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A dual-well step drawdown method for the estimation of linear and non-linear flow parameters and wellbore skin factor in confined aquifer systems

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SUMMARY

In this study a method based on dual-well step drawdown test (i.e. a combination of an aquifer and a well performance test) for the determination of hydrodynamic parameters (namely storage coefficient and hydraulic conductivity), mechanical wellbore finite thickness skin factor, non-linear wellbore and non-linear aquifer parameters in an homogeneous confined aquifer is presented in order to put together aquifer and well tests. The interpretation procedure is based on the application of superposition principle to a large time logarithmic approximation of the solution.

The advantages of this method, that can be considered an extension of Jacob step-test (1947) and Cooper–Jacob approximation (1946), are that: (I) it is possible to determine simultaneously aquifer and well properties in a single test; (II) the method is based on a large time approximation and it is therefore independent from wellbore storage; (III) if the well skin is absent, the aquifer parameters (storage coefficient and hydraulic conductivity) can be derived just from a single-well test; (IV) the interpretation procedure is easy to apply and robust and does not require any specific numeric code or software. The same procedure can be easily adapted to gas well testing.

It is also shown that, even in the presence of linear and non-linear flow, skin effect and wellbore storage, the hydraulic conductivity (and not the storage coefficient) of the aquifer can be correctly estimated by the Cooper and Jacob (1946) method applied to a single-rate pumping test, using exclusively the large time drawdown data measured at the pumping well.

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1. Introduction

Darcy's Law (Darcy, 1856) is the most commonly used equation for describing the flow of a single-phase fluid through a saturated porous medium. This equation predicts a linear relationship between fluid velocity and pressure head gradient. Non-linearities in flow equation can occur for several reasons but they are often due to high fluid velocities. Forchheimer (1901) proposed the first and most widespread empirical model to account for high velocity non-linearities adding a quadratic term to Darcy's equation. Several theories were formulated in order to explain the origin of this effect. Early studies have attributed the non-linearity to turbulent flow even if the phenomenon was observed at relatively low Reynold numbers (Dybbs and Edwards, 1984), other authors imputed it to the influence of microscopic inertial forces (Bear, 1988; Scheidegger, 1974) or to the increased microscopic drag on the pore walls (Firoozabadi and Katz, 1979; Slattery, 1971). Hassanizadeh and Gray (1987) demonstrated that, at Reynolds numbers around 10, the non-linearity arising from macroscopic inertial forces is negligible if compared to the effect of interfacial drag forces. Fourar et al. (2004) showed that Forchheimer's model leads to a good description of the 3D-flow in the non-Darcy regime. Moreover, as the velocity goes to zero, Darcy's law is recovered, so that Forchheimer' law can be, also, successfully used to simulate flows at low Reynolds numbers.

In aquifer systems, but also in oil and in gas reservoirs, non-linear flow can occur due to velocity increase in proximity of the pumped well (Chaudhry, 2003; Di Molfetta, 2002; Jacob, 1947; Muskat, 1982; Singh, 2002). Currently an exact analytical solution of Forchheimer flow towards a well does not exists even if Sen (1988) claims to have derived a similarity solution using the Boltzmann transformation. The use of this transform is rather restrictive because it requires that all the initial and boundary conditions can also be simultaneously transformed into forms only containing the Boltzmann variable. As evidenced by Camacho-V and Vásquez-C (1992), the solution derived by Sen (1988) does not satisfy boundary conditions and thus should be considered an approximation (Wen et al., 2008a). Instead, close to the well Izbash (1931) simplifying hypotheses (being more mathematically tractable than Forchheimer equation) was used to derive, using Laplace

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transform, an analytical solution of the approximated problem in presence of well bore storage (Wen et al., 2008a,b). In a very detailed work Mathias et al. (2008) derived and compared several approximate solutions of non-linear flow in aquifer systems.

Field tests for the determination of non-linear parameters have been developed both in gas reservoir and in groundwater engineering. The determination of gas deliverability of a well is often performed using stabilized data, e.g. flow-after-flow (Rawlins and Schellhardt, 1935), or a combination of transient isochronal and stabilized data, e.g. isochronal (Cullender, 1955), and modified isochronal test (Brar and Aziz, 1978; Katz, 1959). The interpretation of the tests is based on empirical relationships (Rawlins and Schellhardt, 1935) or in a more rigorous way after modifications of the flow equation introducing the concepts of pseudopressure (Al-Hussaini et al., 1966). In the field of groundwater engineering, step drawdown tests are normally undertaken to determine an empirical well-loss coefficient C (Bierschenk, 1964; Rorabaugh, 1953) that can be used to estimate well performance (Walton, 1962) and efficiency (Kruseman and Ridder, 1970). An heuristic graphical method, based on the application of superposition principle to large time logarithmic approximation in step drawdown test, was developed by Eden and Hazel (1973) to derive also the hydraulic conductivity of the aquifer system. Using this method, based on a single-well test, it is not possible to determine aquifer storage coefficient if wellbore skin factor is present, moreover the non-linear coefficient is derived on an empirical basis and therefore cannot be related to any aquifer property. Numerical simulators, instead, can enable the derivation of skin effect and non-linear parameters from one single build-up test (Spivey et al., 2004). If this is true in gas reservoirs where gas compressibility can be determined knowing thermodynamic properties, this test procedure fails in aquifer systems where specific storage should also be determined.

In this study a method which combines an aquifer and a well performance test is presented. The dual-well step drawdown method is used for the simultaneous determination of hydrodynamic parameters of a confined aquifer (namely storage coefficient and hydraulic conductivity), and of mechanical wellbore finite thickness skin factor, non-linear wellbore skin and non-linear aquifer parameters. The interpretation procedure is based on the application of superposition principle to a large time logarithmic approximation of the solution of the Forchheimer flow. The correctness of the method was tested using a numerical model able to simulate the non-linear flow in an aquifer system generated by a fully penetrating well with wellbore storage, mechanical and non-linear skin. The principle applied in the development of this method is that the interpretation procedure should be kept as simple as possible without loosing the ability to capture the characteristics of the problem.

