COMPUTER APPLICATION ON WELL TEST MATHEMATICAL MODEL COMPUTATION OF HOMOGENOUS AND MULTIPLE-BOUNDED RESERVOIRS

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ABSTRACT
This paper presents a reasonable computer application solution of the modern well test analysis mathematical model, concerning the mathematical model and its analytical solution. The works of this paper are conducted to develop a computer program package designed in VISUAL BASIC6, and the utility of this program is to follow the procedure for solving well test mathematical model and establishing the most widely known as theoretical type curves. The appropriate calculations that are provided by the program, are that for each reservoir boundaries' situation the program has an ability to establish multi-type curve for a variety of well and reservoir parameters. Here it is worth mentioning that many publications have no clearly integrated explanation of the mathematical background of establishing well test type curves. Therefore the paper would be considered as a technical guide to what has been done so far regarding the current Computer Aided Application of modern well test theory for the condition of homogeneous and multiple linear boundaries reservoir.

Keyword: *Well test mathematical model; homogeneous; type curves; boundaries; computer application.*

1. INTRODUCTION

The Paper focuses on homogeneous multiple-linear boundaries model, in which the homogeneous reservoir model is considered as a benchmark against which well test data are matched. All methods for the analysis of data from well testing are based on the diffusivity equation for fluid flow through porous media; this work covers the methods of analytical and numerical solution to diffusivity equation. A common method for solving the diffusivity equation is to use the Laplace transformation; the advantages of this method have been described by Van Everdingen (1949). By this method, the equations are transformed into a system of ordinary differential equations, which can be solved analytically. The resulting solution in the transformed space is a function of the Laplace variable. To invert the solution to real time and space, the inverse Laplace transformation is used. Stehfest (1970) presented a numerical inversion algorithm used to invert the Laplace space solution to real space. The analytical solution to diffusivity equation lead to generate Bessel equation, this equation makes use of Bessel functions, Abramowitz (1970) and Stegum present polynomial approximation to compute the modified Bessel function, Giovanni (1990).

Type curves are the graphical representation or numerical solutions to the diffusivity equation for specified case of initial condition, boundary conditions and geometry. According to the literature review, many scientists have presented intensive works on the subject of well test interpretation type curves. To review the tasks of the type curves in well test interpretation, Bourdet (1983) presents a most known set of type curves which simplifies well test analysis and diagnoses.

In modern graphical analyses, the uses of pressure derivative curves have become standard because the curves have greatest precision in the parts of the response of greatest interest and have easily identifiable characteristics.

The introduction of the pressure derivative can be considered as a major breakthrough in the use of type curves, since it facilitates interpretation and uniqueness of match. Modern analysis has been greatly enhanced by the use of the derivative plot introduced by Bourdet(1983) (1989)and Pirard,(1986);E.A.Proano(1986) presented the application of the derivative to bounded reservoirs. Ehlig (1988) showed the visual impression afforded by the log-log presentation has been greatly enhanced by the introduction of the pressure derivative.

Because the presence of a fault in a reservoir is of great importance, considerable numbers of pressure analysis techniques dealing with this situation have been proposed in literature. Djebbar (1979) shows a type curves matching technique for interpretation of the pressure transient behavior of wells located in various Multiple Sealing-fault systems, Tiab and Kumar (1980) presents study on the behavior of a well located between two parallel boundaries.

A.C.Gringarten (1986) discusses the use of computer in analyzing well tests effectively. I.M.Buhidma and W.C.chu (1992) present state-of-the-art computer application of pressure transient analysis. Horne (1994) has summarized modern approaches to well test analysis-using computers.

This paper establishes a computer package application based on the previous conceptions, techniques and methods that are currently applied in well test analysis.

2. MATHEMATICAL MODEL

Based on hypotheses of dimensionless group, the basis flow equations through porous media, and well test mathematical model conditions can be drawn as follows:

Flow control equation
$$\frac{\partial^2 p_D}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial p_D}{\partial r_D}=\frac{\partial p_D}{\partial t_D} \qquad (1)$$

The above diffusivity equation defines how an element of reservoir will react to a local pressure disturbance. To completely define the problem, the following information needs to be defined:

Initial condition:
$$p_D(r_D,0)=0 \qquad (2)$$

Inner boundary condition:
Skin effect condition
$$p_{wd}=\left[p_D-S\left(\frac{\partial p_D}{\partial r_D}\right)\right]_{r_D=1} \qquad (3)$$

Inner boundary condition:
wellbore storage condition
$$C_D\frac{dp_{wD}}{dt_D}-\left[r_D\frac{\partial p_D}{\partial r_D}\right]_{r_D=1}=1 \qquad (4)$$

Outer boundary condition:
$$\lim_{r_D-\infty}[p_D(r_D,t_D)]=0 \qquad (5)$$

3. PRESSURE AFFECTED BY AN OUTER BOUNDARY

The effect of outer boundary on characteristics of well testing type curve can be demonstrated by using the method of Images and the principle of superposition.

Superposition of image wells:

The principle of superposition, applied in time to flow periods, can also be applied in space, in order to reproduce the effects of nearby wells and/or boundaries, this principle is used in interference testing, and can also be used to model sealing and constant pressure boundaries

The assumption is that the pressure responses at the flowing well, due to a nearby fault, and is physically equivalent to the pressure that would be observed at the active well with the presence of a nearby producing well. This is the 'method of images, with a virtual image well replacing the fault and creating the same effect.

