Using Cooper-Jacob Approximation to Take Account of Pumping Well Pipe Storage Effects in Early Drawdown Data of a Confined Aquifer

by Robert P. Chapuis[a]

Abstract

Most hydrogeology textbooks warn the reader that in order to avoid large errors the Cooper-Jacob solution should only be applied when the u-parameter of the Theis solution is less than 0.01 or 0.02. This paper proposes a graphical representation for visualizing and quantifying the difference between the two solutions. The graph of drawdown vs log (time) may be divided into three zones, the early one being influenced by storativity, pumping well pipe capacity and skin effects (among others), the latest by boundary effects, and the intermediate one by the transmissivity and storativity of the aquifer. The differences between the theoretical solutions are maximal in the early data zone. Because both solutions consider a pumping well of infinitesimal diameter, the early time data graph may be distorted by the influence of real well pipe storage capacity and, consequently, may yield a poor estimate of the parameters of the aquifer. The method of Papadopulos and Cooper is already available to take pumping well pipe storage into account in interpreting the pumping well drawdown data, but may result in unreliable S values due to the difficulty in curve matching. A more practical solution, which is also applicable to observation wells, is proposed and demonstrated with a worked example. This solution can be used when the Cooper-Jacob approximation is valid and it does not require curve matching. Early drawdown data in the pumping well cannot provide a reliable estimate of storativity for many reasons. These early data can be used, however, to obtain a better estimate of storativity and transmissivity from the drawdown data of observation wells. The effect of pumping well pipe storage in the early drawdown data may be significant in cases of low transmissivity aquifers and low pumping rates, which are quite common in ground-water remediation.

Introduction

The problem of unsteady-state flow due to a constant pumping rate in a fully confined aquifer was solved by Theis (1935). An approximate solution was proposed by Cooper and Jacob (1946) who stated, "The approximation will be tolerable where u is less than about 0.02." The difference between the Theis and Cooper-Jacob solutions is important if the early time data of an aquifer test are to be used to determine the aquifer storativity, S. In addition, both solutions consider a pumping well of infinitesimal diameter. Actually, the storage capacity of a real well pipe will adversely affect the early measurements of drawdown which are essential in determining the storativity. The method of Papadopulos and Cooper (1967) considers the influence of pumping well pipe storage on time-drawdown relationships in large-diameter pumping wells. This curve-matching method applies only to the pumping well, and the authors stated that "a determination of S by this method has questionable reliability."

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Received January 1991, revised August 1991, accepted August 1991.

Discussion open until November 1, 1992.

Using the Cooper-Jacob approximation, this paper proposes a more practical method which is applicable both to the pumping well and to observation wells. First, a graphical representation is proposed for visualizing and explaining the nature of the Cooper-Jacob approximation, and for quantifying the difference between this and the Theis solution. Then, a worked example is provided to facilitate comparison of the proposed method to that of Papadopulos and Cooper (1967).

Mathematical Development

The Theis Solution

Theis (1935) presented a solution for a vertical well fully penetrating a fully confined horizontal isotropic aquifer of infinite areal extent. When this well is pumped at a constant rate, the influence of the hydraulic head discharge extends outward with time. The problem is axisymmetric around the well axis. The classical conservation equation for ground-water flow assuming isotropic permeability is:

$$\mathrm{div}(\mathrm{grad}\ h) = (S/T)(\partial h/\partial t) \qquad (1)$$

where h is the hydraulic potential (total head), S is the storativity, T is the transmissivity, and t is the time. In polar coordinates (r, θ), it gives:

$$\partial^2 h/\partial r^2 + (1/r)(\partial h/\partial r) = (S/T)(\partial h/\partial t) \qquad (2)$$

Using mathematical developments of heat conduction, Theis (1935) obtained a solution for the following conditions:

a. The aquifer is horizontal, homogeneous, isotropic, infinite in extent and of constant thickness; its hydraulic parameters (transmissivity T and storativity S) are constant;

b. The aquifer is fully confined;

c. All water comes from storage in the aquifer material, and is released instantaneously when pore water pressure drops (assumption of instantaneous transfer of pore water pressure to effective stresses—no consolidation effects—and elasticity of the aquifer solid material);

d. The pumping well fully penetrates the aquifer and is of infinitesimal diameter;

e. The well is pumped at a constant discharge flowrate;

f. The flow is laminar and respects Darcy's law.

