Recitation Session June 6th

Recursive Formulation

$$\max \sum_{e\in E} w_e y_e$$

$$\text{s.t.} \sum_{e\in\delta^-(v)} y_e = f_v^i \quad v\in V \qquad [1]$$

$$\sum_{e\in\delta^+(v)} y_e = f_v^o \quad v\in V$$

$$f_v^o \le f_v^i \le 1 \quad v\in P, \qquad [2]$$

$$f_v^o \le 1 \quad v\in N, \qquad [3]$$

$$\sum_{e\in C} y_e \le |C|-1 \quad C\in C\setminus C_k, \qquad [4]$$

subset of the cycle constraints

$$y_e \in \{0,1\} \quad e\in E. \qquad [5]$$

PC-TSP Formulation

$$\max \sum_{e\in E} w_e y_e + \sum_{C\in\mathcal{C}_k} w_C z_C$$

$$\text{s.t.} \sum_{e\in\delta^-(v)} y_e = f_v^i \quad v\in V$$

$$\sum_{e\in\delta^+(v)} y_e = f_v^o \quad v\in V$$

$$f_v^o + \sum_{C\in\mathcal{C}_k(v)} z_C \le f_v^i + \sum_{C\in\mathcal{C}_k(v)} z_C \le 1 \quad v\in P, \quad (6)$$

$$f_v^o \le 1 \quad v\in N,$$

$$\sum_{e\in\delta^-(S)} y_e \ge f_v^i \quad S\subseteq P,\ v\in S$$

$$y_e \in \{0,1\} \quad e\in E,$$

$$z_C \in \{0,1\} \quad C\in\mathcal{C}_k.$$

Given an integer solution to the recursive formulation, how can we find a violated constraint of type [4]/(6), if one exists?

Due to [1] and [2], the edges selected in the solution form a graph consisting of cycles and paths

↳ Start at an unvisited node, walk along the single possible outgoing edge and cont.

If we end up at the starting node, we have found a cycle.
If we end up at a node w/o outgoing edge, we started in a path

Note: Given a fractional solution, finding a violated [4] inequ. is NP-hard

If there is no cycle created by the edges with $y_e = 1$, we are happy.

Use the procedure to the left, find a cycle if one exists.

Choose S = nodes in cycle and v arbitrary in the cycle.

This gives us a violated inequality of type (6)

How can we do the same for fractional solutions?

If $f_v^i = 0$, the constraint cannot be violated

↳ choose some v with $f_v^i > 0$

We are looking for a set S s.t. $v \in S$, $S \subseteq P$

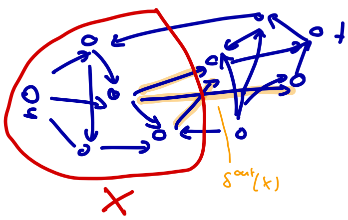
$\Downarrow$

$N \cap S = \emptyset$

and $\sum_{e \in \delta^{in}(S)} y_e < f_v^i$

MIN-s-t-CUT PROBLEM:

$$\min_{\substack{X \subseteq V \\ s \in X \\ t \notin X}} \sum_{e \in \delta^{out}(X)} w_e$$



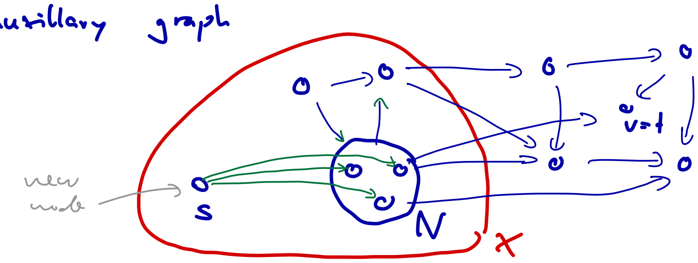
If we can solve $Z = \min_{\substack{S \subseteq P \\ v \in S}} \sum_{e \in \delta^{in}(S)} y_e$,

then there exists a violated cut if and only if

$Z < f_v^i$.

We are looking for a minimum "N-v-cut" with $w_e = y_e$

This problem can be solved by a Min s-t-Cut Problem in an auxillary graph



If we choose $w_e = y_e$ for the blue edges and $w_e = 1$ for the green edges, we can solve the problem.

If the minimum cut has weight $< f_v^i$ simply choose

$\delta^{out}(X) = \delta^{in}(V\setminus X)$

$S = V\setminus X$. We know that $N \in X$ because $f_v^i \leq 1$.

If $N \notin X$ at least one green edge would be part of the cut and then the weight cannot be strictly smaller than $f_v^i \leq 1$.

If the minimum cut has weight $\geq f_v^i$, it is clear that the minimum N-v-cut also has weight $\geq f_v^i$ and thus no violated inequality exists.