Quantum Hall Effect in Graphene

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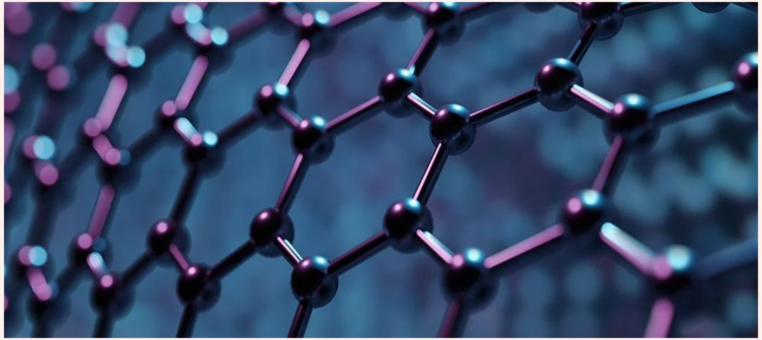
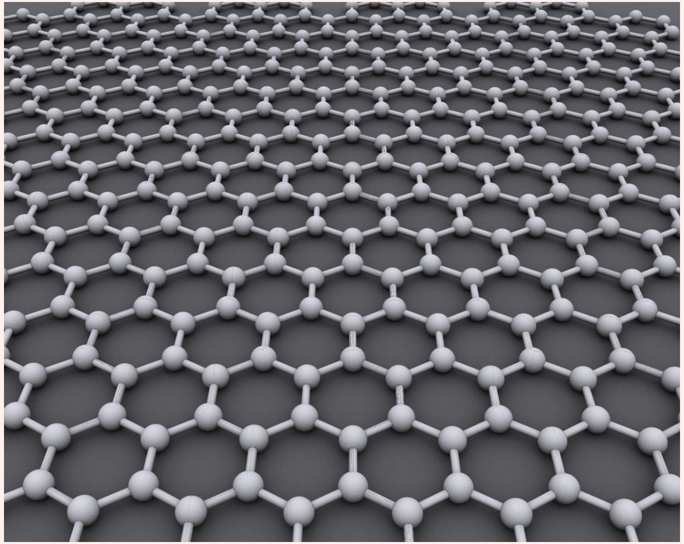
Graphene

- A 2D hexagonal lattice of carbon
- Each carbon bonded to 3 others with sp^2 bonds
- Conductive band structure created by non bonded orbitals