As a result of assumptions a-b-c, the aquifer is an equipotential volume before pumping; other assumptions imply that the flow is also radial and horizontal, which means that this problem is one-dimensional along any r-axis. The solution of (2) was given by Theis as:

$$s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u}\,du \tag{3}$$

where:

$$u = r^2 S/4Tt \tag{4}$$

and s is the drawdown at time t (since start of pumping) and distance r from the pumping well to the point where drawdown s occurs; Q is the constant pumping rate; T is the transmissivity of the aquifer; and S is the storativity of the aquifer.

The integral function in (3) is denoted by W(u) in well hydraulics. In mathematics, this has been known as the exponential integral function Ei (u), since the 18th century, also defined as the series:

$$W(u) = Ei(u) = -\gamma - \ln u + (u/1 \cdot 1!) - (u^2/2 \cdot 2!) + (u^3/3 \cdot 3!) - (u^4/4 \cdot 4!) \ldots \tag{5}$$

where $\gamma = 0.577\,215\,664\,9..$ (the Euler constant).

Equation (2) may be written:

$$s = Q\,W(u)/4\pi T \tag{6}$$

which, in decimal logarithms, gives:

$$\log s = \log (Q/4\pi T) + \log W(u) \tag{7}$$

whereas (4) may be written:

$$r^2/t = 4uT/S \tag{8}$$

which gives:

$$\log (r^2/t) = \log (4T/S) + \log u \tag{9}$$

Consequently, if the test data log s are plotted against log (r^2/t), the resulting data curve will be identical to the theoretical curve of log W(u) against log u, but translated. This is the mathematical basis for the graphical method proposed by Theis: the log W(u) vs log u type curve is superimposed on the experimental data curve (see example in Figure 1) and the parameters T and S are found successively from the displacement values, $(Q/4\pi T)$ and $(4T/S)$, given by (7) and (9).

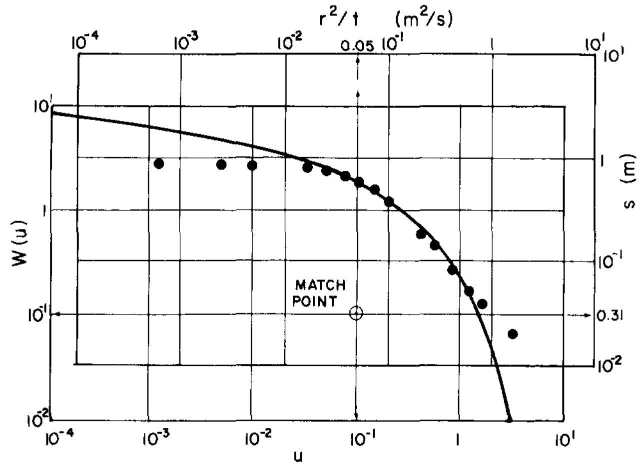

Fig. 1. Theis superposition method for an observation well at r = 7 m of the pumping well in the worked example.

The Cooper-Jacob Approximation

Cooper and Jacob (1946) recognized that the second- and higher-order terms in (5) may be neglected when u is small. Then, (5) becomes:

$$s = (Q/4\pi T) \cdot [\ln (1/u) - 0.5772] = (Q/4\pi T)\,C\text{-}J\,(u) \quad \ldots . (10)$$

where C-J (u) stands for the Cooper-Jacob approximation function, or:

$$s = (Q/4\pi T) \cdot [\ln (4Tt/r^2S) - \ln (1.781)] = (Q/4\pi T) \cdot \ln (2.25\,Tt/r^2S) \tag{11}$$

which in turn may be written in decimal logarithms as:

$$s = (2.30\,Q/4\pi T) \cdot [\log t + \log (2.25\,T/r^2S)] = \Delta s \cdot \log (t/t_0) \tag{12}$$

The latter equation indicates a straight-line relationship between s and log t, with a slope Δs over one time log cycle and a time intercept t_0 at zero drawdown.

