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# An improved neural network approach to the determination of aquifer parameters

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**Abstract**

In this paper, an artificial neural network (ANN) approach to the determination of aquifer parameters is developed. The approach is based on the combination of an ANN and the Theis solution. The proposed ANN approach has advantages over the existing ANN approach. It avoids inappropriate setting of a trained range. It also determines the aquifer parameters more accurately and needs less required training time. Testing the existing and the proposed ANN approaches by 1000 sets of synthetic data also demonstrates these advantages. As to the comparison between the proposed ANN approach and the type-curve graphical method, an application to actual time-drawdown data shows that the proposed ANN approach determines the aquifer parameters more precisely. The proposed ANN approach is recommended as an alternative to the type-curve graphical method and the existing ANN approach.


*Keywords:* Aquifer parameters; Artificial neural network; Back-propagation algorithm; Aquifer test

## 1. Introduction

The determination of aquifer parameters has always been a challenging task for groundwater resources engineers and managers (for example, Jacob, 1940; Hantush, 1956; Walton, 1962; Wikramaratna, 1985; Aziz and Wong, 1992; Zhan et al., 2001; Chen and Chang, 2002; Balkhair, 2002; Chen and Chang, 2003), because it holds a central position in groundwater modeling. The aquifer parameters obtained by the type-curve graphical method (Jacob, 1940) are of questionable reliability (Aziz and Wong, 1992; Balkhair, 2002). In recent years, some convenient and reliable approaches based on artificial neural networks (ANNs) have been developed.

Neural networks, which were devised via imitating brain activity and are capable of modeling and identifying complex systems, provide an alternative approach to the determination of aquifer parameters. Artificial neural networks (ANNs) were first developed in the 1940s (McCulloch and Pitts, 1943). Generally speaking, neural networks are information processing systems. In recent decades, considerable interest has been raised over their practical applications, because the current algorithms can overcome the limitations of early networks. Bypassing the model construction and parameter estimation phases

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adopted by most of the conventional techniques, ANNs can be automatically developed through a simple training process. Such a training process enables the neural system to capture the complex and nonlinear relationships between the known input data and the desired output data that are not easily analyzed using conventional methods.

ANNs have also found increasing applications in various aspects of hydrology because of their ability to model and to identify both linear and nonlinear systems. Previous studies have shown the potential of ANNs for analyzing hydrology and water resource problems (for example, Coulibaly et al., 2001; Hsu et al., 1995; Clair and Ehrman, 1996; Poff et al., 1996; Gumrah et al., 2000; Lin and Chen, 2004a,b). As to the determination of aquifer parameters, Aziz and Wong (1992) and Balkhair (2002) determined aquifer parameters from aquifer test data by an ANN approach, which is referred to as the existing ANN approach herein. They trained their networks through a supervised learning scheme known as back-propagation (Rumelhart et al., 1986).

However, the existing ANN approach has its drawback. It is not capable of producing aquifer parameter values accurately when the desired values are out of the trained range. Hence, the performance of the existing ANN approach is largely determined by the selection of a range of aquifer parameter values in the training phase. However, there is no established methodology for selecting an appropriate trained range especially when there is no prior information of the aquifer parameters available. Such a limitation has prompted a search for an improved ANN approach to estimating aquifer parameters. In this paper, an alternative ANN approach is proposed. The proposed ANN approach has three advantages over the existing ANN approach. First, it avoids the aforementioned problem regarding the selection of an appropriate trained range. Second, it determines the aquifer parameter values more accurately. Finally, the proposed ANN has a simpler structure and is trained more rapidly.

This paper is organized as follows. First, an alternative ANN approach is proposed which is capable of determining the aquifer parameters from aquifer test data. Then two applications are performed and their results are presented to demonstrate the advantages of the proposed approach. Finally, conclusions are drawn.