2. Governing equations

The governing equations of flow to a fully penetrating well with finite thickness skin effect (Fig. 1) an isotropic, confined aquifer are (Novakowski, 1989):

$$S_{ss}\frac{\partial h}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rq)=0 \quad r_w \leqslant r \leqslant r_s \tag{1}$$

$$S_s\frac{\partial h}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rq)=0 \qquad r \geqslant r_s \tag{2}$$

subject to initial and boundary conditions:

$$\begin{aligned} &h(r,0)=0 \quad r \geqslant r_w \\ &h(r_w,t)=h_w \\ &h(\infty,t)=0 \end{aligned} \tag{3}$$

where h is the hydraulic head [L], h_w is the hydraulic head in the wellbore [L], S_s and S_{ss} are the specific storage of the aquifer and of the skin [L^{-1}], r is the radial distance [L], r_w is the radius of the pumping well [L], r_s is the radius of the skin [L], q is water velocity [$L\,T^{-1}$] here modeled according to Forchheimer equation (1901):

$$\frac{1}{K}q+\frac{\beta}{g}|q|q=-\frac{\partial h}{\partial r} \quad r_w \leqslant r \leqslant r_s \tag{4}$$

$$\frac{1}{K_s}q+\frac{\beta_s}{g}|q|q=-\frac{\partial h}{\partial r} \quad r \geqslant r_s \tag{5}$$

where K and K_s are the pseudo hydraulic conductivity of the aquifer and of the skin [$L\,T^{-1}$], β and β_s the non-linear flow coefficients [L^{-1}], g the acceleration of gravity [$L\,T^{-2}$].

The equation for wellbore storage is (Papadopulos and Cooper, 1967):

$$\pi r_c^2\frac{dh_w}{dt}+Q+2\pi b r_w q\Big|_{r=r_w}=0 \tag{6}$$

subject to:

$$h_w(0)=0 \tag{7}$$

where Q is the pumping rate [$L^3\,T$], positive for abstraction, r_c is the equivalent (i.e. considering the presence of the tubing of the pump)

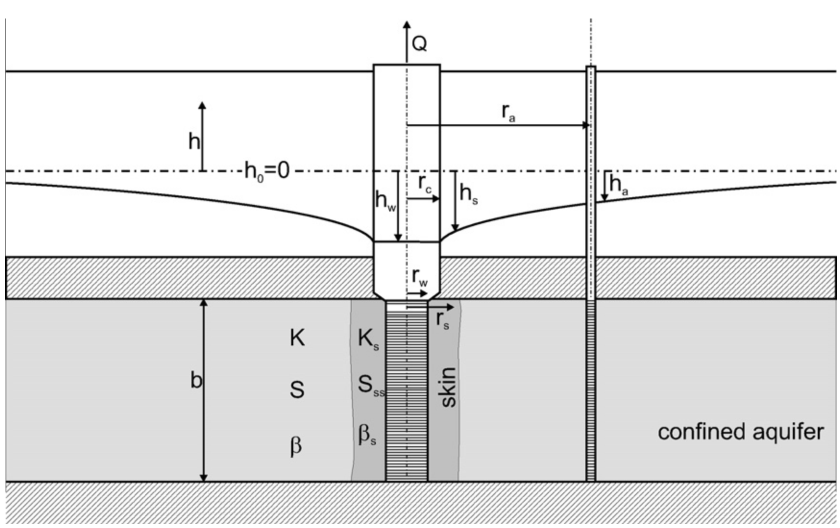

Fig. 1. Schematic representation of a confined aquifer with a piezometer and a fully penetrating well with finite radius and finite thickness skin.

radius of the well casing [L] and b saturated thickness of the aquifer [L].

3. Dimensionless transformation

A dimensionless transformation was performed introducing the following dimensionless groups:

$$h_D = -\frac{2\pi bKh}{Q} \quad h_{wD} = -\frac{2\pi bKh_w}{Q} \quad t_D = \frac{Kt}{S_s b^2} \quad S_{sD} = \frac{S_{ss}}{S_s}$$

$$q_D = -\frac{2\pi b^2 q}{Q} \quad \beta_D = -\frac{Q\beta K}{2\pi b^2 g} \quad K_{sD} = \frac{K_s}{K} \quad \beta_{sD} = -\frac{Q\beta_s K}{2\pi b^2 g}$$

$$r_D = \frac{r}{b} \quad r_{wD} = \frac{r_w}{b} \quad r_{sD} = \frac{r_s}{b} \quad r_{cD} = \frac{r_c}{S_s^{1/2} b^{3/2}}$$

obtaining:

$$S_{sD}\frac{\partial h_D}{\partial t_D} + \frac{1}{r_D}\frac{\partial}{\partial r_D}(r_D q_D) = 0 \quad r_{wD} \leqslant r_D \leqslant r_{sD} \tag{8}$$

$$\frac{\partial h_D}{\partial t_D} + \frac{1}{r_D}\frac{\partial}{\partial r_D}(r_D q_D) = 0 \quad r_D \geqslant r_{sD} \tag{9}$$

$$\frac{1}{K_{sD}}q_D + |\beta_{sD}||q_D|q_D = -\frac{\partial h_D}{\partial r_D} \quad r_{wD} \leqslant r_D \leqslant r_{sD} \tag{10}$$

$$q_D + |\beta_D||q_D|q_D = -\frac{\partial h_D}{\partial r_D} \quad r_D \geqslant r_{sD} \tag{11}$$

