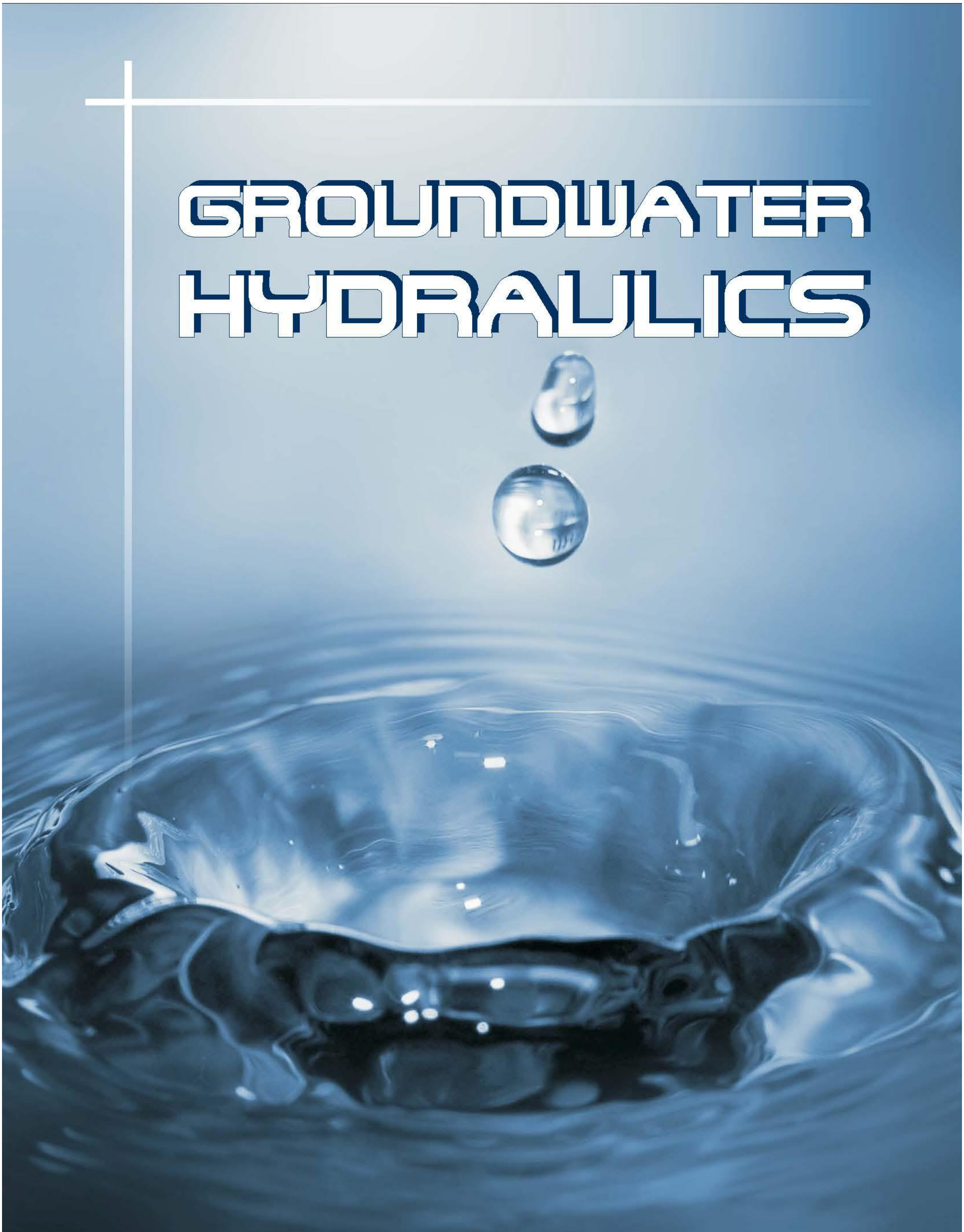
GROUNDWATER HYDRAULICS



C.P. HAZEL

GROUNDWATER HYDRAULICS

by

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Second Edition

June 2009

Lectures presented by C.P.Hazel of the Irrigation and Water Supply Commission, Queensland to the Australian Water Resources Council's Groundwater School, Adelaide in 1973. The lectures were presented initially in Imperial units at the 1973 Groundwater School and then converted to metric units for use at the 1975 and subsequent Groundwater Schools. They were retyped and updated in 2009.

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PREFACE

We have become accustomed to the belief that the most common source of useful water on our planet is from rivers, streams, lakes and dams; a minor source is less obvious being underground. It is, however, extremely important. At any one time less than three percent of the available freshwater on our planet is stored above ground, and more than 97% is stored below ground.

Surface water by virtue of its occurrence, is more easily understood. Groundwater, on the other hand, because of its hidden nature, is shrouded with mystery and superstition.

Minor, though its use may be, it is extremely important, particularly in many parts of Australia where the annual rainfall is strongly seasonal, extremely variable and associated with very high evaporation, and suitable surface water storage sites are not all that abundant.

The science of Groundwater Hydrology is concerned with the occurrence, availability and quality of groundwater. Although many groundwater investigations are qualitative in nature, quantitative studies are necessarily an integral part of the complete evaluation of occurrence and availability. The worth of an aquifer as a source of water depends largely on two inherent characteristics - its ability to store water and to transmit water. As time progresses, its ability to mix waters of different qualities will also become more important.

There are very few places on this planet where one could drill a hole and not encounter water. In many cases, however, it may not be possible to extract water at a useful rate. The problem then is not so much to locate groundwater, but to find a geologic formation which is capable of storing and transmitting the water in useful quantities. Thorough knowledge of the geologic framework is essential before one can hope to understand the operation of the natural plumbing system within it. Fortunately, most workers in the field of groundwater are geologists and geology will only be touched upon incidentally in these notes as it relates to some quantitative problem.

These notes give a brief coverage of the types of groundwater and the properties of water and water bearing materials, but the main emphasis is on the hydraulic properties of the aquifers and the evaluation of groundwater systems.

The principal method of analysis in groundwater hydraulics is the application of equations derived for particular boundary conditions. These are generally applied to field tests of discharging bores. Prior to 1935, such equations were known only for the relatively simple steady-state flow conditions, which incidentally do not occur in nature. The development by Theis (1935) of an equation for the non-steady state flow of groundwater was a milestone in groundwater hydraulics. Since 1935 the number of equations and methods has grown rapidly and steadily. These are described in a wide assortment of publications, some of which are not conveniently available to many engaged in groundwater studies. The essence of many of these will be presented and discussed, but frequent recourse should be made to the more exhaustive treatment given in the references cited. Indeed many more papers have been written since these notes were first prepared and no attempt has been made to include them.

In the years since 1973, when these notes were first prepared, the application of electronic computers has simplified the solution of many difficult equations and computer programmes for most of the analyses presented in this document can be found on the internet. However, I would like to stress the importance of knowing how the manual solutions are carried out before merely accepting the output of a computer programme.

Also since 1973, groundwater modelling has developed significantly and is now used extensively for the solution of many groundwater problems. However, it is very dangerous to attempt to build a groundwater model without a sound understanding of the physical properties and laws which control groundwater storage and movement. When developing a groundwater model, carry out a rough check on the output. If the output is what you would expect and is based on reasonable physical parameters of the aquifer material then use it to determine the temporal and spatial response of the aquifer to the varying stresses placed upon it. If the output is not in accordance with what simple theory predicts or is only able to be calibrated by using unrealistic values for the aquifer parameters, then go back to the drawing board and rebuild it.

To bring out the essential matters and relations and to give a better understanding of the applications and of the limitations of the equations, worked examples and full derivations with their assumptions have been included.

In the limited time available I am able to present only a limited number of analytical methods. However, a more complete coverage of the techniques available is presented in the extremely useful reference "Analysis and Evaluation of Pumping Test Data" by G.P. Kruseman and N.A. de Ridder. The first edition of this reference was first published in 1970. A second edition was published in 1990.

In preparing these notes I have made use of both the above mentioned publication and of the lectures presented by S.W. Lohman to the Australian Water Resources Council Groundwater School, 1967.

I first prepared these notes in 1973 for presentation to the 4th Australian Water Resources Council Groundwater School which was held in Adelaide. They have been used extensively at many Groundwater Schools since that time. I retyped and updated them slightly in 2009 but they are essentially the same as the original notes. I am grateful to the staff of Matrixplus Consulting for formatting the document for me.

Apart from some additions for clarification, the only updates which I have included are minor and deal with flow through anisotropic media, a little more on the relationship between hydraulic conductivity and intrinsic permeability and a brief introduction to the use of groundwater hydraulics in mine dewatering.

Colin P. Hazel

June 2009

SECTION 1: PROPERTIES OF WATER AND WATER BEARING MATERIALS

1.1 INTRODUCTION

The science of Groundwater Hydrology is based upon the fundamental properties of firstly, water itself and secondly, the media through which it moves. At the risk of boring those people who have this basic knowledge at their fingertips, definitions and explanations of terms fundamental to this study are given here. If it serves no other purpose, this will at least collect the terms in a readily accessible place.

1.2 FLUID MECHANICS

It will be recalled that in the first consideration of solid objects they were assumed to be characterised by complete rigidity, i.e. by their ability to transmit shearing stresses.

Water, of course, is not a solid object (if frozen to ice it would be) but is a fluid and we define an ideal fluid as a substance which is incapable of transmitting shearing stresses. Fluids can be divided into gases and liquids but we shall deal specifically with liquids.

The form or shape of any given mass of a liquid is quite indefinable as it conforms to the shape of the containing vessel. However, it does possess a definite volume at a definite temperature and pressure. Liquids (particularly water) are but slightly compressible.

It is true that no real fluid can meet exactly the conditions of an ideal fluid since all fluids exert some shearing stresses. For the moment we will, however, neglect this factor in our consideration of fluids at rest.

1.2.1 Hydrostatics

Hydrostatics is simply the study of fluids at rest. The following definitions are applicable to fluids in general.

However, water is the fluid of prime concern, and the specific properties of water should be borne in mind. Water is the only substance which can exist as solid, liquid or gas at atmospheric pressure.

Mass (or Inertia) (M) is the tendency of a body to resist a change of velocity. It determines the acceleration of a body in response to a given force. Mass has often been explained in terms of "matter" but as the latter lacks any satisfactory definition, so then does the former in terms of it. Mass is a fundamental, easily understood but not so easily adequately defined.

The unit of mass is the kilogram (kg).

Density (ρ) of a substance is its mass per unit volume. It varies with pressure and temperature.

$$\rho = M/V \qquad .....1.1$$

The unit of density is kilogram/cubic metre (kg/m^3). The density of water at 4°C is 1000 kg/m^3.

Densities at other temperatures from 0°C to 100°C are given in **Figure 1-7**.

Specific Gravity (S.G.) or *Relative Density* (R.D.) of a substance is the ratio of its density to the density of water.

Being a ratio, it is dimensionless.

Displacement (s) is a change in position in a specified direction. It is then a vector quantity.

The unit of displacement is the metre (m).

Velocity (v) is the quantitative description of the motion of a body. Since it is related to direction as well as speed of motion, it is a vector quantity. It is a rate of change of position in a specified direction.

Instantaneous velocity = ds/dt

The units of velocity are metres per second (m/sec), or metres per day (m/day).

Acceleration (a) is the time rate of change of velocity, and is a vector quantity.

Instantaneous acceleration = dv/dt

The units of acceleration are metres per second per second (m/sec^2).

Acceleration due to gravity (g) is the acceleration produced on a body by the earth's gravitational field and for most purposes is assumed to be constant at 9.80 m/sec^2. It does, however, vary from place to place on the earth's surface.

Force (F) is that which produces or tends to produce a change in the state of motion of a body.

By Newton's second law of motion:

$$F = Ma$$

The force exerted on a body by the earth's gravitational attraction is referred to as its weight *(W)*:

$$W = Mg \qquad .....1.2$$

The weight of a unit volume is referred to as the specific weight (γ):

$$\gamma = \rho g$$

The adopted unit of force is the Newton, defined as that force which when acting on a mass of 1 kg will produce an acceleration on it of 1 m/sec^2.

A dyne is defined as the force required to move a mass of 1 gm with an acceleration of 1 cm/sec^2.

The weight of a body having a mass of 1 kilogram is then 9.80 newtons (N).

Pressure (p) is defined as the perpendicular force per unit area exerted on a surface with which a fluid is in contact.

The surface with which the fluid is in contact may, of course, be either a solid boundary or an imaginary plane passed through the fluid for purposes of analysis.

$$p = F/A \qquad .....1.3$$

The adopted unit for pressure is the pascal (Pa), which is defined as 1 newton per square metre (1 N/m). The common unit will be the kilopascal (kPa) which is 1000 pascals.

Strictly speaking, the magnitude of pressure should be expressed in terms of pascals above absolute zero. However, it is generally more convenient to use atmospheric pressure as a reference, the relative intensity p then representing the difference between absolute intensity p_{abs} and atmosphere intensity p_{at}.

i.e. $$p = p_{abs} - p_{at} \qquad .....1.4$$

Under normal conditions, p_{at} = 101 kilopascals (k*Pa*).

= 1010 millibars (m*b*) in meteorology.

Pressure may also be expressed in terms of the number of metres of water (or mercury) that a certain pressure would support. Hydraulic head is measured in metres.

The pressure at a point A in a fluid h metres below the surface is given by:

$$p = \rho g h \qquad .....1.5$$

= a head of h metres of water

where:

p = pressure in pascals.

ρ = density in kg/m^3.

h = depth in metres.

g = acceleration due to gravity = 9.8 m/sec^2.

p_{at} is then equal to a head 10.3 metres of water.

Pressure is the same at all points in the same horizontal plane within the fluid at rest. Any increment of pressure applied at any point in a confined fluid is at once transmitted equally to all parts of the fluid.

1.2.2 Hydrodynamics

Hydrodynamics is the study of fluids in motion.

This section deals with the motion of fluids and the various types of flow which may be encountered.

Steady Flow

Consider a fluid in motion in **Figure 1-1** at a given time a particle of fluid at a given point, a, will have a particular velocity v_1, at b a velocity v_2, and at c at velocity v_3. If, at all other times, the velocity of whatever particle of fluid is at the point a, remains constant at v_1, that at b remains v_2 and that at c remains v_3, then the flow is said to be steady.

The path followed by a fluid particle in steady flow is called a streamline. Streamlines are characterised by the property that the tangent at any point on the streamline gives the direction of flow of the fluid at that point. Particles of fluid may not flow from one streamline to another.

A bundle of similar streamlines is called a tube of flow e.g. steady flow in a pipe.

Non-Steady Flow

When the velocity of the particular fluid particle at a given point in a moving fluid varies with time, the flow is described as non-steady.

Continuity Principle

Consider steady flow in a pipe of varying cross-sectional area as in **Figure 1-2**. Let the velocity of flow normal to plane A be v_1 and at B be v_2. Let the cross-sectional area of the pipe at A be A_1, and at B be A_2.

Since the fluid is assumed to be incompressible the mass of fluid passing A in a given time t must equal the mass passing B in the same time, or there would be an accumulation of mass between A and B, i.e. mass passing A in time t = mass passing B in time t:

$$v_1 A_1 t = v_2 A_2 t$$

or

$$v_1A_1 = v_2A_2 \qquad .....1.6$$

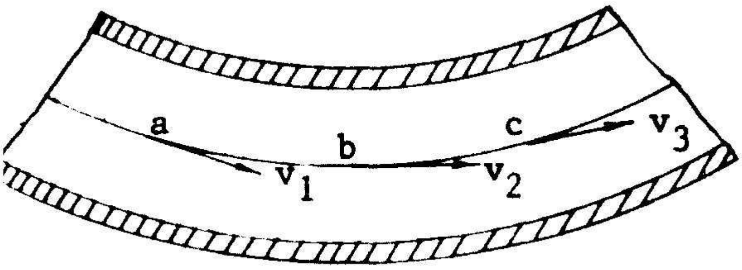


Figure 1-1 Steady flow

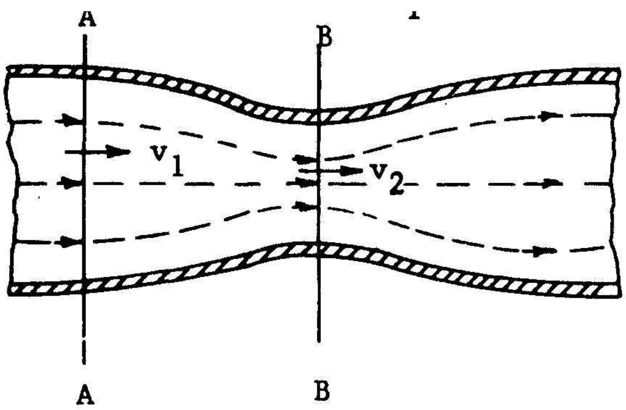


Figure 1-2 Continuity principle

Thus in the case of steady flow in a pipe of varying cross-section, the highest velocity occurs at the smallest section.

Since the fluid undergoes an acceleration in moving from A to B, from the second law of motion (F = Ma) the pressure at A must be higher than the pressure at B.

This principle is used in the venturi which is used to measure flows in pipes. Small tubes tapped into a constricted pipe at positions such as A and B enable the difference in pressure to be recorded. This pressure difference is proportional to the velocity squared, so the meter can be calibrated to read flow directly.

Bernoulli's Theorem

By the application of the principle of conservation of energy to the flow of a fluid in a tube of flow, Bernoulli's Theorem can be derived, viz:

$$\frac{p}{\rho g} + \frac{v^2}{2g} + h = constant \qquad .....1.7$$

or is sometimes written:

$$\frac{p_1}{\rho g} + \frac{v_1^{\,2}}{2g} + h_1 = \frac{p_2}{\rho g} + \frac{v_2^{\,2}}{2g} + h_2 + h_f \qquad .....1.8$$

where:

p = pressure.

ρ = density of fluid.

g = acceleration due to gravity.

v = velocity.

h = potential or elevation head above datum level.

h_f = head lost in overcoming friction when the fluid moves from the point 1 to point 2.

It must be emphasised that Bernoulli's Theorem is strictly applicable only to streamline flow.

In equation 1.7:

$p/\rho g$ = the pressure head.

$v^2/2g$ = the velocity head.

h = the elevation head.

The total head at any point is the summation of these three terms.

The summation of the elevation head and pressure head gives the potentiometric head, and the concept is useful in analysing flows in pipes and flow in underground water.

A popular misconception is that flow takes place from areas of high pressure to areas of low pressure. This is not so. Flow occurs from areas of high potentiometric head to areas of low potentiometric head. With a suitable arrangement of elevation heads it is possible for a fluid to flow from an area of low pressure to one of high pressure, e.g. the pressure in a storage tank located on a hill is smaller than the pressure in a pipeline some distance down the hill. However, because of the greater elevation on the hill, the potentiometric head is greater at the tank site and water is delivered along the pipe.

Viscosity

So far, only "ideal fluids" have been considered, i.e. fluids which cannot transmit shearing stresses and on which no work is done in changing their shape.

In actual fact, no fluid is ideal and all possess, to some degree, the property of viscosity or internal friction.

The coefficient of dynamic viscosity or absolute viscosity (η) is defined as the ratio of the intensity of shear, τ, to the rate of deformation.

i.e. $\eta = \dfrac{\tau}{dv/dy}$ 1.9

where:

η = dynamic viscosity.

τ = intensity of shear.

dv/dy = velocity gradient in the transverse direction.

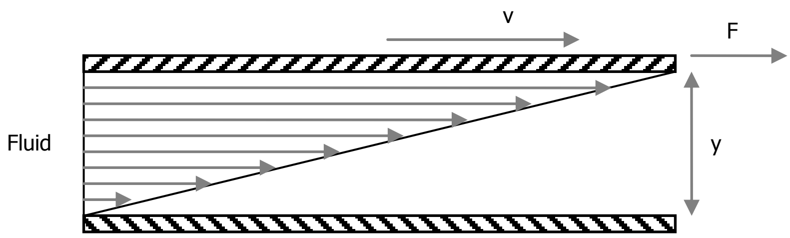


Figure 1-3 Dynamic viscosity

If we consider two plates in **Figure 1-3** of surface area A separated by a thickness y of fluid and moving at a velocity v relative to each other under an imposed force F, then:

$$\eta = \frac{F}{A}.\frac{y}{v}$$ 1.10

where:

F = total tangential force applied, (N).

A = Area over which the force is applied, (m^2).

η = coefficient of dynamic viscosity, in decapoises (Nsm^{-2}).

v = relative velocity, (m/sec).

y = distance between the layers at which velocity is measured, (m).

The coefficient of dynamic viscosity depends on the fluid and on its temperature.

In terms of units:

$$\eta = \frac{force(newtons)}{area(m^2)} x \frac{distance(m)}{velocity(m/sec)}$$

η will have the dimensions of newton second/m^2 and the unit is known as the decapoise.

Another viscosity coefficient, the kinematic coefficient of viscosity is defined by:

Kinematic coefficient of viscosity $\nu = \dfrac{absolute.viscosity(\eta)}{mass.density(\rho)}$ 1.11

Laminar and Turbulent Flow

In laminar flow, the fluid particles move along parallel paths in layers or laminae. The magnitudes of the velocities of adjacent laminae are not necessarily the same. Laminar flow is governed by the equation given as the definition of dynamic viscosity above, i.e.

$$\tau = \eta \frac{dv}{dy}$$ 1.12

The viscosity of the fluid is dominant and thus suppresses any tendency towards turbulence.

Above a certain critical velocity, the viscosity of the fluid is insufficient to damp out turbulence and the fluid particles move in a haphazard fashion where it is impossible to trace the motion of an individual particle. Such motion is called turbulent flow.

The shear stress for turbulent flow can be expressed as:

$$\tau = (\eta + z)\frac{dv}{dy} \qquad .....1.13$$

where z is a factor depending on the density of the fluid and the fluid motion.

Reynold's Number

In pipeflow the transition from laminar to turbulent flow is characterised by well known values of Reynold's Number (N_R) which expresses the ratio of the inertial to viscous forces. Thus, there is a lower limit critical number around 2100 below which flow in pipes is always laminar.

By analogy, in flow through porous media a Reynold's Number has been established as:

$$N_R = \frac{vD}{\upsilon} \qquad .....1.14$$

where:

v = the specific discharge, i.e. discharge per unit area.

D = a characteristic length. In pipe flow D is the internal diameter of the pipe. In flow through porous media D is related to grain size.

υ = the kinematic viscosity of the fluid.

Because the grain sizes *are* so variable in flow through porous media no one value for Reynold's Number can be set as the dividing line between laminar flow and turbulent flow. It has been established that this transition occurs normally for a Reynold's Number in the range 1 to 10.

For non-circular cross-sections (in open channel flow):

$$N_R = \frac{v(4R)}{\nu}$$

where R is the hydraulic radius, and is equal to the ratio of the cross-sectional area to the wetted perimeter.

Surface Tension (σ)

Surface Tension is a property associated with the free surface of any liquid or the interface between any two non-miscible liquids.

It is well known that many insects are able to walk on the surface of liquids in apparent contradiction to Archimedes' Principle. This property tends to suggest that there is a kind of membrane or skin that envelopes all liquids. In fact this very nearly describes what actually the situation is.

A molecule in the interior of a fluid is acted upon by attractive forces in all directions and the vector sum of these is zero. However, at the surface, a molecule is acted upon by a net inward cohesive force perpendicular to the surface. Hence work is required to bring molecules to the surface.

The surface tension of a liquid is the work that must be done to bring enough molecules from inside the liquid to form one new unit area on the surface.

The intermolecular forces also come into play when a liquid is in contact with a solid object - in this case there is an attraction of the molecules of the solid for those of the liquid.

Hence, for example, if pure water is placed in a clean glass container it will be observed that the water surface in contact with the glass turns up and lies flat on the glass. Thus the attraction of glass for water is greater than that of water for itself.

On the other hand mercury placed in a glass container will be observed at the surface contact, to be drawn away from the glass indicating that the attraction of glass for mercury is less than that of mercury for itself.

Capillarity or the rise or fall of a liquid in a capillary tube is caused by surface tension and depends on the relative magnitudes of the cohesion of the liquid and the adhesion of the liquid to the walls of the containing vessel. Liquids rise in tubes they wet (adhesion > cohesion) and fall in tubes they do not wet (cohesion > adhesion).

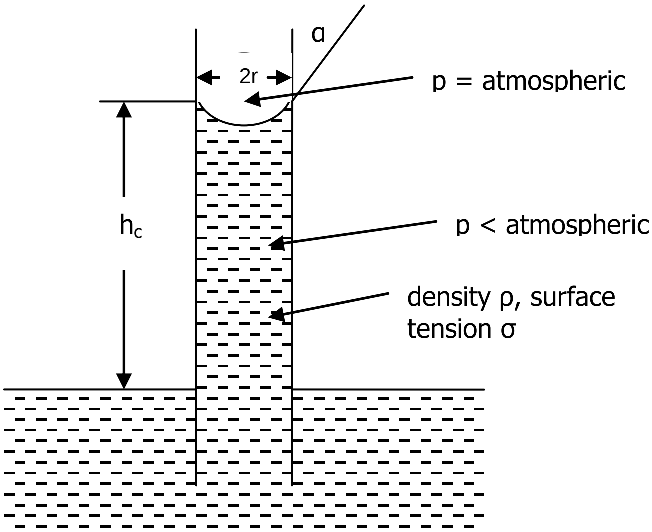


Figure 1-4 Capillary rise

From **Figure 1-4**:

$$\pi r^2 \rho g h_c = 2\pi r \sigma \cos\alpha \qquad .....1.15$$

$$h_c = \frac{2\sigma}{r\rho g}\cos\alpha \qquad .....1.16$$

where:

r = radius of the tube (m).

ρ = density of the fluid (kg/m^3).

g = gravitational acceleration (9.8 m/sec^2).

σ = surface tension (newtons/m).

h_c = capillary rise (m).

α= angle of contact between solid liquid and gas.

For pure water in clean glass:

α = 0 and cos α = 1

At 20˚C:

σ = 0.073 N/m

ρ = 1000 kg/m^3

hence:

h_c = 1.5/r x 10^{-5} m 1.17

where:

h_c = capillary rise in metres.

r = radius of tube in metres.

Surface Tension is dependent on temperature; so then is capillarity.

At 99°C, for pure water in a clean glass:

σ = 0.0591 N/m

ρ = 959 kg/m^3

h_c = (1.26/r) x 10^{-5}

Capillary rise in granular material is comparable to a bundle of capillary tubes of various diameters, see **Figure 1-5**.

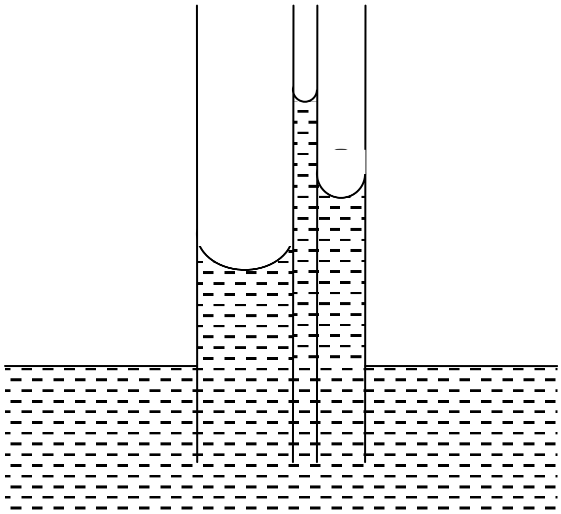


Figure 1-5 Capillary rise in tubes

Capillary rises in samples having essentially the same porosity (41%) after 72 days (Atterberg, cited in Terzaghi, 1942) are given in **Table 1-1**. (Note that h_c is nearly inversely proportional to grain size).

Table 1-1 Capillary rises in granular material

Material	Grain Size (mm)	h_c (cm)
Fine Gravel	5 - 2	2.5
Very Coarse Sand	2 - 1	6.5
Coarse sand	1 - 0.5	13.5
Medium Sand	0.5 - 0.2	24.6
Fine Sand	0.2 - 0.1	42.8
Silt	0.1 - 0.05	105.5
Silt	0.05 - 0.02	*200

* Still rising after 72 days

1.3 SOIL MECHANICS

This study of Soil Mechanics will be limited to those soil properties relevant to groundwater hydraulics. In general then, the study will be limited to porous media. Porous media are comprised of two distinct parts, a granular matrix and interconnected voids. In a saturated porous medium, water (or some fluid) fills all of the voids or pore spaces.

1.3.1 Grain-Void Relationship

Porosity (θ) of the soil mass is defined as the ratio of volume of the voids to the total volume of the mass:

$$\theta = V_v/V_T \qquad .....1.18$$

where:

V_v = volume of voids

V_T = total volume

Primary porosity is related to granular material, while secondary porosity refers to the opening in joints and faults in hard rocks, and solution openings in limestone, dolomite, gypsum or other soluble rocks.

Porosity is commonly expressed as a percentage and has no units.

The porosity of a soil obviously depends on the properties of its constituent grains, some of which are enumerated below.

Shape of the grains: since porosity is a function of the volume of voids, the more intimate the contact between grains the lower the porosity. Angularity tends to increase porosity.

Size of grains: provided grain size is uniform, the actual size will have no effect on the porosity.

Degree of assortment: a wide range of grain sizes will result in a smaller volume of voids and hence a lower porosity. On the other hand, uniform sized grains will produce a higher porosity.

Type of packing or arrangement of grains: consider the idealized case of uniform sized spherical grains. For square packing a porosity of 47.64% is achieved, while for rhombic packing the porosity will be only 25.95%.

In the same way, for random size and shape of grains, the porosity is controlled by the packing c.f. maximum and minimum density.

Voids Ratio (e) is defined as the ratio of the volume of the voids to the volume of solids.

$$e = V_v/V_s \qquad .....1.19$$

1.3.2 Fluid Flow Properties

Coefficient of Permeability or Hydraulic Conductivity (K)

The law governing laminar water flow through soils is Darcy's Law and may be expressed as:- (See Section 4).

$$Q = -KiA \qquad .....1.20$$

where:

Q = rate of flow (in cubic metres per day).

i = hydraulic gradient or head loss per unit distance travelled (non dimensional).

A = the cross-sectional area through which the flow occurs (in square metres).

K = the coefficient of permeability or hydraulic conductivity (in metres/day).

The coefficient of permeability should not be confused with intrinsic permeability (See Section 4).

Darcy's Law may also be written:

$$\frac{Q}{A} = v = -Ki \qquad .....1.21$$

However careful distinction must be made between the superficial velocity v and the actual seepage velocity v_s where:

$$Q = Av = A_v V_s \qquad .....1.22$$

where:

Q = total discharge rate.

A = cross-sectional area of porous medium.

v = average discharge velocity.

A_v = cross-sectional area of voids.

V_s = actual seepage velocity.

Hence:

$$v = \theta v_s \qquad .....1.23$$

where:

θ = porosity.

Hydraulic Conductivity may be determined in the laboratory by the use of permeameters or in the field by pumping tests. Because of the problems associated with obtaining undisturbed samples and of repacking disturbed samples in the laboratory, coefficients of permeability obtained from laboratory tests must be considered unreliable.

In practice, most aquifers are non-homogeneous and anisotropic (i.e. the material does not have like properties on all orientations of planes, generally resulting from stratification) and the hydraulic conductivity will vary with location and direction of flow. The aquifer characteristics obtained from pumping tests represent the average of values around the discharging bore.

Hydraulic conductivity and porosity are essentially unrelated e.g. clay generally has a high porosity and low hydraulic conductivity while sand has a low porosity but high hydraulic conductivity.

Intrinsic Permeability (k)

The velocity of laminar flow through a porous medium may be described almost exactly by the equation:

$$v = k\frac{i_p}{\eta} \qquad \text{.....1.24}$$

where:

i_p = the pressure gradient.

η = the dynamic viscosity of the fluid.

k = the intrinsic permeability (square micrometre).

This may be written:

$$v = \frac{k\rho g i}{\eta} \qquad \text{.....1.25}$$

where:

i = hydraulic gradient.

This equation is of the same form as the Darcy equation:

v = Ki

Hence:

$$K = \frac{\rho g k}{\eta} = \frac{kg}{\upsilon} \qquad \text{.....1.26}$$

The intrinsic permeability (k) is related solely to the properties of the porous medium. The coefficient of permeability or hydraulic conductivity (K) is related not only to the properties of the porous medium but also to the properties of the fluid.

From extensive laboratory testing, Hazen found that the coefficient of permeability of sands in a loose state depended on two quantities he called the effective grain size and the uniformity coefficient.

The effective grain size D_{10} of a sample is a grain size diameter such that 10 percent of the particles are finer and 90 percent coarser.

D_n is defined as that diameter such that n% of the particles in a sample are finer.

The uniformity coefficient U is defined as:

$$U = \frac{D_{60}}{D_{10}} \qquad \text{.....1.27}$$

1.3.3 Soil Pressures

Pore Water Pressure

If the pores or interstices of a porous medium are filled with water then this water is subject to the same principles as outlined in section 1.1.1, hydrostatics.

The hydrostatic pressure in the water is given by:

$$p_w = \rho g h \qquad .....1.28$$

where:

p_w = hydrostatic or pore water pressure.

ρ = density.

g = gravitational acceleration.

h = depth below the potentiometric level.

Intergranular Pressure

If a load is applied to an unsaturated porous medium it is transmitted from grain to grain at the points of contact. The pressure at the points of contact, i.e. the intergranular pressure, is dependent on the force applied and the area of contact. The application of a force results in a larger area of intergranular contact with a resultant slight deformation of the matrix and a reduction in voids ratio.

Total Pressure

If the porous medium is saturated and a confining layer placed over its surface and a load is applied, see **Figure 1-6**, then the load is taken partly by the pore water and partly by the grains.

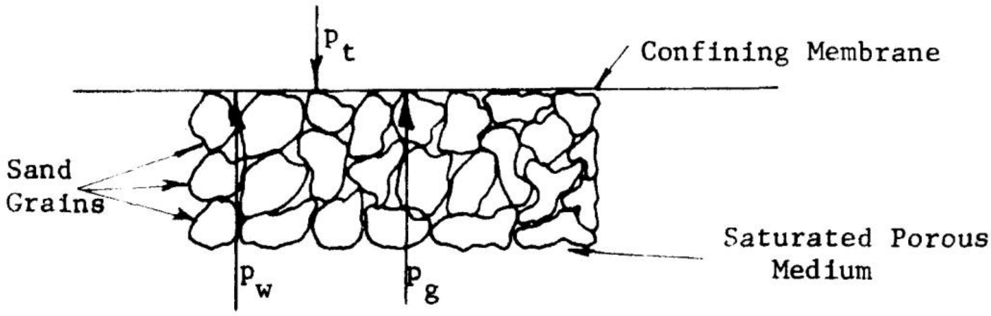


Figure 1-6 Pressure distribution

If the total pressure applied to the porous medium, i.e. force/area, is p_t then at the interface of the porous medium and confining layer:

$$p_t = p_w + p_g \qquad .....1.29$$

where:

p_t = total pressure.

p_w = that part of the pressure borne by water (pore water pressure).

p_g = that part of the pressure borne by grains.

When the applied load is in fact the material overlying an aquifer, then changes in applied load such as atmospheric pressure changes or tidal changes will result in changes in pore water pressure with resultant changes in potentiometric level.

Likewise, a lowering of the potentiometric level by pumping results in a lowering of the pore water pressure and a resultant increase in load to be carried by the grains and a slight compression of the aquifer matrix. An understanding of this transfer of pressures is necessary if the concept of storage coefficient is to be understood.

1.3.4 Properties of Rock Types

The Hydrologic Laboratory of the U.S.G.S. has conducted tests on number of samples in order to determine their properties. **Table 1-2** summarises the arithmetic mean of properties determined by these tests. It must be stressed that the values are only indicative.

The table is taken from "Summary of Hydrologic and Physical Properties of Rock and Soil Materials as Analysed by the Hydrologic Laboratory of the U.S.G.S., 1948 - 1960".

Table 1-2 Summary of the arithmetic mean of properties for all rock types

Strata				Property								
				Hydraulic Conductivity			Dry Unit Weight (g/cc)	S.G. of Solids	Porosity Undisturbed (%)	Porosity Repack (%)	Specific Retention (%)	Specific Yield (%)
				Repack (m/day)	Vert. (m/day)	Horiz. (m/day)						
Sedimentary Rocks	Water-Laid Deposits	Sandstone	Fine		0.2	0.29	1.76	2.65	33		13	21
			Medium		3.1		1.68	2.66	37		10	27
		Siltstone			N		1.61	2.65	35	43	29	12
		Claystone			N		1.51	2.66	43			
		Shale					2.53	2.73	6			
		Clay			N	N	1.49	2.67	42	48	38	6
		Silt			0.0025	0.08	1.38	2.66	46	46	28	20
		Sand	Fine	2.5	3.8		1.55	2.67	43	32	8	33
			Medium	12	14		1.69	2.66	39	35	4	32
			Coarse	45	28		1.73	2.65	39	34	5	30
		Gravel	Fine	450			1.76	2.68		34	7	29
			Medium	273			1.85	2.71		32	7	24
			Coarse	150			1.93	2.69		28	9	21
	Wind-Laid Deposits	Loess			0.08		1.45	2.67	49	46	27	18
		Aeolian Sand			20		1.58	2.66	45	38	3	38
		Tuff			0.16		1.48	2.50	41		21	21
	Ice Laid Deposits	Till	Clay					2.65				
			Silt				1.78	2.70	34		28	6
			Sand	0.5	1		1.88	2.69	31		14	16
			Gravel	30			1.91	2.72		26	12	16
		Washed	Silt		0.2		1.38	2.72	49		9	40
		Drift	Sand	38	14		1.55	2.69	44	36	3	41
			Gravel	204			1.60	2.68	39	41		

Strata			Property								
			Hydraulic Conductivity			Dry Unit Weight (g/cc)	S.G. of Solids	Porosity Undisturbed (%)	Porosity Repack (%)	Specific Retention (%)	Specific Yield (%)
			Repack (m/day)	Vert. (m/day)	Horiz. (m/day)						
Sedimentary Rocks (continued)	Chemical and Organic Deposits	Limestone		1	1.8	1.94	2.75	30		13	14
		Dolomite				2.02	2.69	26			
		Peat		5.7		0.13	1.54	92		49	44
Igneous Rocks	Weathered	Granite		1.4		1.50	2.74	45			
	Weathered	Gabbro		0.16		1.73	3.02	43			
	Basalt			0.008		2.53	3.07	17			
Metamorphic Rocks	Schist			0.16		1.76	2.79	38		17	26
	Slate			N			2.94				

Note: N indicates Negligible

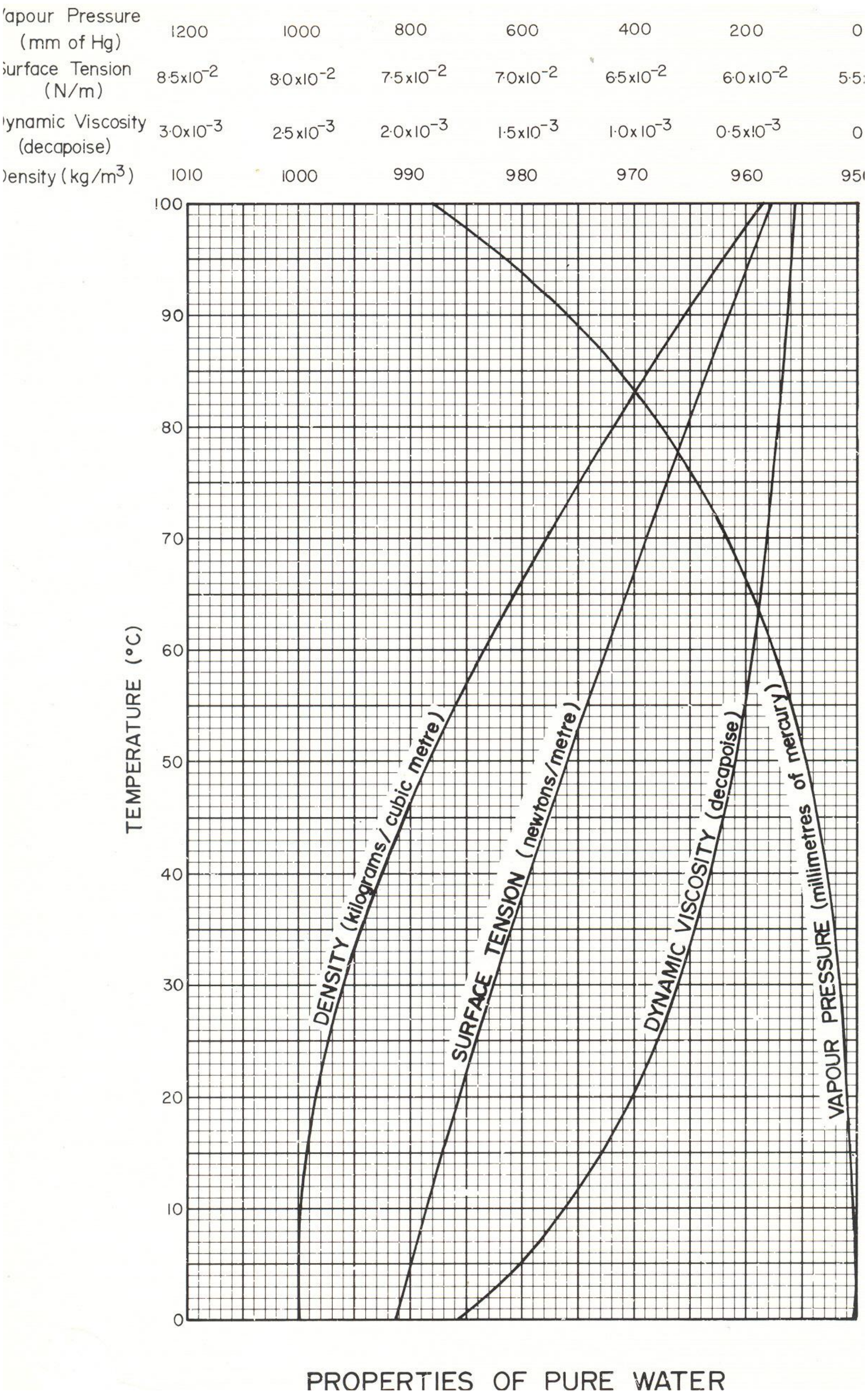


Figure 1-7 Properties of pure water

SECTION 2: OCCURRENCE OF GROUNDWATER

2.1 INTRODUCTION

Natural underground reservoirs have many advantages. They are freely available for storing water without construction expenditure. Commonly, they have enormous capacities and do not become clogged with silt and weeds as do lakes and reservoirs. They are relatively inexpensive to tap; they lose little or no water by evaporation; they can supply water over very large areas without the necessity of building channels, pipelines or other distribution systems; and, if properly managed, their period of usefulness has no foreseeable limit.

Groundwater occurs in the pores and interstices of rocks. In semi-confined aquifers (Section 3) large volumes of water may be stored in the semi-pervious layers above and/or below the main aquifer. With a reduction in pressure this water moves vertically to the aquifer which then transmits it to the bore.

The volume stored in any saturated material is given by:

$$V_v = V_T \theta \qquad \text{.....2.1}$$

And the volume released under gravity drainage is given by:

$$V_D = V_T S_y \qquad \text{.....2.2}$$

where:

V_v = volume of stored water (also volume of interstices).

V_T = total volume of saturated material.

V_D = volume of water released by gravity drainage.

θ = porosity.

S_y = specific yield.

2.2 ORIGIN OF GROUNDWATER

Groundwater may originate in any of three ways:

Juvenile Water has its origin in molten rocks which underlie the earth's crust at great depths. These rocks sometimes find their way to the surface, or near surface, of the earth. Upon cooling of the rock, water may be trapped or given off as steam from a volcanic vent. From the point of view of worthwhile supplies of groundwater, juvenile water has little or no significance.

Connate Water is water trapped in the interstices of a sedimentary rock at the time it was deposited. It may, for example, have been derived from the ocean or from fresh water sources, depending on the locality in which the sedimentary rock was formed. Although of little importance from the point of view of significant quantities of groundwater being obtained from this source, it is nevertheless most important in its effect on water quality in various rocks.

Meteoric Water is water derived from the atmosphere, generally in the form of rain and sometimes snow and hail. It is the basic source from which the great bulk of groundwater is derived.

2.3 HYDROLOGIC CYCLE

The earth's water supplies are being constantly circulated. **Figure 2-1** illustrates this cycle. The sun's heat evaporates water from the seas and lakes covering two-thirds of the earth's surface. This evaporated water is virtually free of saline matter. The moisture forms clouds which precipitate as rain over both the sea and land.

Some is evaporated before reaching the ground; more is intercepted by plant life and returned to the atmosphere. Some rain which reaches the land surface makes its way back to the sea through rivers and lakes. In doing so, it is subject to further evaporation. Some rain finds its way underground and this is the major source of water underground. Part of this water which has entered the soil is used by plant life, and returns to the atmosphere through leaves. However, a proportion does eventually penetrate deep underground and is stored in natural materials. Even this water is acted upon by gravity, and in the long term tends to make its way back to the sea to help recommence the cycle.

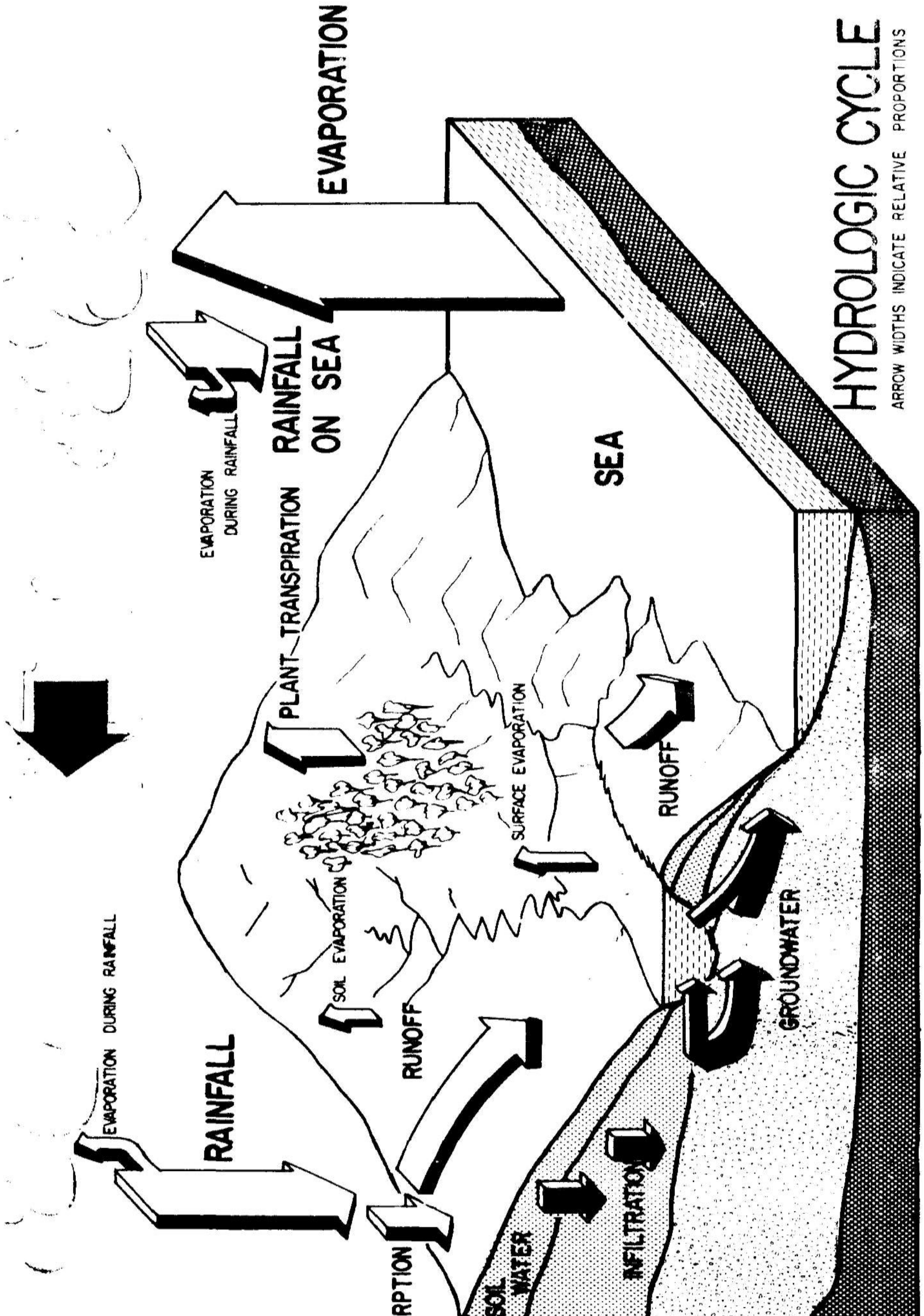


Figure 2-1 Hydrologic cycle

Measurement and recording of precipitation and some of the principles involved in Surface Water Hydrology are dealt with in other lectures. In many instances the demand on the underground water resources of an area exceeds the ability of the aquifer to yield the water. In such cases consideration must be given to the possibility of using surface water in conjunction with groundwater or to the possibility of artificially recharging the aquifer to supplement the inadequate natural recharge.

Both of these considerations will be discussed later in this course.

2.4 FACTORS AFFECTING THE ABSORPTION OF WATER

The amount of rain water reaching aquifers is dependent upon factors which include:

The Type of Rainfall

Heavy rain runs off quickly, steady soaking rain is more likely to be absorbed by the soils.

Topography

Steep slopes aid quick run-off while flat areas allow a greater opportunity for the water to soak underground.

Type of Surface Material

Sandy soils will absorb rainfall but clays prevent easy access for the water.

Type of Vegetation

Plants can greatly reduce water entering the aquifers by transpiring and evaporating it, through their leaves. On the other hand, vegetation can increase infiltration by reducing surface runoff, improving soil structure, protecting the soil from compaction and by providing organic matter - holes.

Climate

A cool climate with frequent rain is more favourable for underground water replenishment than a hot climate and infrequent rain.

2.5 VERTICAL DISTRIBUTION OF SUB-SURFACE WATER

Sub-Surface Water

Water occurring beneath the land surface is called sub surface water. It is in two parts. The lower part within the "Zones of Saturation" is usually referred to as groundwater. Above this, in the "Zone of Aeration", water is held between the particles of soil etc. This is known as "Suspended Water".

Zone of Aeration

As water enters the soil it moves downward due to gravity. However, some of the water is held on the grains and between them by surface tension. This is suspended water, and is found in the "Zone of Aeration" - refer to **Figure 2-2**. This zone extends from the surface to the actual water table. It includes an upper band containing soil water, an intermediate zone and finally, just above the aquifer, a capillary zone. In the soil moisture belt water is evaporated from the soil as well as being used by the plants.

In the intermediate belt the water is held by molecular attraction and little or no movement occurs except when recharge occurs.

The thickness of the capillary zone depends on the nature of the material overlying the water bed. It can vary from a few centimetres to a few metres. The finer the material, the greater the thickness. This zone is recognised by drillers as a sign of the proximity of water.

The study of the flow of water in this unsaturated zone is becoming more widely known as Soil Water Hydrology and will not be touched in any detail in these notes. Soil Water Hydrology can be very important when considering groundwater recharge.

Zone of Saturation

In this zone all the openings in the rocks are completely filled with water. It is from this zone that supplies of groundwater are obtained. Refer to **Figure 2-2**.

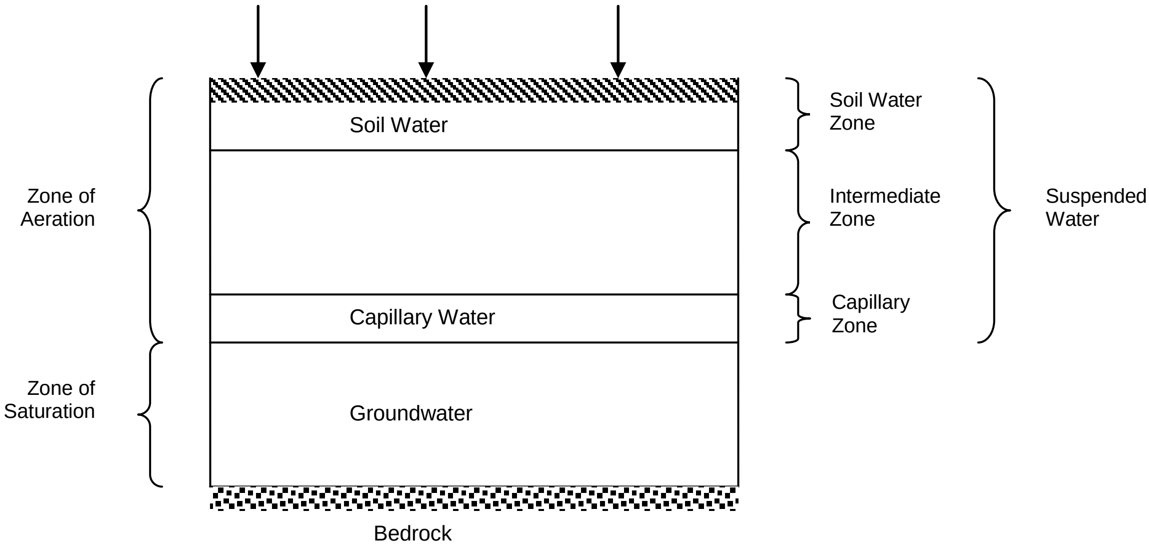


Figure 2-2 Vertical distribution of sub-surface water

In non-pressure or unconfined aquifers, the top of the zone is called the water table. The water table will fluctuate with recharge, use and groundwater flow.

In confined aquifers the water is prevented from rising by a confining layer. If a number of bores were drilled through this confining layer the heights to which the water would rise would represent the potentiometric surface of the aquifer.

For the most part the presence of groundwater is continuous in the zone of saturation. However, its availability depends upon the nature of the rock formation in which it occurs. For example, clay may be saturated but will not release the water to a bore or well. On the other hand, coarse saturated gravel would yield large quantities.

In some cases the rate at which the saturated material will yield water is so slow that it is not immediately obvious in a well or bore and drillers are inclined to say that the hole is dry.

This can explain the situation where a bore is drilled a few feet from a so called dry hole and yields a substantial quantity of water. In actual fact in both cases the material may be saturated but only in the latter case will it yield water at a rate sufficient to give a successful bore.

The schematic representation in **Table 2-1** gives the complete range of divisions. These will all be present in areas of relatively deep water tables after prolonged dry spells. In other areas with higher water tables the uppermost divisions may not be present. Beneath lakes, swamps and streams only the zone of saturation will be present.

Table 2-1 Distribution of sub-surface water

Pressure	Zone	Division[(1)]	Bore
Gas phase, equals atmospheric	Unsaturated Zone	Discontinuous capillary saturation	
Liquid phase, less than atmospheric		Semi-continuous capillary saturation	
Less than atmospheric	Saturated[(2)] Zone	Continuous capillary saturation	
Atmospheric		*water table*	*S.W.L.*
Greater than atmospheric		Unconfined aquifer	

Notes: (1) Capillary "Zones" of Terzaghi (1942)

(2) As redefined by Hubbert (1940). See also Lohman (1965, p.92).

SECTION 3: AQUIFERS

3.1 DEFINITIONS

<u>Aquifer</u>

Lohman and others 1972, have defined an aquifer as a formation, group of formations or part of a formation that contains sufficient saturated permeable material to yield significant quantities of water to bores and springs. Under this definition, an aquifer includes both the saturated and unsaturated part of the permeable formation.

<u>Confined (or Pressure or Artesian) Aquifer</u>

A confined aquifer is a completely saturated permeable formation of which the upper and lower boundaries are impervious layers. Completely impervious layers rarely exist in nature and hence confined aquifers are less common than is often realised.

In a confined aquifer the water is under sufficient pressure to cause it to rise above the aquifer if given the opportunity e.g. if penetrated by a bore. The level to which the water rises is called the potentiometric level, the standing water level (S.W.L.) or static head. Pressure water or confined water relates to water contained in aquifers of this nature.

If the pressure is sufficient to raise the water to the surface when a bore penetrates such an aquifer then the water may flow from the bore without mechanical assistance. This type of bore is called a "flowing artesian bore".

<u>Unconfined (Non Pressure or Water Table) Aquifer</u>

An unconfined aquifer is a permeable formation only partly filled with water and overlying a relatively impervious layer.

It contains water which is not subjected to any pressure other than its own weight. If a bore penetrates such an aquifer the water will rise up the bore no higher than the depth at which it was first encountered.

The level at which water stands in a bore penetrating an unconfined aquifer, i.e. the standing water level, is known as the water table, and is the depth at which the water in the unconfined aquifer is at atmospheric pressure. Water does occur in the aquifer above the water table but at pressures less than atmospheric.

A perched aquifer is an unconfined aquifer separated from an underlying body of groundwater by an unsaturated zone. Its water table is a perched water table. It is supported by a perching bed whose permeability is low. Perched groundwater may be either permanent or temporary.

Water in an unconfined aquifer is called unconfined or phreatic water.

<u>Semi-Confined (or Leaky) Aquifer</u>

The confining layers of many pressure aquifers are not completely impervious. The hydraulic conductivity of the confining layer may be very small when compared with that of the aquifer material, but as the radius of influence of a discharging facility increases, the area through which the confining layer is contributing water becomes very large and the volume of water contributed can be a very significant part of the total water discharged.

Such an aquifer is called a leaky or semi-confined aquifer. The flow of water from the confining layer to the aquifer is assumed to be vertical. The horizontal movement in this layer is negligible.

<u>Semi-Unconfined Aquifer</u>

If the hydraulic conductivity of the fine grained layer in a semi-confined aquifer is so great that the horizontal flow component in the covering layer cannot be ignored, then such an aquifer is intermediate between the semi-confined aquifer and the unconfined aquifer and may be called a semi-unconfined aquifer.

In general, such aquifers do not release their water instantaneously from storage and exhibit what is called delayed drainage. Such aquifers may be called unconfined aquifers exhibiting delayed drainage or delayed yield effects.

Schematic representation of the aquifer types is given in **Figure 3-1**.

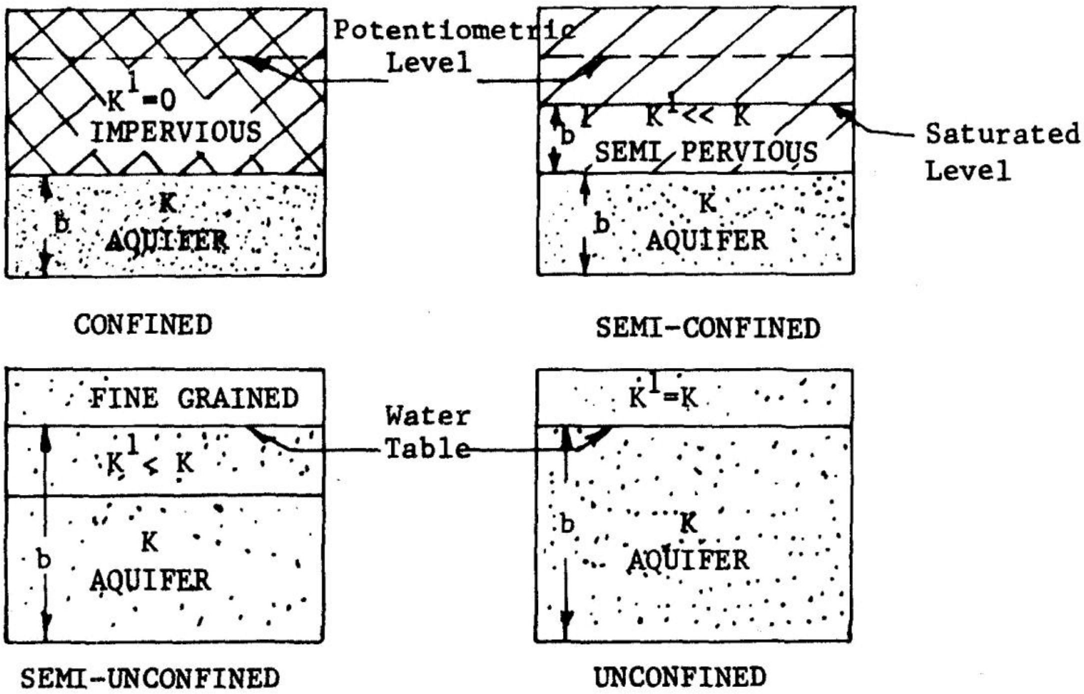


Figure 3-1 Aquifer types

Aquiclude

An aquiclude is an impermeable formation which may contain water, but is incapable of transmitting it in quantities large enough to warrant extraction by a well or a bore.

Aquifuge

An aquifuge is an impermeable formation which neither contains nor transmits water. An example of an aquifuge is solid granite.

Confining Bed

U.S.G.S. Water Supply Paper 1988 suggests that the term "confining bed" should supplant the terms "aquiclude", "aquitard" and "aquifuge". It is a body of impermeable material stratigraphically adjacent to one or more aquifers.

Homogeneous and Isotropic

Two other terms which are frequently used in groundwater hydraulics are "homogeneous" and "isotropic". Homogeneous means "the same at all locations" and isotropic means "the same in all directions".

The drilling logs of bores drilled anywhere in a homogeneous aquifer would be identical both in material encountered and the thickness of material even though a number of different rock types was encountered throughout the depth of the bore.

In a non-homogeneous aquifer the material encountered and the thickness of material could both vary from place to place in the aquifer.

In an isotropic aquifer the type of material and its properties are the same in all directions. In nature, aquifer material has been laid down in such a way that properties vary in different directions e.g. it is normally easier for water to move along the direction of layering in a horizontal direction than it is to move across layers in a vertical direction.

3.2 AQUIFER FUNCTIONS

An aquifer has three important functions namely:

Storage

It stores water as a reservoir. The aquifer characteristic describing the ability of an aquifer to store water is the Porosity. The characteristic describing its ability to release water under gravity drainage is its specific yield. The property describing its elastic storage is the Storage Coefficient or Storativity, and this is related to the elastic properties of both the water and aquifer material.

Transmission

It transmits water like a pipeline. The relevant aquifer characteristics are Hydraulic Conductivity and Transmissivity (called Transmissibility in some references).

Mixing

It mixes water of different qualities. Poor quality water can be injected into an aquifer at one point, mixed with the local groundwater and the mixture withdrawn as useable water at another location.

3.3 TYPES OF AQUIFER FORMATIONS

There are two general classes of formation which store and transmit water. These are:

Porous Rocks

Spaces between grains of sand and gravel - unconsolidated sands and gravels and consolidated sandstones;

Fractured Rocks

These include crevices, joints and fractures in hard rock; and solution channels in limestone, and shrinkage cracks and gas bubbles in basalt type volcanic rocks.

3.4 HYDRAULIC PROPERTIES

There are ten main hydraulic properties of the aquifer which should be understood. These are:

1. Hydraulic Conductivity.
2. Transmissivity.
3. Storage Coefficient.
4. Specific Mass Storativity.
5. Specific Yield.
6. Specific Retention.
7. Hydraulic Resistance.
8. Leakage Coefficient.
9. Leakage Factor.
10. Drainage Factor.

Hydraulic Conductivity (K) (see also Sections 1 and 4)

The hydraulic conductivity of an aquifer is the rate at which water at the prevailing viscosity can be transmitted through a unit area of an aquifer, normal to the direction of flow, under a unit gradient. It is sometimes referred to in engineering literature as the coefficient of permeability.

$$K = \frac{q}{i} \qquad .....3.1$$

where:

q = discharge rate per unit area normal to the direction of flow.

i = hydraulic gradient.

Hydraulic conductivity is dependent primarily on the nature of the pore space, the type of liquid occupying it, and the gravitation attraction.

In a layered anisotropic aquifer the hydraulic conductivity will be different from layer to layer and the average hydraulic conductivity for the aquifer will be a function of the individual hydraulic conductivities of each layer. In addition, the hydraulic conductivity is normally greater along the layering than across it and also greater along the direction of flow for extrusive igneous rocks. Hence, the hydraulic conductivity is generally much greater in the horizontal direction than in the vertical direction. In the absence of more precise data it can be assumed that the horizontal hydraulic conductivity is an order of magnitude greater than the vertical.

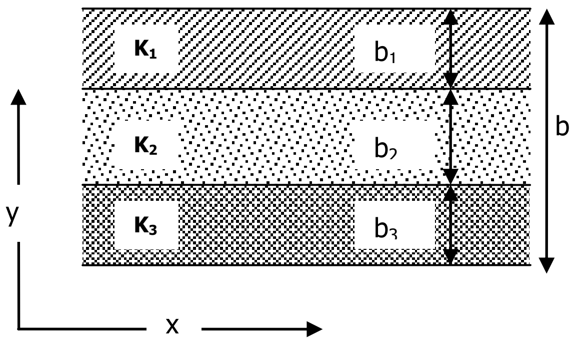


Figure 3-2 Homogeneous anisotropic formation

In **Figure 3-2**, the anisotropic formation comprises 3 layers of thicknesses b_1, b_2, and b_3 with hydraulic conductivities K_1, K_2 and K_3 respectively.

The average horizontal hydraulic conductivity of a formation with n layers is given by:

$$K_h av = \Sigma_{m=1}^{n} \frac{K_{hm} b_m}{b} \qquad .....3.2$$

where:

$K_h av$ = average horizontal hydraulic conductivity.

K_{hm} = the hydraulic conductivity of the m^{th} layer.

b_m = the thickness of the m^{th} layer.

b = the total thickness of the aquifer.

The effective hydraulic conductivity for vertical flow i.e. for flow through the n layers in the y direction is given by:

$$K_v eff = \frac{b}{\Sigma_{m=1}^{n} \frac{b_m}{K_{vm}}} \qquad .....3.3$$

where:

$K_v eff$ = the effective hydraulic conductivity for vertical flow.

K_{vm} = the vertical hydraulic conductivity of the m^{th} layer.

b_m = the thickness of the m^{th} layer.

b = the total thickness of the aquifer.

Equations 3.2 and 3.3 are analogous to the flow of electricity through a series of resistors. In equation 3.2 the resistors are in parallel and in equation 3.3 the resistors are in series.

The dimensions of hydraulic conductivity are Volume/time/area (Length/Time). The units to be used are m/day or m^3/day/m.

Transmissivity (T)

The transmissivity of an aquifer is the rate at which water at the prevailing viscosity can be transmitted through a unit strip of aquifer under a unit gradient.

$$T = Kb \qquad .....3.4$$

where

K = the hydraulic conductivity.

b = aquifer thickness.

The dimensions of transmissivity are Volume/unit time/unit width ($Length^2/Time$). The units to be used are m/day/m or m/day.

Storage Coefficient (S) (Confined Aquifers)

The storage coefficient or storativity of an aquifer is defined as the volume of water which a saturated column of aquifer releases from or takes into storage per unit surface area per unit change in head.

Storage coefficient is related to the elastic properties of the water and of the soil matrix. It is not an indication of the total volume stored in an aquifer.

For a confined aquifer the value of Storage Coefficient ranges from about 10^{-5} to 10^{-3} or about 5 x 10^{-6} per metre of aquifer thickness.

C.E. Jacob (1940, 1950), probably the first to consider flow in an elastic aquifer, derived the equation for Storage Coefficient as:

$$\begin{aligned} S &= b\rho g\theta(\beta + \frac{C\alpha}{\theta}) \\ &= bg\theta(\frac{1}{E_w} + \frac{C}{\theta E_s}) \end{aligned} \qquad .....3.5$$

where:

b = thickness of the aquifer.

ρ = density of the fluid (for water = 1,000 kg/m^3).

g = acceleration due to gravity.

θ = porosity.

β = compressibility of water, 4.8 x 10^{-7} kPa i.e.4.8 x 10^{-10} m^2/N.

E_w = 1/β bulk modulus of water 2.08 x 10^6 kPa.

α = compressibility of soil matrix. Sand and gravel α ~ 10^{-8} m^2/N

E_s = 1/α = bulk modulus of soil matrix.

C = a dimensionless ratio, which may be considered unity in an uncemented granular material. In a solid aquifer such as limestone having tubular solution cavities, C is apparently equal to the porosity. The value for a sandstone ranges between these limits, depending upon the degree of cementation.

Bear (1972, p.207), suggests that the above expression is in error and the correct expression for porous media should be:

$$\begin{aligned} S &= b\rho g(\beta\theta + \alpha(1-\theta)) \\ &= b\rho g(\frac{\theta}{E_w} + \frac{(1-\theta)}{E_s}) \end{aligned} \qquad .....3.6$$

Bear's approach considers a constant control volume with both the fluid and soil matrix being free to move across the boundaries when the porous media is subjected to varying pressures. Jacob considered that the control volume itself undergoes deformation.

In either case, in the absence of other information a minimum value for Storage Coefficient can be determined by using the first part only of equation 3.5, i.e. the compression of water alone.

$$S_{min} = \frac{b\rho\theta g}{E_w} \qquad .....3.7$$

It will be observed that S is directly proportional to b. Storage Coefficient has no dimensions.

From the following it can be seen that Storage Coefficient is best thought of in terms of strain.

The Coefficient of Bulk Compressibility, α_b', may be defined for a saturated porous medium as the fractional change in the bulk volume of the porous medium with a unit change in the external stress (σ) exerted by the formation.

i.e. $$\alpha_b' = \frac{-1}{U_b}\frac{dU_b}{d\sigma} \qquad .....3.8$$

if the hydrostatic pressure (p) is held constant.

where:

U_b = the volume of a fixed mass of porous medium.

Because groundwater is generally associated with a relatively constant external stress (σ) and a variable hydrostatic pressure (p) another coefficient of compressibility, α_b, is often defined with respect to a unit change in p.

$$\alpha_b = \frac{-1}{U_b}\frac{dU_b}{dp} \qquad .....3.9$$

where σ is now held constant.

Two more kinds of compressibility in addition to α_b have been proposed as:

1. rock (or solid) matrix compressibility, α_s, which is the fractional change in volume of the solid matrix (U_s) with unit change in p;

   i.e. $$\alpha_s = \frac{-1}{U_s}\frac{dU_s}{dp} \qquad .....3.10$$

2. pore compressibility, α_p, defined as the fractional change in pore volume (U_p) with unit change in p.

   i.e. $$\alpha_p = \frac{-1}{U_p}\frac{dU_p}{dp} \qquad .....3.11$$

It follows then, since:

$$U_b = U_s + U_p$$

$$\frac{dU_b}{dp} = \frac{dU_s}{dp} + \frac{dU_p}{dp}$$

and since:

$$U_s = (1-\theta)U_b \qquad \text{i.e. } \frac{1}{U_b} = \frac{(1-\theta)}{U_s}$$

and

$$U_p = \theta U_b \qquad \text{i.e. } \frac{1}{U_b} = \frac{\theta}{U_p}$$

then, by definition:

$$\alpha_b = \frac{-1}{U_b}\frac{dU_b}{dp}$$

$$= -\frac{(1-\theta)}{U_s}\frac{dU_s}{dp} - \frac{\theta}{U_p}\frac{dU_p}{dp}$$

$$= -(1-\theta)\alpha_s + \theta\alpha_p \qquad .....3.12$$

From the definition of S (by Bear):

$$S = b\rho g(\beta\theta + \alpha(1-\theta)$$

where:

β is now written for α_p and α is written for α_s.

The major bracketed term is the compressibility of the porous medium, and bρg is in the form of a stress. Since the compressibility is the inverse of the bulk modulus, then the Storage Coefficient is in the form of a strain of the porous medium.

It can be thought of as the amount of deformation of water and matrix per unit increase in hydrostatic pressure, or can be referred to as the elastic storage. The major bracketed term is the compressibility of the porous medium, and bρg is in the form of a stress. Since the compressibility is the inverse of the bulk modulus, then the Storage Coefficient S is in the form of a strain of the porous medium.

It can be thought of as the amount of deformation of water and matrix per unit increase in hydrostatic pressure, or can be referred to simply as the elastic storage.

Specific Mass Storativity or Specific Storage (S_s)

The Specific Mass Storativity (S_s) or Specific Storage is defined as the volume of water released from or taken into storage per unit volume of aquifer per unit change in head.

$$S_s = \rho g(\frac{\theta}{E_w} + \frac{(1-\theta)}{E_s} \qquad .....3.13$$

i.e. $$S_s = \frac{S}{b}$$

where:

S = Storage Coefficient.

b = thickness of aquifer.

Specific Mass Storativity has the dimensions length^{-1}.

It has the units metres^{-1}.

Specific Yield or Phreatic Storage Coefficient (S) (Unconfined Aquifers)

The specific yield of an aquifer is the volume of water which will drain under gravity from a unit volume of aquifer. For a section of aquifer it is the ratio of the drainable water to the saturated volume.

An unconfined aquifer has elastic storage properties as does a confined aquifer, but these are so small in comparison with the non-elastic storage properties that the specific yield is normally referred to as the Storage Coefficient of an unconfined aquifer.

It is used to determine the recoverable volume of water stored between the standing water level and a specified dead storage level.

For an unconfined aquifer the specific yield ranges from about 0.1 to 0.3.

Specific yield has no dimensions.

Specific Retention (R)

The Specific Retention (R) is the volume retained when the specific yield has been released.

$$R = \theta - S \qquad .....3.14$$

Hydraulic Resistance (c)

The hydraulic resistance is a property of the confining layer of a semi-confined aquifer. It is the resistance against vertical flow and is defined as:

$$c = b'/K' \qquad .....3.15$$

where:

b^1 = the saturated thickness of the semi-pervious layer.

K^1 = the hydraulic conductivity of the semi-pervious layer for vertical flow.

If Darcy's law is applied to the confining layer then the hydraulic resistance may be thought of as the drawdown in the aquifer required to give a unit discharge per unit area from the confining layer.

If $c = \infty$ the aquifer is confined.

Hydraulic resistance has dimensions of Time. The units used are days.

Leakage Coefficient

The leakage coefficient is a property of the confining layer of the semi-confined aquifer. It is the inverse of hydraulic resistance and is defined as:

$$\text{Leakage Coefficient} = K'/b' \qquad .....3.16$$

where:

K^1 = hydraulic conductivity of the semi-pervious layer for vertical flow.

b^1 = saturated thickness of the semi-pervious layer.

Leakage coefficient may be defined as the rate at which water will leak from a unit area of the confining layer per unit drawdown in the aquifer proper.

It has the dimensions of 1/time.

The units of leakage coefficient are day $^{-1}$.

Leakage Factor (L)

The leakage factor is a property of the semi-confined aquifer.

It is defined as:

$$L = \sqrt{(Kbc)}$$

$$= \sqrt{(Tc)} \qquad .....3.17$$

where:

c = hydraulic resistance of the semi-pervious layer.

K = hydraulic conductivity of the aquifer material.

b = thickness of the aquifer.

T = transmissivity of the aquifer.

The leakage factor describes the distribution of leakage into a semi-confined aquifer. High values of L indicate that the influence of leakage will be small, i.e. a high resistance of the semi-pervious layer to flow, as compared with the resistance of the aquifer itself.