In an observation well located at a distance r from the pumping well axis, the experimental values of drawdown s plotted against log t (see example in Figure 2) will give T by the slope Δs:

$$T = 2.30\,Q/4\pi\Delta s \tag{13}$$

and S by extrapolating the straight-line portion to the time intercept, t_0:

$$S = 2.25\,Tt_0/r^2 \tag{14}$$

Accuracy of the Cooper-Jacob Approximation

Without demonstration, Cooper and Jacob (1946) stated, "The approximation will be tolerable where u is less than about 0.02." Textbooks (Ferris et al., 1962; Lohman, 1972; Bureau of Reclamation, 1977; Freeze and Cherry, 1979; Kruseman and De Ridder, 1979; Todd, 1980) include the warning that in order to avoid large errors, the Cooper-Jacob solution should be applied only when u is lower than 0.01 or 0.02.

Based on a literature review, apparently the accuracy of the Cooper-Jacob method for the time-drawdown solution was never quantified. When comparing (6) and (10), the relative error, RE, is

$$RE = [W(u) - C\text{-}J(u)]/W(u) \qquad (15)$$

which may be calculated from tables of W(u) available in many mathematical textbooks (for example, Glaisher, 1870).

The approximation of C-J (u) to W(u) may be visualized by drawing W(u) and C-J (u) versus log 1/u on the same graph. In the exact solution:

$$W(u) = s(4\pi T/Q) \qquad (16)$$

whereas in the Cooper-Jacob approximation:

$$C\text{-}J(u) \approx s(4\pi T/Q) \qquad (17)$$

and for both solutions:

$$1/u = t(4T/r^2S) \qquad (18)$$

The coordinates in Figure 3 are proportional to those in the common graph (s versus log t) of the Cooper-Jacob time-drawdown solution. It appears from Figure 3 that the Cooper-Jacob approximation may be considered valid for u values higher than 0.01, as usually quoted: when u = 0.10, the relative error is 5.4% on a small drawdown value, which is scarcely detectable; for u = 0.05, the relative error is 2.0%.

The representation used in Figure 3 enables evaluation of whether or not test data agree with theory. A plot of measured s values versus log t is similar to that in Figure 3, and can be used to show three distinct zones and to analyze whether or not the test data verify the assumptions listed at the beginning of the paper:

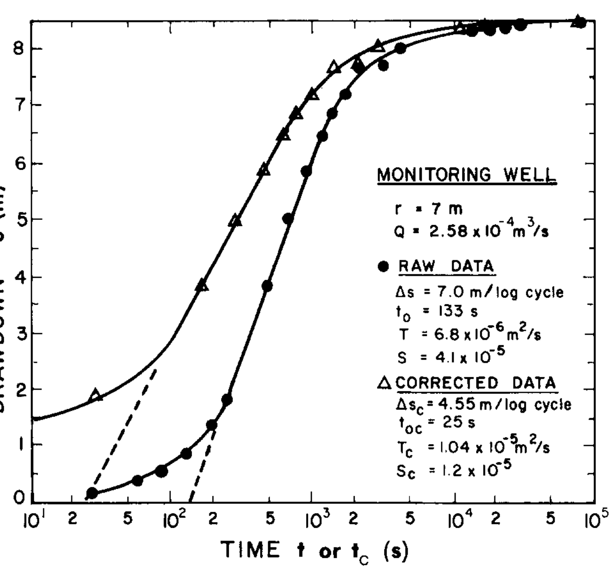

Fig. 2. Cooper-Jacob solution for the observation well of Figure 1.

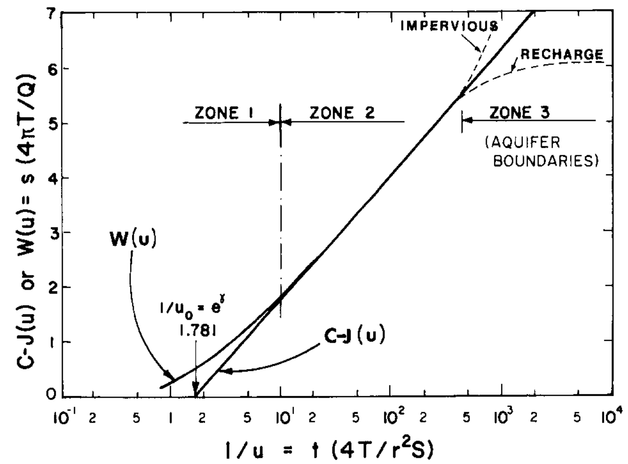

Fig. 3. Plot of W (u) and C-J (u) versus 1/u in semilog graph.

