Zadanie 1

a. Macierz jednostkowa 3x3

In[•]:=
```
(Ma1 = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b. Macierz 3x2

In[•]:=
```
(Ma2 = {{a1,1, a1,2}, {a2,1, a2,2}, {a3,1, a3,2}}) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{pmatrix}$$

c. Macierz 2x4 zawierająca pierwiastki z kolejnych liczb naturalnych

In[•]:=
```
(Ma3 = Table[√(4 i + j), {i, 0, 1}, {j, 1, 4}]) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & \sqrt{2} & \sqrt{3} & 2 \\ \sqrt{5} & \sqrt{6} & \sqrt{7} & 2\sqrt{2} \end{pmatrix}$$

d. Macierz górnotrójkątna 3x3

In[•]:=
```
(Ma4 = {{a1,1, a1,2, a1,3}, {0, a2,2, a2,3}, {0, 0, a3,3}}) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ 0 & a_{2,2} & a_{2,3} \\ 0 & 0 & a_{3,3} \end{pmatrix}$$

Zadanie 2

In[•]:=
```
Ma1 + Ma2; (* Nie można dodać *)
Ma1 - Ma2; (* Nie można odjąć *)
Ma1.Ma2 // MatrixForm
Ma2.Ma1; (* Nie można pomnożyć *)
```

Thread: Objects of unequal length in $\{1, 0, 0\} + \{a_{1,1}, a_{1,2}\}$ cannot be combined.

Thread: Objects of unequal length in $\{0, 1, 0\} + \{a_{2,1}, a_{2,2}\}$ cannot be combined.

Thread: Objects of unequal length in $\{0, 0, 1\} + \{a_{3,1}, a_{3,2}\}$ cannot be combined.

General: Further output of Thread::tdlen will be suppressed during this calculation.

Thread: Objects of unequal length in $\{1, 0, 0\} + \{-a_{1,1}, -a_{1,2}\}$ cannot be combined.

Thread: Objects of unequal length in $\{0, 1, 0\} + \{-a_{2,1}, -a_{2,2}\}$ cannot be combined.

Thread: Objects of unequal length in $\{0, 0, 1\} + \{-a_{3,1}, -a_{3,2}\}$ cannot be combined.

General: Further output of Thread::tdlen will be suppressed during this calculation.

Out[•]//MatrixForm=

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{pmatrix}$$

Dot: Tensors $\{\{a_{1,1}, a_{1,2}\}, \{a_{2,1}, a_{2,2}\}, \{a_{3,1}, a_{3,2}\}\}$ and $\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$ have incompatible shapes.

In[•]:=
```
Ma1 + Ma3; (* Nie można dodać *)
Ma1 - Ma3; (* Nie można odjąć *)
Ma1.Ma3; (* Nie można pomnożyć *)
Ma3.Ma1; (* Nie można pomnożyć *)
```

Thread: Objects of unequal length in $\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\} + \left\{\left\{1, \sqrt{2}, \sqrt{3}, 2\right\}, \left\{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\right\}\right\}$ cannot be combined.

Thread: Objects of unequal length in $\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\} + \left\{\left\{-1, -\sqrt{2}, -\sqrt{3}, -2\right\}, \left\{-\sqrt{5}, -\sqrt{6}, -\sqrt{7}, -2\sqrt{2}\right\}\right\}$ cannot be combined.

Dot: Tensors $\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$ and $\left\{\left\{1, \sqrt{2}, \sqrt{3}, 2\right\}, \left\{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\right\}\right\}$ have incompatible shapes.

Dot: Tensors $\left\{\left\{1, \sqrt{2}, \sqrt{3}, 2\right\}, \left\{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\right\}\right\}$ and $\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$ have incompatible shapes.

```
In[•]:= Ma1 + Ma4 // MatrixForm
        Ma1 - Ma4 // MatrixForm
        Ma1.Ma4 // MatrixForm
        Ma4.Ma1 // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1+a_{1,1} & a_{1,2} & a_{1,3} \\ 0 & 1+a_{2,2} & a_{2,3} \\ 0 & 0 & 1+a_{3,3} \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 1-a_{1,1} & -a_{1,2} & -a_{1,3} \\ 0 & 1-a_{2,2} & -a_{2,3} \\ 0 & 0 & 1-a_{3,3} \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ 0 & a_{2,2} & a_{2,3} \\ 0 & 0 & a_{3,3} \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ 0 & a_{2,2} & a_{2,3} \\ 0 & 0 & a_{3,3} \end{pmatrix}$$