The dimensions of L are in length. The units of L are metres.

Drainage Factor (B)

The drainage factor is associated with the delayed yield in unconfined aquifers and is similar to the leakage factor for semi-confined aquifers.

It is defined as:

$$B = \sqrt{\frac{Kb}{\alpha S_y}}$$

or

$$B = \sqrt{\frac{T}{\beta S_y}} \qquad .....3.18$$

where:

K = hydraulic conductivity of the aquifer.

b = aquifer thickness.

T = transmissivity of the aquifer.

$1/\alpha$ = the Boulton delay index (an empirical constant).

S_y = the specific yield after a long pumping time.

Large values of B indicate a fast drainage. If B = ∞ the yield is instantaneous with the lowering of the water table, so the aquifer would be confined without delayed yield.

The dimensions of B are in length, and the units are metres.

SECTION 4: GROUNDWATER FLOW

4.1 INTRODUCTION

Before a pumping test or groundwater basin can be analysed with any degree of confidence, it is necessary to have a basic understanding of groundwater flow, aquifer types and hydraulic properties of the aquifer.

Groundwater in its natural state is invariably moving and this movement is related to established hydraulic principles which in turn are governed by the geological structure of the material in which the water occurs.

4.2 DARCY'S LAW

Although Hazen (1839) and Poisseuille (1846) found that the rate of flow through capillary tubes is proportional to the hydraulic gradient, Darcy, a French hydraulic engineer, was the first to experiment with sand.

In 1856 Darcy reported as follows, on his investigation of the flow of water through a horizontal sand bed to be used for water filtration; (symbols as used in these notes).

"I have attempted by precise experiments to determine the law of flow of water through filters. The experiments demonstrate positively that the volume of water which passes through a bed of sand of a given nature is proportional to the pressure and inversely proportional to the thickness of the bed traversed; thus, in calling "A" the surface area of a filter, "K" a coefficient depending on the nature of the sand, "l" the thickness of the sand bed, P+H the pressure below the filter bed; one has for the flow of this last condition:

$$Q = \frac{KA}{l}(H + l - H_0) \qquad .....4.1$$

which reduces to:

$$Q = \frac{KA}{l}(H + l) \qquad .....4.2$$

H_0 = 0 when the pressure below the filter is equal to the weight of the atmosphere".

This may be written in the form:

$$q = \frac{Q}{A} = -K\frac{dh}{dl} \qquad .....4.3$$

where:

q = discharge per unit area.

This statement, that the rate of flow of water through porous media is proportional to the head loss and inversely proportional to the length of the flow path, is known universally as Darcy's Law. Darcy's Law forms the basis for the present day knowledge of groundwater flow.

As water percolates through a permeable material, the individual water particles move along paths which deviate erratically but only slightly from smooth curves known as "flow lines". If adjacent flow lines are parallel the flow is said to be "linear" or "laminar".

The hydraulic principles involved in laminar flow are illustrated **Figure 4-1**.

In **Figure 4-1**, the points A and B represent the extremities of the flow lines. At each extremity a stand pipe, known as a potentiometric tube, has been installed to indicate the level to which the water rises at these points. The water level in the tube at B is designated as the potentiometric level at B and the vertical distance from this level to point B is the pressure head at B. The vertical distance between A and B represents the "position head" or difference in elevation heads above a set datum level.

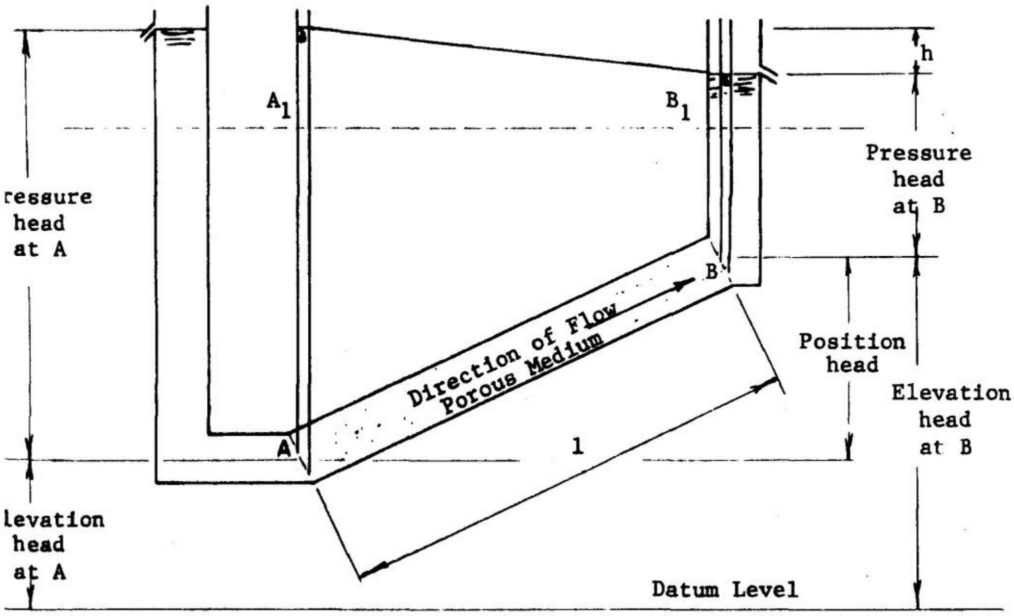


Figure 4-1 Laminar flow in a porous medium

If the water in the hydraulic system stands at the same elevation in the potentiometric tubes at A and B, the system is in a state of rest, regardless of the magnitude of the "position head". Flow can occur only if the potentiometric levels at A and B differ. The difference in potentiometric levels, "h" is known as the "hydraulic head" at A with respect to B, or merely referred to as the difference in potentiometric level between A and B. It can be observed that the difference in potentiometric level is equal to the difference in pressure heads at A and B only if the position head is zero.

In **Figure 4-1**, A and B represent any two points at the same elevation in the potentiometric tubes rising from A and B respectively. Since the unit weight of the water is $\rho_w g$, the hydrostatic pressure at A_1 exceeds that at B_1 by an amount $\rho_w gh$. The difference $\rho_w gh$ between the hydrostatic pressures at the two points located at the same elevation is referred to as "excess hydrostatic pressure". It is this pressure that drives the water through the soil between A and B. The ratio:

$$i_p = \rho_w g \frac{h}{l} = \frac{u}{l} \qquad .....4.4$$

in which "u" is the excess hydrostatic pressure, represents the pressure gradient from A to B. The ratio:

$$i = \frac{i_p}{\rho_w g} = \frac{1}{\rho_w g} \frac{u}{l} = \frac{h}{l} \qquad .....4.5$$

is known as the hydraulic gradient. It is a pure number.

If water percolates through fine saturated sand or other fine grained completely saturated soils without affecting the structure of the soil, the discharge velocity or unit discharge is almost exactly determined by the equation -

$$v = \frac{-k}{\eta} i_p \qquad .....4.6$$

in which:

v = average discharge velocity or unit discharge (m/s).

n = the dynamic viscosity of water (Nsm^2).

k = an empirical constant referred to as the intrinsic permeability (m^2).

i_p = pressure gradient (Pa/m).

The viscosity of water decreases with increasing temperature. The value k is a constant for any permeable material with given porosity characteristics, and is independent of the physical properties of the percolating liquid.

From equation 4.5 and 4.6 we obtain for the discharge velocity the expression:

$$v = \frac{-k}{\eta} \rho_w g i \qquad .....4.7$$

Most seepage problems encountered in Civil Engineering and Groundwater Hydrology deal almost exclusively with the flow of groundwater at moderate depths below the surface and with leakage out of reservoirs. The temperature of the percolating water varies so little that the unit weight $\rho_w g$ is practically constant, and, in addition, the viscosity varies within fairly narrow limits. Therefore, it is customary to substitute in equation 4.7:

$$K = \frac{k \rho_w g}{\eta} \qquad .....4.8$$

Hence:

v = -Ki 4.9

where:

v = average discharge velocity or unit discharge.

K = coefficient of permeability, or hydraulic conductivity.

i = hydraulic gradient.

In Civil Engineering, the value K is commonly called the "coefficient of permeability". In Groundwater Hydrology K is called the hydraulic conductivity. Equation 4.9 is commonly known as Darcy's Law.

Darcy's Law may also be written in the form:

Q = -KiA 4.10

where:

Q = discharge rate through an area A.

i = the hydraulic gradient.

A = the cross-sectional area normal to the direction of flow.

K = the hydraulic conductivity of the material.

4.3 HYDRAULIC CONDUCTIVITY (K)

From equation 4.3 the hydraulic conductivity is expressed by

$$K = -\frac{Q}{A(dh/dl)}$$

which shows that K has dimensions of distance divided by time, i.e. velocity, but is in fact a discharge per unit area. It is defined as the rate at which water is transmitted through a unit area under a unit gradient.

Laboratory tests by the United States Geological Survey have yielded coefficients of permeability or hydraulic conductivity, varying from 9.1×10^{-7} to 4.5×10^{2} metres per day, but for most natural aquifers values range between 0.04 and 25 metres per day.

The hydraulic conductivity of a porous medium refers to the ease with which a fluid will pass through it, and varies with the diameter and "degree of assortment" of the individual particles. A well sorted gravel has a higher hydraulic conductivity than a well sorted coarse sand. However, gravel with a moderate percentage of medium- and fine-grained material may be considerably less permeable than a uniformly sized coarse sand. The reason is that the smaller particles fill the larger pore spaces of the gravel and so reduce the area available for flow of water.

The hydraulic conductivity need not be the same in all directions. Hydraulic conductivity in the vertical direction is normally less than the hydraulic conductivity in the horizontal directions. This is caused by deposition of materials in horizontal layers. An occasional relatively impermeable layer reduces the vertical hydraulic conductivity without appreciably affecting horizontal hydraulic conductivity.

4.4 Relation of K to Particle Velocity

Because K has the dimensions of velocity (length/time), some might mistake this for the actual, or particle velocity, of the water. However, from equation 4.10 we see that before simplification:

$$K = -\frac{m^3}{m^2 day.m.m^{-1}}$$

which is the volumetric rate of flow through a given cross-sectional area under unit gradient. For the average actual or particle velocity, we must know also the porosity of the medium. Thus,

$$Q = vA = -KA\frac{dh}{dl} \qquad .....4.11$$

$$v = -\frac{K}{\theta}\frac{dh}{dl} \qquad .....4.12$$

where:

v = average water particle velocity.

θ = porosity, as a decimal fraction.

dh/dl = hydraulic gradient = i.

4.5 Relation of K to Intrinsic Permeability

Intrinsic permeability is a measure of the relative ease with which a porous medium can transmit a liquid under a potential gradient. It is a property of the medium alone dependent on the size and shape of the pores, and is independent of the nature of the fluid and of the force field causing movement.

Intrinsic permeability, k, has the dimensions length^2, whereas in equation 4.12, hydraulic conductivity, K, has the dimensions, length/time.

The two concepts are related by the following expression:

$$k = \frac{K\eta}{\rho g} \qquad .....4.13$$

or

$$= \frac{K\upsilon}{g} \qquad .....4.14$$

where:

k = intrinsic permeability of the medium (length^2).

K = the hydraulic conductivity (length/time).

η = dynamic viscosity of the fluid (mass/length time).

ρ = the density of the fluid (mass/length^2).

g = acceleration of gravity (length/time^2).

υ = kinematic viscosity.

It should be emphasised that the intrinsic permeability characteristics of a porous material are expressed by k (length^2) and not by K (length per unit time). The value of k is independent of the properties of the liquid, whereas K depends not only on the properties of the porous material, but also on the properties of the liquid.

Groundwater hydraulics deals predominantly with the flow of water, however, the equations used could apply equally as well under certain conditions to the flow of other fluids through porous or fractured media. The application to other fluids would of course have to take into account the relevant viscosity of that fluid.

In the petroleum industry oil and gas are the dominant fluids but on occasions the flow of water needs to be taken into account. In addition, the petroleum industry uses intrinsic permeability to describe the ease with the fluid moves through the solid matrix. The common unit used is the millidarcy. It is important then that the relationship between hydraulic conductivity and the millidarcy is defined in practical terms.

Because of the odd combination of units used at the time of Darcy's experiments the unit of intrinsic permeability is defined using a mixture of units.

A medium with an intrinsic permeability of 1 darcy permits a flow of 1 cm^3/s of a fluid with viscosity of 1 cP under a pressure gradient of 1 atm/cm acting across an area of 1 cm^2. A millidarcy is equal to 0.001 darcy.

Water has a viscosity of 1.0019 cP or 1.0019 x 10^{-3} daP at about room temperature.

Occasions arise where it is desirable to convert intrinsic permeabilities in millidarcys to equivalent hydraulic conductivities for the flow of water. An intrinsic permeability of 1 millidarcy translates to a hydraulic conductivity of 8.64 x 10^{-4} m/day.

Table 4-1 gives an indication of values for different materials.

Table 4-1 Indicative values of intrinsic permeability and hydraulic conductivity

Material	k (millidarcys)	K (m/day)
Clay	10^{-3}- 1	8.64 x 10^{-7} - 8.64 x 10^{-4}
Silt, sandy silts, clayey sands, till	1 - 10^2	8.64 x 10^{-4} - 8.64 x 10^{-2}
Silty sands, fine sands	10 - 10^3	8.64 x 10^{-3} - 8.64 x 10^{-1}
Well sorted sands, glacial outwash	10^3 - 10^5	8.64 x 10^{-1} - 8.64 x 10^1
Well sorted gravel	10^4 - 10^6	8.64 - 8.64 x 10^2

The channels through which the water particles travel in a mass of soil have a variable and irregular cross-section. As a consequence, the real velocity of flow is extremely variable. However, the average flow through such channels is governed by the same laws that determine the rate of flow through straight capillary tubes having a uniform cross-section.

If the cross-section of the tube is circular, the velocity of flow increases with the square of the diameter of the tube. Since the average diameter of the voids in soil at a given porosity increases practically in proportion to the grain size D it is possible to express K as:

$$K = \text{constant} \times D^2 \qquad .....4.15$$

From his experiments with loose filter sands of high uniformity (uniformity coefficient not greater than about 2), Allen Hazen obtained the empirical equation,

$$K(cm/\sec) = C_1 D_{10}^2 \qquad .....4.16$$

in which D_{10} is the effective size in centimetres of the sand and C_1 (1/cm.sec) varies from about 80 to 150. The lower values apply to medium sand, well sorted and the higher values to coarse sand, well sorted. Mid range values apply to poorly sorted coarse sand. It should be noted that equation 4.16 is applicable only to fairly uniform sands in a loose state.

The coefficient of permeability or the hydraulic conductivity of uniform unconsolidated materials may then be thought of as being proportional to the square of the effective grain size.

4.6 REYNOLD'S NUMBER

In pipe flow the transition from laminar to turbulent flow is characterised by well known values of Reynold's Number (N_R) which expresses the ratio of the inertia forces to the viscous forces. There is a lower limit critical number around 2,100 below which flow in pipes is always laminar.

By analogy, in flow through porous media a Reynold's Number has been established as:

$$N_R = \frac{vD}{\upsilon} \qquad .....4.17$$

where:

v = specific discharge i.e. discharge per unit area.

D = characteristic length. In pipe flow D is the internal diameter of the pipe. In flow through porous media D is related to grain size.

υ = kinematic viscosity of the fluid.

Because the grain sizes are so variable in flow through porous media no one value for Reynold's Number can be set as the dividing line between laminar flow and turbulent flow. It has been established that this transition occurs normally for a Reynold's Number in a range 1 to 10.

4.7 RANGE OF VALIDITY OF DARCY'S LAW

It must be emphasised that Darcy's Law is only valid for laminar flow in porous media.

For very low velocities, laminar flow occurs. There is a condition known as prelaminar flow for extremely slow percolation. However, this will not be considered in these notes. The upper limit of the applicability of Darcy's Law is when the Reynold's Number is in the range of 1 to 10. A range rather than a specific number is given because the distribution of grain sizes of natural media for a specified average grain diameter is limitless.

4.8 GROUNDWATER FLOW RATE

The previous sections have shown that when the water table is sloping, or has a gradient, then groundwater movement must exist and the magnitude of the resultant flow is dependant on the hydraulic conductivity of the aquifer material and the magnitude of the gradient.

To obtain some idea of the magnitude of this flow, assume a gradient of 2 metres per kilometre and a hydraulic conductivity of 0.5 m^3/dav/m^2.

v = K dh/dl

= 0.5 (2/1000)

v = 0.001 m/day

Assuming a hydraulic conductivity of 250 m^3/day/m^2.

v = 250 (2/1000)

= 0.5 m/day

As can be seen, the rate of flow of groundwater under natural conditions is relatively small and varies markedly from place to place as the gradient or the hydraulic conductivity varies.

It is possible to measure hydraulic conductivity both in the laboratory and in the field. Laboratory measurements are outside the scope of this course but types of field measurements such as pumping tests will be discussed later.

4.9 FLOW ANALOGIES

The equivalent of Darcy's Law, as expressed in equation 4.3, to the basic laws of heat and electricity flow has permitted the ready adaptation of solutions of complex heat and electrical flow problems to the flow of groundwater. The classic text "Conduction of Heat in Solids" by H.S. Carslaw and J.C. Jaeger has become a source of information for the solution of problems in groundwater hydraulics.

The equivalents of flow of groundwater, heat and electricity is very useful in the modelling of groundwater problems.

Heat Equivalent

Fourier's Law

$$f = -K\ dv/dl \qquad .....4.18$$

where:

f = a flux, or heat flow, per unit area.

K = the thermal conductivity.

v = temperature.

l = length of flow path.

Electrical Equivalent

Ohm's Law

$$I = -C\ (dE/dl)$$

where:

I = current (amperes) per unit area.

C = electrical conductivity (l/R), (mho cm) per unit area.

R = electrical resistance (ohms) per unit area.

E = electrical potential (volts).

l = length of flow path (centimetres).

4.10 TYPES OF GROUNDWATER FLOW

Laminar Flow (Darcy Flow)

Darcy Flow is laminar and any head loss in overcoming frictional resistance for laminar flow is proportional to the discharge. This flow conforms to Darcy's Law. Occasions do arise where the flow is so low that other forces such as surface tension become dominant and Darcy's Law does not apply. These cases are not dealt with in these notes.

Turbulent Flow (Non-Darcy Flow)

If the velocity becomes large enough, and Reynold's Number exceeds some value between 1 and 10, the fluid flow becomes turbulent and Darcy's Law no longer applies. Head loss associated with overcoming frictional resistance for turbulent flow is normally proportional to the discharge squared.

4.11 STATES OF GROUNDWATER FLOW

There are two general states of groundwater flow. These are Steady State flow and Non-Steady State flow.

Steady State Flow

Steady State or Equilibrium flow occurs when the discharge from the bore is in equilibrium with the recharge to the aquifer system.

Examples of when steady state flow might occur are:

1. when the radius of influence from a bore located in the centre of a circular island intersects the sea; or
2. after extensive pumping in a leaky aquifer.

Steady State flow does not occur very frequently. In practice it is assumed to occur when the change of drawdown with times becomes negligible, after corrections have been made for outside influences, such as tides, which might affect the water levels. Some analysts more loosely define steady state as occurring when the rates of drawdown in the pumping bore and the observation bores are the same. This is not really steady state as water is still being drawn from storage from within the radius of influence of pumping.

<u>Non-Steady State Flow</u>

All flow which is not in equilibrium with the recharge to the aquifer is called a Non-Steady State flow. As long as there is a measurable change in drawdown with time the flow is classed as non-steady. In practice, this is the most common type of flow associated with discharging bores.

SECTION 5: BORE DISCHARGE TESTS

5.1 INTRODUCTION

Various methods of estimating the aquifer parameters of hydraulic conductivity, transmissivity and storage co-efficient were presented in Section 4. However, it must be stressed that these were very much estimates and based on such approximations as; the effective grain size of a disturbed sample of aquifer material instead of the undisturbed sample in the aquifer itself, the thickness of the aquifer at one bore site rather than the varying thickness of the aquifer throughout the whole of the groundwater system, an assumed value of porosity throughout the whole system and assumed value of bulk modulus for the solid matrix. Such estimates should be used only in the absence of more accurate information.

By far the most accurate method of determining these aquifer parameters is by means of a pumping test. The pumping test will not give a precise value of the parameters at each any every point within the aquifer. However, because of the large areal extent of the radius of influence of the pumping bore, the drawdown responses observed at the pumping bore or observation bore will represent the average effect of all transmissivities and storage coefficients encountered by that radius of influence. They will be representative of the aquifer as a whole.

This chapter on testing of bores by pumping and by slug tests gives a brief overview of the design and carrying out of pumping tests. The reader is assumed to have a basic understanding of groundwater hydraulics. The main references used for the lecture notes are Kruseman and de Ridder (1990), Dawson and Istok (1991) and Hazel (1975). Further reading material is given in the reference list. You are referred also to the Australian Standard® on Test Pumping of Water Wells (AS 2368-1990).

5.2 BACKGROUND

Generally pumping tests are carried out on bores for one of two reasons:

- to determine the hydraulic characteristics of the aquifer. These aquifer characteristics, such as Transmissivity and Storage Coefficient, are used to determine the ability of the aquifer to store and transmit water and hence assess its response to stresses such as recharge and discharge; or
- to determine the long term pumping capability of the bore itself under sustained pumping. This is important for the correct selection of pumping equipment for commissioning of the bore or for the determination of the number and spacing of bores to achieve a required objective.

Occasionally pumping tests are carried out for other reasons including:

- to determine the existence and location of sub-surface boundaries which may affect adversely or beneficially the long term pumping performance of a particular bore;
- to check on the performance of a particular groundwater basin; or
- to determine the radius of influence of a bore for dewatering or interference purposes.

On occasions it is not feasible or possible to carry out a pumping test. (The bore diameter may be very small and preclude the insertion of pumping equipment or the hydraulic conductivity of the aquifer is such that the pumping rate would be exceedingly small.) However, it may be essential that the hydraulic conductivity be determined. In cases such as this slug tests are frequently used to calculate the hydraulic conductivity in the near vicinity of the bore being tested. Such determinations are representative of area in the immediate vicinity of the hole and not of the aquifer in general.

Whether flowing or non-flowing conditions are concerned, the testing process involves the removal of water from the aquifer at a controlled rate whilst monitoring the head or change in head (water level or potentiometric level) response over time.

5.3 DEFINITIONS

Before proceeding with the various types of tests a number of terms will be defined.

Standing Water Level (S.W.L.)

The depth from ground level to the water level in a non pumping bore outside the area of influence of any pumping bore.

Static Head

This is the flowing bore equivalent of S.W.L. It is the height above ground level that water at a particular temperature would stand if the casing were extended upwards. It is expressed as meters or kilopascals.

Drawdown

The distance that the water level in a bore or well has been lowered from the S.W.L. during pumping. It is measured at specified time after pumping commenced.

Residual Drawdown

The distance that the water level in a bore or well remains lowered from the S.W.L. after pumping ceases. It is measured at specified times after pumping stopped.

Recovery

The amount by which the water level in a bore has risen at a given time after pumping ceased. It is the difference between the Residual Drawdown after the given time and the hypothetical drawdown if pumping had not ceased. When the water level returns to S.W.L. recovery is said to be complete.

Available Drawdown

For a particular pump installation this is the distance between the S.W.L. and the pump suction or the depth of water over the pump suction.

5.4 FLOWING AND NON-FLOWING BORES

For a long time, flowing and non-flowing bores were regarded by some authorities as being completely different from one another and as a result the techniques used in testing and analysing them were different. This idea is entirely wrong. The only basic difference between a flowing bore and a non-flowing bore in a confined aquifer is that the ground level is lower than the standing water level (or static head) in the case of a flowing bore and higher than it in a non-flowing bore as shown in **Figure 5-1**. If, in the case of a flowing artesian bore, the casing were extended high enough above the ground the water would stand in the casing and a pump would have to be installed to discharge the water. The groundwater hydraulics are the same for both types of bores and no differentiation will be made between the hydraulics of flowing and non-flowing bores in later chapters on evaluation of aquifer properties.

It could be concluded that the idea of different of facilities is being proposed in this course because separate sections have been allocated to tests on flowing and non-flowing bores. This is not so. It is true that the methods of measuring pressure in flowing bores can be different from the normal methods used for non-flowing bores and other measurements such as temperature may be required. However, the only reason for having a separate for tests on flowing bores is that they are so much easier to test (no pumping equipment is required) that a multitude of tests can be carried out with a minimum of fuss.

The Australian Standard AS 2368-1990 exists to provide the minimum specification in terms of procedures, measurements and other observations required when designing and performing a pumping test. These procedures were included in Hazel (1975) and are set out at the end of this chapter.

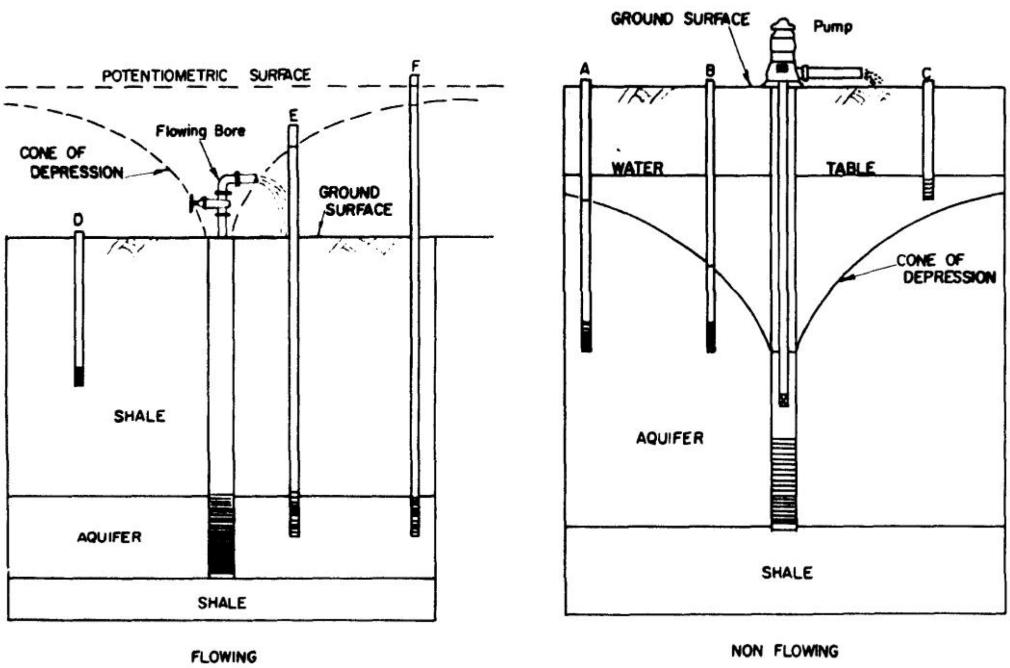


Figure 5-1 Flowing and non-flowing bores

5.5 PLANNING A PUMPING TEST

5.5.1 Test Design

To help in understanding the reasons for a number of the comments below typical cross sections of both a confined aquifer and an unconfined aquifer under pumping conditions are presented in **Figure 5-2** and **Figure 5-3**.

Basically, the field procedure requires that a pump be installed in the bore or for flowing bores, a valve fitted to control flow. Once discharge has commenced observations of water levels from the test and observation bores are made at selected times. Data are obtained in a way that is suitable to plot on a logarithmic time scale. Analysis of plotted data solves for the constants in the flow equation which represent the hydraulic parameters of the aquifer. This may appear to be straightforward, however, many factors including the accuracy and ambiguity of data, and the skill, experience and understanding of the analyst will determine the success of the analysis. Methods of analysis are presented in Chapters 10 and 11.

Proper test design is essential for the successful determination of aquifer properties and for the long term pumping performance of a bore. The steps that follow are presented with the assumption that an initial site investigation has been completed and that the geology and the aquifer types and water levels are known and there are no local authority reasons why a bore should not be drilled or tested.

The test duration could depend on the type of aquifer being tested. If a simple confined aquifer is being tested then the drawdown-log time relationship may settle down very quickly. However, if the aquifer exhibits leakiness, delayed yield, boundary effects or the like then the test duration may have to be extended. These effects will only become apparent during the test and for this reason it is prudent to get into the habit of plotting the test results on semi-logarithmic paper as the test progresses.

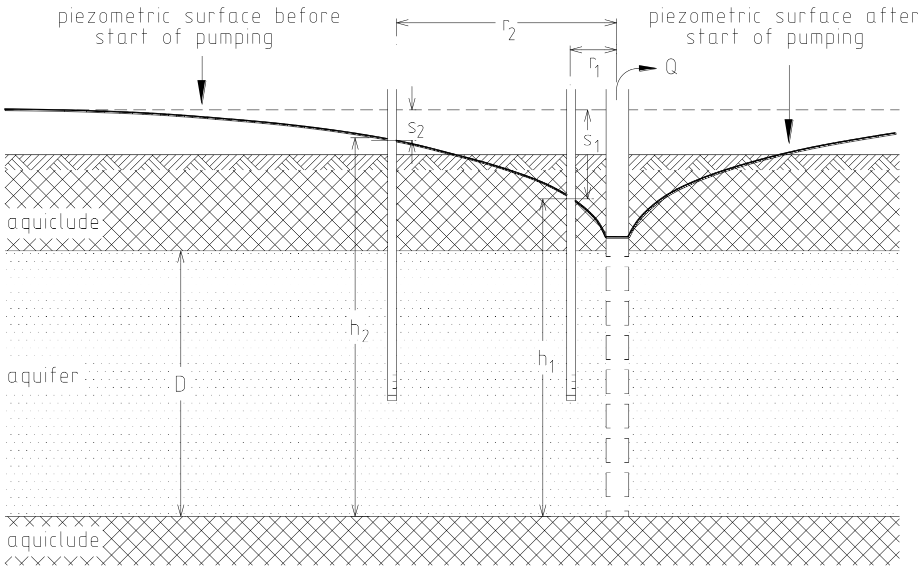


Figure 5-2 Cross section of a confined aquifer (after Kruseman and de Ridder, 1990)

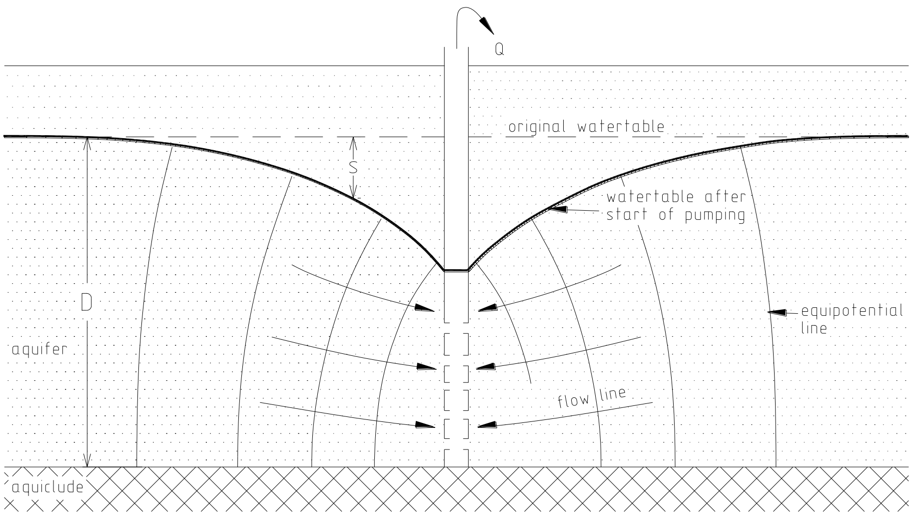


Figure 5-3 Cross section of an unconfined aquifer (after Kruseman and de Ridder, 1990)

5.5.2 Identify Site Constraints

Conditions at the site impose constraints on pumping test design which should be recorded and taken into account prior to the start of the design process. A few examples, some of which are irrelevant if the bore is already constructed, are:

- limitation of placement (e.g. buildings, roads, railway lines or other bores);
- limitation on pumping rate(s);
- limitation on test duration;
- local authority limitation on discharge of water;
- location of the point of discharge for the pumped water to prevent recirculation;
- use of an existing bore (this may limit number and location of observation bores, problems with effects of bore storage, head losses); and
- limitation on the placement of bores due to known presence of aquifer discontinuities, presence of recharge and discharge zones.

These limitations add complexities to the design of pumping tests and to the selection of the best means of analysing them.

5.5.3 Purpose of the Test

If the purpose of the test is to ascertain aquifer parameters then a list of the required aquifer parameters is needed to select an appropriate test type. For the determination of some parameters observation bores will be required.

If the purpose of the test is to determine the long term pumping capability of the bore them a much simpler testing procedure can be carried out and observation bores may not be required.

5.5.4 Specify Test Conditions

With knowledge of why the test is being carried out the test conditions can be specified. These include: type of test, diameter of the pumping bore if the bore not been drilled, the pumping rate, the number and location of observation bores, the depth and screen length of bores, and test duration (Driscoll, 1986; Kruseman and de Ridder, 1990). Some of these conditions are discussed below. Once they have been decided upon and the bore is ready for testing, all of the conditions relating to the bore and ancillary equipment should be recorded on a Bore Setup Sheet, an example of which is given in Attachment A. It should be stressed that the test design is an iterative process.

5.5.5 Pumping rate and bore diameter

If the bore has not yet been drilled it may be necessary to select a bore casing diameter and a likely pumping rate. The selected pumping rate should be large enough to insure that drawdown can be measured accurately in the bores, but the selected rate should not result in excessive drawdown. For unconfined aquifers ideally the water table should not be lowered by more than 25% (Ferris, 1962). However, on occasions this may have to be sacrificed in favour of higher productivity. The pumping rate may be selected in a number of ways:

- by using empirical equations (see Driscoll, 1986);
- by using the analytical solutions of the models to predict drawdown for a range of assumed pumping rates;
- by carrying out short tests before the test proper; or
- by seeking advice from the driller.

A preliminary selection of pumping bore casing diameters is shown in **Table 5-1**.

Table 5-1 Recommended bore casing diameters (after Driscoll, 1986)

Pumping Rate , (m^3/d)	Bore Casing Diameter, (mm)
<545	152 ID
409-954	203 ID
818-1910	254 ID
1640-3820	305 ID
2730-5450	356 OD
4360-9810	406 OD
6540-16400	508 OD
10,900-20,700	610 OD
16,400-32,700	762 OD

5.5.6 Bore Depth and Bore Screen

The depth of a new bore will usually be determined from the log of an exploratory bore or from logs of nearby existing bores. The bore should be drilled to the bottom of the aquifer, if possible.

The bore screen, slots or perforations should be designed to keep the entrance velocity low. This is discussed further in Section 8. As a general rule bores should be screened over at least 80% of the aquifer thickness because at this screen length the groundwater flows towards the bore can be assumed to be horizontal. However, the length screened depends very much on required use for the bore.

Exceptions are:

- in unconfined aquifers, it is common practice to screen only the lower half or lower one-third of the aquifer;
- if the aquifer is relatively shallow and the available drawdown is not large then it may be better to screen the lower one-third of the aquifer of the aquifer to get a better discharge rate from the bore. This may result in higher turbulent head losses but would result in a more productive bore;
- in very thick aquifers, the length of the screen will be less than 80% (too expensive). Such a partially penetrating bore induces vertical flow which can extend outward from the bore to distances about 1.5 the thickness of the aquifer. Within this radius, measured drawdowns have to be corrected. The effect of vertical flow can sometimes be reduced by screening at intervals throughout the aquifer rather than having the total length of the screen in the one section;
- if the bore is to be used for dewatering purposes then the drawdown and pumping rate from the bore have to be optimized; or
- bores in consolidated aquifers may not need a screen.

5.5.7 Observation Bores and Piezometers

One of the methods available to determine an aquifer's ability to store water is to analyse drawdown observations taken in nearby bores or water level measuring points during a pumping test. The storage co-efficient cannot be determined accurately from drawdown data from the pumping bore itself.

Be mindful that most of the information presented in this section is for flow in porous media. It does not necessarily apply to flow in fractured rocks. In particular, measurements of drawdowns in observation bores taken during a pumping test of a bore drilled in a fractured rock aquifer may do little more than indicate what the drawdown effect is in that direction from the bore. It tells nothing about drawdown effects in other directions. The flow in fractured rock aquifers can be very directional rather than radial in nature.

In the case of routine groundwater investigations, observation bores are invariably drilled to allow this information to be obtained. Drawdown readings should be taken at approximately the same intervals as for the production bore. Unless data loggers are used it will not be possible, without excessive use of manpower, to measure drawdowns at exactly the same times in each bore. While attempts should be made to measure them at approximately the same times as drawdowns are measured in the production bore, it is more important that the actual times at which the measurements were taken should be recorded.

When testing a private facility the opportunity should be taken to measure drawdown in adjacent bores. If this is to be done, the S.W.L. should be measured in the adjacent bore before pumping commences.

The location of the observation bore should be determined by GPS if possible. If this is not possible then the distance from the pumping bore should be measured and recorded and the relative locations shown on the bore setup sheet.

In some detailed investigations it may be necessary to use more precise water level measuring points. While these are still water level observation points they are commonly referred to as piezometers. A piezometer is an open-ended pipe (normally of small diameter, say 50mm) with a screen, 0.5 to 1 m long fitted to the bottom. The water levels measured in piezometers represent the average head at the screen of the piezometer. In a heterogeneous aquifer, multiple aquifers or even in a thick homogeneous aquifer, a number (or cluster) of piezometers may be able to be placed at different depths in one bore if the diameter of the bore hole is large enough and the water inlet section of each piezometer is hydraulically isolated from the inlet of each of the others. The hole in which the cluster has been installed is normally is left cased above the uppermost piezometer. In such cases care has to be taken to ensure complete isolation of the separate piezometers or vertical transfer of water can occur and invalidate the measurements. Such an arrangement of piezometers in a homogeneous aquifer may be used for example, to determine the vertical component of flow beneath a source of contamination on the surface.

The number of observation bores depends on the reason for test, the required degree of accuracy and available funds. It is always preferable, but not always possible, to have many observation bores, (at least three are recommended). The advantage is that the drawdowns measured can be analysed in two ways: by time-drawdown relationship and by distance-drawdown relationship. However, if the objective of the test is to determine the long term pumping rate then observation bores may not be required.

An observation bore should never be located closer than one aquifer thickness from the pumping bore. Many suggestions have been made concerning the spacing of observation bores, one of which is that they should be located at distances b, 2b and 4b, from the discharging bore where b is the aquifer thickness. If possible, observation bores should be located at such distances from the pumping bore as to provide a reasonable spread of points when plotted on logarithmically in distance-drawdown plots.

If a substantial gradient exists in the potentiometric surface, then bores should be located up slope and down slope from the discharging bore. In unconfined aquifers two bores are desirable at each distance, one located near the top and the other near the bottom of the aquifer. However, the distances at which observation bores should be placed depends on the type of aquifer, its transmissivity, the duration of pumping, discharge rate, the length of the bore screen and aquifer stratification (for details see Kruseman and de Ridder, 1990 and AS 2368-1990).

The depth of observation bores is also important. Ideally, in an isotropic and homogeneous aquifer, if possible, the depth of the observation bore should coincide with depth to the mid point of the bore screen. However, in some cases existing bores are used as observation bores and as long as they are at a suitable distance from the pumping bore they should be satisfactory. For heterogeneous aquifers the use of a cluster of piezometers is recommended. Observation bores should also be placed in an aquitard to check whether its water level is affected by pumping in the aquifer. This information is needed for leaky aquifer tests (Kruseman and de Ridder, 1990).

5.6 MEASUREMENTS

The adequate evaluation of a pumping test relies very much upon the recording of a number of sets of measurements throughout the test. These include measurements of:

1. Time.
2. Discharge.
3. Water level or Head.
4. Temperature.
5. Water quality.

To analyse a pumping test accurately each set of measurements must include the recording time, head and discharge. The values obtained for the aquifer characteristics cannot be of greater accuracy than that of the basic data. Care should be taken then in the measuring and recording of time discharge and head. For hot flowing bores measurements of temperature should be recorded as well.

Ideally, the natural fluctuation in hydraulic head of the aquifer should be known before the test commences (e.g. hydrographs). This information can be used to correct the drawdown observed during the test.

In coastal aquifers where the hydraulic head is affected by tidal movements, a complete hydrograph should be obtained, including maximum and minimum levels.

For long-term tests (days), the levels of near-by surface waters and any precipitation should also be recorded and, if the aquifer being tested is a confined, the barometric pressure should recorded as well.

5.6.1 Time

The time measurements are normally started from the beginning of the test and may be recorded as time of day or time since the test started It is desirable to record the time of day as this not only gives the location of the set of measurements in any one test, but also indicates the relationship between that test and any other test which may have been carried out on the same day. This allows correction for any antecedent pumping conditions.

Time is recorded as time of day and date. In addition the time in minutes after a particular test starts should be recorded, but if an accurate record of time of day is kept, this is not essential.

5.6.2 Water Levels/Heads

The water levels must be measured many times during a test, and as accurately as possible. The head measurement may be expressed in terms of water level, drawdown, or for a flowing bore, the back pressure. At the beginning of a test, water levels or pressures will drop rapidly and the readings should be made at short time intervals. Suggested time intervals for water level measurements in the pumping bore and observation bore, respectively are given later for each individual test (see also Kruseman and de Ridder, 1990; and AS 2368-1990).

After some hours of pumping, the results can be plotted as time-drawdown curves on log-log and semi-log paper. This can help to evaluate the progression of the test and to decide pump shutdown time. These initial plots will also give an indication of the type of aquifer and the presence of boundaries. When the pump is shut down the water level recovery can be measured in what is known as a recovery test.

Table 5-2 Suggested durations for discharge tests

Bore Use or Reason for Test	Pumping Duration (hours)	Recovery Duration (hours)
Stock and Domestic	4-6	2-4
Irrigation	24	6
Town Water Supply Bore /Industrial	100	24
Remeasurement	24	2

5.6.3 Discharge Rate

Discharge measurements are made using:

1. Time to fill a container of known volume.
2. Weir Boards.
2. Orifice Bucket.
3. Orifice Meter.
4. Flow Meter.

These are written in ascending order of accuracy.

5.6.4 Temperature

This is important in flowing bores only. When a hot bore stops flowing the water cools down and as it cools it becomes denser than the hot water so that a decrease in measured pressure results. This is often the cause of the falling off in pressure during the later stages of a static test. This effect has not been fully appreciated in the past but it is now obvious that a systematic programme of water temperature reading must be incorporated in any test programme for flowing bores.

The temperature of the water should be taken as near to the discharge point as possible and preferably in the mouth of the orifice meter at the following times:

1. at the beginning of the flow recession test, just after opening;
2. at the end of the flow recession test, immediately before closing for the static test;
3. during the first minute of the first stage of the opening dynamic test;
4. at each set of measurements in the first stage of the opening dynamic test; and
5. at the end of each other stage of the opening or closing dynamic test.

5.6.5 Water Quality

It is not enough to know the aquifer's ability to store and transmit water or even its ability to provide water on a sustainable basis. All of this may be to no avail if the water quality is not suitable for the required use. Water samples should be taken during the test to assess its quality.

Samples should be taken at the start and end of the discharge stage of the test. One or both of these samples should be dispatched to the laboratory for the appropriate analysis. Details of sampling, storage, preservation and analyses required are presented in Section 15.

It is also desirable to carry out field measurements of water quality during the test to see if changes in water quality are occurring. Parameters which are easily determined in the field by hand held instruments include electrical conductivity, pH, total dissolved salts and temperature.

5.7 SETUP AND INSTRUMENTATION

The setup stage involves the installation of the pumping and ancillary equipment, and the appropriate instrumentation to measure time, discharge and water level or pressure head. Where the discharge involves hot water (>40^{o}C), temperature should be measured to enable a correction of pressure heads to be made. If observation bores are monitored, the distance and direction to the pumped bore should also be recorded.

For all measurements, the instrumentation selected should be such that a relative level of accuracy is maintained. For example, when measuring time to the minute, an error of less than 5 seconds is acceptable, whereas when measuring hourly, the error allowed is up to one minute. Similarly, the magnitude of discharge will dictate the facility for measurement. The error involved will increase with the 'coarser' devices; however, the relative error should be consistent.

Instantaneous discharge measurements are desirable, so the use of equipment such as the flow meter, orifice meter or orifice bucket is advocated. Data loggers are now used to keep a continuous record of discharge rate. Examples of discharge measuring devices include flow meters and buckets or drums of known volume for smaller flows, to orifice tube with piezometer (also know as an orifice meter or a sharp crested circular weir), flow meters and calibrated sharp crested weirs (e.g. V-notch) for larger flows. Allowable limits for errors are discussed by Stallman (1971). Discharge (10%), water level (5 mm), time (1%) and distance to observation bores (0.05%) are acceptable.

Figure 5-4 illustrates examples of these devices. AS 2368-1990 also gives a diagram of an orifice bucket which is quite accurate and useful for measuring discharges of aerated water.

Water levels in non flowing bores are measured by a variety of apparatus the most common being the type whereby an electrical circuit is completed when the probe touches the water surface. It is commonly referred to as a "dipper". Transducers connected to data loggers are becoming increasingly popular. However, when using transducers check that the sensitivity of the transducer is compatible with the required sensitivity of the water level measurement. It is also possible to measure water levels by means of an air line. If air is bubbled down the line a pressure measuring device at the surface will indicate the head of water being displaced. Remember to keep the air bubbling during the recovery phase otherwise the rising water in the bore will rise up the air line and give a false reading.

In the case of flowing bores it is not feasible to measure water levels. The pressures measured are measurements of back pressure on the inlet side of the control valve.

Most bores have a reasonably high back pressure (which reduces as the flow increases) and the most accurate method of measuring it is by use of a transducer or a pressure gauge. A pressure gauge may need to be calibrated quite regularly between tests.

If the back pressure is very small a water tube could be used i.e. the back pressure is the height that water rises up the tube. Normally back pressures of bores being tested are not low enough for the use of a water tube.

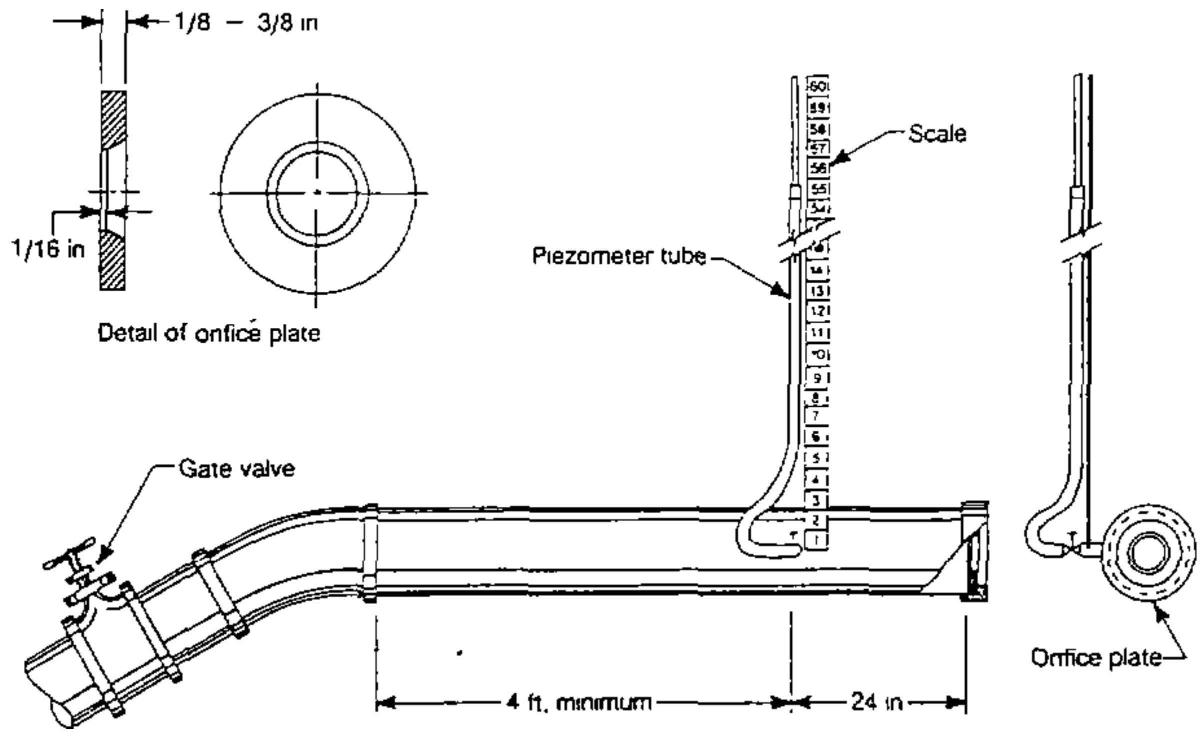
Several common methods for water level (or potentiometric head) measurement are listed below (see **Figure 5-5**):

- Electrical contact probe with tape - a meter or light indicates closed circuit on contact with water. This commonly known as a dipper.
- Acoustic device with tape - "plopper" makes sound on contact with water surface.
- Pressure transducers - the magnitude of the electrical signal response of the device is proportional to its depth of immersion. It may be connected to a data logger for remote and continuous water level recording.
- Pressure gauge - installed at bore head of flowing bore. Need to convert readings to metres head of water.
- Float device with tape - floats on water surface.
- An air line.

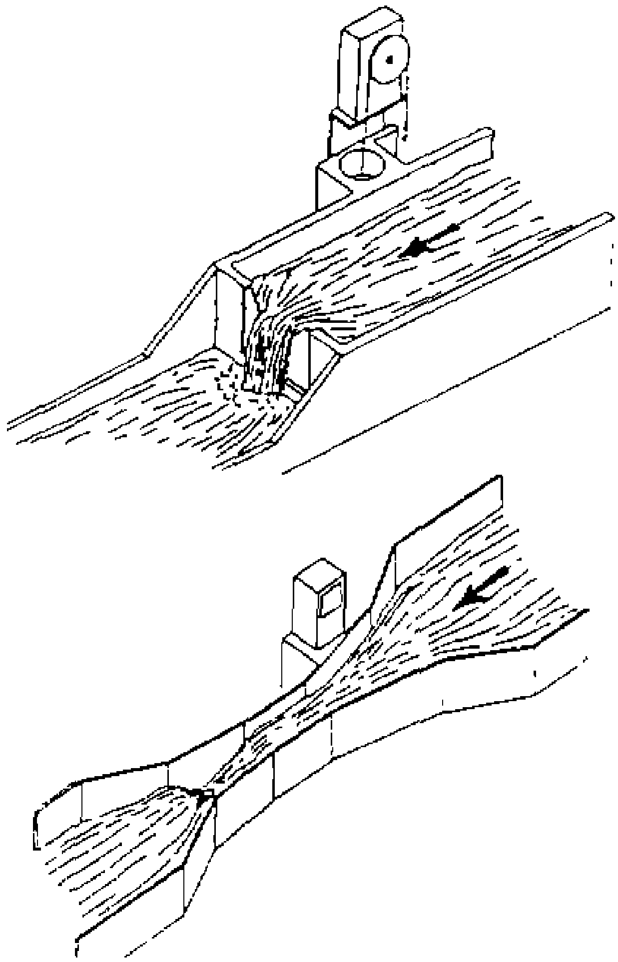
For pumped bores, the type of device utilised will depend firstly on the available clearance in the hole once the pump is installed, and secondly the suitability of the device. The electrical contact device is probably the most common as it is small and inexpensive to make. Conduit to house the device down hole is advised since there is a tendency for the cable to entwine the pump column. Pressure transducers are also popular. For observation bores, pressure transducers may be preferred since once established before commencement of pumping, they can provide continuous reliable data throughout the test and present an avenue for reducing labour intensity on-site. Data stored on a data logger can also be transferred to a computer quite easily for analysis.

Instrumentation should also be available to monitor various aspects of the water chemistry. Parameters such as pH, conductivity, temperature, DO, Eh and dissolved CO2 may require monitoring. Other characteristics of the discharge such as colour, smell and sediment load should be noted.

Primarily, suitable pump selection is based on its capacity to meet the range of drawdown and discharge expected. With appropriate planning, the bore casing size will have been anticipated to adequately house the pump. The nature of the discharge may also need to be considered as some types of pump do not handle heat or sand for example, as well as others. An example of a pumping installation in a borehole is illustrated in **Figure 5-6**.



Orifice Tube and Piezometer (Orifice Meter)



V-Notch Weir and Flume

Figure 5-4 Common discharge measuring devices

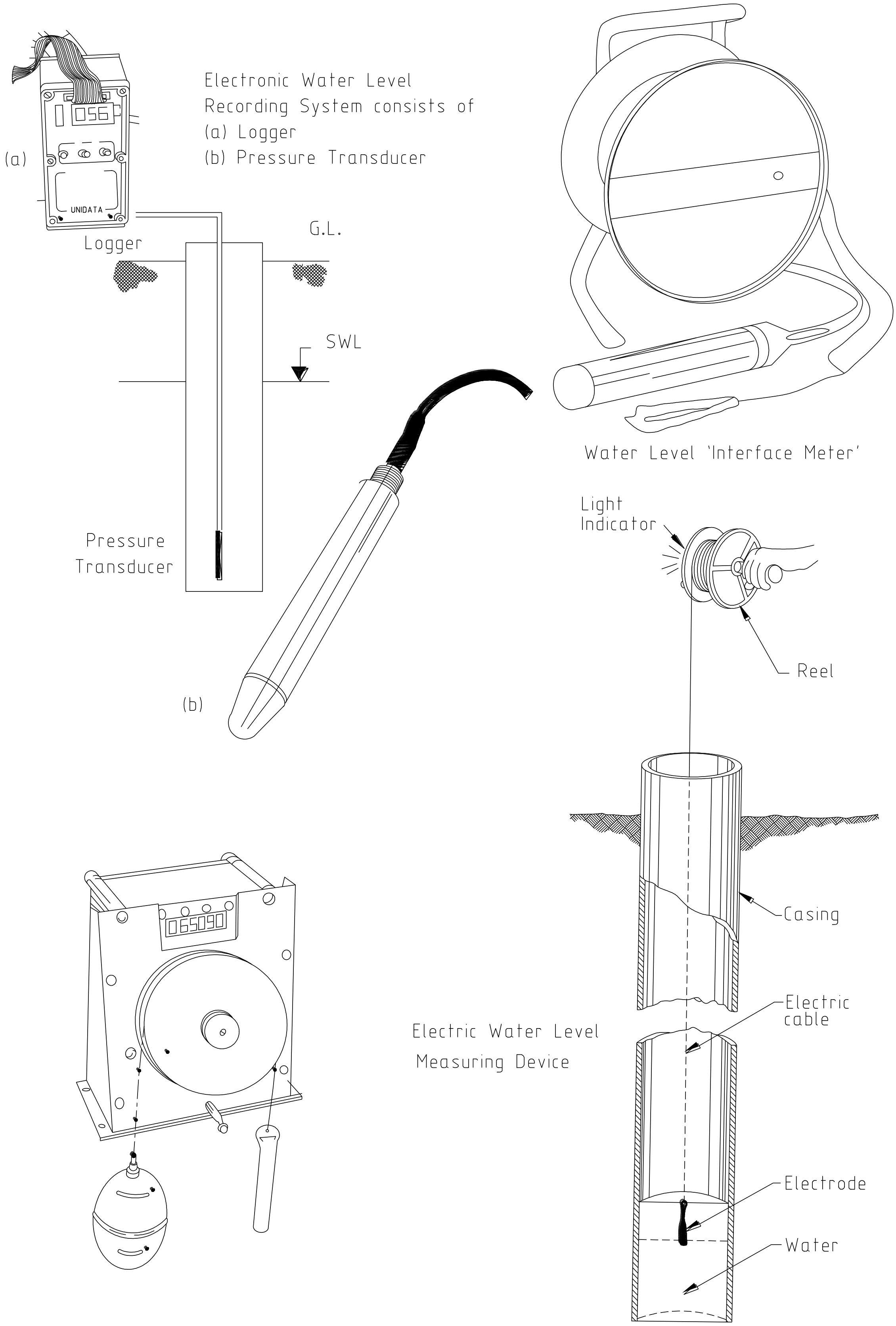


Figure 5-5 Some common water level measuring devices

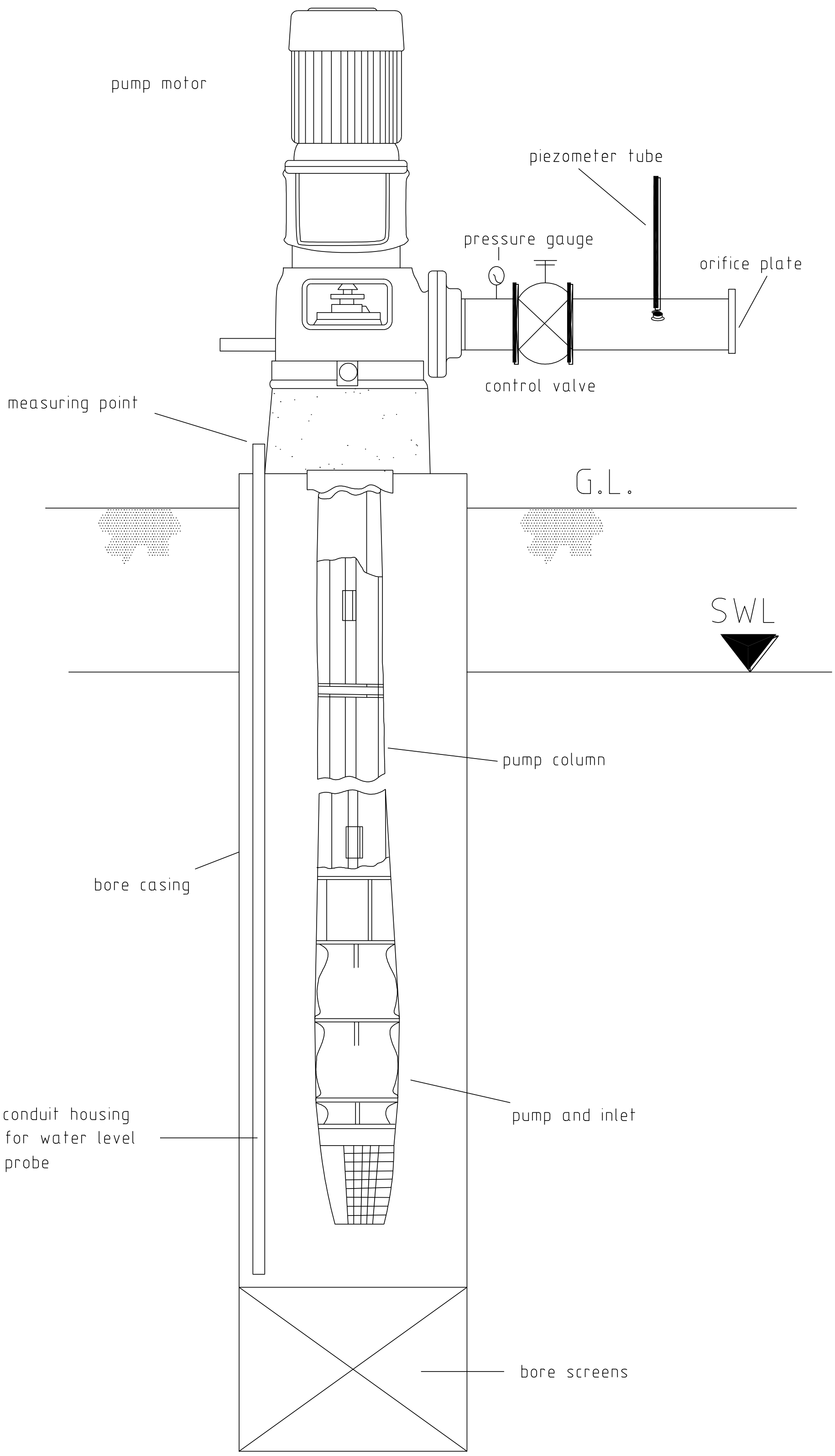


Figure 5-6 Typical bore hole pump installation

5.8 DATA RECORDING AND PRESENTATION

A good quality of presentation of data retrieved from the field is required at all times. It should be set out clearly with all relevant measurements recorded. An example test data sheet is given in Attachment B. Record the precise details of time of measurement (exact time is noted if the prescribed time is not possible or missed), discharge and drawdown measurements are required, as well as SWL, measuring point and observation bore distance. Details of the bore construction and conditions to be encountered should be studied beforehand and may also be listed.

A graphical presentation is the most effective means of data representation as it facilitates most analysis techniques and also immediate visual interpretation. Data may be plotted once the appropriate corrections (if any) have been applied. Graphs of drawdown or recovery versus log time for the pumped bore and each observation bore should be plotted for each test. For observation bores, plotting log drawdown against log time (on the same scale as type curve) will enable comprehensive analysis and determination of transmissivity, storage co-efficient, leakage, drainage and other parameters. Drawdown versus log distance may be plotted if there are a number of observation bores. In addition to this data, plots of barometric pressure, rainfall, river stage, other pumping influences and antecedent water levels may be graphically represented. It is also useful to provide a sketch of the location and distances of observation bores, boundaries and topography or provide their locations with a GPS instrument.

5.8.1 Possible Corrections to Drawdown Data

The field data may have to be converted into appropriate units before they are processed. The units of the International System are recommended.

Before being analysed the observed water levels may have to be corrected for external influences. It is therefore important that the local trend in the hydraulic head or water table is known. These external influences and applied corrections will be discussed in Section 10 and in Section 12 (see also standard hydrogeological textbooks such as Fetter, 1980; Freeze and Cherry, 1979; Todd, 1980). Common influences which may require correction are:

- unidirectional variations such as natural recharge and discharge;
- tidal and barometric fluctuations of hydraulic head and momentary fluctuations caused by passing trains;
- drawdowns causing dewatering of an unconfined aquifer;
- temperature variations in a hot flowing bore; or
- unique fluctuations such as a sudden rise or fall of nearby surface waters which are hydraulically connected with the aquifer, heavy rains.

5.9 TESTING NON-FLOWING BORES

5.9.1 Antecedent Conditions

It is most important for the analysis of the test of a bore to know what variations had taken place in the discharge during the 24 hours or so previous to the test, as such variations may continue to have some effect on the bore and these effects must be accounted for in the analysis. Before doing anything to the bore, therefore it is necessary to record the condition of the bore as it is found – has it been pumped in the last 24 hours? What is the standing water level? Are there any bores in the near vicinity that are pumping or have been pumped in the last 24 hours? Are there any rivers or lakes in the near vicinity? Has there been any substantial rainfall during this period? Any information obtained should be recorded and reported.

5.9.2 Constant Discharge Test

As the name implies this test involves pumping the bore at a constant discharge rate and measuring the varying drawdown throughout the test.

With the discharge held constant, drawdown measurements are taken during the test at the following times after pumping commenced - 1, 2, 3, 4, 6, 8, 10, 20, 25, 30, 45, 60, 75, 90, 100, 120 minutes then each half-hour to six hours and then hourly until the end of the test. If for any reason a reading at any of the above times is missed, then a reading should be taken as soon as possible thereafter and the actual time of this reading should be recorded.

Rate measurements are taken at least at the start of the test, after 15 minutes, 30 minutes and every half-hour thereafter, but at more frequent intervals if possible. However, care should be taken to maintain the discharge rate constant throughout the test by regular inspections of the tube on the orifice meter or orifice bucket if these are being used.

5.9.3 Recovery Test

At the end of the constant discharge test residual drawdown measurements are taken, if possible, at the following times after pumping ceases - 1, 2, 3, 4, 6, 8, 10, 15, 20, 25, 30, 45, 60, 75, 90, 100, 120 minutes then hourly until the water level has come back to within 15 centimetres of the standing water level to 80% of recovery. If for any reason a reading at any of the above times is missed, then a reading should be taken as soon as possible thereafter and the actual time of this reading should be recorded.

5.9.4 Constant Drawdown Test

In this type of variable discharge test the drawdown is held at a constant value and variations in discharge are measured. This type of test may be required when it is not possible to measure drawdowns. The drawdown could be maintained at a constant level with the pump.

The drawdown is held constant by making sure that the pump breaks suction soon after the test begins. The water level is then maintained at the pump suction throughout the test.

Because of the air/water mixture for this type of test an orifice meter cannot be used with any reasonable degree of accuracy for the measurement of discharge and it is preferable to use a container of known volume or, if one is available, an orifice bucket.

If an orifice bucket is used, rate measurements should be taken at the same intervals as drawdowns were taken during the constant discharge test, i.e. 1, 2, 3, 4, 6, 8, 10, 15, 20, 25, 30, 45, 60, 75, 90, 100, 120 minutes then each half-hour to six hours and then hourly until the end of the test.

Regular checks should be made to see if the pump is maintaining a constant drawdown, i.e. the pump is breaking suction throughout the test.

On the completion of the pumping test, residual drawdown should also be taken, if possible, at 1, 2, 3, 4, 6, 8, 10, 15, 20, 25, 30, 45, 60, 75, 90, 100, 120 minutes then hourly until the water level comes to within 15 centimetres of the standing water level or 80% of recovery.

However, in many cases, a constant drawdown test is only carried out because it is not possible to measure drawdowns and in these cases, so it may not be possible to measure residual drawdowns either.

If a container of known volume is used to measure the discharge rate then measurements should be taken as soon as the pump breaks suction then at 5, 15, 30, 45, 60 minutes and every half-hour during the remainder of the test. The size of container should be suited to the likely discharge, since it is important to complete the measurement in as short a time as accuracy permits, e.g. a 500 litre tank is unsuitable for a discharge of 5 cubic metres per day - a 1 litre container would be a better choice.

As the duration of the measurements may have a bearing on the plotting and analysis of the test, the actual time at which the measurements were commenced should be recorded in the remarks column.

5.9.5 Step Drawdown Test

In this type of variable discharge test the discharge is varied in controlled stages. The discharge rate is maintained at a constant value within each stage. The discharge could be increased or decreased.

The advantage of this test is that the relationship between drawdown, laminar flow and so called, turbulent flow can be determined accurately and the satisfactory pumping rate for any specified drawdown can be ascertained. This knowledge is particularly desirable where the test rate is considerably less than the rate at which the bore is to be equipped.

The test is normally carried out in steps, either increasing or decreasing the discharge from one step to another. Drawdown measurements should be taken throughout each of the steps, with each step regarded as a new stage. For example, if the first step ended at 60 minutes, then drawdown measurements in the second step should be taken at 61, 62, 63 minutes etc. In other words, the time intervals between drawdown readings in each step should be the same as for the Constant Discharge Test, but the actual time since commencement of the test should be recorded. Actual clock time for the commencement of the test should be recorded also.

On completion of the Step Drawdown Test, residual drawdown measurements should be taken as for the Constant Discharge Test.

5.9.6 Step Drawdown Test (Extended First Step)

One of the main reasons for testing a potential irrigation or town water supply bore is to determine its long term pumping rate. If this rate is in excess of twice the test discharge rate then non-linear head loss could become a very significant part of the drawdown and a straight comparison of test drawdown and available drawdown for estimation of long term yield is not valid. A long period of pumping at constant rate is desirable to determine the likely effect of delayed yield in unconfined aquifers, or the presence of boundaries.

One possible way of determining these factors by using one only 24 hour test is to carry out the test on the basis of a normal constant discharge pump test up until 23 hours and at that time reduce the discharge rate in steps, each of 20 minutes duration, until the discharge rate is zero. At this stage a normal recovery test is conducted.

While the above paragraph and the following test procedure suggest that the shorter steps need only be of 20 minute duration it may be necessary on occasions to increase this time. This should be determined after an examination of the drawdown performance in the early stages of the extended first step. The hydrologic conditions which influence this first stage will have exactly the same influence on the early stages of each step. The length of each step should be determined on the basis of how long it took the first step to settle down. This particularly important when testing bores in fractured rock aquifers and in aquifers exhibiting delayed yield

If this type of test is carried out then all particulars of the bore including non-linear head loss or possible delayed yield can be determined and more accurate estimate of long term pumping rate assessed.

When testing an irrigation bore, industrial bore or other bore with high capital cost equipment the following test procedure is advocated:

1. Conduct a normal constant discharge test for 23 hours (i.e. 1,380 minutes), at discharge rate Q.
2. At 1380 minutes reduce the rate to 3/4Q and hold constant.
3. Take residual drawdown readings at 1381, 1382, 1383 etc. minutes to 1400 minutes.
4. At 1400 minutes reduce the discharge rate to Q/2 and hold constant.
5. Take residual drawdown readings at 1401, 1402 etc. minutes to 1420 minutes.
6. At 1420 minutes reduce the discharge rate to Q/4 and hold constant.
7. Take residual drawdown readings at 1421, 1422 etc. minutes as for a normal recovery test.
8. At 1440 minutes reduce the discharge rate to zero.
9. Take residual drawdown readings at 1441, 1442 etc. minutes as for a normal recovery test.
10. Ensure that the residual drawdown readings are taken at 1380, 1400, 1420, 1440 minutes prior to the change in discharge rate.
11. Ensure that the final stage, i.e. step 9, is not omitted.

Recovery readings should be taken for a duration of some 6 hours in the same manner as was indicated in the recovery section of the constant discharge test.

It is also acceptable to carry out the tests with increasing rather than decreasing steps. The radius of influence of the pumping bore is dependent only on the transmissivity and storage co-efficient and independent of discharge rate. The discharge rate only determines the magnitude of the drawdown within the radius of influence.

When testing procedures require a 100 hour test the same procedure as above could be used except that the first step would be 99 hours duration.

5.9.7 Variable Discharge/Variable Drawdown Test

A variable discharge test can be carried out in which both the drawdown and the discharge are allowed to vary at random. This test has not been recommended in the past but now can be analysed by computer.

While it is generally advisable to adopt a constant drawdown or discharge or a standard step drawdown test, the electronic computer now permits variable readings to be analysed. For this reason, even if difficulty is encountered in maintaining, say, a constant discharge, provided accurate measurements of drawdown and discharge can be made at given times, a test need not be abandoned.

5.9.8 Multiple Aquifer Testing

Occasions arise where more than one aquifer is encountered when drilling a bore and each aquifer has access to the bore by slotted casing or screens adjacent to it.

The available drawdown is different for each aquifer and the contribution from each aquifer should be considered in the analysis. More important still, a variation in S.W.L. will result in a proportionally greater reduction in available drawdown for the shallow aquifer than for the deeper ones.

One way of testing a bore which has encountered multiple aquifers is to carry out a number of tests on the bore. The number of tests should equal the number of aquifers contributing directly to the bore.

Each test should be of the constant drawdown type. The first test should bring the water level below the bottom of the top aquifer. The second test should bring it below the bottom of the aquifer second from the top.

This process is carried out for each aquifer except the deepest one. In that case the water is drawn down to the top of the aquifer.

The test on the deepest aquifer should be of 24 hours duration plus 6 hours recovery. The tests on each of the others need only be of 6 hours duration plus 2 hours recovery.

The series of tests can then be analysed to determine the contribution from each aquifer.

5.9.9 Applicability of Testing Procedures

A suggested programme to be carried out on various types of bores is detailed below and summarised in **Table 5-3**.

Table 5-3 Recommended pumping test applications

Type of Bore	Type of Test
Stock and domestic	Constant discharge or constant drawdown (4-6 hours)
Irrigation	Step drawdown (extended first step) (24 hours)
Investigation	As for irrigation bores
Town water supply and industrial	As for irrigation bores but 100 hour duration

5.9.10 Pump Stoppages

It may happen that the discharge rate is stopped at some point during a pumping test on a bore. Such a stoppage could be planned, such as a normal maintenance stoppage, such as an oil change during a 100 hour test, or accidental, such as mechanical or power failure.

If the test is temporarily suspended, the time at which the stoppage occurred should be recorded as should the time when pumping is recommenced. Accurate measurements of recovery should be made for the non-pumping period. Every attempt should be made to record them as this could assist in the analysis of the test. It is important, however, that the time when the breakdown occurred and when the pump was restarted are accurately recorded.

If the non-pumping period is less than 4 hours, the test should be recommenced and the test duration extended by an amount equal to the non-pumping period. The actual time of pumping does not alter. The non pumping period can be treated as a recovery time within the test and the test analysed accordingly.

If the non-pumping period exceeds 4 hours the test should be abandoned and a new 24 hour test, or 100 hour test, started after the breakdowns are rectified. Should this stoppage occur during the supervisor's absence from the site and time of breakdown is missed then the test should be abandoned and a new test started the next day.

5.9.11 Slug Tests

On occasions it is not feasible or possible to carry out a pumping test. (The bore diameter may be very small and preclude the insertion of pumping equipment; the hydraulic conductivity of the aquifer is such that the pumping rate would be exceedingly small or the water in the aquifer may be contaminated and should not be discharged to the surface for test purposes.) However, it may be essential that the hydraulic conductivity be known. In cases such as this slug tests are frequently used to determine the hydraulic conductivity in the near vicinity of the bore being tested. Such determinations are representative of area in the immediate vicinity of the hole and not of the aquifer in general.

A slug test involves inducing a rapid change in water level in the test bore and recording the recovery in water level with time. Water level changes are induced by quickly adding or removing a slug (a volume of water or a solid mass) to/from the bore, or using compressed air to displace the column of water.

The means of water column displacement are generally not significant in the analysis, as long as the water level recovery can be measured accurately. However, it is important that the slug be added or removed quickly. There have been instances where some operators have been told to fill the bore to the top and then measure the rate of recovery. Unfortunately in some cases, the hydraulic conductivity of the material outside the bore is such that it takes a long time for the bore to be filled. Such a test is meaningless as the water begins to discharge into the aquifer from the moment that the water level begins to rise and not when the bore is full.

A *rising head slug test* is one where a slug of water is removed from the bore and the water levels are measured as the water rises to its original standing water level.

A *falling head slug test* is one where a slug of water is deposited in the bore and the water levels are measured as the water falls to its original standing water level.

The "Slug Test" data can be analysed using methods such as the Bouwer and Rice or the Hvorslev methods (for details refer to Freeze and Cherry – 1979, Weight WD and Sonderegger -2000 or Sanders – 1998).

To carry out a Slug Test the following data need to be collected:

H = the initial head of water (water table or standing water level) prior to the test.

H_0 = the head of water immediately after adding (or removing) the slug of volume V.

h = head of water in the bore at time t after the slug of water was deposited (or removed).

r = radius of the bore screen (or screen plus filter pack).

t = time since the slug of water was deposited (or removed).

L = the length of bore screen below the water table.

This information can then be plotted and the hydraulic conductivity calculated.

Because the volume involved are very small the measurements need to be taken at more frequent intervals. Once the slug has been inserted or withdrawn from the bore recovery measurements should be taken at the following times after pumping commenced - 1, 2, 3, 4, 6, 8, 7,10, 20, 25, 30, 45, 60 seconds and each 20 seconds thereafter until 80% water level recovery has occurred. The duration of the test will depend very much on the hydraulic conductivity of the material being tested; a highly conductive material will recover much quicker than a tight material.

The test can be of very short duration so it is best to maximize the amount of data obtained during such a short period. This is best done by using a transducer and data logger combination for recording water levels. Additionally because the slug is being introduced or withdrawn from the top of the water the transducer is best installed well below the level traversed by the slug.