Zone 1: The left and theoretically curved portion of the graph is related to short-term effects: storativity plays a major part if the aquifer is fully confined. Other phenomena, like pumping well pipe capacity and skin effects, shape this portion of the curve. If the aquifer is unconfined or not fully confined (leaky confining beds), or in the case of partial penetration, this portion of the curve will be strongly modified (Boulton, 1963; Bureau of Reclamation, 1977; Ferris et al., 1962; Hantush, 1956; Jacob, 1950; Kruseman and De Ridder, 1979; Lohman, 1972; Neuman, 1972, 1974; Neuman and Witherspoon, 1972; Todd, 1980).

Zone 2: The linear portion is related to medium-term effects: its slope is related only to the transmissivity of the aquifer, whereas the time intercept is related only to the aquifer storativity. Even if the early test data are missing, the Cooper-Jacob representation in Figure 3 makes it possible to obtain both T and S, whereas the Theis superposition method leads to multiple solutions because the test data can slide along the type curve. This is one advantage of the Cooper-Jacob representation. All the numerous phenomena influencing the early time test data of a pumping test introduce at least a translation along the time axis (change of the time intercept t_0).

Zone 3: The right portion is related to long-term effects: aquifer boundaries are important and can be detected.

Correction for Pumping Well Capacity

The difference between the Theis and the Cooper-Jacob solutions is important for the early data of a pumping test which are useful in determining the storativity S of the aquifer when the Theis superposition method is used. In this paper, only the influence of storage in the pumping well pipe will be examined.

In the Theis solution, it is assumed that the pumping well is of infinitesimal diameter. The solution for a pumping well of finite diameter was developed by Hantush (1964) who assumed that the pumping flowrate from the well is equal to the aquifer flowrate entering the well. Actually, the storage capacity of a real pumping well pipe will adversely affect the early measurements of drawdown which are essential in determining the storativity. One method has already been proposed to take into account the influence of well pipe storage in evaluating the parameters of the aquifer from the drawdown in a large-diameter pumping well (Papadopulos and Cooper, 1967). This method is discussed below, and then a new alternative solution is proposed.

The Papadopulos and Cooper Solution (1967)

These authors used an existing analytical (closed-form) solution for an analogous heat-flow problem to solve (2) with the boundary conditions more representative of an actual pumping well:

- the drawdown in the well is a constant;
- at any time the drawdown at the face of the well is equal to that in the well (full efficiency);
- at any time the constant rate of discharge of the well is equal to the sum of the rate of the flow of water into the well and the rate of decrease in volume of water within the well.

Papadopulos and Cooper (1967) provided tables for comparison of their solution to the Theis solution for the drawdown in the well itself. They found that the effect of well pipe storage is always negligible after a time t_l given by:

$$t_l = 250\, r_w^2 / T \tag{19}$$

where r_w is the diameter of the pumping well. They proposed new type curves which, by their own admission, cannot give reliable estimates of the S value. They stated, "However, because the matching of data plot to the type curves depends on the shape of the type curves, which differ only slightly when α differs by an order of magnitude, a determination of S by this method has questionable reliability" (see Figure 5 for the definition of α).

The graphs of the Theis and Cooper-Jacob solutions can also be used after the time t_l. Many data cannot be used, however, and, if some boundary condition is detected at a time of the same order of magnitude as t_l, it may be impossible to obtain a reliable estimate of the S value.

This solution cannot be used to analyze the influence of water storage effects in the pumping well pipe on the measured drawdown in an observation well.

Proposed Solution

At any time t, when the drawdown in the pumping well is s_w, the pump has taken a time t_w to remove the water in the "well space." This is the annular space between the inner wall of the well and the outer wall of the pump riser pipe, minus any volume occupied by instruments, for example, in the interval over which the water level declines in the well. The "well space" is characterized by an equivalent diameter D.

