Technische Universität München, TUM School of Management
Lehrstuhl für Operations Research

Applied Discrete Optimization

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Assignment 3

Exercise A3.1 (Path-Based Formulation for KEP) [10]

In addition to the numerous reasons why a donor-recipient pair might drop out of the exchange pool, compatibility issues might be detected only very shortly before transplantation. In graph terms, this means that some edges might fail. If this occurs early in a long chain, it can lead to the whole chain failing. Therefore, in practice, shorter parallel chains are favorable. Thus, we want to also include an upper bound on the chain length in the model.

Please provide a path/cycle-based formulation of the Kidney Exchange Problem with bounds k_{cycle} and k_{chain}. As this formulation has many variables that we do not want to enumerate a priori, please also provide the corresponding pricing problem for column generation.

Hint: Please provide IP formulations for the pricing problems. For general k, they are NP-hard.

Exercise A3.2 (Separation of Subtour Inequalities) [10]

Consider the following IP formulation for the (symmetric) Traveling Salesperson Problem on the complete graph $G = (V, E)$:

$$\begin{aligned} \min \quad & \sum_{e \in E} c_e x_e \\ \text{s.t.} \quad & \sum_{e \in \delta(\{i\})} x_e = 2, \text{ for all } i \in V, && (1) \\ & \sum_{e \in \delta(S)} x_e \geq 2, \text{ for all } S \subset V,\ S \neq \emptyset, && (2) \\ & x_e \in \{0, 1\} \end{aligned}$$

There are exponentially many constraints of type (2). Similar to the recursive algorithm we have seen in the lecture, assume that we have only added a subset of the inequalities (2). First, provide an easy procedure that, given an **integer** solution that respects (1) and the subset of (2), finds a violated constraint of type (2), if one exists. Then provide a procedure that, given a **fractional** solution that respects (1) and the subset of (2), finds a violated constraint of type (2), if one exists.

Hint: For the second part, assume that $v \in S$ and $w \notin S$.

Exercise A3.3 (Implementation: KEP models) [10]

In this exercise, we will implement the two models for KEP that we have seen in the lecture. First, we want to implement the recursive model as has been described in the lecture. That is, we solve the IP (with only a subset of the cycle constraints) to optimality and then check whether the solution contains long cycles. If so, we add the corresponding constraint to the model and repeat. Afterwards, we hope to be a little bit smarter and add the cycle constraints as lazy constraints. This means that Gurobi will check every integral solution it encounters and add the corresponding constraint on the fly. Lastly, we will implement the PC-TSP formulation that we have seen in the lecture. For simplicity, we will consider the cut constraints also as lazy constraints.

Due by Monday, June 17, 1 p.m.