The volumes involved are very small so the volume of the bore casing may need to be taken into account.

5.10 TESTING FLOWING BORES

5.10.1 Antecedent Conditions

It is most important for the analysis of the test of a bore to know what variations had taken place in the discharge during the 24 hours or so previous to the test, as such variations may continue to have some effect on the bore and these effects must be corrected for in the analysis. Before doing anything to the bore, therefore it is necessary to record the condition of the bore as it is found - what is the flow if any, and what is the back pressure if any? Any substantial alteration in the flow of the bore during the previous 24 hours should be determined also possibly by enquiry. These alterations should be recorded and reported. If it is not possible to measure either the discharge or the back pressure the reasons for this must be stated and the best approximation made of the missing data.

5.10.2 Risk in Closing Low Pressure Bores

It is generally prudent not to close completely a bore with a pressure of less than 70 kPa especially if the temperature exceeds about 45°C. Whenever a bore is closed down the pressure should be kept under observation. If, in a low pressure bore, the pressure stops rising and begins to fall the bore valve should immediately be opened fully as this would indicate the development of conditions dangerous to the restoration of the flow. If the pressure in a high pressure bore begins to fall it is of no consequence and can probably be attributed to the cooling of the hot water. In such cases continuation of the test will give information about this cooling effect.

5.10.3 Flow Recession Test (Constant Drawdown)

5.10.3.1 Purpose of Test

The flow recession test is carried out on an artesian bore which has been closed down completely or partially for a period. This test is identical with a constant drawdown test in a pumped bore and, as in such a test, the head loss or drawdown remains constant while the discharge becomes less with time. In this case however, the discharge is not mixed with air and can be measured by an orifice meter or other type of flow meter. The purpose of the test is to record measurements of discharge and back pressure at intervals of time after the bore is opened. In this case the back pressure will usually be zero but this cannot be taken for granted and must be checked.

5.10.3.2 Procedure for Testing

The equipment necessary to measure flow and pressure is installed, and the flow and pressure of the bore as it is found is measured and recorded. The bore is then opened to allow it to flow freely through the orifice meter or bucket or over the weir board. The time of opening should be recorded. Measurement of the flow should be made at standard intervals of time after the bore is opened. The usual intervals are 1, 2, 5, 10, 15, 20, 30, 60, 90 and 120 minutes. As each measurement of flow is made the back pressure is also recorded, together with the time. If it is not possible to measure the back pressure this should be stated. A period of 120 minutes is generally sufficient for a flow recession test but in some circumstances it may be prolonged. If, for example, the bore is visited late in the afternoon it could be allowed to flow freely all night, proceeding to the next stage in the test programme on the following morning. In this case the last reading must be made immediately before the flow is altered for the next test.

5.10.3.3 *Points to Remember*

1. The back pressure and flow immediately before opening up the bore must be recorded, whether or not the actual flow is zero. If this information is not available the test cannot be analysed and provides little information, although it Is still a useful and necessary preliminary to the static test.
2. Changes in back pressure during the flow recession must be recorded.
3. Any sign of mud, drill cuttings or gas in the flow must be reported.

5.10.4 Static Test (Recovery)

5.10.4.1 *Purpose of the Test*

The static test in an artesian bore is identical with a water level recovery test in a pumped bore and is carried out by closing down the bore after it has been discharging at a reasonably constant rate for some time. A static test is made as the first operation when the bore is flowing freely when visited or, in a bore which is found partially or completely closed, following the flow recession test. The Static Test is the best means available to obtain a value of Transmissivity for a flowing bore.

When the bore is closed down the pressure will increase with time. The purpose of the static test is to record the back pressure at intervals of time. During a static test the flow is usually zero but this is not always so and when the flow is not zero it must be measured at the same time as the pressure.

5.10.4.2 *Flow Prior to Static Test*

It is desirable that the flow of the bore be in a reasonably steady state before the bore is closed down for a static test. If the bore is found already closed down when visited, partly or completely, a flow recession test must be carried out so that a condition of reasonably steady flow may be achieved. Even quite a long period of flow recession may not result in a completely stable condition but the usual 2 hours is generally sufficient and if the flow recession test has been properly carried out any necessary correction can be made in the analysis. If the bore is flowing freely when found it may be closed down for the static test immediately without the necessity of a prior period of flow recession. The measuring equipment should be fitted to the bore prior to the static test being carried.

Before closing down an artesian bore for a static test it is essential to measure and record the back pressure and flow immediately before closure. In the absence of this data the analysis becomes very much harder and sometimes impossible. Every effort should therefore be made to obtain this information. When this information is not recorded the reason should be clearly stated.

5.10.4.3 *Rate of Closure*

When an artesian bore is closed down it must be done slowly and steadily to avoid excessive water hammer which could damage the bore or the measuring equipment. A column of water in a bore one thousand metres deep has a mass of the order of 10-20 tonnes and may be moving with a velocity of some two metres per second. This momentum cannot safely be checked suddenly and the water must be brought to a stop gently. Observation of the gauge is the best guide to the smoothness of the operation. It is suggested that the closure time be approximately one to three minutes and that the time of the beginning of the test be taken from the time of complete closure. A comment should be made in the report indicating the time taken to close the bore.

5.10.4.4 *Static Test with Some Discharge*

While a static test usually follows the complete closure of a bore, it can also be carried out with only partial closure. This can be done if it is imprudent to close the bore completely due to a fear of damage or where complete closure cannot be affected because of defective head works. There are two ways of carrying out a static test with partial closure, either keeping the discharge constant by manipulation of the valve or allowing it to decrease with time normally. The first is preferable if it can be managed but this cannot always be done. In either case, it is essential that a measure of the flow be made and recorded whenever the pressure is measured. This will show the analyst whether the flow was constant or, if not, how it varied.

5.10.4.5 Frequency of Measurements

Following the closure of the bore the pressure should be measured at standard time intervals, usually 1, 2, 3, 6, 10, 15, 20, 30, 45, 60, 90 and 120 minutes after closure. This period of 2 hours is sufficient for most static tests although in some cases a longer period may be specifically requested. Where the static test is to be followed by other tests and the static test would finish late in the afternoon it may be desirable to leave the bore closed down all night and begin the next test in the morning. Periodic readings of the pressure should be made during this extra period but certainly the final pressure immediately before the next test must be recorded.

5.10.4.6 Gaseous Bores

The presence of gas in a bore can be a source of error in a static test involving complete closure. An accumulation of gas in the top of the bore will force the water back down the bore. The pressure measurement made by the manometer is the pressure at the water surface and will be in error. If the bore is only partially closed or if there is a leakage past the gate valve the gas will not accumulate. This suggests that where the presence of gas is known or suspected a slight intentional leak should be allowed to permit the gas to escape. Such an operation should, of course be reported and the amount of the leak reported.

5.10.4.7 Points to Remember

1. The flow and back pressure immediately before closure must be recorded.
2. Any leakage during the test and any change in the leakage must be measured and recorded.
3. Every measurement must include observations of flow, if any, back pressure and time of measurement.
4. The actual time of measurement must be recorded, not the time at which it should have been made.

5.10.5 Dynamic (Step Drawdown) Tests

5.10.5.1 General

The name Dynamic Test is retained for this type of test for historical reasons as the original purpose of the test was to determine the horse power available from a large artesian bore. Its modern use is different and it will be quite obvious that a dynamic test in an artesian bore is identical with that known as a step drawdown test in a pumped bore and much of the procedure will be found to be similar. A dynamic test is in fact, a measurement of the different head loss caused by different rate of discharge. The test involves, therefore, a series of stages during each of which the discharge is held constant at a different rate while the back pressure is read at suitable intervals. A dynamic test is done either by opening the bore up in stages from a closed down condition or by closing it down in stages from a free flowing condition. The former is known as an Opening Dynamic Test while the latter is known as a Closing Dynamic Test. Either one or both may be included in a test programme.

5.10.5.2 Pressure Control

The back pressure may change rapidly during the various stages of the dynamic test. In order to ensure that pressure being read is the correct one a useful trick is to install a petcock between the bore and the pressure measuring device. The petcock is closed at the time of the reading thus holding the pressure steady. Once the measurement is taken the petcock is again opened to expose the measuring device to the bore. It should be opened well in advance of the next reading.

5.10.6 Opening Dynamic Test

The opening dynamic test is the more usual. It can follow a static test or it can be the first test in a programme in the case of a bore which is found closed when visited. In the former case, the pressure measuring equipment will have been already connected to the bore for the purpose of the static test. In the latter case it should have been connected in order to read the pressure "as found". In either case the flow is zero or a small amount which will also have been measured and recorded.

5.10.6.1 Testing Procedure

The bore is opened up to allow a predetermined discharge. This discharge should be approximately one fifth of the estimated free flow. The bore should be allowed to discharge at this rate for 20 minutes although in very hot bores it may be desirable to extend this first stage for a longer period. During the whole of the stage the discharge should be kept constant by the manipulation of the valves to compensate for the progressive reduction in flow which would normally take place. The pressure should be read at standard intervals of time after the commencement of the stage. Customary intervals are 1, 2, 5, 10, 15 and 20 minutes.

Following the last reading of the first stage the bore is opened up to increase the discharge to the amount determined for the second stage which will be approximately 2/5 of the maximum expected free flow or twice the amount of the first stage. This discharge is again held constant at this value for another 20 minutes with readings of pressure at the standard 2, 5, 10, 15 and 20 minute intervals of time. After the completion of the second stage the discharge is again increased by the same amount for the third stage and so on.

It may not be possible to make the amount of the last stage an even multiple like the others. As it is desirable to keep the discharge constant and as it is therefore undesirable to run out of pressure during the last stage, it is better to select a discharge for the last stage which is somewhat less than the usual increment to ensure that it can be maintained constant throughout the stage.

5.10.7 Closing Dynamic Test

The closing dynamic test may be made as the first test in the programme if the bore is found flowing freely but this is not usual. It is more usual for a closing dynamic test, when it is included in a programme, to follow immediately after the opening dynamic test. It is particularly desirable that a closing dynamic test be carried out on very hot bores (say 50°C plus) to help overcome the effect of cooling on back pressure. In the former case the back pressure "as found" should be noted, and the discharge read either by orifice meter or bucket or by other method. If an orifice meter is used, however, it must be remembered that its connection may cause a slight increase in back pressure and resultant slight decrease in flow. The flow measurement by the orifice meter may then not be the correct "as found" flow. It is important therefore to record the back pressure immediately before and immediately after the diversion of the flow through the orifice meter. This will allow any necessary correction to be made. The time of all these readings must of course be recorded.

5.10.7.1 Testing Procedure

In either case the pressure and flow are observed and the bore closed until the discharge is that selected for the first stage, i.e. about 4/5 of the free flow. This discharge is kept constant for 20 minutes by the manipulation of the valve. This will result in a progressive increase in the back pressure which must be read at the appropriate intervals of time as in the opening dynamic test. Note that pressures fall during an opening dynamic test but rise during a closing dynamic test.

At the end of the 20 minutes of the first stage the bore is closed further until the discharge is the amount selected for the second stage, i.e. about 3/5 of the free flow. This discharge is maintained constant for 20 minutes while the pressure is read at the appropriate times. Stages 3 and 4 follow in the same way. The last stage will be carried out by completely closing the bore. In the unusual case where the closing dynamic test is carried out directly from the initial "as found" condition, the last stage should be treated as a static test and continued for 2 hours or preferably longer. In the more usual case in which the closing dynamic test follows the opening dynamic test this last stage need not extend beyond the usual 20 minutes.

5.10.7.2 Discharge Control

In either type of dynamic test it is important that the discharge remain constant during each stage but if this cannot be done, perhaps because of insensitive control of the flow, the next best thing is to record all measurable changes in the discharge together with the time at which they are observed and the back pressure at that time. This will make an analysis possible still, although more difficult.

5.10.7.3 Points to Remember

1. Discharge and back pressure immediately before the beginning of the first alteration in flow must be recorded.
2. The time, discharge and pressure must be recorded for each observation.
3. The discharge should be constant during each stage.
4. Any mud, drill cuttings or gas in the flow should be watched for and recorded.
5. The temperature of the water must be recorded as set out in Section 5.4.4.

5.10.8 Order of Tests

Assuming that pressure tests are to be carried out, the order in which they are done depends on the condition of the bore as found. The bore may be flowing freely or slightly controlled, it may be completely or almost completely shut down or it may be in some intermediate stage of control.

5.10.8.1 Free Flowing Bore

When the bore is found to be flowing freely or almost so the test programme is normally a static test followed by an opening dynamic test followed, if time permits or if specially instructed, by a closing dynamic test. Each of these tests should be carried out in accordance with the detailed instructions given for the operation of whatever measuring devices are used.

5.10.8.2 Closed Bore

When the bore as found is closed or substantially closed the normal programme is a flow recession, followed by a static test followed by an opening dynamic test, followed, if time permits, by a closing dynamic test. Each of these tests should be carried out in accordance with the detailed instructions for each and in accordance with the detailed instructions given for the operation of the particular measuring devices used.

5.10.8.3 Partially Closed Bore

If the bore is found to be in an intermediate stage of control, between fully closed and fully open, a decision has to be made whether to close down at once for a static test or open up first for a recession test. It may be stated as a general rule however, that, when in doubt, the bore should be opened to free flow. Only if the flow as found is a substantial fraction, at least two thirds of the estimated free flow, is there any case for closing it down for the static test without an intervening period of free flow. The programme of testing will in either case be identical with that outlined in the two preceding paragraphs according to whether the static test comes first or whether it is preceded by a recession test.

5.11 DISINFECTION

Because of the risk cross contamination from one aquifer to another the down hole pumping equipment should be disinfected thoroughly before and after a pumping test. This is normally done with a chlorine solution to kill any bacteria which may be present. The details of disinfection will not be covered in these documents but can be found in references such as Driscoll, 1988.

SECTION 6: EVALUATION OF AQUIFER PROPERTIES USING OBSERVATION BORES

6.1 INTRODUCTION

It is now possible to apply the information which has been presented in the previous sections to a particular situation i.e. the evaluation of aquifer parameters. With an understanding of Darcy's Law and the law of conservation of mass, equations governing groundwater flow can be developed. With the use of field data collected from pumping tests these equations can be solved to determine accurately the aquifer parameters which govern that flow. In addition, by the application of these equations the long term pumping performance of bores can also be determined.

The field data can be collected from the discharging bore itself or from observation bores in the vicinity of the discharging bore. The evaluation of aquifer properties has been sub-divided into test data from observation bores, test data from discharging bores, and data without pumping tests at all.

This Section presents methods for evaluating these parameters using data collected from observation bores. Section 7 presents methods to determine some of these parameters using data collected from the pumping bore itself and how to use this information to develop the equation to drawdown for the bore under a wide range of discharge rates. It also presents some methods to estimate some parameters if pumping tests are not available and to set upper and lower limits on them. Section 7 shows how some of these parameters are used to solve frequently encountered groundwater problems.

Software is available to solve many of these equations. However, without a solid understanding of how they were developed, the assumptions made, and the ability to solve them manually, indiscriminate use of the software can be fraught with danger. To provide this understanding the derivation and solutions of the equations for each type of analysis are presented together with the procedure for applying them and worked examples. Where possible the assumptions made regarding hydraulic conductivity, types of flow and boundary conditions have been placed at the end of the relevant sub-section. Although such solutions often only approximate field conditions, they provide valuable insight into the intricacies of groundwater flow. However, the extensive development of groundwater supplies and the need to understand groundwater systems makes it important that practical solutions to groundwater problems be obtainable.

Because the following Steady State flow equations and the solutions of the Non-Steady State flow equations using type curves (with logarithmic plots) do not account for non-linear head losses at or near the bore they cannot be used to analyse data from the pumped bore if non-linear losses are present. They should be used only to analyse data from observation bores.

The Modified Non-Steady State flow equations, using semi-logarithmic plots can be used to analyse data obtained from the pumping bore itself.

6.2 SELECTING THE TYPE OF ANALYSIS

Before attempting to analyse the results of a pumping test which has been carried out on a bore it is prudent to examine the log of the bore and determine the type of aquifer with which you are dealing. Different aquifer types can and do respond differently when subjected to pumping. Typical aquifer responses are shown in **Figure 6-16** at the end of this Section.

Having determined the type of aquifer, the method of analysis can be selected. If there is some doubt as to the type of aquifer, then methods appropriate to each possible type should be used. For any one type of aquifer it is also desirable to analyse the test by a number of different methods, if different methods exist, and select the solution after comparing results. The following pages are written on the assumption that the type of aquifer has been determined.

Methods of solution are presented for each type of aquifer, i.e. confined, semi-confined and unconfined. Not all methods of analysis are able to be presented in these notes and you are referred to the references for other methods if required.

Where possible, for each type of aquifer, solutions are given for both Steady State flow and Non-Steady State flow.

Literature deals mainly with cases in which non-linear head loss is absent. In most cases, this applies only to observation bores some distance from the discharging bore. Methods of analysing data from discharging bores as well as variable discharging pumping tests on bores and analyses for intermittent pumping conditions are provided in Section 7.

6.3 CONFINED AQUIFER TEST ANALYSIS

6.3.1 Constant Discharge Tests

When a bore is pumped, water is removed from the aquifer surrounding the bore and the potentiometric surface is lowered.

The drawdown at a given time is the distance or depth the water level is lowered from its original position.

A drawdown curve shows the variation of drawdown with distance from the bore and, to conform to Darcy's Law, the hydraulic gradient decreases as distance from the well increases i.e. as the area through which the water is moving increases.

The head provided by lowering the water level in the bore will balance the frictional resistance of the aquifer to the passage of the water at the pumped rate.

In three dimensions the drawdown curve describes the shape more like a vortex than a cone, but it is commonly referred to as "the cone of depression". The outer limits of the cone describe what is called the "area of influence" of the bore. The radius of this "area of influence" is known as the "radius of influence".

6.3.1.1 Steady State Flow (Thiem Equation)

A pumping test under steady state conditions will give no indication of the storage coefficient of the aquifer since it is assumed that the change in storage is negligible once steady state conditions have been achieved.

Storage coefficient can not be obtained from analysis of steady state flow conditions.

Derivation

Figure 6-1 represents half the cross-section of the cone of depression in the confined aquifer around a bore that has been pumped at a constant discharge rate Q for a period sufficiently long that steady state flow is being closely approximated, and the volume of water being derived from storage is negligible compared with the volume of water moving towards the bore.

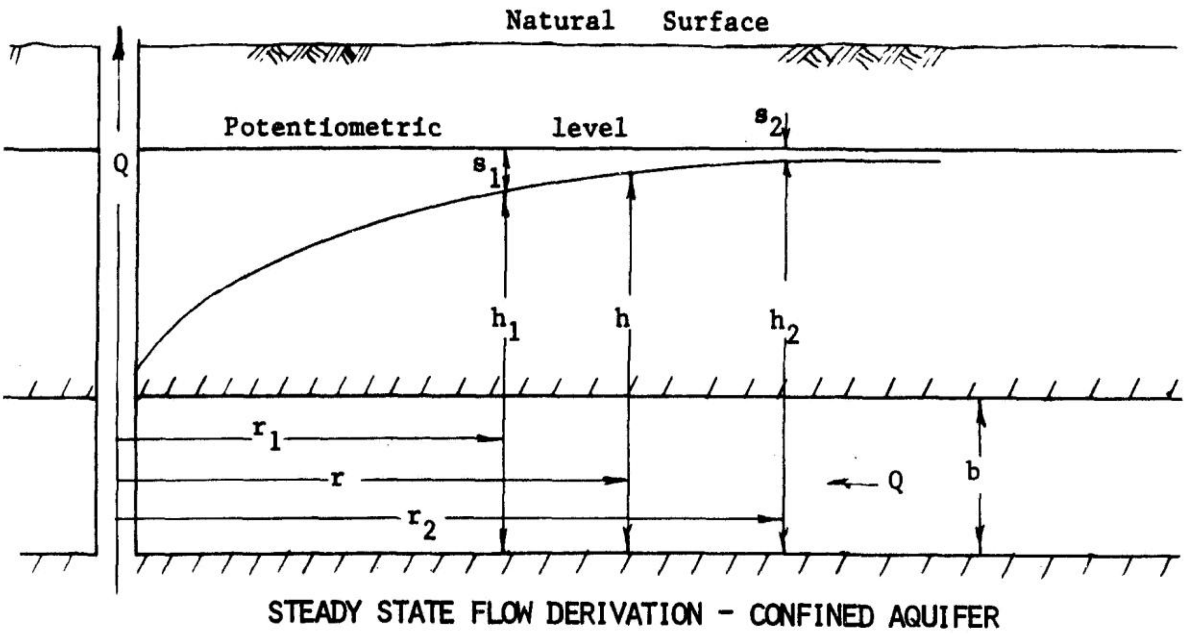


Figure 6-1 Steady state flow derivation – confined aquifer

By the principle of conservation of mass the volume of water flowing radially towards the bore through any two concentric cylinders within the cone of depression is equal to the volume of water being discharged from the bore.

If these concentric cylinders have radii of r_1 and r_2 then observation bores placed r_1 and r_2 from the centre of the discharging bore will reflect water level changes in each of these cylinders.

If Darcy's Law is expressed as a first order ordinary differential equation in cylindrical co-ordinates, as:

$$Q = -KA\frac{dh}{dr} \qquad \text{.....6.1}$$

where:

A = 2πrb.

b = the aquifer thickness.

then:

$$Q = -K2\pi rb\frac{dh}{dr} \qquad \text{.....6.2}$$

Separating variables:

$$\frac{dr}{r} = \frac{-2\pi b}{Q}Kdh$$

Integrating between r_1 and r_2 and h_1 and h_2:

$$\int_{r_1}^{r_2}\frac{dr}{r} = -\frac{2\pi b}{Q}\int_{h_1}^{h_2}dh$$

$$\left[\log_e r\right]_{r_1}^{r_2} = \frac{-2\pi Kb}{Q}\left[h\right]_{h_1}^{h_2}$$

$$\log_e\frac{r_2}{r_1} = \frac{-2\pi Kb}{Q}(h_2 - h_1)$$

Hence:

$$K = \frac{-2.3\log_{10}(r_2/r_1)}{2\pi b(h_2 - h_1)} \qquad \text{.....6.3}$$

or

$$T = Kb = \frac{-2.3\log_{10}(r_2/r_1)}{2\pi(h_2 - h_1)} \qquad \text{.....6.4}$$

Because:

$h_1 + s_1 = h_2 + s_2$

then:

$s_1 - s_2 = h_2 - h_1$

and

$$T = \frac{-2.3Q}{2\pi}\frac{\log_{10}(r_2/r_1)}{(s_1 - s_2)} \qquad \text{.....6.5}$$

If readings are taken in two observation bores during a pumping test on a bore then equation 6.5 may be used to determine the transmissivity of the aquifer. However, a straight line solution does exist which will allow the computation of transmissivity by graphical means and incorporates the drawdowns in any number of observation bores.

Straight Line Relationship - For Steady State Flow

Equation 6.5 may be written as:

$$s_2 - s_1 = \frac{2.3Q}{2\pi T}(\log_{10} r_2 - \log_{10} r_1) \qquad .....6.6$$

Equation 6.6 is now in the form:

$$y_2 - y_1 = m(x_2 - x_1)$$

This is an equation to a straight line with slope m.

Thus a plot of drawdown "s" against "$\log_{10}$r" under steady state conditions will result in a straight line with slope.

$$\frac{-2.3Q}{2\pi T}$$

If, on the plot, r_2 is taken as $10r_1$ then $\log_{10}r_2 - \log_{10}r_1 = 1$ and

$$s_2 - s_1 = \frac{2.3Q}{2\pi T}$$

or

$$s_1 - s_2 = \frac{-2.3Q}{2\pi T}$$

$$= \Delta s' \qquad .....6.7$$

where Δs' is the change in drawdown per log cycle from the distance-drawdown plot.

From Equation 6.7:

$$\Delta s' = \frac{-2.3Q}{2\pi T}$$

and

$$T = \frac{-2.3Q}{2\pi \Delta s'} \qquad .....6.8$$

The slope is negative because the drawdown, "s", decreases as distance "r" increases, or, stating in another way, if r_2 is greater than r_1, in the above expression then $s_2 - s_1$ is always negative. T will always be positive.

This is a very convenient method of determining transmissivity under Steady State conditions.

Procedure

1. On semi-logarithmic graph paper plot the drawdowns (on natural scale) against the radial distance from the pumping bore at which the drawdowns were measured (on logarithmic scale).
2. Fit a straight line through the plotted points.
3. From the plot read the drawdown per log cycle, Δs'.
4. Using equation 6.8 calculate T, from T = $\frac{-2.3Q}{2\pi \Delta s'}$

Example

Table 6-1 presents data which was obtained by S.W. Lohman from a 3 day pumping test in 1937, near Wichita, Kansas U.S.A. The aquifer in this case is actually unconfined but under steady state conditions provides a suitable example. The test results were in imperial units but have been converted for these notes. The discharge rate Q was 5450 m^3/day. The initial saturated thickness was 8.17m, and six observation bores were used, three on a line extending north from the pumped bore, and three to the south. Steady State conditions can be assumed. Determine the transmissivity of the aquifer.

In **Figure 6-2** the values of drawdown, s, (col. 4), are plotted against corresponding distances from the discharging bore, r, (col. 3) on semi-logarithmic graph paper.

A straight line has been drawn through the graphical averages of drawdowns for bores N-1 and S-1, N-2 and S-2, N-3 and S-3. Transmissivity has been calculated using equation 6.8.

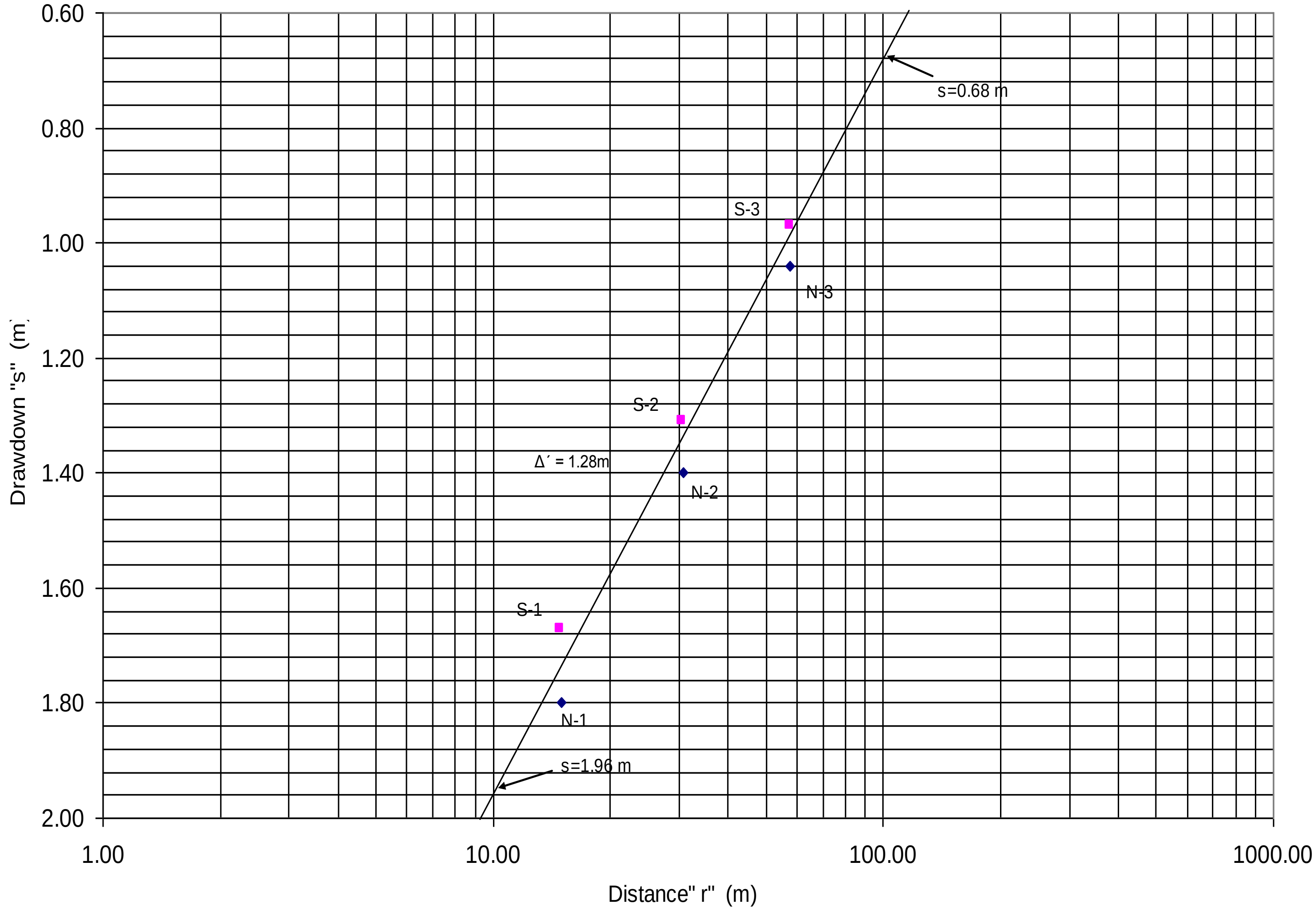
Table 6-1 Data for steady state analysis

1	2	3	4	5	6
Line	**Bore**	**r (m)**	**s (m)**	**$s^2/2b$ (m)**	**$s-s^2/2b$ (m)**
N	1	15.0	1.80	0.20	1.60
	2	30.7	1.40	0.12	1.28
	3	57.7	1.04	0.07	0.97
S	1	14.9	1.67	0.17	1.50
	2	30.6	1.31	0.11	1.20
	3	57.9	0.97	0.06	0.91

Assumptions

In deriving the Thiem equation, the following assumptions were made:

1. the aquifer is homogeneous, isotropic, and of infinite areal extent;
2. the discharging bore penetrates and receives water from the entire thickness of the aquifer;
3. the transmissivity is constant at all places and all times;
4. discharge has continued at a constant rate for a time sufficient for the hydraulic system to reach a steady flow condition;
5. flow lines are radial; and
6. flow is laminar.



Example

Using data from Table 6-1:

Using equation 6.8:

$T = 2.3\ Q / (2\ \pi\ \Delta s')$

$Q = 5450\ m^3/day$

$\Delta s' = 1.28m$

$T = 1600\ m^2/day$

Figure 6-2 Steady state flow example, confined aquifer

6.3.1.2 Non-Steady State Flow

Derivation of Partial Differential Equation

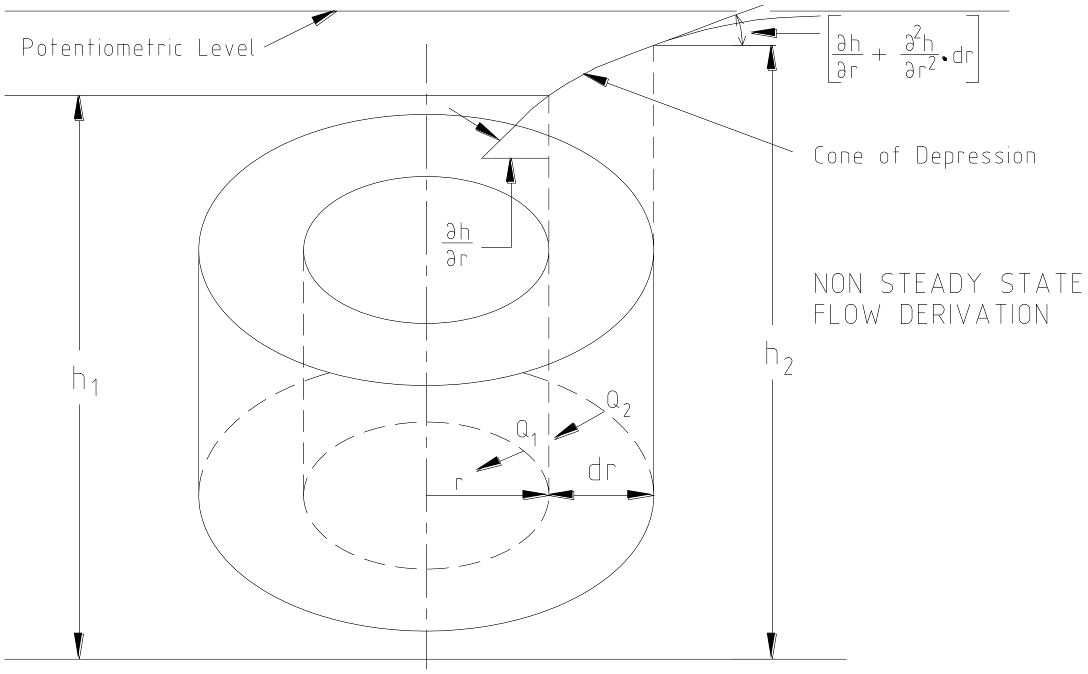


Figure 6-3 Non-steady state flow derivation - confined aquifer

A generalised free body diagram is given in **Figure 6-3** in the vicinity of a discharging bore. Assuming impermeable planes bound the system on top and bottom and the flow is radial, we find, by the principle of conservation of matter, that the difference in the rate of flow through the inner face (Q_t) and the outer face (Q_2) of the cylinder must be drawn from storage within the annulus.

$$Q_1 - Q_2 = \frac{dV}{dt} \qquad .....6.9$$

where $\frac{dV}{dt}$ is the change in volume of water between h_2 and h_1 with time.

The flow through the inner face is:

$$Q_1 = -Ti_1W_1$$

$$= -T2\pi r\frac{\partial h}{\partial r} \qquad .....6.10$$

where:

T = transmissivity.

i = hydraulic gradient (at the inner face) = $\frac{\partial h}{\partial r}$

W = the width of flow across the section (at the inner face equals the circumference of the circle = 2πr).

Since the second derivative defines the rate of change of slope the slope or gradient of the potentiometric surface at the outer face of the cylinder is:

$$i_2 = i_1 + \frac{\partial^2 h}{\partial r^2} dr$$

$$= \frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr \qquad .....6.11$$

Then the flow through the outer face is:

$$Q_2 = -T(\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr) 2\pi (r + dr) \qquad .....6.12$$

The rate of change of volume within the cylindrical shell is expressed as:

$$\frac{dV}{dt} = 2\pi r dr \frac{\partial h}{\partial t} S \qquad .....6.13$$

where:

S = coefficient of storage.

For unconfined aquifer (or water table) conditions, S is equivalent to the specific yield of the material dewatered by pumping. For pressure conditions where water is drawn from storage by compression of the aquifer, S is the storage coefficient. Specific yield and storage coefficient have been defined previously.

Substituting the above values in equation 6.9:

$$-T \frac{\partial h}{\partial r} 2\pi r + T(\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} dr).2\pi (r + dr) = 2\pi r dr \frac{\partial h}{\partial t} S$$

i.e. $$T \frac{\partial h}{\partial r} 2\pi r + T(2\pi r \frac{\partial h}{\partial r} + 2\pi r dr \frac{\partial^2 h}{\partial r^2} + 2\pi dr \frac{\partial h}{\partial r} + 2\pi (dr)^2 \frac{\partial^2 h}{\partial r^2}) = 2\pi r dr \frac{\partial h}{\partial t} S$$

Dividing through by 2πrTdr and neglecting differentials higher than first order:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \qquad .....6.14$$

Equation 6.14 is the partial differential equation for non-steady state radial flow.

This equation may be written in cartesian co-ordinates as:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T} \frac{\partial h}{\partial t}$$

<u>Note</u>: when, $\frac{\partial h}{\partial t} = 0$ the entire right hand side of equation 6.14 is zero indicating that there are no changes in storage in the aquifer and steady state flow occurs.

The equation for steady state radial flow is then:

$$\frac{1}{r} \frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2} = 0 \qquad .....6.15$$

Solution of Equation

Equation 6.14 for Non-Steady State Flow at constant discharge includes time "t" as a variable. The solution may be applied both to conditions where dh/dt is finite and, for large values of "t", to conditions approaching steady state flow.

Through analogy with the mathematical theory of heat conduction, Theis (1935) with the assistance of Clarence Lubin developed the following solution of equation 6.14, which was later developed using entirely hydrologic concepts by Jacob (1940).

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-x}}{x}dx \qquad .....6.16$$

where, in consistent units:

s = drawdown in an observation bore in the vicinity of the discharging bore, in metres.

Q = the discharge rate (constant) of discharging bore, in cubic metres per day.

T = transmissivity of the aquifer, in square metres per day.

r = distance from discharging bore to observation bore, in metres.

S = storage coefficient, expressed as a decimal fraction, dimensionless.

t = time of discharge and observation, in days.

u = lower limit of integration, $\frac{r^2S}{4Tt}$

Equation 6.16 involves a number of assumptions which are listed further on in this section.

These restrictive assumptions make equation 6.16 strictly applicable only to a confined aquifer of rather unusual attributes not found in nature. However, when used with caution and judgment, the equation can be applied successfully to many problems of groundwater flow that do not fully meet all the assumptions. When "t" is sufficiently large it can even be applied to unconfined aquifers. Under the latter conditions, the specific yield S is used instead of the storage coefficient.

The integral expression in equation 6.16 cannot be integrated directly, but its value is given by the infinite series derived as follows:

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-x}}{x}dx$$

$$\frac{e^{-x}}{x} = \frac{1}{x} - 1 + \frac{x}{2!} - \frac{x^2}{3!} + \frac{x^3}{4!}$$

$$\int_u^\infty \frac{e^{-x}}{x}dx = \left[\log_e - x + \frac{x^2}{2.2!} - \frac{x^3}{3.3!} + \frac{x^4}{4.4!} - ....\right]_u^\infty$$

$$= (-0.577216 - \log_e u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - .....)$$

and

$$s = \frac{Q}{4\pi T}(-0.577216 - \log_e u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - ......) \qquad .....6.17$$

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-x}}{x}dx$$

$$s = \frac{Q}{4\pi T}W(u)$$

where:

$$u = \frac{r^2S}{4Tt}$$

The value of the series is commonly expressed as W(u), the "well function of "u".

Values of W(u) for values of u from 10^{-15} to 9.9 are given in tabular form in Appendix 6-A. Some of these values are presented in graphical form in **Figure 6-4**. However, if the value of W(u) is required then in most cases sufficient accuracy is obtained by calculating it using the first two terms of the series (i.e. $W(u) \approx -0.577216 - \log_e u$). For a given value of u, T may be determined from:

$$T = \frac{Q}{4\pi s} W(u) \qquad .....6.18$$

And S may be determined from:

$$S = 4Ttu/r^2 \qquad .....6.19$$

Theis Type Curve Solution

Theis devised a simple graphical method of superposition that makes it possible to obtain solutions of equation 6.18 and 6.19.

By re-arranging equations 6.18 and 6.19 and taking logarithms of both sides, the following relationships are obtained:

$$\log s = \left\{ \frac{\log Q}{4\pi T} \right\} + \log W(u) \qquad .....6.20$$

and

$$\log \frac{r^2}{t} = \left\{ \frac{\log 4T}{S} \right\} + \log u \qquad .....6.21$$

If the discharge is held constant, the bracketed parts of the equations 6.20 and 6.21 are constant, and s is related to r^2/t in the same manner as W (u) is related to u. If the values of W (u) and u were plotted on logarithmic paper, a type curve for the relationship between s and r^2/t would result. Values of "s" could then be plotted against r^2/t on transparent logarithmic paper to the same scale as the type curve and would be the same shape as the type curve, but would be displaced by an amount $Q/4\pi T$ on the "s" and "W (u)" axes, and by an amount 4T/S on the " r^2/t " and "u" axes.

The plot of "s" versus r^2/t could be moved over the type curve, keeping the axes parallel, until its shape was matched with a section of the type curve.

For this matching position, corresponding values of "W (u)" and "u", "r^2/t" and "s" are recorded for any point on the graphs. The point selected is called the "Match Point". This match point does not have to lie on the plotted curve. In fact, for convenience of calculations, a match point is frequently chosen so that "W (u)" = 1 and "u" = 1 and the values of s and r^2/t are read.

Transmissivity and storage coefficient are then calculated from equations 6.18 and 6.19 making sure that the units used are consistent.

If readings from only one observation bore are used then "s" could be plotted against "1/t" and "r^2" would be introduced in equation 6.19. This eliminates the necessity for calculating many values of r^2/t.

However, it is recommended that the alternative method presented below be used in practice.

Alternative Type Curve Solution of Equation

The above method involves the calculation of the reciprocal for every "time" measurement, and apart from the extra calculations, additional sources of error are introduced because of these calculations.

A more convenient method is to plot "s" against "t" on logarithmic paper, and superimpose this on a type curve. This can be done for each observation bore. It is also possible to plot "s" against "t/r^2" for all observation bores.

Following the same reasoning as previously, "s" has the same relationship to "t/r^2" as the well function "W (u)" has to "1/u".

The type curve in this case is then a plot of W (u) against 1/u on logarithmic paper. A stylised curve of W (u) against 1/u is given in **Figure 6-4.**

The uppermost curve in this family of curves is in fact the curve of "W(u)" versus "I/u" (i.e. no leakage), or the Theis curve. If a deviation from the W (u) versus I/u curve does occur it can be interpreted from the same type curve if the deviation is caused by leakiness.

Procedure

Summarising the type curve solution, the procedure to be followed is:

1. Plot drawdown versus time for the observation bore, on transparent logarithmic paper with time on the horizontal axis.
2. Fit the plotted curve to the type curve of W (u) versus 1/u (**Figure 6-4**).
3. Select a match-point and read off values of W (u), (1/u), s and t,
4. Calculate the transmissivity, T, and storage coefficient, S, from equations 6.18 and 6.19.
5. Before calculating storage coefficient, S, from equation 6.19, ensure that the units of T and t are such that S will be dimensionless.

The scale of the logarithmic paper on which drawdown and time are plotted must of course be the same as the scale of the type curve paper.

If the recorded time units are not consistent with the required time units in equation 6.19 plot the recorded time values and make one only conversion to the match-point value when solving equation 6.19.

Table 6-2 Values of (Wu) for values of u between 10^{-15} and 9.9

N\u	$Nx10^{-15}$	$Nx10^{-14}$	$Nx10^{-13}$	$Nx10^{-12}$	$Nx10^{-11}$	$Nx10^{-10}$	$Nx10^{-9}$	$Nx10^{-8}$	$Nx10^{-7}$	$Nx10^{-6}$	$Nx10^{-5}$	$Nx10^{-4}$	$Nx10^{-3}$	$Nx10^{-2}$	$Nx10^{-1}$	N
1.0	33.9615	31.6590	29.3564	27.0518	24.7512	22.4486	20.1460	17.8435	15.5409	13.2383	10.9357	8.6332	6.3315	4.0379	1.8229	0.2194
1.1	33.8662	31.5637	29.2611	26.9585	24.6559	22.3533	20.0507	17.7482	15.4456	13.1430	10.8404	8.5379	6.2363	3.9436	1.7371	0.1860
1.2	33.7792	31.4767	29.1741	26.8715	24.5689	22.2663	19.9637	17.6611	15.3586	13.0560	10.7534	8.4509	6.1494	3.8576	1.6595	0.1584
1.3	33.6992	31.3966	29.0940	26.7914	24.4889	22.1863	19.8837	17.5811	15.2785	12.9759	10.6734	8.3709	6.0695	3.7785	1.5889	0.1355
1.4	33.6251	31.3225	29.0199	26.7173	24.4147	22.1122	19.8096	17.5070	15.2044	12.9018	10.5993	8.2968	5.9955	3.7054	1.5241	0.1162
1.5	33.5561	31.2535	28.9509	26.6483	24.3458	22.0432	19.7406	17.4380	15.1354	12.8328	10.5303	8.2278	5.9266	3.6374	1.4645	0.1000
1.6	33.4916	31.1890	28.8864	26.5838	24.2812	21.9786	19.6760	17.3735	15.0709	12.7683	10.4657	8.1634	5.8621	3.5739	1.4092	0.0863
1.7	33.4309	31.1283	28.8258	26.5232	24.2206	21.9180	19.6154	17.3128	15.0103	12.7077	10.4051	8.1027	5.8016	3.5143	1.3578	0.07465
1.8	33.3738	31.0712	28.7686	26.4660	24.1634	21.8608	19.5583	17.2557	14.9531	12.6505	10.3479	8.0455	5.7446	3.5481	1.3080	0.06471
1.9	33.3197	31.0171	28.7145	26.4119	24.1094	21.8068	19.5042	17.2016	14.8990	12.5964	10.2939	7.9915	5.6906	3.4050	1.2649	0.05620
2.0	33.2684	30.9658	28.6632	26.3607	24.0581	21.7555	19.4529	17.1503	14.8477	12.5451	10.2426	7.9402	5.6394	3.3547	1.2227	0.04890
2.1	33.2196	30.9170	28.6145	26.3119	24.0093	21.7067	19.4041	17.1015	14.7989	12.4964	10.1938	7.8914	5.5907	3.3069	1.1829	0.04261
2.2	33.1731	30.8705	28.5679	26.2653	23.9628	21.6602	19.3576	17.0550	14.7324	12.4498	10.1473	7.8449	5.5443	3.2614	1.1454	0.03719
2.3	33.1286	30.8261	28.5235	26.2209	23.9183	21.6157	19.3131	17.0106	14.7080	12.4054	10.1028	7.8004	5.4999	3.2179	1.1099	0.03250
2.4	33.0861	30.7835	28.4809	26.1783	23.8758	21.5732	19.2706	16.9680	14.6654	12.3628	10.0603	7.7579	5.4575	3.1763	1.0762	0.02844
2.5	33.0453	30.7427	28.1401	26.1375	23.8349	21.5323	19.2298	16.9272	14.6246	12.3220	10.0194	7.7172	5.4167	3.1365	1.0443	0.02491
2.6	33.0060	30.7035	28.1009	26.0983	23.7957	21.4931	19.1905	16.8880	14.5854	12.2828	9.9802	7.6779	5.3776	3.0983	1.0139	0.02185
2.7	32.9683	30.6657	28.3631	26.0806	23.7580	21.4554	19.1528	16.8502	14.5476	12.2450	9.9425	7.6401	5.3400	3.0615	0.9849	0.01918
2.8	32.9519	30.6294	28.3268	26.0242	23.7216	21.4190	19.1164	16.8138	14.5113	12.2087	9.9061	7.6038	5.3037	3.0261	0.9573	0.01686
2.9	32.8968	30.5943	28.2917	25.9891	23.6865	21.3839	19.0813	16.7788	14.4762	12.1736	9.8710	7.5687	5.2687	2.9920	0.9309	0.01482
3.0	32.8629	30.5604	28.2578	25.9552	23.6526	21.3500	19.0474	16.7440	14.4423	12.1397	9.8371	7.5348	5.2349	2.9591	0.9057	0.01305
3.1	32.8302	30.5276	28.2250	25.9224	23.6198	21.3172	19.0146	16.7121	14.4095	12.1069	9.8043	7.5020	5.2022	2.9273	0.8815	0.01149
3.2	32.7984	30.4958	28.1932	25.8907	23.5880	21.2855	18.9829	16.6803	14.3777	12.0751	9.7726	7.4703	5.1706	2.8965	0.8583	0.01013
3.3	32.7676	30.4651	28.1625	25.8599	23.5573	21.2547	18.9521	16.6495	14.3470	12.0444	9.7418	7.4395	5.1399	2.8668	0.8361	0.008939
3.4	32.7378	30.4352	28.1326	25.8300	23.5274	21.2249	18.9223	16.6197	14.3171	12.0145	9.7120	7.4097	5.1102	2.8377	0.8147	0.007891
3.5	32.7088	30.4062	28.1036	25.8010	23.4985	21.1959	18.8933	16.5907	14.2881	11.9855	9.6830	7.3807	5.0813	2.8099	0.7942	0.006970
3.6	32.6806	30.3780	28.0755	25.7729	23.4703	21.1677	18.8651	16.5625	14.2599	11.9574	9.6548	7.3526	5.0532	2.7827	0.7745	0.006160
3.7	32.6532	30.3506	28.0481	25.7455	23.4429	21.1403	18.8377	16.5351	14.2325	11.9300	9.6274	7.3252	5.0259	2.7563	0.7554	0.005448
3.8	32.6266	30.3240	28.0214	25.7188	23.4162	21.1136	18.8110	16.5085	14.2059	11.9033	9.6007	7.2985	4.9933	2.7306	0.7371	0.004820
3.9	32.6006	30.2980	27.9954	25.6928	23.3902	21.0877	18.7851	16.4825	14.1799	11.8773	9.5748	7.2725	4.9735	2.7056	0.7194	0.004267

N\u	Nx10^{-15}	Nx10^{-14}	Nx10^{-13}	Nx10^{-12}	Nx10^{-11}	Nx10^{-10}	Nx10^{-9}	Nx10^{-8}	Nx10^{-7}	Nx10^{-6}	Nx10^{-5}	Nx10^{-4}	Nx10^{-3}	Nx10^{-2}	Nx10^{-1}	N
4.0	32.5753	30.2727	27.9701	25.6675	23.3649	21.0623	18.7598	16.4572	14.1546	11.8520	9.5495	7.2472	4.9482	2.6813	0.7024	0.003779
4.1	32.5506	30.2480	27.9454	25.6428	23.3402	21.0376	18.7351	16.4325	14.1299	11.8273	9.5248	7.2225	4.9236	2.6576	0.6859	0.003349
4.2	32.5265	30.2239	27.9213	25.6187	23.3161	21.0136	18.7110	16.4084	14.1058	11.8032	9.5007	7.1985	4.8997	2.6344	0.6700	0.002969
4.3	32.5029	30.2004	27.8978	25.5952	23.2926	20.9900	18.6874	16.3884	14.0823	11.7797	9.4771	7.1749	4.8762	2.6119	0.6546	0.002633
4.4	32.4800	30.1774	27.8748	25.5722	23.2696	20.9670	18.6644	16.3619	14.0593	11.7567	9.4541	7.1520	4.8533	2.5899	0.6397	0.002336
4.5	32.4575	30.1549	27.8523	25.5497	23.2471	20.9446	18.6420	16.3394	14.0368	11.7342	9.4317	7.1295	4.8310	2.5684	0.6253	0.002073
4.6	32.4355	30.1329	27.8303	25.5277	23.2252	20.9226	18.6200	16.3174	14.0148	11.7122	9.4097	7.1075	4.8091	2.5474	0.6114	0.001841
4.7	32.4140	30.1114	27.8088	25.5602	23.2037	20.9011	18.5985	16.2959	13.9933	11.6907	9.3882	7.0860	4.7877	2.5268	0.5979	0.001635
4.8	32.3929	30.0904	27.7878	25.4852	23.1826	20.8800	18.5774	16.2748	13.9723	11.6697	9.3671	7.0650	4.7667	2.5068	0.5848	0.001453
4.9	32.3723	30.0697	27.7672	25.4646	23.1620	20.8594	18.5568	16.2542	13.9516	11.6491	9.3465	7.0444	4.7462	2.4871	0.5721	0.001291
5.0	32.3521	30.0495	27.7470	25.4444	23.1418	20.8392	18.5366	16.2340	13.9314	11.6289	9.3263	7.0242	4.7261	2.4679	0.5598	0.001148
5.1	32.3323	30.0297	27.7271	25.4246	23.1220	20.8194	18.5168	16.2142	13.9116	11.6091	9.3065	7.0044	4.7064	2.4491	0.5478	0.001021
5.2	32.3129	30.0103	27.7077	25.4051	23.1026	20.8000	18.4974	16.1948	13.8922	11.5896	9.2871	6.9850	4.6871	2.4306	0.5362	0.0009086
5.3	32.2939	29.9913	27.6887	25.3861	23.0835	20.7809	18.4783	16.1758	13.8732	11.5706	9.2681	6.9659	4.6681	2.4126	0.5250	0.0008086
5.4	32.2752	29.9726	27.6700	25.3674	23.0648	20.7622	18.4596	16.1571	13.8545	11.5519	9.2494	6.9473	4.6495	2.3948	0.5140	0.0007198
5.5	32.2568	29.9542	27.6516	25.3491	23.0465	20.7439	18.4413	16.1387	13.8361	11.5336	9.2310	6.9289	4.6313	2.3775	0.5034	0.0006409
5.6	32.2388	29.9362	27.6336	25.3310	23.0285	20.7259	18.4233	16.1207	13.8181	11.5155	9.2130	6.9109	4.6134	2.3604	0.4930	0.0005708
5.7	32.2211	29.9185	27.6159	25.3133	23.0108	20.7082	18.4056	16.1030	13.8004	11.4978	9.1953	6.8932	4.5958	2.3437	0.4830	0.0005085
5.8	32.2037	29.9011	27.5985	25.2959	22.9934	20.6908	18.3882	16.0856	13.7830	11.4804	9.1779	6.8758	4.5785	2.3273	0.4732	0.0004532
5.9	32.1866	29.8840	27.5814	25.2789	22.9763	20.6737	18.3711	16.0685	13.7659	11.4633	9.1608	6.8588	4.5615	2.3111	0.4637	0.0004039
6.0	32.1698	29.8672	27.5646	25.2620	22.9595	20.6569	18.3543	16.0517	13.7491	11.4465	9.1440	6.8420	4.5448	2.2953	0.4544	0.0003601
6.1	32.1533	29.8507	27.5481	25.2455	22.9429	20.6403	18.3378	16.0352	13.7326	11.4300	9.1275	6.8254	4.5283	2.2797	0.4454	0.0003211
6.2	32.1370	29.8344	27.5318	25.2293	22.9267	20.6241	18.3215	16.0189	13.7163	11.4138	9.1112	6.8092	4.5122	2.2645	0.4366	0.0002864
6.3	32.1210	29.8184	27.5158	25.2133	22.9107	20.6081	18.3055	16.0029	13.7003	11.3978	9.0952	6.7932	4.4963	2.2494	0.4280	0.0002555
6.4	32.2053	29.8027	27.5001	25.1975	22.8949	20.5923	18.2898	15.9872	13.6846	11.3820	9.0795	6.7775	4.4806	2.2346	0.4197	0.0002279
6.5	32.0898	29.7872	27.4846	25.1820	22.8794	20.5768	18.2742	15.9717	13.6691	11.3665	9.0640	6.7620	4.4652	2.2201	0.4115	0.0002034
6.6	32.0745	29.7719	27.4693	25.1667	22.8641	20.5616	18.2590	15.9564	13.6538	11.3512	9.0487	6.7467	4.4501	2.2058	0.4036	0.0001816
6.7	32.0595	29.7569	27.4543	25.1517	22.8491	20.5465	18.2439	15.9414	13.6388	11.3362	9.0337	6.7317	4.4351	2.1917	0.3959	0.0001621
6.8	32.0446	29.7421	27.4395	25.1369	22.8343	20.5317	18.2291	15.9265	13.6240	11.3214	9.0189	6.7169	4.4204	2.1779	0.3883	0.0001448
6.9	32.0300	29.7275	27.4249	25.1223	22.8197	20.5171	18.2145	15.9119	13.6094	11.3608	9.0043	6.7023	4.4059	2.1632	0.3810	0.0001293

N\u	Nx10^{-15}	Nx10^{-14}	Nx10^{-13}	Nx10^{-12}	Nx10^{-11}	Nx10^{-10}	Nx10^{-9}	Nx10^{-8}	Nx10^{-7}	Nx10^{-6}	Nx10^{-5}	Nx10^{-4}	Nx10^{-3}	Nx10^{-2}	Nx10^{-1}	N
7.0	32.0156	29.7131	27.4105	25.1079	22.8053	20.5027	18.2001	15.8976	13.5950	11.2924	8.9899	6.6879	4.3916	2.1508	0.3738	0.0001135
7.1	32.0015	29.6989	27.3963	25.0937	22.7911	20.4885	18.1860	15.8834	13.5808	11.2782	8.9757	6.6737	4.3775	2.1376	0.3668	0.0001032
7.2	31.9875	29.6849	27.3823	25.0797	22.7771	20.4746	18.1720	15.8694	13.5668	11.2642	8.9617	6.6598	4.3636	2.1246	0.3599	0.00009219
7.3	31.9737	29.6711	27.3685	25.0659	22.7633	20.4603	18.1582	15.8556	13.5530	11.2504	8.9479	6.6460	4.3500	2.1118	0.3532	0.00008239
7.4	31.9601	29.6575	27.3549	25.0523	22.7497	20.4472	18.1446	15.8420	13.5394	11.2368	8.9343	6.6324	4.3364	2.0991	0.3467	0.00007364
7.5	31.9467	29.6441	27.3415	25.0389	22.7363	20.4337	18.1311	15.8286	13.5260	11.2234	8.9209	6.6190	4.3231	2.0867	0.3403	0.00006583
7.6	31.9334	29.6308	27.3282	25.0257	22.7231	20.4205	18.1179	15.8153	13.5127	11.2102	8.9076	6.6057	4.3100	2.0744	0.3341	0.00005886
7.7	31.9203	29.6173	27.3152	25.0126	22.7100	20.4074	18.1048	15.8022	13.4997	11.1971	8.8946	6.5927	4.2970	2.0623	0.3280	0.00005263
7.8	31.9074	29.6048	27.3023	24.9997	22.6971	20.3945	18.0919	15.7893	13.4868	11.1842	8.8817	6.5798	4.2842	2.0503	0.3221	0.00004707
7.9	31.8947	29.5921	27.2895	24.9869	22.6844	20.3818	18.0792	15.7766	13.4740	11.1714	8.8689	6.5671	4.2716	2.0386	0.3163	0.00004210
8.0	31.8821	29.5795	27.2769	24.9744	22.6718	20.3692	18.0666	15.7640	13.4614	11.1589	8.8563	6.5545	4.2591	2.0269	0.3106	0.00003767
8.1	31.8697	29.5671	27.2645	24.9619	22.6594	20.3568	18.0542	15.7516	13.4490	11.1464	8.8439	6.5421	4.2468	2.0155	0.3050	0.00003370
8.2	31.8574	29.5548	27.2523	24.9497	22.6471	20.3445	18.0419	15.7393	13.4367	11.1342	8.8317	6.5298	4.2346	2.0042	0.2996	0.00003015
8.3	31.8453	29.5427	27.2401	24.9375	22.6350	20.3324	18.0298	15.7272	13.4246	11.1220	8.8195	6.5177	4.2226	1.9930	0.2943	0.00002699
8.4	31.8333	29.5307	27.2282	24.9256	22.6230	20.3204	18.0178	15.7152	13.4126	11.1101	8.8076	6.5057	4.2107	1.9820	0.2891	0.00002415
8.5	31.8215	29.5189	27.2163	24.9137	22.6112	20.3086	18.0060	15.7034	13.4008	11.0982	8.7957	6.4939	4.1990	1.9711	0.2840	0.00002162
8.6	31.8098	29.5072	27.2046	24.9020	22.5695	20.2969	17.9943	15.6917	13.3891	11.0865	8.7840	6.4822	4.1874	1.9604	0.2790	0.00001936
8.7	31.7982	29.4957	27.1931	24.8905	22.5879	20.2853	17.9827	15.6801	13.3776	11.0750	8.7725	6.4707	4.1759	1.9498	0.2742	0.00001733
8.8	31.7868	29.4842	27.1816	24.8790	22.5765	20.2739	17.9713	15.6687	13.3661	11.0635	8.7610	6.4592	4.1646	1.9393	0.2694	0.00001552
8.9	31.7755	29.4729	27.1703	24.8678	22.5652	20.2626	17.9600	15.6574	13.3548	11.0523	8.7497	6.4480	4.1534	1.9290	0.2647	0.00001390
9.0	31.7643	29.4618	27.1582	24.8566	22.5540	20.2514	17.9488	15.6462	13.3437	11.0411	8.7386	6.4368	4.1423	1.9187	0.2602	0.00001245
9.1	31.7533	29.4507	27.1481	24.8455	22.5429	20.2404	17.9378	15.6352	13.3326	11.0300	8.7275	6.4258	4.1313	1.9087	0.2557	0.00001115
9.2	31.7424	29.4398	27.1372	24.8346	22.5320	20.2294	17.9268	15.6243	13.3217	11.0191	8.7166	6.4148	4.1205	1.8987	0.2513	0.000009988
9.3	31.7315	29.4290	27.1264	24.8238	22.5212	20.2186	17.9160	15.6135	13.3109	11.0083	8.7058	6.4040	4.1098	1.8888	0.2470	0.000008948
9.4	31.7208	29.4183	27.1157	24.8131	22.5105	20.2079	17.9053	15.6028	13.3002	10.9976	8.6951	6.3934	4.0992	1.8791	0.2429	0.000008018
9.5	31.7103	29.4077	27.1051	24.8025	22.4999	20.1973	17.8948	15.5922	13.2896	10.9870	8.6845	6.3828	4.0887	1.8595	0.2387	0.000007185
9.6	31.6998	29.3972	27.0946	24.7920	22.4895	20.1869	17.8843	15.5817	13.2791	10.9765	8.6740	6.3723	4.0784	1.8599	0.2347	0.000006439
9.7	31.6894	29.3868	27.0843	24.7817	22.4791	20.1765	17.8739	15.5713	13.2688	10.9662	8.6637	6.3620	4.0681	1.8505	0.2308	0.000005771
9.8	31.6792	29.3766	27.0740	24.7714	22.4688	20.1663	17.8637	15.5611	13.2585	10.9559	8.6534	6.3517	4.0579	1.8412	0.2269	0.000005173
9.9	31.6690	29.3664	27.0639	24.7613	22.4587	20.1561	17.8535	15.5509	13.2483	10.9458	8.6433	6.3416	4.0479	1.8320	0.2231	0.000004637

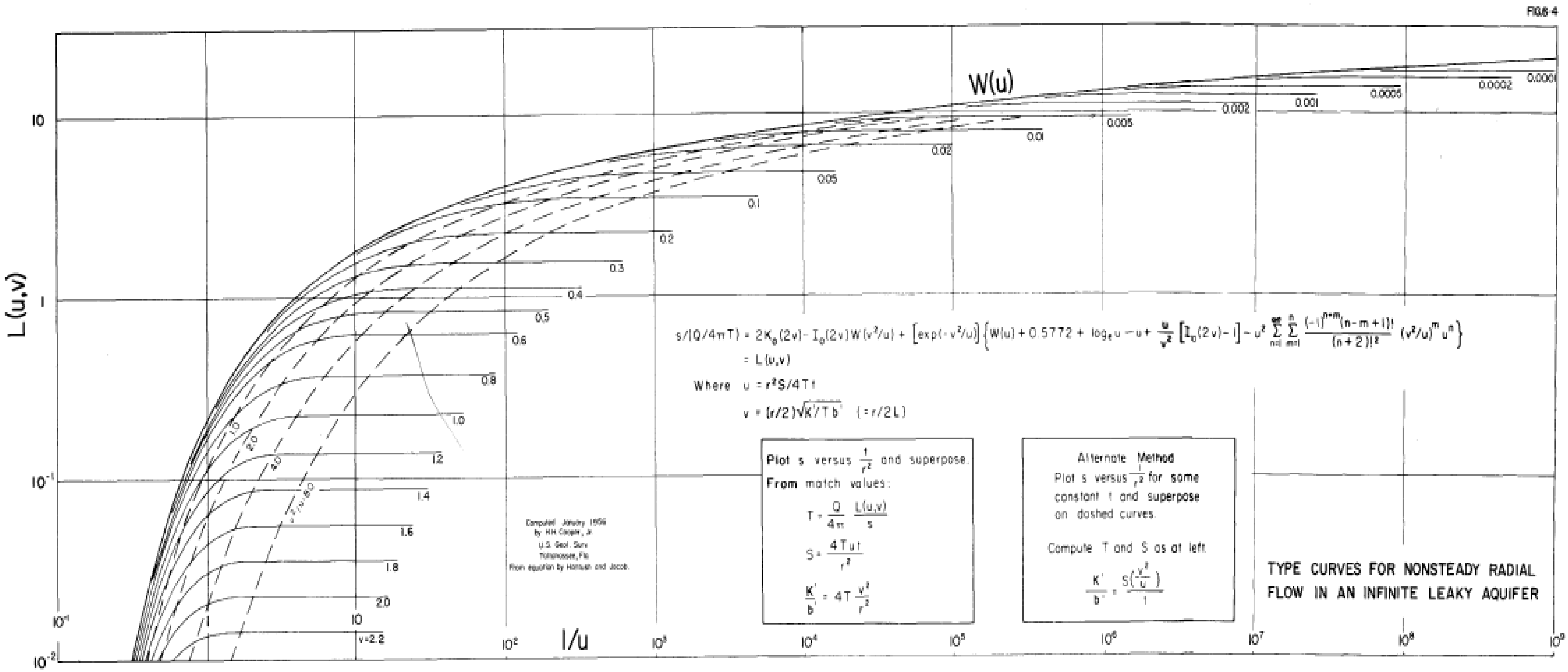


Figure 6-4 Type curves for non-steady state flow in leady aquifer

Example

Table 6-3 presents data supplied by J.G. Ferris and presented at the A.W.R.C. 1967 Ground Water School, by S.W. Lohman. The test results were in imperial units but have been converted to metric for these notes. **Table 6-3** gives drawdowns in three bores in an aquifer at varying distances from a bore being pumped at a rate of 2720 m^3/day.

Determine the transmissivity and storage coefficient for the aquifer using equations 6.18 and 6.19.

Table 6-3 Data for non-steady state flow analysis

Time since pumping started	Bore N-1 r = 61 m	Bore N-2 r = 122 m	Bore N-3 r = 244 m
t_m (mins)	Drawdown "s" (m)	Drawdown "s" (m)	Drawdown "s" (m)
1	0.20	0.05	0.00
1.5	0.27	0.08	0.01
2	0.30	0.12	0.01
2.5	0.34	0.14	0.02
3	0.37	0.16	0.03
4	0.41	0.20	0.05
5	0.45	0.23	0.07
6	0.48	0.27	0.08
8	0.53	0.30	0.11
10	0.57	0.34	0.14
12	0.60	0.37	0.16
14	0.63	0.40	0.18
18	0.67	0.44	0.22
24	0.72	0.48	0.27
30	0.76	0.52	0.29
40	0.81	0.57	0.34
50	0.85	0.61	0.37
60	0.88	0.64	0.40
80	0.93	0.68	0.45
100	0.96	0.73	0.49
120	1.00	0.76	0.52
150	1.04	0.80	0.56
180	1.07	0.83	0.59
210	1.10	0.86	0.62
240	1.12	0.88	0.64

(Drawdowns in observation bores N-1, N-2, N-3 at distance r from bore pumped at constant rate of 2720^3/day.)

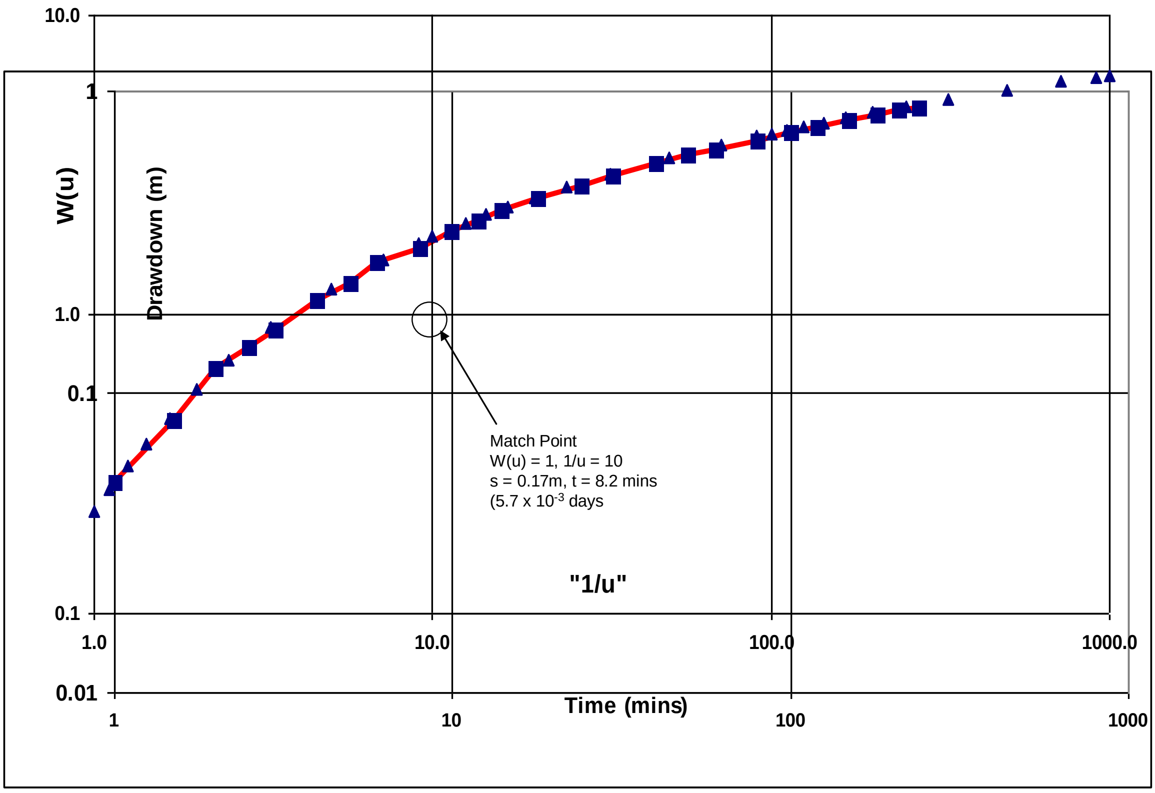


Figure 6-5 Type curve solution, confined aquifer, non-steady state, constant Q

The alternative type curve solution is shown is **Figure 6-5**, as a plot of drawdown versus time (in minutes) on logarithmic paper. There is no need to convert the time units from minutes to days before plotting. The conversion can be done prior to the calculation of S from equation 6.19. The analysis has been carried out for one bore only (bore N-2). Plots for N-1 and N-3 could be carried out separately or plotted as one curve as t/r^2.

<u>Solution</u>

From equation 6.18:

$$T = \frac{Q}{4\pi}\frac{W(u)}{s}$$

Q = 2720 m^3/day

From match point:

W(u) = 1, s = 0.17m

$$T = \frac{2720x1}{4\pi 0.17}$$

= 1270 m^2/day

From equation 6.19:

$$S = \frac{4Tut}{r^2}$$

$r^2 = 1.5 \times 10^4$ m^2

From match point:

T = 5.7×10^{-3} days, u = 0.1

S = 1.93×10^{-4}

Assumptions

In deriving the equations for non-steady state flow the following assumptions were made:

1. the aquifer is homogeneous and isotropic;
2. the aquifer has infinite areal extent;
3. the discharging bore penetrates the entire thickness of the aquifer;
4. the bore has an infinitesimal (very small) diameter; and
5. the water removed from storage is discharged instantaneously with decline in head.

6.3.1.3 Modified Non-Steady State Flow Equations

Under certain circumstances the non-steady state flow equations can be modified to give straight line solutions.

Jacob found that if "u" is very small (he suggested "u" is ≤ 0.01), then all terms beyond and including "u" in equation 6.17 can be neglected. However, Driscoll suggests, and the author has found, that the assumption is still valid for higher values of "u" with the error increasing as "u" increases. The smaller the value of "u" the smaller is the error involved.

Kruseman and de Ridder gives the following comparisons:

An error less than	1%	2%	5%	10%
For u smaller than	0.03	0.05	0.1	0.15

For these notes it is assumed that the Modified Non-Steady State Flow equations apply if "u" is ≤ 0.05. It can also be shown that, for an observation bore, "u"≤ 0.05 when s/Δs ≥ 1.05. This relationship is developed in the derivation of equation 6.40.

Drawdown Method - Constant r, Varying t

For small values of "u" equation 6.17 reduces to:

$$s = \frac{Q}{4\pi T}(-0.577216 - \log_e \frac{r^2 S}{4Tt}) \qquad .....6.22$$

This may be rewritten as:

$$T = \frac{Q}{4\pi s}(\log_e 0.562 + \log_e \frac{4Tt}{r^2 S} S)$$

$$= \frac{2.3Q}{4\pi s}(\log_{10} \frac{2.25Tt}{r^2 S}) \qquad .....6.23$$

If a series of drawdown measurements are taken at one observation bore at various times during a constant discharge pump test, then all terms in equation 6.23 are constant except "s" and "t".

Equation 6.23 can be rearranged to give:

$$s = \frac{2.3Q}{4\pi T}(\log_{10} \frac{2.25Tt}{r^2 S} + \log_{10} t)$$

$$= aQ + bQ \log_{10} t \qquad .....6.24$$

where:

$$b = \frac{2.3}{4\pi T} = \frac{\Delta s}{Q} \quad = \text{constant}$$

$$a = b \log_{10} \frac{2.25T}{r^2 S} \quad = \text{constant}$$

A plot of drawdown "s" versus "$\log_{10}$t" will give a straight line with slope "bQ" and, provided non linear head loss is absent, an intercept of "aQ".

"bQ" is called "Δs" and is defined as the drawdown per log cycle.

Transmissivity

In practice "s" is plotted against "t" on semi-logarithmic graph paper with "t" on the logarithmic scale.

$$bQ = \Delta s = \frac{2.3Q}{4\pi T} \qquad .....6.25$$

and

$$T = \frac{2.3Q}{4\pi \Delta s} \qquad .....6.26$$

Storage Coefficient (drawdown method)

From:

$$s = \frac{2.3Q}{4\pi T}(\log_{10}\frac{2.25Tt}{r^2 S})$$

$$= \Delta s \log_{10}\frac{2.25Tt}{r^2 S} \qquad .....6.27$$

i.e.

$$\frac{s}{\Delta s} = \log_{10}\frac{2.25Tt}{r^2 S}$$

and

$$10^{\frac{s}{\Delta s}} = \frac{2.25Tt}{r^2 S}$$

$$S = \frac{2.25Tt}{r^2 10^{\frac{s}{\Delta s}}} \qquad .....6.28$$

Storage Coefficient (zero drawdown intercept)

Consider the case when the drawdown in the observation bore is zero, i.e. immediately prior to drawdown commencing. Let this point occur t_o days after the commencement of pumping.

Equation 6.28 then reduces to:

$$S = \frac{2.25Tt_0}{r^2 10^0}$$

$$= \frac{2.25Tt_0}{r^2} \qquad .....6.29$$

t_0 is determined by extending the "time-drawdown" curve back until it intersects the zero drawdown line.

Procedure

1. Plot on semi-logarithmic graph paper, drawdown versus time, with time on the logarithmic scale.
2. Calculate Δs, the drawdown per log cycle,
3. Transmissivity is determined using equation 6.26.
4. Storage Coefficient is determined either from equation 6.28 or 6.29.
5. In equation 6.29, "t_0" is obtained by extrapolating the straight line plot back to intersect the zero drawdown line.

Note: equation 6.26 can be used to determine transmissivity using data from either the pumped bore or observation bore. The presence of non linear head loss does not affect the slope on a semi-log plot, merely its location.

From equation 6.26 it can be seen that Δs is proportional to Q, and that "b" is the drawdown per log cycle per unit discharge.