## 2. The proposed ANN approach

The proposed ANN approach is developed based on the combination of an ANN and an analytical solution that can calculate the drawdown from known aquifer parameters. Theis (1935) presented an analytical solution to calculate the drawdown $s$ at a distance $r$ from the transmissivity $T$, the storage coefficient $S$ and the discharge $Q$. The Theis solution can be written as

$$s = \frac{Q}{4\pi T} W(u) \tag{1}$$

where

$$W(u) = \int_u^\infty \frac{1}{y} \exp(-y) \mathrm{d}y \tag{2}$$

and

$$u = \frac{r^2 S}{4Tt} \tag{3}$$

The type-curve graphical method is developed based on the Theis solution. In the type-curve graphical method for the determination of aquifer parameters, one has to fit observed time-drawdown data to a type curve of $W(u)$ versus $1/u$ and find a match point. In this paper, the above procedures are done by a three-layered ANN (Fig. 1). An ANN is composed of a number of interconnected processing elements. These elements, called neurons, are joined

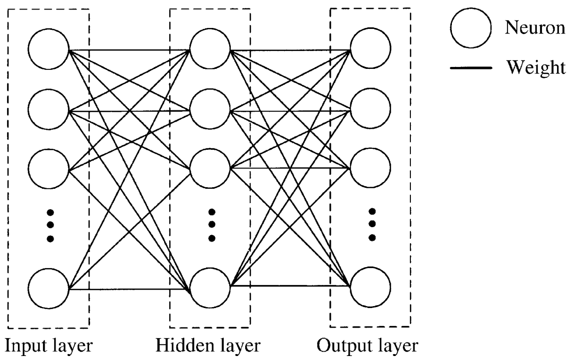


Fig. 1. Architectural graph of a three-layered ANN.

together with weighted connections. The parameters associated with each of these connections are called weights.

In order to determine the aquifer parameters from a set of $N$ observed time-drawdown data, the proposed ANN includes $N-1$ neurons in the input layer. These $N-1$ neurons are designed to process an input vector consisting of $N-1$ components. The components of the input vector are obtained from $N$ observed time-drawdown data as

$$x_i = \log(s_{i+1}) - \log(s_1) = \log\left(\frac{s_{i+1}}{s_1}\right),\quad i = 1, 2, \ldots, N-1 \tag{4}$$

where $s_i$ is the drawdown observed at time $t_i$. The proposed ANN is designed to produce the $1/u$ coordinate of the match point ($1/u_\mathrm{m}$) as

$$\hat{y} = \log\left[\frac{1}{u_\mathrm{m}}\right] \tag{5}$$

The components of input vector and the output of the proposed ANN are shown in Fig. 2.

An ANN has to be trained so as to produce the desired outputs when the associated inputs are received. The training process is an iterative process for determining appropriate weights that minimize the objective function $F$

$$F = \frac{1}{2}\left[\sum_{i=1}^{N_\mathrm{train}}\sum_{j=1}^{N_\mathrm{out}}(d_{ij} - y_{ij})^2\right] \tag{6}$$

where $d_{ij}$ is the desired output, $y_{ij}$ is the actual output, $N_\mathrm{train}$ is the number of patterns in the training data set and $N_\mathrm{out}$ is the number of neurons in the output layer. Learning continues until $F$ converges to a convergence criterion, which is an acceptably small value.

Training of the proposed ANN consists of generating the training patterns and adjusting the weights and biases of each neuron by the back-propagation algorithm. A training pattern includes an input vector and a target output vector. To generate training patterns, a trained range of the output values must be specified. Then the values selected from the trained range are used during the generation process. According to the type curves presented by Walton (1962), log ($1/u$) is always greater than $-0.5$. When log ($1/u$) is greater than 4.0, log $W(u)$ approaches to a constant. Hence the trained range of the proposed ANN output, log ($1/u_\mathrm{m}$), is selected from $-0.5$ to 4.0. Once a specific log ($1/u_\mathrm{m}$) is chosen, the $W(u_\mathrm{m})$ value is calculated. Then, the input vector components are generated as

$$x_i = \log\left[\frac{W(u_\mathrm{m}t_1/t_{i+1})}{W(u_\mathrm{m})}\right],\quad i = 1, 2, \ldots, N-1 \tag{7}$$

Fig. 3 illustrates the input vector and target output of the training pattern.

