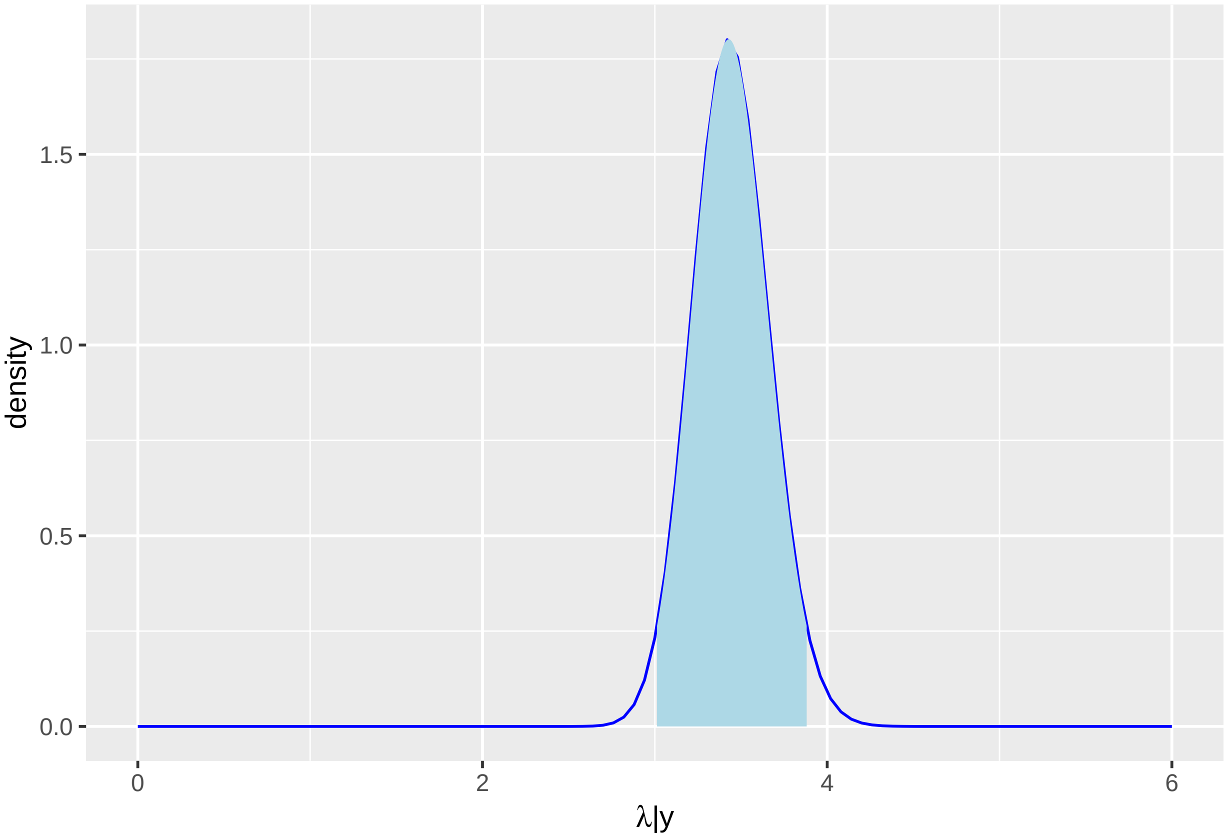
```r
# plot 95% equal-tail credible interval
ggplot(data = df, aes(x = xrange)) +
  geom_function(fun = posterior_dist, aes(color = "posterior")) +    # posterior
  stat_function(fun = posterior_dist,
                xlim = c(qgamma(0.025, a_post, b_post), qgamma(0.975, a_post, b_post)),
                fill = "lightblue", geom = "area") +
  labs(x = expression(paste(lambda, "|y")), y = "density") +
  theme(legend.title = element_blank())
```



```r
# HPD interval (highest posterior density)
library(HDInterval)

hdi(qgamma, 0.95, shape=a_post, rate=b_post)
```

```
##    lower    upper 
## 3.012764 3.880983 
## attr(,"credMass")
## [1] 0.95
```

```r
L <- hdi(qgamma, 0.95, shape=a_post, rate=b_post)[1]    # lower bound
U <- hdi(qgamma, 0.95, shape=a_post, rate=b_post)[2]    # upper bound

ggplot(df, aes(x = xrange)) +
  stat_function(fun = posterior_dist, color = "blue") +
  stat_function(fun = posterior_dist,
                xlim = c(L, U), fill = "lightblue", geom="area") +