Consequently, a practical solution for eliminating pumping well pipe storage effects in the interpretation of the data is to correct all time readings by a variable time correction t_w to get the corrected time $t_c = t - t_w$ really spent to withdraw water from the aquifer. This time t_c is the only time to be considered in interpreting the drawdown data of the pumping well itself and of any observation well.

If Q is the pumped constant flowrate, the total volume of water extracted at time t is:

$$V = Qt \tag{20}$$

and the portion of this volume coming from pumping well pipe storage is:

$$V_w = (\pi/4)\, D^2 s_w = Q t_w \tag{21}$$

where s_w is the measured drawdown in the pumping well at time t.

Consequently, the time t_c spent to withdraw water from the aquifer is:

$$t_c = (V - V_w)/Q = (Qt - V_w)/Q \tag{22}$$

The proposed time correction is theoretically valid only when the Cooper-Jacob approximation is valid (zone 2 of Figure 3), provided the pumping rate is constant. If it is not constant, the proposed time correction method is also applicable when the Cooper-Jacob approximation is valid, after the drawdown s is replaced by the specific drawdown s/Q, which is also equivalent to using a weighted logarithmic mean time (Kruseman and De Ridder, 1979). Then the usual Cooper-Jacob graph may be used, where the drawdown data s (r, t_c) are plotted versus the corrected time t_c to obtain T from the straight-line approximation and S from the extrapolated corrected time t_{c0}.

This practical method is illustrated in the next section with a worked example (real case).

Worked Example
Theis and Cooper-Jacob Methods

Using the Cooper-Jacob graph, Josseaume (1970) interpreted the broken shape AB and BC (Figure 4) as resulting from the influence of pumping well pipe storage.

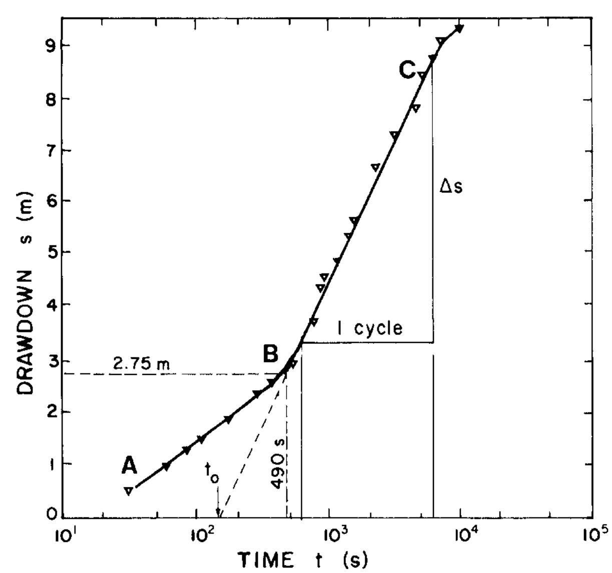

Fig. 4. Pumping well drawdown versus log (time) indicates the influence of pumping well pipe storage (from Josseaume, 1970).

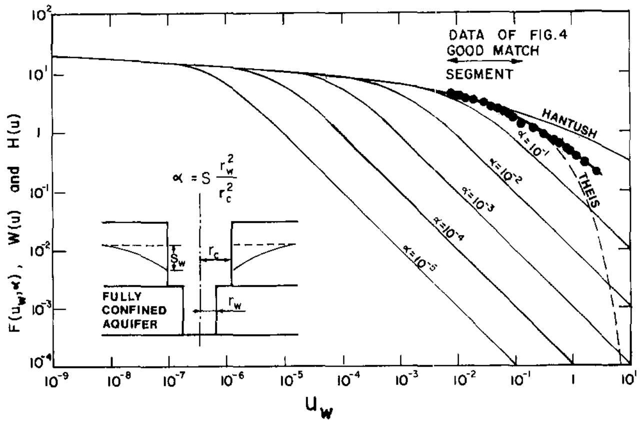

Fig. 5. Type curves of Theis and Hantush with those of Papadopulos and Cooper for the drawdown in pumping wells of large diameter and/or aquifers of low transmissivity. Data for the pumping well of the example.