The proposed ANN is trained with training patterns by the back-propagation algorithm, which is the most popular algorithm available for adjusting the weight coefficient $w_{ij}^{k-1,k}$ between neuron $i$ in layer $k-1$ and neuron $j$ in layer $k$. The output $u_j^k$ from neuron $j$ in layer $k$ can be obtained from

$$u_j^k = \sum_{i=1}^{m} w_{ij}^{k-1,k} u_i^{k-1} \tag{8}$$

where $m$ is the total number of inputs applied to neuron $i$ in layer $k-1$. The change of weights is

$$\Delta w_{ij}^{k-1,k}(n+1) = \eta\delta_j^k u_i^{k-1} + \alpha\Delta w_{ij}^{k-1,k}(n) \tag{9}$$

where $n$ is the iteration number, $u_i^{k-1}$ is the output from neuron $i$ in layer $k-1$, $\delta_j^k$ is the local gradient for neuron $j$ in layer $k$, $\eta$ is the learning-rate parameter and $\alpha$ is the momentum constant. The $\delta_j^k$ can be calculated by

$$\delta_j^k = \begin{cases} e_j\dfrac{\mathrm{d}f(u_j^l)}{\mathrm{d}u_j^l}, & \text{for neuron } j \text{ in output layer } l \\ \dfrac{\mathrm{d}f(u_j^k)}{\mathrm{d}u_j^k}\displaystyle\sum_{r=1}^{m_{k+1}}\delta_r^{k+1}w_{jr}^{k,k+1}, & \text{for neuron } j \text{ in hidden layer } k \end{cases} \tag{10}$$

where $f$ is the output function for each neuron, $m_{k+1}$ is the total number of inputs applied to neuron $r$ in layer $k+1$, and $e_j$ is the error at the output of neuron $j$. The most often used output function for ANN is the sigmoid function:

$$f(u_j^k) = \frac{1}{1+\exp(-u_j^k)} \tag{11}$$

Once the ANN is well trained, it is capable of producing an output when an input vector transformed