$$h_D(r_D, 0) = 0 \quad r_D \geqslant r_{wD} \tag{12}$$

$$h_D(r_{wD}, t) = h_{wD} \tag{13}$$

$$h_D(\infty, t_D) = 0 \tag{14}$$

$$\frac{r_{cD}^2}{2}\frac{dh_{wD}}{dt_D} - 1 + r_{wD}q_D\Big|_{r_{wD}} = 0 \tag{15}$$

$$h_{wD}(0) = 0 \tag{16}$$

expliciting q_D from Eqs. (10) and (11), (8) and (9) become (analogously to Jamiolahmady et al. (2007)):

$$S_{sD}\frac{\partial h_D}{\partial t_D} + \frac{1}{r_D}\frac{\partial}{\partial r_D}\left(-\frac{2r_D}{\frac{1}{K_{sD}} + \sqrt{\frac{1}{K_{sD}^2} + 4|\beta_{sD}|\left|\frac{\partial h_D}{\partial r_D}\right|}}\frac{\partial h_D}{\partial r_D}\right) = 0 \quad r_{wD} \leqslant r_D \leqslant r_{sD} \tag{17}$$

$$\frac{\partial h_D}{\partial t_D} + \frac{1}{r_D}\frac{\partial}{\partial r_D}\left(-\frac{2r_D}{1 + \sqrt{1 + 4|\beta_D|\left|\frac{\partial h_D}{\partial r_D}\right|}}\frac{\partial h_D}{\partial r_D}\right) = 0 \quad r_D \geqslant r_{sD} \tag{18}$$

This set of non-linear partial differential Eqs. (17), (18) and boundary conditions were solved using implicit finite differences scheme applying a Newton's method in order to generate some synthetic cases for the application of the dual-step-test procedure described in the following paragraphs.

4. Large time solution

Because early time data, during a fully transient pumping test, are influenced by many factors (also by storage inside the pumping well and by specific storage of the skin) that can lead to a difficult estimation of linear and non-linear parameters of interest – even in presence of an exact solution – it can be interesting to derive a large time solution of the two regions flow problem. This can be attained by using the method of matched asymptotic expansions as described in Mathias et al. (2008) and the steady state solution derived in Appendix A:

$$h_{wD} \approx \frac{1}{2}\left(\ln\frac{4t_D}{r_D^2} - \gamma\right) + \sigma_{wD,s,M} + \sigma_{wD,s,NL} + \sigma_{wD,a,NL} \quad r_D = r_{wD} \tag{19}$$

represents the non-dimensional drawdown inside the well (γ is the Euler constant). For a generic r_D it is possible to express the drawdown in the skin or in the aquifer system according to the following expressions:

$$h_{s,D} \approx \frac{1}{2}\left(\ln\frac{4t_D}{r_D^2} - \gamma\right) + \sigma_{D,s,M} + \sigma_{D,s,NL} + \sigma_{D,a,NL} \quad r_{wD} \leqslant r_D \leqslant r_{sD} \tag{20}$$

$$h_{a,D} \approx \frac{1}{2}\left(\ln\frac{4t_D}{r_D^2} - \gamma\right) + \sigma_{D,a,NL} \quad r_D \geqslant r_{sD} \tag{21}$$

$\sigma_{D,s,M} = \left(\frac{1}{K_{sD}} - 1\right)\ln\frac{r_{sD}}{r_D}$ is the non-dimensional drawdown due to the mechanical (linear) skin effect; $\sigma_{D,a,NL} = |\beta_D|\left(\frac{1}{r_D} - \frac{1}{r_{eD}}\right) \approx \frac{|\beta_D|}{r_D}$ is the non-dimensional drawdown due to non-linear losses in the aquifer system; $\sigma_{D,s,NL} = |\beta_D|\left(\frac{|\beta_{sD}|}{|\beta_D|} - 1\right)\left(\frac{1}{r_D} - \frac{1}{r_{sD}}\right)$ is the non-dimensional drawdown due to the non-linear losses in the finite thickness skin.

Substituting r_{wD} to r_D in the above expressions it is possible to get the additional drawdowns in the well: $\sigma_{wD,s,M}$, $\sigma_{wD,s,NL}$, $\sigma_{wD,a,NL}$.

Reverting back to dimensional parameters the drawdown inside the well, in the skin and in a generic position of the aquifer lead to the following expressions:

$$s_w = -h_w \approx \frac{1}{4\pi bK}\left[\ln\left(4\frac{Kt}{r^2S_S}\right) - \gamma + 2\sigma_{w,s,M}\right]Q + (F_{w,s,NL} + F_{w,a,NL})|Q|Q \quad r = r_w \tag{22}$$

$$s_s = -h_s \approx \frac{1}{4\pi bK}\left[\ln\left(4\frac{Kt}{r^2S_S}\right) - \gamma + 2\sigma_{s,M}\right]Q + (F_{s,NL} + F_{a,NL})|Q|Q \quad r_w \leqslant r \leqslant r_s \tag{23}$$

$$s_a = -h_a \approx \frac{1}{4\pi bK}\left[\ln\left(4\frac{Kt}{r^2S_S}\right) - \gamma\right]Q + F_{a,NL}|Q|Q \quad r \geqslant r_s \tag{24}$$

where

$$\sigma_{s,M} = \left(\frac{K}{K_s} - 1\right)\ln\frac{r_s}{r} \quad \text{is the mechanical (linear) finite thickness skin factor;} \tag{25}$$

$$F_{a,NL} = \frac{\beta}{g}\left(\frac{1}{2\pi b}\right)^2\left(\frac{1}{r}\right) \quad \text{is the coefficient of non-linear loss in the aquifer system;} \tag{26}$$

$$F_{s,NL} = \frac{\beta}{g}\left(\frac{1}{2\pi b}\right)^2\left(\frac{\beta_s}{\beta} - 1\right)\left(\frac{1}{r} - \frac{1}{r_s}\right) \quad \text{is the coefficient of non-linear loss in the skin.} \tag{27}$$

Substituting r_w to r in the above expressions it is possible to obtain the values of the mechanical skin factor $\sigma_{w,s,M}$ and of the non-linear loss coefficients $F_{w,s,NL}$ and $F_{w,a,NL}$ calculated at the wellbore. It is interesting to notice that at large times, the influences of wellbore storage in Eq. (15) and of specific storage of the skin is negligible.

