HW 5

2a. $p(Y_i | \lambda_i) = \dfrac{e^{-(N\lambda_i)} (N\lambda_i)^{Y_i | \lambda_i}}{(Y_i | \lambda_i)!}$ $\propto \dfrac{(N\lambda_i)^{Y_i}}{(Y_i \quad)!}$

$p(\lambda_i | \tau) = \dfrac{\tau}{\Gamma(1)} (\lambda_i | \tau)^0 \cdot e^{-\tau(\lambda_i | \tau)}$ $\propto e^{-\tau(\lambda_i \quad)}$

$p(\tau) = \dfrac{\beta_0^{\alpha}}{\sqrt{\alpha_0}} \tau^{\alpha_0 - 1} e^{-\beta_0 \tau}$ $\propto \tau^{\alpha_0 - 1} e^{-\beta_0 \tau}$

the λ's have similar full conditional distributions

$p(\lambda_j) \propto \left(\prod_{i=1}^{4} \dfrac{(N\lambda_i)^{Y_i}}{(Y_i)!} \right) \cdot e^{-4\tau(\lambda_j)} \cdot \tau^{\alpha_0 - 1} e^{-\beta_0 \tau}$

$$\boxed{\propto \frac{(256\lambda_j)^{171} \cdot (256\lambda_j)^{152} \cdot (256\lambda_j)^{123} \cdot (256\lambda_j)^{139}}{(171)! \quad (152)! \quad (123)! \quad (139)!} \times e^{(\lambda_j)}}$$

$p(\tau) \propto \left(\prod_{i=1}^{4} e^{-\tau(\lambda_i)} \right) \cdot \left(\tau^{\alpha_0 - 1} e^{-\beta_0 \tau} \right)$

$= e^{-4\tau} e^{\sum_{i=1}^{4}(\lambda_i \quad)} \tau^{\alpha_0 - 1} e^{-\beta_0 \tau}$

$= e^{-\tau(4+\beta_0)} e^{\sum_{i=1}^{4}(\lambda_i)}$

$\boxed{\propto e^{-\tau + \sum_{i=1}^{4}(\lambda_i)}}$

3a. $p(y_i | \theta) = \dfrac{1}{y_i!} \left(e^{-\theta M_i} \cdot (\theta M_i)^{y_i} \right)$

$p(\theta) = \dfrac{1}{5\sqrt{2\pi}} e^{-\frac{1}{50}(\theta - 2)^2}$

$p(\theta | y) \propto \left(\dfrac{1}{5 y_i! \sqrt{2\pi}} (\theta M_i)^{y_i} e^{-\frac{1}{50}(\theta-2)^2 - \theta M_i} \right)$

plugging in y_1, y_2, M_1, M_2

$\rightarrow = \dfrac{1}{5 \cdot 5! \sqrt{2\pi}} (\theta 1)^5 e^{-\frac{1}{50}(\theta-2)^2 - \theta} \cdot \dfrac{1}{5 \cdot 8! \sqrt{2\pi}} (\theta 2)^8 e^{-\frac{1}{50}(\theta-2)^2 - 2\theta}$

$= \dfrac{2^8}{5! 8! 50\pi} \theta^{13} \cdot e^{-3\theta - \frac{1}{25}(\theta-2)^2}$

$\boxed{\propto \theta^{13} e^{-3\theta - Y_i (\theta - 2)^2}}$ actually, let's not substitute the data in so that we can get a general function for the code, so $\boxed{\propto (\theta M_1)^{y_1} \cdot e^{-\theta \cdot M_1} \cdot (\theta M_2)^{y_2} \cdot e^{-\theta \cdot M_2} \cdot e^{-\frac{1}{50}(\dots)}}$