The well of diameter $D = 0.20$ m was pumped at a constant flowrate $Q = 0.93\ m^3/h = 2.58 \times 10^{-4}\ m^3/s$. At point B ($t = 490$ s and $s = 2.75$ m in Figure 4), Josseaume (1970) calculated that the total pumped volume was:

$$V = Qt = 2.58 \times 10^{-4}\ m^3/s \times 490\ s = 0.127\ m^3 \quad (23)$$

whereas the volume extracted from the pumping well pipe storage was:

$$V_W = (\pi/4)\, D^2 s_W = (\pi/4) \times 0.2^2 \times 2.75 = 0.086\ m^3 \quad (24)$$

showing that AB (Figure 4) represents mainly the drawdown during the extraction of water stored in the pumping well pipe.

Branch BC (Figure 4) has a slope $\Delta s = 5.4$ m/cycle and an intercept $t_0 = 145$ s. These values give the following transmissivity and storativity: $T = 8.8 \times 10^{-6}\ m^2/s$ and $S = 0.286$. This storativity value is much too large for a confined aquifer. Data for an observation well at a distance $r = 7.0$ m from the pumping well gave average values: $T = 6.8 \times 10^{-6}\ m^2/s$; $S = 4.1 \times 10^{-5}$ (see Figures 1 and 2). The T values are similar, but there is a large difference in S values.

The Theis superposition method is shown in Figure 5 which also includes the functions $F(u, \alpha)$ as defined by Papadopulos and Cooper (1967) and the function of Hantush (1964). As could be expected from Figure 4, the earlier data cannot be matched with the Theis curve. However, data for $t > 500$ s can slide along some portion of the Theis type curve, yielding $T = 8.5$ to $9.8 \times 10^{-6}\ m^2/s$ and $S = 0.23$ to 0.34, in reasonable agreement with the Cooper-Jacob method.

Papadopulos and Cooper Method

According to Papadopulos and Cooper (1967), their function $F(u_W, \alpha)$ that accounts for pumping well pipe storage effect is close to the Theis function $W(u)$ after a time $t_1 = 250\ r_W^2/T = 250 \times 0.1^2/9 \times 10^{-6} = 278{,}000$ s $= 3.2$ days, whatever the value of α. The fact that t_1 largely exceeds the observation time for the examined well does not justify an automatic rejection of data for times $t < t_1$, but it does indicate that matching should be tried with the curves $F(u_W, \alpha)$.

Actually, the drawdown data for the pumping well cannot be superposed on any α-curves (Figure 5), but they follow the Theis curve fairly well after some time. Consequently, the Papadopulos and Cooper method is reduced to the Theis method for $t > 500\ s = t_1/556$, a value significantly lower than the theoretical time t_1.

Proposed Method

(a) The first step in the proposed method is to compare the data for the total volume of water extracted at time t, and the volume coming from pumping well pipe storage, as shown in Figure 6 where the straight line represents the equality of equations (20) and (21).

Calculations indicate that for times $t \leq 180$ s, the volume coming from pumping well pipe storage is higher than the total volume pumped out at a constant flowrate $Q = 2.58 \times 10^{-4}\ m^3/s$, which is clearly impossible. In order to have equal volumes, the origin of the abscissa must be moved to the left as shown in Figure 6. It means that pumping must be considered to have started 60 s before the indicated time. Consequently, 60 s was added to time t—which became t^* ($t^* = t + 60$ s)—in equation (22) to calculate the real time t_c spent to withdraw water from the aquifer (Table 1).

Several factors may explain the need for this correction to make the data agree with the theoretical solution in which the well discharge is assumed to be constant. Actually, the well discharge may vary initially as the pump adjusts itself to the changing pressure, or there may be a time-lag. Another common reason for a time correction is starting the chronometer when water flows out of the discharging pipe rather than when the pump is started.