  labs(x = expression(paste(lambda, "|y")), y = "density")
```



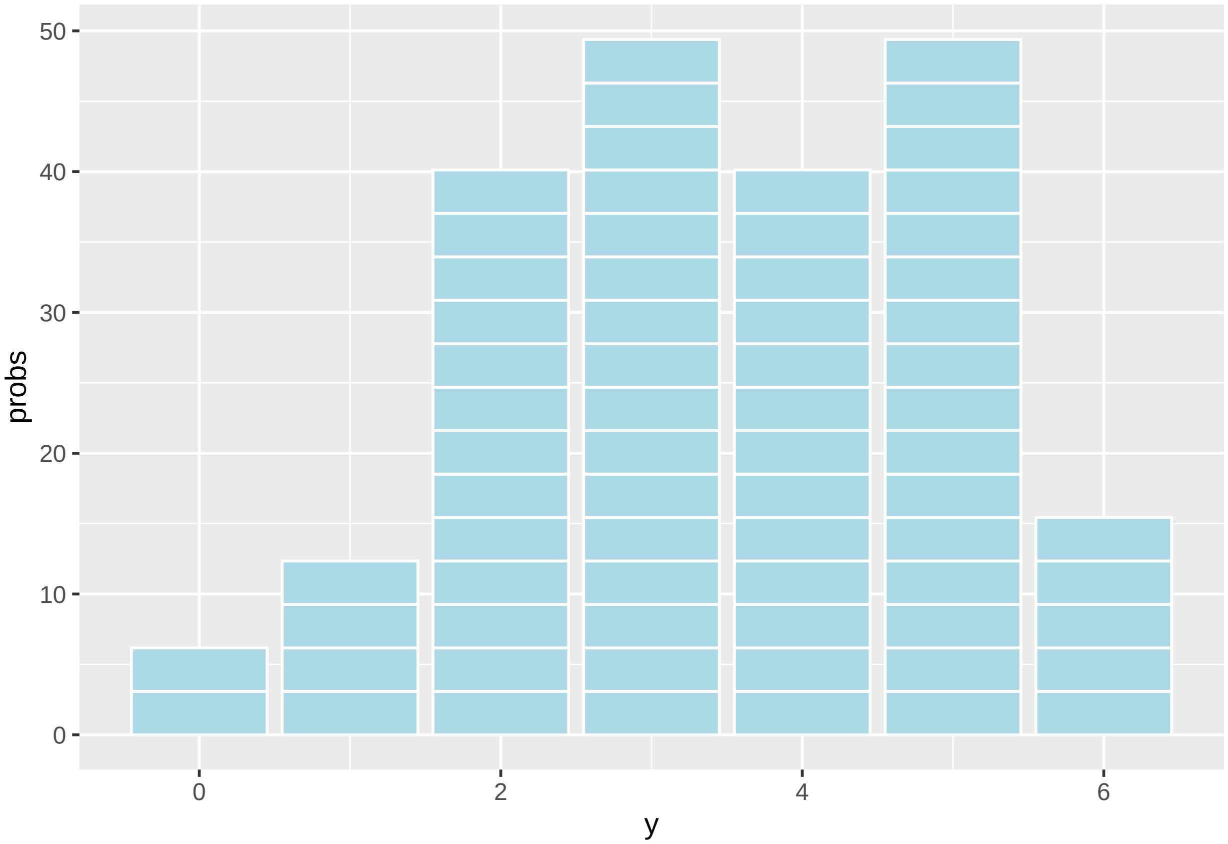
```r
#==============================================================================

# posterior predictive distribution

library(ProbBayes)
```

```
## Loading required package: LearnBayes
## Loading required package: gridExtra
##
## Attaching package: 'gridExtra'
##
## The following object is masked from 'package:dplyr':
##
##     combine
##
## Loading required package: shiny
```

```r
qgamma(0.95, shape = a_post, rate = b_post, lower.tail = FALSE)
```

```
## [1] 3.086372
```

```r
# plot posterior predictive distribution
probs <- qgamma(0.95, shape = a_post, rate = b_post, lower.tail = FALSE)   # posterior parameters, numb
df <- data.frame(y, probs)
ggplot(data = df, aes(x = y, y = probs)) +
  geom_bar(stat = "identity", col = "white", fill = "lightblue")
```


50
40
30
20
10
0
probs
0
2
4
6
y

```
#==========================================================================
```