4.1

The speedup, depending on the number of n processors, can be determined from Amdahl's Law.

$$\begin{aligned}
speedup &= \frac{1}{(1-p)+\frac{p}{n}} \\
&= \frac{1}{(1-0.6)+\frac{0.6}{n}} \\
&= \frac{1}{0.4+\frac{0.6}{n}} \\
&= \frac{n}{0.4n+0.6}
\end{aligned}$$

The maximum theoretical speedup happens when n approaches inifinity, making the parallel portion take no time.

$$speedup_{max} = speedup_{n\to\infty} = \frac{1}{0.4+0} = 2.5$$

4.2

We recalculate the speedup now with a 30% sequential program.

$$\begin{aligned}
s_n &= \frac{1}{(1-p)+\frac{p}{n}} \\
&= \frac{1}{(1-0.7)+\frac{0.7}{n}} \\
&= \frac{1}{0.3+\frac{0.7}{n}} \\
&= \frac{n}{0.3n+0.7}
\end{aligned}$$

The new speedup is double the original speedup, and the new sequential portion is decreased by a factor of k.

$$\begin{aligned}
s'_n &\geq 2s_n \\
\frac{n}{\frac{0.3n}{k}+0.7} &\geq \frac{2n}{0.3n+0.7} \\
\frac{1}{\frac{0.3n}{k}+0.7} &\geq \frac{2}{0.3n+0.7} \\
\frac{0.3n}{k}+0.7 &\leq \frac{0.3n}{2}+\frac{0.7}{2} \\
\frac{0.3n}{k} &\leq 0.15n-0.35 \\
k &\geq \frac{0.3n}{0.15n-0.35}
\end{aligned}$$

4.3

The sequential portion can be written as f, and the parallel portion as $1-f$. The new sequential portion is now $f/3$, which affect the new parallel portion of now $1-f/3$.

$$\begin{aligned}
s'_n &= 2s_n \\
\frac{1}{\frac{f}{3}+\frac{1-f/3}{n}} &= \frac{2}{f+\frac{1-f}{n}} \\
\frac{f}{3}+\frac{1-\frac{f}{3}}{n} &= \frac{f}{2}+\frac{1-f}{2n} \\
2f+\frac{6(1-\frac{f}{3})}{n} &= 3f+\frac{3(1-f)}{n} \\
2f+\frac{6-2f}{n} &= 3f+\frac{3-3f}{n} \\
\frac{6-2f}{n}-\frac{3-3f}{n} &= f \\
\frac{6-2f-3+3f}{n} &= f \\
\frac{3+f}{n} &= f \\
3+f &= fn \\
3 &= fn-f = f(n-1) \\
f &= \frac{3}{n-1}
\end{aligned}$$