Equations 6.28 and 6.29 are influenced by the location on the plot. The drawdown in the pumped bore is influenced by non linear head loss, and the effective radius of the bore is unknown. Storage coefficient should only be calculated from data obtained from an observation bore if it is to be calculated from pumping tests.

Example

Using the Modified Non Steady State Flow equations 6.26, 6.28 and 6.29 calculate Transmissivity and Storage Coefficient from the test data for Bore N-2 in **Table 6-3**.

The semi-logarithmic plot and analysis are presented on **Figure 6-6**.

Drawdown Method - Constant "t" varying "r"

This situation occurs when drawdowns are measured at the same time "t" in a number of observation bores at varying distances "r" from the discharging bore.

Equation 6.23 reduces to:

$$s = \frac{2.3Q}{4\pi T}(\log_{10}\frac{2.25Tt}{S} - 2\log_{10} r)$$

$$= \Delta s D - 2\Delta s \log_{10} r \qquad .....6.30$$

Equation 6.30 is an equation to a straight line. A plot of drawdown "s" against "r" will give a straight line with slope -2Δs i.e. twice the slope of the time-drawdown curve and in the opposite direction. As "r" increases "s" decreases.

$$D = \log_{10}\frac{2.25Tt}{S} = constant$$

In practice, drawdown "s" is plotted against "r" on semi-logarithmic graph paper.

If the slope of the distance drawdown plot is called Δs' , then:

Δs' = -2Δs

= -2 x 2.3 Q/4πT

and

T = -2.3 Q/2π Δs' 6.31

Δs' is always negative so T is always positive.

In the same manner as was used to derive equation 6.28:

$$S = \frac{2.25Tt}{r^2 10^{(-2s/\Delta s')}} \qquad .....6.32$$

Transmissivity and storage coefficient are then calculated using equations 6.31 and 6.32 respectively.

Procedure

1. Plot on semi-logarithmic graph paper, drawdown in the observation bores, at time "t", versus the distance of the observation bores from the pumping facility "r". The distance should be plotted on the logarithmic scale.
2. Determine the drawdown per log cycle Δs'.
3. Calculate the Transmissivity T from equation 6.31.
4. Calculate the Storage Coefficient from equation 6.32.

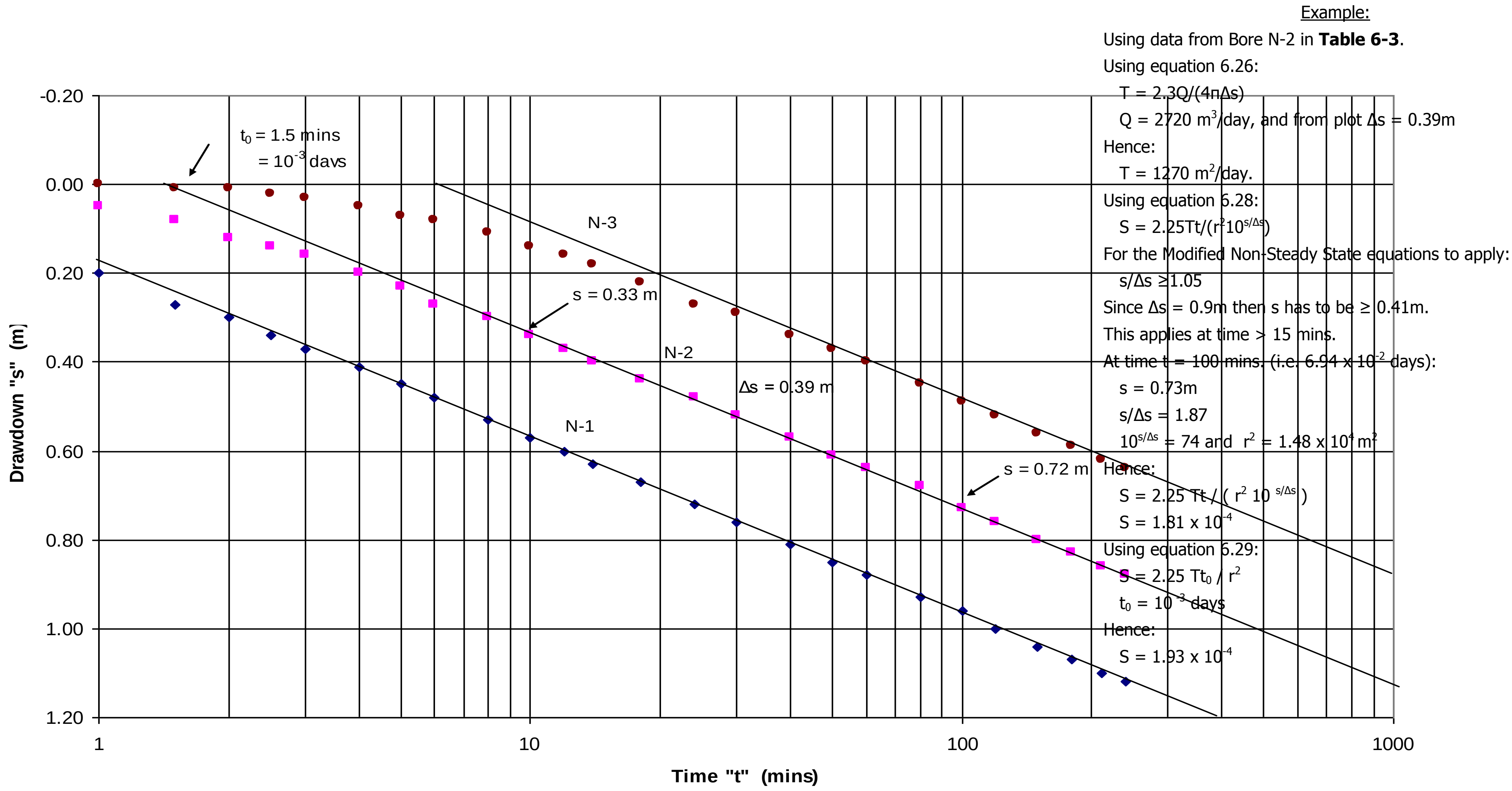


Figure 6-6 Modified non-steady state flow example – confined aquifer, constant Q, constant r, varying t

Example

Using the Modified Non-Steady State Flow equation 6.31 and 6.32 calculate Transmissivity and Storage Coefficient from the test data presented in **Table 6-3** Use the drawdown data at time 240 minutes.

The semi-logarithmic plot and analysis are presented on **Figure 6-7**.

Residual Drawdown Method

When discharge from a bore ceases, the water level in the bore begins to rise, or recover. This process of recovery is best understood if it is thought of in terms of the following analogy.

Cessation of discharge from a bore is analogous to continuing pumping but having the water returned to the bore at the same rate by means of a recharge pump. The net discharge is then zero.

It has been shown previously that the increase in drawdown in a bore is proportional to the logarithm of time. Under the influence of the discharging pump the drawdown continues to increase but the time for which the pump has been operating is relatively long.

The recharging bore has the effect of causing a rise in water level. Since it has only just started to operate and is in an earlier part of the log scale, the rise in water level is going to be much faster than the increase in drawdown caused, at this particular time, by the discharging bore. The net result is that the water level in the bore rises.

The net drawdown in the bore, or residual drawdown, is equal to the summation of the effects of the hypothetical pumps operating during the recovery phase.

The influence of each of these hypothetical pumps is given by the following equations.

$$s_1 = \frac{2.3Q_1}{4\pi T}\log_{10}\frac{2.25Tt_1}{r^2}S \quad \text{(discharging at } Q_1\text{)} \qquad .....6.33$$

and

$$-s_2 = \frac{-2.3Q_2}{4\pi T}\log_{10}\frac{2.25Tt_2}{r^2 S} \quad \text{(recharging at } Q_2\text{)} \qquad .....6.34$$

where:

t_1 = time since pumping began.

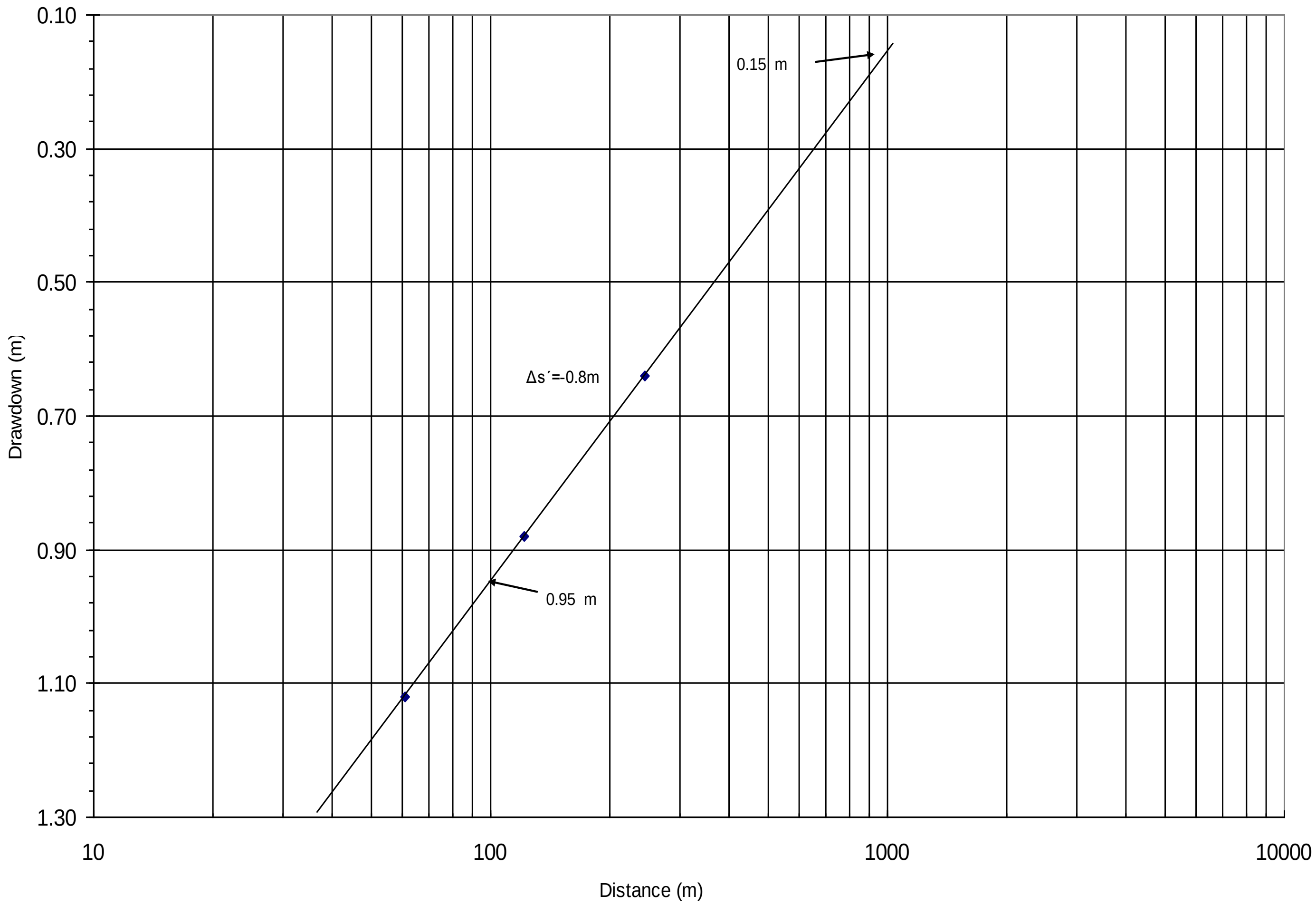
t_2 = time since pumping stopped.

To obtain a rise in water level during pumping it is not necessary to have a complete cessation of pumping. Consider initially the case where the rate of recharge from the recharging bore is equal to the rate of discharge from the discharging bore.

Let Q_1 = Q = Q

Residual drawdown = s_1 + ($-s_2$)

$$= \frac{2.3Q}{4\pi T}\log_{10}\frac{t_1}{t_2} \qquad .....6.35$$



Example:

Using data at 240 mins from **Table 6-3**.

From equation 6.31:

$T = -2.3\ Q / (2\ \pi\ \Delta s')$

$Q = 2720\ m^3/day$, and $\Delta s' = 0.8m$ (from plot)

Hence:

$T = 1250\ m^2/day$

From equation 6.32:

$S = 2.25\ Tt /(\ r^2\ 10^{(-2s/\Delta s')}\ )$

$t = 240$ mins (0.167 days)

Using the drawdown at 50m:

$s = 1.19m$, $r^2 = 2500m^2$ and $-2s/\Delta s = 2.98$

Hence:

$S = 1.88 \times 10^{-4}$

Figure 6-7 Modified non-steady state flow example – confined aquifer, constant Q, constant t, varying r

If Q, T, S, and r are all constant:

residual drawdown = constant $\log_{10} t_1/t_2$

A plot of residual drawdown versus $\log_{10} t_1/t_2$ results in a straight line with a slope of $\frac{2.3Q}{4\pi T}$ hence:

$$T = \frac{2.3Q}{4\pi\Delta s(residual drawdown)} \quad .....6.36$$

where:

Δs (residual drawdown) is the slope of the residual drawdown versus $\log_{10} t_1/t_2$.

If no interference effects are present, then the line extrapolated should intersect the zero residual drawdown line at $t_1/t_2 = 1$.

If a recharge boundary has been encountered then the line will intersect the zero residual drawdown line at a value of t_1/t_2 which is > 1.

If dewatering has occurred or an impermeable boundary has been encountered then the line will intersect the zero residual drawdown line at a value of t_1/t_2 which is < 1.

From equation 6.35 it can be seen that the residual drawdown has no storage coefficient term in it and storage coefficient cannot be determined by means of residual drawdown analysis.

Procedure

1. Calculate t_1/t_2 (t_1 = time since pumping began and t_2 = time since pumping stopped).
2. Plot, on semi-logarithmic graph paper, residual drawdown, on the natural scale, versus t_2/t_2, on the logarithmic scale.
3. Determine Δs (residual drawdown).
4. Calculate the Transmissivity from equation 6.36.

Recovery Method

During the recovery period following a pumping test on a bore residual drawdowns are invariably measured. The true recovery i.e. s_2 in equation 6.34, is equal to the difference between the extended drawdown effect and the residual drawdown at that particular time.

The extrapolated drawdown could be read directly from the graph or determined by the following method:

Extrapolated drawdown = drawdown when pumping ceased + $\Delta s \log_{10} \frac{t_1}{t_1 - t_2}$

Hence:

Recovery = drawdown when pumping ceased + $\Delta s \log_{10} \frac{t_1}{t_1 - t_2}$ - residual drawdown

Transmissivity

Equation 6.34 has exactly the same form as equation 6.33 which is the equation to drawdown. A plot of recovery versus $\log_{10} t_2$ will give a straight line with slope Δs, which will be the same as the slope of the time-drawdown curve.

The transmissivity is given by:

$T = 2.3\ Q / 4\pi\Delta s$

If the pumping rate were reduced, but not to zero, then the recharge to the bore by means of the hypothetical recharge pump would be less than the initial discharge from the bore.

If the recharge to the bore, i.e. the reduction in discharge rate, is given as Q_2, then the recovery in the bore will be given by equation 6.34 and the transmissivity given by:

$$T = \frac{2.3Q_2}{4\pi\Delta s_{(recovery)}} \quad .....6.37$$

where Δs (recovery) is the slope of the recovery-time line following the reduction in discharge rate.

Storage Co-efficient

While the storage coefficient cannot be obtained by an analysis of the residual drawdown plot it can however, be obtained from the recovery plot. In fact, in many cases, a more accurate value of storage coefficient can be obtained by using recovery figures.

In some cases there is a residual drawdown or an error in standing water level in the observation pipes prior to commencement of pumping which has not been allowed for in the analysis. The analysis of recovery is a difference between two measured water levels during a pumping test and any initial error will be compensated for.

The recovery at time t_2 after pumping ceased is:

$$-s_2 = \frac{-2.3Q}{4\pi T}\log_{10}\frac{2.25Tt_2}{r^2 S}$$

The storage coefficient is given by:

$$S = \frac{2.25Tt_2}{r^2 10^{(s_2/\Delta s)}} \qquad .....6.38$$

which has been derived in the same manner as equation 6.28.

If the recovery line is extrapolated back to zero recovery point to intersect the zero recovery line at t_0, then:

$$S = \frac{2.25Tt_0}{r^2} \qquad .....6.39$$

These equations are identical to those for determination of T and S using drawdowns.

When determining transmissivity, either residual drawdown can be plotted against t_1/t_2 or recovery data can be plotted against the time since pumping stopped. In some cases the standing water level, or static head, is unknown and recovery i.e. static head - residual drawdown, can be plotted against t_1/t_2 to allow for antecedent conditions.

The slope of such a plot is of course in the opposite direction to the slope of residual drawdown against t_1/t_2.

The value of T which is used in the equations for calculating storage coefficient is determined from drawdown or recovery measurements in the observation bore being considered and not from the pumping bore.

The reason for this is that the aquifer may not be homogeneous and the transmissivity may not be the same in all directions from the pumping bore. The transmissivity as indicated by the pumping bore is an average transmissivity for all materials encountered by the radius of influence. The drawdowns in an observation bore, however, are related to the transmissivity in that direction alone.

It has been shown that the drawdown varies with the logarithm of time since pumping started. Significant errors can result in determining long term bore yield from a short duration pumping test. Any natural scale plot of drawdown level versus pumping time, will invariably give the deceptive picture of the approach to a fixed pumping level.

The flattening of slope is indicative of a logarithmic relationship when plotted to rectangular co-ordinates. If the pumping levels for a long duration test were plotted on the logarithmic scale it becomes apparent that the drawdown is still increasing and so the bore has not reached an equilibrium position and theoretically, never will.

Check on the Applicability of the Equations

The straight line approximation used in the Modified Non-Steady State Equation is valid for values of "u" greater than or equal to 0.05.

This can be developed into simpler terms.

The equation for storage coefficient using the Modified Non-Steady State Flow Equations is given by equation 6.28 as:

$$S = \frac{2.25Tt}{r^2 10^{(s/\Delta s)}}$$

where:

T = transmissivity.

t = time since pumping began (or in the case of recovery data, time since pumping stopped).

s = drawdown (or recovery) in the observation bore.

Δs = drawdown per log cycle.

r = distance of the observation bore from the discharging bore.

For the straight approximation to be valid "u" ≤ 0.05

i.e.

$$\frac{r^2 S}{4Tt} \leq 0.05$$

or

$$S \leq \frac{0.2Tt}{r^2}$$

From the Modified Non-Steady State flow equation the storage coefficient is given by:

$$S = \frac{2.25Tt}{r^2 10^{s/\Delta s}}$$

If the Modified Non-Steady State flow equations are to be applicable:

$$\frac{2.25Tt}{r^2 10^{s/\Delta s}} \leq \frac{0.2Tt}{r^2}$$

i.e.

$2.25 / 10^{(s/\Delta s)} \leq 0.2$

or

$10^{(s/\Delta s)} \geq 11.25$

or

$(s/\Delta s) \geq 1.05$ 6.40

For the Modified Non-Steady State Flow equations to be applicable the drawdown in the observation bore must be greater than 1.05 Δs.

This is a very quick and easy check on the applicability of the Modified Non-Steady State Flow Equations, for radial flow in confined aquifers. Occasions will arise where the straight line approximation is valid even where the drawdown is less than 1.05 Δs. However, this will be apparent in the plot, and it will be permissible to use the Modified Non-Steady State Flow Equations.

6.3.2 Variable Discharge Tests

Inherent in the previous equations is the fact that the discharge was held constant throughout the test. This is not always possible and in some cases it is desirable to allow the discharge to change throughout the test. The type of test in which the discharge is allowed to vary is called a variable discharge pumping test. Variable discharge tests are normally carried out to determine the pumping bore's long term performance but the measurements from observation bores can still be analysed.

The two main types of variable discharge pumping tests are:

1. constant drawdown test. In this type of test the drawdown is held constant and the discharge is allowed to vary; and
2. the step drawdown test. In this type of test the discharge is allowed to vary in steps but is held constant during each of the steps and measurements of drawdown are recorded at specified times.

Generally the step drawdown test is carried out such that the rate increases during the test. However, it is not necessary to carry out the test in this manner, in fact one of the most useful types of tests is one in which the test is run at a constant discharge for a reasonably long period of time, say in excess of 24 hours, to determine the existence of boundaries or delayed drainage, and then to allow the discharge to be reduced in steps so that the magnitude of any non-linear head loss can be determined. The last step in this particular type of the test is the recovery stage i.e. the discharge has been reduced to zero.

Extending this a little further it is apparent that stoppages, planned or accidental, during a pump test can be allowed for in the analysis without a need to repeat the test.

The method of analysing variable discharge pumping tests is covered in Section 11 and will not be treated here. For most observation bores the non-linear head loss term is absent and can be ignored. The Eden-Hazel Method, Chapter 11 can be used to analyse variable discharge test data whether the tests are constant drawdown or variable drawdown.

6.3.3 Other Methods

Other methods, such as the Chow Nomogram, are available for analysing Non-Steady State flow in confined aquifers and can be found in the references cited. They will not be presented in this course but this does not mean that they are not suitable methods. Full advantage should be taken of all available methods for analysing pumping tests and each of the methods compared before the final conclusions are drawn.

6.4 SEMI-CONFINED AQUIFER TEST ANALYSIS

Much of the information presented in this section on Semi-Confined aquifers can be found in G.P. Kruseman and N.A. De Ridder, 1991.

For a more complete coverage of available methods of analyses the reader is referred to that publication.

6.4.1 General

In nature, perfectly confined and perfectly unconfined aquifers are less frequently found than semi-confined (or leaky) aquifers. In general, the latter are a common feature of many alluvial areas such as: deltas, coastal plains, river valleys, former lake basins etc.

The water associated with a semi-confined aquifer is stored not only in the aquifer material itself but also in the semi-pervious overlying or underlying material. It is quite probable that in this case the whole of the saturated material, including both the coarse transmitting material and the finer semi-pervious material, should be grouped together as the aquifer. However, only the coarse material should be regarded as having any significant transmitting characteristics.

When a semi-confined aquifer is pumped, water is contributed by both the aquifer proper and the semi-pervious materials.

Water is drawn from the aquifer in accordance with the principles and equations given for confined aquifers, but because of the increasing contribution from the semi-pervious layer as the drawdown increases and as the radius of influence increases, the contribution from the aquifer itself becomes smaller. The situation can arise where the contribution from the semi-pervious material is equal to the discharge from the bore and steady state flow will occur.

As a result of pumping, the potentiometric head in the aquifer is lowered, creating a difference in groundwater head between the aquifer and the semi-pervious material.

This pressure differential causes a vertical flow of water from the semi-pervious material to the aquifer. The amount of flow per unit area is directly proportional to the head differential and inversely proportional to the hydraulic resistance of the saturated part of the semi-pervious layer.

$$q = \frac{h' - h_{pot}}{c} \qquad .....6.41$$

where:

q = flow per unit area in the semi-pervious layer.

h^1 = the head in the semi-pervious layer (assumed to be constant).

h_{pot} = the potentiometric head in the aquifer at a particular time.

c = the hydraulic resistance of the semi-pervious layer.

It is assumed that the head of water in the semi-pervious layer is constant. Of course, with long periods of pumping the semi-pervious layer can become drained and this assumption would not be valid.

The leakage coefficient is defined as the reciprocal of the hydraulic resistance (c).

If h^1 remains constant then $h' - h_{pot}$ at a particular point is equal to the drawdown in the aquifer at that radial distance from the discharging bore at a particular time.

From equation 6.41:

$$q = \frac{s}{c} \qquad .....6.42$$

where:

1/c = the leakage coefficient.

s = the drawdown at a particular point in the aquifer.

Leakage coefficient is defined as the rate of leakage from the semi-pervious layer per unit area per unit drawdown in the aquifer proper.

The total contribution from the semi-pervious layer is the summation of all terms such as equation 6.42 over the total area influence by pumping.

The contribution per unit area of the semi-pervious layer may be small, and thought to be negligible. However, the contribution is over such a vast area, that, it can not only be significant, but may, in fact, be the major source of water during a particular pump test.

It was stated previously that water was contributed by the aquifer in accordance with the principles and equations controlling flow in confined aquifers. This is true, but the actual amount of water being contributed by the aquifer proper at any one time is not equal to the discharge from the bore. Thus an attempt to utilise the confined aquifer equations with the bore discharge rate from a leaky aquifer will give erroneous results. The effect of leakage from the semi-pervious material must be taken into account.

6.4.2 Constant Discharge

In these notes constant discharge test analyses only will be presented. The methods outlined in Section VII for analysing variable discharge test analyses may be applied to tests on observation bores also.

6.4.2.1 Steady State Flow

A number of methods are available for analysing steady state flow in semi-confined aquifers but only one will be presented here.

This does not mean that the others should be ignored.

<u>Method of Analysis (Hantush-Jacob's Method)</u>

Unaware of previous work done by De Glee, Hantush and Jacob (1955) derived the equation:

$$s = \frac{Q}{2\pi T} K_0(2v) \qquad \text{.....6.43}$$

where:

K_0 = the modified Bessel function of the second kind of zero order

$$v = \frac{r}{2L}$$

Hantush and Jacob noted that if r/L is small (r/L < 0.05) then the steady state drawdown in a semi-confined aquifer can be approximated by:

$$s_m = \frac{2.3Q}{2\pi T}(\log_{10}\frac{1.12L}{r}) \qquad \text{.....6.44}$$

where:

s_m = maximum (or steady state) drawdown at distance r from the pumping bore.

Q = discharge rate.

T = the transmissivity of the aquifer.

L = the leakage factor of semi-pervious material = √(Tc).

r = distance of observation bore from pumping bore.

Thus a plot of "s" versus "r" on semi-logarithmic paper, with "r" on the logarithmic scale, will result in a straight line in the range where r/L is small.

In the range where r/L is large, the points fall on a curve that approaches the zero drawdown axis asymptotically.

The slope of the straight line portion of the curve, i.e. the drawdown per log cycle of r, is expressed as:

$$\Delta s_m = \frac{2.3Q}{2\pi T} \qquad \text{.....6.45}$$

This is of the same form as equation 6.8 for confined aquifers.

The extended straight line portion of the curve intersects the r axis where the drawdown is zero at the point r_0. At this point s = 0, and r = r_0, and equation 6.44 reduces to:

$$0 = \frac{2.3Q}{2\pi T}(\log_{10}\frac{1.12L}{r_0})$$

From which it follows that:

$$\frac{1.12L}{r_0} = \frac{1.12}{r_0}\sqrt{Tc} \qquad \text{.....6.46}$$

= 1

And hence:

$$c = \frac{(r_0 / 1.12)^2}{T} \qquad .....6.47$$

Procedure

1. Plot on semi-logarithmic paper "s_m" versus "r" ("r" on the logarithmic scale), i.e. the maximum or steady state drawdown measured in each observation bore against the corresponding distance of the observation bore from the pumped bore.
2. Draw the straight line of best fit through the points which appear to fall on a straight line.
3. Determine the slope of the line of best fit, i.e. the drawdown per log cycle.
4. Using the discharge rate Q calculate T from equation 6.45.
5. Extend the straight line to intersect the r axis and record this intercept as r_o.
6. Calculate the hydraulic resistance "c" of the semi-pervious layer by substituting r and T into equation 6.47. The leakage coefficient can be calculated from the reciprocal of the hydraulic resistance.

Table 6-4 Data for semi-confined aquifer test analysis

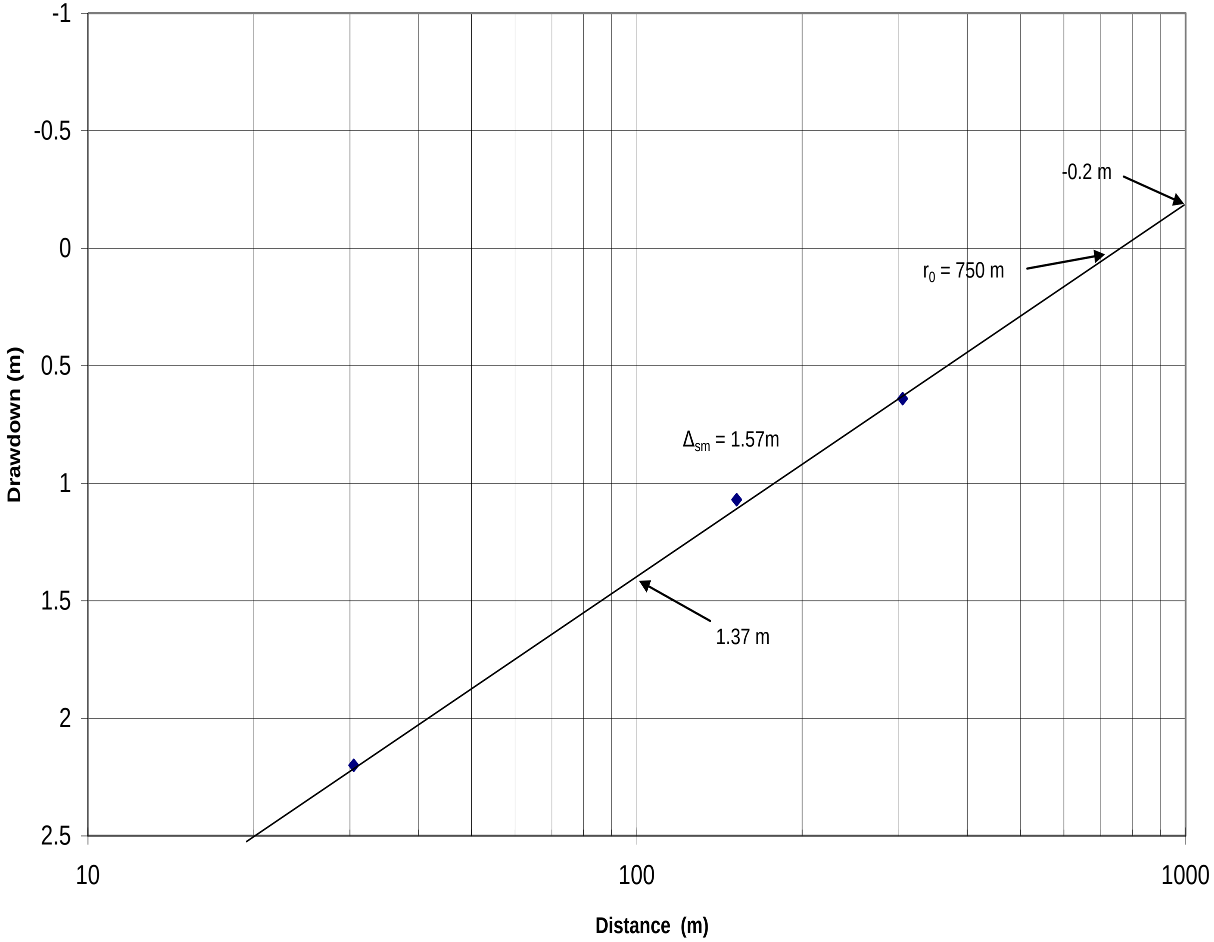
Time Since Pumping Began		Drawdown in metres		
Mins	Days	Bore 1 r = 30.5 m	Bore 2 r = 152 m	Bore 3 r = 305 m
0.2	0.000139	0.54	0.00	0.00
0.5	0.000347	0.84	0.04	0.00
1	0.000694	1.09	0.14	0.00
2	0.00139	1.30	0.28	0.04
5	0.00347	1.61	0.54	0.17
10	0.00694	1.80	0.71	0.30
20	0.0139	1.97	0.87	0.45
50	0.0347	2.11	1.01	0.59
100	0.0694	2.17	1.07	0.64
200	0.139	2.20	1.07	0.64
500	0.347	2.20	1.07	0.64
1000	0.694	2.20	1.07	0.64

Example

Table 6-4 (taken from Cooper, 1963 p. C54) gives postulated drawdowns in observation bores at various distances from a bore which discharges at a constant rate of 5 450 m^3 /day for 1,000 minutes from a leaky artesian aquifer.

Determine transmissivity and hydraulic resistance for the aquifer utilising equation 6.45 and the drawdown data at 1,000 minutes.

The semi-logarithmic plot and analysis are given in **Figure 6-8**.



Example

Using data from **Table 6-4**.

Transmissivity:

Using equation 6.45:

$T = 2.3\ Q / 2\ \pi\ \Delta s$

$Q = 5450\ m^3/day$

From the plot:

$\Delta s = 1.37m$.

Hence:

T = 1270 m2/day

Hydraulic Resistance: (see note)

Using equation 6.47:

$c = ((r_0/1.12)^2\ )\ /\ T$

From the plot:

$r_0 = 750m$

Hence:

c = 353 days

Leakage Coefficient: (see note)

Leakage Coefficient =1/c

$= 2.83 \times 10^{-3}$ days

NOTE: the values derived for c and 1/c are approximate only. One assumption for the straight line solution is r/L < 0.05.

From equation 3.15:

$L = \sqrt{(Tc)}$

= 670m

i.e. only r values < 0.05 x 670 (i.e. 34m) can be used.

More reliance should be placed on the Non-Steady State solution given in Figure 6.9

Figure 6-8 Steady state flow example – semi-confined aquifer

Assumptions

The assumptions made in deriving the Steady State flow equations presented above are:

1. The aquifer is infinite in areal extent.
2. The aquifer is homogeneous, isotropic and of uniform thickness.
3. Prior to pumping the potentiometric surface was (nearly) horizontal.
4. The aquifer is pumped at a constant discharge rate.
5. The pumped bore penetrates the entire aquifer and receives water from the entire thickness of the aquifer by horizontal flow.
6. The aquifer is semi-confined.
7. Flow to the bore is in a steady state.
8. The potentiometric head in the semi-pervious material remains constant, so that leakage through the covering layer is proportional to the drawdown of the potentiometric head in the aquifer.
9. L > 3b.
10. r/L < 0.05.

6.4.2.2 Non-Steady State Flow

Before a state of equilibrium is reached the drawdown of the potentiometric head increases with time.

The flow equation will not be identical to the Theis equation because the contribution from the aquifer is not constant but decreases as the contribution from the semi-pervious layer increases.

However, a flow equation similar to the Theis equation would be expected.

Method of Analysis (Hantush-Jacob Method)

According to Hantush and Jacob the drawdown in a semi-confined aquifer can be described by the following equation:

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{1}{y} e^{(-y-\frac{r^2}{4L^2 y})} dy$$

$$= \frac{Q}{4\pi T}(2K_0(2v) - \int_{v^2/u}^\infty \frac{1}{v^2/u} e^{(-y-\frac{v^2}{y})} dy)$$

$$s = \frac{Q}{4\pi T} L(u,v)$$

$$u = \frac{r^2 S}{4Tt} \qquad .....6.48$$

Equation 6.48 has the same form as the Theis equation for unsteady state flow in a confined aquifer, but there are two parameters in the integral, u and v (v = r/2L).

Values of L(u,v) for certain values of "v", as "u" varies, were computed by Cooper.

Utilising tabulated values of L(u,v) and u, solutions for T, S and c can be obtained. However, a type curve solution would be preferable.

A method of solution along the same lines as was followed by the Theis type curve method has been developed, but instead of one type curve, there is a family of type curves, a type curve for each value of "v".

This family of type curves is given in **Figure 6-4**.

Procedure

1. Plot on transparent double logarithmic paper, of the same scale as the type curve plot, drawdown (on the vertical scale) versus time (on the horizontal scale).
2. Superimpose the plot on the type curve **Figure 6-4**, keeping the axes of the two plots parallel, and fit it to one of the type curves.
3. Select the match point and record values of L(u,v), 1/u,s and t.
4. Record the v value of the type curve fitted.
5. Compute transmissivity from Equation 6.48:

$$T = \frac{Q}{4\pi}\frac{L(u,v)}{s}$$

6. Compute storage coefficient from:

$$S = \frac{4Tut}{r^2} \qquad .....6.49$$

7. Compute hydraulic resistance from:

   C = b'/K'

   $= r^2/4Tv^2$ 6.50

   If required the leakage coefficient could have been calculated as:

   $1/c = 4Tv^2/r^2$

8. Make sure that the units used are consistent.

Example

Using the data presented in **Table 6-4** for bore 3; determine the transmissivity storage coefficient, hydraulic resistance and leakage coefficient for the aquifer.

The plot and type curve analysis are presented in **Figure 6-9**.

The test data is plotted on double logarithmic paper and overlaid on **Figure 6-4**.

Assumptions

The assumptions made in deriving the Non-Steady State flow equations presented above are:

1 to 6 from section 6.4.2.1

7. The flow to the bore is in a non-steady state i.e. the drawdown differences with time are not negligible nor is the hydraulic gradient constant with time.
8. The water removed from storage is discharged instantaneously with decline in head.
9. The bore diameter is very small, so the storage in the bore can be neglected.

6.5 UNCONFINED AQUIFERS WITHOUT DELAYED YIELD

6.5.1 Constant Discharge

In this sub-section constant discharge test analysis only will be dealt with. The methods outlined in Section 7 for analysing variable discharge tests may be applied.

6.5.1.1 General

There are some basic differences between unconfined and confined aquifers when they are pumped.

A confined aquifer is depressurized but not dewatered during pumping. This creates a cone of depression in the potentiometric surface water pumped from a confined aquifer comes from expansion of the water in the aquifer due to a reduction in pressure, and from the compaction of the aquifer material due to increased effective stress flow towards a fully penetrating well in a confined aquifer remains horizontal.

Pumping from an unconfined aquifer causes a dewatering of the aquifer and creates a cone of depression in the water table. As pumping progresses, flow towards the well deviates to include a vertical flow component.

In unconfined aquifers, the water levels near the bore tend to decline at a slower rate than described by the Theis equation. The time-drawdown curves on log-log paper show a typical S-shape, from which three distinct segments can be recognised. The commonly used explanation is based on the concept of 'delayed yield' (Boulton, 1954). This concept was further developed by Neuman (1972) and Streltsova (1976).

They explain the three segments of the curve as follows:

- the steep first segment covers the first minutes of the test and the unconfined aquifer reacts in the same way as a confined aquifer (Theis curve); water is instantaneously released from storage;
- the flat intermediate part reflects the effect of dewatering that accompanies the decline in water table elevation; and
- the steep late segment shows the situation where the flow in the aquifer is horizontal again and the curve tends to conform to the Theis curve.

The derivation of equations for radial flow in any unconfined aquifer is extremely difficult. Dupuit showed that in order to derive them it is necessary to assume that the flow is horizontal and uniform everywhere in a vertical section through the axis of the bore. This assumption is not too bad when the magnitudes of the gradients involved are considered.

A concentric recharge boundary would have to surround the bore if steady state flow is to be achieved. However, without such a boundary, steady state flow is approximated when the drawdown differences become negligible with time if the pumping period is sufficiently long.

FIG. 6.9

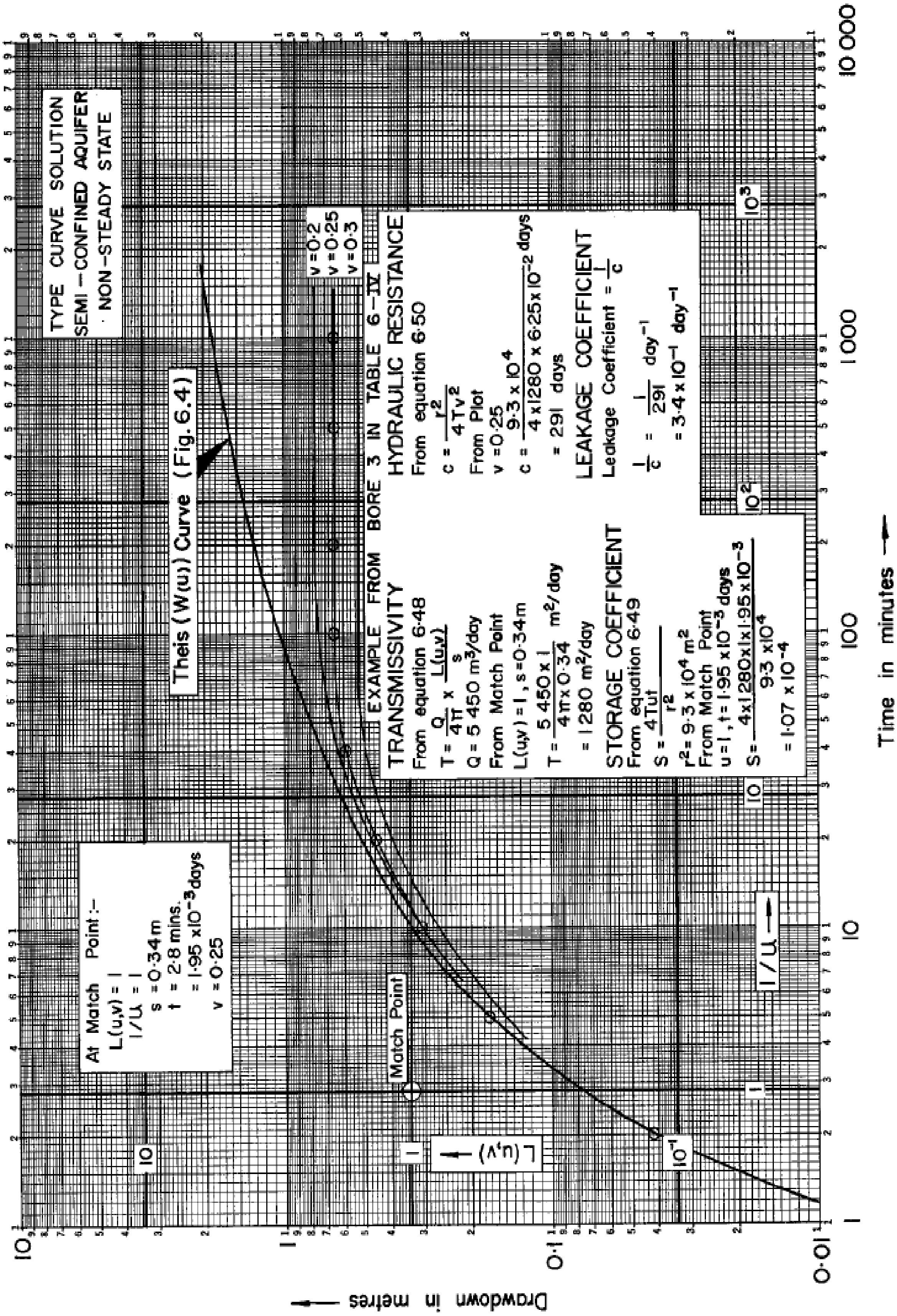


Figure 6-9 Type curve solution, semi-confined aquifer, non-steady state

Let two such concentric cylinders have radii r_1 and r_2.

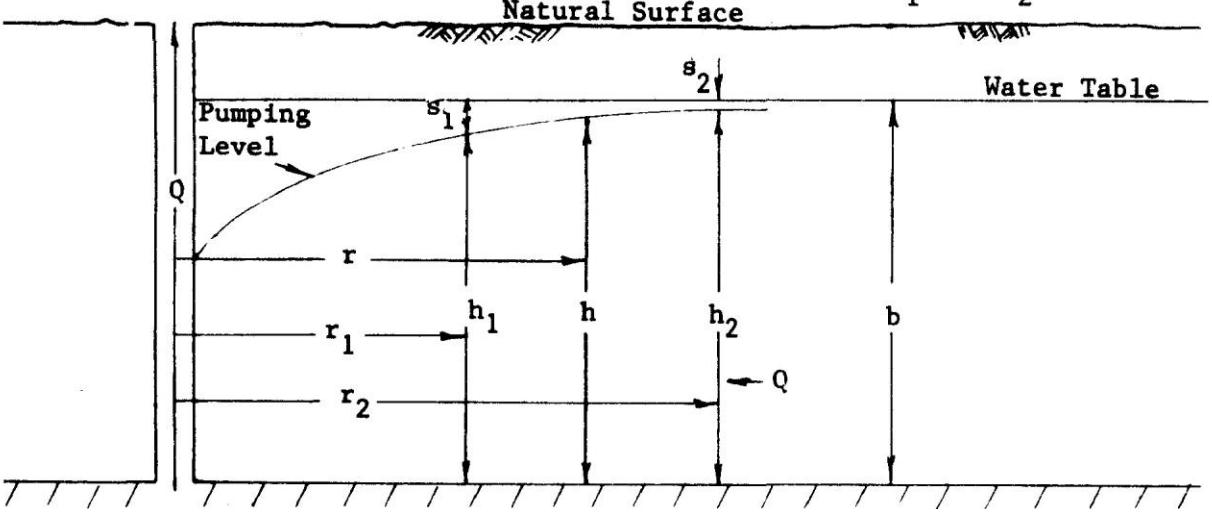


Figure 6-10 Unconfined aquifer, steady state flow derivation

6.5.1.2 Steady State Flow (Thiem-Dupuit Method)

Derivation

To derive the Thiem equation for steady state flow in an unconfined aquifer let **Figure 6-10** represent half the cross-section of the cone of depression in an unconfined aquifer around a discharging bore that has been pumped at a constant rate Q long enough for a steady state flow to be closely approached, and the volume of water being derived from storage is negligible compared with the volume of water moving towards the bore.

If the material is reasonably homogeneous, and if the base of the aquifer and the undisturbed water table are assumed to be parallel and horizontal then, by the law of conservation of matter, provided changes in storage are negligible, equal quantities of water are discharged from the bore and flow radially towards the bore through any two concentric cylinders within the cone of depression.

Under these conditions Darcy's Law may be expressed as first order ordinary differential equation in cylindrical co-ordinates.

$$Q = -KA\frac{dh}{dr}$$

where:

Q = the discharge rate from the bore.

K = the hydraulic conductivity of the aquifer.

A = the area of flow at radius "r".

= $2\pi rh$.

then:

$$Q = -K2\pi rh\frac{dh}{dr}$$

Separating variables:

$$\frac{dr}{r} = -2\pi\frac{K}{Q}hdh$$

Integrating between r_1 and r_2, and h_1 and h_2:

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{-2\pi K}{Q} \int_{h_1}^{h_2} h\,dh$$

$$\left[\log_e r\right]_{r_1}^{r_2} = \frac{-2\pi K}{Q} \left[\frac{h^2}{2}\right]_{h^1}^{h_2}$$

$$\log_e \frac{r_2}{r_1} = -\frac{\pi K}{Q}(h_2^2 - h_1^2)$$

.....6.52

Hence:

$$K = \frac{-2.3Q\log_{10}(r_2/r_1)}{\pi(h_2^2 - h_1^2)}$$

.....6.53

where:

K = the hydraulic conductivity.

Q = the discharge rate.

h_2 = the potentiometric head (during pumping) at distance r_2 from discharging bore.

h_1 = the potentiometric head (during pumping) at distance r_1 from the discharging bore.

In thick unconfirmed aquifers (where drawdown is negligible compared to the aquifer thickness, b):

$h_1 + h_2 = 2b$

and

$(h_2^2 - h_1^2) = 2b\,(h_2 - h_1)$

If s_1 and s_2 are drawdowns at distances r_1 and r_2 from the pumping bore, then:

$h_2 - h_1 = s_l - s_2$

And equation 6.53 reduces to:

$$T = \frac{-2.3Q\log_{10}(r_2/r_1)}{2\pi(s_1 - s_2)}$$

.....6.54

which is the same as the Thiem equation for confined aquifers, and the straight relationship derived for confined aquifers can be applied.

<u>Example</u>

Re-analyse the data presented in **Table 6-1**, using the corrected values of $(s - s^2)/2b$ (see Section 6.4.2 below for rationale) and determine transmissivity. Compare the result with the results obtained using the uncorrected values. The plot and analyses are given in **Figure 6-12**.

6.5.1.3 Non-Steady State Flow

Provided the drawdowns are not excessive, and corrections applied as shown in section 6.4.2, then the same equations can be used to solve for non-steady state flow in unconfined aquifers, as were used for non-steady state flow in confined aquifers.

6.5.2 Jacob's Corrections for Drawdowns in Thin Unconfined Aquifers

Jacob (U.S.G.S. Water Supply Paper 1536-1, p.p. 253-254) showed that for thin unconfined aquifers in which drawdown is an appreciable proportion of the aquifer thickness, a correction has to be applied to the observed drawdowns before the equation for confined aquifers can be used to obtain transmissivity and storage coefficient.

The corrected drawdown is:

$s' = s - (s^2/2b)$ 6.55

where:

s' = the corrected drawdown

s = the observed drawdown

b = the aquifer thickness

Derivation

This correction is derived in the following manner.

If:

b = saturated aquifer thickness prior to pumping

s = observed drawdown

h = saturated thickness during pumping then

h = b - s

and equation 6.52 can be rewritten as:

$$Q = \frac{-\pi K \{(b - s_2)^2 - (b - s_1)^2\}}{\log_e (r_2 / r_1)} \qquad .....6.56$$

If the right hand side is expanded and multiplied by unity in the form 2b/2b:

$$Q = \frac{2\pi K b \left\{ (s_1 - \frac{s_1^2}{2b}) - (s_2 - \frac{s_2^2}{2b}) \right\}}{\log_e (r_2 / r_1)} \qquad .....6.57$$

If equation 6.57 is written as:

$$Q = \frac{-2\pi T (s_1^{'} - s_2^{'})}{\log_e (r_2 / r_1)} \qquad .....6.58$$

where:

$s_1^{'} = (s_1 - \frac{s_1^2}{2b})$ and $s_2^{'} = (s_2 - \frac{s_2^2}{2b})$

It is now in the same form as the Thiem equation for radial flow to a bore in a confined aquifer, equation 6.5, except that:

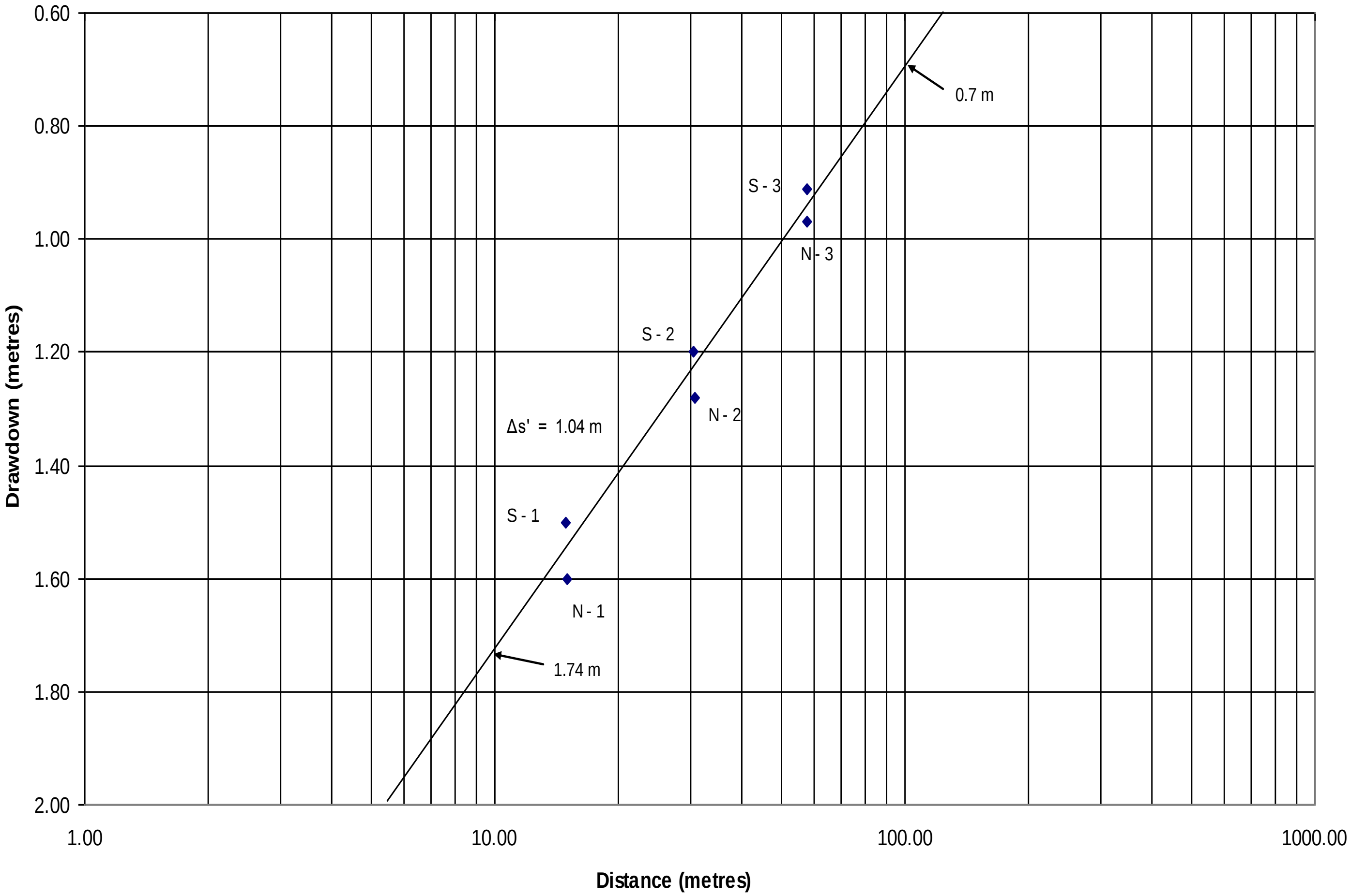
$(s_1 - \frac{s_1^2}{2b})$ and $(s_2 - \frac{s_2^2}{2b})$ are written instead of s_1 and s_2 respectively.

Transmissivity

Using the corrected values for drawdowns the transmissivity can be calculated in the same manner as was used for the Thiem equation for Steady State flow confined aquifers, Section 6.2.1.1, and the various equations for Non-Steady State flow.

Storage Coefficient

The apparent value of storage co-efficient can be determined using corrected values of drawdown and transmissivity.



Example

Using data from **Table 6-1**.

Transmissivity:

Using equation 6.8:

$T = 2.3Q / (2\pi\Delta s')$

$Q = 5450\ m^3/day$

From the plot:

$\Delta s' = 1.04m$

Hence:

$T = 1920m^2/day$

Figure 6-11 Steady state flow example – unconfined aquifer

The true value of S is then obtained by applying the following correction:

$$S = \frac{b-s}{b} S'$$ 6.59

where:

S = the corrected value of Storage Coefficient.

b = aquifer thickness.

s = observed drawdown.

S' = apparent storage coefficient calculated using corrected values of drawdown and Transmissivity.

Remarks

When using equation 6.56 or 6.57, the drawdown should be small in relation to the saturated thickness of the aquifer, or the assumption that the thickness of the aquifer is no longer valid.

There is no hard and fast rule stating just how small this drawdown should be, however, it would not be wise to use these equations if the drawdown were greater than 10 percent of the saturated thickness of the aquifer.

It is suggested that if $s^2/2b$ exceeds the accuracy of water level measurement, use it; if not neglect it. It should be used in testing thin unconfined aquifers if it meets this criterion, regardless of the method or equation used.

The equations given above for determining T and S under conditions of both near steady state flow and non steady state flow are founded upon the assumptions pertaining to radial flow in confined aquifers.

Some of these equations can be successfully applied to tests on unconfined aquifers provided that certain precautions are undertaken.

Among the assumptions involved in the Theis non-steady state flow equations is that the water removed from storage is discharged instantaneously with decline in head, as seems to happen in a reasonably elastic confined aquifer. However, in an unconfined aquifer water drains very slowly from the cone of depression, and to obtain reasonably complete gravity drainage may require pumping the well many days or even weeks, according to the character of the material. Boulton has provided one method of analysing tests on aquifers exhibiting delayed drainage and this is presented in Section 6.5.2. If the above equations are used, however, to analyse tests on unconfined aquifers exhibiting slow drainage then we may determine the apparent value of S at intervals of say 2 days after pumping was started, and plot S against t. If this is done we would obtain a relation something like that shown in **Figure 6-12**.

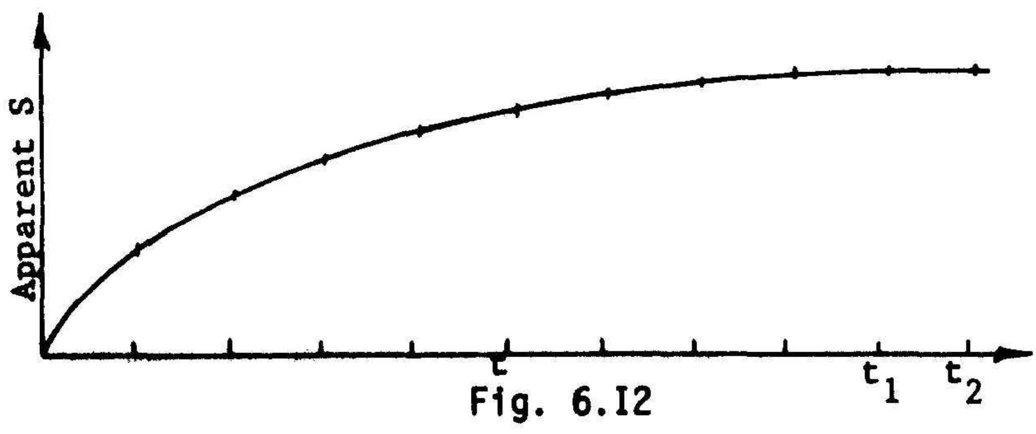


Figure 6-12 Unconfined aquifer, variation of S with time

The last two values of the apparent S, at t_1 and t_2, are the same, and suggest that drainage is reasonably complete, that the value of S (specific yield) has approached or reached the true value, and that steady state flow has been approached.

Then, the obvious conclusion is that T and S should be determined for t_1 or t_2, and as these values of time are constant for all observation wells, this eliminates from choice any of the flow equations in which the t is a variable.

Of the equations given above this leaves only those in which s and r are the variables, or are considered the variables (this includes equations containing t/r^2 or r^2/t, when t is considered constant, and r is variable).

For thin unconfined aquifers, Jacob's corrections for s and S also should be used, and, if needed, corrections for partial penetration should be made. Of the equations above that meet these qualifications, obviously the simplest to use are the Thiem equations, the equivalent straight line solutions of the Theis equation for constant t, and solutions for S at constant t.

6.6 UNCONFINED AQUIFERS WITH DELAYED YIELD AND SEMI-UNCONFINED AQUIFERS

For this section on delayed yield you are referred to Kruseman and De Ridder.

6.6.1 General

The water pumped from an unconfined aquifer is derived from storage from:

1. gravity drainage;
2. compaction of the aquifer; and
3. expansion of the water itself.

In some fine grained unconfined aquifers, the assumption that the water is released instantaneously from storage is not valid. There is some delay between the pressure reduction and the final drainage of water from the aquifer.

Such aquifers are said to exhibit delayed drainage or delayed yield.

It will be noted that delayed yield occurs not only in homogeneous fine grained aquifers, but also in stratified unconfined aquifers. In these coarse grained aquifers one or more layers of fine sand are intercalated. The simplest form is that of a homogeneous coarse grained aquifer, which is bounded below by an impervious layer and above by a fine grained layer whose hydraulic conductivity is notably lower than that of the material itself, but not so low that it can be classified as semi-pervious.

In fact such an aquifer is intermediate between the semi-confined and the "true" unconfined aquifer and will therefore be called a semi-unconfined aquifer. If such an aquifer is pumped, the water table in the covering layer will also drop, though initially less than the potentiometric head in the underlying pumped aquifer. Since the drawdown of the phreatic surface is not negligible, a horizontal flow component exists in the covering fine grained layer and should be taken into account.

Obviously the condition for a semi-confined aquifer, that the phreatic surface in the covering semi-pervious layer is not affected by pumping of the aquifer, is not satisfied. Therefore the methods of analysing pumping test data from semi-confined aquifers are not applicable here.

Boulton (1963) (see also Pricket 1965) introduced a type curve method of analysing pumping test data from unconfined aquifers, in which allowances are made for the delayed yield from storage due to slow gravity drainage. It can, for practical purposes, also be used to analyse the steady flow in a pumped semi-unconfined aquifer, as described above, Boulton's method is described below. This is the only method which will be dealt with in this document but this does not mean that other methods are not available or that they should not be used.

6.6.2 Boulton's Method

The time-drawdown curve of an observation bore in a pumped unconfined aquifer with delayed yield can be divided into three distinct segments (**Figure 6-14**):

1. The first segment, covering a short period after pumping has begun indicates that an unconfined aquifer reacts initially in the same way as does a confined aquifer. Water is released instantaneously from storage by the compaction of the aquifer and by the expansion of the water itself. Gravity drainage has not yet started. Under favourable conditions the transmissivity of the aquifer can be calculated by applying the Theis method to the first segment of the time-drawdown curve, which may cover little more than the first few minutes of data. Only nearby bores can be used because the drawdown in distant bores during the first minutes of pumping is often too small to be measured. Moreover, the storage coefficient computed from this segment cannot be used to predict the long term drawdown of the water table.
2. The second segment of the time-drawdown curve shows a decrease in slope because of the replenishment by gravity drainage from the interstices above the cone of depression. During this time, there is a marked discrepancy between the observed data curve and the type curve for Non-Steady State flow.
3. The third segment, which may start from several minutes to several days after pumping has begun, again conforms closely to the Theis type curve.

In the third segment there is equilibrium between the gravity drainage and the rate of fall of the water table, and hence the error between the observed data and the theoretical data according to the Theis equation becomes progressively smaller.

It can be shown that the effective storage coefficient is:

$$S_A + S_Y = \gamma S_A \qquad .....6.60$$

where:

S_A = the volume of water instantaneously released from storage per unit drawdown per unit horizontal area (equals effective early time storage coefficient).

S_Y = total volume of delayed yield from storage per unit drawdown per unit horizontal area (equals specific yield).

$$\gamma = 1 + S_Y/S_A \qquad .....6.61$$

The general solution of the flow equation is a rather complicated differential equation which symbolically, and in analogy to the Theis equation, may be written as:

$$s = \frac{Q}{4\pi Kb} W(u_{AY,} r/B) \qquad .....6.62$$

$W(u_{AY,}r/B)$ may be called the "well function of Boulton".

Under early time conditions this equation describes the first segment of the time-drawdown curve and equation 6.62 reduces to:

$$s = \frac{Q}{4\pi Kb} W(u_A, r/B) \qquad .....6.63$$

where:

$$u_A = \frac{r^2 S_A}{4Kbt} \qquad .....6.64$$

Under later time conditions equation 6.62 describes the third segment of the time-drawdown curve and reduces to:

$$s = \frac{Q}{4\pi Kb} W(u_Y, r/B) \qquad .....6.65$$

$$u_Y = \frac{r^2 S_Y}{4Kbt} \qquad .....6.66$$

However, the above mentioned formulae are only valid if γ tends to infinity; in practice this means that γ > 100. If 10 < γ < 100, the second segment of the time-drawdown curve is no longer horizontal, as it is when γ > 100, but the Boulton method still gives a fairly good approximation.

If γ tends to infinity, the second segment is described by:

$$s = \frac{Q}{2\pi Kb} K_0(r/B) \qquad .....6.67$$

where:

$K_Q(r/B)$ is the modified Bessel function of the second kind and zero order.

In analogy to the leakage factor L from the semi-confined aquifer, the element B may be called the drainage factor. It is defined:

$$B = \sqrt{\frac{Kb}{\alpha S_Y}} \qquad .....6.68$$

$$B = \sqrt{\frac{T}{\alpha S_Y}} \qquad .....6.69$$

and is expressed in length units (metres).

The value 1/α is called the "Boulton delay index" and is an empirical constant. It is expressed in time units (days) and is used combination with the "Boulton delay index curve" (**Figure 6-14**) to determine the time t_{wt} that the delayed yield ceases to affect the drawdown.

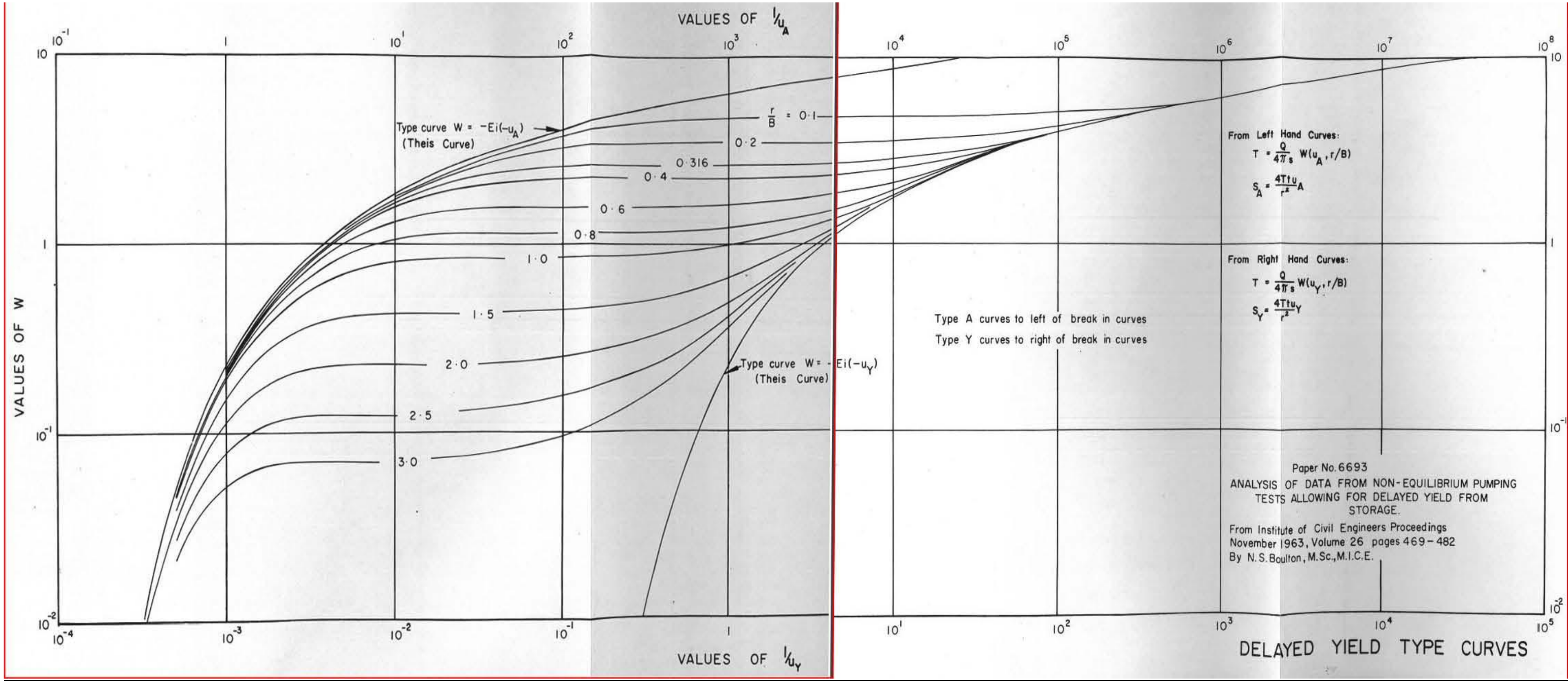


Figure 6-13 Delayed yield type curves

Procedure

1. A family of "Boulton type curves" is constructed by plotting $W(u_{AY},r/B)$ versus I/u_A and I/u_y for a practical range of values of r/B on logarithmic paper. These values are plotted in **Figure 6-14**. The left hand portion of **Figure 6-14** are the "type-A" curves, ($W(u_A,r/B)$ versus I/u.); the right hand portion shows the "type-Y" curves (u_Y, r/B) versus I/u_y).
2. Prepare the observed data curve on another sheet of double logarithmic paper of the same scale as that used for the type curves, by plotting the values of drawdown s against the corresponding time t for a single bore at a distance r from the pumped bore.
3. Superimpose the observed data plot on the type-A curves and, while keeping the co-ordinate axes of the curves parallel, adjust until as much as possible of the early time field data fall on one of the type-A curves. Note the r/B value of the selected type-A curve.
4. Select an arbitrary point A on the overlapping portion of the two sheets of graph paper and note the values of s, t, $1/u_A$ and $W(u_A, r/B)$ for this point A.
5. Substitute these values into equations 6.63 and 6.64. Since Q is also known, calculate T and S_A.
6. Shift the observed data curve until as much as possible of the later time field data falls on the type-Y curve with the same r/B values as the selected type-A curve.
7. Select an arbitrary point Z on the superimposed curve and note the values of s, t, $1/u_Y$ and $W(u_Y, r/B)$ for this match point Z.
8. Substitute these values into equations 6.65 and 6.66 and because Q is also known, calculate T and S_Y. The two calculations should give approximately the same values for T.
9. Calculate $1/\alpha$ by first determining the value of B from the value of r/B and the corresponding value of r and substituting the values of B, S_Y, and T into equation 6.69.
10. Eventually the effects of delayed gravity drainage become negligible, and the type-Y curve merges with the Theis curve. Determine the merging points of the type-Y curve for a particular value of r/B by measuring the value of αt_{wt} for the particular value of r/B on the vertical axis of the "Boulton delay index curve" (Fig 6-15). Because $1/\alpha$ is known, calculate t_{wt}. The factor t_{wt} is the time coordinate of the merging points of the time-drawdown curve that matches the type curve with the particular value of r/B on the right hand type curve.
11. Repeat the procedure with the observed data from each of the available observation bores. The calculations of T, S_A and S_Y from the data of the different observation bores should give approximately equal results.

Remarks

1. It should be noted that, for values of $\gamma > 100$, the slope of the line joining the corresponding type-A and type-Y is essentially zero. For values $10 < \gamma < 100$ the slope of this line is small and can be approximated by a line tangent to both curves. The points on the observed data curves that could not be superimposed on the type-A or type-Y curve should fall along the tangent (Boulton 1964).
2. If no influence of delayed yield is apparent the observed data will fall completely along the left hand type curve.
3. If sufficient observations are made after the delayed yield has ceased to influence the time-drawdown curve, the observed data for $t > t_{wt}$, together with the right hand type curve, can be used to calculate T and S_Y.
4. If the Boulton method is applied to pumping tests in semi-confined aquifers, it gives no information about the properties of the covering layer because B is defined in properties of an unconfined aquifer.

Example

Table 6-5 Data for delayed yield analysis

Time (mins)	Drawdown (m)	Time (mins)	Drawdown (m)
0	0	41	0.128
1.17	0.004	51	0.133
1.34	0.009	65	0.141
1.7	0.015	85	0.146
2.5	0.030	115	0.161
4.0	0.047	175	0.161
5.0	0.054	260	0.172
6.0	0.061	300	0.173
7.5	0.068	370	0.173
9.0	0.064	430	0.179
14	0.090	485	0.183
18	0.098	665	0.182
21	0.103	1340	0.200
26	0.110	1490	0.203
31	0.115	1520	0.204

Table 6-5 presents data collected at an observation bore during pumping test at "Vennebulton".

The drilling log and the performance of a shallow observation bore during the test Indicated that the test should be analysed using the Boulton Method.

The bore was pumped for 25 hours at 873 m^3/day.

The observation bore is 90 m from the discharging bore.

The logarithmic plot and match points are shown on **Figure 6-15**.

The left hand portion of the time-drawdown curve is superimposed on the family of Boulton type curves and adjusted, while keeping the co-ordinate axes parallel, until a good matching position with one of the left hand type-curve portions is found. This is the case with the left hand portion of the curve for r/B = 0.6.

A match point (A) is chosen. This point is characterised by the following co-ordinate.

$I/u_A = 10$

$W(u_A, r/B) = 1$

$s = 0.070$ m

$t = 16$ min

$= 1.11 \times 10^{-2}$ days

Substitution of these values into equations 6.63 and 6.64 yields.

Now the right hand portion is superimposed on the right hand portion of type curve r/B = 0.6 and again a match point (Z) is chosen. This match point is characterised by the following co-ordinate:

$I/u_Y = 1$

$W(u_Y, r/B) = 1$

$s = 0.105$ m

$t = 250$ min $= 1.74 \times 10^{-1}$ days

Substitution of these values into equation 6.65 and 6.66 yields:

$$T = \frac{Q}{4\pi s} W(u_Y, r/B)$$

$$= \frac{873}{4\pi 0.105}$$

= 660 m^2/day

$$S_Y = \frac{u_Y 4Tt}{r^2}$$

$$= \frac{1x4x660x1.74x10^{-1}}{90}$$

$$= 5.7x10^{-2}$$

Because r = 90m it follows that:

$$B = \frac{r}{r/B} = \frac{90}{0.6} = 150m$$

Using equation 6.69 α is calculated:

$$\alpha = \frac{T}{S_Y B^2}$$

$$= \frac{660}{5.7x10^{-2} x150^2}$$

$$= 0.51 day^{-1}$$

Because r/B = 0.6 we can find on the Boulton Delay-Index Curve, that αt_{wt} = 3.6, hence:

t_{wt} = 3.6/0.51

= 7.0 day.

Finally with equation 6.61:

$$\gamma = 1 + \frac{S_Y}{S_A}$$

$$= 1 + \frac{5.7x10^{-2}}{5.4x10^{-4}}$$

$$= 1 + 104$$

$$= 105$$

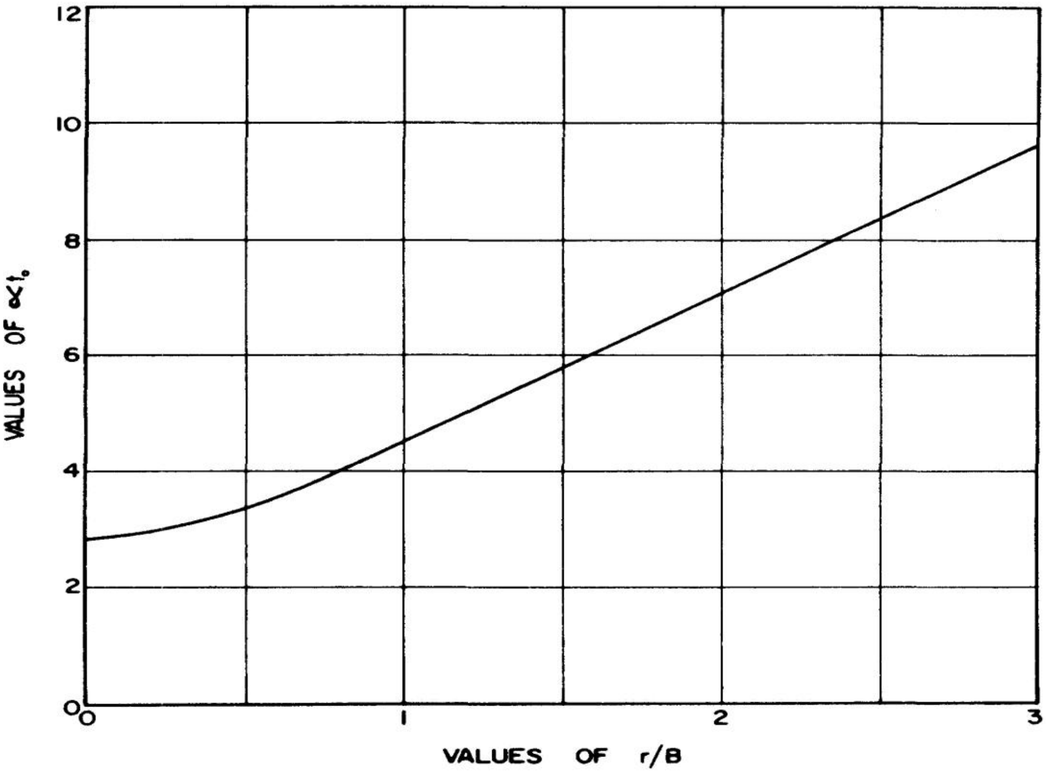
This procedure should be carried out for all available observations.