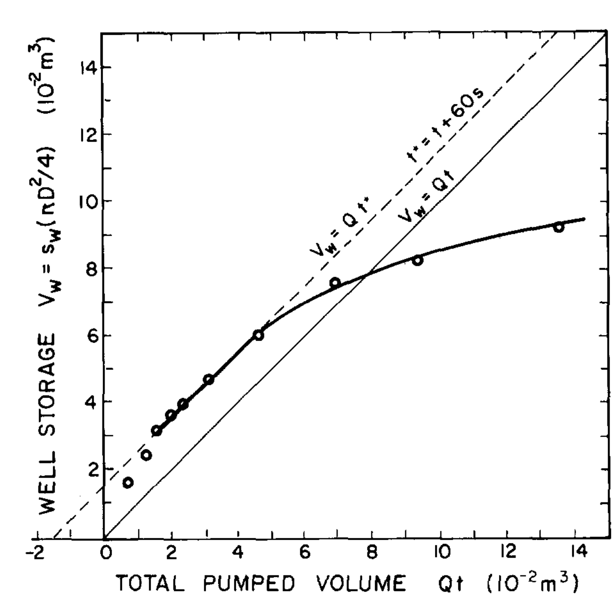

Fig. 6. Plot of total pumped volume (Qt) versus pumping well storage (V_W) assuming $D = D_W$.

Table 1. Example of Time Correction for the Pumping Well Pipe Storage Effect

Time t (s)	*Time* t* t* = t + 60 s	*Drawdown* s_w (m)	Qt* (m^3)	V_w (m^3)	$t_c = (Qt^* - V_w)/Q$ (s)
00 030	00 090	0.50	2.33×10^{-2}	1.57×10^{-2}	29
00 045	00 105	0.75	2.71×10^{-2}	2.36×10^{-2}	14
00 060	00 120	1.00	3.10×10^{-2}	3.14×10^{-2}	−2
00 075	00 135	1.15	3.49×10^{-2}	3.61×10^{-2}	−5
00 090	00 150	1.25	3.88×10^{-2}	3.93×10^{-2}	−2
00 120	00 180	1.50	4.65×10^{-2}	4.71×10^{-2}	−2
00 180	00 240	1.90	6.20×10^{-2}	5.97×10^{-2}	9
00 270	00 330	2.40	8.52×10^{-2}	7.54×10^{-2}	38
00 360	00 420	2.60	1.09×10^{-1}	8.17×10^{-2}	104
00 540	00 600	2.95	1.55×10^{-1}	9.27×10^{-2}	241
00 720	00 780	3.75	2.02×10^{-1}	1.18×10^{-1}	324
00 840	00 900	4.40	2.33×10^{-1}	1.38×10^{-1}	365
00 900	00 960	4.60	2.48×10^{-1}	1.45×10^{-1}	401
01 200	01 260	4.90	3.26×10^{-1}	1.54×10^{-1}	664
01 500	01 560	5.40	4.03×10^{-1}	1.70×10^{-1}	903
02 100	02 160	6.70	5.58×10^{-1}	2.10×10^{-1}	1 345
03 000	03 060	7.40	7.90×10^{-1}	2.32×10^{-1}	2 160
04 800	04 860	7.80	1.256	2.45×10^{-1}	3 911
05 400	05 460	8.40	1.411	2.64×10^{-1}	4 438
06 000	06 060	9.25	1.566	2.91×10^{-1}	4 935
08 000	08 060	9.60	2.082	3.02×10^{-1}	6 893
10 000	10 060	9.80	2.599	3.08×10^{-1}	8 868

$Q = 0.93\ m^3/h = 2.58 \times 10^{-4}\ m^3/s$; $V_w = \pi D^2 s_w/4$; $D = 0.20$ m.

Table 2. Summary of Results

Method		*Pumping well*	*Observation well*
Theis	T (m^2/s)	$8.5\text{-}9.8 \times 10^{-6}$	6.6×10^{-6}
	S (—)	0.23-0.34	5.3×10^{-5}
Cooper-Jacob	T (m^2/s)	8.8×10^{-6}	6.8×10^{-6}
	S (—)	0.286	4.1×10^{-5}
Papadopulos and Cooper	T (m^2/s)	poor fitting	not applicable
	S (—)	≡ Theis	not applicable
Proposed with C-J	T (m^2/s)	1.13×10^{-5}	1.04×10^{-5}
	S (—)	0.10	1.2×10^{-5}