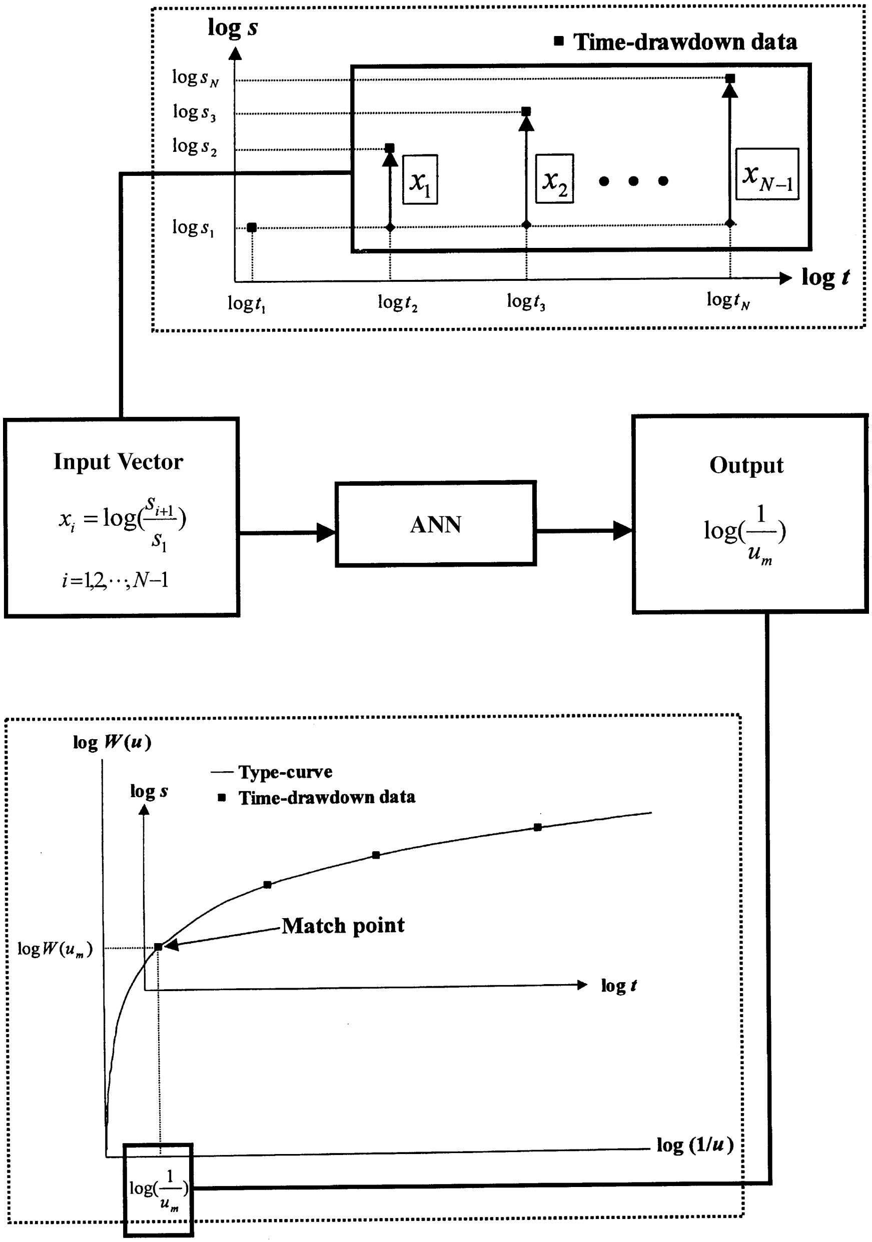


Fig. 2. The components of input vector and the output of the proposed ANN.

from time-drawdown data is processed. Then the coordinates of the match point $W_m$, $s_m$, $1/u_m$ and $t_m$ are determined from the output and time-drawdown data as follows