<u>Assumptions</u>

The Boulton method can be used for analysing pumping test data if the following conditions are satisfied:

1. The aquifer has a seemingly infinite areal extent.
2. The aquifer is homogeneous, isotropic and of uniform thickness over the area influenced by the pumping test.
3. Prior to the pumping, the potentiometric surface and/or phreatic surface are (nearly) horizontal over the area influenced by the pumping test.
4. The aquifer is pumped at a constant discharge rate.

5. The pumped bore penetrates the entire aquifer and thus receives water from the entire thickness of the aquifer by horizontal flow.
6. The aquifer is unconfined but showing delayed yield phenomena or the aquifer is semi-unconfined.
7. The flow to the bore is in an unsteady state.
8. The diameter of the bore is small, i.e. the storage in the bore can be neglected.



Curve for estimating Time, t_0, when Delayed Yield ceases to influence Drawdown.

Figure 6-14 Boulton's delay index curve

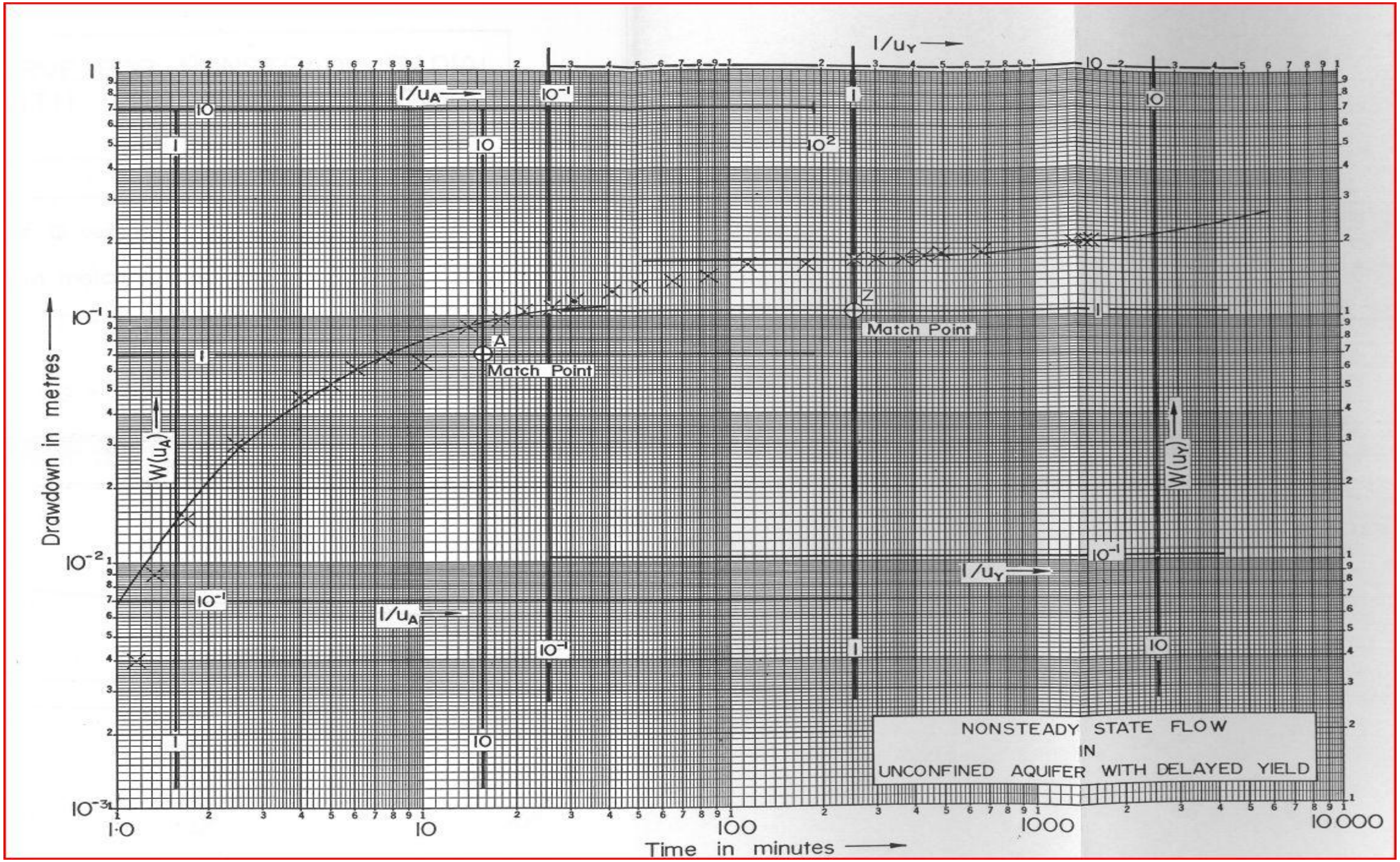


Figure 6-15 Unconfined aquifer with delayed yield, non-steady state flow example

6.7 SOFTWARE

A large range of software currently exists for the analysis of pumping tests (e.g. ADEPT, 1994; Dawson and Istok, 1991; Walton, 1988; see also catalogs of Scientific Software and Rockware; also explore Web sites). Included are programs for step drawdown test analysis for non linear head loss to sophisticated type curve match programs via which a comprehensive range of parameters may be determined. It should be emphasised that computer software exists simply to aid in the process of presenting and analysing data. The prerequisite for meaningful analysis of pumping test data is still an understanding and appreciation of the thought processes described in this chapter. The recognition of aquifer types and the selection of the analysis technique applicable to the particular situation is still required before use of the computer is contemplated. Although software is abundant and in most cases inexpensive, some are not considered user friendly. Before embarking on the purchase of a suite of analysis software, its applicability to the desired analysis, its user friendliness and layout as well as the compatibility to your computer should be checked.

6.8 IDENTIFYING AQUIFER TYPE FROM TEST DATA

It is often difficult to identify an appropriate analytical method to use to interpret drawdown data. The best way to identify an aquifer system and hence the appropriate method of analysis is to compare the drawdown pattern of the aquifer with that of the various theoretical models. **Figure 6-16** shows typical semi-logarithmic drawdown response patterns for different types of aquifers. (Kruseman and de Ridder, 1990; Dawson and Istok, 1991). Dawson and Istok also presents a table indicating the type of analysis most appropriate to determine various aquifer parameters for different types of aquifers together with the data requirements to obtain those parameters. The information presented in this chapter will enable most analyses to be carried out but the reader is encouraged to read the references.

System identification includes the construction of logarithmic plots of the drawdown vs. time since pumping started. In a number of cases, a semi-log plot of drawdown vs. time has more diagnostic value. In practice, the effects of aquifer characteristics may appear much less clearly than as shown in **Figure 6-16**. Data scatter and overlapping obscure the idealized trends shown.

A troublesome fact remains, that the theoretical solutions to a well-flow problem are not unique. Some models, developed for different aquifer systems, yield similar responses to a given stress exerted on them. In many cases, uncertainty as to which analysis to select will remain.

Figure 6-16a shows the straight-line method when $u < 0.05$. For a given pumping rate the slope of the line is inversely proportional to the value of T. A larger T will have a smaller slope. **Figure 6-16b** and **Figure 6-16c** show the effects of a leaky aquitard after some pumping. The effect of partial penetration on a semi-log plot is complex (**Figure 6-16d**). Initially, flow is horizontal, and early drawdown will be similar to a fully penetrating pumping well. With time, groundwater from below the screen contributes well discharge and additional head loss increase the slope of the curve. **Figure 6-16e** shows that as pumping continues, well storage is depleted and the curve joins the curve predicted for no well storages. The effects of aquifer boundaries are seen as a change in slope (**Figure 6-16f**). Discharge boundaries increase the slope and recharge boundaries reduce the slope. In an unconfined aquifer the curve shows a *delayed yield* component as pumping continues and the groundwater is withdrawn from the pore space (**Figure 6-16g** and **Figure 6-16h**).

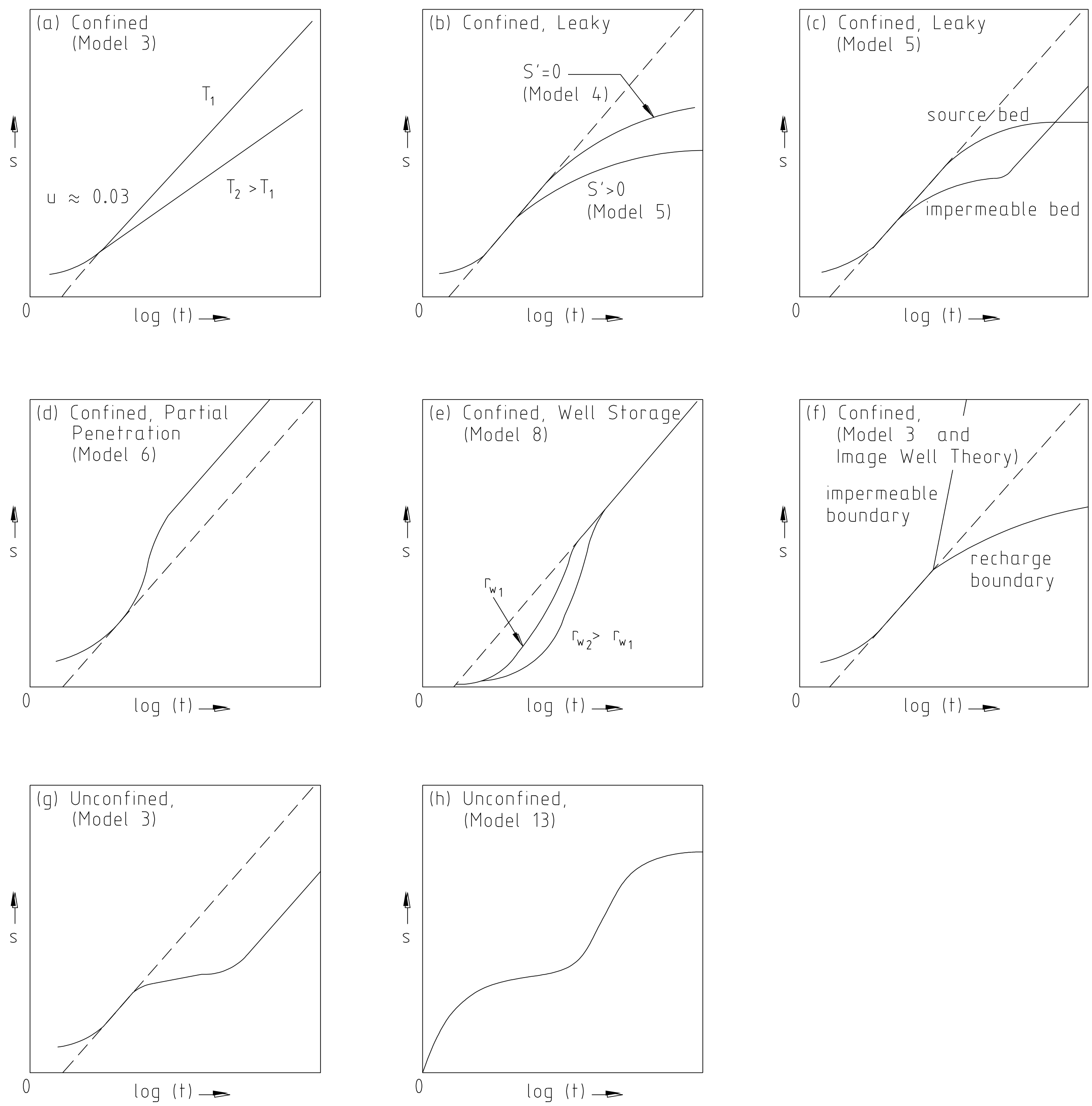


Figure 6-16 Typical response curves for different aquifer types

SECTION 7: BORE PERFORMANCE TESTS

7.1 INTRODUCTION

The flow equations presented so far have been based on the assumption that flow is laminar. This is true for an observation bore located some distance from the discharging facility and the equations presented so far have been based on the use of data collected from observation bores which are remote from the pumping bore. They have also been developed with the sole purpose of determining the aquifer parameters.

While the determination of aquifer parameters is essential if we are to have an adequate understanding of the aquifer response to various stresses it is not the only reason for carrying out pumping tests on bores. Frequently bores are tested to determine how they will perform under periods of sustained pumping. The only effective way to determine this is to analyse data collected from the pumping bore itself.

Additionally, in many cases the only test data available are drawdown observations from within the discharging bore itself and these are the only data available for determination of aquifer parameters.

However, the flow through the bore screens, up the bore casing to the pump and in the aquifer adjacent to the screens may not be laminar and new techniques must be developed to cope with this situation. The cause of non-linear flow is not fully understood. Some investigators refer to it as turbulent head loss. However, an examination of Reynold's Number does not always support the existence of turbulent flow. Some have suggested that it is the result of inertia forces which are also quadratic. In this document they will be referred to simply as non-linear head losses.

The Non-Steady State flow equations developed by Theis relied on the magnitude of drawdown for the determination of the aquifer characteristics of transmissivity and storage coefficient. A change in the magnitude of this drawdown by addition of a non-linear head loss would change the location and, because the scale is logarithmic, the shape of the data curve. This would result in the data curve being fitted at an incorrect location on the type curve with subsequent errors in the evaluation of transmissivity and storage coefficient.

Clearly then, the type curve analyses relying on the magnitude of the drawdown cannot be used to analyse data obtained from the pumping bore itself, unless the non-linear head loss has been subtracted from the total drawdown, leaving only the laminar flow drawdown for analysis.

The Modified Non-Steady State flow equations for determining transmissivity rely only on a linear change in drawdown with the log cycle of time, and are independent of the magnitude of the drawdown. The Modified Non-Steady State flow equations may then be used to analyse test data obtained from within the pumping to determine transmissivity or to predict long term performance of the bore.

The only analyses which will be presented in this section will be based on the Modified Non-Steady State flow equations.

7.2 EQUATION TO DRAWDOWN

It is generally agreed that the drawdown in a pumped facility is comprised of three terms, one non-linear flow term and two linear (or laminar) flow terms, one of which is time dependent while the other is independent of time (but is dependent on the units of time).

The non-linear flow is constant for a particular value of discharge. The magnitude of each of the three components is governed by characteristics in the aquifer remote from the bore which are unlikely to be changed. However, they are also governed by factors closer to the bore which can be changed and which can have very significant impacts on the drawdown in the bore. These changeable factors include such things as incomplete development, poor gravel pack material, head loss through the bore screens and up the bore casing and non-Darcy flow regimes.

If the long term performance of a bore under different pumping regimes is to be evaluated then the equation to drawdown has to be determined. It is not necessary that each separate component be identified and evaluated but it is important that all similar types are grouped e.g. all non-linear losses be grouped together, all time dependent linear losses be grouped and all time independent linear losses be grouped.

Using the modified non-steady state flow equations, the drawdown in the discharging bore can be expressed as:

$$s_{wt} = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_w^2} + CQ^n$$

$$= (a + b\log t)Q + CQ^n \qquad .....7.1$$

Generally n = 2 and the more common form of the equation to drawdown is:

$$= (a + b\log t)Q + CQ^2 \qquad .....7.2$$

where:

s_{wt} = drawdown in the discharging bore at time "t" after discharge commenced.

$b = \frac{2.3}{4\pi T}$ = a constant in most cases. 7.3

$a = b\log\frac{2.25T}{r_w^2 S}$ = a constant. 7.4

C = a constant.

T = transmissivity.

Q = discharge rate.

r_w = effective radius of the bore.

t = time since discharge began at discharge rate Q.

S = storage coefficient.

n = a constant (normally 2).

CQ^n = non-linear head loss at discharge rate Q.

The value of "b" and hence transmissivity, can be determined from a single test.

To determine "a" and "C" and fully describe the equation to drawdown in a discharging bore it is necessary to solve at least two, and preferably more, simultaneous equations. This can be done arithmetically or graphically.

This then requires two or more tests which are carried out at different discharge rates, or a variable discharge test such as a step drawdown test.

7.3 EVALUATION OF AQUIFER PARAMETERS

In general, it is not possible to determine the storage coefficient using drawdown observations from within the pumping bore, by methods presented so far because the effective radius of the bore is not known. For a bore finished in rock the effective radius may approach the nominal radius. However, the existence of large crevices, the over-reaming action of the drill bit, or local cementation of the bore face may result in an effective radius greater or smaller than the nominal diameter. For a bore finished in unconsolidated materials, the water level in the bore during pumping is lower than the water level in an equivalent uncased hole by an amount of the entry loss through the screen. If development of the bore is incomplete, the packing of fine material in the formation adjacent to the screen can greatly reduce the hydraulic conductivity and result in an effective radius which is considerably less than the nominal drilled size. An approximate method of determining the effective radius of any bore was been developed by Jacob (1947).

7.3.1 Constant Discharge Test Analysis

Transmissivity

Since the non-linear head loss, CQ^n is a constant for a particular discharge rate then the magnitude of the drawdown, as predicted by the Modified Non-Steady State flow equations for laminar flow alone, will be increased by a constant value regardless of the time since pumping commenced. This then means that a plot of drawdown versus logarithm of time on semi-logarithmic paper, with the time on the logarithmic scale, will result in a drawdown curve which has the same slope as that for laminar flow alone but is displaced on the drawdown axis by a constant value, CQ^n.

The evaluation of the aquifer characteristic, transmissivity is dependent only on the slope Δs, of the drawdown-time curve on semi-logarithmic paper. The techniques used to evaluate transmissivity utilising test data from an observation bore may be used then to evaluate transmissivity using test data from the discharging bore.

T is given by equation 10.26 as:

$$T = \frac{2.3Q}{4\pi T}$$

Procedure

1. Plot on semi-logarithmic graph paper, drawdown versus time, with time on the logarithmic scale.
2. Calculate Δs, the drawdown per log cycle.
3. Calculate the transmissivity by:

$$T = \frac{2.3Q}{4\pi \Delta s}$$

7.3.2 Variable Discharge Test Analysis

7.3.2.1 Constant Drawdown Test

Type Curve Solution

Jacob and Lohman (1952) derived an equation for determining T and S from tests in which the drawdown is constant and the discharge varies with time. These conditions are met when a flowing artesian bore is closed long enough for the artesian head to recover, then the bore is opened and allowed to flow for a period of 3 or 4 hours, during which time discharge measurements are made of the declining flow. They are met also when because, drawdown measurements are unable to be obtained in a discharging bore, the water is pumped at such a rate as to maintain the water level at pump suction. The equation, based upon the assumptions that the aquifer is extensive in area and that T and S are constant at all times and places, was developed from analogous thermal conditions in an equivalent thermal system. This analysis does not account for non-linear head losses and is not recommended for the analysis of tests where non-linear head losses are significant.

The equation, which is another solution of the partial differential equation to radial flow of groundwater, is:

$$T = \frac{Q}{2\pi G(\alpha) s_w} \quad .....7.5$$

where:

$$\alpha = \frac{Tt}{Sr_w^2} \quad .....7.6$$

s_w = the constant drawdown in the discharging bore.

r_w = radius of the discharging bore (effective radius).

G (α) = the G function of α

$$G(\alpha)=\frac{4\alpha}{\pi}\int_0^{\infty} xe^{-\alpha x^2}\{\frac{\pi}{2}+\tan^{-1}\frac{Y_0(x)}{J_0(x)}\}dx$$ 7.7

$J_0(x)$ and $Y_0(x)$ are Bessel functions of zero order of the first and second kinds respectively.

Transmissivity and Storage Co-efficient

Equation 7.7 is not tractable by integration, but the integral was replaced by a summation and solved by numerical methods. The resulting values of G(α) for corresponding values of α are given in **Table 7-1**, and can be plotted in the same way as W (u) was plotted against u. The plot is not given in this document but can be plotted if required from **Table 7-1**.

To determine T and S, a type curve solution is used and transmissivity calculated from equation 7.5. The value of storage coefficient determined from a type curve solution for data collected from a discharging bore must be considered to be very approximate and has not been calculated here.

Table 7-1 G(α) for values of α between 10^{-4} and 10^{12}

α	10^{-4}	10^{-3}	10^{-2}	10^{-1}	1	10^{1}	10^{2}	10^{3}
1	56.9	18.34	6.13	2.249	0.985	0.534	0.346	0.251
2	40.4	13.11	4.47	1.716	.803	.461	.311	.232
3	33.1	10.79	3.74	1.477	.719	.427	.294	.222
4	28.7	9.41	3.30	1.333	.667	.405	.283	.215
5	25.7	8.47	3.00	1.234	.630	.389	.274	.210
6	23.5	7.77	2.78	1.160	.602	.377	.268	.206
7	21.8	7.23	2.60	1.103	.589	.367	.263	.203
8	20.4	6.79	2.46	1.057	.562	.359	.258	.200
9	19.3	6.43	2.35	1.018	.547	.352	.254	.198
10	18.3	6.13	2.25	0.985	.534	.346	.251	.106
α	10^{4}	10^{5}	10^{6}	10^{7}	10^{8}	10^{9}	10^{10}	10^{11}
1	.1964	.1608	.1360	.1177	.1037	.0927	.0838	.0764
2	.1841	.1524	.1299	.1131	.1002	.0899	.0814	.0744
3	.1777	.1479	.1266	.1106	.0982	.0883	.0801	.0733
4	.1733	.1449	.1244	.1089	.0968	.0872	.0792	.0726
5	.1701	.1426	.1227	.1078	.0958	.0864	.0785	.0720
6	.1675	.1408	.1213	.1066	.0950	.0857	.0779	.0716
7	.1654	.1393	.1202	.1057	.0943	.0851	.0774	.0712
8	.1636	.1380	.1192	.1049	.0937	.0846	.0770	.0709
9	.1621	.1369	.1184	.1043	.0932	.0842	.0767	.0706
10	.1608	.1360	.1177	.1037	.0927	.0838	.0764	.0704

(From Jacob and Lohman, 1952)

Procedure

1. On transparent logarithmic paper, to the same scale as the type curve, plot Q/s_w against t/r_w^2 (or values of Q for a single well may be plotted against values of t).
2. Fit the plotted curve over the type curve.
3. Select a match-point and read off values of Q/s_w, t/r^2, G(α) and α.
4. Calculate T, from:

$$T=\frac{Q}{2\pi G(\alpha)s_w}$$

5. Calculate S, from:

$$S = \frac{Tt}{r_w^2 \alpha} \qquad .....7.8$$

Because the type Curve is very flat a worked example has not been included but rather the rationale behind the derivation has been used to develop a more easily applied straight line solution.

7.3.2.2 Straight Line Solutions

Reciprocal of Discharge Method

Jacob and Lohman (1952) showed that for relatively large values of t, the function G (α) can be closely approximately by 2/W (u). It was shown above that for sufficiently small values of u; W(u) can be closely approximated by:

$$W(u) = 2.3 \log \frac{2.25Tt}{r^2 S} \qquad .....7.9$$

Making these substitutions in equation 7.5 and adding the subscript w to s and r^2, we obtain, if non-linear head losses are absent:

$$T = \frac{2.3Q}{4\pi s_w} \log \frac{2.25Tt}{r^2 S} \qquad .....7.10$$

This is identical to equation 10.23 except for the subscripts.

In this case Q is a variable and s_w is a constant.

Rearranging equation 7.10:

$$\frac{1}{Q} = \frac{2.3}{4\pi s_w T} \log \frac{2.25Tt}{r_w^2 S} \qquad .....7.11$$

$$= E \log \frac{2.25T}{r_w^2 S} + E \log t$$

$$= E \log F + E \log t \qquad .....7.12$$

where:

$E = \dfrac{2.3}{4\pi T s_w}$ = constant = Δ1/Q.

$F = \dfrac{2.25T}{r_w^2 S}$ = constant.

Equation 7.12 is in the form of an equation to a straight line with 1/Q as the ordinate and log t as the abscissa. A plot of 1/Q against log t will yield a straight line with slope E. this slope is the change in 1/Q per log cycle and is called Δ1/Q.

Transmissivity

Rearranging equation 7.12 and substituting Δ l/Q for E we obtain:

$$T = \frac{2.3}{4\pi s_w \Delta(1/Q)} \qquad .....7.13$$

where:

s_w = constant drawdown in the discharging bore.

Δ(1/Q) = change in 1/Q per log cycle of time.

Transmissivity is then calculated using equation 7.13. As explained later this value of transmissivity will frequently be too small.

Storage Co-efficient

Unless the non-linear head loss is taken into account the storage coefficient cannot be determined accurately from drawdown measurements in a pumped bore. An attempt to determine storage coefficient from a discharging bore also assumes that the value of effective radius of the bore is known. If there is any doubt at all of the value of the effective radius of the bore, owing to well construction, well development, or caving, do not try to determine storage coefficient by these means.

Procedure

1. On semi-logarithmic graph paper, plot the reciprocal of discharge (1/Q), on the natural scale, against time since the start of pumping, on logarithmic scale.
2. Determine Δ (1/Q), the slope of the reciprocal of discharge per log cycle.
3. Calculate transmissivity from equation 7.13.

Example

Table 7-2 presents field data for a test carried out on Richmond Town Bore No. 3 on the 3rd July 1969. Richmond is a small town in North West Queensland overlying the Great Artesian Basin. The bore is 405 m deep, has a water temperature of 41.7°C, a static head of 26.91 metres and is drilled into the Hooray Sandstone. The bore had been shut down for a long period prior to the test.

These test data have been used to analyse the constant drawdown (flow recession) first part of the test using the straight line approximation; to analyse the stepped part of the test following the recovery (static test) using the Hazel Graphical method and finally to analyse the test as a whole from the opening of the bore through the flow recession, static and stepped section using the Eden-Hazel method of analysis. The data in columns 6 and 9 relate to the Eden-Hazel analysis.

The test was conducted by C.P. Hazel. Static head prior to test: 26.57m = 261 kPa.

The bore casing radius: r_w = 0.15 m.

(The data were analysed using the type curve to give a transmissivity of approximately 400 m^2/day but is not presented here.)

Analyse the constant drawdown component of the test data presented in **Table 7-2** (from the start of the test to 240 minutes) using the straight line solution.

The analysis is given in **Figure 7-1**.

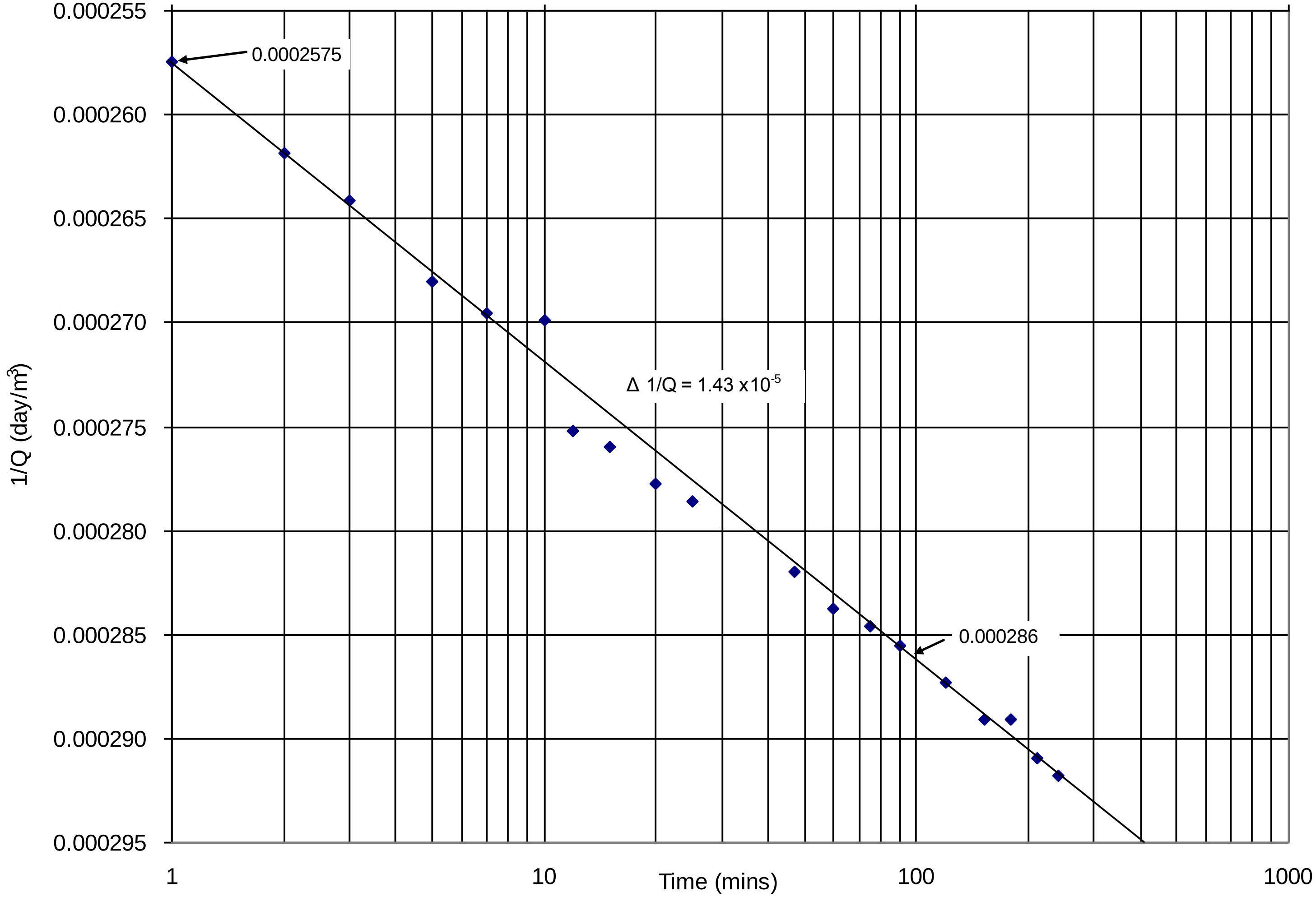
Table 7-2 Richmond town bore no. 3 – test data

Time Period (i)	Step No.	Time From		Q_i (m^3/day)	ΔQ_i (m^3/day)	Back Pressure	Drawdown	$\Sigma_{i=1}^{i=n} \Delta Q_i \log(t-t_i)$
		Start of test t (mins)	End of Static Test (mins)			p_{wt} (m)	s_{wt} (m)	
Start of Flow Recession (Constant drawdown) section of Test at 0 minutes. The bore had been shut down for a long period of time prior to the Flow Recession Test.								
t_1	1	0		0		26.57	0.00	0.00
t_2	2	1		3884	3884.0	2.28	24.29	0.00
t_3	3	2		3819	-64.8	2.24	24.33	1169.20
t_4	4	3		3786	-33.1	2.20	24.37	1833.63
t_5	5	5		3731	-54.7	2.06	24.51	2643.53
t_6	6	7		3709	-21.6	2.16	24.41	3169.37
t_7	7	10		3705	-4.3	2.16	24.41	3728.90
t_8	8	12		3633	-72.0	2.16	24.41	3995.83
t_9	9	15		3623	-10.1	2.07	24.50	4317.14
t_{10}	10	20		3600	-23.0	2.07	24.50	4734.09
t_{11}	11	25		3590	-10.1	2.03	24.54	5062.17
t_{12}	12	47		3547	-43.2	2.03	24.54	5964.54
t_{13}	13	60		3524	-23.0	2.03	24.54	6306.33
t_{14}	14	75		3514	-10.1	1.99	24.58	6624.87
t_{15}	15	90		3502	-11.5	1.95	24.62	6881.35
t_{16}	16	120		3480	-21.6	1.95	24.62	7274.62
t_{17}	17	152		3459	-21.6	1.95	24.62	7588.74
		180		3459	0.0	1.91	24.66	7833.30
t_{18}	18	210		3437	-21.6	1.91	24.66	8027.00
t_{19}	19	240		3427	-10.1	1.91	24.66	8202.19
Start of Static (Recovery) section of test at 240 minutes								
		242		0	-3427.2	23.00	3.57	7182.22
		243		0		23.34	3.23	6584.56
		246		0		24.04	2.53	5570.22
		250		0		24.46	2.11	4832.76
		255		0		24.79	1.78	4257.39
		260		0		24.95	1.62	3856.86
		270		0		25.24	1.33	3307.32
		285		0		25.57	1.00	2781.49
		300		0		25.74	0.83	2427.31
		330		0		25.91	0.66	1961.95
		352		0		26.03	0.54	1730.35
		385		0		26.11	0.46	1476.76
		473		0		26.24	0.33	1072.43
		543		0		26.32	0.25	884.38
		1170		0		26.57	0.00	349.78
t_{20}	20	1320	0	0		26.69	-0.12	306.05

Time Period (i)	Step No.	Time From: Start of test t (mins)	Time From: End of Static Test (mins)	Q_i (m^3/day)	ΔQ_i (m^3/day)	Back Pressure p_{wt} (m)	Drawdown s_{wt} (m)	$\Sigma_{i=1}^{i=n} \Delta Q_i \log(t-t_i)$
Start of Dynamic (Step Drawdown) section of Test at 1320 minutes								
		1322	2	1135	1135.0	21.93	4.76	647.21
		1323	3	1135		21.84	4.85	846.82
		1324	4	1135		21.80	4.89	988.37
		1325	5	1135		21.76	4.93	1098.11
		1327	7	1135		21.68	5.01	1263.46
		1330	10	1135		21.60	5.09	1438.52
		1332	12	1135		21.55	5.14	1527.89
		1335	15	1135		21.47	5.22	1637.14
t_{21}	21	1340	20	1135		21.43	5.26	1777.71
		1341	21	1640	505.0	18.69	8.00	1801.51
		1342	22	1640		18.61	8.08	1976.21
		1343	23	1640		18.57	8.12	2086.81
		1344	24	1640		18.53	8.16	2170.63
		1345	25	1640		18.49	8.20	2239.45
		1347	27	1640		18.40	8.29	2350.69
		1350	30	1640		18.32	8.37	2480.13
		1352	32	1640		18.32	8.37	2551.44
		1355	35	1640		18.28	8.41	2643.83
t_{22}	22	1360	40	1640		18.24	8.45	2771.55
		1361	41	2180	540.0	14.80	11.89	2794.18
		1362	42	2180		14.71	11.98	2978.58
		1363	43	2180		14.55	12.14	3094.78
		1364	44	2180		14.55	12.14	3182.68
		1365	45	2180		14.47	12.22	3254.80
		1367	47	2180		14.38	12.31	3371.55
		1370	50	2180		14.30	12.39	3508.10
		1372	52	2180		14.30	12.39	3583.88
		1375	55	2180		14.22	12.47	3682.81
t_{23}	23	1380	60	2180		14.13	12.56	3821.29
		1381	61	2725	545.0	10.24	16.45	3846.07
		1382	62	2725		10.07	16.62	4034.11
		1383	63	2725		10.03	16.66	4153.32
		1384	64	2725		9.99	16.70	4243.97
		1385	65	2725		9.95	16.74	4318.70
		1387	67	2725		9.95	16.74	4440.41
		1390	70	2725		9.82	16.87	4584.02
		1392	72	2725		9.78	16.91	4664.34
		1395	75	2725		9.74	16.95	4769.92
t_{24}	24	1400	80	2725		9.66	17.03	4919.10

Time Period (i)	Step No.	Time From: Start of test t (mins)	Time From: End of Static Test (mins)	Q_i (m^3/day)	ΔQ_i (m^3/day)	Back Pressure p_{wt} (m)	Drawdown s_{wt} (m)	$\sum_{i=1}^{i=n} \Delta Q_i \log(t-t_i)$
Start of Dynamic (Step Drawdown) section of Test at 1320 minutes (continued)								
		1401	81	3270	545.0	5.47	21.22	4945.96
		1402	82	3270		5.31	21.38	5136.08
		1403	83	3270		5.22	21.47	5257.35
		1404	84	3270		5.18	21.51	5350.04
		1405	85	3270		5.06	21.63	5426.80
		1407	87	3270		5.02	21.67	5552.52
		1410	90	3270		5.02	21.67	5702.05
		1412	92	3270		4.97	21.72	5786.25
		1415	95	3270		4.85	21.84	5897.54
		1420	100	3270		4.77	21.92	6056.00

(Tested by C.P.Hazel, 3rd July 1969)



<u>Example</u>

Using data from Table 7-2.

For this example the "flow recession" section of the test, i.e. the first 240 mins, has been analysed.

Using equation 7.13:

$T = 2.3 / (4\pi s_w \Delta 1/Q)$

$s_w = 26.5m$

From the plot:

$\Delta 1/Q = 1.43 \times 10^{-5}$

Hence:

$T = 482\ m^2/day$

Figure 7-1 Constant drawdown example - straight line solution

Alternative Method - Time to Fill Container

In many cases the discharge during a constant drawdown test is measured by noting time t" to fill a container of known volume V. This coupled with a task of calculating I/Q for each value of Q has given rise to the following form of equation 7.11.

By replacing Q with V/t" in equation 7.11:

$$t''/V = \frac{2.3}{4\pi T s_w}\log\frac{2.25Tt}{r_w^2 S} \qquad .....7.15$$

or

$$t'' = \frac{2.3V}{4\pi T s_w}\log\frac{2.25T}{r_w^2 S} + \frac{2.3V}{4\pi T s_w}\log t \qquad .....7.16$$

This again is in the form of an equation to a straight line, ordinate t" and abscissa log t.

If t" (time to fill a container of volume V at time t after pumping commenced) is plotted against time since pumping commenced, on semi-logarithmic graph paper, with t" on natural scale, a straight line will result with slope $\frac{2.3V}{4\pi T s_w}$ provided that the change in non-linear head loss is negligible as the discharge rate varies.

If the change in non-linear head loss is significant and the transmissivity is low then a curve and not a straight line will result.

The slope, by the same reasoning as in previous cases, is then the change in t" per log cycle of t and is designated by Δ t". Then:

$$\Delta t'' = \frac{2.3V}{4\pi T s_w} \qquad .....7.17$$

This is now in a form which is convenient to use and does not rely on the calculation of I/Q.

Transmissivity

The transmissivity is determined from:

$$T = \frac{2.3V}{4\pi s_w \Delta t''} \qquad .....7.18$$

where:

V = volume of container.

Δ t" = slope of the "time to fill container" versus log t.

s_w = constant drawdown in the bore.

It should be noted that the value of transmissivity calculated from a constant drawdown test is nearly always lower than the true value. This is because the presence of non-linear head loss in the measured drawdown in the discharging bore makes its actual value larger than the theoretical value of drawdown in the bore.

Procedure

1. Plot on semi-logarithmic graph paper, time to fill a container of volume V against logarithm of time since pumping commenced.
2. Determine Δt'', the change in t'' per log cycle of time.
3. Calculate transmissivity from:

$$T = \frac{2.3V}{4\pi s_w \Delta t''}$$

Applicability of Straight Line Solutions

The straight line approximation of the constant drawdown condition is valid "for relatively large values of t". It is desirable to have a yard-stick to show when the straight line approximation is valid.

A plot of α versus 1/G(α) on semi-logarithmic paper shows that the straight line approximation is valid for values of $\alpha > 10^2$ i.e. $1/\alpha < 10^{-2}$.

Using the same process as was used to determine the s/Δs limit for u ≤ 0.05:

$$\frac{1}{\alpha} = \frac{r_w^2 S}{Tt} \le 10^{-2} \qquad .....7.19$$

$$\frac{s_w}{Q} = \frac{2.3}{4\pi T}\log\frac{2.25Tt}{r_w^2 S}$$

$$\Delta(s_w / Q) = \frac{2.3}{4\pi T}$$

$$\frac{s_w / Q}{\Delta(s_w / Q)} = \log\frac{2.25Tt}{r_w^2 S} \qquad .....7.20$$

$$S = \frac{2.25Tt}{r_w^2 10^{(1/Q)/\Delta(1/Q)}}$$

From equation 7.19:

$$S \le \frac{10^{-2} Tt}{r_w^2}$$

i.e. $\dfrac{2.25}{10^{(1/Q)/\Delta(1/Q)}} < 10^{-2}$

This results in:

$$(1/Q)/\Delta(1/Q) > 2.35 \qquad .....7.21$$

Hence for the straight line approximation to be applicable the reciprocal of the discharge divided by the change in reciprocal of the discharge per log cycle must be greater than 2.35.

Limitation of the Straight Line Solutions

The above straight line approximations have been based on the assumption that the non-linear head loss is negligible. They assume only head losses in the aquifer due to laminar flow. In practice we have to consider also losses in the casing slots, screens and casing, due to turbulent flow and other non-linear losses. A more general form of the basic equation therefore, allowing for casing etc. friction as well as aquifer friction loss would be:

$$s_{wt} = (a + b \log t) Q + CQ^2 \qquad .....7.22$$

In the constant drawdown test this reverts to:

$$\frac{1}{Q} = \frac{a + b\log t}{s_{wt}} + \frac{CQ}{s_{wt}} \qquad .....7.23$$

The variable Q appears on both sides of the equation and a plot of 1/Q against log t will not therefore give us a straight line in this case unless C is very small. This means in practice that the usefulness of the straight line approximation for constant drawdown analysis is limited to small flows and shallow bores where C is in fact negligible.

Eden-Hazel Analysis

The Eden-Hazel Analysis as presented in detail in Section 7.2.4 is a valid analysis of a constant drawdown test and takes into account the effect of each change in discharge. It must be remembered, however, that the Eden-Hazel Analysis only applies when b, i.e. Δs/Q is constant throughout the test. It can be used to determine if b is constant or to determine whether or not boundary effects have been encountered during the test.

The constant drawdown data presented in **Table 7-2** together with the recovery (static test) component (from 240 minutes to 1320 minutes) has been analysed using the Eden-Hazel analysis, with particular emphasis on the recovery component, and is presented on **Figure 7-2**.

Remarks

While the solutions obtained are similar, it would not be wise to use the type curve solution involving G(α) and (α) by itself. The curve G(α) versus (α) is so very flat that, for low values of T, a large range of possible match points can be obtained.

In the case of the Eden-Hazel analysis, the observed head in column 7 of **Table 7-2** is plotted against H, column 5, in **Figure 7-2**.

The equation for the line of best fit is:

$$p_w = -aQ - bH - CQ^2 + h_{static}$$

where:

H (as explained in Section 7.2.4) = $\sum_{i=1}^{i=n} \Delta Q_i \log(t - t_i)$.

p_w = back pressure in the bore at the surface (i.e. static head - residual drawdown).

h_{static} = static head (or standing water level).

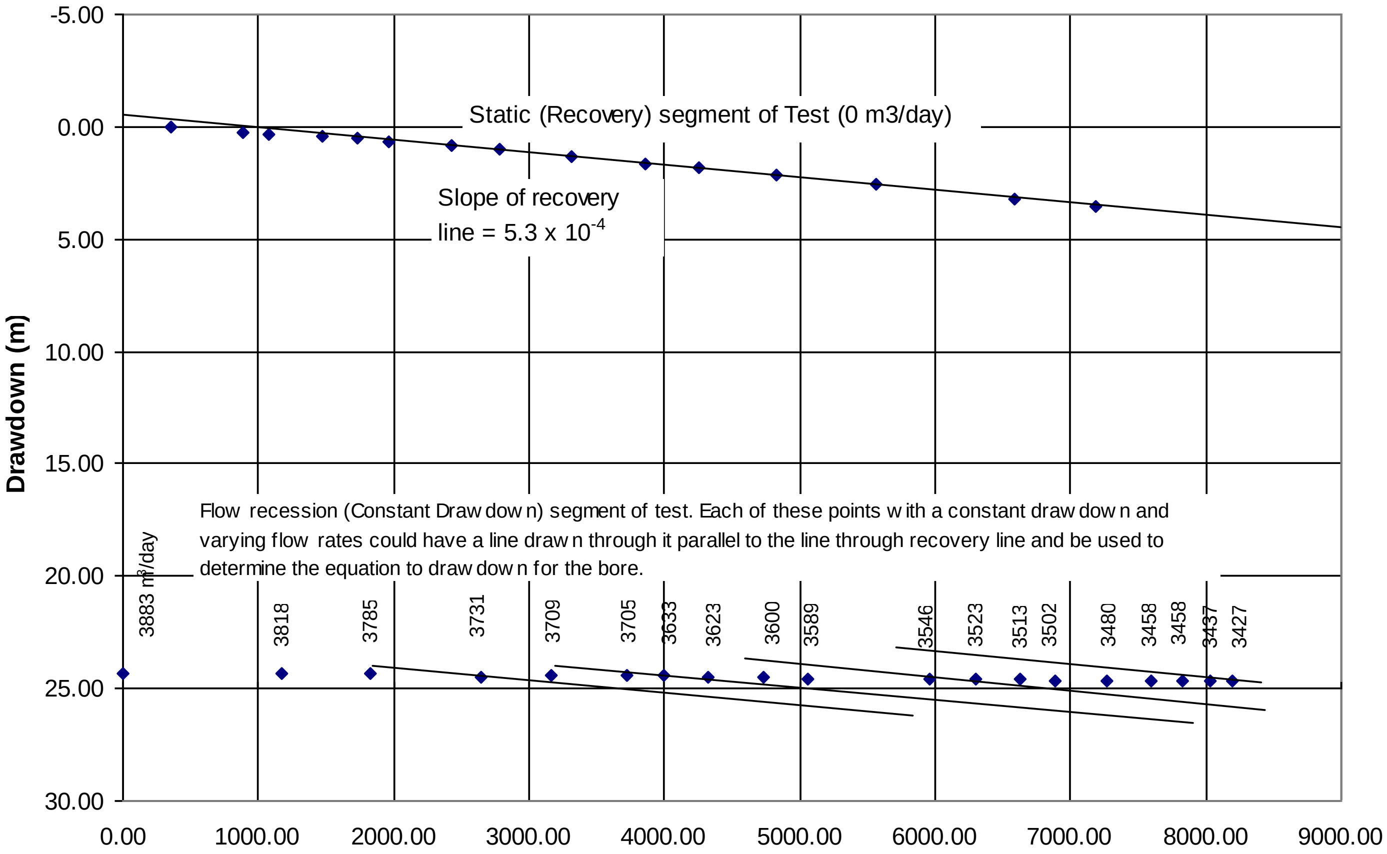
h_{static} - p_w = drawdown.

Note: negative values occur in the above equation because back pressures have been used in the drawdown equation. These have an opposite sign to drawdown.

T is determined from the slope of the line which is equal to "b". The slope in this occasion has been determined from the recovery (static test) stage of the test. The same slope can be applied to every point on the constant drawdown (flow recession) section and the non linear head loss could be calculated. In this case however, we will only calculate the transmissivity. The full equation to drawdown will be determined later in this chapter when the Eden-Hazel method of analysing step drawdown tests is presented.

It will be noted that there is a difference between the transmissivity values determined using the reciprocal of discharge and the Eden-Hazel method. This difference is probably due to a combination of a number of factors. One factor is the water temperature. The bore is warm and had been closed down for a long period before the flow recession test (more than a week). The water in the bore would have cooled and the static head would have been lower on arrival than after the bore had been flowing for an extended period. This not only effects the drawdown value used in the calculation but also the measured discharge rates at the varying temperatures as the water heats up. The water temperature is only 47.1°C which is not terribly hot, so this is not expected to be a large contributing factor. Another factor is that the straight line solution assumes that there is no non-linear head loss. This not the case in this bore and the difference in non-linear head loss between the start and end of the flow recession test can cause a significant difference in discharge rate.

The more accurate determination of transmissivity from this test is to be obtained from the recovery section following the test using the Eden-Hazel method.



$$\Sigma_{i=1}^{i=n} \Delta Q_i \log(t - t_i)$$

Example:

The test data are taken from Table 7-2.

The constant drawdown (flow recession) segment is from 0 - 240 mins and the recovery (static) section is from 240 - 1320 mins.

Emphasis has been placed on the recovery segment but the Eden-Hazel analysis enables the antecedent conditions of the flow recession segment to be taken into account.

The slope of the lines = 5.3×10^{-4}

T = 2.3 / (4πb)

= 345 m^2/day

Figure 7-2 Constant drawdown test example using Eden-Hazel method

7.4 EVALUATION OF NON-LINEAR HEAD LOSSES

7.4.1 Drawdown Method

The general equation for the drawdown in a discharging facility can be written as:

$$s_{wt} - (a + b \log t) Q + CQ^n$$

The value of b can be determined from a constant discharge test but evaluation of a and C requires test data at more than one discharge rate.

The solution of the simultaneous equations to drawdown can be determined graphically in the following manner:

<u>Case 1:</u> n = 2

When n = 2, the general equation reduces to:

$$s_{wt} = (a + b \log t) Q + CQ^2 \qquad .....7.24$$

Dividing both sides by Q:

$$\frac{s_{wt}}{Q} = (a + b\log t) + CQ \qquad .....7.25$$

which is an equation to a straight line.

If s_{wt}/Q is plotted against Q, then a straight line will result with slope C and intercept of (a + b log t) on the s_{wt}/Q axis.

The value of "b" can be determined from the plot of drawdown versus log time on semi-logarithmic paper.

Knowing the time "t" for which the various values of s_{wt} were determined, "a" can be evaluated.

If the values of s_{wt}/Q were determined at t = 1, then the intercept on the s_{wt}/Q axis would be equal to "a".

<u>Case 2:</u> n ≠ 2

When n ≠ 2, a plot of s_{wt}/Q versus Q will result in a curve which intersects the s_{wt}/Q axis at (a + b log t).

$$\frac{s_{wt}}{Q} = (a + b\log t) + CQ^{n-1} \qquad .....7.26$$

The value of the intercept, (a + b log t), is then subtracted from each value of s_{wt}/Q to give:

$$s_{wt} - (a + b \log t) = CQ^{n-1} \qquad .....7.27$$

Taking log of both sides:

$$\log (s_{wt} - (a + b \log t)) = \log C + (n-1) \log Q$$

A plot of (s_{wt}/Q - (a + b log t)) versus Q on logarithmic graph paper will yield a straight line with slope (n-1) and intercept C on the (s_{wt}/Q - (a + b log t)) axis.

The values of a, b, C and n are then readily evaluated.

Although an expression involving n ≠ 2 may occasionally appear to give a better fit to field data, it is doubtful whether this refinement is ever justified. It is suggested that departures from a straight line of the plot s /Q against Q are far more likely to be due to departures from the laminar flow component from aQ than to departures of the non-linear flow component from CQ^2.

Data which do not conform to a straight line plot of s_{wt}/Q against Q should, indeed, be viewed with the utmost suspicion and the possibilities of delayed drainage, anisotropic conditions and boundary effects, should be sifted thoroughly before the solution is accepted that in this particular facility the non-linear head loss is not proportional to Q^2. It is not an infrequent occurrence for the physical properties of an aquifer adjacent to the bore to alter during pumping. This can result in a change in the value of C during the test and this also should be investigated before the Q^2 relationship is abandoned.

7.4.2 Pressure Differential Method

In some cases the magnitude of the drawdown may be incorrect. This could be a direct result of an incorrect reading of standing water level prior to the test or due to the fact that the standing water level was not available. The case may arise also where a set of drawdown readings for one test of a set of tests is incorrect and these values need to be isolated and removed.

When n = 2, equation 7.24 applies:

$$s_{wt} = (a + b \log t)\, Q + CQ^2$$

If two tests are carried out with discharge rates Q_1 and Q_2 then the drawdowns at time t after pumping commenced will be s_1 and s_2. These drawdowns are expressed by the following equations:

$$s_1 = (a + b\log t)Q_1 + CQ_1^2$$

$$s_2 = (a + b\log t)Q_2 + CQ_2^2$$

By subtraction:

$$s_1 - s_2 = (a + b \log t)\,(Q_1 - Q2) + C\,(Q_1^2 - Q_2^2)$$

Dividing by $(Q_1 - Q_2)$:

$$\frac{s_1 - s_2}{Q_1 - Q_2} = (a + b\log t) + C(Q_1 + Q_2) \qquad \text{....7.28}$$

This is an equation to a straight line.

A plot of $\frac{s_1 - s_2}{Q_1 - Q_2}$ versus $(Q_1 + Q_2)$ will result in a straight line with slope C and intercept of (a + b log t) on the $\frac{s_1 - s_2}{Q_1 - Q_2}$ axis.

This same process can be used to compare all drawdowns at a particular time "t" with drawdowns resulting from all other tests which have been carried out on a particular bore.

It is not necessary to use drawdowns in this analysis.

If:

h_0 = the potentiometric head prior to pumping (s.w.l. or static head).

h_1 = the potentiometric head at time "t" after commencement of pumping at discharge rate Q_1.

h_2 = the potentiometric head at time "t" after commencement of pumping at discharge rate Q_2.

then:

$$s_1 = h_0 - h_1$$

$$s_2 = h_0 - h_2$$

and

$$s_1 - s_2 = h_2 - h_1 \qquad \text{.....7.29}$$

This means then that water levels or, in the case of artesian bores, back pressures, may be inserted in the equation instead of draw-down values.

Remember, however, that the order is reversed. That is $h_2 - h_1$ is inserted in place of $s_1 - s_2$.

In the case of artesian bores which are being pressure tested the low flow stable pressure head may not be known and back pressure differentials can be used to analyse step drawdown tests.

Table 7-3 Format for pressure differential analysis

1	2	3	4	5	6	7
Stage (n)	Q	p	p_1-p_n = s_n-s_1	Q_n-Q_1	Qn+Q_1	sn-s1/Qn-Q1
1						
2						
3						
4						
5						

In an important analysis it may be desirable not only to compute all differences in head loss between Stage 1 and other stages as in **Table 7-3**, but to carry out this procedure using each of the stages in turn as a base of reference.

In each case data from column 6 is plotted as abscissa against data from column 7 as ordinate. Each plotted point should be labelled as to its source.

Sometimes it will be found that all combinations involving a certain observation are in disagreement with those for all other combinations and this may be adequate reason for rejection of the incompatible observation as being probably in error.

This pressure differential analysis has the advantage of giving quite a number of points but they are plotted at a relatively high value of discharge. Where possible the drawdown method and the pressure differential method should be used in conjunction with one another. More points will be available for plotting and more confidence can be placed in the analysis.

7.4.3 Range of Intercepts

In some cases when plotting s_{wt}/Q versus Q a scatter of points results and difficulty is experienced in selecting the best straight line to fit the points.

It would be desirable to have one end of the required straight line fixed - if not exactly at least within certain limits.

The intercept is given as:

Intercept = (a + b log t) 7.30

"b" is able to be determined from the semi-logarithmic plot as Δs/Q, and the time "t" is known. The only unknown in the intercept is "a".

A range of intercepts can then be obtained by determining a range of values for "a".

<u>Example</u>

Take the case of an artesian bore:

Assume for the selected artesian bore the storage coefficient S lies between 10^{-3} and $10^{-5.}$

All practical values of r for a 152 mm diameter bore should lie between 10^{-2} and 1 metre.

Therefore all practical values of r^2S should lie between 10^{-3} and 10^{-7}.

Since:

$$a = b \log \frac{2.25T}{r^2 S}$$

then:

$$b \log \frac{2.25T}{10^{-3}} < a < b \log \frac{2.25T}{10^{-7}} \qquad .....7.31$$

$$b \log 2.25T10^{3} < a < b \log 2.25T10^{7} \qquad .....7.32$$

and "a" must fall within these limits.

This example has been worked for the case of a confined aquifer and values for S and r have been assumed. Other values of storage coefficient and effective radius could be inserted for other aquifer conditions and a range of values for "a" determined for these conditions. The principle only is outlined in this example.

7.4.4 Step Drawdown Test Analysis

Inherent in the analyses of section 7.2.1 and 7.2.2 is the assumption that the drawdown s_{wt} is measured at the same time "t" for each separate rate.

This situation can be achieved by carrying out a number of pumping tests at different rates.

However, this is time consuming and expensive. It is preferable to obtain all the required information during one test.

A means of doing this is by conducting a step drawdown test, or a variable discharge-variable drawdown pumping test.

A step drawdown test is one in which the discharge rate is changed, normally (but not necessarily) increased, in controlled stages. This type of test is used extensively to determine the non-linear head loss associated with the discharging bore. This then enables the complete equation to drawdown to be determined for a range of discharge rates and also for a range of pumping periods.

In some cases it is desirable to carry out a long duration pumping test to determine the existence of, distance to, and location of any hydrological boundaries. Determination of the existence of these boundaries is quite important especially for irrigation bores, town water supply or industrial bores. With these bores it is also important to know the magnitude of non-linear head loss so that the long term pumping rate can be determined.

In order to avoid the necessity of carrying out an additional step drawdown test or additional constant discharge test, a convenient method of testing is to reduce the discharge rate in steps during the final stages of a long duration pumping test. If the aquifer is showing a straight line relationship between drawdown and log time, then this stepped component need only be done over the last hour of a long term test. Depending on the type of aquifer being tested the stages can be as short as 20 minutes. The last stage of course would be complete closure, that is, the recovery.

This type of test can be used to determine not only the existence of hydrological boundaries, during the long period of constant discharge, but also to ascertain the complete equation to drawdown in the bore. If subterranean hydrological boundaries exist then the recharge part of the test (that is that section near the end of the test where the discharge rates are being decreased) will be affected in the same manner as was the initial part of the test.

For all steps, other than the first, the drawdowns recorded during a step drawdown test will be influenced by the effects of the previous steps. The drawdowns measured during these later steps will not be the drawdowns which would have resulted at that time after pumping, if the bore had been pumped at that particular step rate from the commencement of the test.

The drawdown figures must then be corrected to allow for antecedent pumping conditions.

Consider the following step drawdown test:

- Step 1- Discharge Rate Q_1 for time t_1.
- Step 2 - Discharge Rate Q_2, for time $(t_2 - t_1)$.
- Step 3 - Discharge Rate Q_3, for time $(t_3 - t_2)$.

The total duration of the test is t_3, and at time t_1, the discharge rate was changed from Q_1 to Q_2 and at time t_2 the discharge rate was changed from Q_2 to Q_3.

This is analogous to:

- Pump number one discharges Q_1 from the bore for time t_3.
- At time t_1, an additional pump is added which discharges at $(Q_2 - Q_1)$ for time $(t_3 - t_1)$.
- At time t_2 an additional pump is added and discharges at $(Q_3 - Q_2)$ for time $(t_3 - t_2)$.

There are two important points which arise from the above analogy:

1. when each additional pump is added, its drawdown effect, in relation to time, starts from the time when it was bought into operation; and
2. the hypothetical pumps are considered so that the true location of the drawdown curves can be determined. The hypothetical pump discharging at $Q - Q_{n-1}$ should not be analysed as a pump in its own right, because the non-linear head losses can cause complications.

The non-linear head loss associated with Q_{n-1} is CQ_{n-1}^2.

When the rate is changed to Q_n the non-linear head loss changes to CQ_n^2. The increase in non-linear head loss as a result of the change in rate from Q_{n-1} to Q_n is then $C\,(Q_n^2 - Q_{n-1}^2)$.

This is not equal to $C(Q_n - Q_{n-1})^2$ which would be the non-linear loss associated with a pump of $Q_n - Q_{n-1}$.

It is stressed then, that the hypothetical pump analogy should be used only to locate the drawdowns associated with the relevant discharge rate, and the non-linear head loss is the head loss associated with the real pumping rate and not the hypothetical rates.

The plot of drawdown versus time on semi-logarithmic paper for the entire step drawdown test shows the true position and variation of drawdown for rate Q_1, but incorrect drawdowns and time variations for Q_2 and Q_3. The drawdowns recorded for Q_2 and Q_3 are the summations of separate pumping tests starting at different times.

Therefore, before the test can be used effectively the true drawdowns for Q_2 and Q_3 must be calculated.

If the discharge rate is decreased, this is analogous to inserting a recharge pump in the bore.

A graphical method of correcting drawdowns, in the step drawdown test, for antecedent conditions, is given below.

7.4.5 Graphical Analysis

Lennox (1966) refined the analysis of step drawdown tests by assuming that each step was a new test and then plotted the measured drawdowns on semi-logarithmic paper at times corresponding to the time from the start of that particular step. This refinement is still incorrect because the drawdowns measured during the second and subsequent steps are equal to the summation of the drawdown effect of the first step plus the drawdown effect of second and subsequent steps. These drawdowns are measured at different time intervals.

The correction which should be applied is to take the increase in drawdown from the start of the second and subsequent steps and add this to the drawdown at the corresponding time in the first and subsequent steps. This correction will then give the true drawdown of a constant discharge test starting at that particular step rate.

Hantush and Bierschenk in 1964 refined this approach taking into account that the drawdowns in each step are a summation of the effects of all incremental steps but commencing at different times. They extend each step and determine the incremental drawdown at a set time after that step was started and determine the equation to drawdown for a set time after pumping commenced.

The following procedure, developed by Hazel in 1966 unaware of the work done by Hantush and Bierschenk, follows the same methodology as that of Hantush and Bierschenk but extends each step back to begin at 1 minute. The resultant plot is then a series of drawdown curves, one curve for each discharge rate of the test. From such a plot it is a simple matter to determine a general equation to drawdown for the bore and apply this for any time period or discharge rate required.

Procedure

1. Plot on semi-logarithmic graph paper, drawdown versus time, with time on the logarithmic scale.
2. Extend the first step by drawing a straight line through the plotted points. This indicates how the bore would have behaved if the rate had not been altered.
3. Record the time when the rate was first changed, "t_1". Using dividers or computer techniques mark off the increments between step 2 and step 1 (extended) at time "t_{1+x}" and plot this increment as an addition to the drawdown in Step 1 at time "x". This is then the true drawdown for rate Q_2 at time "x". For instance, if the first step had ended at 60 minutes and the first drawdown reading in the second step was taken at 61 minutes, then a line would be drawn through the drawdown points for the first step and the difference between that line extended, and the 61 minute drawdown reading would be plotted at 1 minute as an addition to the 1 minute drawdown for step 1. The same thing would apply to other time intervals.
4. Repeat step 3 for all points of step 2.
5. Draw and extend a straight line through the corrected points for step 2.
6. Extend the curve for step 2 of the test beyond "t_2" (i.e. the end of step 2) by adding to step 1 (extended) the appropriate increments between step 1 (extended) and corrected step 2 (extended), e.g. to determine a point on the extended step 2 curve at time t_{2+y}, mark off the distance between the step line and the corrected step 2 line at time y, and add this to the step 1 line (extended) value at time t_{2+y}. The same procedure would apply at other time intervals.
7. Using dividers or computer techniques in the same manner as in step 3 above, step off the incremental drawdowns from step 2 (test results extended) and step 3 at time "t_{2+x}" and plot these times at "x" as additional drawdowns to step 2 (corrected). This gives a corrected straight line for step 3.
8. This process is continued for any additional steps.
9. Determine Δs for steps 1, 2 (corrected) and step 3(corrected) and check to see if they are in proportion to the rates.
10. Determine the drawdown at a selected time "t" for each of the corrected steps.
11. Carry out the graphical analysis as outlined in section 7.2.1 or 7.2.2 to determine the equation for drawdown in the bore.

Remarks

This method of analysis is dependent on the drawdowns in the first, and in fact every step, being accurate and correctly plotted. If the drawdowns, and slope, in the first step are incorrect then so is the rest of the analysis. The Eden-Hazel method described later removes this problem and provides more certainty to the analysis. It also enables rogue steps to be identified and allowed for or discarded without abandoning the test as a whole.

Example

The data from the test which was carried out on the Richmond Town Bore No. 3 are tabulated in **Table 7-2**. Following a closure of some 1080 minutes an opening dynamic test (step drawdown test) was carried out. The effect of additional recovery on the dynamic test was small.

Analyse the step drawdown test component from 1320 minutes using the Graphical method of analysis.

The semi-logarithmic plot and analysis are presented in **Figure 7-3** and **Figure 7-4**.

The values of drawdown at 1 minute have been extracted from **Figure 7-3** and the following tables, **Table 7-4** compiled. **Figure 7-4** gives the plot of $s_{w\ 1min}$ versus Q, and $\dfrac{s_n - s}{Q_n - Q} versus Q_n + Q.$

The corrected drawdowns for each step were determined by the method set out in the procedure for this type of analysis and explained on the right hand side of **Figure 7-3**.

Table 7-4 Analysis of step drawdown test

Using 1 Minute Drawdowns					
Step (n)	Q	$s_{w\ 1min}$	$s_{w\ 1min}/Q$	Δs (m)	Δs/Q (m/m3/day)
1	1135	4.59	0.00404	0.52	0.00046
2	1640	7.25	0.00442	0.82	0.00050
3	2180	10.6	0.00486	1.23	0.00056
4	2725	14.41	0.00529	1.54	0.00056
5	3270	18.62	0.00570	1.82	0.00056

Using Pressure Differentials						
Step (n)	Q	$s_{w\ 1min}$	s_n-s_1	Q_n-Q_1	Q_n+Q_1	s_n-s_1/Q_n-Q_1
1	1135	4.59				
2	1640	7.25	2.66	505	2775	0.00527
3	2180	10.6	6.01	1045	3315	0.00575
4	2725	14.41	9.82	1590	3860	0.00618
5	3270	18.62	4.03	2135	4405	0.00657
Step (n)	Q	$s_{w\ 1min}$	s_n-s_1	Q_n-Q_1	Q_n+Q_1	s_n-s_1/Q_n-Q_1
2	1640	7.25				
3	2180	10.6	3.35	540	3820	0.0062
4	2725	14.41	7.16	1085	4365	0.0066
5	3270	18.62	11.37	1630	4910	0.00698
Step (n)	Q	$s_{w\ 1min}$	s_n-s_1	Q_n-Q_1	Q_n+Q_1	s_n-s_1/Q_n-Q_1
3	2180	10.6				
4	2725	14.41	3.81	545	4905	0.007
5	3270	18.62	8.02	1090	5450	0.00736
Step (n)	Q	$s_{w\ 1min}$	s_n-s_1	Q_n-Q_1	Q_n+Q_1	s_n-s_1/Q_n-Q_1
4	2725	14.41				
5	3270	18.62	4.21	545	5995	0.00771

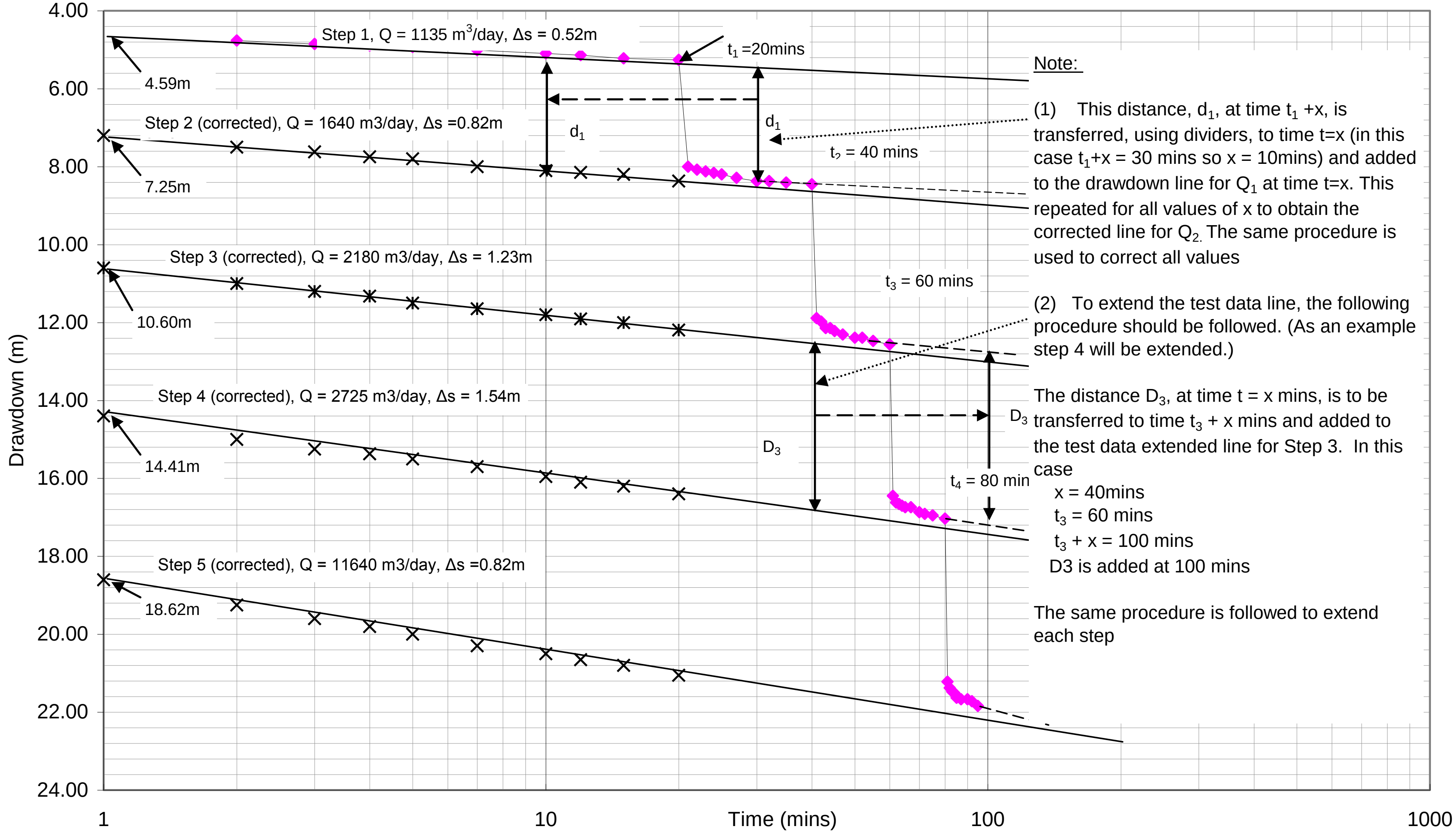


Figure 7-3 Step drawdown test – graphical analysis example

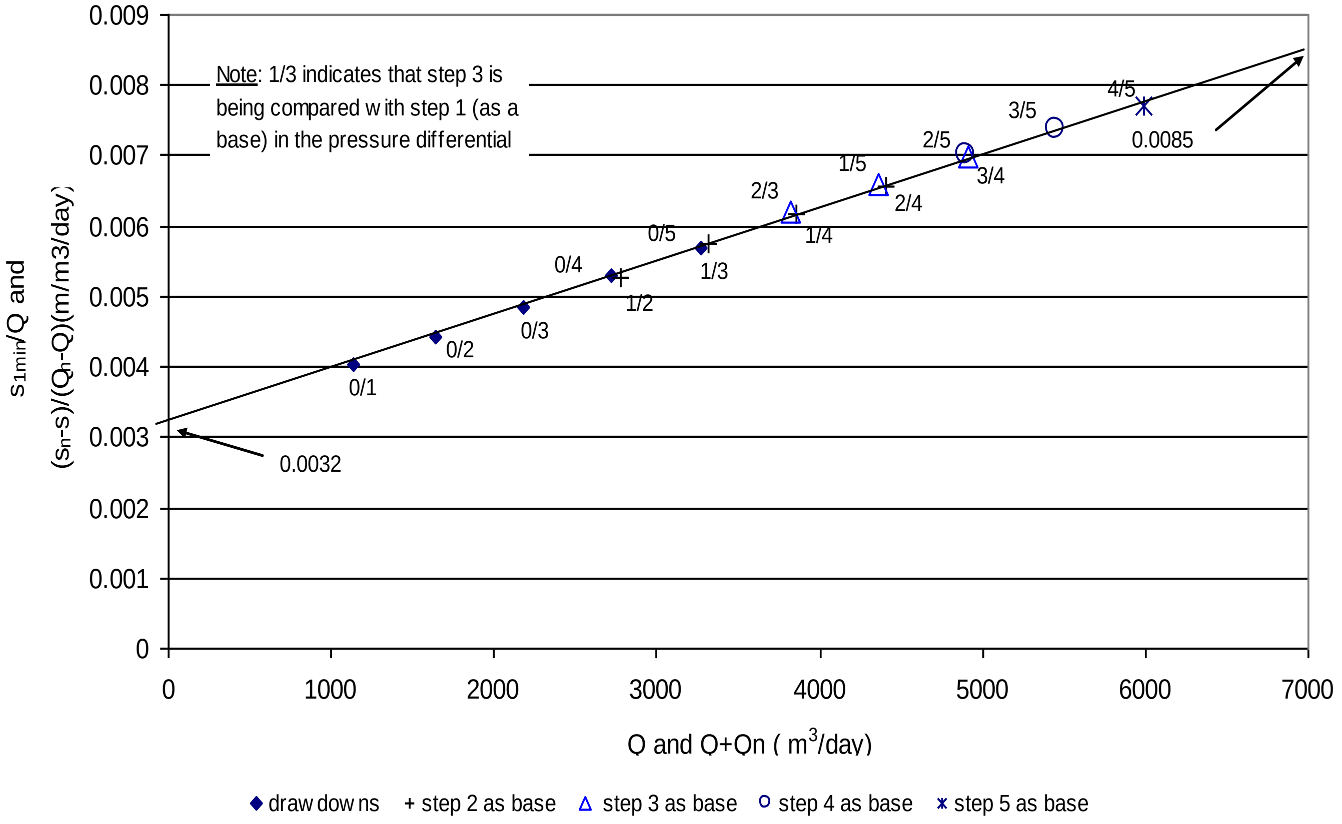


Figure 7-4 Step drawdown test – graphical analysis, determination of "a" and "C"

<u>Example</u>

To determine the equation to drawdown:

$s_w = (a + b \log t) Q + C Q^2$

It is necessary to determine the values of "a", "b" and "C".

"b" is determined from Figure 7-3.

From the semi logarithmic plot the average value of "b" is 0.000528 (Δs/Q in Table 7 4):

then T = 345 m^2/day

Values for "a" and "C" are obtained from Figure 7-4.

"a" is the intercept on the sw / Q axis and "C" is the slope of the line s_w/Q v Q.

From the plot:

"a" = 0.0032

and

"C" = (0.0085 - 0.0032) / 7000

= 7.6×10^{-7}

The equation to drawdown is:

$s_w = (3.21 \times 10^{-3} + 5.29 \times 10^{-4} \log t) + 7.60 \times 10^{-7} Q^2$

where:

s_w = drawdown in metres at time t.

t = time in minutes since discharge began.

Q = discharge rate in m^3/day.

<u>Note:</u> if time in days is used in the equation then the value of "a" will have to be adjusted accordingly.

7.4.6 Eden-Hazel Analysis

Eden and Hazel (1973) provided a reliable method of analysing variable discharge pumping test data collected at the discharging bore which enables non-linear head losses to be calculated and the relationship between drawdown and discharge to be determined for any period of pumping. With such a relationship it is possible not only to calculate the aquifer parameters but also to determine the long term pumping rate for a bore for any set period of pumping. The method also has the advantage of catering for any change of discharge even pump stoppages during the test.