The dimensionless bottom hole pressure (p_{wD}) for well testing of homogenous reservoir with outer boundary effect can be expressed as follows:

$$p_{wD}=p_{wDi}+p_{wDb} \qquad (6)$$

p_{wDi}, dimensionless bottom hole pressure infinite homogenous reservoir

p_{wDb}, dimensionless bottom hole pressure affected by outer boundary

The p_{wDb} is dimensionless line source solution pressure of infinite homogenous reservoir, which is a function in the principle of superposition.

The expression of dimensionless line source solution pressure of infinite homogenous reservoir given by follows:

$$p_D(r_D,t_D)=-\frac{1}{2}Ei\left(-\frac{r_D^2}{4t_D}\right) \qquad (7)$$

Where (Ei) is a mathematical function known as Exponential Integral Function, for this reason the line source solution sometimes is called the Exponential Integral solution.

The Ei ($-x$) is exponential integral function, can be expressed as: $-Ei(-x)=\int_x^\infty \frac{e^{-u}}{u}du$

3.1 The Equations of Pressure Affected By An Outer Boundary

When more than one sealing boundary or constant pressure boundary is presented, the solution to pressure affected by outer boundary p_{wDb} can involve a large number (hundreds) of image wells, depending upon the geometry of the system. This will lead to the generation of a flow of theoretical solution equations, and it can describe the geometry of the system.

Here we are just to deal with the flow of equations that can be use to describe the most common geometrical system of faults and constant pressure:

3.1.1 Linear System

Linear sealing fault

$$p_{wDb} = -0.5Ei\left(-\frac{L_D^2}{t_D}\right) \; L_D = \frac{L}{r_w} \qquad (8)$$

Where: L the distance from well to boundary, (m) r_w Wellbore radius, m

Linear constant pressure

$$p_{wDb} = 0.5Ei\left(-\frac{L_D^2}{t_D}\right) \; L_D = \frac{L}{r_w} \qquad (9)$$

3.1.2 Two - Perpendicular System

Perpendicular-sealing faults

$$p_{wDb} = \frac{1}{2}\left\{Ei\left[-\frac{L_{1D}{}^2}{t_D}\right] + Ei\left[-\frac{L_{2D}^2}{t_D}\right] + Ei\left[-\frac{L_{1D}^2 + L_{2D}^2}{t_D}\right]\right\} \qquad (10)$$

Perpendicular constant pressures

$$p_{wDb} = -\frac{1}{2}\left\{Ei\left[-\frac{L_{1D}{}^2}{t_D}\right] + Ei\left[-\frac{L_{2D}^2}{t_D}\right] + Ei\left[-\frac{L_{1D}^2 + L_{2D}^2}{t_D}\right]\right\} \qquad (11)$$

Perpendicular mixed boundaries

$$p_{wDb} = -\frac{1}{2}\left\{Ei\left[-\frac{L_{1D}{}^2}{t_D}\right] - Ei\left[-\frac{L_{2D}^2}{t_D}\right] - Ei\left[-\frac{L_{1D}^2 + L_{2D}^2}{t_D}\right]\right\} \qquad (12)$$

3.1.3 Two – Parallel System

Parallel sealing faults

$$p_{wDb} = -\frac{1}{2}\sum_{j=1}^{\infty}\left\{\begin{aligned}&Ei\left[-\frac{\left(int\left((j+1)/2\right)L_{1D} + int(j/2)L_{2D}\right)^2}{t_D}\right]\\ &+ Ei\left[-\frac{\left(int\left((j+1)/2\right)L_{2D} + int(j/2)L_{1D}\right)^2}{t_D}\right]\end{aligned}\right\} \qquad (13)$$

Parallel constant pressures

$$p_{wDb} = -\frac{1}{2}\sum_{j=1}^{\infty}(-1)^j\left\{\begin{aligned}&Ei\left[-\frac{\left(int\left((j+1)/2\right)L_{1D} + int(j/2)L_{2D}\right)^2}{t_D}\right]\\ &+ Ei\left[-\frac{\left(int\left((j+1)/2\right)L_{2D} + int(j/2)L_{1D}\right)^2}{t_D}\right]\end{aligned}\right\} \qquad (14)$$

Parallel mixed boundaries

$$p_{wDb} = -\frac{1}{2}\sum_{j=1}^{\infty}\left\{\begin{aligned}&(-1)^{\frac{j}{2}}Ei\left[-\frac{\left(int\left((j+1)/2\right)L_{1D} + int(j/2)L_{2D}\right)^2}{t_D}\right]\\ &+ (-1)^{\frac{j+1}{2}}Ei\left[-\frac{\left(int\left((j+1)/2\right)L_{2D} + int(j/2)L_{1D}\right)^2}{t_D}\right]\end{aligned}\right\} \qquad (15)$$

3.1.4 Intersecting System

Intersecting faults

$$p_{wDb} = -\frac{1}{2}\sum_{j=1}^{5}\left\{Ei\left[\frac{a_j{}^2}{2t_D}\right]\right\} \qquad (16)$$

Intersecting constant pressures

$$p_{wDb} = -\frac{1}{2}\sum_{j=1}^{5}(-1)^j\left\{Ei\left[\frac{a_j{}^2}{2t_D}\right]\right\} \qquad (17)$$

Intersecting mixed boundaries $$p_{wDb} = -\frac{1}{2}\sum_{j=1}^{5}(-1)^{j+1}\left\{Ei\left[\frac{a_j^{\ 2}}{2t_D}\right]\right\},\quad a_j = \frac{1-\cos(j\theta)}{\sin^2\left(\frac{\theta}{2}\right)}L_D^2,\ \text{j = 1,2,3,4,5}\quad \theta = 600 \quad (18)$$

4. THE ANALYTICAL SOLUTION TO THE WELL TEST MODEL

Based on an analytical solution to the diffusivity equation, on which all methods for the analysis of data from well testing are based. The concept of dimensionless parameters is used to generalize the solutions. The boundary conditions are applied to obtain solutions applicable to well test pressure analysis. A brief discussion on the application of Laplace transforms, and inverse Laplace transforms (Stehfest numerical inversion) will be presented in obtaining a line source solution for infinite homogeneous reservoirs. Skin effect and storage effect are also included as a condition applied to obtain solutions applicable to well test pressure analysis.