$$\frac{1}{u_m} = 10^{\hat{y}} \tag{12}$$

$$W_m = W(u_m) = W\left(\frac{1}{10^{\hat{y}}}\right) \tag{13}$$

$$s_m = s_1 \tag{14}$$

and

$$t_m = t_1 \tag{15}$$

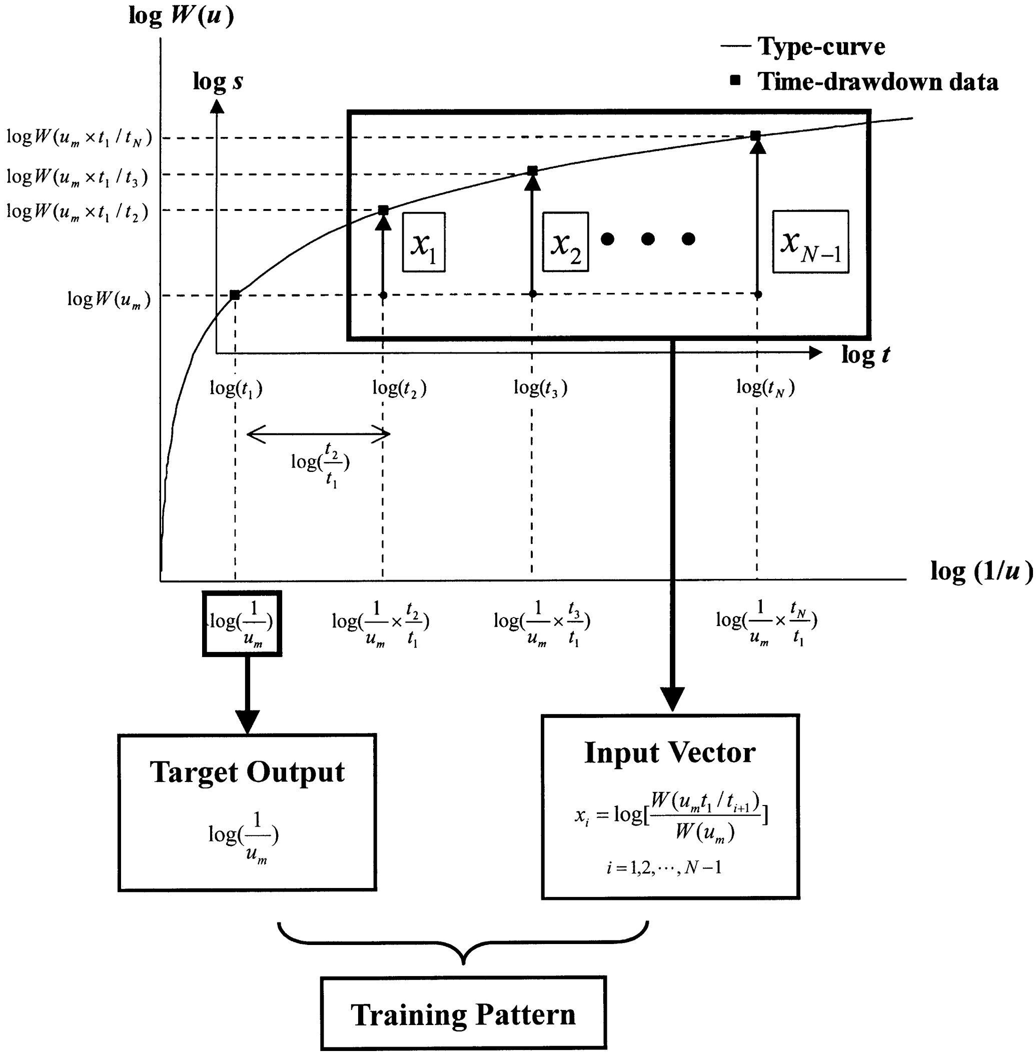


Fig. 3. The input vector and target output of the training pattern.

Finally, transmissivity $T$ and storage coefficient $S$ are determined by

$$T = \frac{QW_{\mathrm{m}}(u)}{4\pi s_{\mathrm{m}}} \tag{16}$$

and

$$S = \frac{4Tu_{\mathrm{m}}t_{\mathrm{m}}}{r^2} \tag{17}$$

The major difference between the proposed and the existing ANN approach is the design of input and output components. Fig. 4 shows the structures of the proposed and the existing ANNs. Our design makes the proposed ANN approach successfully avoid the problem regarding the selection of an appropriate trained range. Moreover, the proposed ANN has a simpler structure and is trained more rapidly than the existing ANN. These advantages will be further demonstrated in the next section.

## 3. Application and discussion

In this section, the proposed approach is applied to two examples. In the first example, 1000 sets of

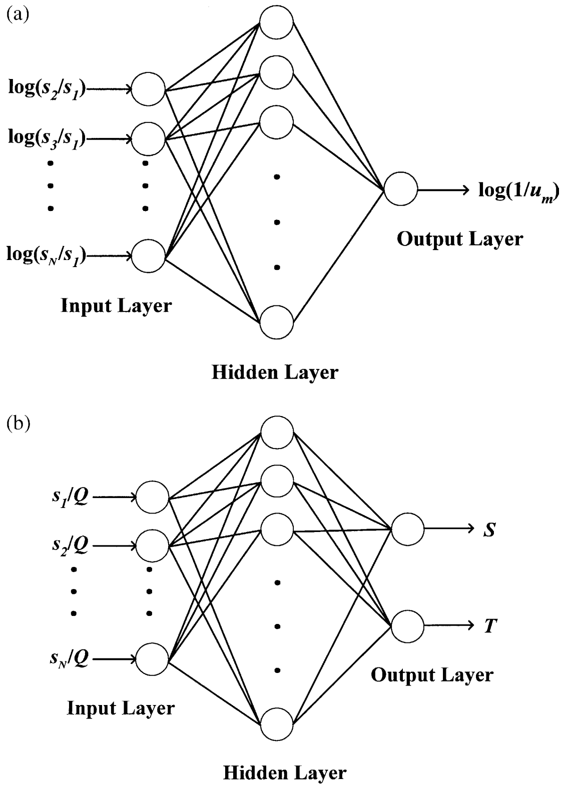


Fig. 4. Structures of (a) the proposed and (b) the existing ANNs.