Eq. (22) describes the evolution, at large times, of the drawdown inside the wellbore and is made of two terms, one showing a linear and the other a quadratic dependence on the pumping rate, in full agreement with the work of Jacob (1947). Eq. (24) shows, instead, the evolution (at large times) of the drawdown at an observation piezometer outside the skin. Because of the hyperbolic dependence of $F_{a,NL}$ with the distance from the well, the second term (due to non-linear loss in the aquifer system) evanishes at sufficient distances, and the equation will turn into the Cooper–Jacob model (1946).

5. Test description and interpretation procedure

According to the derived large time solutions Eqs. (22)–(24) it is not possible to estimate the linear and non-linear parameters using a single-rate drawdown test whether the heads are measured at the well itself or at an observation piezometer. In principle, the estimation of the parameters would, instead, be possible by fitting the experimental data to a fully transient solution of the problem. From the practical point of view, this latter procedure, specially at small times, would be strongly affected by the high number of parameters, and of local minima involved in the fitting process. Therefore, there are here presented (i) a dual-well step-test procedure, which can be used to estimate the aquifer and skin parameters using the derived large time solution or, eventually, to increase the robustness of the fitting using fully transient solutions and (ii) an interpretation technique, that can be used for an efficient estimation of the parameters using the large time solutions.

The proposed procedure is a dual-well step drawdown test performed, in confined aquifers, pumping the well at increasing discharge rates (at least three) while measuring the drawdowns at the well itself and at an observation piezometer. Each pumping step should be long enough (not necessary the same duration) to permit the application of the logarithmic approximation of Eqs. (22), (23) or (24) to every step, in both the well and the observation points. Therefore if t_i is the starting time of each step, $t - t_i > 12.5 S_s r^2/K$. An example of synthetic data generated over time using the numerical simulator at the pumping well and at two different observation points are reported in Fig. 2.

Before describing the interpretation procedure, it is more convenient to rewrite the drawdown equations inside the well ($r = r_w$) (22) and in the piezometer (usually $r \geqslant r_s$) (24) as follows:

$$s_w = -h_w \approx \frac{2.30}{4\pi bK}\left[\log\left(2.25\frac{K}{r_w^2 S_S}\right) + \frac{2}{2.30}\sigma_{w,s,M} + \log t\right]Q + (F_{w,s,NL} + F_{w,a,NL})|Q|Q \quad r = r_w \tag{28}$$

$$s_a = -h_a \approx \frac{2.30}{4\pi bK}\left[\log\left(2.25\frac{K}{r^2 S_S}\right) + \log t\right]Q + F_{a,NL}|Q|Q \quad r \geqslant r_s \tag{29}$$

This formulation can be written, both for the well and for the piezometer, in a similar way to Eden and Hazel (1973):

$$s_j \approx (A_j \log t + B_j)Q + C_j Q|Q| \tag{30}$$

where the subscript j refers to the well w, or to the aquifer system a and therefore:

$$A = A_w = A_a = \frac{2.30}{4\pi bK} \tag{31}$$

$$B_w = \frac{2.30}{4\pi bK}\left[\log\left(2.25\frac{K}{r_w^2 S_S}\right) + \frac{2}{2.30}\sigma_{w,s,M}\right] = \frac{2.30}{4\pi bK}\left[\log\left(2.25\frac{K}{r_w^2 S_S}\right) + \frac{2}{2.30}\left(\frac{K}{K_s} - 1\right)\ln\frac{r_s}{r_w}\right] \tag{32}$$

$$B_a = \frac{2.30}{4\pi bK}\left[\log\left(2.25\frac{K}{r^2 S_S}\right)\right] \tag{33}$$

$$C_w = F_{w,s,NL} + F_{w,a,NL} = \frac{\beta}{g}\left(\frac{1}{2\pi b}\right)^2\left[\frac{1}{r_w} + \left(\frac{\beta_s}{\beta} - 1\right)\left(\frac{1}{r_w} - \frac{1}{r_s}\right)\right] \tag{34}$$

$$C_a = F_{a,NL} = \frac{\beta}{g}\left(\frac{1}{2\pi b}\right)^2\left(\frac{1}{r}\right) \tag{35}$$

Applying the superposition principle to the linear part of the drawdown, to every constant rate step, it is possible to write the specific drawdown (at the well and at the piezometer) according to the following expression:

$$\frac{s_{i,j}}{|Q_i|} \approx AZ_i + B_j + C_j Q_i = AZ_i + \frac{\hat{s}_{0,i,j}}{|Q_i|} \tag{36}$$

where Q_i is the pumping rate during the ith step, $\Delta Q_i = Q_i - Q_{i-1}, Z_i = \sum_{k=1}^{i}\frac{\Delta Q_k}{|Q_i|}\log(t - t_k)$ is the new independent variable, $\hat{s}_{0,j,i}/|Q_i|$ is the extrapolation on the y axis at $Z = 0$ of the specific drawdown for every observation point and discharge rate, and t_k is the starting time of each pumping step.