The method uses the same analogy of hypothetical discharging or recharging bores as presented in section 7.3.4.

The drawdown at any time "t" is a summation of the individual drawdowns of each of the hypothetical pumps.

The non-linear head loss at time "t" is the non-linear head loss associated with the actual discharge at "t" and not a summation of the individual non-linear head losses associated with the hypothetical pumps.

Derivation

The general form of the modified non-steady state flow equation is:

s_{wt} - (a + b log t) Q + CQ^n

This equation applies to each and every constant discharge test which is carried out on the bore.

If "m" such constant discharge tests are conducted simultaneously on a bore but the starts of the tests are staggered such that:

Test (Step) No.	Discharge	Start Time	Analysis Time	Duration of Test
1	$\Delta Q_1 = Q_1$	$t_1 = 0$	t	$t - t_1 = t$
2	$\Delta Q_2 = Q_2 - Q_1$	t2	t	$t - t_2$
3	$\Delta Q_3 = Q_3 - Q_2$	t3	t	$t - t_3$
m-1	$\Delta Q_{m-1} = Q_{m-1} - Q_{m-2}$	t_{m-1}	t	$t - t_{m-1}$
m	$\Delta Q_m = Q_m - Q_{m-1}$	t_m	t	$t - t_m$

Then the drawdown in the bore at time "t" after the start of the first step is equal to the sum of the individual effects from each step. That is:

s_{wt} = (a-b log (t-t_1) ΔQ_1 + (a-b log (t-t_2) ΔQ_2 + (a-b log (t-t_3) ΔQ_3 +.....+(a-b log (t-t_i) ΔQ_i +...+.....+ (a-b log (t-t_m) ΔQ_m + CQ_m^n 7.33

i.e.

$$s_{wt} = \sum_{i=1}^{i=m}(a + b\log(t - t_i))\Delta Q_i + CQ_m^n$$

.....7.34

$$s_{wt} = \sum_{i=1}^{i=m} a\Delta Q_i + \sum_{i=1}^{i=m} b\log(t - t_i)\Delta Q_i + CQ_m^n$$

or

$$s_{wt} = aQ_m + b\sum_{i=1}^{i=m}\log(t - t_i)\Delta Q_i + CQ_m^n$$

.....7.35

Since a, b and C are constants, then a plot of s_{wt} versus $\sum_{i=1}^{i=m} b\log(t - t_i)\Delta Q_i$ on natural scale graph paper will give a series of parallel straight lines (one for value of Q_i) with slope "b" and intercepts of (aQ_i + CQ_i^n) on the s_{wt} axis.

If these intercepts are divided by Q_i, then "a" and "C" can be determined in the same manner as was presented in sections 7.4.1 and 7.4.2.

This method of analysis gives a very ready means of allowing for antecedent pumping conditions, and for pump stoppages. It is also is in a form which is readily amenable to computer programming or for spreadsheet analysis. For simple tests manual computation is possible. This analysis is not dependent on the antecedent drawdowns, as is the method outlined in section 7.4.4, but rather on antecedent discharges.

If the pump stops during a step for which the discharge rate had been Q_n, then a recharge bore delivering Q_n to the bore is superimposed in the analysis. A similar operation is carried out for any change in rate, so all rate changes are able to be accounted for in the analysis.

By means of a regression analysis, the constants a, b, C are able to be calculated and the equation to drawdown can be determined for any required time of discharge. This technique is used extensively in Queensland to analyse pumping tests.

Procedure

The analysis is amenable to spreadsheet analysis and where possible it, including the plot of pressure or drawdown versus $\sum_{i=1}^{i=m} b\log(t-t_i)\Delta Q_i$ should be carried out by computer. However, as is the case with all pumping test analyses, the computer regression analysis output should not be accepted on face value. The plots should always be scrutinised visually to ensure that that the results are acceptable. If the plots appear to be straight forward then accept the computer regression analysis output. If the plots appear to have some inconsistencies then accept the mathematical and graphical output but analyse the plots manually.

Occasions will arise, however, where a computer is not available or the analysis is so simple that a manual operation may be preferable. In this case the following procedure should be followed, using the format suggested in **Table 7-5**. The same procedure is used to set up the spreadsheet analysis.

1. The steady state conditions, either complete shutdown unaffected by previous discharge or steady flow for a previous long period, are recorded i.e. pressure, flow and time of measurement.
2. The incremental changes in discharge ΔQ_1, ΔQ_2 etc are recorded.
3. The times when the incremental discharges occurred, t_1,t_2, t_3 etc are recorded.
4. The time since the start of the test, the discharge rate and pressure (or head or drawdown) are tabulated in columns 1, 2 and 3 of **Table 7-5**.
5. All positive values of $t-t_1$, $t-t_2$, $t-t_3$ etc are listed in columns 4, 7, 10 etc respectively.
6. The logarithms of $(t-t_1)$, $(t-t_2)$, $(t-t_3)$ etc are listed in columns 5, 8, 11 etc respectively.
7. The values of $\Delta Q_1 \log (t-t_1)$, $\Delta Q_1 \log (t-t_1)$, $\Delta Q_1 \log (t-t_1)$, etc are listed in columns 6, 9, 12 etc respectively.
8. For each value of "t", columns 6, 9, 12 etc are added and the sum i.e. $\sum_{i=1}^{i=n} \Delta Q_i \log(t-t_i)$ is recorded in the last column (column 13 in this case since only three steps are being considered).
9. Column 3 is plotted against the last column; i.e. pressure (head or drawdown) is plotted against $\sum_{i=1}^{i=n} \Delta Q_i \log(t-t_i)$ to give a series of parallel straight lines, one line for each value of Q (as recorded in column 2).
10. The slope of the parallel lines is calculated and recorded. This slope is the value "b' i.e. $\Delta s/Q$.
11. The intercepts are recorded. These intercepts record the drawdown at one time unit (i.e. 1 minute, 1 day or whichever time unit is used in column 1). If the pressure differential method is to be used to analyse the results, the distance between the straight lines would be recorded.
12. The intercepts divided by the corresponding values of Q are plotted, on natural scale graph paper, against the values of Q (or the corresponding pressure differential method can be used).

13. The slope of the line in (xii) gives the value of C, while the intercept on the intercept/Q axes gives the value of a.
14. The equation to drawdown for the bore at any time "t" is then s_{wt} - (a + b log t) Q + CQ^2.

<u>Example</u>

Analyse the test data in **Table 7-2** for the flowing artesian bore at Richmond, in North West Queensland. The bore is 405 metres deep, the water temperature 41.7°C and the static head 26.91 metres. The bore had been shut down for a long period prior to the test.

In this case the analysis was carried out using a spreadsheet but, as there were some 24 changes in discharge, use of a computer program would have hastened the analysis.

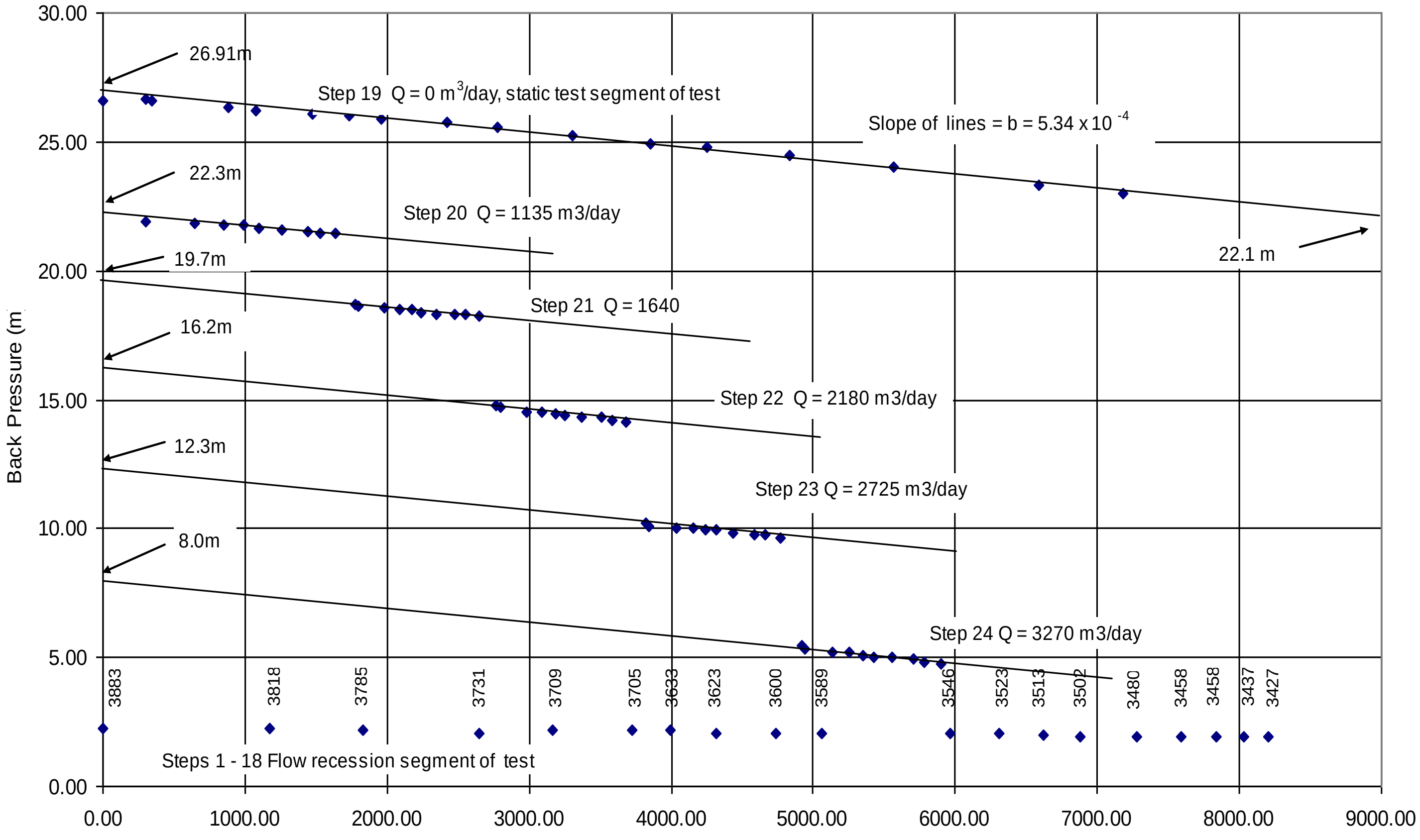
The plots and analysis are given in **Figure 7-5** and **Figure 7-6**.

In **Figure 7-5**, back pressures at the bore head, i.e. static head -drawdown, are plotted as well as drawdowns listed in **Table 7-2**. This is similar to plotting negative values for drawdowns and negative slopes result. This method of analysis is convenient when accurate values of static head, or S.W.L., are not known. In this case the static test crossed the zero $\sum_{i=1}^{i=n} \Delta Q_i \log(t - t_i)$ line at 26.91 m indicating that the correct static head for the bore was 26.91m and not 26.57m which was recorded at the start of the test in **Table 7-2**. The cooling effect in the column of water in the bore prior to testing was then some 0.34m. The equation to drawdown has been derived on the basis that the drawdown in each step was 26.91m minus the back pressure during the step.

It will be noticed that the equation to drawdown is slightly different from that obtained in the Step Drawdown Test Analysis, **Figure 7-3** and **Figure 7-4**, when the antecedent flow recession and static tests were ignored.

Table 7-5 Format for Eden-Hazel spreadsheet analysis

t	Q_t	s_{wt}	ΔQ_1 = t_1 =			ΔQ_2 = t_2 =			ΔQ_3 = t_3 =			$\Sigma_{i=1}^{i=n} \Delta Q_i \log(t - t_i)$
			t - t_1	log (t - t_1)	ΔQ_1 log (t - t_1)	t - t_2	log (t - t_2)	ΔQ_2 log (t - t_2)	t - t_3	log (t - t_3)	ΔQ_3 log (t - t_3)	
1	2	3	4	5	6	7	8	9	10	11	12	13



$$\Sigma_{i=1}^{i=n} \Delta Q_i \log(t - t_i)$$

Example:

The data are taken from Table 7.4, the test on Richmond Town Bore No. 3.

Details of the analysis are given in Table 7.6

From the plot:

b = (-22.1 - (- 26.91)) / 9000

= (26.91 -22.1) / 9000

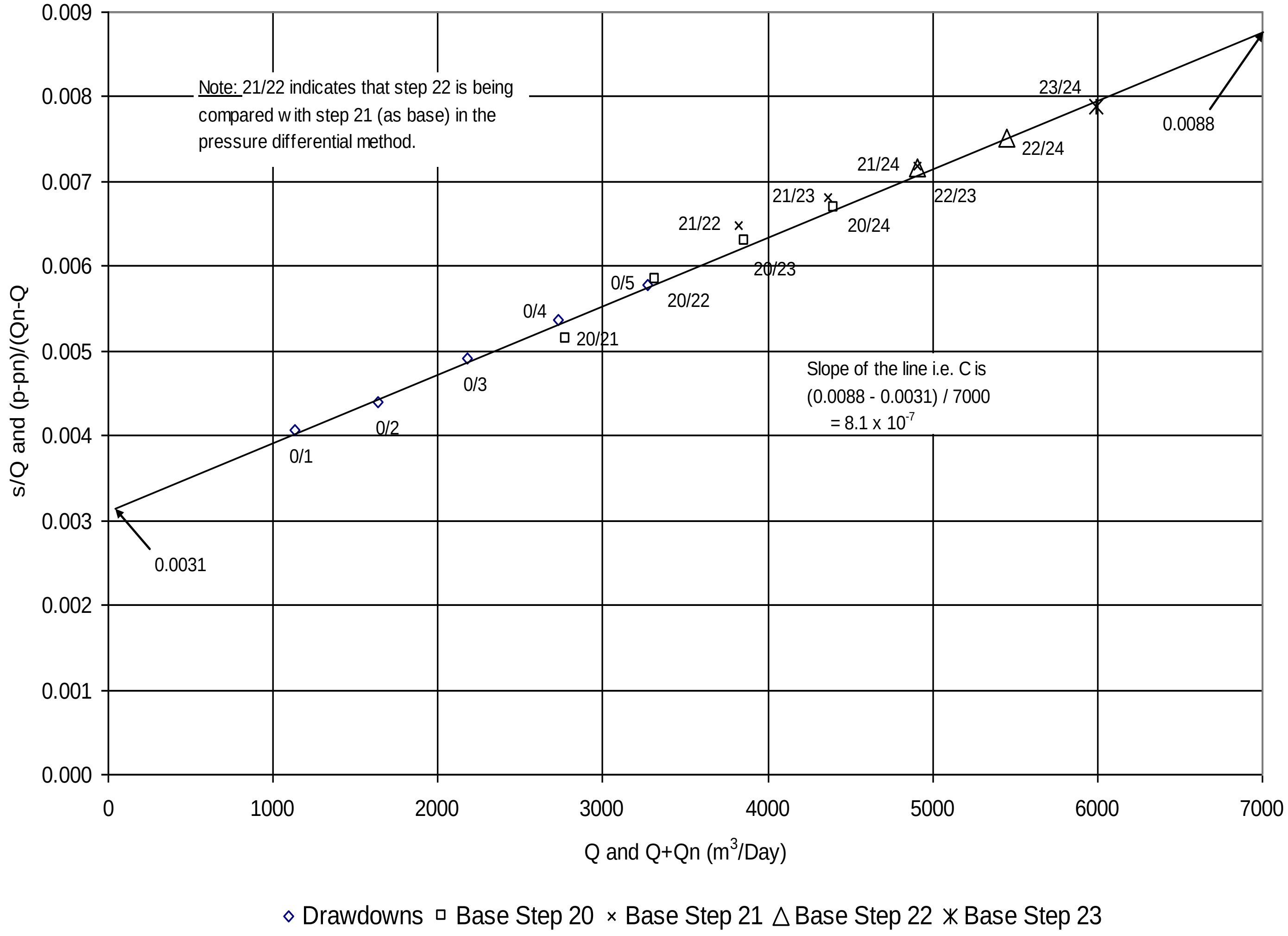
= 5.34 x 10^{-4} m/m^3/day

The negative sign occurs because back pressure is used in the plot instead of drawdown.

(p = static head - drawdown)

This also accounts for the negative slope.

Figure 7-5 Step drawdown test – Eden-Hazel analysis



Example:

To determine the equation to drawdown:

$s_w = (a + b \log t)\ Q + C\ Q^2$

it is necessary to determine the values of "a", "b" and "C".

The value of "b" from Figure 7-5 is 5.34×10^{-4}.

then T = 343 m^2/day

Values for "a" and "C" are obtained from Figure 7-5.

"a" is the intercept on the sw / Q axis and "C" is the slope of the line s_w /Q v Q.

From the plot:

"a" = 0.0031

and

"C" = (0.0085 - 0.0032) / 7000

= 8.1×10^{-7}

The equation to drawdown is:

$s_w = (3.1 \times 10^{-3} + 5.34 \times 10^{-4} \log t) + 8.10 \times 10^{-7}\ Q^2$

where:

s_w = drawdown at time t in metres.

t = time from start of discharge in mins.

Q = discharge rate in m^3/day.

Note: If time in days is used in the equation then the value of "a" will have to be adjusted accordingly.

Figure 7-6 Step drawdown test – Eden-Hazel analysis, determination of "a" and "C"

Table 7-6 lists the 1 min. drawdown divided by discharge rate as well as the analysis using the pressure differential method for various discharge rates.

There are 24 different separate steps in the test - 18 in the flow recession (constant drawdown) section, 1 static (recovery) section and 5 steps in the dynamic (step drawdown) section. All sections have been taken into account in the analysis which has resulted in the lot in **Figure 7-7** and straight lines could be drawn through every point plotted to get 24 separate intersects on the drawdown or back pressure axis. However, in this case the lines through the static test section and through the step drawdown sections give sufficient control to enable the determination of the constants for the equation to drawdown.

Table 7-6 Eden-Hazel test analysis

Using Drawdowns			
Discharge Rate Q (m^3/day)	**Back pressure Intercept at 1 minute (m)**	**Drawdown Intercept at 1 minute ($s_{w\,1\,min}$) (m)**	**$s_{1\,min}$ /Q (m/m^3/day)**
0	26.91	0	
1135	22.30	4.61	0.00406
1640	19.70	7.21	0.00440
2180	16.20	10.71	0.00491
2725	12.30	14.61	0.00536
3270	8.00	18.91	0.00578

Note: the drawdown intercept at 1 minute can be obtained as the distance between the zero discharge line and the relevant discharge line.

Using Pressure Differentials					
Base Step 20					
Step (n)	**Qn (m^3/day)**	**$p_{20} - p_n$**	**$Q_n - Q_{20}$**	**$Q_n + Q_{20}$**	**$\frac{p_{20} - p_n}{Q_n - Q_{20}}$**
20	1135				
21	1640	2.6	505	2775	0.00515
22	2180	6.1	1045	3315	0.00584
23	2725	10.0	1590	3860	0.00629
24	3270	14.3	2135	4405	0.00670
Base Step 21					
Step (n)	**Qn (m^3/day)**	**$p_{21} - p_n$**	**$Q_n - Q_{21}$**	**$Q_n + Q_{21}$**	**$\frac{p_{21} - p_n}{Q_n - Q_{21}}$**
21	1640				
22	2180	3.5	540	3820	0.00648
23	2725	7.4	1085	4365	0.00682
24	3270	11.7	1630	4910	0.00718
Base Step 22					
Step (n)	**Qn (m^3/day)**	**$p_{22} - p_n$**	**$Q_n - Q_{22}$**	**$Q_n + Q_{22}$**	**$\frac{p_{22} - p_n}{Q_n - Q_{22}}$**
22	2180				
23	2725	3.9	545	4905	0.00716
24	3270	8.2	1090	5450	0.00752

Using Pressure Differentials					
Base Step 23					
Step (n)	Qn (m^3/day)	p_{23} - p_n	Q_n - Q_{23}	Q_n + Q_{23}	$\frac{p_{23} - p_n}{Q_n - Q_{23}}$
23	2725				
24	3270	4.3	545	5995	0.00789

For demonstration purposes both drawdown and pressure differential methods were used. In this case and in most cases it is sufficient to use only the drawdown method.

The values of "a", "b" and "C" have now been determined and the equation to drawdown is:

$$s_{wt} = (a + b \log t)\, Q + C\, Q^2$$

$$= (3.1 \times 10^{-3} + 5.34 \times 10^{-4} \log t)\, Q + 8.1 \times 10^{-7}\, Q^2$$

where:

Q = discharge rate in m^3/day.

t = time in minutes since discharge began.

Note: time is in minutes in this equation because the time units used to calculate $\sum \Delta Q_i (\log t - t_i)$ for plotting were in minutes. If "day" time units had been used in the calculations and plotting we would have had 1 day as the unit time intercept and the "a" term would have changed accordingly. It would have been increased by "b log1440". The constants "b" and "C" terms in the equation to drawdown would be unaltered and the time units in the equation to drawdown would have to be days.)

This equation can now be used to produce drawdown versus discharge curves for various periods of continuous pumping.

By way of illustration two curves have been drawn; one for a period of 10^5minutes (approximately 2 months) and one for a period of 10^6 minutes (approximately 2 years). These are given in **Figure 7-7**.

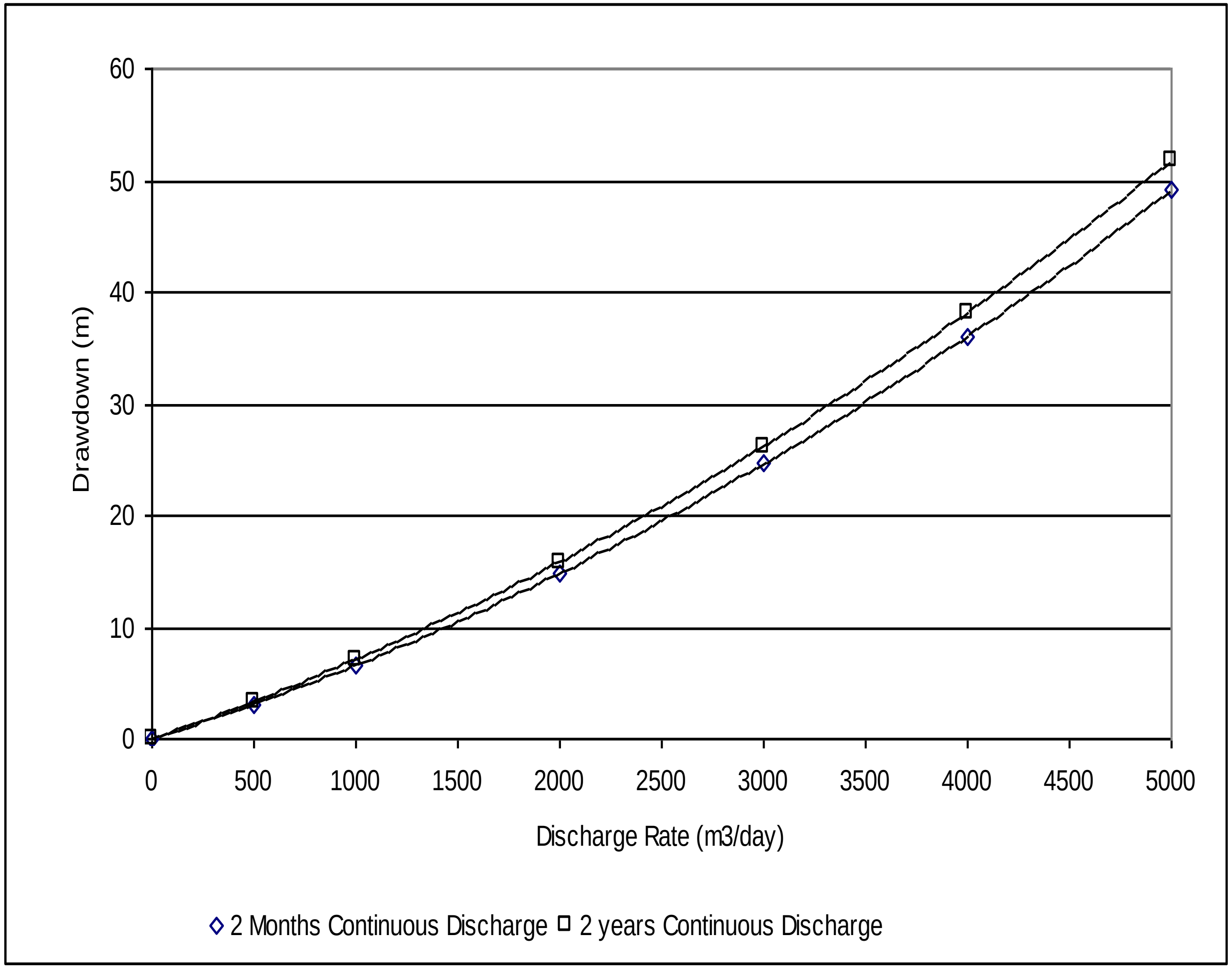


Figure 7-7 Drawdown versus discharge curves for various times of discharge

Example

Having derived the equation to discharge for a bore in the form:

$s_{wt} = (a + b \log t) Q + C Q^2$

it is now possible to determine the drawdown for any duration "t" of constant discharge "Q".

Likewise it is also possible to what rate of discharge will result in a particular drawdown after a given duration of discharge.

If we take the equation to discharge for the Richmond Town Bore No. 3:

$s_{wt} = (3.1 \times 10^{-3} + 5.34 \times 10^{-4} \log t) Q + 8.1 \times 10^{-7} Q^2$

where:

drawdown s_{wt} is in metres.

discharge Q is in m^3/day.

time t is in minutes.

Note: the equation has been derived for time in minutes. If time in days is used in the equation then the value of "a" will have to be adjusted accordingly.

The plot shows the drawdown discharge relationship for times of 2 months and 2 years continuous discharge.

7.5 INTERMITTENT PUMPING TEST ANALYSIS

The preceding drawdown equations refer to conditions during a continuous pumping test and any extrapolation of the drawdown plots from the above must be made on the assumption that pumping is continuous.

In many cases continuous pumping is not required. A typical one being the case of a bore which is equipped with a windmill or a stock bore which is only pumped for, say 6 hours per day. In order to extrapolate drawdowns fairly accurately under these conditions it becomes necessary to derive an equation for drawdown in a bore which is subjected to intermittent pumping. The derivation of such an equation is set out below.

Because of the variation in non-linear head loss with discharge rate a correct solution really requires that a step drawdown test or a number of constant discharge tests be carried out on the bore. However, if the extrapolated rate is reasonably close to the test rate then quite reasonable results can be obtained by carrying out a single pumping test.

It is assumed that intermittent pumping is carried out for a period of t = "n" time units (these may be days, months, years or any other time unit) at a discharge rate of Q.

The intermittent pumping is such that the bore is only pumped for a fraction "p" of a time unit during each time unit. It is also assumed that pumping commences at the same position in each time unit. Under these conditions an expression is required for the maximum drawdown at the end of the last pumping cycle which occurs during the nth time unit at t = (n - 1 + p).

The following analysis, developed by Hazel in 1965, accounts for recovery by the analogy that the act of stopping the discharge from a bore and allowing the water level to recover, is the same as allowing the bore to continue discharging and superimposing a recharge pump which recharges the bore at the same rate as the discharge pump discharges and thus gives a net zero discharge.

Derivation

At the beginning of the pumping period a pump is started and discharges at a rate Q and continues to pump for (n - 1 + p) time units.

At time "p", a second pump, a recharge pump, is started and recharges the bore at a rate Q and continues to recharge for (n - 1) time units.

At time t = 1, a third pump, a discharge pump, is started and discharges at a rate Q from the bore and continues to discharge for the remaining (n -2 + p) time units.

At t = (1 + p) a fourth pump, a recharge pump is started and recharges at a rate Q and continues to recharge for (n - 2) time units.

This process is then continued until t = (n - 1 + p) so that at the beginning of each pumping cycle a discharge pump is started and at the beginning of each recovery cycle a recharge pump is started. The situation at t = (n - 1 + p) is analogous to having n pumps discharging and (n - 1) pumps recharging.

The drawdown at t = (n - 1 + p) is then given by:

$$s_{w,t=(n-1)+p} = \sum_{t=1+p}^{t=(n-1+p)} s_{wt} - \sum_{t=1}^{t=(n-1)} s_{rt} + s_{w,t=p} \qquad .....7.36$$

where:

s_{wt} = the drawdown in the bore at time t resulting from the discharging pumps.

= the recovery in the recharged bore at time t.

s_{rt} = the recovery in the recharged bore at time t.

$s_{w,t=p}$ = the drawdown at t = p, i.e. at the end of the first pumping cycle.

Σs_{wt} = the sum of the individual drawdowns resulting from each of the discharging pumps.

Σs_{rt} = the sum of the individual recoveries resulting from each of the recharging pumps.

Utilizing the modified non-steady state flow equations, the effect of each discharging bore is such that:

s_{wt} = (a + b log t) Q + C Q^2

= aQ + bQ log t + C Q^2

= Δs a/b + Δs log t + C Q^2

The sum of the drawdown effects from each discharging pump at t = (n - 1 + p) is:

$$\sum_{t=p}^{t=n-1+p} s_{wt} = (n-1)(CQ^2 + \Delta s\frac{a}{b}) + \sum_{t=1+p}^{t=n-1+p} \Delta s \log t + s_{w,t=p} \quad .....7.37$$

The effect of recharging bore is such that:

s_{rt} = (a + b log t) Q +CQ^2

= a Q + b Q log t + C Q^2

= Δs a/b + Δs log t + C Q^2

And the sum of the recovery effects at t = (n - 1 + p) is given by:

$$\sum_{t=1}^{t=n-1+p} s_{rt} = (n-1)(CQ^2 + \Delta s\frac{a}{b}) + \sum_{t=1}^{t=n-1} \Delta s \log t \quad .....7.38$$

The net drawdown at t = (n - 1 + p) is then:

$$s_{w,t=(n-1+p)} = \sum_{t=p}^{t=(n-1+p)} s_{wt} - \sum_{t=1}^{t=n-1} s_{rt}$$

$$= \sum_{t=(1+p)}^{t=(n-1+p)} s_{wt} - \sum_{t=1}^{t=(n-1)} s_{rt} + s_{w,t=p} \quad .....7.39$$

The non-linear head loss (CQ^2) is considered to be identical for discharge and recharge conditions. Then:

$$s_{w,t=(n-1+p)} = (n-1)(CQ^2 + \Delta s\frac{a}{b}) + \sum_{t=(1+p)}^{t=(n-1+p)} \Delta s \log t + s_{w,t=p} - (n-1)(CQ^2 + \Delta s\frac{a}{b} - \sum_{t=1}^{t=(n-1)} \Delta s \log t$$

$$= \sum_{t=(1+p)}^{t=(n-1+p)} \Delta s \log t + s_{w,t=p} - \sum_{t=1}^{t=(n-1)} \Delta s \log t$$

$$= \Delta s\{\sum_{t=(1+p)}^{t=(n-1+p)} \log t - \sum_{t=1}^{t=(n-1)} \log t\} + s_{w,t=p}$$

$$= \Delta s(\log\frac{(n-1+p)!}{p} - \log(n-1)!) + s_{w,t=p}$$

$$= \Delta s \log\frac{(n-1+p)!}{(n-1)!\,p} + s_{w,t=p} \quad .....7.40$$

where, for convenience the following has been written:

(n-1+p)! = (n-1+p)(n-2+p)(n-3+p).....(1+p)p

then:

$$s_{w,t=(n-1+p)} = F\Delta s + s_{w,t=p} \quad .....7.41$$

where:

$s_{w.(t=(n-1+p)}$ = the maximum drawdown to be expected during the nth time unit of intermittent pumping at discharge rate Q.

p = fraction of the time unit for which pumping takes place.

Δs = drawdown per log cycle of the "log time versus drawdown" curve for discharge rate Q.

$s_{w,t=p}$ = drawdown at the end of the first pumping cycle. The inclusion of this term also accounts for the non-linear head loss during discharge.

F = $\log\frac{(n-1+p)!}{(n-1)!\,p}$

Values of F have been computed for a number of "n" and "p". Curves of "F" versus "p", and "F" versus "n" for various "n" and "p" respectively have been plotted in **Figure 7-8** and **Figure 7-9**

<u>Procedure</u>

1. Plot the test data on semi-logarithmic graph paper, drawdown versus logarithm of time.
2. Determine Δs, the drawdown per log cycle for discharge rate Q.
3. Determine the length of the pumping cycle, e.g. 1 day, 1 week etc.
4. Determine the fraction, p, of the pumping cycle for which the bore is being pumped.
5. Determine the number of cycles, n, to be analysed.
6. From the test data read off the drawdown at the end of a pumping duration "p" at discharge rate Q, i.e. $s_{w,t=p}$. (If the equation to drawdown has been determined the drawdown at time t = p can calculated for any discharge rate.)
7. From **Figure 7-8** read the value of F for the values of "p" and "n" selected.
8. From equation 7.41 determine the drawdown at the end of pumping during the nth cycle, i.e. $s_{w,t=(n-1+p)}$.

<u>Example</u>

A bore is to be pumped 6 hours per day every day. A pumping test was carried out at 200 m^3/day and Δs was calculated as 0.5m per log cycle. The drawdown at 6 hours was 10 m.

Find the maximum drawdown at the end of 70 days intermittent pumping at 200 m^3 /day.

<u>Solution</u>

The time unit is taken as 1 day.

p = 6/24 = 0.25

n = 70

The drawdown at the end of the final pumping cycle i.e. time (n -1 + p), is given by equation 7.41.

i.e. $s_{w,t=(n-1+p)} = F\Delta s + s_{w,t=p}$

From **Figure 7-8**:

For p = 0.25 and n =70

F = 0.5

i.e. Max. Drawdown = 0.5 x 0.5 + 10

=10.25 m

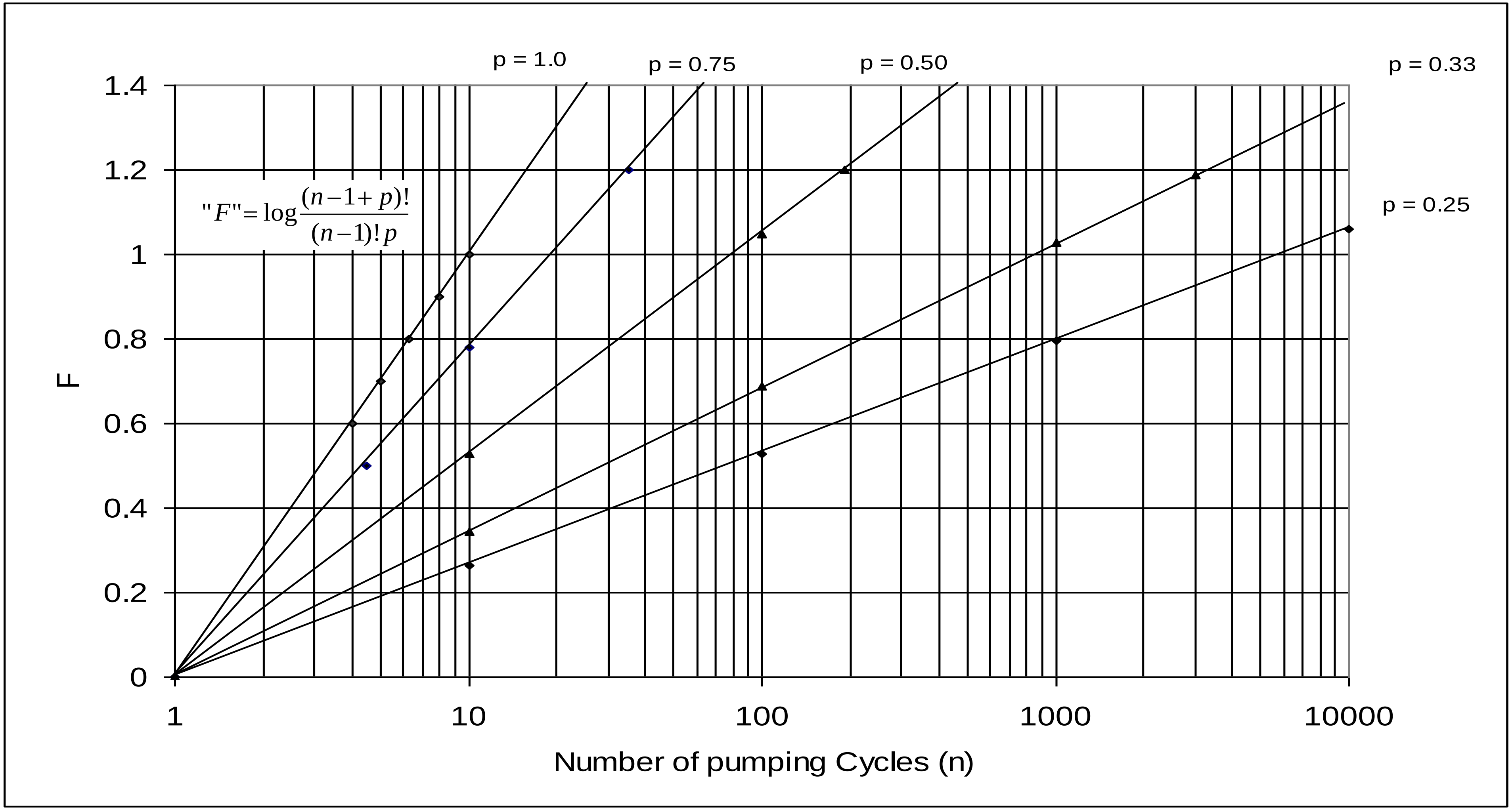


Figure 7-8 Intermittent pumping – “F” versus “n” curves

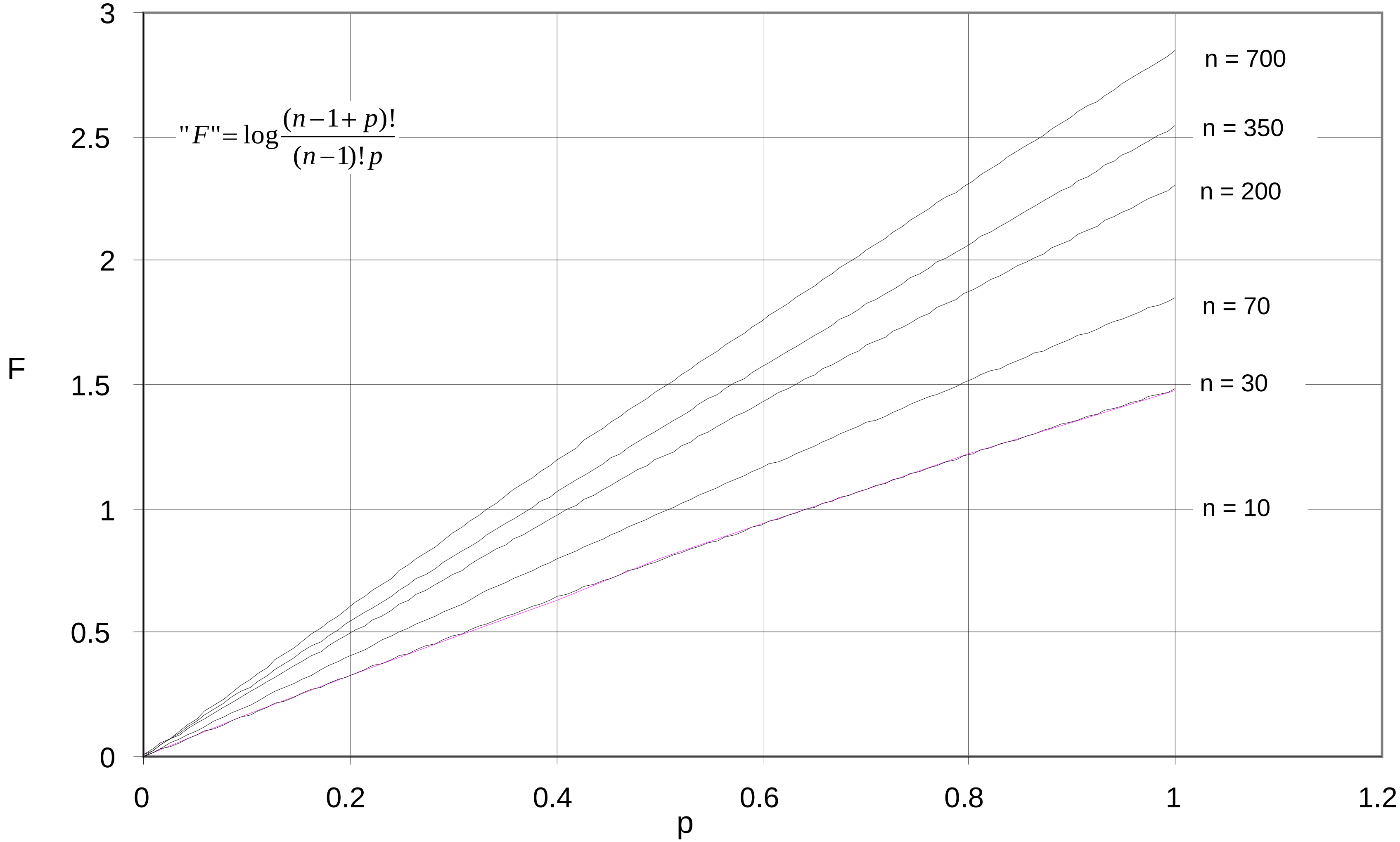


Figure 7-9 Intermittent pumping – "F" versus "p" curves

7.6 EVALUATION OF LONG TERM PUMPING RATE

Frequently the objective of carrying out a pumping test on a bore is to determine its long term pumping. This needs to be evaluated firstly to see if the bore is capable of supplying the required volume and secondly to ensure that the correct pumping equipment is purchased and installed.

This section gives some indication of how such a rate can be estimated and some of the pitfalls to be avoided when such estimates are made.

Constant Discharge Pumping Test

The constant discharge pumping test with recovery is the most common type of tests used and is suitable for the estimation of long term pumping rates which are of the same magnitude as the test rate. The method of analysis uses the Modified Non-Steady State flow equations.

The following relates to the practical problems of estimating the long term pumping rate of a particular bore using data obtained from one or more tests of this type- data being available only from the pumping bore. The main assumption in the analysis is that the bore will continue to perform, under sustained pumping, in the same manner as indicated during the test.

Procedure

1. The drawdown-time data are plotted on semi-logarithmic graph paper with time on the logarithmic scale. It is desirable to show the measured discharge rates at the relevant times on this plot.
2. The recovery-time, (or residual drawdown-t/t′), data are plotted on the same sheet to the same scale.
3. The depth to pump suction is decided upon. Normally the pump suction is placed just above the seal on the screen assembly.
4. The maximum available drawdown is obtained by subtracting the depth to standing water level from the depth to pump suction.
5. The working drawdown is obtained by subtracting from the maximum available drawdown an allowance for anticipated drop in standing water level as a result of seasonal conditions and other reasons.
6. By examination and computation from (i) and (ii) the rate which will give a drawdown after 100,000 mins (70 days approximately), or any other predetermined period of anticipated demand, which is equal to the working drawdown is called the long term pumping rate.

At first the method may seem relatively simple but it is beset with certain difficulties and in most cases a certain amount of judgement has to be applied.

Step 6 is of course an important step and some explanation is necessary.

Assume an ideal case where, after an initial curved portion, the drawdown-log time curve becomes a straight line. If the discharge rate is held constant and there are no hydrologic boundaries, this straight line should extend to the end of the test. It can be extrapolated also to any time desired, in this case 100,000 minutes has been adopted. In the case of a town water supply, or any other situation where very long term continuous pumping is required, a period of 1,000,000 minutes i.e. 2 years, would be a better figure. In systems of very high storage/flow ratio a longer period may be warranted e.g. a bore of relatively low transmissivity being used for a town water supply in the Great Artesian Basin.

The long term pumping rate is obtained by multiplying the test rate by the ratio of the working drawdown to the extrapolated drawdown at 100,000 minutes during the test.

$$Q_L = Q_T \frac{s_w}{s_{10^5}} \qquad .....7.42$$

where:

Q_L = long term pumping rate.

Q_T = test rate.

s_w = working drawdown.

s_{10^5} = test drawdown (extrapolated) to 10^5 minutes, or to the time required.

This rule should be applied with caution particularly in those cases where non-linear head losses are present. It assumes, if non-linear head loss is present, that it is proportional to the discharge whereas in fact the non-linear head loss is proportional to the discharge squared. In general it is considered most unwise to predict a long term pumping rate beyond the constant discharge test rate unless you are very familiar with the conditions in the area and the efficiency of the contractor.

The straight comparison of drawdowns to discharge is most useful when "ironing out" variations in drawdown caused by relatively small variations in pumping rate during a pumping test.

The figure of 100,000 minutes is more or less an arbitrary end point which seems reasonably convenient to use. The previous period is too short a time and the subsequent log period may be too long a time for a normal irrigation cycle. A longer period is used of course for Town Water Supply or industrial requirements.

<u>Example</u>

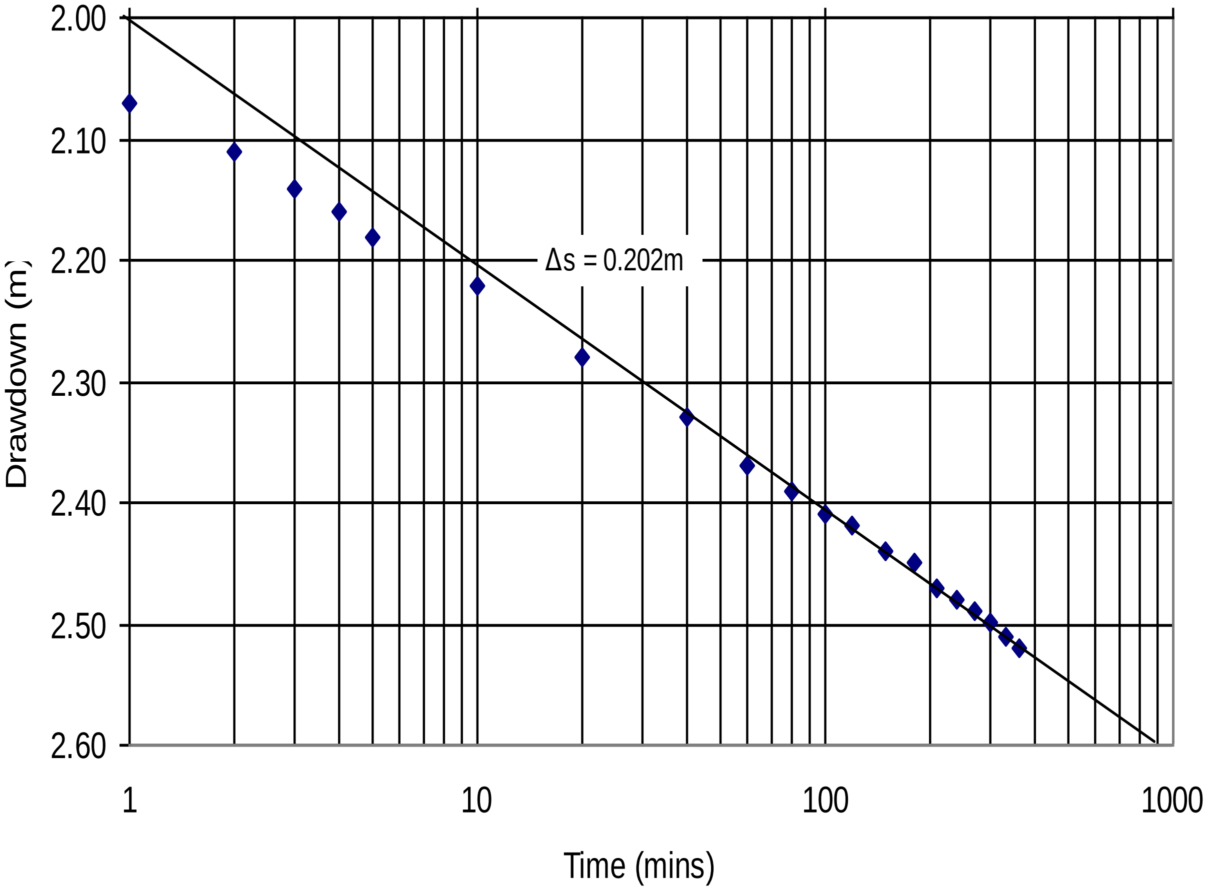
Table 7-7 presents test data obtained by the Queensland Irrigation and Water Supply Commission in 1962.

The bore is located at Biloela in the Callide Valley of Central Queensland.

The semi-logarithmic plot is shown in **Figure 7-10** and the analysis is given below.

Table 7-7 Test data from Biloela, Callide Valley

Time (mins)	Drawdown (m)	Discharge Rate (m^3/day)
1	2.07	2860
2	2.11	
3	2.14	
4	2.16	
5	2.18	2860
10	2.22	
20	2.28	
40	2.33	
60	2.37	2860
80	2.39	
100	2.41	
120	2.42	
150	2.44	2860
180	2.45	2860
210	2.47	
240	2.48	2860
270	2.49	
300	2.50	
330	2.51	2860
360	2.52	



<u>Example</u>

The test data used in this example are from Table 7.7.

<u>Analysis</u>

Long Term Pumping Rate:

Date Tested: August 1962

Top of Packer: 15.5mm BGL

Place Suction 0.15 m above top of packer: 15.35m BGL

Standing Water Level (SWL): 9.95mm BGL

Available Drawdown: 5.40m

Seasonal variation in SWL: 2m

Working Drawdown: 3.40m

At 2860 m^3/day, Δs: 0.202m

Drawdown at 1,000 minutes: 2.61m

Drawdown at 100,000 minutes: 2.61 + 2 x Δs

= 3.014 m

Estimated Long Term Pumping Rate:

= (3.40/3.014) x 2860 m^3/day

= 3220 m^3/day

= 37.2 litres / sec

Transmissivity:

T = 2.3 Q/4πΔs

= 260 m^2/day

Figure 7-10 Determination of long term pumping rate

Variable Discharge Test

If the long term pumping rate for the bore seems to be much greater than the pumping ability of the test pump then it is desirable to carry out a variable discharge pumping test to enable the non-linear head losses to be determined.

If a variable discharge test, such as a step drawdown test, is carried out then it is possible to determine the equation to drawdown for the bore for a certain period of discharge. Once the equation to drawdown is determined the long term pumping rate can be calculated.

It is desirable to plot the equation to drawdown, at the time corresponding to the required pumping duration, in the form of drawdown versus discharge on natural scale graph paper. If a non-linear loss is present this will result in a curved line.

The working drawdown is determined in the same way as described in the previous section.

Working drawdown = Depth to pump suction – Standing water level - seasonal variation.

The working drawdown is then read into the curve of drawdown versus discharge and the rate corresponding to the working drawdown is the long term pumping rate.

The determination of long term pumping rates by using variable discharge tests is desirable, as it accounts for non-linear head loss. This enables the analyst to estimate with some degree of confidence, long term rates in excess of the test rate.

7.7 SPECIFIC CAPACITY

The Specific Capacity of a discharging bore at a particular time is defined as the ratio of the discharge rate to the drawdown in the bore at that time. It is then the discharge per unit drawdown for a discharging bore. Its units are m^3/day/m.

It is not a fixed value. It varies with both discharge rate and duration of pumping. However, if calculated at the same time it can give an indication of the changing efficiency of a bore say during development. Some analysts put a good deal of faith in it. If there is a reasonable degree of confidence in the values of drawdown and discharge being used, then in the absence of more accurate information, specific capacity can be a useful tool for estimating transmissivity.

Several methods for estimating transmissivity from specific capacity have been published, some of which are cited below. If we solve equation 10.23 for Q/s (specific capacity), using s as the drawdown in the discharging bore, and r_w as the radius of the bore, and assuming that the bore is 100 percent efficient, we obtain:

$$\frac{Q}{s_w} = \frac{4\pi T}{2.3\log 2.25Tt/r_w^2 S} \qquad .....7.43$$

which shows the manner in which Q/s_w is approximately related to the other constants (T, and S) and variables (r,t). As r_w is constant for a particular bore being pumped, it can be seen that Q/s_w is nearly proportional to T at a given value of t, but gradually diminishes as t increases. Thus, for a given bore, considered 100 percent efficient, and assuming that water is discharged instantaneously from storage with decline in head, we may symbolise the foregoing statements by the following equation:

where:

$$\frac{Q}{s_w} \approx \frac{B}{\log t} \qquad .....7.44$$

B = a constant for the bore, and includes the other terms in equation 7.43.

Driscoll simplifies this further and suggests inserting typical values of T, S, t, and r_w into equation 7.43 to obtain an approximate value of transmissivity.

If typical values are assumed for the variables in the log function of equation 7.43 such that:

$t = 1$ day;

$T = 400\ m^2/day$;

$r_w = 0.15m$;

$S = 10^{-3}$ for a confined aquifer; and

$S_y = 10^{-1}$ for an unconfined aquifer;

then the specific capacity at 1 day has the following approximate relationships:

for a confined aquifer:

$$Q/s_w = 0.72\ T \qquad .....7.45$$

and for an unconfined aquifer:

$$Q/s_w = 0.98\ T \qquad .....7.46$$

It may appear to be presumptuous to use an average transmissivity or even to assume a transmissivity value at all before one is known. However, because it appears in the log term of equation 7.43, its effect on the value of the divisor in either aquifer case is minimal. For example, if a transmissivity of 1600 m^2/day is used then the value for the confined aquifer would change from 0.72 T to 0.66 T a difference of about 8% for a 300% increase in the value of T.

No bore is 100 percent efficient, but, according to design, construction, age, etc., some bores are more efficient than others.

Thus we see that Q/s_w diminishes not only with time but with pumping rate Q. In unconfined aquifers it may be necessary to take into account delayed yield from storage.

7.8 EVALUATION OF BORE EFFICIENCY

While the radial flow equations provide a very good approximation to actual drawdowns in observation bores remote from the pumping bore, very seldom if ever, does the actual drawdown in a pumping bore equal that predicted by the radial flow equations. Invariably the drawdown in the pumping bore is greater, sometimes much greater than that theoretically predicted.

There are many reasons for this increase in drawdown. Some relate to the design of the bore, some to bore construction and others simply to normal groundwater flow which does not always comply with the assumptions made for the derivation of the equations.

The design factors include:

1. Poor choice of screen apertures, resulting in higher than acceptable flow velocities at the screen.
2. Less than full screen exposure to the flow resulting in longer flow lines and additional head loss. This reduced screen exposure could be a consequence of either only part of the aquifer depth being screened (partial penetration) or less than complete screen exposure around the circumference of the bore.
3. Incorrect filter pack material being used giving rise to greater head loss.

The construction factors would include inadequate development of the bore or screening the wrong formation.

The hydraulic factors could result from the aquifer parameters being such that the flow near the screen is no longer linear and additional non-linear head losses come into play.

The ratio of the theoretical drawdown to the actual drawdown is called the bore efficiency.

Regardless of the cause of the difference between the theoretical and actual drawdown in the bore it is desirable to minimise the difference and make the bore as efficient as possible, and can provide optimal performance.

Sometimes some of the factors have to be compromised to achieve optimal production from the bore. For example, it would not be sensible to fully screen a shallow unconfined aquifer and so limit the depth at which the pump can be set. It would be more prudent in such a case to screen only the bottom one third of the aquifer and allow additional head loss while at the same time achieving a much greater working drawdown. The pump suction could then be set at two thirds of the aquifer depth. This would then achieve optimal sustained yield while sacrificing efficiency.

If bore efficiency is so important then it is important to have a way to measure it.

The higher the bore efficiency the smaller is the drawdown required in the bore to achieve a certain rate of discharge. This is important for a number of reasons, not the least of which is the energy costs to pump water.

Jacob was one of the first to try to determine the efficiency of a bore. He assumed that the bore efficiency was reduced solely by the presence of the non-linear head loss CQ^2. He defined the efficiency of the bore as the ratio of the laminar flow component of the drawdown to the total drawdown. He called the laminar flow term the aquifer loss and the turbulent flow head loss the well loss.

His was the first step towards determining bore efficiency but it was not correct. While the turbulent head loss is a contributing factor towards a reduction in bore efficiency it is not the sole contributing factor. Some of the turbulent losses are in the aquifer itself and may not be able to be removed but more importantly some of the factors contributing to reductions in efficiency are in the laminar flow component.

7.8.1 When the Equation to Drawdown is Known

A more correct approach is to compare the actual drawdown in the bore with the drawdown would have occurred if the radial flow equations applied right up to the bore. It is also significant that any such determination will result in changes in bore efficiency with time as well as discharge rate.

The theoretical drawdown at the bore for a 100% efficient bore will be:

$$s_{tw} = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_w^2 S} \qquad .....7.47$$

And the actual drawdown in the pumping bore is:

$$s_{aw} = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_w^2 S} + CQ^2$$

i.e. $$s_{aw} = (a + b\log t)Q + CQ^2 \qquad .....7.48$$

where:

s_{tw} = the theoretical drawdown in a 100% efficient bore.

s_{aw} = the actual drawdown in the bore.

The bore efficiency is then obtained by comparing equation 7.47 with 7.48.

The efficiency η of the bore is then:

s_{tw}/s_a

$$\eta = \left(\frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_w^2 S}\right) / \ ((a + b\log t)Q + CQ^2) \qquad .....7.49$$

To make such a comparison the true values of T, t, S and r should be inserted into equation 7.47. The actual value of T can be obtained from the pumping tests on the bore and the bore radius is known. The value of S will have to be assumed for confined and unconfined aquifers if it is not known. This will not present a major error since the range of S would be known and the S value is contained within the log term. The required value of t can also be inserted.

The values of "a", "b" and "C" in equation 7.48 can be obtained from the analysis of step drawdown tests so s_w in equation 7.47 is known for any required value of t.

Note: remember that the "a" value in equation 7.48 is dependent on the time units used for t in the equation. It is quite acceptable to use minutes as the time unit in the equation to drawdown if the equation is to be used on its own. The use of days for the units is dimensionally correct but all that the conversion from minutes to days does is to change the value of the "a" term by a constant "b x log 1440". However, if we are comparing equations as we must do in determining the bore efficiency then the units must be consistent. The "t" in equation 7.48 must be in days and the "a" term must be consistent. If the "a" term had been calculated on the basis of a 1 minute intercept and "t" had been expressed in minutes in equation 7.48 then to convert "t" to day units simply add "b x log 1440" i.e. 3.16 x b to the "a" term.

If we divide both sides of equations 7.47 and 7.48 by Q then the efficiency of the bore at one unit of time can be expressed as:

$$\eta = \left(\frac{2.3}{4\pi T}\log\frac{2.25Tt}{r_w^2 S}\right) / \left(a + b\log t + CQ\right) \qquad .....7.50$$

If the time at which the efficiency is being determined is1 then (b log t) in equation 7.48 becomes 0.

The transmissivity of the aquifer is constant and 2.3/(4πT) in equation 7.47 is equal to "b" which has been determined from the pumping tests.

For convenience, in the absence of more reliable information, S could be taken as 10^{-3} for a confined aquifer and 10^{-1} for an unconfined aquifer.

<u>Example</u>

Calculate the efficiency of the Richmond Town Bore No. 3 for discharge rates 1000 m3/day and 3000 m3/day at times 1 minute, 1 day and 1 week after pumping discharge commenced.

The equation to drawdown for the bore has been determined previously as:

$s_{wt} = (a + b \log t)\, Q + C\, Q^2$

$= (3.1 \times 10^{-3} + 5.34 \times 10^{-4} \log t)\, Q + 8.1 \times 10^{-7}\, Q^2$

Since the time units in this equation are minutes they will have to be changed to days so that the equation can be compared with equation 7.43. The "a" term has to be changed by adding "b x log 1440". The "a" term now becomes 4.78 x 10-3. All other terms remain the same.

The equation then becomes:

$s_{wt} = (4.78 \times 10^{-3} + 5.34 \times 10^{-4} \log t)\, Q + 8.1 \times 10^{-7}\, Q^2$

where "t" is now in days.

The radius of the bore is 0.075m.

It is in a confined aquifer so we can take S as 10^{-3} and T (2.3/(4πb) is 345 m^2/day.

<u>Case 1 (at time 1 minute)</u>

For Q = 3000 m^3/day

$b\log\frac{2.25Tt}{r_w^2 S}$ = 0.000534 x log {(2.25 x 345) / (1440 x 0.075^2 x 0.001)}

= 0.00266

a + CQ = 4.78 x 10^{-3} + 8.1 x 10^{-7} x 3000

= 0.00553

η = 0.00266 / 0.00553

= 48%

The same procedure can be used to determine the efficiencies at the other times and rates to give the results tabulated in **Table 7-8**.

Table 7-8 Bore efficiencies for Richmond Town bore no. 3

Time (mins)	(Days)	Discharge Rate (m^3/day)	Bore Efficiency (η) (%)
1	0.000694	1000	67.70
1	0.000694	3000	47.85
1440	1	1000	77.25
1440	1	3000	59.89
10080	7	1000	78.90
10080	7	3000	62.22

From **Table 7-8** it can be seen that in this case the efficiency decreases with an increase in discharge rate but increases with the duration of pumping. If continuous pumping is not required then in some instances it may be more economical to pump at lower rates for longer periods.

7.8.2 When the Equation to Drawdown is Not Known

The above procedure is able to be used if the equation to drawdown for the bore is known. In many instances the equation is not known and only a single pumping test is available. In such cases the following procedure can be used.

The efficiency is determined using the same comparison as in the previous section i.e. the drawdown in the bore operating at 100% efficiency is compared with the actual drawdowns in the pumping bore.

The theoretical drawdown at the bore for a 100% efficient bore will be:

$$s_{tw} = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r_w^2 S}$$

and the actual drawdown in the pumping bore is taken from a plot of the test results. This analysis is exactly the same as the previous case but it is limited to the test discharge rate. It does not give the ability to determine the efficiency for rates other than the test rate.

Once again T can be calculated from the test data, r_w is known and S can be assumed for confined and unconfined conditions.

Driscoll uses more approximations still. Using the approximations to specific capacities given in equation 7.45 and equation 7.46 for confined and unconfined aquifer conditions respectively, he compares the specific capacity of a theoretical bore of 100% efficiency with the specific capacity (Q/s_w) of the pumping bore at time 1 day.

This is a very quick method and gives a good indication of the efficiency of the bore.

By way of comparison, the values of corrected drawdowns for Richmond Town Bore No. 3 as plotted in **Figure 7-3** were extrapolated to 1 day. The specific capacities were calculated for each of the steps and the bore efficiencies estimated using Driscoll's approximation. The bore efficiencies for each of the steps were also calculated using equation 7.50. **Table 7-9** gives the efficiencies calculated using each method.

Table 7-9 Comparison of efficiency calculations

Time (day)	Discharge Rate (m^3/day)	Efficiency (%)	
		Using Equation 7-50	Using Approximate Method
1	1135	76	72
1	1640	71	67
1	2180	66	61
1	2725	62	57
1	3270	58	54

SECTION 8: EVALUATION OF AQUIFER PROPERTIES WITHOUT PUMPING TESTS

8.1 INTRODUCTION

The most accurate method of determining aquifer parameters is to carry out a properly conducted pumping test on a bore with measurements of drawdown and discharge rate at selected times after pumping commenced. However, in many instances data from pumping tests are not available and aquifer parameters have to be estimated. The methods outlined below provide some means of obtaining reasonably reliable results.

8.2 AREAL METHODS

8.2.1 Numerical Analysis

The partial differential equation for two dimensional non-steady state flow can be expressed as:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad .....8.1$$

where:

h = the head at any point whose coordinates are x,y.

If we let the infinitesimal lengths ∂x and ∂y be expanded so that each may be equivalent to a finite length "a" and similarly let ∂t be considered equivalent to Δt. A plan representation of the region of flow to be studied may then be subdivided by two systems of equally spaced parallel lines at right angles to each other. One system is oriented in the x direction, the other in the y direction, and the spacing of lines equals the distance "a". A set of 5 gridline intersections, or nodes (observation bores), as shown in **Figure 8-1**, is called an array.

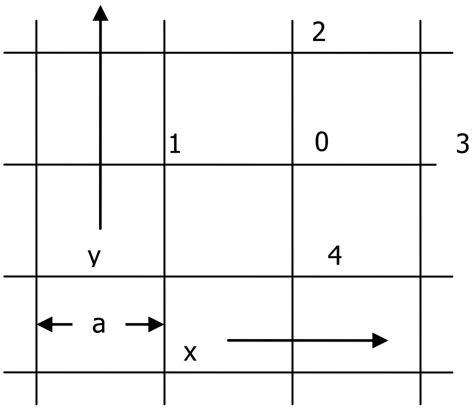


Figure 8-1 Numerical analysis array

The first two differentials in equation 8.1 can be expressed in terms of the head values at the nodes (bores) in the array as:

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{h_1 + h_3 - 2h_0}{a^2} \qquad .....8.2$$

and

$$\frac{\partial^2 h}{\partial y^2} \approx \frac{h_2 + h_4 - 2h_0}{a^2} \qquad .....8.3$$

where the subscripts refer to the numbered nodes in **Figure 8-1**.

Substituting these closely equivalent expressions in equation 8.1, and letting $\frac{\partial h}{\partial t}$ be considered equivalent to $\Delta h_0/\Delta t$, we obtain:

$$h_1 + h_2 + h_3 + h_4 - 4h_0 \approx \frac{a^2 S}{T}\frac{\Delta h_0}{\Delta t} \qquad .....8.4$$

where:

Δh_0 is the change in head at node (bore) 0 during the time interval Δt.

This is merely an introduction to numerical analysis.

Example

This data was presented originally in imperial units but has been converted for these notes.

Stallman tried this method successfully on several such arrays in the Arkansas Valley, Colorado, during the winter of 1965-66. Bores 1- 4 were spaced 304m apart, so that "a" (304/√2) was equal to 216m.

From the estimated values of T and S, "a" normally is determined from the convenient empirical relation a^2S/T =10± days but in the Arkansas Valley, nearby boundaries made it necessary to use a^2S/T = 4± days. The elevations of the measuring points at each of the five wells were determined above a convenient arbitrary datum, and the water levels, measured in metres above datum, were obtained from automatic water level sensors. The winter data from the Nevius site near Lamar, Colorado were plotted as shown in **Figure 8-2**, in which Σh, in mm, represents the left hand side of equation 8.4, and $\Delta h_0/\Delta t$ is in mm/day.

From equation 8.4 note that the slope of the straight line, which Stallman drew form a least-squares fit, is $\Sigma h/(\Delta h_0/\Delta t)$ = 3.99 days.

Hence:

a^2S/T = 3.99 days

The value T/S is known as the Hydraulic Diffusivity.

S was known from neutron moisture probe tests, made during periods of both high and low water levels, to be about 0.18.

$a^2 = 4.7 \times 10^4\ m^2$

then:

$T = (a^2S/\Sigma h)/(\Delta h_0/\Delta t)$

$= (4.7 \times 10^4 \times 0.18)\ 3.99$

$= 2.12 \times 10^3\ m^2$ / day

This value of T was in close agreement with the results of a nearby pumping test.

Conversely, if the value of T were known, the storage coefficient could have been determined.

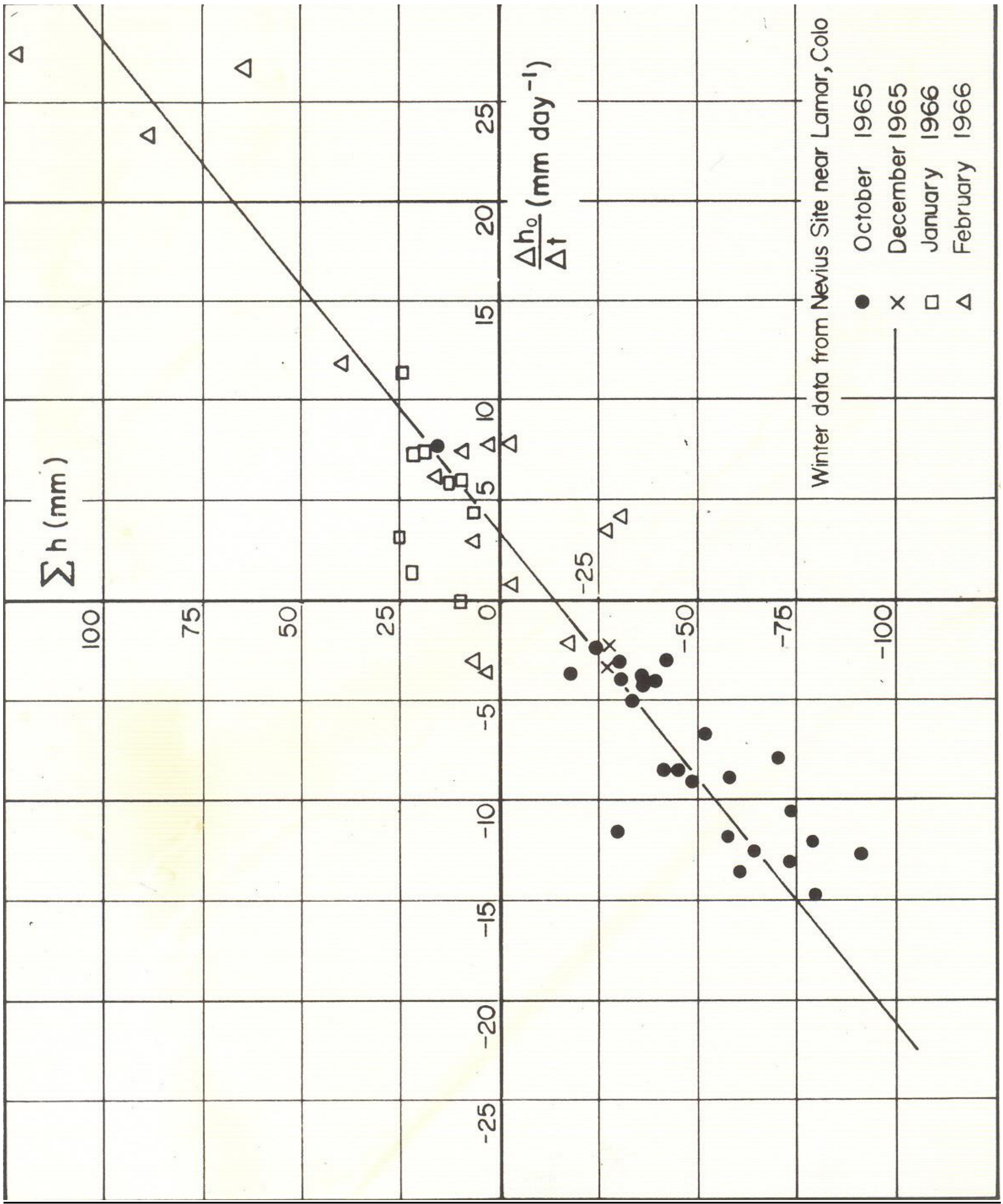
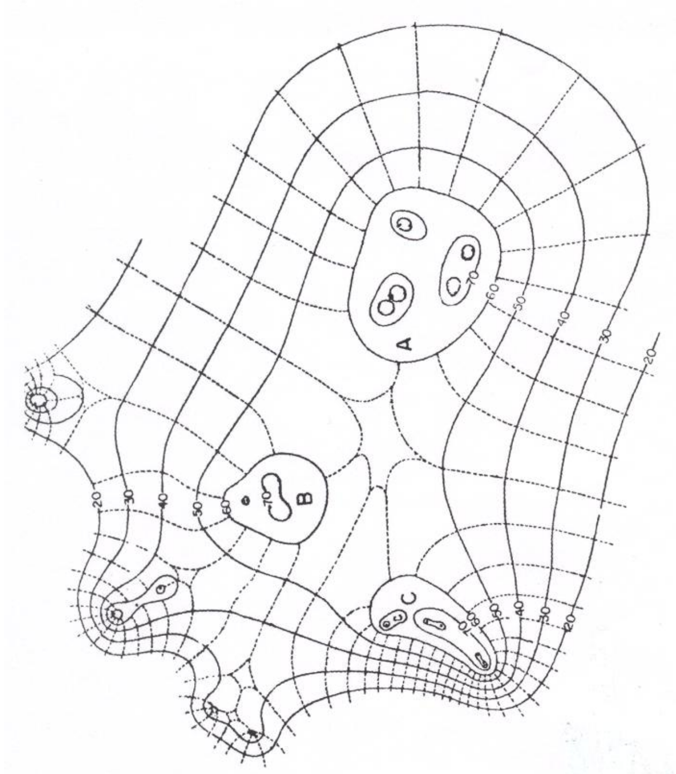


Figure 8-2 Numerical analysis example

8.2.2 Flow-Net Analysis

In analysing problems of groundwater flow, a graphical representation of the flow pattern can be of considerable assistance and may provide solutions to problems not readily amenable to mathematical solution. This is a logical, and on occasions eye-opening, follow-on from the normal water level or potentiometric level contours which are frequently drawn during groundwater investigations. They can be used to determine aquifer characteristics of transmissivity and storage coefficient, or, knowing these, the volume extracted. Recharge areas can be detected also by this means.

The first significant development in graphical analysis of flow patterns was made by Forchheimer (1930), but additional information was given by Casagrande (1937, p. 136, 137) and Taylor (1948).



Legend:

___20___ depth below sea level in metres

- . - . - . - . - flow line

Figure 8-3 Typical flow net

A "flow net", which is a graphical solution of a flow pattern, is composed of two families of lines or curves (see **Figure 8-3**). One family of curves, termed equipotential lines (solid lines on map), represent contours of equal head in the aquifer, and may be contours of the potentiometric surface or of the water table. Intersecting the equipotential lines at right angles (in isotropic aquifers) is another family of curves (dashed lines on map) representing the streamlines, or flow lines, where each curve indicates the path followed by a particle of water as it moves through the aquifer in the direction of decreasing potentiometric head.

Although the real flow pattern contains an infinite number of flow and equipotential lines, it may be represented conveniently by constructing a net that uses only a few such lines, the spacing being conveniently determined by the contour interval of the equipotential lines. The contour interval indicates that the total drop in head in the system is evenly divided between adjacent pairs of equipotential lines; similarly the flow lines are selected so that the total flow is equally divided between adjacent pairs of flow lines. The movement of each particle of water between adjacent equipotential lines will be along flow paths involving the least work, hence it follows that, in isotropic aquifers, such flow paths will be normal to the equipotential lines, and the paths are drawn orthogonal to the latter.

The net is constructed so that the two sets of lines form a system of "squares". Note on the map that some of the lines are curvilinear, but that the "squares" are constructed so that the sum of the lengths of each line in one system is closely equal to the sum of the lengths in the other system.

Let **Figure 8-4** represent one idealised "square" of **Figure 8-3**, whose dimensions are Δw and Δl.