synthetic data are used to test the accuracy of the existing ANN approach and the proposed ANN approach. In the second example, actual time-drawdown data are used to test the applicability and reliability of the proposed ANN approach. The time-drawdown data, which are taken from Walton (1962), were measured during pumping at a constant discharge of 1199 $m^3$/day and the distance between observed well and pumping wells is 502 m.

### 3.1. Example 1: testing the existing and the proposed ANN approaches using synthetic data

In the existing ANN approach, a three-layered ANN is designed to produce values of $T$ and $S$ when the values of $s/Q$ are received. The components of the input and output vectors are shown in Fig. 4. To generate training patterns, a trained range of the ANN output values for both $T$ and $S$ must be specified and then combinations of these values are used during the generation process. Once a specific combination of $T$ and $S$ values is chosen, the corresponding input data are calculated using the Theis solution, Eq. (1). In this example, 22 and 2 neurons are constructed in the input and output layers, respectively. Then the ANN is trained with 10,201 training patterns which are generated using $T$ values ranging from 100 to 1000 $m^2$/day with a step size of 9.0 $m^2$/day and $S$ values ranging from $10^{-6}$ to $10^{-5}$ with a step size of $9.0\times10^{-8}$.

The proposed ANN includes 21 and 1 neurons in the input and output layers, respectively. To train the proposed ANN, a total of 10,201 training patterns are generated using $\log(1/u)$ values ranging from $-0.5$ to 4.0 with a step size of $4.41\times10^{-4}$. The existing and the proposed ANN are trained with the same influential parameters and the same number of training patterns. The ANN influential parameters used during training are given in Table 1.

After the ANNs are trained with the training patterns, the trained ANNs are capable of yielding the corresponding estimated output values when the input values of the training patterns are processed. Given a set of estimated $T$ and $S$ values, the estimated drawdown can be calculated by the Theis solution, Eq. (1). Then, the relative root mean square error (RRMSE) of the estimated drawdown is computed according to

$$\mathrm{RRMSE} = \sqrt{\frac{1}{N_s}\sum_{i=1}^{N_s}\left(\frac{\hat{s}_{\mathrm{i}} - s_{\mathrm{i}}}{s_{\mathrm{i}}}\right)^2} \tag{18}$$

where $\hat{s}_{\mathrm{i}}$ is the estimated drawdown, $s_i$ is the observed drawdown and $N_s$ is the number of drawdowns. Table 2 summarizes the ANN structures and the required training times for the existing and proposed ANN approaches. The required training times for the existing and the proposed ANN approaches are 76 and

Table 1
The ANN influential parameters used during training

| Parameter | Value |
|---|---|
| Learning rate | 0.5 |
| Momentum constant | 0.6 |
| Convergence criterion | 0.001 |
| Maximum training cycle | 10,000 |
| Number of training patterns | 10,201 |

Table 2
The numbers of ANN neurons and the required training times for the existing and the proposed ANN approaches

| ANN approach | Number of ANN neurons | | | Required training time (min) |
|---|---|---|---|---|
| | Input layer | Hidden layer | Output layer | |
| Existing | 22 | 32 | 2 | 76 |
| Proposed | 21 | 32 | 1 | 17 |

17 min on a 1.8 GHz personal computer, respectively. The proposed ANN approach yields a training time reduction of 78% as compared to the existing ANN approach. The proposed ANN is trained more rapidly than the existing ANN because of its simpler structure. Table 3 summarizes the RRMSE values for the existing and proposed ANN approaches during the training process. The RRMSE values for the existing and proposed ANN approaches during the training process are 8 and 1%, respectively. The proposed ANN approach yields a RRMSE reduction of 87.5% as compared to the existing ANN approach.