4.1 Laplace Transforms

The Laplace transform of a function (here the dimensionless pressure) is the convolution of this function with a negative exponential function. The Laplace transform function represented by the same symbol of a dash"—"above the function.

$$\overline{p}(u) \quad = \int_0^\infty e^{-ut}\, p(t)dt \qquad (19)$$

We will not go into details here, but we can present the result of applying this tool .The effect of applying the Laplace transform to the terms of a partial differential equation is to convert the equation to ordinary differential equation in which term (Laplace Space), Reasonable solution equations and mathematical derivation using Laplace transformation was presented by Van Everdingen (1949).The main advantage of the Laplace transform is that it will simplify the equation and that once the solution is found in Laplace space the reverse process is possible, either analytically for simple solutions, or numerically using algorithms such as Stehfest(1990).

4.2 Modified Bessel Function

The modified Bessel functions $K_0(x)$, $K_1(x)$, $I_0(x)$ and $I_1(x)$, are generated as a result of the solution of the mathematical model, and the calculations of these functions are based on the polynomial method of computing Bessel functions, which is presented by Abramowitz(1970) and Stegum, Giovanni(1990)

4.3 Stehfest's Numerical Inversion Method

The previous section introduced the application of Laplace transformation on solving the diffusivity equation. Because the use of Laplace in this way is very complicated, and therefore, using any analytical methods to convert Laplace space is even more complicated. Nowadays, in common use is a numerical empirical formula provided by Stehfest (1970) and is used to transform the solution from "Laplace space" to "real space". This method is considered as a simple way among several applicable methods for transforming Laplace space" to "real space".

The main formula is as follow:

$$f(t) = \frac{\ln(2)}{t}\sum_{i=1}^{N} V(i)\,\tilde{f}(s) \qquad (20)$$

Where $f(t)$ the transformed real space function , $\tilde{f}(s)$ The function in Laplace space

$$V(i) = (-1)^{N/2+i}\sum_{k=(i+1)/2}^{\min(i,N/2)}\frac{k^{N/2}(2k)!}{(N/2-k)!k!(k-1)!(k-1)!(i-k)!(2k-i)!},\quad s = i\,\frac{\ln(2)}{t} \quad (21)$$

Where（S）is Laplace transform factor. (K,i,N) an integers for summation

By using the main formula and giving values to t and i, the values of $f(t)$ And Vi can be calculated. Therefore, the value of the real space function $f(t)$ can be solved out.

5. COMPUTER APPLICATIONS

This section describes and lists basic computer programs in visual basic language that solve the general behavior of homogeneous affected by outer linear boundaries. The appropriate result to be obtained by this program that a friendly user interfaces can be used to create a various type curves for the mathematical model and also can create the most common theoretical type curves used in well test interpretation. The programs provide appropriate development of color visualizations for type curves that can be easily created and easily identified.

5.1 The Computer Program Calculation Procedures

According to an analytical expression of $\overline{p}_{wD}$ where obtained by using Laplace transform as a method of solution. In general the solution in Laplace space involves the computation of the modified Bessel function (K_0, K_1) Second kind as follows:

$$\overline{p}_{wD} = \frac{K_0(\sqrt{u}) + S\sqrt{u}K_1(\sqrt{u})}{u\{C_D u[K_0(\sqrt{u}) + S\sqrt{u}K_1(\sqrt{u})] + \sqrt{u}K_1(\sqrt{u})\}} \quad (22)$$

$$\overline{p}_{wD} = \frac{K_0(\sqrt{u/C_D})}{u\{\sqrt{u/C_D}K_1(\sqrt{u/C_D}) + uK_0(\sqrt{u/C_D}\}} \quad (23)$$

To obtain p_{wD} as a function of t_D, what we need to do is to invert $\overline{p}_{wD}$ to real space. This can be done in several ways. The simplest numerical method that is going to be used here is an algorithm for the approximate numerical inversion of the Laplace transforming solution. The algorithm already discussed above, which has been presented by Stehfest (1970) is coded as a function to find the inverse of Laplace space, see (Appendix B)
The program includes the calculation of the function p_{wD} (t_D) in two Public Functions, Pwds and PwdR. The calculation procedure of these two functions achieves related to the above questions of Laplace space solution (22) - (23) (see Appendix C). These two functions compute the Laplace transform inverse of $\overline{p}_{wD}(u)$.
These pressure functions make use of the functions BessI0, BessK0, BessI1, and BessK1, where they are called the modified Bessel functions, and a simple algorithm is presented by Abramowitz(1970) and Stegum to evaluate this modification, Giovanni (1990).The polynomial approximations for the modified Bessel functions can be used to calculate the Bessel function, where it is coded as a functions (Appendix D) to be called for the solution of the homogenous reservoir with the conditions of wellbore storage, skin effect and boundary effect, and bases on applying of these conditions the calculation model yield either as families of type curves like Ramey, Gringarten, Bourdet and combined type curves or as a single type curve. See the Figure (2) the program's reproduced type curves. Note: The value of (ε) in modified Bessel function is too small, and so it can be neglected.