The interpretation procedure, using the large time solutions presented, is the following:

- calculate for each data point and observation point Z_i;
- plot $s_{i,j}/|Q_i|$ as a function of Z_i (Fig. 3);
- for each observation point and step fit late time data using parallel lines with the same slope A;
- for each observation point and step determine the intercept $\hat{s}_{0,j,i}/|Q_i|$ of the fitting line on the y axis;
- for each observation piezometer, plot $\hat{s}_{0,j,i}/|Q_i|$ as a function of Q_i (Fig. 4);

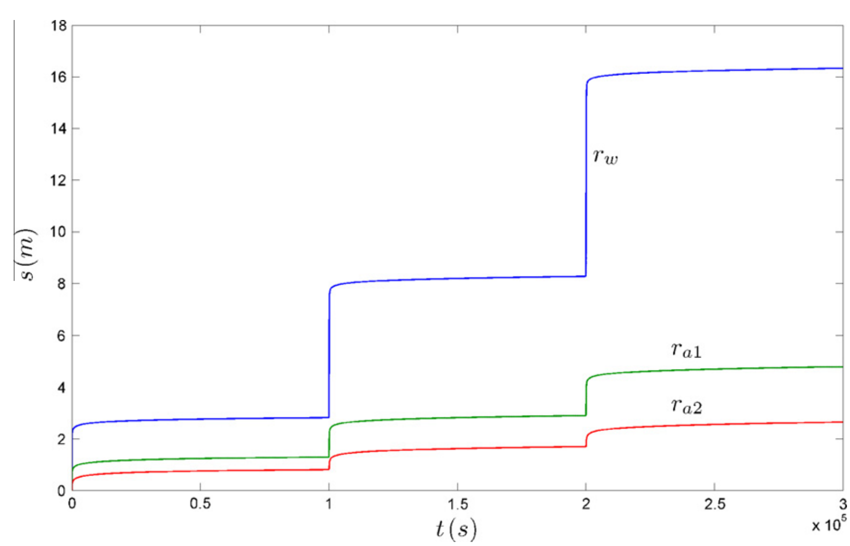

Fig. 2. Drawdown vs time generated at the pumping well and at two different observation points during a multi-well step drawdown test. The curves are generated using the numerical simulator.

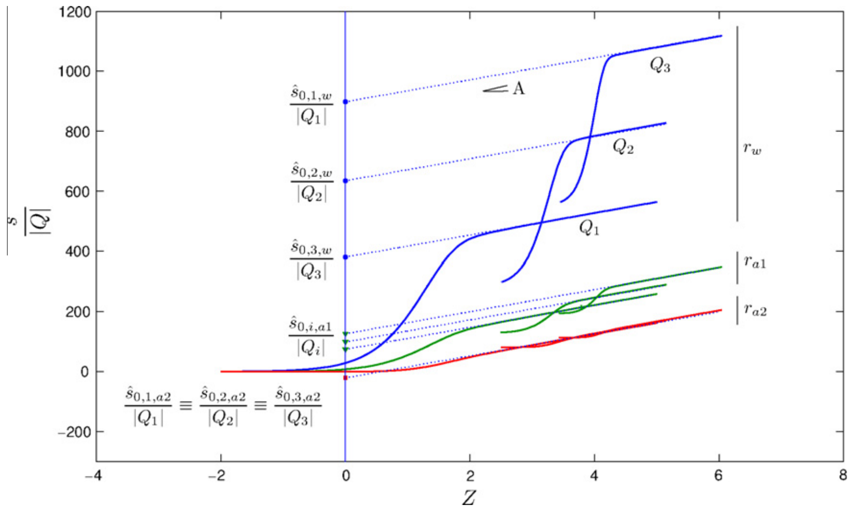

Fig. 3. Graphical method for the interpretation of the multi-well step drawdown test.

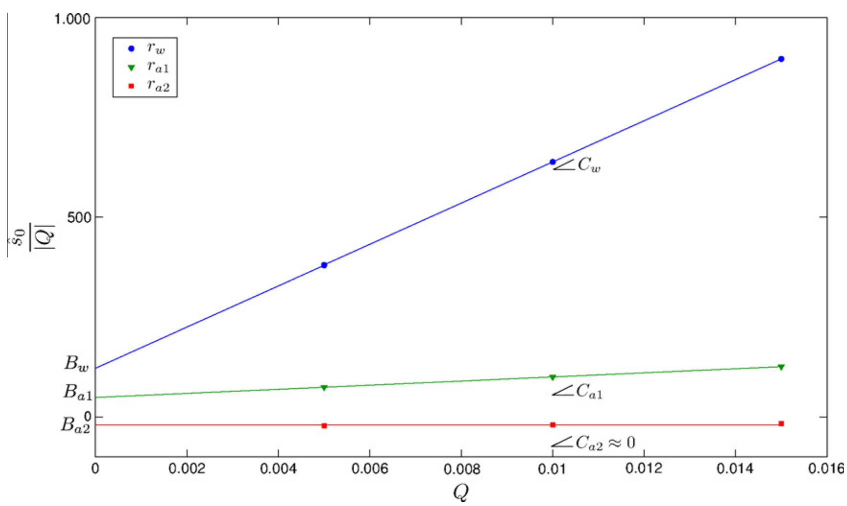

Fig. 4. Determination of the linear and non-linear coefficients B and C.

- for every observation piezometer, in a similar way to the procedure described by Jacob (1947), fit the corresponding set of data using a straight line of equation: $\frac{\hat{s}_{0,j}}{|Q|} = B_j + C_j Q$.

The unknown parameters A, B_j, C_j can be straightforward calculated using the described graphical technique or by an automatic fitting algorithm (e.g. non-linear least square minimization). At least four cases are worth discussing in the estimation of the skin and aquifer parameters.