```
In[•]:= Ma2 + Ma3; (* Nie można dodać *)
        Ma2 - Ma3; (* Nie można odjąć *)
        Ma2.Ma3 // MatrixForm
        Ma3.Ma2; (* Nie można pomnożyć *)
```

••• Thread: Objects of unequal length in $\{\{a_{1,1}, a_{1,2}\}, \{a_{2,1}, a_{2,2}\}, \{a_{3,1}, a_{3,2}\}\} + \left\{\left\{1, \sqrt{2}, \sqrt{3}, 2\right\}, \left\{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\right\}\right\}$ cannot be combined.

••• Thread: Objects of unequal length in $\{\{a_{1,1}, a_{1,2}\}, \{a_{2,1}, a_{2,2}\}, \{a_{3,1}, a_{3,2}\}\} + \left\{\left\{-1, -\sqrt{2}, -\sqrt{3}, -2\right\}, \left\{-\sqrt{5}, -\sqrt{6}, -\sqrt{7}, -2\sqrt{2}\right\}\right\}$ cannot be combined.

Out[•]//MatrixForm=

$$\begin{pmatrix} a_{1,1}+\sqrt{5}\,a_{1,2} & \sqrt{2}\,a_{1,1}+\sqrt{6}\,a_{1,2} & \sqrt{3}\,a_{1,1}+\sqrt{7}\,a_{1,2} & 2\,a_{1,1}+2\sqrt{2}\,a_{1,2} \\ a_{2,1}+\sqrt{5}\,a_{2,2} & \sqrt{2}\,a_{2,1}+\sqrt{6}\,a_{2,2} & \sqrt{3}\,a_{2,1}+\sqrt{7}\,a_{2,2} & 2\,a_{2,1}+2\sqrt{2}\,a_{2,2} \\ a_{3,1}+\sqrt{5}\,a_{3,2} & \sqrt{2}\,a_{3,1}+\sqrt{6}\,a_{3,2} & \sqrt{3}\,a_{3,1}+\sqrt{7}\,a_{3,2} & 2\,a_{3,1}+2\sqrt{2}\,a_{3,2} \end{pmatrix}$$

••• Dot: Tensors $\left\{\left\{1, \sqrt{2}, \sqrt{3}, 2\right\}, \left\{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\right\}\right\}$ and $\{\{a_{1,1}, a_{1,2}\}, \{a_{2,1}, a_{2,2}\}, \{a_{3,1}, a_{3,2}\}\}$ have incompatible shapes.

In[•]:=
```
Ma2 + Ma4; (* Nie można dodać *)
Ma2 - Ma4; (* Nie można odjąć *)
Ma2.Ma4; (* Nie można pomnożyć *)
Ma4.Ma2 // MatrixForm
```

Thread: Objects of unequal length in $\{a_{1,1}, a_{1,2}\} + \{a_{1,1}, a_{1,2}, a_{1,3}\}$ cannot be combined.

Thread: Objects of unequal length in $\{a_{2,1}, a_{2,2}\} + \{0, a_{2,2}, a_{2,3}\}$ cannot be combined.

Thread: Objects of unequal length in $\{a_{3,1}, a_{3,2}\} + \{0, 0, a_{3,3}\}$ cannot be combined.

General: Further output of Thread::tdlen will be suppressed during this calculation.

Thread: Objects of unequal length in $\{a_{1,1}, a_{1,2}\} + \{-a_{1,1}, -a_{1,2}, -a_{1,3}\}$ cannot be combined.

Thread: Objects of unequal length in $\{a_{2,1}, a_{2,2}\} + \{0, -a_{2,2}, -a_{2,3}\}$ cannot be combined.

Thread: Objects of unequal length in $\{a_{3,1}, a_{3,2}\} + \{0, 0, -a_{3,3}\}$ cannot be combined.

General: Further output of Thread::tdlen will be suppressed during this calculation.

Dot: Tensors $\{\{a_{1,1}, a_{1,2}\}, \{a_{2,1}, a_{2,2}\}, \{a_{3,1}, a_{3,2}\}\}$ and $\{\{a_{1,1}, a_{1,2}, a_{1,3}\}, \{0, a_{2,2}, a_{2,3}\}, \{0, 0, a_{3,3}\}\}$ have incompatible shapes.

Out[•]//MatrixForm=

$$\begin{pmatrix} a_{1,1}^2 + a_{1,2}\,a_{2,1} + a_{1,3}\,a_{3,1} & a_{1,1}\,a_{1,2} + a_{1,2}\,a_{2,2} + a_{1,3}\,a_{3,2} \\ a_{2,1}\,a_{2,2} + a_{2,3}\,a_{3,1} & a_{2,2}^2 + a_{2,3}\,a_{3,2} \\ a_{3,1}\,a_{3,3} & a_{3,2}\,a_{3,3} \end{pmatrix}$$

In[•]:=
```
Ma3 + Ma4; (* Nie można dodać *)
Ma3 - Ma4; (* Nie można odjąć *)
Ma3.Ma4; (* Nie można pomnożyć *)
Ma4.Ma3; (* Nie można pomnożyć *)
```

Thread: Objects of unequal length in $\{\{1, \sqrt{2}, \sqrt{3}, 2\}, \{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\}\} + \{\{a_{1,1}, a_{1,2}, a_{1,3}\}, \{0, a_{2,2}, a_{2,3}\}, \{0, 0, a_{3,3}\}\}$ cannot be combined.

Thread: Objects of unequal length in $\{\{1, \sqrt{2}, \sqrt{3}, 2\}, \{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\}\} + \{\{-a_{1,1}, -a_{1,2}, -a_{1,3}\}, \{0, -a_{2,2}, -a_{2,3}\}, \{0, 0, -a_{3,3}\}\}$ cannot be combined.

Dot: Tensors $\{\{1, \sqrt{2}, \sqrt{3}, 2\}, \{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\}\}$ and $\{\{a_{1,1}, a_{1,2}, a_{1,3}\}, \{0, a_{2,2}, a_{2,3}\}, \{0, 0, a_{3,3}\}\}$ have incompatible shapes.

Dot: Tensors $\{\{a_{1,1}, a_{1,2}, a_{1,3}\}, \{0, a_{2,2}, a_{2,3}\}, \{0, 0, a_{3,3}\}\}$ and $\{\{1, \sqrt{2}, \sqrt{3}, 2\}, \{\sqrt{5}, \sqrt{6}, \sqrt{7}, 2\sqrt{2}\}\}$ have incompatible shapes.

Zadanie 3

In[•]:=
```
Det[Ma1]
Tr[Ma1]
Inverse[Ma1] // MatrixForm
Inverse[Ma2]; (* Macierz odwrotna nie istnieje *)
(Ma5 = Transpose[Ma2.Ma3]) // MatrixForm
```

Out[•]=
```
1
```

Out[•]=
```
3
```

Out[•]//MatrixForm=
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse: Argument $\{\{a_{1,1}, a_{1,2}\}, \{a_{2,1}, a_{2,2}\}, \{a_{3,1}, a_{3,2}\}\}$ at position 1 is not a non-empty square matrix.

Out[•]//MatrixForm=
$$\begin{pmatrix} a_{1,1} + \sqrt{5}\, a_{1,2} & a_{2,1} + \sqrt{5}\, a_{2,2} & a_{3,1} + \sqrt{5}\, a_{3,2} \\ \sqrt{2}\, a_{1,1} + \sqrt{6}\, a_{1,2} & \sqrt{2}\, a_{2,1} + \sqrt{6}\, a_{2,2} & \sqrt{2}\, a_{3,1} + \sqrt{6}\, a_{3,2} \\ \sqrt{3}\, a_{1,1} + \sqrt{7}\, a_{1,2} & \sqrt{3}\, a_{2,1} + \sqrt{7}\, a_{2,2} & \sqrt{3}\, a_{3,1} + \sqrt{7}\, a_{3,2} \\ 2\, a_{1,1} + 2\sqrt{2}\, a_{1,2} & 2\, a_{2,1} + 2\sqrt{2}\, a_{2,2} & 2\, a_{3,1} + 2\sqrt{2}\, a_{3,2} \end{pmatrix}$$

In[•]:=
```
Ma5[[2, 3]]
Ma5[[3]]
Ma5[[All, 1]] // MatrixForm
```

Out[•]=
$$5\sqrt{2} + 6\sqrt{6}$$

Out[•]=
$$\{\sqrt{3} + 2\sqrt{7},\ 3\sqrt{3} + 4\sqrt{7},\ 5\sqrt{3} + 6\sqrt{7}\}$$

Out[•]//MatrixForm=
$$\begin{pmatrix} 1 + 2\sqrt{5} \\ \sqrt{2} + 2\sqrt{6} \\ \sqrt{3} + 2\sqrt{7} \\ 2 + 4\sqrt{2} \end{pmatrix}$$

Zadanie 4