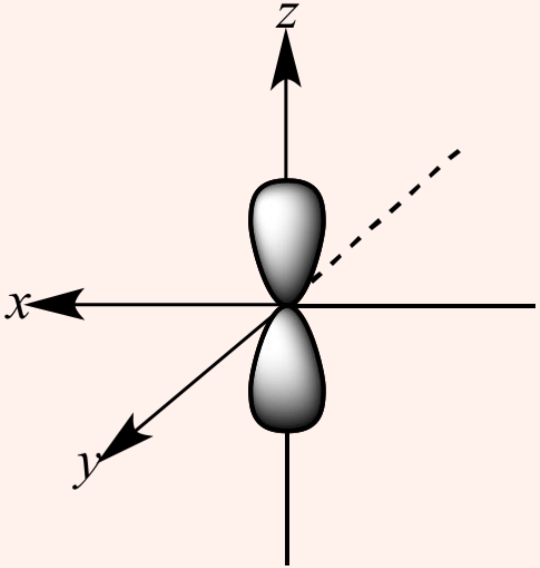


Bandstructure

- Basis method
- Tight-binding model

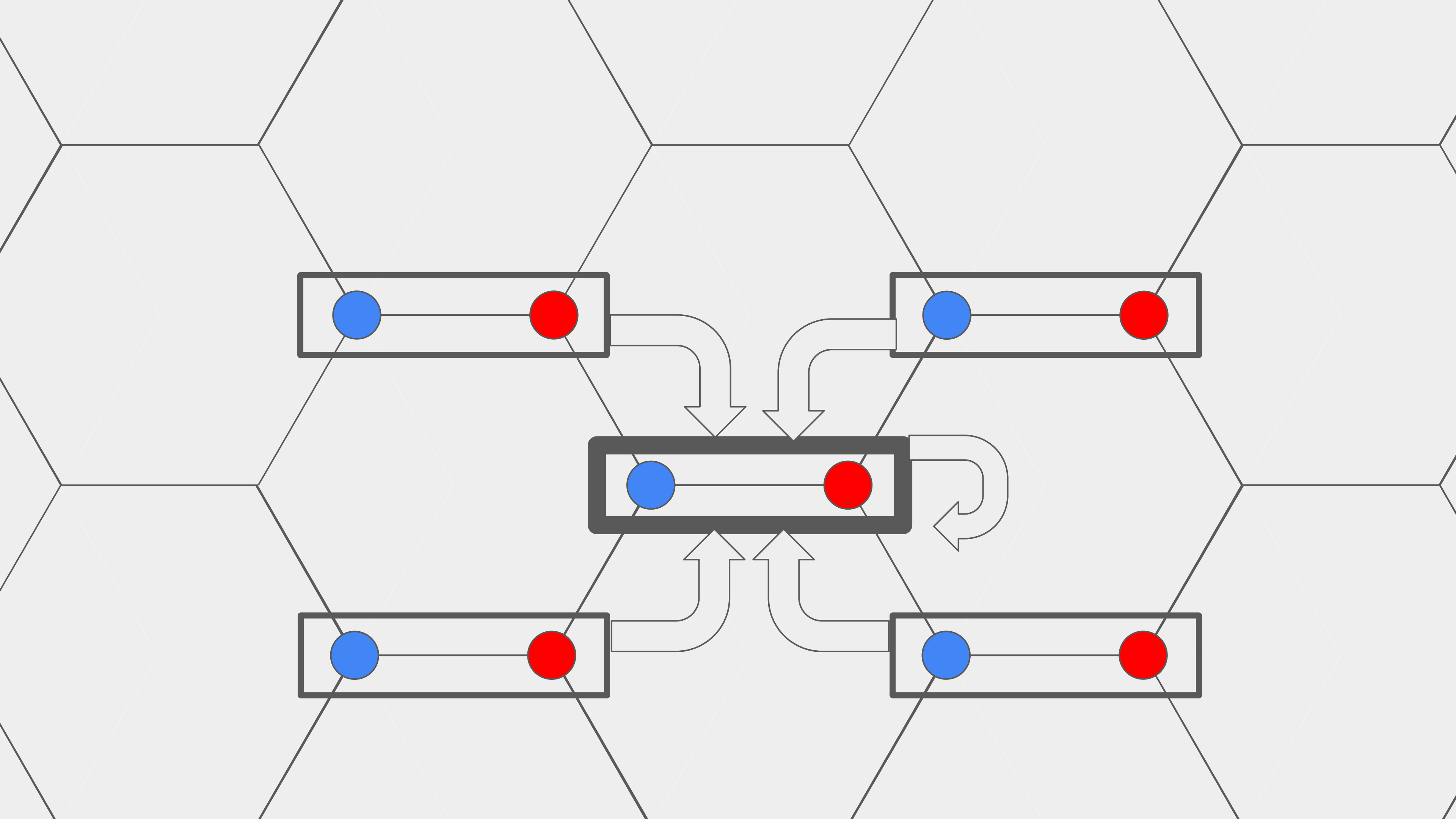


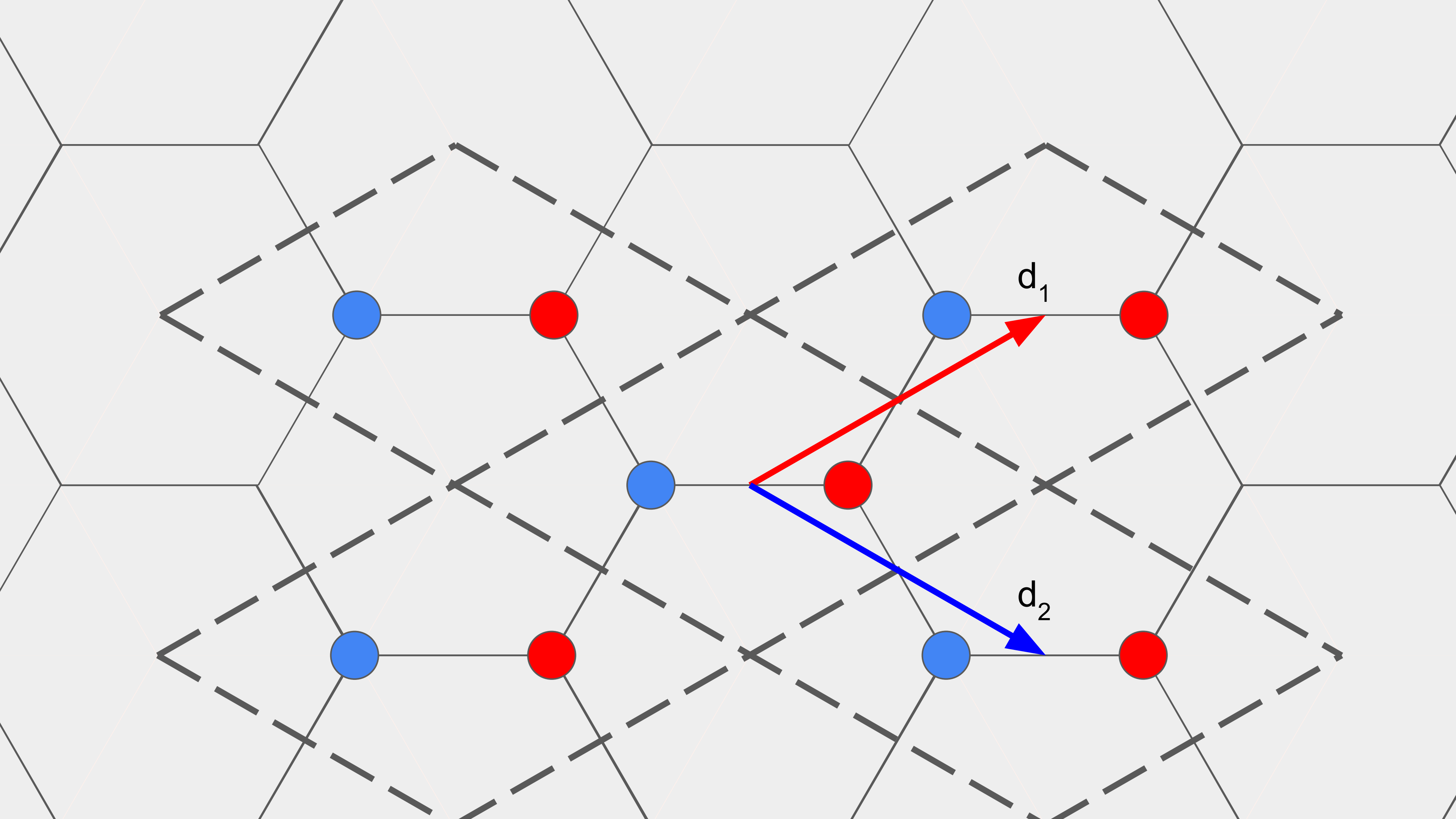
Hexagonal structure



$2p_z$ orbitals

Derivation of the graphene band structure





d_1
d_2

$$\mathbf{H}\psi = E\psi \Longrightarrow \mathbf{H}(\vec{k})\vec{\phi}_0 = E\vec{\phi}_0$$

$$\mathbf{H}(\vec{k}) = \sum_{m} H_{nm} e^{i\vec{k}\cdot(\vec{d}_m - \vec{d}_n)}$$