(i) If the radius of the skin can be determined or estimated (i.e. from the radius of the drilling tool) it is possible to determine hydraulic conductivity of the aquifer K from A, specific storage S_s from B_a, skin factor $\sigma_{w,s,M}$ and hydraulic conductivity of the skin K_s from B_w, non-linear coefficients due to flow in the aquifer $F_{a,NL}$ and β from C_a in the piezometer, non-linear coefficients due to flow in the aquifer and in the skin $F_{w,s,NL}F_{w,a,NL}$ and β_s from C_w in the well, as follows:

$$K = \frac{2.30}{4\pi bA} \tag{37}$$

$$S_s = 10^{-\frac{4\pi bK}{2.30}B_a + \log\left(2.25\frac{K}{r^2}\right)} \tag{38}$$

$$\sigma_{w,s,M} = 2\pi bKB_w - \frac{2.30}{2}\log\left(2.25\frac{K}{r_w^2 S_S}\right) \tag{39}$$

$$K_s^{-1} = \left(K\ln\frac{r_s}{r_w}\right)^{-1}\left[2\pi bKB_w - \frac{2.30}{2}\log\left(2.25\frac{K}{r_w^2 S_S}\right)\right] + K^{-1} \tag{40}$$

$$\beta = gC_a r(2\pi b)^2 \tag{41}$$

$$\beta_s = \beta\left[\frac{g}{\beta}(2\pi b)^2 C_w - \frac{1}{r_w}\right]\left(\frac{1}{r_w} - \frac{1}{r_s}\right)^{-1} + \beta \tag{42}$$

(ii) If the radius of the skin cannot be determined, most of the parameters can still be retrieved (i.e. K, S_s, β, $\sigma_{w,s,M}$, $F_{w,s,NL}$ and also $F_{w,a,NL}$) except K_s and β_s. This is a demonstration of the robustness of the procedure.

(iii) For a given β and large r, Eq. (41) shows that C_a becomes small and, thus, difficult to estimate as shown in Fig. 4. This reflects the limited effect of non-linear flow far from the pumping well where q is small. In this case it is not possible to calculate independently the values of β, β_s, $F_{w,s,NL}$ and $F_{w,a,NL}$ but just the sum $F_{w,s,NL} + F_{w,a,NL}$.

Table 1
Application of the interpretation procedure to some synthetic scenarios generated by the numerical simulator: results and comparison.

r (m)	Numerical simulator data					Dual-well test results								Comparison				
	Aquifer data			Skin data		Fitting coefficients			Aquifer data			Skin data		Δ Aquifer data			Δ Skin data	
	Ss (1/m)	K (m/s)	β (1/m^4)	Ks (m/s)	βs (1/m^4)	A (s/m^2)	B (s/m^2)	C (s^2/m^5)	Ss (1/m)	K (m/s)	β (1/m)	Ks (m/s)	βs (1/m)	Ss (1/m)	K (m/s)	β (1/m)	Ks (m/s)	βs (1/m)
CASE 1 (HOMO)																		
0.1						36.66	122.8	5158.2	1.00E−05	1.00E−04	4.99E+08	1.00E−04	4.99E+08	0%	0%	0%	0%	0%
1						36.66	49.5	515.3	1.00E−05	1.00E−04	4.99E+08	1.00E−04	4.99E+08	0%	0%	0%	0%	0%
5	1.00E−05	1.00E−04	5.00E+08	1.00E−04	5.00E+08	36.66	−1.7	102.6	1.00E−05	1.00E−04	4.97E+08	1.00E−04	5.00E+08	0%	0%	1%	0%	0%
10						36.66	−23.8	51.0	1.00E−05	1.00E−04	4.94E+08	1.00E−04	5.00E+08	0%	0%	1%	0%	0%
100						36.62	−97.0	4.8	9.97E−06	1.00E−04	4.69E+08	1.00E−04	5.03E+08	0%	0%	6%	0%	−1%
CASE 2 (/10)																		
0.1						36.66	782.4	979.6	–	1.00E−04	–	–	–	–	0%	–	–	–
1						36.66	49.5	515.3	1.00E−05	1.00E−04	4.99E+08	1.00E−05	4.99E+07	0%	0%	0%	0%	0%
5	1.00E−05	1.00E−04	5.00E+08	1.00E−05	5.00E+07	36.66	−1.7	102.6	1.00E−05	1.00E−04	4.97E+08	1.00E−05	5.02E+07	0%	0%	1%	0%	0%
10						36.66	−23.8	51.0	1.00E−05	1.00E−04	4.94E+08	1.00E−05	5.05E+07	0%	0%	1%	0%	−1%
100						36.62	−97.0	4.8	9.97E−06	1.00E−04	4.68E+08	1.00E−05	5.34E+07	0%	0%	6%	0%	−7%
CASE 3 (/100)																		
0.1						36.71	7378.6	561.3	–	9.99E−05	–	–	–	–	0%	–	–	–
1						36.69	49.4	515.1	1.01E−05	1.00E−04	4.99E+08	1.00E−06	4.97E+06	−1%	0%	0%	0%	1%
5	1.00E−05	1.00E−04	5.00E+08	1.00E−06	5.00E+06	36.69	−1.8	102.4	1.01E−05	1.00E−04	4.96E+08	1.00E−06	5.29E+06	−1%	0%	1%	0%	−6%
10						36.68	−23.9	50.8	1.01E−05	1.00E−04	4.92E+08	1.00E−06	5.70E+06	−1%	0%	2%	0%	−14%
100						36.65	−97.1	4.6	1.00E−05	1.00E−04	4.49E+08	1.00E−06	1.04E+07	0%	0%	10%	0%	−109%
*CASE 4 (*10)*																		
0.1						36.66	56.8	46944.2	–	1.00E−04	–	–	–	–	0%	–	–	–
1						36.66	49.5	515.1	1.00E−05	1.00E−04	4.99E+08	1.00E−03	4.99E+09	0%	0%	0%	0%	0%
5	1.00E−05	1.00E−04	5.00E+08	1.00E−03	5.00E+09	36.66	−1.7	102.8	1.00E−05	1.00E−04	4.96E+08	1.00E−03	5.00E+09	0%	0%	1%	0%	0%
10						36.66	−23.8	51.0	1.00E−05	1.00E−04	4.93E+08	1.00E−03	5.00E+09	0%	0%	1%	0%	0%
100						36.62	−97.0	4.9	1.00E−05	1.00E−04	4.60E+08	1.00E−03	5.00E+09	0%	0%	8%	0%	0%
*CASE 5 (*100)*																		
0.1						36.77	50.1	464802.5	–	9.99E−05	–	–	–	–	0%	–	–	–
1						36.71	49.4	513.7	1.01E−05	9.99E−05	4.97E+08	1.10E−02	4.99E+10	−1%	0%	1%	−10%	0%
5	1.00E−05	1.00E−04	5.00E+08	1.00E−02	5.00E+10	36.71	−1.8	101.0	1.00E−05	9.99E−05	4.89E+08	1.14E−02	4.99E+10	0%	0%	2%	−14%	0%
10						36.70	−23.8	49.4	1.00E−05	9.99E−05	4.79E+08	1.16E−02	5.00E+10	0%	0%	4%	−16%	0%
100						36.67	−97.0	0.0	1.00E−05	9.99E−05	3.16E+08	1.29E−02	5.00E+10	0%	0%	37%	−29%	0%