5.2 The Calculation Of Pressure Affected By Outer Boundary:

Basically, the input parameters to calculate Pwd and PwdR functions are also a function of which the inverse transform is desired to be calculated. These functions are written as Pwds (cd, td, S, n), PwdR (cd, td, S, n) and PwdK (cd, td, S, Boud, n) (see Appendix B), where PwdK is a function designed to compute the summation of both dimensionless bottom hole pressure for infinite homogenous reservoir (p_{wDi}) and dimensionless bottom hole pressure affected by outer boundary (p_{wDb}) as expressed in equation (24) and illustrated in the flow chart figure (1).

$$p_{wD} = p_{wDi} + p_{wDb} \quad (24)$$

(p_{wDb}) Calculated as a function of dimensionless time, dimensionless distance Ld and *Ei* function, and all these function are coded as Bou*(td, Ld). Note: as briefly summarizing the calculation the sign (*) that attached with (Bou) function is indicated to the letter or number, which can describe the specific geometrical boundary condition for all equations from (8) to (18), for example Bou1 (td, Ld) to describe the effect of linear constant pressure boundary, Bou2c (td, Ld1, Ld2) to describe the effect of Two-Perpendicular sealing faults, etc. As example (Appendix C) shows the calculation code of Bou, Bou1 functions, which are represent the closed and constant pressure linear boundary respectively.

$$p_D(r_D, t_D) = -\frac{1}{2}Ei(-\frac{r_D^2}{4t_D}) \quad (25)$$

The calculation of (p_{wDb}) is line source solutions, which make use of *Ei* (x) function; equation (25) represents the exponential integral function. Therefore, the calculations involve the evaluation of *Ei* (x) function in all equations that designed to evaluate the pressure affected by the geometrical condition of the reservoir boundary. These equations illustrated before where numbered from (8) to (18).

5.3 The Exponential Integral Function and Polynomial Approximations

The exponential integral function of order n, written as a function of a variable x, is defined as

$$Ei_n(x) = \int_1^{\infty} t^{-n} e^{-xt}\, dt$$: Let Ei1(x) be the Ei_n with n = 1: $$Ei_n(x) \equiv \int_1^{\infty} \frac{e^{-tx}}{t} dt = \int_x^{\infty} \frac{e^{-u}}{u} du$$

Then define the exponential integral Ei1(x) by: E_i1(x)=- Ei1(-x) $$Ei(x) = -\int_{-x}^{\infty} \frac{e^{-t} dt}{t}$$

To evaluate Ei(x), the program utilizes Abramowitz(1970) and Stegun Rational Approximations that presented by E.E.Allen for 0 < x < 1 and C.Hastings for 1< x < ∞, the calculation of *Ei* (x) function coded in (Appendix A)

For 0 < x < 1

$$Ei(x) + \ln x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \in(x) : \left| \in(x) \right| = 2 \times 10^{-7}$$

a_0 = -0.57721566, a_1= 0.99999193, a_2= -0.024991055,
a_3= 0.05519968, a_4= -0.00976004, a_5= 0.00107857.

For 1< x < ∞

$$xe^x Ei(x) = \frac{x^4 + a_1 x^3 + a_2 x^2 + a_3 x + a_4}{x^4 + b_1 x^3 + b_2 x^2 + b_3 x + b_4} + \in(x)$$

a_1= 8.5733287401, a_2= 18.0590169730, a_3=8.6347608925, a_4= 0.2677737343
b_1= 9.5733223454, b_2= 25.6329561486, b_3=21.0996530827, b_4= 3.9584969228

5.4 Derivative Pressure Methods

The analysis of the differential of pressure, which has been introduced by D.Bourdet (1983)(1989), T.M.Whittle in(1983), From a practical point of view, it was founded that a preferable way to plot the type curves as:

$$p_D'(t_D / C_D) \text{ Versus } (t_D / C_D), \qquad p_D'(t_D / C_D) = \Delta t \Delta p' kh / (141.2 qB\mu)$$

5.5 Three Point's Derivative Algorithm

The algorithm presented here is simple, and is best adapted to test interpretation needs. This differentiation algorithm reproduces the type curve over the complete time interval better than others. It uses one point before and one point after the point of interest, to calculate the corresponding derivatives, and places their weighted mean at the point considered as derived in the below equation.

$$(dp/dX)_i = [(\Delta p_1 / \Delta X_1)\Delta X_2 + (\Delta p_2 / \Delta X_2)\Delta X_1] / (\Delta X_1 + \Delta X_2)$$

6. THE PROGRAM'S REPRODUCED TYPE CURVES

Type curves are reproduced as output of this program package, which derived from mathematical solutions to the flow equations under specific geometrical boundaries conditions. For the sake of generality, type curves are usually presented in dimensional less terms, here we deal with dimensionless pressure vs. a dimensionless time, a given interpretation model yield either as a single type curve or one or more families of type curves like Ramey, Gringarten, Bourdet and combined type curve. See the program's reproduced type curves diagram figure (2).
The program uses the same theoretical model as the pressure and derivative type curves of Gringarten and Bourdet and combines them into a group set of type curves figure(3)as known by combined type curves.