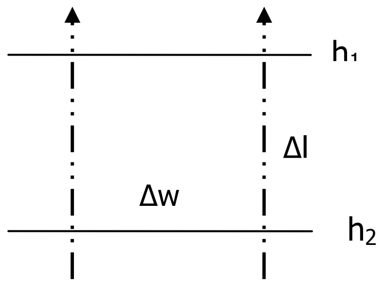


Figure 8-4 Elemental square

Then, by rewriting Darcy's Law as a finite-difference equation for the flow, ΔQ, through this elemental "square" of thickness b, we obtain:

$$\Delta Q = -K\, b\, \Delta w\, (\Delta h/\Delta l) \qquad .....8.5$$

Since, by construction, Δw = Δl, then:

$$\Delta Q = -T\Delta h \qquad .....8.6$$

If:

n_f = number of flow channels;

n_d = number of potential drops; and

Q = total flow

then:

$$Q = n_f\, \Delta Q \text{ or } \Delta Q = Q/n_f \qquad .....8.7$$

and

$$h = n_d\, \Delta h \text{ or } \Delta h = h/n_d \qquad .....8.8$$

Substituting equations 8.7 and 8.8 in equation 8.6 we obtain:

$$Q = -T\,(n_f/n_d)h \qquad .....8.9$$

or

$$T = -\frac{Q}{(n_f / n_d)h} \qquad .....8.10$$

Conversely Q can be determined if T is known.

Example

Figure 8-3 presents the flow net for a hypothetical situation in which it is known that the discharge from sub-area A is 100 000m^3/day. It shows 15 flow channels surrounding the sub-area, hence n_f = 15. The number of equipotential drops between the 30- and 60- m contours is 3, so n_d = 3. The total potential drop between the 30 and 60 m contours is 30 m, so h = 30 m.

Then, from equation 8.10:

$$T = -\frac{100{,}000}{(15/3)(-30)}$$

= 667 m^2/day

The value of T thus determined is for a much larger sample of the aquifer than that determined by a pumping test on a single bore. This method has been largely neglected, but is deserving of more widespread application.

8.3 ESTIMATING TRANSMISSIVITY

8.3.1 General

In some ground-water investigations, such as those of a reconnaissance type, it may be necessary to estimate the transmissivity of an aquifer from the specific capacity (yield per unit of drawdown) of bores, as the determination of T by use of some of the equations discussed above may not be feasible. On the other hand, some of our modern quantitative studies, such as those for which electric-analog models or mathematical models are constructed, require a sufficiently large number of values of T so that transmissivity-contour maps (T maps) may be constructed.

In unconfined aquifers, such T maps generally require also the construction of water table contour maps and bedrock-contour map, from which may be obtained maps showing lines of equal saturated thickness, b, for we have seen that T = Kb. For example, a quantitative investigation of a 240 kilometre reach of the Arkansas River Valley, in eastern Colorado, required a T map based upon about 750 values, or about 0.65 values per square kilometre. About 25 of these values were obtained from pumping tests, selected as reliable tests from a greater number of tests conducted. About 200 values of T were estimated from the specific capacity of bores, by one of the methods to be described. About 525 values were estimated by geologists from studies of logs of bores and test holes, by methods to be described. Thus, only about 3 percent of the values were actually determined from pumping tests.

8.3.2 Specific Capacity of Bores

The Specific Capacity of a discharging bore at a particular time is defined as the ratio of the discharge rate to the drawdown in the bore at that time. It is then the discharge per unit drawdown for a discharging bore. Its units are m^3/day/m.

It is not a fixed value. It varies with both discharge rate and duration of pumping. However, if calculated at the same time it can give an indication of the changing efficiency of a bore say during development.

Some analysts put a good deal of faith in it. If there is a reasonable degree of confidence in the values of drawdown and discharge being used, then in the absence of more accurate information, specific capacity can be a useful tool for estimating transmissivity.

Several methods for estimating transmissivity from specific capacity have been published, some of which are cited below. If we solve equation 6.23 for Q/s (specific capacity), using s as the drawdown in the discharging bore, and r_w as the radius of the bore, and assuming that the bore is 100 percent efficient, we obtain:

$$\frac{Q}{s_w} = \frac{4\pi T}{2.3 \log 2.25 Tt / r_w^2 S} \qquad .....8.11$$

which shows the manner in which Q/s_w is approximately related to the other constants (T,S) and variables (r,t). As r_w is constant for a particular bore being pumped, we see that Q/s_w is nearly proportional to T at a given value of t, but gradually diminishes as t increases by the amount log t. Thus, for a given bore, considered 100 percent efficient, and assuming that water is discharged instantaneously from storage with decline in head, we may symbolise the foregoing statements by the following equation:

$$\frac{Q}{s_w} \approx \frac{B}{\log t} \qquad .....8.12$$

where:

B = a constant for the bore, and includes the other terms in equation 8.11.

Driscoll simplifies this further and suggests inserting typical values of T, S, t, and r_w into equation 8.11 and obtaining an approximate value of transmissivity.

If typical values are assumed for the variables in the log function of equation 8.11 such that:

t = 1 day;

T = 400 m^2/day;

r_w = 0.15m;

$S = 10^{-3}$ for a confined aquifer; and

$S_y = 10^{-1}$ for an unconfined aquifer;

then the specific capacity at 1 day has the following approximate relationships:

for a confined aquifer:

$$Q/s_w = 0.72\ T \qquad .....8.13$$

and for an unconfined aquifer:

$$Q/s_w = 0.975\ T \qquad .....8.14$$

It may appear to be presumptuous to use an average transmissivity or even to assume a transmissivity value at all before one is known. However, because it appears in the log term of equation 8.11, its effect on the value of the divisor in either aquifer case is minimal. For example, if a transmissivity of 1600 m^2/day then the value for the confined aquifer would change from 0.72 T to 0.66 T a difference of about 8%.

No bore is 100 percent efficient, but, according to construction age, etc., some bores are more efficient than others. Jacob (1947, p. 1048) has approximated the head loss resulting from the relatively high velocity of water entering a bore or bore screen as being proportional to some higher power of the velocity approaching the square of the velocity, which in turn is nearly proportional to Q^2; thus:

$$\text{head loss} = CQ^2$$

where:

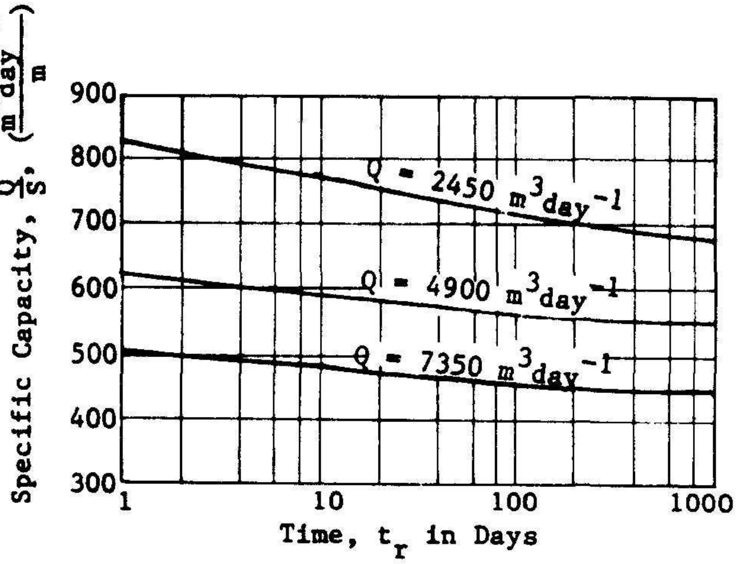
C = a constant of proportionality.

Adding this to equation 8.12:

$$\frac{Q}{s_w} \approx \frac{B}{\log t}\left(1 - \frac{CQ^2}{s_w}\right) \qquad .....8.15$$

Thus we see that Q/s_w diminishes not only with time but with pumping rate Q. In unconfined aquifers it may be necessary to adjust factor B to further account for delayed yield from storage.

The relation of specific capacity to discharge and time for a particular bore in a confined aquifer is shown in **Figure 8-5** (Jacob, 1947, Figure 5; converted to metric for these notes).



TYPICAL SPECIFIC CAPACITY - TIME - DISCHARGE CURVES
Fig. 8.5

Figure 8-5 Typical specific capacity - time - discharge curves

In an uncased bore in, say, sandstone, r_w may be assumed equal to the radius of the bore, but in screened bores in unconsolidated material, in which the finer particles have been removed near the screen by bore development, or in gravel-packed bores, the effective r_w generally is larger than the screen diameter. Jacob (1947) described a method for determining the effective r_w and the well loss (CQ^2) from a step drawdown test, and other methods are given earlier in this chapter.

Most other investigators have neglected well loss in their equations, which are equations for bores of assumed 100 percent efficiency, such as equation 8.12, but some have adjusted for this loss by selection of an arbitrary constant for bores of similar construction in a particular area or aquifer, which generally gives satisfactory results when used with caution.

Theis (1963a, p. 331-336) gave equations and a chart, based upon the Theis equation, for estimating T from specific capacity for constant S and variable t, with allowance for variable bore diameter but not bore efficiency. Brown (1963, p. 336-338) showed how Theis' results may be adapted to artesian aquifers. Meyer (1963, p. 338-340) gave a chart for estimating T from the specific capacity at the end of one day of pumping, for different values of S and for bore diameters of 0.5 (0.15), 1.0 (0.30) and 2.0 (0.61) ft (m). Bedinger and Emmett (1963, p. C188-C190) gave equations and a chart for estimating T from specific capacity, based upon a combination of the Thiem and Theis equations, and upon average values of T and S for a specific area, for bore diameters of 0.5 (0.15), 1.0 (0.3) and 2.0 (0.61) ft. (m). Hurr (1966) gave equations and charts based upon the Theis and Boulton (1954a) equations, which allow for delayed yield from storage, for determining T from specific capacity at different values of t for a bore 1.0 ft (0.3 m) in diameter. None of the methods just cited includes corrections for bore efficiency, but this can be added in an approximate manner.

8.3.3 Rough Method

Using the Steady State Flow Equation, Equation 6.5:

$$T = -\frac{2.3Q}{2\pi(s_1 - s_2)}\log\frac{r_2}{r_1} \qquad .....8.16$$

Let:

r_1 = the effective radius of the bore, in this case 0.3 m;

r_2 = radius of influence of bore, assume 300 m;

s_1 = drawdown in the bore (one only drawdown, no test has been carried out); and

s_2 = 0.

then:

$$T = -\frac{2.3Q}{2\pi s_1}\log 1000$$

$$= -\frac{2.3*3}{2\pi}(\frac{Q}{s_1})$$

If r^2 were taken as 3000 m, then log r_2/r_1 would only be increased to 4. This is a 25% alteration but the order of T has not changed. Apart from the assumed values of r the solution must be approximate as the bore has been assumed to be 100% efficient.

8.3.4 Logs of Bores

As noted in Section 8.3.1, about 525 values of t out of 750 total values, in the Arkansas Valley of Eastern Colorado, were estimated by geologists from studies of logs of bores, and from drill cuttings from test holes. Wherever possible pumping tests were carried out on bores for which, or near which, logs were available; otherwise, test holes were drilled near the bore tested. From several of many such pumping accompanied by logs, the values of T were carefully compared against the water-bearing bed or beds, and, as T = Kb, the total T was distributed by cut and try among the several beds, according to the following equation:

$$T = \sum_{1}^{n} K_m b_m = K_1 b_1 + K_2 b_2 + K_3 b_3 . + ... + K_n b_n \qquad .....8.17$$

From this the following table was prepared, comparing average values of K for different alluvial materials in the valley.

The same geologist who prepared **Table 8-1** then carefully examined the logs of other bores and test holes, for which no pumping tests were available. He assigned values of K to each bed of known thickness, on the basis of the descriptive words used by the person who prepared the log. The values of K assigned may have been equal to, or more or less than values given in the table (depending upon cleanliness, sorting, mixing, etc), and thus necessarily involved subjective judgement; however, as experience is gained, the geologist can estimate K and T with fair to good accuracy. The T values from all sources also are compared carefully with the saturated-thickness map.

This method for estimating T has been used successfully in the Arkansas Valley in Colorado, in the Arkansas Valley in Arkansas and Oklahoma (Bedinger and Emmett, 1963), in Nebraska, in California, and can be used elsewhere.

8.3.5 Laboratory Analysis

Laboratory determinations for K of cores of consolidated rocks, such as partly to well cemented sandstone, may be used in place of estimates. Reconstitution of disturbed samples of unconsolidated material is not possible, however, so laboratory determinations for K generally do not give reliable values. However, they may be very useful in indicating relative values. **Table 1-2** at the end of Section 1 gives indicative values of hydraulic conductivity for various rock types.

The above methods may also be used by the geologist in measuring exposed sections of rocks containing water-bearing beds.

Table 8-1 Average values of hydraulic conductivity of alluvial material in the Arkansas Valley, Colorado

Material	Hydraulic Conductivity (m/day)
Gravel, coarse	305
Gravel, medium	286
Gravel, fine	265
Sand, very coarse to gravel	246
Sand, very coarse	204
Sand, coarse to very coarse	143
Sand, coarse	73
Sand, medium to coarse	33
Sand, medium	16.5
Sand, fine to medium	8.2
Sand, fine	4.1
Sand, very fine to fine	1.7
Sand, very fine	0.8
Clay	0.4

(Courtesy R. T. Hurr)

8.4 ESTIMATING STORAGE COEFFICIENT AND SPECIFIC YIELD

8.4.1 Confined Aquifers

In measuring sections of exposed rock which dip down beneath confining beds to become confined aquifers, or in examining logs of bores or test holes in confined aquifers, S can be determined fairly closely from **Table 8-2**. The table is merely the calculation of the first term of equation 3.6 for compression of water alone:

i.e. $$S = b\rho g\left(\frac{\theta}{E_w} + \frac{(1-\theta)}{E_s}\right)$$

Table 8-2 Storage coefficient approximation

b (m)	S	S/b (m^{-1})
1	5×10^{-6}	5×10^{-6}
10	5×10^{-5}	5×10^{-6}
100	5×10^{-4}	5×10^{-6}
1000	5×10^{-3}	5×10^{-6}

One may either multiply the thickness in metres by 5×10^{-6} or interpolate between values in the first two columns, thus for b = 300m, S = 1.5×10^{-3}, etc. Values thus obtained are not absolutely correct; they represent the minimum value of S as no adjustments have been made for porosity or for compressibility of the aquifer. However, the values so obtained are reliable estimates.

8.4.2 Unconfined Aquifers

It is more difficult to estimate the specific yield of unconfined aquifers than the storage coefficient of confined aquifers, but it can be done. It should be remembered that the specific yield applies only to the material in the zone of water-table fluctuation and to the material within the cones of depression of pumping bores. In general the specific yield ranges between 0.1 and 0.3 (10-30 percent), and long periods of pumping may be required to drain water-bearing material.

Thus, in the absence of any determinations whatever, as in a rapid reconnaissance study, we would not be far off by assuming that, for supposed long periods of pumping, the specific yield of an unconfined aquifer is about 0.2. If logs of bores or tests holes are available, careful study of the grain sizes present, degree of sorting, and cleanliness, might allow a better estimate of S, which might then be more or less than 0.2. A few laboratory determinations of the specific yield of disturbed samples may refine the estimate and allow extrapolation to similar material in the logs of other wells or test holes, but such determinations are more likely to be too large than too small. The values of specific yield obtained from reliable long pumping tests would greatly refine the estimates, and could be extrapolated to similar types of material elsewhere in the same aquifer. The values obtained from neutron-moisture probes (Meyer, 1962) could be similarly extrapolated.

8.4.3 Water Balance

In any aquifer system a water balance must exist. This balance is expressed very simply by the following equation:

Recharge+Groundwater Inflow = Draft+Groundwater Outflow +/- Change in Storage ...8.18

The Recharge includes such items as natural recharge from rainfall, or streamflow, artificial recharge and excess irrigation.

The Draft includes evapotranspiration, withdrawals for irrigation, industrial, Town Water Supplies, stock and domestic uses.

The change in storage is expressed as $AS\Delta h$ where:

A = surface area of the water table.

S = storage coefficient or specific yield.

Δh = change in water level during the period of analysis.

This is probably the most accurate means of determining a regional storage coefficient.

8.4.4 Barometric Efficiency

The storage coefficient of a confined aquifer may be approximated by utilising the barometric efficiency (BE) of the aquifer.

A reduction in barometric pressure results in a rise in ground-water levels. The ratio of the change in groundwater pressure to the change in atmospheric pressure is called the barometric efficiency.

The barometric efficiency is defined as:

$$BE = \rho g \Delta h / \Delta p_a \qquad .....8.19$$

where:

ρ = density of water.

g = gravitational acceleration.

Δh = change in potentiometric level due to Δp_a.

Δp_a = change in atmospheric pressure.

The barometric efficiency can be interpreted as a measure of the competence of the overlying confining beds to resist pressure changes; thick impermeable confining strata are associated with high barometric efficiencies, whereas thinly confined aquifers will display low values.

The barometric efficiency of unconfined aquifers is zero, i.e. a change in atmospheric pressure does not result in a change in water level.

It can be shown that the barometric efficiency is related to the storage coefficient of an aquifer by the following equation:

$$S = \frac{\rho g b \theta}{E_w BE} \qquad .....8.20$$

where:

S = storage coefficient.

θ = porosity.

ρ = density of water.

g = gravitational acceleration.

b = thickness of the aquifer.

E_w = bulk modulus of elasticity of water.

BE = barometric efficiency.

Thus, from the barometric efficiency of a confined aquifer, an estimate of its storage coefficient can be obtained.

8.4.5 Tidal Efficiency

Just as atmospheric pressure changes produce variations of potentiometric levels, so do tidal fluctuations; both earth tides and ocean tides. The ocean tides vary the load on confined aquifers extending under the ocean floor. The earth tides result from varying gravitational forces on the aquifer from both the moon and the sun. These forces can cause expansion and contraction of the aquifer which result changes in porosity and accompanying changes in groundwater levels.

Unlike the atmospheric pressure effect, tidal fluctuations are direct; i.e. as the sea level increases, so too does the ground water level. The ratio of potentiometric level amplitude to tidal amplitude is known as the tidal efficiency of the aquifer.

Jacob showed that tidal efficiency (TE) is related to barometric efficiency BE by:

$$TE = 1 - BE \qquad .....8.21$$

Thus, tidal efficiency is a measure of the incompetence of overlying confining beds to resist pressure changes.

The storage coefficient of an aquifer can be computed from observations of the tidal efficiency by replacing BE by 1 - TE in equation 8.20.

It has been further shown that the amplitude h of groundwater fluctuation at a distance x from the shore is given by:

$$h_x = h_0 e^{-x\sqrt{\pi S / t_0} T} \qquad .....8.22$$

where:

h_x = the amplitude of groundwater fluctuations at distance x from shore.

h_0 = the amplitude of tidal fluctuations at the shore.

x = distance of the observation bore from the shore.

t_0 = the tidal period.

T = the transmissivity.

It has been further shown that the time lag t_L of a given maximum or minimum water level in a bore after high or low tide occurs can be obtained from:

$$t_L = x\sqrt{(t_0 S / 4\pi T)} \qquad .....8.23$$

If the storage coefficient has been estimated from the tidal efficiency then a value of transmissivity could also be estimated from equation 8.21.

SECTION 9: CORRECTIONS AND EFFECTS TO BE ALLOWED FOR WHEN ANALYSING

9.1 GENERAL

Many of the assumptions which have been made in developing the equations presented previously are not met in nature and corrections have to be made to the data before they can be applied.

Some of the effects which have to be taken into account are:

- delayed yield in unconfined aquifers;
- increased drawdown caused by dewatering in unconfined aquifers;
- anomalies in drawdown readings;
- partial penetration;
- antecedent pumping conditions;
- possible development during pumping;
- proximity of recharging or discharging boundaries;
- water temperature variations in hot bores;
- anisotropy, non-homogeneity and finite bore diameter;
- variations in barometric pressure; and
- tidal effects.

9.2 DELAYED YIELD FROM STORAGE

This effect is taken into account quite adequately by Boulton's analysis for unconfined aquifers exhibiting delayed yield. However if other equations are to be used then the value of storage coefficient which has been calculated may have to be calculated many times throughout the test until the stable storage coefficient, or in this case specific yield, has been determined.

9.3 INCREASED DRAWDOWN CAUSED BY DEWATERING

Jacob suggested a correction to allow for dewatering which was presented in Section 9.4.3. The correction applies to drawdowns measured in observation bores during a pumping test. These drawdowns should be small in relation to the saturated thickness of the aquifer.

Jacob's correction to drawdown is:

$$s' = s - s^2/2b$$

where:

s' = the corrected drawdown.

s = the observed drawdown.

b = the aquifer thickness.

The corrected value is then used instead of the measured drawdown in the flow equations for confined aquifers.

A correction should also be applied to the storage coefficient which is obtained when using these equations. The correction to storage coefficient which has been applied after the correction to drawdown is as follows:

$$S = (b-s)/b \times S'$$

where:

S = the corrected value of Storage Coefficient.

b = aquifer thickness.

s = observed drawdown.

S^1 = apparent storage coefficient calculated using corrected drawdowns and transmissivity.

In many cases when pumping a bore the aquifer adjacent to the bore is dewatered causing a reduction in saturated thickness of the aquifer and hence a reduction in the apparent transmissivity at that point.

If the Modified Non-Steady State Flow Equations are used to analyse such a test the plot results in a curve which increases in slope as time progresses. This increase in slope is caused by a continuing reduction in aquifer thickness. A correction should be made to allow for its reduction if transmissivity is to be calculated from information from the discharging bore alone. The correction presented by Jacob (Section 9.2) would be a good first approximation. The corrected drawdowns should not be used when determining long term pumping rates.

9.4 ANOMALIES IN DRAWDOWN READINGS

The data being analysed in any pumping test are subject to human error. Anomalies being encountered in the data should be carefully weighed before attributing them to a boundary condition. If they are not consistent with other readings it is possible that they should be discarded. However, they should not be discarded too quickly or before an investigation is carried out as to why the anomaly has occurred.

9.5 PARTIAL PENETRATION

A bore whose length of water entry is less than the aquifer thickness which it penetrates is known as a partially penetrating bore. For such a bore the flow pattern differs from that for a fully penetrating bore with the same discharge because the average length of flow line is longer and so a greater resistance to flow occurs.

For practical purposes, this results in the following relationships between two similar bores, one partially and one fully penetrating the same aquifer.

If $Q_p = Q$, then, $s_{wp} > s_w$,

and

if $s_{wp} = s_w$, then, $Q_p < Q$

where:

Q = bore discharge for the fully penetrating bore.

s_w = drawdown in a fully penetrating bore.

s_{wp} = drawdown in a partially penetrating bore.

Q_p = discharge from a partially penetrating bore.

The alteration to the flow lines and so the magnitude of the drawdown applies not only to the bore but to conditions in the aquifer at a considerable distance from the bore, which will give erroneous values of storage coefficient in observation bores installed in this effected zone.

The drawdowns in observation bores inserted into the top and the bottom of the aquifer respectively and at the same distance from the discharging bore will be different if they are affected by partial penetration.

It has been found, however, that the effect of partial penetration is negligible beyond a distance of twice the saturated thickness from the bore. For this reason, observation bores which are to be used in the calculation of storage coefficient should not be located closer to the partially penetrating pumping bore than twice the saturated thickness of the aquifer.

The equations for determining the hydraulic characteristics of an aquifer are based upon the assumption that the discharging bore taps the full thickness; observed water level drawdowns in bores tapping less than the full thickness should be corrected before they are used in the equations. Equations and values are given by Jacob.

Such corrections generally are unnecessary if the observation bores are placed in pairs, one of each pair tapping the top part of the aquifer just below the cone of depression, and the other tapping the bottom part of the aquifer. The drawdowns in each bore of each pair (corrected by subtracting $s^2/2b$ if necessary) are plotted, then averaged graphically, not arithmetically (in a semi-logarithmic plot the arithmetic and graphical average is the same but this is not so in a logarithmic plot). It has been found that for relatively thin unconfined aquifers, it is convenient to place three pairs of observation bores at distances from the discharging bore of b, 2b, 4b.

It is necessary to adjust for partial penetration effects if type curve solutions on log-log paper or "distance-drawdown" curves on semi-logarithmic paper are to be used. In the case of "time-drawdown" curves on semi-logarithmic paper the correction is necessary for the calculation of S but, for a sufficiently long pumping period it may not be necessary in computing T.

9.6 ANTECEDENT CONDITIONS

The standing water level in a bore may vary prior to the commencement of a test. Such variations may be caused by tides, barometric pressure changes or other conditions. It is desirable to monitor the water level in the bore for a period prior to the commencement of the test and extend the variation trend through the test. If possible pre-test observations are compared with records of barometric pressure and, if applicable, tidal variations.

Correction of the data from a static or recovery test is generally necessary and occasions arise where other pumping test data has to be corrected for antecedent pumping conditions. The only time that the uncorrected data can be used is when the test follows a condition of a steady flow. The meaning of steady flow in this context is a flow condition which could not be expected to change by a measurable amount over a period equal to that covered by the test in question, if the test had not been carried out. This period includes the no flow condition.

Depending on the particular bore this would generally require that the flow of the bore had not been altered in any way for at least 24 hours prior to the new test. If there is any doubt as to whether any residual effect remains from earlier alteration in discharge, this can be checked quickly by evaluating the maximum correction, which will be required at the end of the test, to see if this is significant.

Corrections for variation in discharge are computed by use of the equation:

$$\delta s = \Delta Q b \log_{10} t_1/t_2$$

where:

δs = change in head caused by an increase (or decrease) in discharge of ΔQ between t_1 and t_2 after this increase (or decrease) commenced to operate.

b = constant ($\Delta s/Q$), a function of the transmissivity of the bore.

ΔQ = change in discharge rate.

An increase, or positive value of delta Q, produces an increase in head loss or a reduction in pressure while a decrease or negative value of delta Q produces a decrease in head loss or an increase in pressure. The effects of all previous increases or decreases in discharge can be added or subtracted to give the net effect at any given point of time.

To compute this net correction it is obviously necessary to know the value of b. However, an approximate value only is necessary and this can be obtained from the early portion of the test being analysed, before the effect of antecedent variations become apparent. If the value of b from the corrected data differs significantly from that used as a first approximation in computing the correction, this new value of b may be substituted in the correction equation as a second approximation. This is rarely necessary as quite a large change in b does not make a very large change in the correction.

If the test results are analysed using the Eden-Hazel method as presented in Section 7, the corrections for antecedent conditions are carried out automatically.

9.7 POSSIBLE DEVELOPMENT DURING PUMPING

Pumping is a form of mechanical development and it is possible that during a pumping test the bore is still developing. Smaller drawdowns will be recorded in the pumping bore during the latter stages of such a test than would be expected if the aquifer conditions immediately adjacent to the bore had remained constant throughout the test.

It is possible also that a collapse in the aquifer may have occurred during the pumping test and this would result in a much larger drawdown during the final stages of the test than that which would be expected. If this is the case then this final condition would control the future performance of the bore, unless the bore was brought back to the original condition. It would not be valid to discard this as being a rogue value and determine the equation to drawdown for the bore from all previous drawdowns.

Negative values of C, the non-linear head constant, would indicate development during the test.

On occasions, development during testing, or collapses in the aquifer during testing, have been attributed to other causes.

9.8 PROXIMITY OF BOUNDARIES

So far, the aquifers we have considered in flow equations have all been assumed to be of infinite areal extent. Some aquifers are sufficiently large to satisfy this assumption reasonably well, at least during, say, the period of a pumping test. Many aquifers have hydrologic boundaries, such as a nearby stream or lake, and (or) geologic boundaries, such as the relatively impermeable bedrock wall of an alluvium-filled valley, or a fault. Such boundaries, if close enough to a discharging bore, may invalidate the results obtained by use of the flow equations, unless suitable adjustments are made.

9.8.1 Method of Images

The method of images used for the solution of boundary problems in the theory of heat conduction in solids has been adapted to the solution of boundary problems in groundwater flow. In this method, imaginary bores or streams, referred to as images, are arbitrarily placed at proper locations to duplicate hydraulically the effect on groundwater flow caused by the real hydrologic or geologic boundary.

Following heat-flow terminology, a discharging image bore is regarded as a point sink, a recharging image bore as a point source; a discharging image stream (drain) is regarded as a line sink, a recharging image stream as a line source. By use of various combinations of such sinks and sources, corrections for almost every conceivable type and shape of boundary have been made so as to permit solution of the appropriate groundwater flow equation. We will take up a few of these; for others see Ferris (1959), Ferris and others (1962), Walton (1962), and Kruseman and De Ridder (1970).

9.8.1.1 Recharging Boundary

If a bore in an unconfined aquifer near a large perennial stream hydraulically connected to the aquifer is pumped, obviously the cone of depression cannot extend beyond the stream as the water level in such a stream remains relatively constant. Such a situation is shown in **Figure 9-1A** and **Figure 9-1B** (Ferris and others, 1962).

In **Figure 9-1A** the nearly straight partly penetrating perennial stream is assumed to be straight and fully penetrating and, hence, equivalent to a line source at constant head. The hydraulic counterpart in an infinite aquifer is shown in **Figure 9-1B**, where a recharging image bore, or point source, has been placed on a line connecting the real and image bores, at right angles to the stream, at the same distance, a, from the stream. The recharge and discharge rates, Q, are assumed equal. The resulting cone of depression (heavy solid line) is the algebraic sum of the cone of depression of the real bore (dashed line) and the cone of impression of the recharging image bore (dashed line) and the former intersects the level of the stream as it should. The flow net for these conditions is shown in **Figure 9-2** - note that if the image bore and its flow net are removed, the flow net on the left is that of the real bore obtaining water by induced infiltration from the streams.

In most field situations streams only partly penetrate the aquifer and the cone of depression may extend beneath and beyond the stream bed. Hence when analysing pumping test data, when the distance to the recharge boundary is determined, this distance may be only the effective distance to a fully penetrating stream which has the same hydraulic effect as the real stream.

One of the first applications of the image-bore theory to groundwater flow was made by Theis (1941), who developed an equation and presented a graph for computing the percentage of the water pumped from a bore near a stream that 1s diverted from the stream at a known distance from the well (see also Theis, 1963b, p. C101-C105, Theis and Conover, 1963, p. C106-C109).

In 1938 Theis (Wenzel and Sand, 1942, p. 45) developed a formula for determining the decline in artesian head at any distance from a drain (line sink) discharging water at a constant rate.

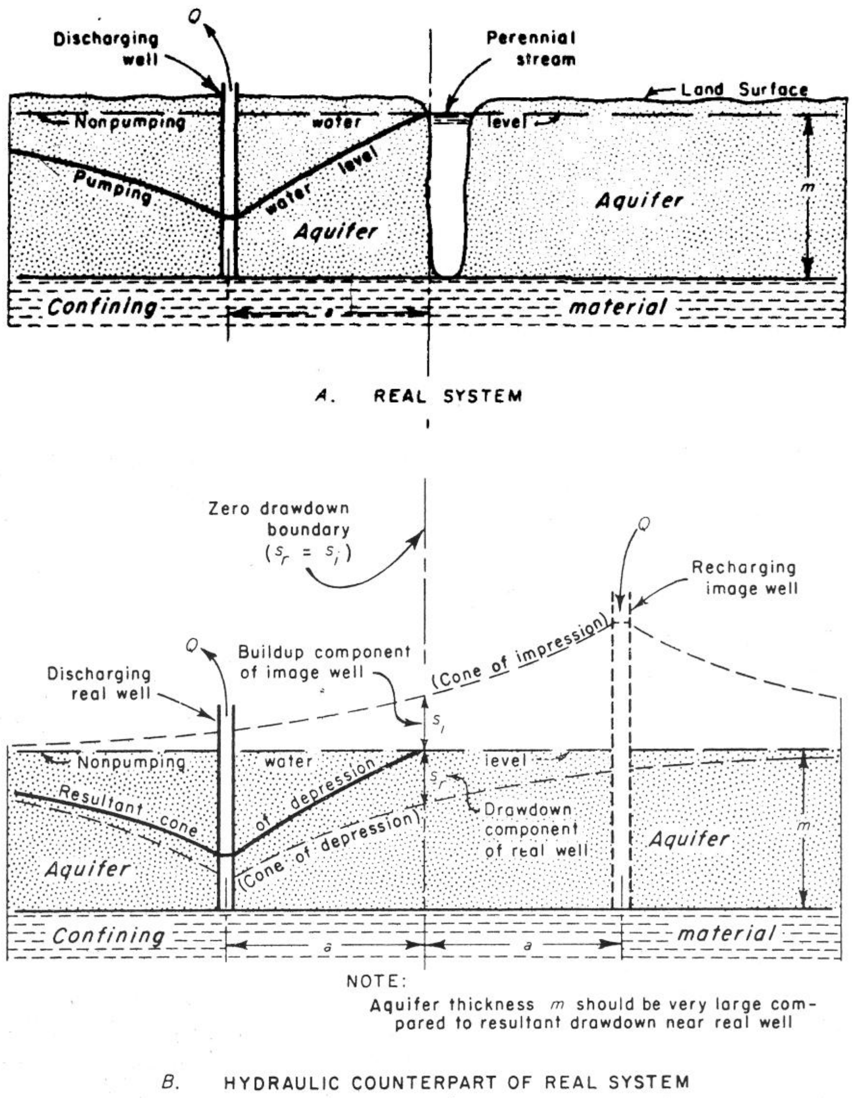


Figure 9-1 Idealised section views of a discharging well in a semi-infinite aquifer bounded by a perennial stream, and of the equivalent hydraulic system in an infinite aquifer

Ferris (1950) developed the same formula and gave data for plotting a type curve of his drain function, C(u) versus u^2.

Stallman (Ferris and others, 1962, p. 126-131) developed equations and a type curve for determining the decline in artesian head at any distance from a linear stream or drain of constant water level (head).

Jacob (1943) developed methods based upon an unconfined aquifer subject to a constant rate of recharge (W) and bounded by two parallel fully penetrating streams. From the shape of the water table, as determined from water-level measurements in bores, may be estimated the base flow (groundwater runoff) of the streams or the average rate of groundwater recharge.

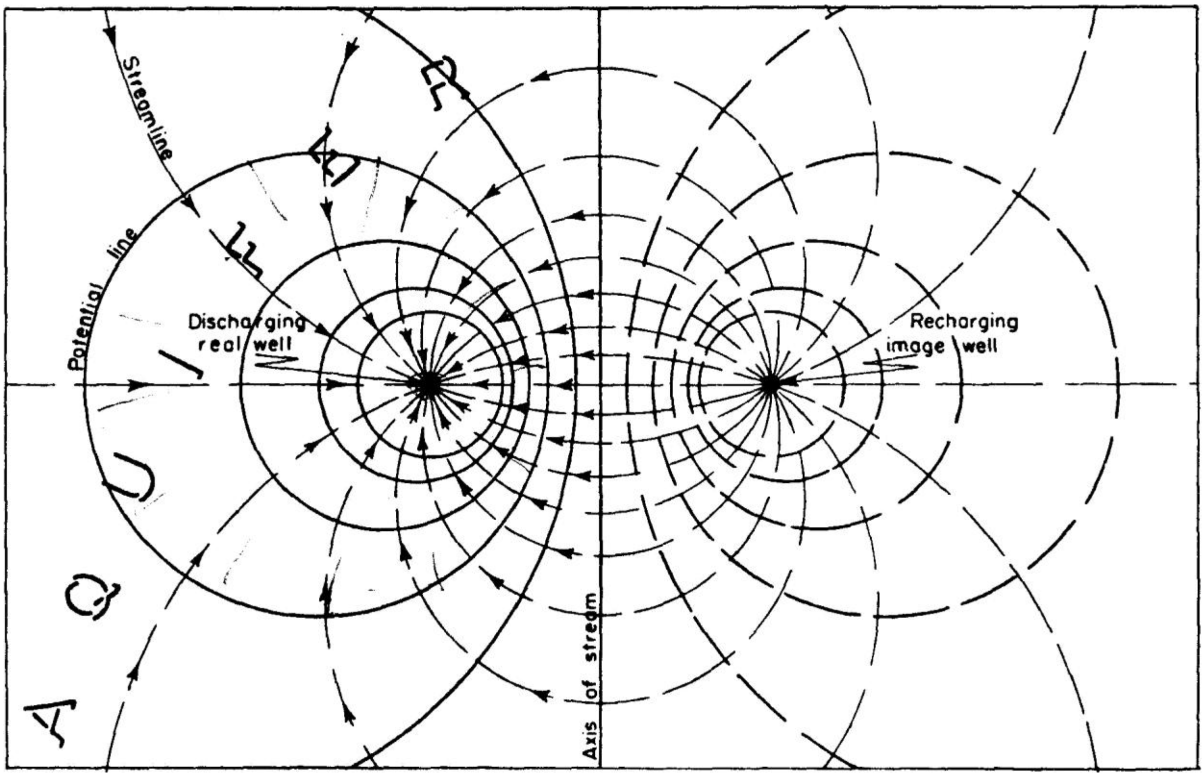


Figure 9-2 Generalised flow net showing stream lines and potential lines in the vicinity of a discharging well dependent upon induced infiltration from a nearby stream

Rorabaugh (1964) gave methods, equations, and charts for estimating the aquifer constant T/S from natural fluctuations of water levels in observation bores in finite aquifers having parallel boundaries. Examples of such aquifers are: a long island or peninsula, an aquifer bounded by parallel streams, and an aquifer bounded by a stream and a valley wall. For similar finite aquifers, Rorabaugh (1964) also developed methods for estimating groundwater outflow (as into streams) and for forecasting streamflow recession curves. The component of outflow related to bank storage 1s computed from river fluctuations; the component related to recharge from irrigation and precipitation is computed from water levels in a bore. Rorabaugh's methods should have widespread application 1n areas having the required boundary conditions.

9.8.1.2 "Impermeable" Barrier

Figure 9-3A below (Ferris and others, 1962, fig. 37) shows a discharging well 1n an aquifer bounded on the right by a barrier of relatively impermeable material. Hence it is assumed that no groundwater can flow across the barrier. The image system in the hydraulic counterpart which permits a solution of the real problem by use of the flow equations, is shown in **Figure 9-3B**. Here an image bore with discharge equal to that of the real bore is situated the same distance (a) from the barrier. The dashed theoretical cones of depression of the real and image bores intersect to form a groundwater divide at the barrier, across which no flow can take place, thus satisfying the conditions. The real resultant cone of depression (heavy line) is the algebraic sum of the dashed theoretical cones of depression. **Figure 9-4** depicts the flow nets of the two-bore system. If the image bore and flow net are removed, the flow net on the left is the one that would be observed for a discharging bore near the "impermeable" boundary.

In the case of two parallel impermeable boundaries (a channel aquifer) the effects of image bores will be cumulative and the number of image bores required is never ending. This results in a progressive steepening of the semi-logarithmic plot. Particular reference to a bore with two parallel boundaries can be found in U.S.G.S. Water Supply Paper I536-E p.p. 156, 157, and in Kruseman and De Ridder, 1970.

9.8.1.3 Drawdown Interference From Discharging Bores

Regardless of the extent of an aquifer, other discharging bores within the area of influence (circular area of cone of depression at radius where drawdown in an observation bore would be negligible at time t, say, <0.01 m), or within overlapping area of influence, constitute boundaries of the point-sink type. Likewise, a recharging bore would be a boundary of the point-source type.

If T and S are known, as from discharging bore tests, then the effect of one discharging bore upon a non-discharging bore is readily obtained by use of equation 6.18, equation 6.19 solved for $u = r^2S/4Tt$, and a table of W(u) v (u), for any distance r for known or assumed values of t and Q, or for any time t and for known or assumed values of r and Q. In this manner, families of semi-logarithmic curves showing drawdowns at various distances from the discharging bore for various discharge rates can be drawn.

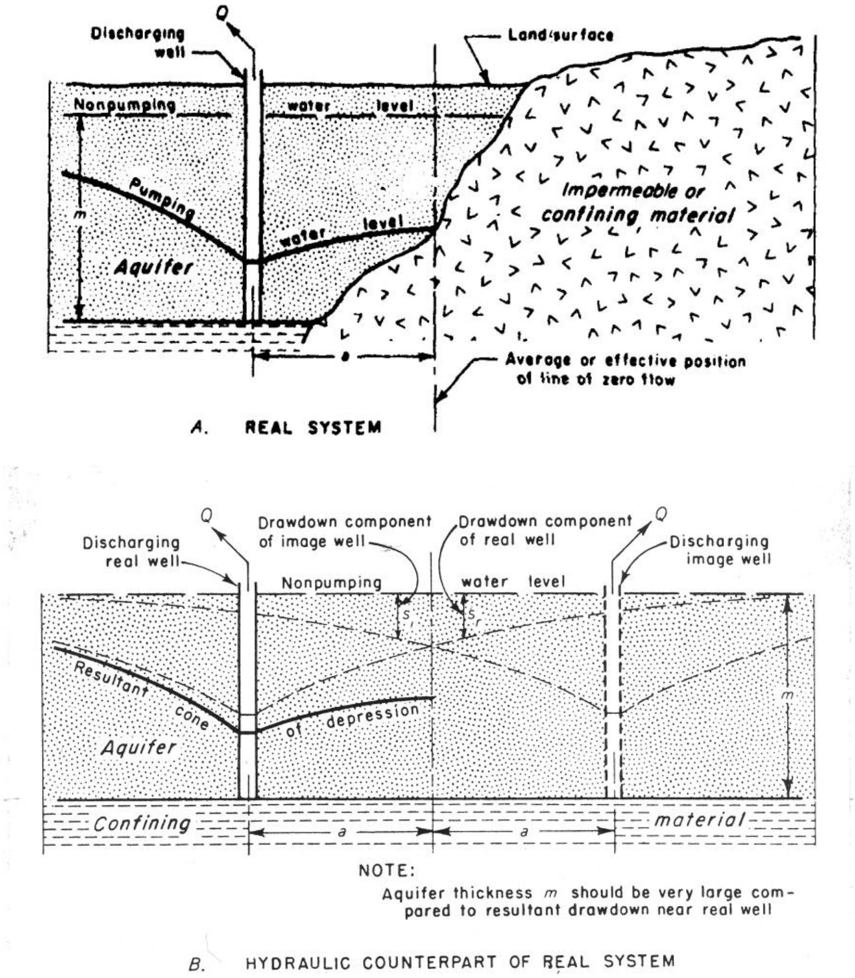


Figure 9-3 Idealised section views of a discharging bore in a semi-infinite aquifer bounded by an impermeable formation, and of the equivalent hydraulic system in an infinite aquifer

In preparing families of curves of this type, note that once the curves become straight lines, only a few points are required to define the straight parts. Theis (1963c, p. C10-C15) has prepared a simple chart for the computation of drawdowns in the vicinity of a discharging bore.

Where many discharging bores are mutually interfering with each other, the problem becomes much more complex and is best handled by an electric analog model or digital computer. If the aquifers are artesian, and have low values of T and S, the interference effects spread far 1n relatively short periods and increase relatively rapidly with time at a given distance.

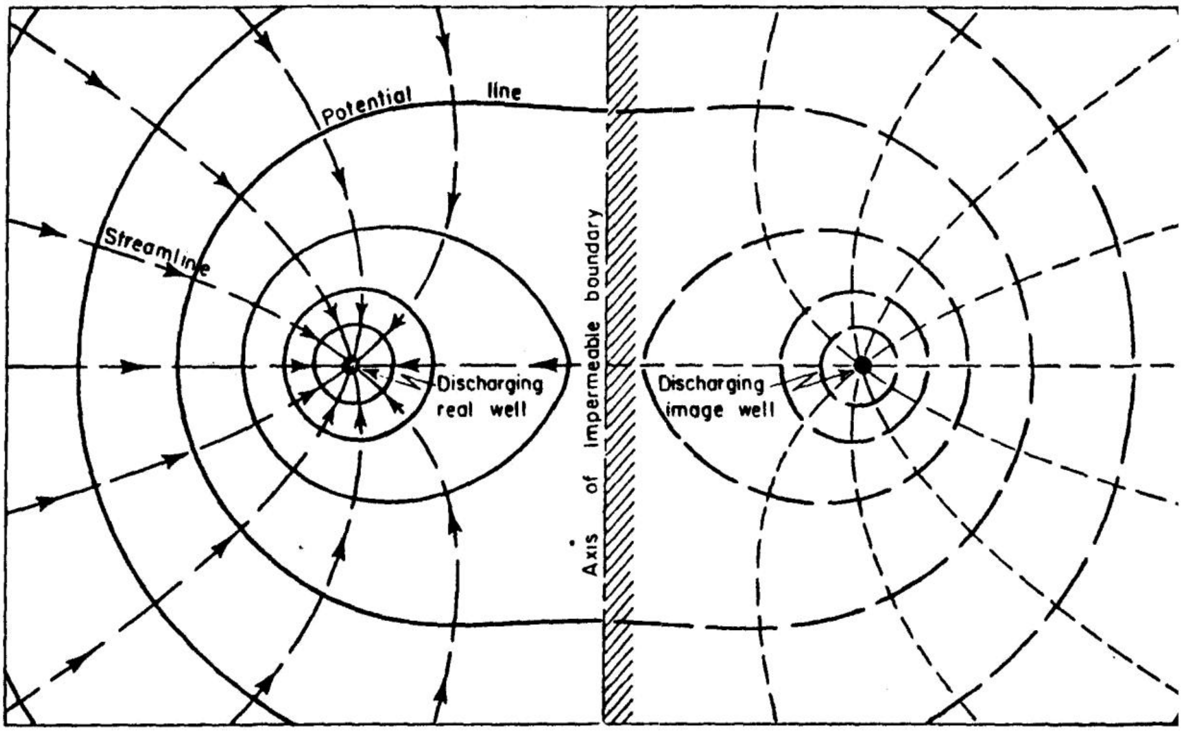


Figure 9-4 Generalised flow net showing stream lines and potential lines in the vicinity of a discharging well near an impermeable boundary

9.8.1.4 Application of Image-Bore Theory

Now that we have discussed several types of hydrogeologic boundaries and their simulation by the method of images, what do we do with them, and how do they affect the results of pumping tests? From the preceding discussions, it should be evident that for a discharging bore in an aquifer bounded by a relatively impermeable barrier, as in Fig. 9.4 a time versus drawdown plot for an observation bore near the pumped bore will be steepened (greater drawdown) at the value of t after the cone of depression reaches the barrier. Conversely, if the discharging bore were near a perennial stream, as in Fig. 9.2, the curve for an observation bore near the pumped bore would be flattened (less drawdown) at the value of t after the stream is reached. Computation of T and S by the Theis or similar equations would then be valid only for the early data before the boundary effects changed the slope of the curve. This would offer no problem for a confined aquifer; but, in an unconfined aquifer, sufficient time might not have elapsed to allow for reasonably complete drainage from storage, as discussed earlier.

Ferris and others (1962, p. 161, 162) described a method of plotting s versus t or r^2/t for matching with the Theis type curve that permits of a solution for T and S and also for the distance from the observation bore to the image bore. If the boundary is concealed, as a hidden fault, three or more observation bores are required to locate the boundary (see Ferris and others, 1962, p. 164-166; Moulder, 1963).

Rather than describe the somewhat laborious method referred to in the first part of the preceding paragraph, we will take up a much simpler method devised by Stallman (1963b), for the solution of single-boundary problems involving either a source or a sink. From **Figure 9-2** and **Figure 9-4**,it is evident that if s_0 is the drawdown is an observation bore, and s_p and s_i are the components of that drawdown caused, respectively, by the pumped bore and by the discharging or recharging image bore, then s_0 is the algebraic sum of s_p and s_i , or:

$$s_0 = s_p +/- s_i \qquad .....9.9$$

For this condition, equations 10.18 and 10.19 may be rewritten, respectively:

$$s_0 = \frac{Q}{4\pi T}(W(u)_p \pm W(u)_i) + \frac{Q}{4\pi T}(W(u)) \qquad .....9.10$$

and

$$u_p = r_p^2 S/4Tt, \text{ and } u_i = r_i^2\ S/4Tt \qquad .....9.11$$

where r_p is the distance from the pumped bore to the observation bore, and r_i is the distance from the observation bore to the image bore.

In equation 9.11, u_p and u_i are seen to be related thus:

$$u_i = (r_i/rp)^2 u_p, \text{ or } u_i = K_1^2 u_p \qquad .....9.12$$

where:

$$K_1 = r_i/r_p \qquad .....9.13$$

Note: the K_1 in equations 9.12 and 9.13 of Stallman is simply a constant of proportionality and is not to be confused with the K used for the hydrologic conductivity.

Stallman plotted a family of logarithmic type curves of W(u) versus 1/u for many values of his $K_1 = r_1/r_p$ as shown in **Figure 9-5**.

In this analysis the drawdown in the observation bore, s_0, is plotted against time since pumping started, t, on logarithmic graph paper. For an aquifer in which a single boundary is suspected, the plotted curve is superimposed on the family of type curves in **Figure 9-5**, and a match point found for values of ΣW(u) and i/u_p, and corresponding values of s_0 and t respectively, in the same way as matching the Theis type curve. Equation 9.10 can then be used to solve for T, after which equation 9.11 can be used to solve for S. (Note: By writing equations 9.10 and 9.11 for a particular curve that is followed by the observed data, the value of r_i can be calculated from equation 9.13).

If the suspected boundary is absent, and hence the aquifer is extensive, the observed data should follow the heavy parent type curve which is the Theis type curve. If a boundary exists, the observed data will follow the parent curve until the boundary is first "felt", then it will deviate from the parent curve along one of the modified curves. Deviations below the parent curve are caused by recharging images; those above by discharging images.

FIG. 9-6

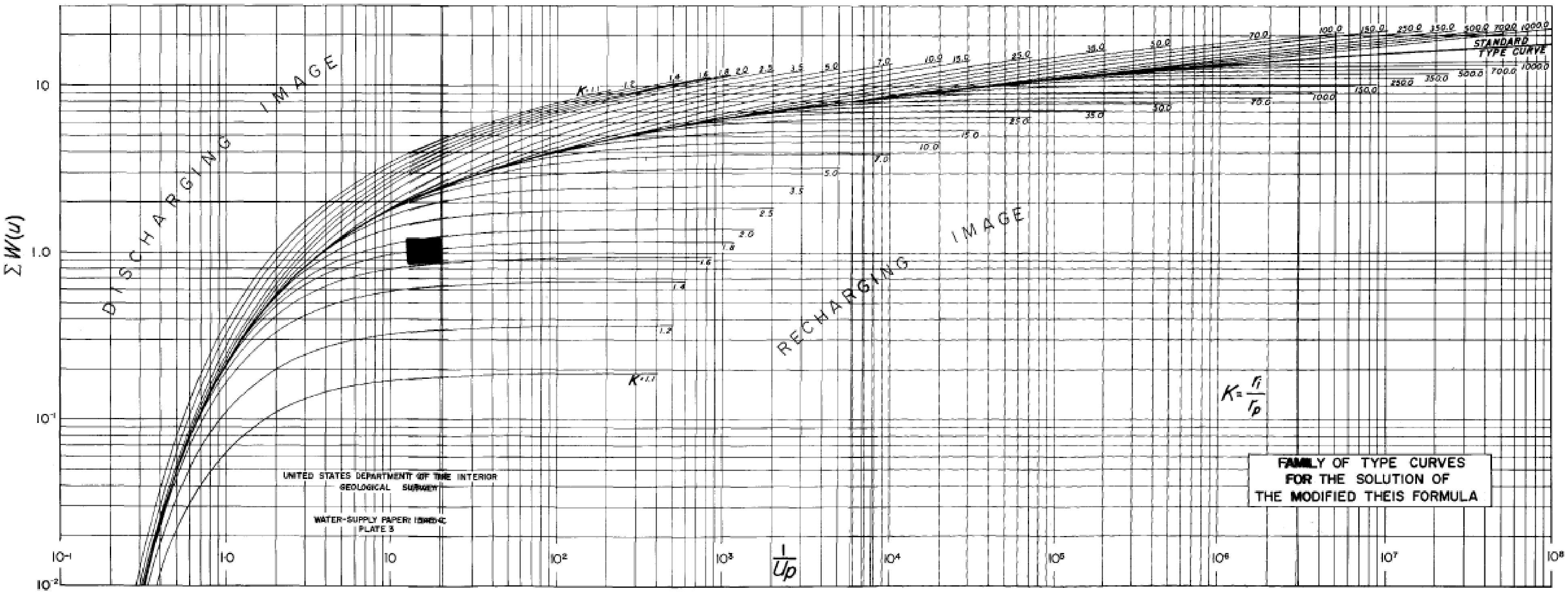


Figure 9-5 Family of type curves for the solution of the modified Theis formula

9.9 WATER TEMPERATURE VARIATIONS IN HOT BORES

By virtue of the aquifer depth the water in many flowing artesian bores is extremely hot. One bore at Birdsville in southwest Queensland is some 1,220 metres in depth and has a temperature of 99°c at the surface. The back pressure at the surface on this bore is some 1210 kPa with zero flow.

It will be appreciated that the density of the water at this very high temperature is much lower than what it would be at a lower temperature. With this lower density the head of water to maintain the pressure in the aquifer will be much higher.

If the flow from such a bore is stopped for any significant period then heat will be lost from the water to the surrounding ground, and the water cools. The most significant drop in temperature of course is near the surface of the bore with a less significant drop as the aquifer is approached.

With the lowering in temperature the density of the water increases and the apparent pressure at the surface decreases.

If a static test (recovery test) is carried out on such a bore then the pressure rises during the early stages of the test but the rate of rise does not conform with the theoretical rate as the water begins to cool. In fact in some cases the pressure may begin to drop. The analyst must be aware of such conditions and how to overcome them.

Unfortunately there is no hard and fast rule for correcting for this condition as yet but a rough approximation based solely on densities indicates that the difference in head for the Birdsville bore at 99°C and 37.8°C is some 30 metres. This assumed a linear variation in temperature from the surface to the aquifer which is unlikely to be correct.

9.10 VARIATIONS IN ATMOSPHERIC PRESSURE

A change in atmospheric pressure results in a change in the force applied to both the aquifer matrix and the water within the aquifer.

The effect of this change in applied force is zero in an unconfined aquifer and there is no change in water level. However, in a confined aquifer, while the competence of the overlying confining material is able to absorb the increase in load, such changes can and do have a significant impact on groundwater levels. The way in which the water level responds to changes in atmospheric pressure is directly related to the storage coefficient of the aquifer. The relationship between water level change and change barometric pressure is called the barometric efficiency and will be discussed in more detail in Section 8. The barometric efficiency can be used to determine storage coefficient.

Suffice to say at this stage that a drop in atmospheric pressure results in a rise in groundwater level in a confined aquifer. In the Great Artesian Basin, some bores which have recently stopped flowing will start to flow again if a low pressure event such as a cyclone passes over the area. Of course such a recommencement of flow will be temporary.

9.11 TIDAL EFFECTS

Tidal effects can influence water levels in confined aquifers. The effect of tides is the opposite from that of changes in atmospheric pressure. Rising pressure caused by higher tides result in rises in groundwater levels.

There are two types of tidal effects which need to be taken into account; ocean tides and earth tides. Both of these effects are sinusoidal in nature.

The rising and falling sea levels impose varying loads on the confining layer of the aquifer where it extends out under the sea. The change in groundwater level is greatest near the coast and attenuates as the distance from the coast increases.

Earth tides are a result of the gravitational attraction, predominantly between the moon and the earth. These gravitational forces act on the matrix of the aquifer alternately stretching it and compressing it. These changes in structure cause changes in groundwater levels.

9.12 OTHER FACTORS TO BE CONSIDERED

Other factors such as anisotropy, non-homogeneity and finite diameter of the facility should also be taken into account but will not be discussed here. Methods of allowing for anisotropy and finite diameter of the bore can be found in the references.

SECTION 10: APPLICATION OF AQUIFER PROPERTIES

10.1 INTRODUCTION

Knowledge of aquifer properties is a necessary pre-requisite for the solution of most groundwater problems. A few of the everyday problems are presented in this section together with an indication of how aquifer parameters are used to solve them.

10.2 VOLUME IN STORAGE

The total volume of water which is stored in a saturated material is given by:

$$V_T = V'_S\theta \quad .....10.1$$

where:

V_T = total volume of water stored.

V'_T = total saturated volume.

θ = porosity.

However, not all of this water is available for extraction. Some water is held by the soil matrix under gravity drainage, and additional water is unavailable as it is needed to provide the necessary gradient for groundwater flow. When considering the extractable volume of any groundwater basin some lower limit, or dead storage level, should be determined and only water stored above that level considered in the calculations.

On this basis the total extractable volume stored in a ground-water basin is given by:

$$V_E = V_S \text{ x } S_Y \quad .....10.2$$

where:

V_E = total extractable volume.

V_S = saturated volume above dead storage level.

S_Y = specific yield of the material.

The volume of saturated material can best be obtained by obtaining average saturated thicknesses throughout the area, normally by an examination of private facilities or investigation facilities, and multiplying this average figure by the total area being investigated. Shape factors of the basin may have to be taken into account.

Storage coefficient is not used to determine the total volume stored in an aquifer.

10.3 VOLUME REMOVED FROM STORAGE

In a case of a confined aquifer the net volume of water which has been removed from the aquifer may be obtained by multiplying the storage coefficient by the change in potentiometric head during that period. The total volume which has been removed, of course, will have to take into account any recharge effects which occurred during the period. If the water level fell below the top of the confined aquifer then the specific yield would have to be used for that period.

In the case of an unconfined, semi-unconfined, or semi-confined aquifer a drop in water level would normally be associated with drainage of some of the saturated material. In this case the net volume removed during a certain period is obtained by multiplying the specific yield of the dewatered material by the change in potentiometric head during that period.

$$\text{Volume removed from storage} = S\,\Delta h \quad .....10.3$$

where:

S = storage coefficient for a confined aquifer.

S = specific yield of dewatered material for any other type of aquifer.

Δh = change in head during the period considered.

If the volumes removed for irrigation, stock, industrial and own water supplies are negligible or are known from metering, then the volume removed by evapotranspiration can be computed.

It must be remembered that this is a net volume removed. To get the total volume removed any recharge events and down valley groundwater flow will have to be taken into account.

10.4 GROUNDWATER FLOW

The groundwater flow in any section of aquifer can is obtained by using the transmissivity, the hydraulic gradient, and the area through which the water is moving.

In the case of leaky aquifers the main transmission system is the aquifer proper even though the semi-pervious material which overlies the aquifer acts as a storage. The transmissivity which is obtained from pumping tests is invariably the transmissivity of the aquifer proper.

The groundwater flow through an aquifer is obtained from:

$$Q = TiW \qquad \text{.....10.4}$$

where:

Q = total groundwater flow through the section considered.

T = the average transmissivity in that section.

i = the hydraulic gradient (i.e. the slope of the potentiometric surface).

W = width of the section being considered.

The value of transmissivity varies with the saturated thickness in the alluvium and for this reason the transmissivity values may not be constant across a particular cross-action. In cases such as this the cross-section is divided into smaller sections and groundwater flow computed for each of these smaller sections. The total groundwater flow in the cross-section is then the sum of the flows in each of the smaller sections.

A special case of groundwater flow is the determination of flow into excavations which is important for the mining industry and for construction operation which involves excavation below the water table. This situation is covered briefly in section 10.7.

Total flow in an aquifer can also be obtained by utilising flow nets.

10.5 LEAKAGE

From the analysis of tests carried out on leaky aquifers the leakage coefficient, (1/c), can be determined.

From this, the rate of leakage from an overlying aquifer to a lower aquifer can be computed by using Darcy's Law.

Darcy's Law may be written as:

$$Q = -KiA$$

where:

K = the hydraulic conductivity.

i = the hydraulic gradient.

A = area through which the flow is occurring.

For vertical flow in the semi-confining layer or leaking layer Darcy's Law can be written as:

$$Q = -K^{|}\frac{s}{b^{|}}A \qquad \text{.....10.5}$$

where:

K^l= vertical hydraulic conductivity of the semi-pervious layer.

B^l= saturated thickness of the leaking layer.

S = drawdown in the main aquifer.

A = area through which leakage is occurring.

The leakage coefficient (K'/b1) is then the rate of flow from the leaking zone to the aquifer per unit drawdown per unit area.

If the average leakage coefficient is obtained for an area then for each drawdown in the main aquifer the volume of water which will migrate to the lower aquifer per unit time per unit area can be computed. Summing this over the total area the total volume which has migrated to the aquifer is able to be computed.

This is important if an unconfined aquifer overlies a leaky aquifer and is separated by a leaky material. The major recharge to the system could occur in the unconfined zone and migrate to the lower aquifer through this leaking zone.

By analysing the demands which are placed on the upper unconfined aquifer the area to be covered by a well field tapping the lower aquifer and still maintaining equilibrium with the natural recharge conditions can be determined.

10.6 DRAWDOWN INTERFERENCE EFFECTS

10.6.1 Drawdown Within the Area of Influence

Regardless of the extent of an aquifer, other discharging bores within the area of influence or within overlapping areas of influence, constitute boundaries, of the point-sink type. Likewise, a recharging bore would be a boundary of the point-source type. If T and S are known, from discharging bore tests, then the effect of one discharging bore upon a non-discharging bore is readily obtained by use of equation 6.18 and equation 6.19 (solved for $u = r^2S/4Tt$, and then calculate W(u) as the first two terms of the series i.e. $W(u) = -0.577216 - \log_e u$) or equation 6.24 and for any distance "r"< for known or assumed values of t and Q, or for any time "t" for known or assumed values of r and Q.

<u>Example</u>

A bore, in an aquifer with transmissivity of 400 m^2/day and a storage coefficient of 10^{-3}, is pumped at 1000 m^3/day. What is the drawdown in a bore 200m from the pumping bore after 1 day?

From equation 6.19:

$u = r^2S/4Tt$

= 0.025

$W(u) = (-0.577216 - \log_e u)$

= 3.112

From equation 6.18:

$$s = \frac{Q}{4\pi T} W(u)$$

= 0.619m

Or using equation 6.24:

$$s = \frac{2.3Q}{4\pi T} \log_{10} \frac{2.25Tt}{r^2 S}$$

= 0.619m

The drawdown at any point in an aquifer system is a summation of the drawdown effects from all discharging bores within that system.

10.6.2 Comparative Spread of Area of Influence

Lohman (1965, p. 109, 110) showed that:

$$r^2 = \frac{2.25Tt}{S10^{4\pi T/2.3Q)}}$$

$$= \frac{2.25Tt}{S10^{(s/\Delta s)}} \qquad .....10.6$$

For a given set of conditions, all terms except r and S may be considered constant; then, using C as a constant of proportionality:

$r^2 = C/S$

If we multiply both sides by π then:

$\pi r^2 = A = C'/S$ 10.7

Equation 10.7 may be used to compare the area of influence in a confined aquifer, A_1 having a storage coefficient of, say, 5 x 1O-5 with the area of influence in an unconfined aquifer, A_2 having a specific yield of, say, 0.20. Assuming that T, Q, and s are the same for both aquifers, and that t also is the same and long enough that u ≤ 0.05 & that the cone of depression in the unconfined aquifer has had time to be drained, then:

$$\frac{A_1}{A_2} = \frac{C'/5*10^{-5}}{C'/0.2} = \frac{2*10^4}{5} = 4*10^3$$

Thus under the assumed conditions, the area of influence in the confined aquifer is 4,000 times larger than that in the unconfined aquifer, or, the ratio of the radius extending to the circumference of negligible drawdown in the confined aquifer, to that in the unconfined aquifer, is:

$$\frac{\pi r_1^2}{\pi r_2^2} = 4*10^3$$

whence:

$$\frac{r_1}{r_2} = \sqrt{4*10^3} = 63.2$$

Thus, changes in artesian head or pressure in a confined aquifer spread outward very readily from the discharging bore, although it requires time for the drawdown to be measurable at a given distance from a discharging bore. By contrast, in an unconfined aquifer, changes in water level occur very slowly as gravity drainage takes place, so the cone of depression enlarges very slowly.

10.6.3 Determination of Radius of Influence

If it is assumed that we are dealing with an aquifer in porous media and that the assumptions for the modified non-steady state equations apply then the drawdown at any point within the area of influence at a given time t is given by:

$$s = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{r^2 S}$$

The radius of influence at any time t is the radius at which the drawdown is zero.

For this to apply the log term must be zero i.e. the value within the log term must equal 1.

Thus at the radius of influence:

$$r_0 = 1.5\sqrt{\frac{Tt}{S}} \qquad .....10.8$$

where:

r_0 = radius of influence at time t after pumping commenced.

T = transmissivity.

T = time since pumping commenced.

S = storage coefficient or specific yield, depending on the aquifer type.

From the above it can be seen that the radius depends only on time transmissivity and storage coefficient and is completely independent of discharge rate. The discharge rate determines the magnitude of the drawdown within the cone of depression but not the areal extent.

This is a very simple means of determining the radius of influence and, if used with care and common sense, can be used to provide a reasonable approximation even for those aquifers which are not in porous media.

10.7 DRAINAGE PROBLEMS

Water tables near ground surface can be controlled by construction of drains or bores to maintain level at or below specified depths.

Drains are designed in several ways. Some are composed of coarse sand or gravel so that their permeability is higher than the surrounding porous media. Water flows readily through them and they serve as outlets for draining surrounding groundwater. Horizontal lines of open jointed tile or perforated pipe are widely employed for drains. Other drains are simply open ditches which intercept ground-water whenever the water table rises above the bottom of the ditch.

Drains have many applications, only a few of which can be mentioned here. An earth dam usually contains drains near its toe to prevent saturation of the downstream face. Most foundations of structures contain drains around their perimeters to reduce hydrostatic pressure or water entrance. Modern highways often contain sub-drains to avoid saturation of highway grade. On agricultural lands, adequate drainage systems are essential for stabilising water tables below the root zone. High water tables may result naturally in flat lands bordering rivers, lakes, or the ocean, or may be produced artificially by percolation from excess irrigation water.

To regulate water levels within narrow limits over a large area, drains are made in parallel lines at depths and spacing governed by local crop and soil or aquifer conditions.

The design and construction of drains for controlling ground-later levels will not be covered in this text. Basic principles involved, however, are the same as those described in preceding pages.

Pumping bores also may control water levels, the process being identical to bores providing water supplies. Where drainage bores are employed on agricultural lands, the extracted water may be reapplied to the land for irrigation or may be wasted depending on the salinity of the water. A battery of spears, i.e., a line of small diameter bores, is often installed for dewatering construction sites. Relief bores are placed near the toes of dams and levees to lower water tables and thereby reduce uplift pressures produced by seepage under the structure.

10.7.1 Mine Dewatering

Mine dewatering is a special case of drainage and is not covered fully in these notes. However, there are applications of hydraulic parameters which can be of assistance in many cases and some of these are given below.

There are two main types of mining operations namely:

- open cut mining; and
- underground mining.

In the case of open cut mining all aquifers overlying the material being mined are intersected and inflows from all aquifers have to be taken into account in dewatering calculations.

However, in many underground mining operations it is not necessary to remove all water overlying the zone being mined. For instance a coal seam may need to be dewatered to enable safe and economic working conditions. The coal seam may be overlain by other shallower aquifers which are not hydraulically connected to the coal seam itself. It is not necessary to dewater these overlying aquifers as long as a connection does not exist.

The removal of water from the seam itself involves two separate processes:

- depressurising the seam by removing water from elastic storage; and
- dewatering the seam by draining water from the pores of the matrix.

The first process involves the use of the storage coefficient and the second involves the use of the specific yield.

If there is no hydraulic connection between the seam being mined and the overlying aquifers then both of these processes can be carried out without affecting the water levels in the overlying aquifers.

In isotropic aquifers it is possible to calculate discharge and drawdown for individual bores, multiple bores (borefields), open pits and strip mines using simple analytical methods. Some of these methods are presented below.

10.7.1.1 Single Bore Dewatering System

On some occasions it is possible to dewater a working area by pumping from a single bore. If the bore is deep enough, the pumping rate is high enough and the transmissivity and storage coefficient are suitable then the cone of depression may be such that the water level will be drawn down below the bottom of the working area. A pumping test should be carried out on the bore to determine the aquifer parameters and assess its long term pumping capacity. Since dewatering is normally a long term process it is reasonable to assume that steady state conditions can be approximated. However, it would be prudent to find out at what time after pumping began that steady state conditions would be approached. This can be done by either by plotting the time-drawdown on natural scale and selecting a time when the rate of drawdown increase appears to be very small or by plotting time-drawdown on a semi-logarithmic scale and choosing a time from that. Under equilibrium conditions the rate at which a bore will have to pump to produce a certain drawdown is given by equations 10.9 to 10.11 for unconfined and confined aquifers respectively:

unconfined aquifer:

$$Q = \frac{\pi K (H^2 - h_w{}^2)}{2.3 \log R / r_w} \qquad .....10.9$$

confined aquifer:

$$h = \sqrt{(H^2 - \frac{2.3 Q \log R / r}{\pi K})} \qquad ....10.10$$

$$Q = \frac{2\pi K b (H - h)}{2.3 \log r_0 / r_w} \qquad ....10.11$$

$$s = \frac{2.3 Q \log r_0 / r}{2\pi K b} \qquad ....10.12$$

where:

Q = discharge rate (m3/day).

K = hydraulic Conductivity (m/day).

H = potentiometric head in bore before pumping (m).

h_w = potentiometric head in bore during pumping (m).

h = potentiometric head at distance r from the bore (m).

r_0 = radius of cone of depression (radius of influence) (m).

r_w = effective radius of bore (m).

r = radial distance from bore (m).

s = drawdown at distance r from the bore (m).

10.7.1.2 Equivalent Bores

Equations 10.9 to 10.12 for individual bores can on occasions be applied to circular and some rectangular arrays of bores around a similar shaped area to be dewatered. On such occasions, the arrays of bores can be replaced by one bore (or equivalent circular pit) with a radius equivalent to the effective radius of the array and having an equivalent drawdown effect similar to the array of bores. This equivalent radius is then used in the equations 10.9 to 10.12. The effective radius r_e is the radius of a circle having the same area as the base of the rectangle.

If the rectangular array is such that the base length of the pit (y) is ≤ 1.5 times the base width (x) the pit can be regarded as an equivalent bore with an equivalent bore radius of $re = \sqrt{(xy/\pi)}$.

For cases where (y) > 1.5 (x) the flow can be treated as parallel flow through 2 sides and 2 ends plus radial flow at each corner. This case is considered in Strip Pits below.

10.7.1.3 Multiple Bores - Bore Fields

In most cases a single bore will not be sufficient to provide adequate dewatering and a number of bores will be required around the working area. The cones of depression from each of the bores will intersect and the drawdown at any point within the area of influence will be the sum of the individual drawdowns from each bore.

The first step in this process is to identify a number of control points within the working area and the drawdown necessary at each of those points to dewater the working area.

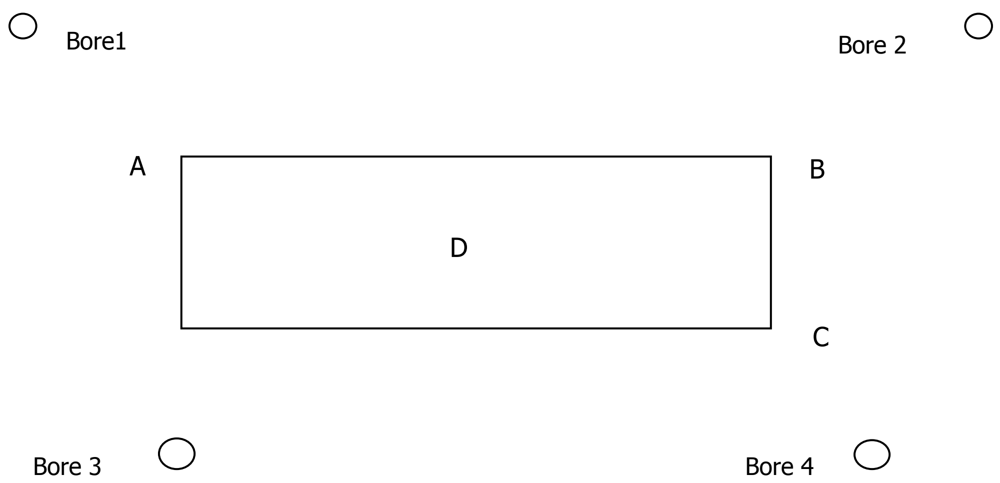
Next the number, location and pumping rates for each of the dewatering bores have to be determined. The locations of the bores may be influenced by physical features such as roads or buildings. It is prudent to construct one bore and carry out a pumping test to calculate the transmissivity and storage coefficient for the area as well as a range of possible pumping rates for the bore before attempting to determine the number and locations of the remaining bores.

Once these parameters have been obtained the locations of the bores can be set and drawdowns calculated at each of the control points. Once again equations 10.9 to 10.12 can be used to calculate rates and drawdowns. A spreadsheet can be used to prepare a table such as **Table 10-1**. The spreadsheet enables various combinations of the number of bores, bore locations and individual pumping rates to be trialled before the final bore installation begins. Using different combinations, the drawdowns at each of the control points are determined and checked against the design requirements.

<u>Example of borefield design</u>

To enable safe excavation at the site below the groundwater levels at points A, B, C and D need to be drawn down 15m below the present standing water level when excavation begins in 4 months time. This will be achieved by pumping from bores 1, 2, 3 and 4. At what rates do the bores have to be pumped to achieve the required water levels?

The transmissivity of the aquifer being dewatered is 100 m^2/day and the storage coefficient is 0.01.



The drawdown "s_w" at distance "r" from a pumping bore at time "t" after pumping commenced at discharge rate "Q" is given by:

$$s_w = \frac{2.3Q}{4\pi s}(\log_{10}\frac{2.25Tt}{r^2 S})$$

The drawdown at any point in the aquifer is the summation of the individual drawdowns resulting from each pumping bore.

The pumping bores must of course be drilled deep enough to enable such drawdowns to occur.

Table 10-1 is a spreadsheet showing the design of the borefield.

Table 10-1 Drawdowns at control points during dewatering

Pumping Bore	Pumping Rate (m^3/day)	Control Point A		Control Point B		Control Point C		Control Point D	
		Distance From Pumping Bore (m)	Drawdown (m)	Distance From Pumping Bore (m)	Drawdown (m)	Distance From Pumping Bore (m)	Drawdown (m)	Distance From Pumping Bore (m)	Drawdown (m)
1	1300	35	5.58	122	2.99	134	2.80	87	3.69
2	1600	146	3.23	55	5.71	77	4.86	108	4.00
3	1200	59	4.15	117	2.84	105	3.05	63	4.03
4	1100	130	2.42	63	3.69	34	4.77	80	3.27
Total Drawdown			15.38		15.23		15.48		14.99
Design Drawdown			15.00		15.00		15.00		15.00

Occasionally it is necessary to dewater a working area while still maintaining the groundwater levels on one or more zones outside of the working area. One way of achieving this is to pump water from the dewatering bores and then recharge the water back into the aquifer where required. This technique is illustrated in **Figure 10-1**.