(iv) On the contrary, if the skin is homogeneous with the aquifer system (i.e. $K = K_s$ and $\beta = \beta_s$), the linear and non-linear skin factors ($\sigma_{s,M}$ and $F_{s,NL}$) are identically zero and therefore the aquifer parameters can be calculated just from the data measured in the pumped well according to the following Eq. (37) together with the following expressions:

$$S_s = 10^{-\frac{4\pi bK}{2.30}B_w+\log\left(2.25\frac{K}{r_w^2}\right)} \tag{43}$$

$$\beta = gC_w r_w (2\pi b)^2 \tag{44}$$

From Eq. (28) it is also interesting to notice that even in presence of linear and non-linear flow, skin factor and wellbore storage, the hydraulic conductivity (and not the specific storage) of the aquifer can be correctly estimated from the pumping well data alone, using Eq. (37). Therefore, if a single-rate, single-well pumping test is performed, this procedure is equivalent to the use of the Cooper–Jacob large times semi-log analysis (1946) for the determination of the hydraulic conductivity. A similar consideration has already been reported for linear flow only towards a well with skin by Butler (1988).

6. Application to synthetic cases

The aforementioned method was tested on some synthetic cases generated by the numerical simulator described before. The simulated aquifer is characterized by a saturated thickness $b = 50$ m, an hydraulic conductivity $K = 1 \times 10^{-4}$ m/s, a specific storage $S_s = 1 \times 10^{-5}$ 1/m, a non-linear coefficient $\beta = 5 \times 10^8$ 1/m. The well is fully penetrating the aquifer and is characterized by a radius $r_w = 0.1$ m, the skin radius is $r_s = 1$ m. The pumping rate steps are $Q_1 = 5 \times 10^{-3}$ m^3/s ($0s < t < 1 \times 10^5 s$), $Q_2 = 10 \times 10^{-3}$ m^3/s ($1 \times 10^5 s < t < 2 \times 10^5$ s) and $Q_3 = 15 \times 10^{-3}$ m^3/s (1×10^5 s $< t < 1 \times 10^6 s$). Because the maximum distance at which the linear approximation is valid for every pumping step is $r = 282$ m, the observation points are placed at 1, 5, 10 and 100 m from the well; the domain limit is 10,000 m distant from the well.

Five different test cases were generated using the numerical simulator (Table 1):

- Case 1: The well skin and aquifer system are characterized by the same values of linear, non-linear hydrodynamic parameters and specific storage.
- Cases 2 and 3: The well skin is characterized by values of hydrodynamic and storage parameters respectively ten or hundred of times smaller than the aquifer parameters.
- Cases 4 and 5: The well skin is characterized by values of hydrodynamic and storage parameters respectively ten or hundred of times bigger than the aquifer parameters.

The results coming from the application of the dual-well step drawdown test to the simulated drawdowns are reported in Table 1. In the same table, for each case, a calculation of the relative error between model data and data derived from the interpretation procedure is reported.

The analysis of the results coming from the described test cases confirms that:

- in absence of skin (Case 1), it is possible to derive correctly the linear, non-linear hydrodynamic parameters and storage coefficient just from the drawdown inside the well;
- the accuracy of the method is very good at small to average distances (in this case $1 < r < 10$ m) from the pumping well. The maximum reported error (−16%, Case 5, $r = 10$ m) is still small;
- if the observation well is very far from the pumping well (e.g. $r = 100$ m) the estimation of C can be inaccurate (as described in the previous paragraph, due to the limited effects of the non-linear flow at large distances) and therefore also the estimation of non-linear parameter of the aquifer and of the skin are affected by considerable errors.

If we remember that in study the errors include both the uncertainties deriving from the numerical simulator and from the interpretation procedure we can conclude that the presented method is able to provide excellent to good results in the determination of the parameters at small to average distances to the pumping well.

7. Conclusions

In this study a combination of an aquifer and a well performance test is presented based on dual-well step drawdown method for the simultaneous determination of hydrodynamic parameters (namely storage coefficient and hydraulic conductivity), mechanical wellbore finite thickness skin factor, non-linear wellbore and non-linear aquifer parameters in a confined aquifer is presented. The interpretation procedure is based on the application of superposition principle to a large time logarithmic approximation of the solution but also an automatic fitting algorithm (e.g. non-linear least square minimization) can be used.