6.1 Reproduced Single Type Curve

The appropriate calculations that are provided by the program, is that for each reservoir boundaries situation the program has ability to establish multi type curve for a variety of well and reservoir parameters, based on the time rate of change of dimensionless pressure for interpreting the homogenous behavior of well located in various multiple-sealing-fault system combined with constant pressure boundary system. Figure (4) illustrates the program user interface for establishing type curve for a variety of well and reservoir Parameters, the parameters including skin factor, distances between well and outer boundary and wellbore radius were entered as input data in an input dialog box, When the single type curve was established, the program simultaneously shows the shape of location in distance (meter) of the well in the reservoir geometrical boundary system. See Figure (5).

7. CONCLUSION

This computer application can be considered as a technical guide to those who are interested in knowing the

mathematical background of reproduce theoretical type curves, the program reflect the establishment of the most important bases concepts for the analyzing well test software.
Several publications do not exclusively discuss the computation of the pressure response affected by outer boundary, and the program addresses the investigation of the type curve based on source solution of pressure affected by the outer boundary, and provides reasonable calculation for establishing theoretical type curves that can be easily used in determining the distance between well and outer boundary.
Using VISUAL BASIC6 provides good capability in graphics and color visualization, which enables the best presentation of the type curves, and can also be used for future development of the type curves matching process.

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Appendix (A)

********Stehfest (Laplace transforms numerical inversion method) ********

```
Public Function Li (ByVal k As Integer) As Double
Dim i As Integer
  Li = 1
  For i = 2 To k Step 1
  Li = Li * i
  Next i
End Function
Public Function Vi (ByVal i As Integer, ByVal n As Integer) As Double
Dim j, k, min As Integer
Dim M As Double          j = Int ((i + 1) / 2)
  M = 0
  If i <= (n / 2) Then min = i Else min = (n / 2)
  For k = j To min Step 1
     Vi = (k ^ (n / 2) * Li (2 * k)) / (Li (n / 2 - k) * Li (k) * Li (k - 1) * Li (i - k) * Li (2 * k - i))
    M = M + Vi
 Next k
  Vi = (-1) ^ (n / 2 + i) * M
End Function
```

******** (The calculations of Ei (x) function) ********

```
Public Function Ei (ByVal x As Double) As Double
     Dim x2, x3, x4, x5 As Double
     Dim MM, NN As Double
     x2 = x * x,   x3 = x2 * x,   x4 = x3 * x,    x5 = x4 * x
     If x > 1# Then
          MM = 0.2677737343 + 8.6347608925 * x + 18.059016973 * x2 + 8.5733287401 * x3 + x4
          NN = 3.9584969228 + 21.0996530827 * x + 25.632956486 * x2 + 9.5733223454 * x3 + x4
          Ei = Exp(-x) * MM / (x * NN)
        Else
         x = Abs(x)
Ei = -Log(x) - 0.57721566 + 0.99999193 * x - 0.24991055 * x2 + 0.05519968 * x3 - 0.00976003999 * x4 + 0.00107857 * x5
End If
Ei = -Ei
End Function
```

Appendix (B)

******** (The Calculation of the Pressure drop for Homogeneous with skin equal zero) ********

```
Public Function Pwds (ByVal cd As Double, ByVal td As Double, ByVal S, ByVal n As Integer) As Double
     Dim i As Integer,      Dim a, R, M, Pwd1, Cd2s, u As Double, Pwds = 0
       For i = 1 To n Step 1
```

```
        u = i * Log (2) / td
        Cd2s = cd * Exp (2# * S)
      If Cd2s >= 100 Then
        a = Log10 (2# / (1.781 * Sqr(u / Cd2s)))
        R = u * u + u /              M = 1# / R
        Pwds = M
      Else
         a = Sqr (u / Cd2s)    R = a * BessK1 (a) / BessK0 (a)        M = 1 / (u * R + u * u)
         Pwds = M
      End If
         Pwds = Vi(i, n) * M + Pwds        ' transform to real space
     Next i
        Pwds = (Log(2) / td) * Pwds        'calculation of  log
    End Function
```

******** (The Calculation Of the Pressure drop for Homogeneous with skin) ********

```
Public Function PwdR( ByVal cd As Double, ByVal td As Double, ByVal S, ByVal n As Integer) As Double
    Dim i As Integer,        Dim p1, p2, u As Double, PwdR = 0
    For i = 1 To n Step 1
      u = i * Log(2) / td  : 'The solution of homogeneous model in Laplace space

      p1 = BessK0(Sqr(u)) + S * Sqr(u) * BessK1(Sqr(u))

     p2 = u * Sqr(u) * BessK1(Sqr(u)) + cd * u ^ 2 * BessK0(Sqr(u)) + cd * u ^ 2 * S * BessK1(Sqr(u)) * Sqr(u)
      PwdR = Vi(i, n) * (p1 / p2) + PwdR ' transform to real space
       Next i
      PwdR = (Log(2) / td) * PwdR  'calculation of  log
End Function
```

Appendix (C)

******** (The calculation code of the closed and constant pressure linear boundary functions) ********

```
  Public Function Bou(ByVal td As Double, ByVal Ld As Double) As Double
    'Closed linear Boundary
  g = -(Ld ^ 2 / td)
    Bou = -0.5 * Ei(g)
    Bou = Bou
  End Function
  Public Function Bou1(ByVal td As Double, ByVal Ld As Double) As Double
   'Constant Pressure linear Boundary
   g = -(Ld ^ 2 / td)
    Bou1 = 0.5 * Ei(g)
    Bou1 = Bou1
      End Function
```