Hamiltonian

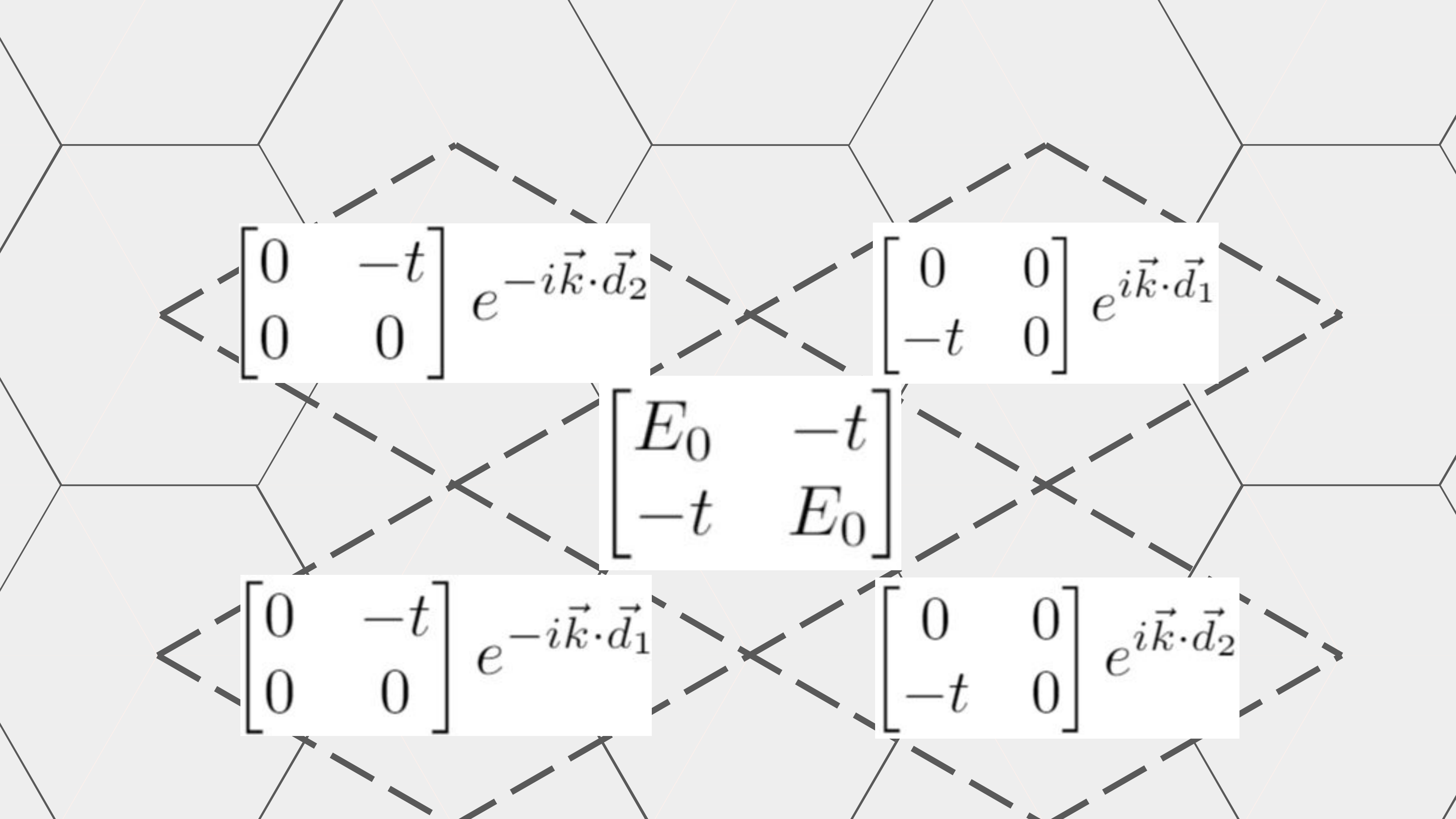
Momentum

Neighboring unit cells

Interaction between unit cells

$$\int u_i^*(\vec{r})\mathbf{H}u_j(\vec{r})d\vec{r}$$

Vector joining unit cells


$\begin{bmatrix} 0 & -t \\ 0 & 0 \end{bmatrix} e^{-i\vec{k}\cdot\vec{d}_2}$
$\begin{bmatrix} 0 & 0 \\ -t & 0 \end{bmatrix} e^{i\vec{k}\cdot\vec{d}_1}$
$\begin{bmatrix} E_0 & -t \\ -t & E_0 \end{bmatrix}$
$\begin{bmatrix} 0 & -t \\ 0 & 0 \end{bmatrix} e^{-i\vec{k}\cdot\vec{d}_1}$
$\begin{bmatrix} 0 & 0 \\ -t & 0 \end{bmatrix} e^{i\vec{k}\cdot\vec{d}_2}$

$$\begin{bmatrix} E_0 & -t(1+e^{-i\vec{k}\cdot\vec{d}_1}+e^{-i\vec{k}\cdot\vec{d}_2}) \\ -t(1+e^{i\vec{k}\cdot\vec{d}_1}+e^{i\vec{k}\cdot\vec{d}_2}) & E_0 \end{bmatrix}$$

$$= \begin{bmatrix} E_0 & h_0^* \\ h_0 & E_0 \end{bmatrix}$$

$$E(\vec{k}) = E_0 \pm |h_0|$$

$$\vec{k} = k_x\hat{x} + k_y\hat{y}$$

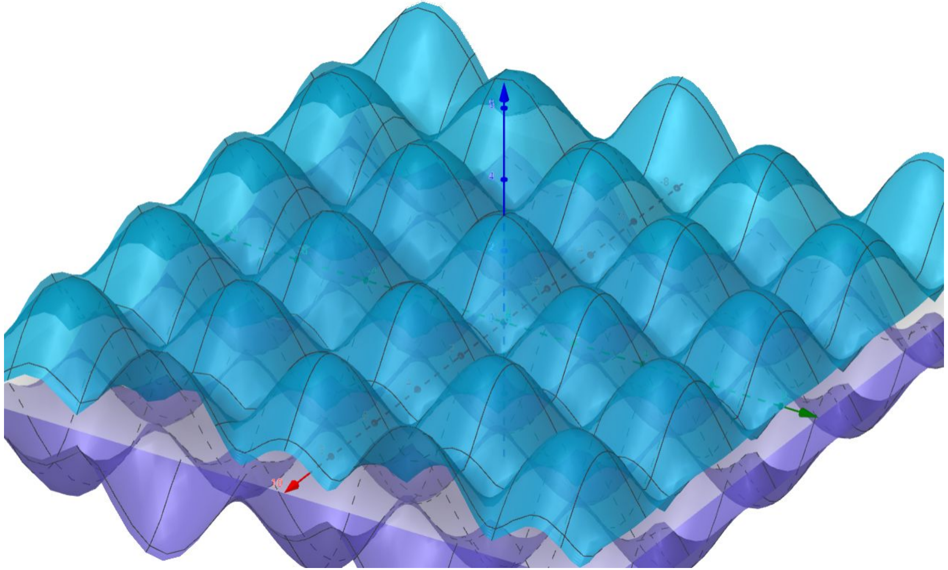
$$\vec{d}_1 = \frac{3}{2}a\hat{x} + \frac{\sqrt{3}}{2}a\hat{y}$$