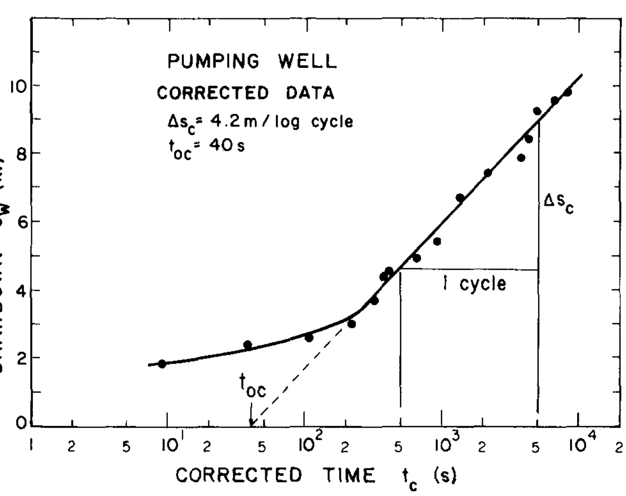

Fig. 7. Drawdown in the pumping well versus the log of corrected time t_c.

(b) The second step in the proposed method is to plot the pumping well drawdown data against the log of the corrected time t_c (Figure 7). This gives the usual trend which is expected with the Cooper-Jacob representation. The straight portion (Figure 7) has a slope $\Delta s = 4.2$ m/cycle and an intercept $t_0 = 40$ s. These values give a transmissivity $T = 1.13 \times 10^{-5}\ m^2/s$ which is slightly higher (30%) than previously calculated (see Table 2) with the raw data, and a storativity $S = 0.10$ which once again is much too high for the confined aquifer.

(c) The third step in the proposed method is to apply the same time correction to the interpretation by the Cooper-Jacob method of the drawdown data of the observation well, as shown in Figure 2 (corrected data). After correction, the observation well has a transmissivity $T = 1.04 \times 10^{-5}\ m^2/s$ and a storativity $S = 1.2 \times 10^{-5}$, a reasonable value for a confined aquifer. Thus, the pumping and observation wells have similar T values as indicated in Table 2 which summarizes the results of the different interpretations.

Discussion and Conclusion

The poor estimate of S (Table 2) given by the pumping well data must be related to short-term effects in the most perturbed zone of the aquifer (adjacent to the well) where there are high gradients and positive or negative skin effects. In addition, there may be a non-negligible time-lag between the head decline and the release of stored water (assuming that S is a time-independent constant is a simplification of the real behavior of soils and rocks). This time-lag has a major influence on early time data, but its influence progressively vanishes at some distance and at larger times when the Cooper-Jacob approximation becomes valid. These different reasons can explain the inadequate S value deducted from the pumping well drawdown data.

In a sense, this confirms the common opinion (Kruseman and De Ridder, 1979) that it is highly preferable to evaluate the aquifer parameters from drawdown observations in piezometers, because they are not influenced by anomalous properties in the vicinity of the pumping well. Furthermore, it was recently demonstrated by Butler (1990) that "the further an observation well is from the pumping well, the less the drawdown is impacted by the properties of material in the immediate vicinity of the observation well."

In conclusion, the effect of real pumping well pipe storage capacity on the drawdown data of any observation well may be taken into account by using a variable time correction which is determined from the pumping well drawdown data. This proposed method is valid when the Cooper-Jacob approximation is valid, and it does not use curve matching. Because it may be used for the pumping well and any observation well, it seems to have a definite advantage over the Papadopulos and Cooper method (1967) which applies only to the pumping well.

An example of the application of the proposed method to a pumping well and an observation well provides two corrected T values closer than those calculated from the raw data. It confirms, however, that early drawdown data in the pumping well itself cannot provide a reliable estimate of S for many reasons. These data are useful, however, in determining the variable time correction, and in obtaining a better estimate of transmissivity and storativity from the drawdown data in observation wells. The effect of pumping well pipe storage in the early drawdown data may be significant in cases of low transmissivity aquifers and low pumping rates, which are quite common in ground-water remediation. However, in situations where the pumping rate is high and the aquifer has a high transmissivity, the pumping well pipe storage effects may be insignificant.

Acknowledgments

This paper is the result of a research program involving both theoretical analysis and fieldwork designed to improve the reliability of permeability and pumping tests. These studies have been sponsored by the Natural Sciences and Engineering Council of Canada under grant URF0043967. This financial assistance is gratefully acknowledged.

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