Appendix (D)

******** (Modified Bessel function First Kind Zero Order (I_0)) ********

```
Public Function BessI0 (ByVal x As Double) As Double
Dim ax, ax1，  y, ans As Double
If Abs(x) < 3.75 Then
  y = x * x / (3.75 * 3.75)
  ans = 1# + y * (3.5156229 + y * (3.0899424 + y * (1.2067492 + y * (0.2659732 + y * (0.0360768 + y * 0.0045813)))))
Else
  ax = Abs(x)，ax1 = Exp (ax)，  y = 3.75 / ax
  ans = (ax1 / Sqr(ax)) * (0.39894228 + y * (0.01328592 + y * (0.00225319 + y * (-0.00157565 + y * (0.00916281 + y * (-
0.02057706 + y * (0.02635537 + y * (-0.01647633 + y * 0.00392377))))))))
End If
BessI0 = ans
End Function
```

******** (Modified Bessel function Second Kind Zero Order (K_0)) ********

```
Public Function BessK0 (ByVal x As Double) As Double
Dim ax, ax1 ， y, ans  As Double
If x <= 2# Then ,  y = x * x / 4#
  ans = (-Log(x / 2#) * BessI0(x)) + (-0.57721566 + y * (0.4227842 + y * (0.23069756 + y * (0.0348859 + y * (0.00262698 + y * (0.0001075 + y * 0.0000074))))))
Else  , y = 2# / x
  ans = (Exp(-x) / Sqr(x)) * (1.25331414 + y * (-0.07832358 + y * (0.02189568 + y * (-0.01062446 + y * (0.00587872 + y * (-0.0025154 + y * 0.00053208))))))
End If
BessK0 = ans
End Function
```

*******(Modified Bessel function First Kind First Order (I_1))*******

```
Public Function BessI1 (ByVal x As Double) As Double
Dim ax, ax1 ， y, ans As Double
If Abs(x) < 3.75 Then   y = x * x / (3.75 * 3.75)
   ans = x * (0.5 + y * (0.87890594 + y * (0.51498869 + y * (0.15084934 + y * (0.02658733 + y * (0.00301532 + y * 0.00032411))))))
Else          ax = Abs(x) ，     ax1 = Exp(ax) ，   y = 3.75 / ax
ans = 0.02282967 + y * (-0.02895312 + y * (0.01787654 - y * 0.00420059))
ans = 0.39894228 + y * (-0.03988024 + y * (-0.00362018 + y * (0.00163801 + y * (-0.01031555 + y * ans))))
ans = (ax1 / Sqr(ax)) * ans
End If
BessI1 = ans
End Function
```

********(Modified Bessel function Second Kind First Order (K_1))********

```
Public Function BessK1(ByVal x As Double) As Double
Dim ax, ax1, y, ans As Double
If x <= 2# Then
   y = x * x / 4#
   ans = (Log(x / 2#) * BessI1(x)) + (1# / x) * (1# + y * (0.15443144 + y * (-0.672778579 + y * (-0.18156897 + y * (-0.01919402 + y * (-0.00110404 + y * (-0.00004686)))))))
Else
   y = 2# / x
   ans = (Exp(-x) / Sqr(x)) * (1.25331414 + y * (0.23498619 + y * (-0.0365562 + y * (0.01504268 + y * (-0.00780353 + y * (0.00325614 + y * (-0.00068245)))))))
End If
BessK1 = ans
End Function
```