To assess the generalization performances of the existing and the proposed ANN approaches, 1000 tested patterns that are not used during the training process are employed. The tested patterns are randomly generated from combinations of idealized $T$ and $S$ values ranging from $10^2$ to $10^6$ $m^2$/day and $10^{-2}$ to $10^{-6}$, respectively. Figs. 5 and 6 show the scatter plots for the idealized and estimated aquifer parameters, $T$ and $S$, obtained by the proposed and the existing ANN approaches. As shown in Figs. 5 and 6, the existing ANN approach is capable of accurately estimating aquifer parameters only over the trained range. When the tested values fall outside the trained range, the existing ANN approach cannot produce desired output. The performance of the existing ANN approach is largely determined by the selection of the trained range. Moreover, there is no established methodology for selecting an appropriate trained range especially when there is no prior information

Table 3
The RRMSE values for the existing and the proposed ANN approaches during the training process

| ANN approach | RRMSE (%) |
|---|---|
| Existing | 8 |
| Proposed | 1 |

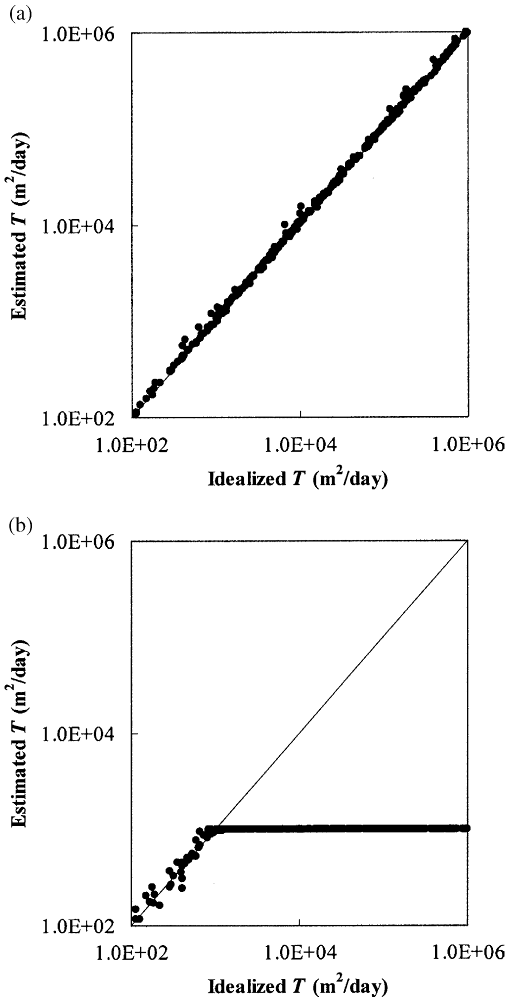


Fig. 5. Idealized $T$ values versus estimated $T$ values obtained by (a) the proposed and (b) the existing ANN approaches.

of the aquifer parameters available. Hence, the existing ANN approach has the problem of selecting an appropriate trained range.

On the contrary, almost all the estimated aquifer parameters obtained by the proposed ANN approaches match the idealized aquifer parameters perfectly over the whole tested range (Figs. 5 and 6). The proposed ANN approach can accurately estimate aquifer parameters over a wide tested range. It can also avoid the problem of selecting an appropriate trained range. Hence, the proposed ANN approach is a better method as compared to the existing ANN approach.

