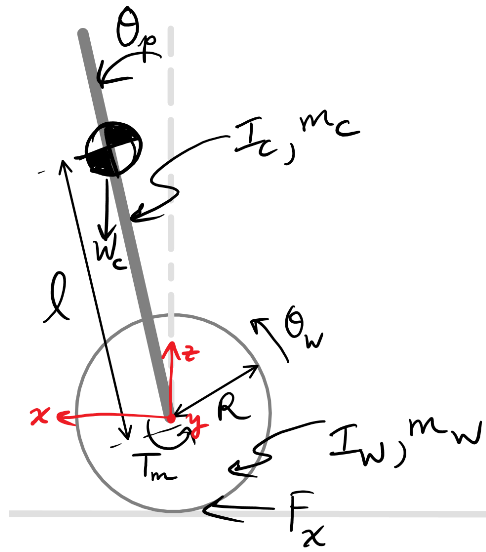
Vehicle Parameters:

Stepper Motor Mass, m_{SM}	300 g (each)
Motor Rotor Inertia, I_R	54 g·cm^2 (each)
Tire Mass, m_T	52 g (each)
Tire Inertia, I_T	520 g·cm^2 (each)
Tire Radius, R	4 cm
Frame Mass, m_F	160 g
Arduino and Breadboard, m_A	100 g
Battery Mass, m_B	120 g



Combined Parameters:

Chassis Combined Mass, m_C	980 g
Chassis Combined Inertia about Axle, I_C	92,336 g·cm^2
Distance from Axle to CG of Chassis, ℓ	6 cm
Wheel and Rotor Combined Mass, m_W	150 g
Wheel and Rotor Combined Inertia, I_W	1150 g·cm^2

The linearized equation of motion for the self-balancing robot is given by

$$\alpha\ddot{\theta}_p - \beta\theta_p = -\gamma\ddot{\theta}_w \tag{1}$$

where

1. $\alpha = I_C + m_C R\ell$
2. $\beta = m_C g\ell$
3. $\gamma = I_W + (m_W + m_C)R^2$

Project Deliverables:

1. Derive the A_c and B_c matrices from the continuous-time state-space model for the system, assuming that the system states are given by
$$\mathbf{x} = \begin{bmatrix} \theta_p \\ \dot{\theta}_p \end{bmatrix} \tag{2}$$

2. Using your result from the previous part, derive the A and B matrices for the discrete-time state-space model, assuming that the sampling frequency is $f_s = 50$ Hz.

3. Add the wheel velocity, $\dot{\theta}_w(k)$ as an additional state to your discrete-time state-space model, where
$$\dot{\theta}_w(k) = \dot{\theta}_w(k-1) + \ddot{\theta}_w(k-1) \cdot T \tag{3}$$
Call your new A and B matrices A_{AUG} and B_{AUG} (AUG for augmented), respectively.

4. Now, define
$$\ddot{\theta}_w(k) = -K \begin{bmatrix} \theta_p(k) \\ \dot{\theta}_p(k) \\ \dot{\theta}_w(k) \end{bmatrix} = -K\mathbf{x}_{AUG} \tag{4}$$
and calculate K such that the closed-loop system settling time is less than one second. (There is not one unique solution: there is design flexibility here!)

5. Next, we need to design an estimator to give $\hat{\theta}_p(k)$, $\hat{\dot{\theta}}_p(k)$, and $\hat{\dot{\theta}}_w(k)$ at each time step. We measure $\dot{\theta}_p(k)$ with a gyroscope, and let us assume that we also measure $\dot{\theta}_w(k)$ (we know the wheel velocity fairly accurately because we regulate the stepper motor timing). Therefore, assume that our measurement equation is
$$\mathbf{y}(k) = \begin{bmatrix} \dot{\theta}_p(k) \\ \dot{\theta}_w(k) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_p(k) \\ \dot{\theta}_p(k) \\ \dot{\theta}_w(k) \end{bmatrix} = C_{AUG}\mathbf{x}_{AUG} \tag{5}$$
Given C_{AUG}, use the `place` command in MATLAB to calculate the L matrix so that the state estimation error has an overall settling time of less than 0.25 s. (The reason we use the `place` command instead of `acker` is that `acker` only works for systems with one output. Here, we have two outputs.)

6. Given your controller and your estimator, simulate the closed-loop system response in MATLAB. Assume that the true initial state vector is
$$\mathbf{x}_{AUG}(0) = \begin{bmatrix} \theta_p(0) \\ \dot{\theta}_p(0) \\ \dot{\theta}_w(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.2 \\ 0 \end{bmatrix} \tag{6}$$
and that the estimated initial state vector is
$$\mathbf{x}_{AUG}(0) = \begin{bmatrix} \hat{\theta}_p(0) \\ \hat{\dot{\theta}}_p(0) \\ \hat{\dot{\theta}}_w(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$
For this simulation, include the following plots:

(a) The three estimated states as a function of time.

(b) The state estimation error, $\tilde{\mathbf{x}}(k) = \hat{\mathbf{x}}(k) - \mathbf{x}(k)$, as a function of time.

Be sure to print out and attach any code that you use to complete this homework!