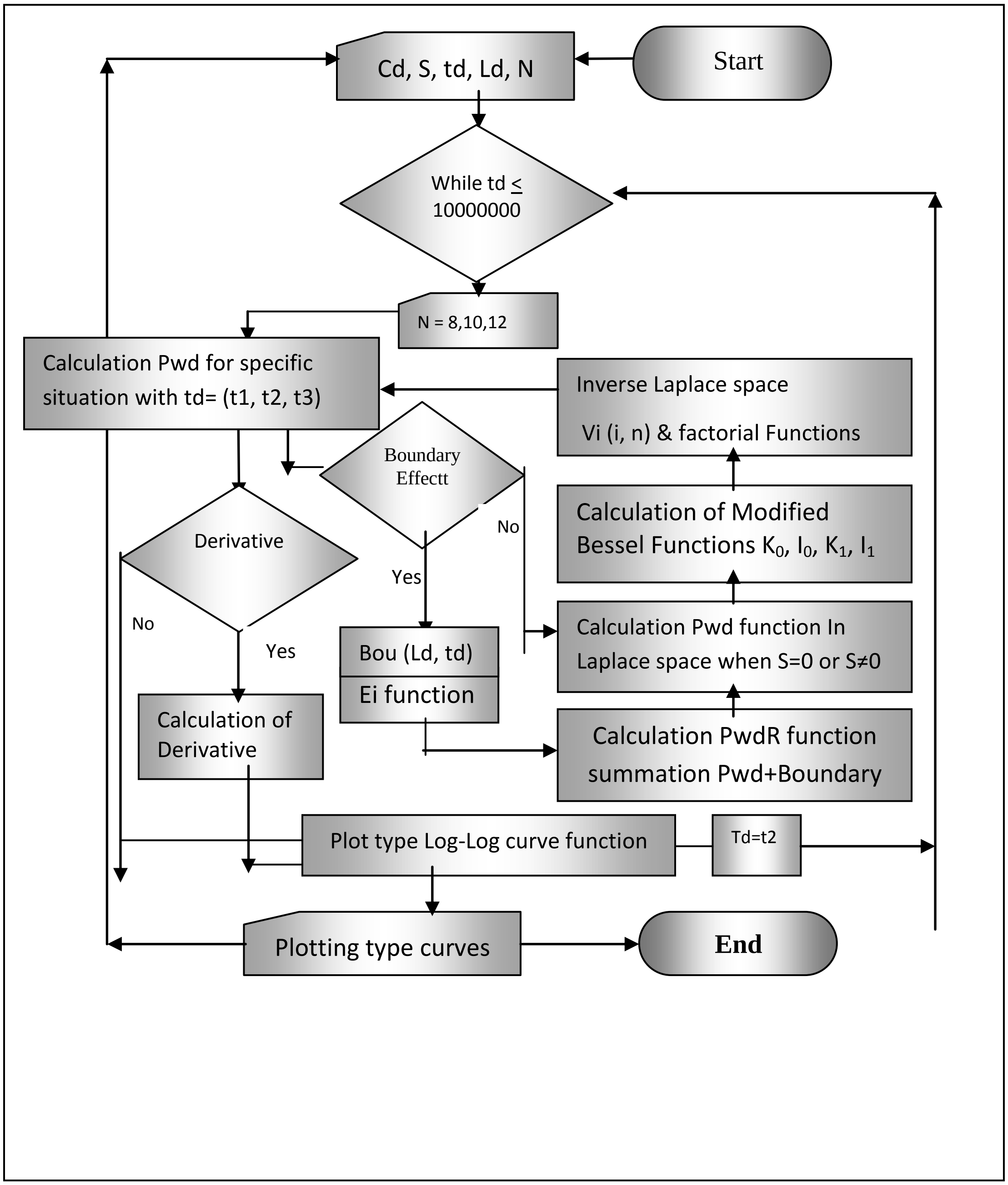

Figure (1) the main calculation steps flow chart

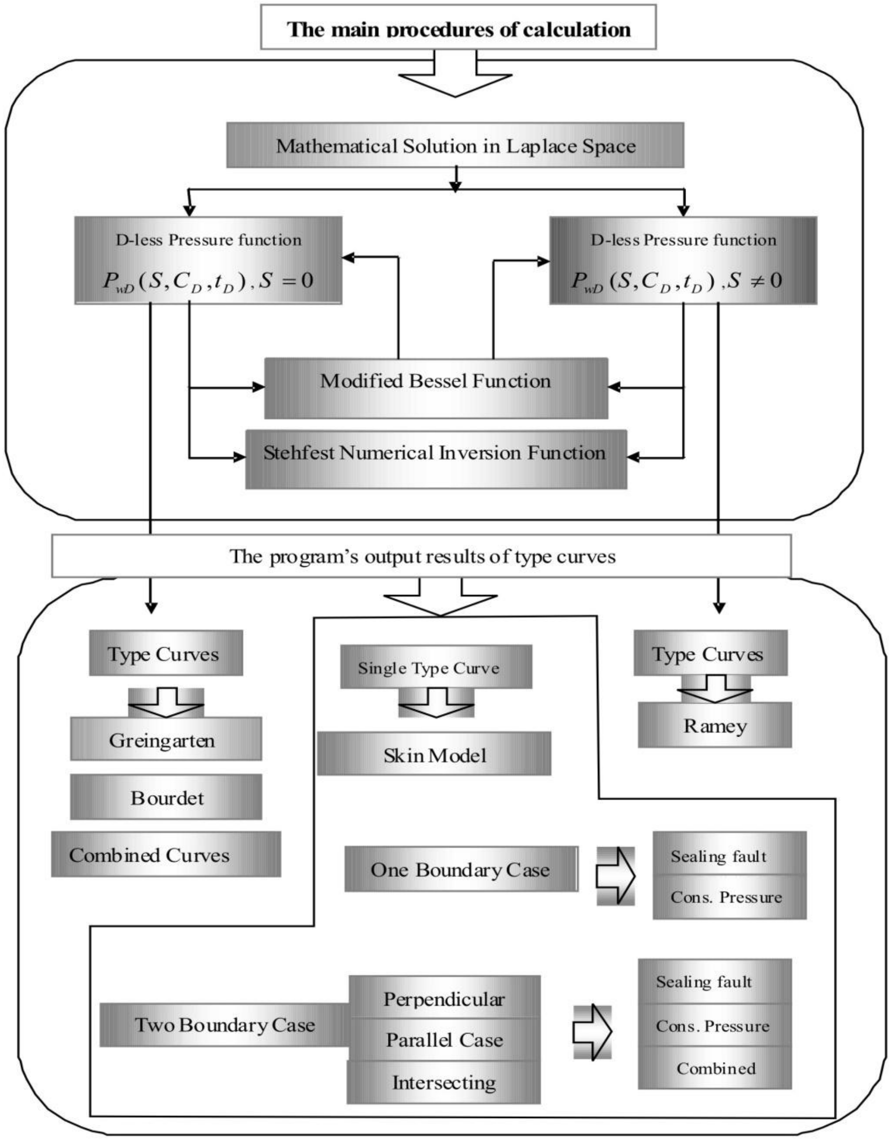

Figure (2) the program's reproduced type curves

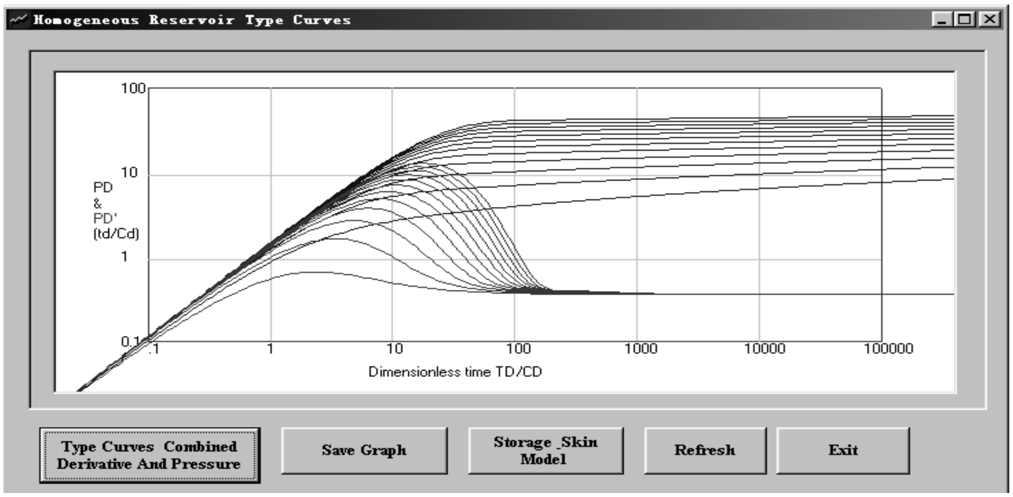

Figure (3) Homogeneous with skin and wellbore storage Combined type curves