```
In[•]:= Ma6 = Table[Ma1[[4 - j]] + Sqrt[j^2 + 1], {j, 3}];
        Ma6 = Append[Ma6, ConstantArray[0, 3]]; (* Dodanie wiersza zer *)
        Ma6 = Map[Append[#, 0] &, Ma6]; (* Dodanie kolumny zer *)
        Ma6 // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \sqrt{2} & \sqrt{2} & 1+\sqrt{2} & 0 \\ \sqrt{5} & 1+\sqrt{5} & \sqrt{5} & 0 \\ 1+\sqrt{10} & \sqrt{10} & \sqrt{10} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Zadanie domowe

Macierz Ma7

```
In[•]:= w1 = Range[1, 5];
        w2 = 2^Range[1,5];
        w3 = Range[1, 9, 2]^2;
        w4 = Range[2, 10, 2]!;
        w5 = ConstantArray[0, 5];

        (Ma7 = {w1, w2, w3, w4, w5}) // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 8 & 16 & 32 \\ 1 & 9 & 25 & 49 & 81 \\ 2 & 24 & 720 & 40\,320 & 3\,628\,800 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Trójkąt Pascala

```
In[•]:= PascalTriangle[n_] := Table[Binomial[i, j], {i, 0, n - 1}, {j, 0, i}]

        (* Test dla n=7 *)
        PascalTriangle[7] // MatrixForm
```

Out[•]//MatrixForm=

$$\begin{pmatrix} \{1\} \\ \{1, 1\} \\ \{1, 2, 1\} \\ \{1, 3, 3, 1\} \\ \{1, 4, 6, 4, 1\} \\ \{1, 5, 10, 10, 5, 1\} \\ \{1, 6, 15, 20, 15, 6, 1\} \end{pmatrix}$$