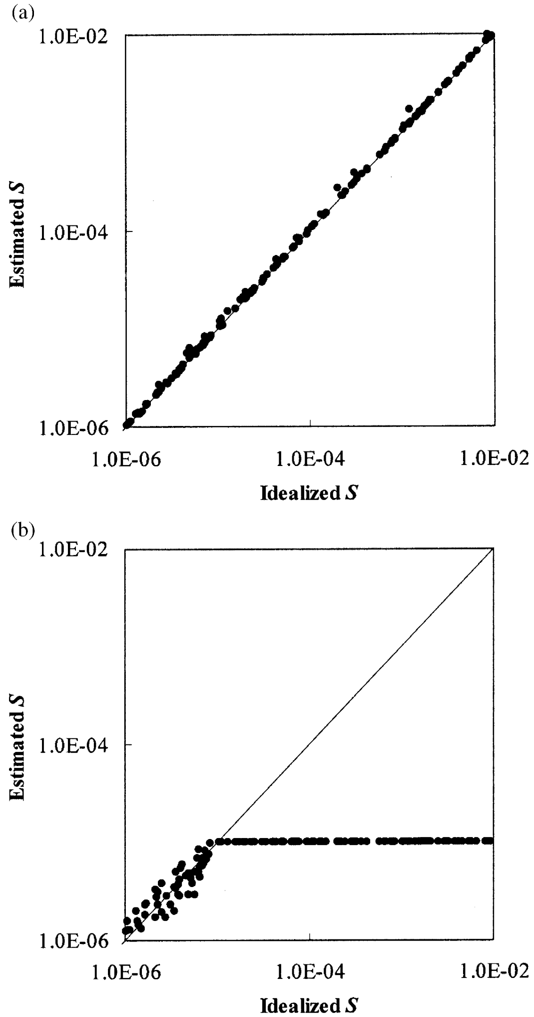


Fig. 6. Idealized $S$ values versus estimated $S$ values obtained by (a) the proposed and (b) the existing ANN approaches.

### 3.2. Example 2: testing the proposed ANN approach using field data

When the existing ANN approach is applied to this example, it is very difficult to select an appropriate trained range because there is no prior information of the aquifer parameter available. Hence, only the proposed ANN approach is applied herein. Testing the trained proposed ANN with the field data results in $T$ and $S$ values of 119 m$^2$/day and $2.220\times10^{-5}$, respectively. The values of $T$ and $S$ obtained by the type-curve graphical method are 125 m$^2$/day and $2.0\times10^{-5}$ (Walton, 1962). Table 4 summarizes the estimated aquifer parameters and the RRMSE values for the proposed ANN approach and the type-curve graphical method. The RRMSE values for the ANN approach and the type-curve graphical method are 3 and 10%, respectively. The proposed ANN approach yields a RRMSE reduction of 70% as compared to the type-curve graphical method.

Table 4
The estimated aquifer parameters and RRMSE values for the proposed ANN approach and the type-curve graphical method

| Method | Aquifer parameter | | RRMSE (%) |
|---|---|---|---|
| | $T$ (m$^2$/day) | $S$ ($10^{-5}$) | |
| Proposed ANN approach | 119 | 2.220 | 3 |
| Type-curve graphical method | 125 | 2.000 | 10 |

## 4. Summary and conclusions

The objective of this paper is to develop an alternative approach to the determination of aquifer parameters from aquifer test data. ANNs provide an effective approach, but the existing ANN approach has a problem of selecting an appropriate trained range. In this paper, an alternative ANN approach to the determination of aquifer parameters from aquifer test data has been proposed. The proposed ANN approach is based on the combination of an ANN and the Theis solution. The major difference between the existing and the proposed ANN approaches is the design of ANN input and output components. Our design of ANNs has three advantages over the existing ANN approach. First, the proposed ANN approach avoids the problem of selecting an appropriate trained range. Second, the proposed ANN approach determines the aquifer parameter values more accurately. Third, the proposed ANN has a simpler structure and needs less training time. Testing the existing and the proposed ANN approaches by 1000 sets of synthetic data also demonstrates these three advantages. As to the comparison between the proposed ANN approach and the type-curve graphical method, an application to actual time-drawdown data shows that the proposed ANN approach performs better than the type-curve graphical method. The proposed ANN approach is recommended as an alternative to the existing methods (the type-curve graphical method and the existing ANN approach) because of its simpler ANN structure, less required training time, higher accuracy. In addition, the proposed methodology can be applied to more complex aquifers by combining an ANN and other analytical solutions, which may describe the physical mechanisms of the more complex aquifers, instead of the Theis solution.

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