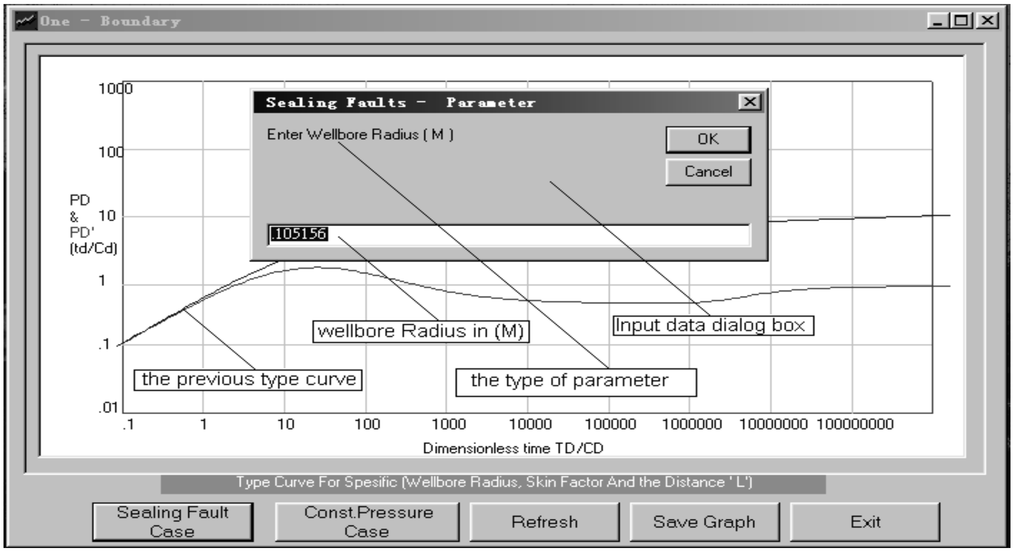

Figure (4) the user input dialog box for a variety conditions of well and reservoir parameters

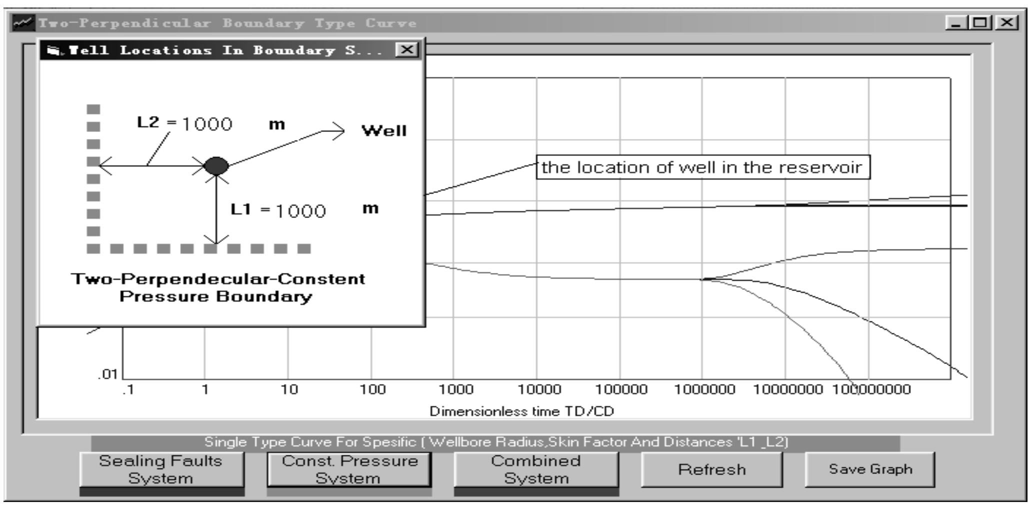

Figure (5) the distance between well and outer boundaryL1, L2 [by default set in (1000m)]

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