The interpretation procedure was tested under five scenarios using different ratio between aquifer and skin parameters generating the drawdown with a numerical simulator. The results of the interpretations are in good agreements with the input of the numerical simulator in particular at small to average distances from the pumping well. Discrepancies can be present at observations points very far from the well where also fully transient models are likely to produce inaccurate results.

The advantages of this method, that can be considered an extension of Jacob step-test (1947) and Cooper–Jacob approximation (1946), are that: (I) it is possible to determine simultaneously aquifer and wellbore properties in a single test, also when the radius of the skin cannot be estimated correctly; (II) the method is based on a large time approximation and it is therefore independent from the wellbore storage; (III) if the well skin is absent, the aquifer parameters can be derived just from a single-well test; (IV) the interpretation procedure is easy and robust for small and average distances from pumping well and do not require any specific numeric code or software. The same procedure, which does not require an a priori estimation of the storage coefficient, can be easily adapted to gas well testing.

It was also shown that, even in the presence of linear and non-linear flow, skin effect and wellbore storage, the hydraulic conductivity of the aquifer can be correctly estimated using the Cooper and Jacob (1946) method from a long lasting, single-rate pumping test using exclusively the large time drawdown values measured at the pumping well.

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Appendix A. Steady state solution

The steady state solution of the problem can be derived as follows:

$$\frac{1}{K_{sD}}q_D + |\beta_{sD}||q_D|q_D = -\frac{\partial h_D}{\partial r_D} \quad r_{wD} \leqslant r_D \leqslant r_{sD} \tag{10}$$

$$q_D + |\beta_D||q_D|q_D = -\frac{\partial h_D}{\partial r_D} \quad r_D \geqslant r_{sD} \tag{11}$$

The wellbore drawdown in the system is:

$$\int_{r_{wD}}^{r_{eD}} -\frac{\partial h_D}{\partial r_D} dr_D = \int_{rwD}^{rsD} \left(\frac{1}{K_{sD}}q_D + |\beta_{sD}||q_D|q_D\right) dr_D + \int_{rsD}^{reD} (q_D + |\beta_D||q_D|q_D) dr_D \tag{45}$$

Under steady state conditions it is possible to write the continuity equation as follows:

$$q_D = r_D^{-1} \tag{46}$$

and substituting into Eq. (45) leads to:

$$\int_{r_{wD}}^{r_{eD}} -\frac{\partial h_D}{\partial r_D} dr_D = \int_{rwD}^{rsD} \left(\frac{1}{K_{sD}}r_D^{-1} + |\beta_{sD}|r_D^{-2}\right) dr_D + \int_{rsD}^{reD} (r_D^{-1} + |\beta_D|r_D^{-2}) dr_D \tag{47}$$

and, after integration the drawdown is:

$$h_{rwD} - h_{reD} = \frac{1}{K_{sD}} \ln\frac{r_{sD}}{r_{wD}} - |\beta_{sD}|\left(\frac{1}{r_{sD}} - \frac{1}{r_{wD}}\right) + \ln\frac{r_{eD}}{r_{sD}} - |\beta_D|\left(\frac{1}{r_{eD}} - \frac{1}{r_{sD}}\right)$$

choosing r_{eD} such that h_{reD} = 0 (i.e. the initial condition of the problem) and after some algebra:

$$h_{rwD} = \ln\frac{r_{eD}}{r_{wD}} + \left(\frac{1}{K_{sD}} - 1\right)\ln\frac{r_{sD}}{r_{wD}} + |\beta_D|\left(\frac{1}{r_{wD}} - \frac{1}{r_{eD}}\right) + (|\beta_{sD}| - |\beta_D|)\left(\frac{1}{r_{wD}} - \frac{1}{r_{sD}}\right) \quad r_D = r_{wD} \tag{48}$$

More in general, for a generic r_D it is possible to get:

$$h_{s,D} = \ln\frac{r_{eD}}{r_D} + \left(\frac{1}{K_{sD}} - 1\right)\ln\frac{r_{sD}}{r_D} + |\beta_D|\left(\frac{1}{r_D} - \frac{1}{r_{eD}}\right) + (|\beta_{sD}| - |\beta_D|)\left(\frac{1}{r_D} - \frac{1}{r_{sD}}\right) \quad r_{wD} \leqslant r_D \leqslant r_{sD} \tag{49}$$

$$h_{a,D} = \ln\frac{r_{eD}}{r_D} + |\beta_D|\left(\frac{1}{r_D} - \frac{1}{r_{eD}}\right) \quad r_D \geqslant r_{sD} \tag{50}$$

Or in terms of dimensional parameters the drawdown is:

$$s_s = -h_s = \frac{1}{2\pi bK}\left[\ln\frac{r_e}{r} + \left(\frac{K}{K_s} - 1\right)\ln\frac{r_s}{r}\right]Q + \frac{\beta}{g}\left(\frac{1}{2\pi b}\right)^2\left(\frac{1}{r} - \frac{1}{r_e}\right)|Q|Q + \frac{\beta}{g}\left(\frac{1}{2\pi b}\right)^2\left(\frac{\beta_s}{\beta} - 1\right)\left(\frac{1}{r} - \frac{1}{r_s}\right)|Q|Q \quad r_w \leqslant r \leqslant r_s \tag{51}$$

$$s_a = -h_a = \frac{1}{2\pi bK}\ln\frac{r_e}{r}Q + \frac{\beta}{g}\left(\frac{1}{2\pi b}\right)^2\left(\frac{1}{r} - \frac{1}{r_e}\right)|Q|Q \quad r \geqslant r_s \tag{52}$$

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