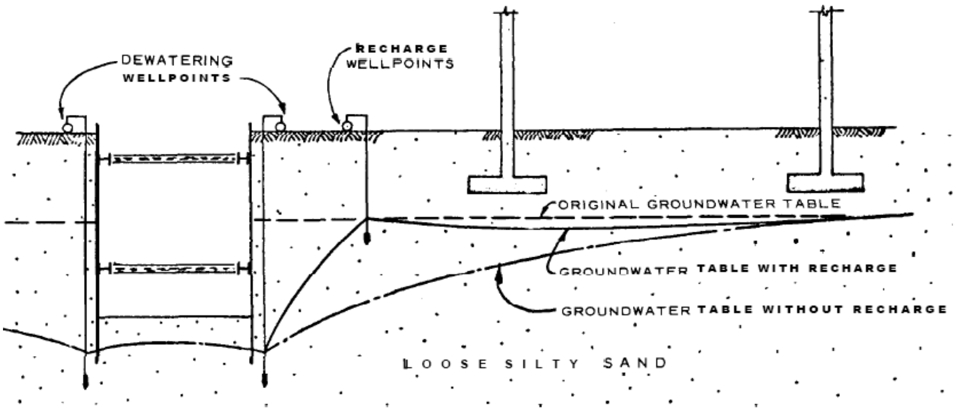


Figure 10-1 Recharging pumped water to maintain water levels in sensitive areas

10.7.1.4 Strip Pits

In strip pits (such as open cut mines) which intersect aquifers, water flows into the pit as parallel flow through each of the four sides and as radial flow through the corners as shown in **Figure 10-2**. This model is equivalent to the discharge from one bore, with a quarter of the bore at each corner of the pit, and parallel flow through two sides and two ends. Equations 10.13 and 10.14 give the relevant equations for this model for the unconfined and confined aquifer situations.

Figure 10-3 illustrates flow into a strip pit which intersects more than one aquifer. It highlights the fact that, regardless of depth of the pit, the maximum drawdown that can occur in any aquifer is to the bottom of that aquifer. In the multi aquifer case each aquifer must be treated separately.

At any specified time t the lateral extent of the parallel flow can be considered to be equal to the radius of influence of the bore quadrant located at each corner. Using the Modified Non-Steady State flow equations, the radius of influence is defined as that distance where the drawdown is zero.

The radius of influence is determined by equation 10.9 as $r = 1.5 \sqrt{Tt/S}$.

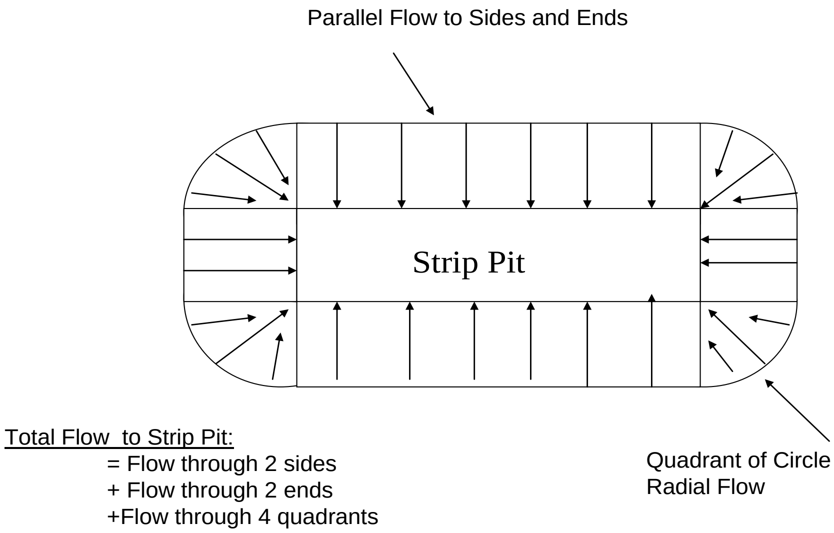


Figure 10-2 Groundwater flow into a strip pit

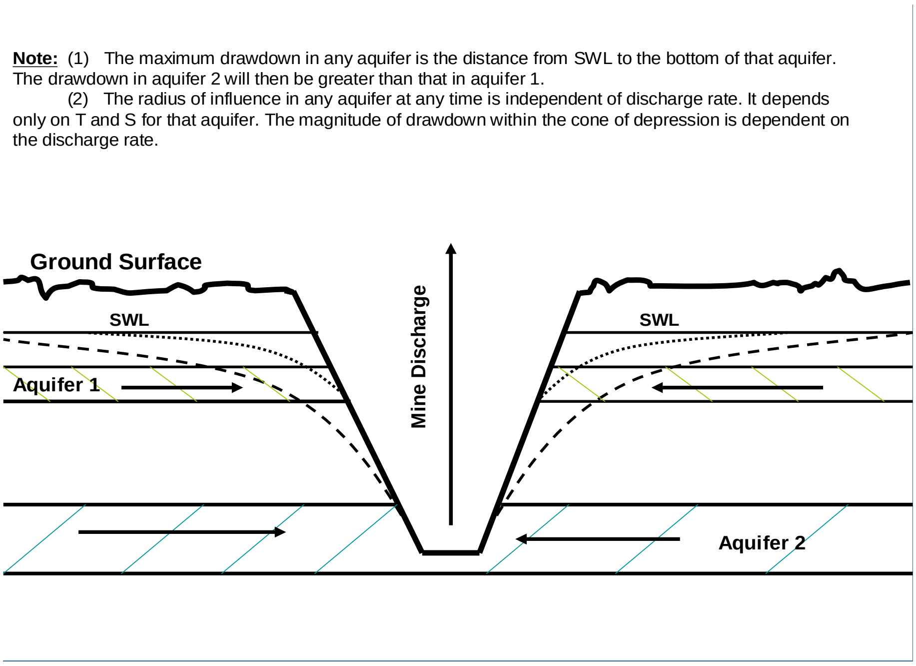


Figure 10-3 Multiple aquifer flow into a strip pit

Knowing the radius of influence at time t and knowing the drawdown in aquifer at the pit face at the same time it is possible to determine the drawdown at any distance r from the pit at that time. This can be done using the equations or simply by using semi logarithmic paper and plotting drawdown against logarithm of distance. A straight line joining drawdown in the pit (plotted as 1 metre from the side of the pit) and zero drawdown at the radius of influence. It is important to remember that the water level in an aquifer cannot be drawn down below the base of the aquifer. Hence, in the case of multiple aquifers the maximum drawdown in each aquifer is determined by the base of the aquifer and not necessarily to the water level in the pit itself.

Equations 10.13 and 10.14 can be used to calculate the flow into the pit:

unconfined aquifer:

$$Q = \frac{\pi K(H^2 - h^2)}{2.3\log(r_0 / r_w)} + 2\frac{(x+y)K(H^2 - h^2)}{2L_0} \qquad \text{....10.13}$$

confined aquifer:

$$Q = \frac{2\pi K(H - h)}{2.3\log(r_0 / r_w)} + 2\frac{(x+y)Kb(H - h)}{L_0} \qquad \text{....10.14}$$

where:

Q = discharge rate (m^3/day).

K = hydraulic Conductivity (m/day).

H = potentiometric head in pit before pumping (m).

h_w = potentiometric head in pit during pumping (m).

h = potentiometric head at distance r from the pit (m).

r_0 = radius of cone of depression (radius of influence) (m).

r_w = effective radius of bore (say 1m) at corner quadrant (m).

r = radial distance from pit (m).

L_0 = maximum length of drawdown influence from side of pit (use r_0) (m).

s = drawdown at distance r from the pit (m).

x = width of base of pit (m).

y = length of base of pit (m).

If the pit is of constant depth during its life then the drawdown in the pit will remain constant and the inflow rate will decline in the same manner as for a constant drawdown pumping test. However, in mining practice, while the pit depth does remain relatively constant for long periods, the pit area continues to expand. This expansion continues to intercept more and more aquifer and new drawdown conditions have to be taken into account. This can be done in discrete time steps and does not have to be a precise and continuous operation.

SECTION 11: GROUNDWATER MANAGEMENT

11.1 GROUNDWATER YIELD ANALYSIS

11.1.1 Bore Yields

Quite frequently the objective of carrying out a pumping test on a particular bore is to determine a long term pumping rate at which the bore can be equipped for irrigation, town water supply or some other purpose.

The following paragraphs give some indication of how such a rate can be estimated and some of the pitfalls to be avoided when such estimates are made.

11.1.1.1 Constant Discharge Pumping Test

The constant discharge pumping test with recovery is suitable for the estimation of long term pumping rates which of the same magnitude of the test rate. The method of analysis uses the Modified Non-Steady State flow equations.

The following relates to the practical problems of estimating the long term pumping rate of a particular bore using data obtained from one or more tests of this type- data being available only from the pumping bore. The main assumption in the analysis is that the bore will continue to perform, under sustained pumping, in the same manner as indicated during the test.

Procedure

1. The drawdown-time data are plotted on semi-logarithmic graph paper with time on the logarithmic scale. It is desirable to show the measured discharge rates at the relevant times on this plot.
2. The recovery-time, (or residual drawdown-t/t′), data are plotted on the same sheet to the same scale.
3. The depth to pump suction is decided upon. Normally the pump suction is placed just above the seal on the screen assembly.
4. The maximum available drawdown is obtained by subtracting the depth to standing water level from the depth to pump suction.
5. The working drawdown is obtained by subtracting from the maximum available drawdown an allowance for anticipated drop in standing water level as a result of seasonal conditions and other reasons.
6. By examination and computation from (i) and (ii) the rate which will give a drawdown after 100,000 mins (70 days approximately), or any other predetermined period of anticipated demand, which is equal to the working drawdown is called the long term pumping rate.

At first the method may seem relatively simple but it is beset with certain difficulties and in most cases a certain amount of judgement has to be applied.

Step 6 is of course an important step and some explanation is necessary.

Assume an ideal case where, after an initial curved portion, the drawdown-log time curve becomes a straight line. If the discharge rate is held constant and there are no hydrologic boundaries, this straight line should extend to the end of the test. It can be extrapolated also to any time desired, in this case 100,000 minutes has been adopted. In the case of a town water supply, or any other situation where very long term continuous pumping is required, a period of 1,000,000 minutes i.e. 2 years, would be a better figure. In systems of very high storage/flow ratio a longer period may be warranted e.g. a bore of relatively low transmissivity being used for a town water supply in the Great Artesian Basin.

The long term pumping rate is obtained by multiplying the test rate by the ratio of the working drawdown to the extrapolated drawdown at 100,000 minutes during the test.

$$Q_L = Q_T \frac{s_w}{s_{10^5}} \qquad .....11.1$$

where:

Q_L = long term pumping rate.

Q_T = test rate.

s_w = working drawdown.

$s_{10}{}^5$ = test drawdown (extrapolated) to 10^5 minutes, or to the time required.

This rule should be applied with caution particularly in those cases where non-linear head losses are present. It assumes, if non-linear head loss is present, that it is proportional to the discharge whereas in fact the non-linear head loss is proportional to the discharge squared. In general it is considered most unwise to predict a long term pumping rate beyond the constant discharge test rate unless you are very familiar with the conditions in the area and the efficiency of the contractor.

The straight comparison of drawdowns to discharge is most useful when "ironing out" variations in drawdown caused by relatively small variations in pumping rate during a pumping test.

The figure of 100,000 minutes is more or less an arbitrary end point which seems reasonably convenient to use. The previous period is too short a time and the subsequent log period may be too long a time for a normal irrigation cycle. A longer period is used of course for Town Water Supply or industrial requirements.

<u>Example</u>

Table 11-1 Pumping test data

Time (mins)	Drawdown (m)	Discharge Rate (m^3/day)
1	2.07	2860
2	2.11	
3	2.14	
4	2.16	
5	2.18	2860
10	2.22	
20	2.28	
40	2.33	
60	2.37	2860
80	2.39	
100	2.41	
120	2.42	
150	2.44	2860
180	2.45	2860
210	2.47	
240	2.48	2860
270	2.49	
300	2.50	
330	2.51	2860
360	2.52	

Table 11-1 presents test data obtained by the Queensland Irrigation and Water Supply Commission in 1962.

The bore is located at Biloela in the Callide Valley of Central Queensland.

The semi-logarithmic plot is shown in **Figure 11-1** and the analysis is given below.

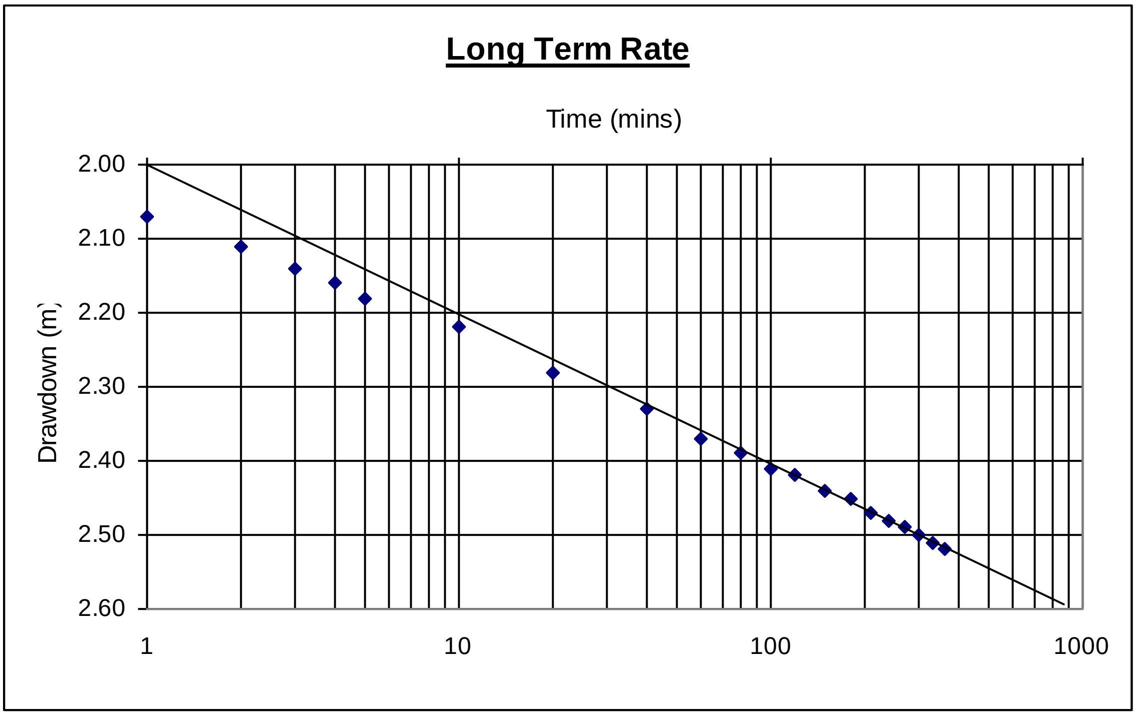


Figure 11-1 Constant discharge test – Callide Valley

Analysis

Long Term Pumping Rate

Date Tested	August 1962
Top of Packer	15.5 m BGL
Place Suction 0.15 m above top of packer	15.35 m BGL
Standing Water Level (SWL)	9.95 m BGL
Available Drawdown	5.40 m
Seasonal variation in SWL	2 m
Working Drawdown	3.40 m
At 2860 m^3/day, Δs	0.202 m
Drawdown at 1,000 minutes	2.61 m
Drawdown at 100,000 minutes	2.61 + 2 x Δs = 3.014 m
Estimated Long Term Pumping Rate	(3.40/3.014) x 2860 m^3/day
	= 3220 m^3/day
	= 37.2 litres / sec

Transmissivity

$T = 2.3\ Q/4\pi\Delta s$

$= 260\ m^2/day$

11.1.1.2 Variable Discharge Test

If the long term pumping rate for the bore seems to be much greater than the pumping ability of the test pump then it is desirable to carry out a variable discharge pumping test to enable the non-linear head losses to be determined.

If a variable discharge test, such as a step drawdown test, is carried out then it is possible to determine the equation to drawdown for the bore for a certain period of discharge. The Eden-Hazel method of analysis is given in Section 7.

Once the equation to drawdown is determined the long term pumping rate can be calculated.

It is desirable to plot the equation to drawdown, at the time corresponding to the required pumping duration, in the form of drawdown versus discharge on natural scale graph paper. If a non-linear loss is present this will result in a curved line.

The working drawdown is determined in the same way as described in the previous section.

working drawdown = depth to pump suction - standing water level - seasonal variation

The working drawdown is then read into the curve of drawdown versus discharge and the rate corresponding to the working drawdown is the long term pumping rate.

The determination of long term pumping rates by using variable discharge tests is desirable, as it accounts for non-linear head loss. This enables the analyst to estimate with some degree of confidence, long term rates in excess of the test rate.

11.1.2 Aquifer Yields

11.1.2.1 General

Before a dam is constructed, the hydrology of the stream and the losses from the dam must be evaluated if the long availability of water from the dam is to be determined.

In like manner the hydrology of any groundwater basin must be thoroughly understood before the sustainable yield from the basin can be determined. This sustainable yield has had many names in past years. Terms such as "Safe Yield" and "Annual Yield" are a couple that have been used.

In 1967 Stan Lohman, from the United States Geological Survey, was invited to be the principal lecturer at the Groundwater School which was run under the auspices of the Australian Water Resources Council. During his lectures he defined "Safe Yield" as "the amount of water one can withdraw without getting into trouble". Withdrawal may mean from flowing or pumped bores, and may mean continuously, as for industrial or municipal supplies, or seasonally, as for irrigation. Trouble may mean anything under the sun, such as

1. running out of water;
2. drawing in salt water;
3. getting shot, or shot at, by irate bore owner or land owner;
4. getting sued by less irate neighbour; or
5. getting sued for depleting the flow of a nearby stream for which the water rights have been appropriated, or over-appropriated.

Lohman's definition may sound facetious to some, but remembering that he would not attempt to put a number on it before development or in the early stages of development, especially if he did not know where and how the withdrawal would be made, it actually makes a lot of sense. David Todd defines it as "the amount of water which can be withdrawn annually without producing an undesired result".

Occasions arise when the volume taken from storage far exceeds the sustainable yield of the basin. In many circumstances the use has to be controlled or reduced or surface water used in conjunction with the groundwater or the groundwater recharged by artificial means. Sewage effluent could be reused by pumping back into the aquifers after treatment or poor quality water in some aquifers could be mixed with better quality water in other parts of the aquifer to give usable water in larger quantities.

There some who believe that a figure should not be put on sustainable yield. However, when dealing with practical water supply problems it is necessary to know how much water can be withdrawn annually with some reasonable assurance of being recharged over a specified period of time.

There many ways to evaluate the sustainable yield of an aquifer but I will present only one and even it must be considered to be approximate since aquifer recharge is literally as variable as the weather.

11.1.2.2 Evaluation

A realistic but conservative approach is to examine the records available for water levels, rainfall, and streamflow and determine what the "critical period" is for the area and examine the aquifer performance over that period. The "critical period" for an aquifer could be taken as the maximum period that the water level has been below the full supply level. The full supply level determined may be an arbitrarily defined level.

The lower level to which the water level is to be allowed to fall should also be determined. This is called the dead storage level. The sustainable yield can then be determined by a water balance approach.

In order to be able to evaluate and operate this water balance it is necessary to have accurate records of rainfall, water level measurements, volumes used (including irrigation, town water supply, industrial use, stock and domestic use and evapotranspiration) for the period being analysed. It is also important to know other groundwater requirements such as environmental requirements, groundwater dependent ecosystems and sub-marine groundwater discharges. Since a programme of water level measurement and water quality sampling is required, a network of water monitoring bores is then a prerequisite to any groundwater basin evaluation and an integral part of groundwater management.

<u>Procedure</u>

1. Determine the recharge mechanism. An examination of water level response to likely recharge events such as rainfall and streamflow may allow a "rainfall-recharge" or "streamflow recharge" relationship to be determined.

2. Assess the availability of recharge water by preparing a Rainfall Residual Mass Curve for rainfall stations in the area. Quite frequently rainfall records are extensive and go back many years. They not only present longer periods of record but are also more frequently read than the normal groundwater level records that are available. Normally a Rainfall Residual Mass Curve based on monthly rainfalls is adequate for groundwater analyses.

   The <u>*Rainfall Residual Mass Curve*</u> is determined in the following manner:

   For the entire period of rainfall records calculate the average rainfall for each month, i.e. the average for January, February, March, etc.

   Starting at the beginning of the rainfall records subtract the average rainfall for that month from the actual rainfall in that month and add that difference to the cumulative value of actual monthly rainfall minus average for all preceding months.

   Plot the cumulative residual rainfall as a mass curve.

   The *Rainfall Residual Mass Curve* represents the availability of water for recharge. It does not represent the magnitude of recharge but rather the likelihood that recharge will occur.

   The magnitude of recharge will depend not only on the availability of water but also on the fullness of the aquifer and the ability of the material in the recharge area to absorb the water into the aquifer. However, there is more likelihood that recharge will occur if water is available than if it is not. The Rainfall Residual Mass Curve is a very powerful tool for groundwater analyses.

   An examination of the Curve will show that a wet period (above average rainfall) is associated with a rising limb of the Curve and a dry period (below average rainfall) is associated with a falling limb. In many cases the groundwater levels mirror the shape of the Curve but at a different scale. Rainfall Residual Mass Curves can reveal if a declining water level is result of overuse or merely a climatic trend.

3. Determine the critical period. Having determined which is the major contributing recharge mechanism, locate the critical period of groundwater availability from either rainfall or streamflow records or both for the area under investigation.

4. Define the groundwater depletion curve. Any groundwater movement is associated with a groundwater gradient. A change in water level will result in a change in gradient. A higher water level is normally associated with a steeper gradient (and higher groundwater flow) while a lower groundwater level is normally associated with a lower gradient (and a lower groundwater flow).

   For water levels in a particular bore the rate of fall is greater when the water level is higher than it is when the water level is low. Hence a plot of water level variation with time for this bore will yield a logarithmic variation. Such a curve can be called a Depletion Curve. For a particular water use the water level will fall in accordance with the Depletion Curve.

   The shape of the Depletion Curve depends, among other things, on the natural depletion of groundwater storage by groundwater flow, and on the volume extracted by other means such as bores and evapotranspiration.

   An examination of water levels in a number of bores may allow a depletion curve to be drawn for each of the bores. The depletion curve for each bore may be entirely different from that for each of the other bores. Recharge events will of course have to be removed from the depletion curve when it is drawn.

5. Determine the water use, other than by natural means, during the period for which the depletion curve has been drawn.

6. Draw the water level behaviour curve for the critical period. Starting at full supply level at the beginning of the critical period draw the depletion curve from that water level until a recharge event occurs. From the relationship derived for the recharge mechanism, evaluate the magnitude of the recharge and apply this water level rise at the time when the recharge event occurs.

   Apply the depletion curve again at the new level and trace the water level depletion until the next recharge event occurs.

   This procedure is continued until the end of the critical period. The rainfall residual mass curve should show the same trends as this curve and provide a check on the validity of the result.

7. Determine the levels which are to be adopted as full supply level and dead storage level.

8. Calculate the sustainable yield. The water level is not to fall below the dead storage level. If it falls below such a level then the system fails.

   The water level curve drawn during the critical period is for water use at the time for which the depletion curve was drawn.

Using the storage volume per unit depth and from the difference between the lowest level reached and the dead storage level a volume either additional or reduced is available over the critical period. The yield which can be sustained on an annual basis is then the annual use during the period when the depletion curve was drawn plus or minus the additional volume to bring the curve to dead storage level, divided by the duration of the critical period.

Likewise, if the water level at the end of the critical period fails to recover to full supply level, or rises above full supply level, yield which can be sustained on an annual basis is determined as the annual use during the period when the depletion curve was drawn plus or minus the volume required to bring the water level to full supply level divided by the duration of the critical period.

This method is rather rough but gives reasonable results.

The sustainable yield determined above is as stated previously, conservative. It allows for zero failure during the worst historical situation and does not represent the best use of water resources.

It would seem more attractive to base the annual use on the sustainable yield and on the water level situation at the beginning of each year being considered. If the water level is high a use higher than the sustainable yield could be permitted. If the water level is following the critical curve then the use could conform to the sustainable yield. If the water level is low then restrictions could apply.

It could also be argued that using the worst historical period for analysis is too conservative. Maybe the second worse or third worse period could be adopted and accept that failure periods will occur and restrictions will be necessary on occasions.

11.2 CONTROL OF GROUNDWATER USE

The control of groundwater use must be carried at Government level. The first step is to introduce legislation which proclaims the relevant area as one in which licences are required to drill bores and to extract groundwater. These licences would have certain conditions in them pertaining to water use and depths to be drilled etc.

One of the necessary conditions of such licences is that the drilling logs and water samples etc. be forwarded to the relevant authority so that a better understanding of the groundwater resource can be obtained.

A desirable second step is to install water meters on the bores. In this manner the total volume of water which is being extracted from the aquifer can be determined. People tend to use less water if meters are installed.

Neither of these control methods is popular but in many cases they are necessary if the resource is to be managed responsibly.

11.3 CONJUNCTIVE USE OF GROUNDWATER AND SURFACE WATER

In many areas maximum water development can only be achieved by conjunctive utilisation of both surface and groundwater reservoirs. Essentially this requires that surface water impound streamflow which is then transferred at an optimum rate to the groundwater storage.

Surface storage could supply most annual water requirements while the groundwater reservoirs, generally being many times larger, can be retained primarily for cyclic storage covering a series of years having low rainfall. Thus, the groundwater levels would be lowered during a cycle of dry years and raised during wet periods. The optimum rate of transfer from surface to groundwater storage must be large enough so that the surface water reservoir will be drawn enough to retain the next surface water runoff. To have a maximum feasible transfer, water must be artificially recharged into the ground.

Depending on the nature of the existing development, it may be preferable to extract all supplies direct from groundwater and use the surface water storage to replenish the groundwater storage when necessary.

The basic difference between the usual separate developments of surface water and groundwater resources and the conjunctive use operation of both storages is that the conjunctive yield of the system is greater than the sum of the individual yields of the two systems when operating separately.

In the Callide Valley in Central Queensland (refer Ashkanasy and Hazel, 1974) the main source of irrigation water has been from the shallow alluvial aquifers of numerous creeks in the valley. One surface water storage, Callide Dam, had been built on one of the creeks in area to supply water to Callide Power Station. The demand on the groundwater resources in that area has been greater than the aquifers' ability to supply the water. Parts of the district have been overdeveloped and many sections of the aquifers completely dewatered.

The Queensland Government decided to increase the capacity of Callide Dam and use the additional stored volume to augment the groundwater supplies in the valley.

One of the main problems to be overcome in a conjunctive operation such as proposed in the Callide Valley is to determine when the water is to be released from the surface storage to be stored underground. If the water is released too early the aquifers may be full when a potential recharge event occurs which then bypasses the aquifer system. This could occur in the Callide Valley because Callide Dam is located on one creek but the water is being diverted from the dam to replenish the alluvium associated with at least three creeks. On he other hand, if water is left in the dam for too long, evaporation losses become very significant and the advantages of conjunctive use are reduced. It is necessary then to establish an operating rule for releases.

Advantages of Underground Storage

There are many advantages to be gained by storing water underground, a few of which are:

- vast storage volumes are available;
- evaporation losses are negligible so water can be stored for long periods with minimum loss;
- the underground storage acts as a transmission system and avoids the use of costly pipelines; and
- turbid water can be filtered before use.

11.4 GROUNDWATER RECHARGE

11.4.1 What is Recharge?

In order to appreciate fully the process of recharge it is necessary to have a good understanding of the hydrologic cycle which accounts for the occurrence of fresh water on our planet and see how recharge fits into that cycle.

Groundwater recharge is as natural a part of the Hydrologic cycle as precipitation, evaporation or surface water runoff. However, it is not quite as easily understood as it is limited by more factors than those which control the other parts of the cycle.

The process of percolation through the soil to increase the volume stored in an aquifer is called "recharge".

11.4.2 Definitions

Recharge is considered to be that component of the hydrologic cycle which increases the net volume stored in an aquifer. It is not always accompanied by a rise in water level. For example if the recharge event is superimposed on a rapidly declining water level situation, the water level may not rise but because of the recharge the rate of decline may be reduced.

Recharge is variously expressed as a volume, a depth of water and as a depth of saturated aquifer. These are all expressed over a given time period.

Recharge differs from infiltration in that:

- infiltration is the amount of water per unit area that enters the soil profile over a given period; and
- not all of the infiltrated water reaches the aquifer to become recharge; much of it will be utilized by the processes of evaporation and transpiration. The component which finally becomes recharge can vary from 0.1% under some native vegetation to approximately 30%.

Recharge also differs from deep drainage in that:

- deep drainage is that component of infiltration that moves vertically downwards below the zone of evapotranspiration (or root zone). It may or may not become recharge. If there is a low permeability layer below the root zone, some of the deep drainage may move laterally and enter the stream system without entering the aquifer of interest.

In a multiple aquifer system, some of the recharge will occur via leakage from other aquifers. In this chapter, we will deal with recharge from the land surface to the unconfined aquifer in that area.

11.4.3 Necessity for Recharge

Each one of us is familiar with the operation of a normal house-hold tank. If water is required from the tank a tap is opened and water gushes forth. The successful long term operation of such a tank, however, depends upon the volume in storage and replenishment from rain which has fallen on the catchment area. Without recharge to the tank, by rain, its long term yield would be a very small quantity and would depend solely on the capacity of the tank.

In the same manner, the long term yield from any aquifer must depend upon its ability to be recharged or to have the water which is being removed replenished from some external source. If too much recharge occurs then the groundwater storage overflows, sometimes uncontrollably and in areas where it is not required. This can give rise to salinisation and the associated problems.

In some cases, such as Australia's Great Artesian Basin and a number of unconsolidated throughout Australia, the draft from the aquifer far exceeds the volume of natural replenishment. In all of these cases this overdraft results in a steady drop in pressure and a resultant reduction in discharge; with some bores in the Great Artesian Basin ceasing to flow.

This decline in pressure and discharge continues until the recharge to the basin is equal to draft from it. In this case of the Great Artesian Basin, the balance will be established by nature and the pastoral industry will not be endangered because of lack of water.

Sometimes the water pumped from aquifers can be replaced by other than natural means and this is called artificial recharge or managed recharge. Artificial recharge has been practiced for decades and is on the increase.

11.4.4 Natural Recharge

Most aquifers receive fairly regular recharge under natural conditions.

The natural recharge of any groundwater storage depends upon a number of factors which include:

- the availability of water. The recharge water is normally provided by rainfall, unregulated streamflow or regulated streamflow. In some irrigation areas recharge also occurs as result of excess irrigation water percolating down to the aquifer;
- the ability of the surface material in the recharge area to accept the water. A sandy soil will accept water more readily than a clayey soil. With prolonged infiltration the surface material may become clogged. This will result in a reduction in infiltration rate. Recharge areas require regular maintenance if high acceptance rates are to be maintained;
- the groundwater levels at the time of the potential recharge. If the aquifers are empty more water can be accepted than when the water levels are high and the aquifers are full; and
- the ability of the aquifer to distribute the water once it has been accepted. If the water can be transmitted quickly from the recharge area a greater volume of water can be accepted.

Natural recharge normally takes the form of infiltration resulting from:

- rainfall; and
- streamflow.

Rainfall is by far the most common source of natural recharge to all of our Australian aquifer systems. The operation is as explained briefly in the hydrologic cycle. Rain falls upon the ground, filters into the ground and thence to the aquifer system causing rises in the water level below the ground. In some cases, there is not an easy access and water cannot percolate quickly to the aquifer. For instance, there may be a clay layer between the surface and the aquifer, in which case, water may take a long time to penetrate to the aquifer or it may not penetrate at all but may run off in streams.

In some unconfined aquifers the aquifer material extends to the surface over its entire area and recharge by rainfall is appreciable.

In the case of confined aquifers only part of the aquifer extends to the surface and infiltration occurs over a small portion of its total area. This intake area may be higher than most of the aquifer and the water in the rest of the aquifer is under pressure. The recharge effect at the exposed area is then transmitted as a pressure effect and causes water level rises throughout the aquifer.

Streamflow. In many cases the beds of rivers are directly connected with aquifer systems. Flow in the river promotes infiltration through the bed of the river and replenishment of the aquifer system over a large area. This type of recharge is not generally as effective as rainfall recharge. The river cuts through the area like a ribbon and beneficial effects are felt only in a small area adjacent to the river while the effect further from the river is less significant.

Because our Australian rainfall occurs on a seasonal basis, most of our rivers do not flow continuously. The recharge occurrence from the river is then only on an intermittent basis and this type of natural recharge could only be significant when the river flow is high. Such recharge could also be hampered by the fact that many of our rivers are dirty when flowing and the infiltration rate into the aquifers through the bed of the river could be impeded by sediments which are carried in the water.

11.4.5 Artificial or Managed Recharge

Occasions arise where it is desirable to inject water into an aquifer system to achieve one or more objectives which may include:

- to stem the decline in water levels;
- to supplement existing supplies;
- to remove suspended solids by filtration through the soil;
- to inhibit sea water intrusion that threatens to ruin freshwater bores in coastal areas by:
    1. preventing the annual draft from lowering the water levels below sea level; and
    2. creating a barrier to prevent the landward movement of sea water thus enabling greater use of the ground-water reservoir;
- to store cyclic water surpluses for use in dry periods;
- to prevent the subsidence of land surface resulting from excessive groundwater or oil extraction;
- to use the aquifer as a distributory system e.g. in densely populated areas recharge in one area and withdraw in another.

Cases, such as the Burdekin Delta and the Condamine Area, arise here the draft of water from the underground source far exceeds the water which is being replenished on a natural basis. This could be attributed to:

1. overdevelopment in an area where potential recharge water is inadequate; or
2. inadequate means of natural recharge in an area where the water available for recharge is sufficient.

Situation 2 could be caused by the existence of clay seams between the surface and the aquifers, preventing effective recharge.

In such cases, it is necessary to examine ways of increasing this natural recharge by artificial means.

<u>Source of Recharge Water</u>

A prerequisite to any artificial recharge scheme is of course to have a source of water available for artificial recharge purposes. Primary sources are:

- surface runoff;
- effluent or waste waters; and
- imported waters.

<u>Methods of Artificial Recharge</u>

A number of methods are available for recharging groundwater systems. Those which will be discussed briefly include:

Recharge Pits

In recharge pits, an area is normally excavated such that the bottom and sides of the excavation are in contact with the aquifer system over quite a large area. Water is applied to the pit and percolates from it into the aquifer system.

Recharge Trenches

In areas where the aquifer is deep and with side slopes the excavation is excessive it is not economically feasible to excavate down to the aquifer. In such cases it is sometimes possible to recharge the area by utilizing natural drainage paths which are or can be connected to the aquifer. One means of effecting such a connection is by using gravel filled trenches extending from the bed of the channel to the aquifer. Water is normally diverted from an external source into these channels and percolates down to the aquifer either through the bed of the channel or through the gravel filled trenches. This is one method of artificial recharge being used at present in the underground water reservoir in the Burdekin Delta in North Queensland.

Recharge Bores

In some cases it is also possible, provided the water has been suitably treated, to inject water directly into aquifer systems by means of bores and wells. Basically, injection bores are similar to normal producing bores. Owing to the relatively small area of aquifer in contact with the injection well, clogging of the aquifer is to be avoided carefully. Injection water must therefore be of a high standard. Suspended material should be less than 1 p.p.m. Other factors contributing to clogging of the aquifers are:

- chemical incompatibility between recharge water and the water in the aquifer or the aquifer itself;
- bacterial clogging of the aquifer; and
- dissolved air or other gases in the water which can reduce the aquifer's hydraulic conductivity.

The high cost of injection bores and water treatment usually restricts this form of recharge to particular projects such as the creation of sea water barriers.

In order to stem the decline in water levels, it must be remembered that the amount of water to be injected must be at least equal to the overdraft, i.e. the difference between the draft and the annual yield for the area.

It may be argued that if the water has to be supplied to these bores, it would probably be more economical to supply the water directly to the areas in which the excess bores are located. This does not necessarily follow as injection of water into underground facilities also injects the water into natural transmission systems, namely the aquifer. This then removes the necessity for installation of expensive pipelines.

Overseas, recharge bores have been very successful in controlling the sea water intrusion into coastal aquifers.

Stream Bed Spreading

In Queensland most of the rain falls during a very short interval and rivers and creek beds which are potentially good recharge areas are not exposed to water for a sufficiently long period for significant recharge to occur.

The recharge can be increased in these cases by building weirs cross the streams to pond the water and expose it to the aquifer for longer periods.

0ffstream Water Spreading

Offstream water spreading can be considered as essentially the same principal as building weirs across streams. Bays are constructed away from the main course of the stream and water of a predetermined quality is pumped into them to promote recharge. The same effect can be obtained by building contour levees in the catchment area of streams in an area of natural recharge. This reduces the runoff and increases recharge.

To maintain reasonable infiltration rates it is customary to spell these bays periodically to allow them to dry out. Weed growth should be controlled also.

Problems Associated with Artificial Recharge

Artificial recharge is not a straight forward process and is fraught with many difficulties. I will not attempt to deal with all of the problems which are likely to be encountered but will discuss briefly the most common of these which are:

Sedimentation

The sediments carried by recharge water, especially those associated with surface runoff can be appreciable and present a major problem to the operation of a recharge system. In muddy water, these sediments can contain as much as 2.1 tonne of mud per megalitre of water and can seal the surface through which recharge is taking place or they could seal the aquifer itself.

Most of these sediments can be removed by filtration. However, colloidal clays present the major problem as they resist almost all forms of filtration, clarification or settlement.

Because of the costs involved settling basins are normally used. This does not result in settlement of all sediments and the recharge surface has to be cleared on a regular basis.

Algae

Algae should be controlled by use of chemicals, such as chlorine.

Potential Pollution of the Aquifer

If the recharge water carried chemical or biological compounds derived from fertilizers etc. there is a possibility of pollution of the aquifer. This possibility is more pronounced when the water is injected directly into the aquifer by recharge bores. While filtering will remove suspended matter and will often remove bacteria, it will not remove compounds which are chemically bound in the water e.g. salt in saline intrusion.

Pollution appears to be a more serious problem in fractured rock or cavernous aquifers than in porous rocks or unconsolidated sands and gravels.

Precautions must be taken to prevent the build up of bacteria which could reduce the infiltration rate, especially for recharge bores.

It is also good practice in the case of recharge bores to have a pump installed in the bore to enable the bore to be pumped periodically and to clean the aquifer material adjacent to the screens.

Maintenance

To achieve a reasonably uniform infiltration rate the recharge facilities require varying degrees of maintenance. The maintenance required depends on the type of facility used.

Recharge bores need to be pumped periodically to clean the aquifer adjacent to the bore. The maintenance costs involved reduce if thorough treatment of the recharge water is carried out before recharge.

Maintenance of recharge spreading areas and pits can be relatively simple. In some cases it involves periodic "resting" of the area while in others tilling of the area may be necessary. Other conditions may warrant the removal of the silt from the area. Regular maintenance can be costly but is vital for the successful operation of any artificial recharge scheme.

Economic Feasibility

The provision of water by artificial recharge must of course be economically attractive.

Monetary expenditures in installation of the various recharge systems can be significant particularly when maintenance costs are considered. For this reason an economic feasibility study must be an integral part in the design of any artificial recharge system.

Costs of Recharge and Recharge Works (note: these costs are as at the 1975)

Costs for both capital works and operation vary widely.

United States of America

Very little recharge work has been carried out in Australia to date but works in the United States of America indicate that in the 1960's:

- Recharge Barriers could cost $U.S.700,000 per mile (1.61 km) to construct and $U.S. 120,000 per mile (1.61 km) per annum to finance and operate.
- Other forms of recharge range from $U.S. 40.70 per megalitre for interest, redemption, maintenance and water cost to $U.S. 1.30 per megalitre for maintenance only.
- Major factors influencing costs are high land acquisition charges and high water purchase costs.

Burdekin Delta - North Queensland

The capital cost, to the end of 1970/71, involved in implementing the Burdekin Delta Artificial Recharge Scheme was of the order of $1,200,000. The average recharge benefit gained from the scheme from July 1966 to July 1971 was of the order of 61,500 megalitres per annum. The average cost per megalitre of water artificially recharged, including interest, redemption and operating costs, is of the order of $4.00 per megalitre for the area south of the Burdekin River and $2.30 per megalitre for the area north of the Burdekin River.

Examples of Artificial Recharge Investigations in Queensland

Burdekin Delta

The Burdekin Delta, in northern Queensland is one of the largest users of groundwater in the State. The prime source of natural recharge is from rainfall although contribution from streamflow can be significant. The main use is for irrigation of sugar cane. Investigation found that the quantity of water which could be removed safely on an annual basis from the Burdekin Delta was of the order of 197,000 megalitres. The actual use, in 1963/64 when the detailed analysis was carried out, was in excess of the safe yield by some 77,500 megalitres. This resulted in a serious decline in water levels and water quality. If the irrigation were to be continued at that rate then it was obviously necessary to replenish the underground water system be some artificial means. In fact, indications were that the area irrigated was expected to rise to 28,400 hectares in the mid sixties and the water requirement would then be 320,000 megalitres and the overdraft some 123,000 megalitres would need to be provided by artificial recharge.

As a result of the investigation artificial recharge of the underground water resources in the Burdekin Delta is being practised at present. The recharge scheme was implemented in the 1965/66 financial year and the total capital cost for the area to the end of 1970/71 was of the order of $1,200,000.

The average volume recharged during the period July 1966 to July 1971 was of the order of 61,500 megalitres per annum. Infiltration rates of the order of 6m/day are obtained in the offstream recharge areas. In the natural depressions recharge rates of 2500 m3/day are common.

In this case, water is pumped from the unregulated flow in the Burdekin River into recharge channels which wind their way throughout the Burdekin Delta both on the north and south sides of the Burdekin River. On the north side, water is pumped into two natural creeks, namely Sheep-station Creek and Plantation Creek, and finds its way into the aquifer system through the beds of these creeks. On the south side water is pumped into artificial channels. In some areas these artificial channels intersected the aquifers and water migrated down into them. In others, the channels did not intersect the aquifer and trenches were dug in the beds of the channels and backfilled with gravel to provide access to the aquifer.

Bundaberg Area

The draft in the Bundaberg area was also found to be more than the annual yield. This area also relies heavily on underground water to irrigate sugar cane. An investigation was carried out into ways of replenishing this underground water by artificial means. However, unlike the Burdekin Delta, trials indicated the artificial recharge of the underground reservoir was not feasible and the excess has to be provided from a surface storage. One of the main reasons that artificial recharge was not feasible was that quite a thick blanket of clay existed between the aquifer system and the surface.

In the Bundaberg area also, the main source of recharge is from rainfall.

Condamine Area

A detailed analysis of the underground water system in the area has shown that the volume of water which can be removed from the aquifer system with a reasonable assurance of being replaced by natural recharge mechanisms is of the order of 18,400 megalitres per annum. The difference between the draft in 1971 and the annual yield is of the order of 73,800 megalitres per annum. That is, 73,800 megalitres per annum is being drawn from the aquifer system which has no assurance of being recharge by natural means. This conclusion is supported by the continual decline in water levels in the area, even though the potential for recharge with the exemption of one drought year - 1970, has been equal to or above average since 1964. In fact a 1 in 100 year flood was experienced during that period.

As a result of this overexploitation of the groundwater there is a possibility that in the long term, many supplies from the shallower bores in the area will be cut off altogether. Such a system is then equivalent to the mining of water.

If steps were not taken to supply water by artificial recharge to overcome this imbalance then the situation could arise where some 16,000 hectares which are currently being irrigated will no longer be able to be irrigated.

Investigations have shown that a blanket of clay exists between the surface and the aquifer. It appears to be thinner near the river and increases in thickness as distance from the river increases. This clay blanket prevents natural recharge from rainfall.

The main recharge for the area is derived from flow in the Condamine River.

11.5 SEA WATER INTRUSION IN COASTAL AQUIFERS

11.5.1 General

Coastal aquifers may come in contact with the ocean and, under natural conditions, fresh groundwater is discharged into the ocean. With the increased demand for groundwater in many coastal areas, however, the seaward flow of groundwater has been decreased or even reversed, allowing saltwater to enter and move inland in aquifers. This problem is called sea water intrusion.

If the salt water travels inland to bore fields, underground water supplies become useless; moreover, the aquifer becomes contaminated with salt which may take years to remove even if adequate fresh groundwater is available to flush out the saline water. The importance of protecting coastal aquifers against this continual threat has lead to investigations pointing towards methods of prevention or control of sea water intrusion.

Two techniques for determining the location of the salt water/ fresh water interface are presented. The first, which is known as the Ghyben-Herzberg concept is based on hydrostatics i.e. no flow. The existence of a wedge must necessarily indicate the presence of a ground-water gradient which must be associated with a flow. Hence any hydrostatic approach must be an approximation only.

The second method, the Dynamic concept does take into account such a flow condition.

11.5.2 Ghyben-Herzberg Concept

Two investigators, Ghyben and Herzberg, working independently became aware of the fact that in coastal aquifers, salt water existed below the fresh water. The depth at which it is encountered is dependent on the head of fresh water above mean sea level at that point. They showed that, generally, it could be expected to be encountered first at a depth below mean sea level which is approximately 40 times the head of fresh water above mean sea level, i.e. for every 1m of fresh water above mean sea level there should be approximately 40m of fresh water below mean sea level before the salt water is encountered. The depth to the salt water/ fresh water interface is based on, among other things, the densities of the two fluids. In the evaluation of the relationship above it has been assumed that the density of fresh water is 1000 kg/m3 and that of the sea water is 1025 kg/m3. Before applying the equations applicable to salt water intrusion it would be necessary to ascertain the densities for the area being investigated.

<u>Derivation</u>

The necessity for the existence of a salt water wedge is based purely on density considerations as can be seen from

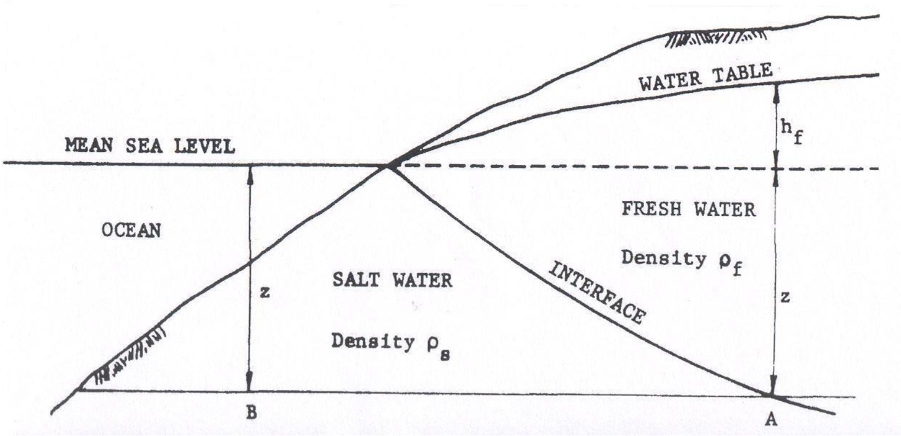


Figure 11-2.

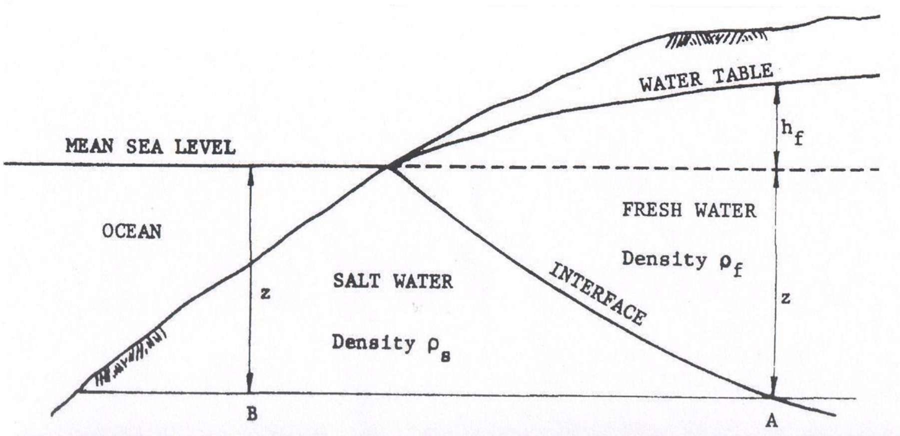


Figure 11-2 Stable saltwater interface

At point A on the interface, for equilibrium to exist, the pressure on the salt water side of the interface (p_{As}) must equal the pressure on the fresh water side (p_{Af}).

From hydrostatics:

$$p_{As} = \rho_s g z \qquad .....11.2$$

and

$$p_{Af} = \rho_f g z + \rho_f g h_f \qquad .....11.3$$

where:

p_{As} = pressure at A on the salt water side.

p_{Af} = pressure at A on the fresh water side.

ρ_f = density of fresh water.

ρ_s = density of salt water.

h_f = head of fresh water above mean sea level.

z = depth to the interface below mean sea level.

Since the pressure at A is the same on both sides of the interface:

$$\rho_s g z = \rho_f g z + \rho_f g h_f$$

or

$$z = \frac{\rho_f}{(\rho_s - \rho_f)} h_f \qquad .....11.4$$

<u>Example</u>

If in an investigation area, the densities of fresh water and salt water 1000 kg/m3 and 1026 kg/m^3, determine the depth below mean sea level at which the salt water interface will be encountered at a point here the fresh water stands 0.5m above mean sea level.

Solution

From equation 11.4:

$$z = \frac{\rho_f}{(\rho_s - \rho_f)} h_f$$

$$= \frac{1000}{1026-1000} 0.5$$

= 19.2m below mean sea level

At this point the salt water interface would be expected to be 19.7m below the standing water level.

Remarks

Although the Ghyben-Herzberg concept, which is based on hydrostatics, implies no flow of the interface, groundwater in coastal areas is invariably moving, and a dynamic concept is required.

Without fresh water flow i.e. no water level gradient, a horizontal interface would develop with the fresh water floating on salt water.

A more correct picture of salt water intrusion is given in

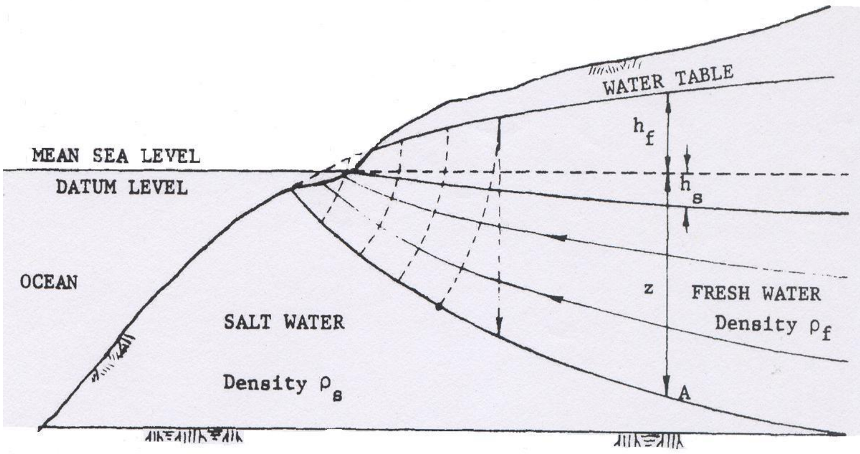


Figure 11-3. Where the flow lines have a vertical component, the Ghyben-Herzberg concept gives too small a depth to salt water. Further inland, where the flow lines are nearly horizontal, the error is negligible.

The Ghyben-Herzberg concept is also applicable, with the same limitations, to confined aquifers, where the potentiometric surface replaces the water table.

It should be remembered that sea water intrusion is quite natural and cannot be overcome completely. What has to be determined is the magnitude of intrusion which will be acceptable in a particular area. A fresh water flow to sea is necessary to stabilise the interface, and the position of the wedge depends on the magnitude of the flow to the sea. A reduction in the magnitude of fresh water flow will result in a movement of the toe of the wedge inland until stability is again achieved. A smaller fresh water flow is associated with a smaller gradient which in turn results in a flatter wedge. If the magnitude of the fresh water flow is increased, then the toe of the wedge moves seaward until stability is achieved and a steeper wedge results. The operation of sea water intrusion is then a management problem.

11.5.3 The Dynamic Concept

In many areas of sea water intrusion the depth of salt water computed from the Ghyben-Herzberg relation differs markedly from observations. Often these discrepancies can be attributed to assumptions that the head in the salt water is at mean sea level and that the fresh and salt waters are static. An expression taking into account flows in each fluid can be derived by considering the fluid heads.

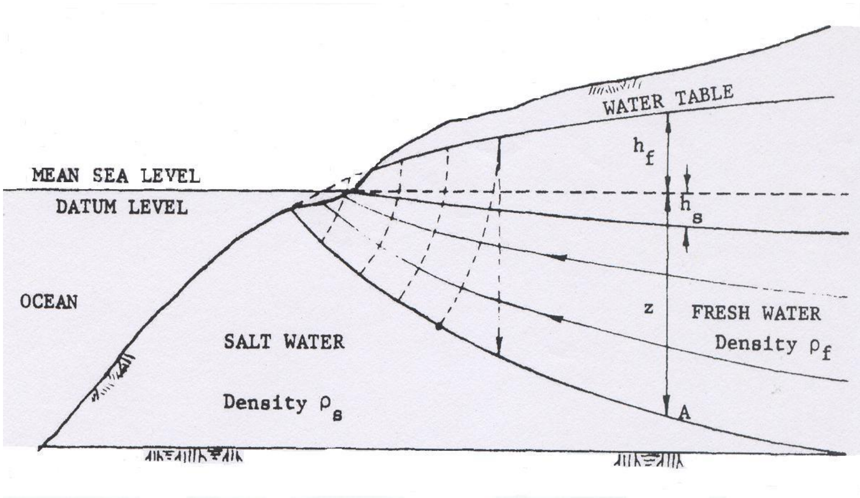


Figure 11-3 The dynamic saltwater interface

<u>Derivation</u>

At point A in

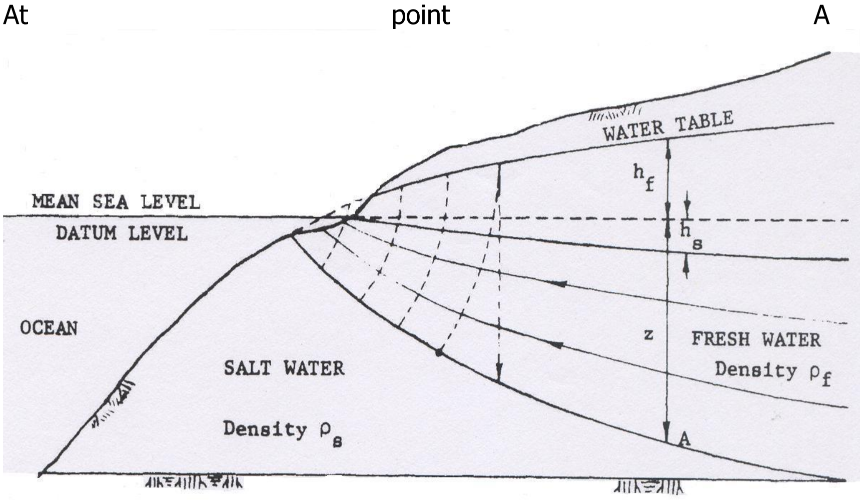

WATER TABLE
h_f
MEAN SEA LEVEL
DATUM LEVEL
h_s
OCEAN
z
FRESH WATER
Density ρ_f
SALT WATER
Density ρ_s
A

Figure 11-3 h_f and h_s are potentiometric heads, related to a particular datum, in a region occupied by fluids of densities ρ_f and ρ_s respectively. The potentiometric head is the summation of the elevation head and the pressure head. As mean sea level has been adopted as datum level, then the following relationships apply.

For fresh water, the potentiometric head, hf, is given by:

$$h_f = z + p/(\rho_f g) \qquad .....11.5$$

and

$$h_s = z + p/(\rho_s g) \qquad .....11.6$$

where:

p = pressure at A.

z = elevation at A above the datum level.

ρ_f = density of fresh water.

ρ_s = density of salt water.

h_f = potentiometric head of fresh water.

h_s = potentiometric head of salt water.

Since there is no flow across the interface the pressure (p) at A on the salt water side of the interface must equal the pressure (p) at A on the freshwater side of the interface.

Equations 11.5 and 11.6 may then, at point A, be reduced to:

$$z = \frac{\rho_s}{\rho_s - \rho_f} h_s - \frac{\rho_f}{\rho_s - \rho_f} h_f \qquad .....11.7$$

This then enables the depth to salt water, above datum, to be defined by the heads and densities across the interface.

If h_s = 0 then equation 11.7 reduces to equation 11.4, the Ghyben-Herzberg relation.

If h_s = 0, i.e. there is no potentiometric head in the salt water then no movement of salt water occurs and the wedge position is table.

However, if h_s is negative, i.e. below mean sea level, then a hydraulic gradient exists from the sea to the land and the wedge will move inland until stability is achieved.

If h_s is positive, i.e. above mean sea level, then a hydraulic gradient exists from the land to the sea and the wedge will move towards the sea until stability is achieved.

Example

Measurements of potentiometric heads in observation bores in a coastal aquifer reveal that at a particular location the fresh water potentiometric head is 2m above mean sea level and the salt water potentiometric head is 1m below mean sea level. If at this location, the densities of fresh water and salt water are 1000 kg/m^3 and 1026 kg/m^3 respectively, calculate the depth of the salt water interface.

Solution

From equation 11.7:

$$z = \frac{\rho_s}{\rho_s - \rho_f} h_s - \frac{\rho_f}{\rho_s - \rho_f} h_f$$

$$= \frac{1026}{(1026-1000)}(-1) - \frac{1000}{1026-1000}(2)$$

= -39.5 - 76.9 m

= -117.4m above mean sea level

The interface is then located 119.4m below the fresh water potentiometric level.

If the salt water head had been neglected the depth to the interface would be given by:

$$z = -\frac{1000}{(1026-1000)} 2$$

= -76.9m above mean sea level

This indicates the invalidity of the Ghyben-Herzberg relation for general applications in intrusion areas.

Tidal fluctuations can cause fluctuations in potentiometric heads in coastal aquifer. Since the above expressions are related to mean sea level, care should be taken to use the mean potentiometric head during a tide cycle when applying these equations.

11.5.4 Location of the Interface

Take the case of a static salt water wedge where the origin is taken at the toe of the wedge.

In this static case the potentiometric head of the salt water is zero.

In practice there must be an outlet, where the aquifer is in contact with the sea, through which fresh water flows to the sea. This outlet is shown on the following figures but the error involved in taking the total length of the wedge, as shown on the figures to be the length of the wedge from shoreline is considered to be negligible.

11.5.4.1 Confined Aquifer

The situation which occurs in a confined aquifer having a stable salt water wedge is as shown in

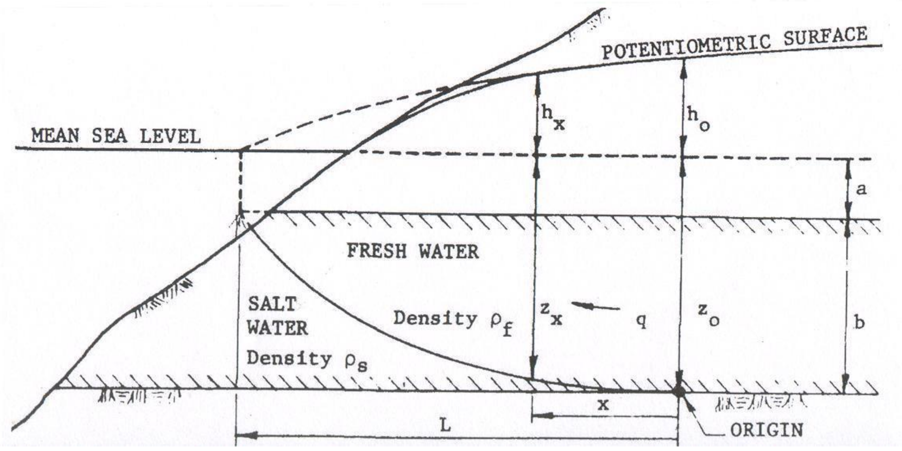


Figure 11-4.

The fresh water discharge rate (q) through a unit width of the aquifer, required to maintain a stable wedge is given by:

$$q = -K(z_x - a)\frac{dh_x}{dx} \qquad .....11.8$$

From equation 11.7 (with $h_s = 0$ and new origin):

$$z = \frac{\rho_f}{(\rho_s - \rho_f)} h_x$$

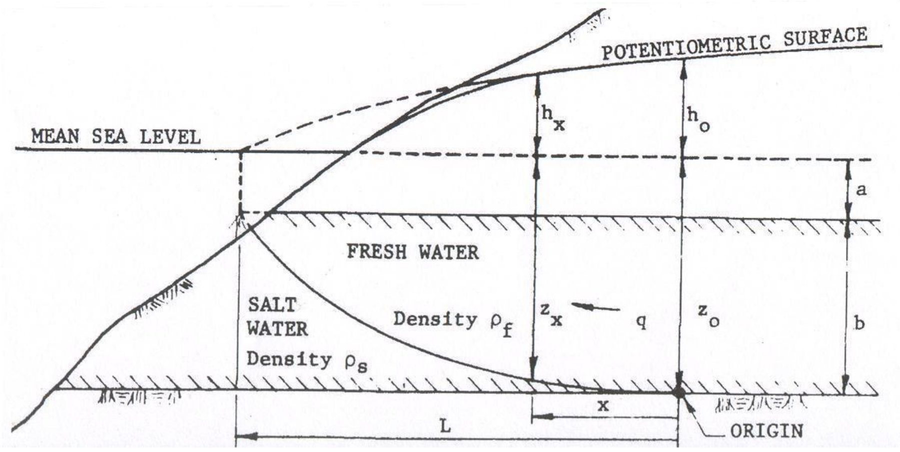
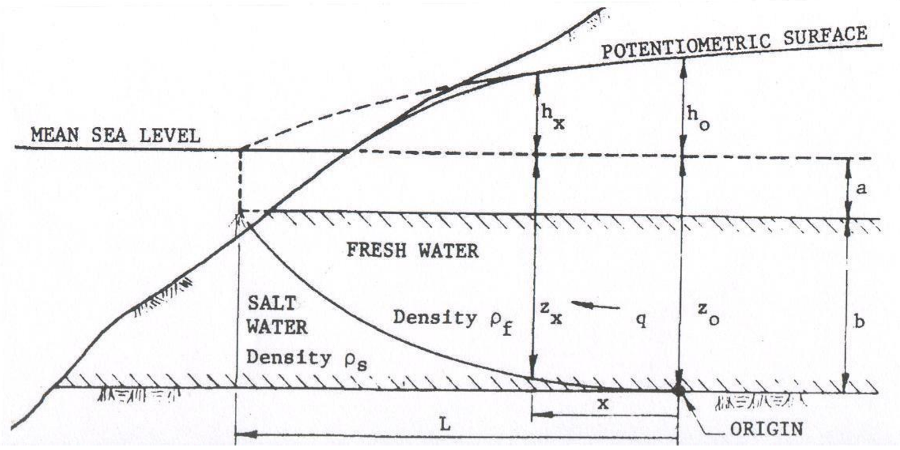


Figure 11-4 Saltwater wedge in a confined aquifer

As the land surface beneath the sea is in fact an equipotential line the salt water interface should not be extended to intersect the mean sea level for this analysis. It should be extended only to intersect the extension of the bottom of the confining layer. The variables and limits integration should then be related to the confined aquifer and not to depths below mean sea level.



In
Figure 11-4 the saturated thickness of the wedge is given by:

$$m_x = z_x - a$$

$$= \frac{\rho_f}{(\rho_s - \rho_f)} h_x - a$$

i.e. $$d_m = \frac{\rho_f}{(\rho_s - \rho_f)} dh_x$$

and $$dh_x = \frac{(\rho_s - \rho_f)}{\rho_f} dm_x$$

Equation 12.30 can be written as:

$$q = -Km_x \frac{dm_x}{dx} \qquad .....11.9$$

Integrating between x = 0 and x = L, and m_x = b and m_x = 0

$$\left[qx\right]_0^L = -\left[\frac{K(\rho_s - \rho_f)}{2\rho_f} m_x^2\right]_b^0$$

$$qL = \frac{K(\rho_s - \rho_f)}{2\rho_f} b^2 \qquad ....11.10$$

where:

q = discharge rate per unit width of aquifer.

L = distance from shoreline to toe of wedge.

K = hydraulic conductivity of aquifer.

b = thickness of confined aquifer.

a = distance from top of confined aquifer to mean sea level.

Since all terms on the right hand side of equation 11.10 are constants, it may be concluded that the length of the wedge is dependent on the unit discharge rate. If q increases, L must decrease and the wedge moves seaward; if q decreases, L must increase and the wedge moves inland.

It will be observed also from the above equations that the shape of the wedge is parabolic.

11.5.4.2 Unconfined Aquifer

The saltwater wedge in an unconfined aquifer is illustrated in **Figure 11-5**.

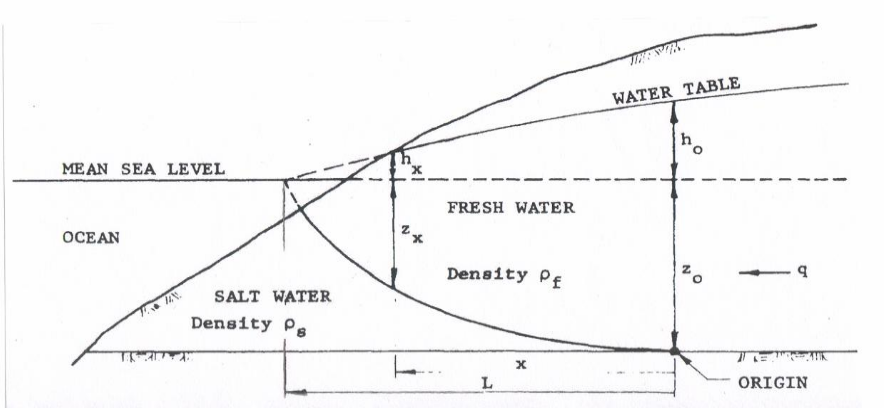


Figure 11-5 Saltwater wedge in an unconfined aquifer

The fresh water discharge (q) through a unit width of aquifer required to maintain a stable wedge is given by:

$$q = -K(z_x + h_x)\frac{dh_x}{dx} \qquad \text{....11.11}$$

From equation 12.29 (with h_s =0 and new origin):

$$z_x = \frac{\rho_f}{(\rho_s - \rho_f)} h_x$$

and $$q = -Kh_x(1 + \frac{\rho_f}{(\rho_s - \rho_f)})\frac{dh_x}{dx}$$

i.e. $$qdx = -Kh_x(1 + \frac{\rho_f}{\rho_s - \rho_f})dh_x$$

Integrating from x = 0 to x = L, and from h_x = h_o to h_x = 0.

$$\left[qx\right]_0^L = -\left[Kh_{h_0}^2(1 + \frac{\rho_f}{\rho_s - \rho_f})\right]_{h_0}^0$$

$$qL = \frac{Kh_0^2}{2}(1 + \frac{\rho_f}{\rho_s - \rho_f}) \qquad \text{....11.12}$$

From equation 11.7:

$$h_0 = z_0 \frac{(\rho_s - \rho_f)}{\rho_f}$$

and equation 11.12 becomes:

$$qL = \frac{Kz_0^2}{2}(\frac{\rho_s - \rho_f}{\rho_f})^2(1 + \frac{\rho_f}{\rho_s - \rho_f}) \qquad \text{....11.13}$$

If h_0 is very small compared with z_0 then h_o + z_0 (i.e. the saturated thickness "b") can be substituted for z_0 in equation 11.13 to give:

$$qL = \frac{Kb^2}{2}\frac{\rho_s}{\rho_f}(\rho_s - \rho_f) \qquad \text{....11.14}$$

or $$qL = \frac{Tb}{2}\frac{\rho_s}{\rho_f}(\rho_s - \rho_f)$$

It can be seen from equation 11.14 and equation 11.10 that qL for an unconfined aquifer is ρ_s times qL for the confined aquifer.

where:

q = discharge rate per unit width of aquifer.

L = length of wedge.

K = hydraulic conductivity of the aquifer.

T = transmissivity of the aquifer.

z_o = depth to bottom of aquifer below mean sea level.

b = saturated thickness.

The shape of the wedge is again parabolic. Again, since all terms on the right hand side of equation 11.13 are constants then L is dependent on q.

11.5.5 Structure of the Interface

Since the two liquids are miscible the interface is not an ideal flow line of zero thickness but a transition zone in which the water density varies from that of sea water to that of fresh. Field measurements of interfaces have revealed a mixing zone ranging from a metre or so to some hundred metres.

Some of the factors which can affect the location of the zone include tidal fluctuations, pumping and natural recharge and discharge of fresh water. These influences cause the interface to shift continually toward a new equilibrium position. Each movement, however, causes dispersion to occur, and a transition zone with a salinity gradient is established.

Dispersion depends on the co-efficient of dispersion of the aquifer and the distance traversed by the groundwater. The thickness of the transition zone at any location depends upon the co-efficient of dispersion, the unsteady fresh water flow field, the hydraulic conductivity and the tidal pattern. One could expect a thinner transition zone where the tidal range is small and thicker zone where the tidal range is large.

11.5.6 Control of Intrusion

Once sea water intrusion develops in a coastal aquifer, it is not easy to control. The slow rates of groundwater flow, the density differences between fresh and sea waters, and the flushing required usually mean that contamination, once established, may require years to remove under natural conditions.

Several methods have been suggested to control intrusion. These include the reduction of pumping or modification of pumping practices, artificial recharge to create a mound parallel to the coast or the establishment of a pumping trough parallel to the coast.

However, as stated previously control of sea water intrusion is basically a management problem. The allowable magnitude of intrusion must be decided upon and entered as a constraint on the system when determining annual yield.

Bear in mind that 1 m of water applied to an aquifer will have an entirely different effect on the salt water wedge than would a 1m rise in water level.

Remarks

It should be remembered that, while a certain fresh water flow is required to maintain a stable wedge it is not always necessary that this flow be lost to the seas. Much of it could be intercepted by a series of collector trenches parallel to the coast and utilised inland.

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GLOSSARY OF SYMBOLS USED

a	-	acceleration (m/sec^2)
	-	constant in equation to drawdown
A	-	area (m^2) (cm^2)
b	-	thickness of confined aquifer (m)
	-	saturated thickness of unconfined aquifer (m)
	-	constant in equation to drawdown
B	-	drainage factor (m)
c	-	hydraulic resistance (day)
C	-	constant in equation to drawdown
d	-	differential
D	-	diameter (m) (cm) (mm)
$D_{\underline{n}}$	-	grain size such that % of sample is finer
e	-	voids ratio
E	-	a constant
E_s	-	bulk modulus of soil matrix
E_w	-	bulk modulus of elasticity of fluid
F	-	force (Newtons) (dynes)
	-	constant
g	-	acceleration due to gravity (9.8 m/sec)
$G_{(\alpha)}$	-	G function of (α) in constant drawdown analysis
h, H	-	head (m)
h_c	-	capillary rise (cm)
h_f	-	friction head loss (m)
i	-	hydraulic gradient (dimensionless)
i_p	-	pressure gradient (gm/cc) (kg/m^3)
J_0	-	Bessel function
K & K'	-	hydraulic conductivity or coefficient of permeability (m/day)
K_0	-	Bessel function
k	-	intrinsic permeability (cm^2)
l	-	length (m)
L	-	Leakage factor (m)
M	-	mass (kg)
n	-	constant
N_R	-	Reynold's number
p, P	-	pressure (kPa)
q	-	specific discharge
Q	-	discharge rate (m^3/day) or (m^3/sec)
r	-	radius (m)

R	-	hydraulic radius (m)
	-	specific retention (dimensionless)
R.D.	-	relative density
s	-	displacement (m)
	-	drawdown (m)
S	-	Storage coefficient (and specific yield)
S_s	-	Specific mass storativity or specific storage
S_y	-	Specific yield
S.G.	-	Specific gravity
T	-	time (sec) (day)
T	-	Transmissivity (m^2/day)
u	-	excess hydrostatic pressure
u	-	$\frac{r^2 S}{4Tt}$
U	-	uniformity coefficient
	-	bulk volume
v	-	velocity (m/sec) (m/day)
	-	a leakage function
v_s	-	seepage velocity (m/sec) (m/day)
V	-	volume (m^3)
W(u)	-	well function of u
y	-	a dimensionless factor
	-	distance (cm) (m)
	-	thickness of fluid in viscosity (m)
Y_0	-	Bessel function
z	-	a dimensionless factor
α (alpha)	-	compressibility of soil matrix
	-	reciprocal of Boulton Delay Index
	-	function in constant drawdown analysis
B (beta)	-	compressibility of water $(kPa)^{-1}$
γ (gamma)	-	specific weight of fluid (use not recommended)
Δ(delta)	-	incremental value
η (eta)	-	dynamic viscosity (poise) (decapoise)
θ (theta)	-	porosity (dimensionless)
ν (nu)	-	kinematic viscosity (stokes)
π (pi)	-	3.1416
ρ (rho)	-	density (kg/m^3)
σ (sigma)	-	surface tension (N/m)
Σ (sigma)	-	summation
τ (tau)	-	intensity of shear

∂	-	partial differential
$>$	-	greater than
$<$	-	less than
∞	-	infinity
$\approx$	-	approximately equal to

METRIC MULTIPLES

Symbol	Designation		Factor
T	tera-	10^{12}	1 000 000 000 000
G	giga-	10^{9}	1 000 000 000
M	mega-	10^{6}	1 000 000
k	kilo-	10^{3}	1 000
h	hecto-	10^{2}	100
da	deca-	10^{1}	10
d	deci-	10^{-1}	0.1
c	centi-	10^{-2}	0.01
m	milli-	10^{-3}	0.001
μ	micro-	10^{-6}	0.000 001
n	nano-	10^{-9}	0.000 000 001
p	pico-	10^{-12}	0.000 000 000 001

THE GREEK ALPHABET

Greek Character		Greek Name	English Equivalent	
Upper Case	Lower Case		Upper Case	Lower Case
Α	α	alpha	A	a
Β	β	beta	B	b
Γ	γ	gamma	G	g
Δ	δ	delta	D	d
Ε	ε	epsilon	Ĕ	ĕ
Ζ	ζ	zeta	Z	z
Η	η	eta	Ē	ē
Θ	θ	theta	Th	th
Ι	ι	iota	I	i
Κ	κ	kappa	K	k
Λ	λ	lambda	L	l
Μ	μ	mu	M	m
Ν	ν	nu	N	n
Ξ	ξ	xi	X	x
Ο	ο	omicron	Ŏ	ŏ
Π	π	pi	P	p
Ρ	ρ	rho	R	r
Σ	σ	sigma	S	s
Τ	τ	tau	T	t
Υ	υ	upsilon	Y	y
Φ	φ	phi	Ph	ph
Χ	χ	chi	Ch	ch
Ψ	ψ	psi	Ps	ps
Ω	ω	omega	Ō	ō

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