$$\vec{d}_2 = \frac{3}{2}a\hat{x} - \frac{\sqrt{3}}{2}a\hat{y}$$

$$\begin{aligned}
h_0 &= -t\left(1 + \exp\left(ia(\frac{3}{2}k_x + \frac{\sqrt{3}}{2}k_y)\right) + \exp\left(ia(\frac{3}{2}k_x - \frac{\sqrt{3}}{2}k_y)\right)\right) \\
&= -t\left(1 + \exp\left(\frac{3}{2}iak_x\right)\left(\exp(\frac{\sqrt{3}}{2}iak_y) + \exp(-\frac{\sqrt{3}}{2}iak_y)\right)\right) \\
&= -t\left(1 + 2\exp\left(\frac{3}{2}iak_x\right)\cos\left(\frac{\sqrt{3}}{2}ak_y\right)\right) \\
&= -t\big(1 + 2\exp(i\theta_1)\cos\theta_2\big)
\end{aligned}$$

$$\theta_1 = \tfrac{3}{2}ak_x, \theta_2 = \tfrac{\sqrt{3}}{2}ak_y$$

$$\begin{aligned}
E &= E_0 \pm |h_0| \\
&= E_0 \pm \sqrt{h_0 h_0^*} \\
&= E_0 \pm \sqrt{t^2\big(1 + 2\exp(i\theta_1)\cos\theta_2\big)\big(1 + 2\exp(-i\theta_1)\cos\theta_2\big)} \\
&= E_0 \pm t\sqrt{1 + 2\cos\theta_2(\exp(i\theta_1) + \exp(-i\theta_1)) + 4\cos^2\theta_2} \\
&= \boxed{E_0 \pm t\sqrt{1 + 4\cos\theta_1\cos\theta_2 + 4\cos^2\theta_2}}.
\end{aligned}$$

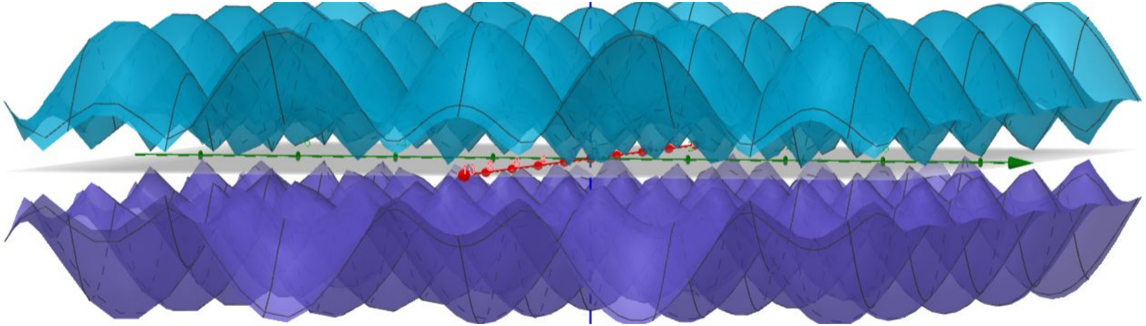


$$4\cos^2\theta_2 + 4\cos\theta_2 + 1 = 0$$

$$(2\cos\theta_2 + 1)^2 = 0$$

$$\cos\theta_2 = -\frac{1}{2}$$

$$K = (0, \frac{4\pi\sqrt{3}a}{9}), K' = (0, -\frac{4\pi\sqrt{3}a}{9})$$



$$\begin{bmatrix} E_0 & h_0^* \\ h_0 & E_0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = (E_0 \pm |h_0|) \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

$$\implies \begin{cases} h_0^* \phi_2 = \pm |h_0| \phi_1 \\ h_0 \phi_1 = \pm |h_0| \phi_2 \\ \phi_1^2 + \phi_2^2 = 1 \end{cases}$$

$$\phi_1^2 = c h_0^*, \phi_2^2 = c h_0$$

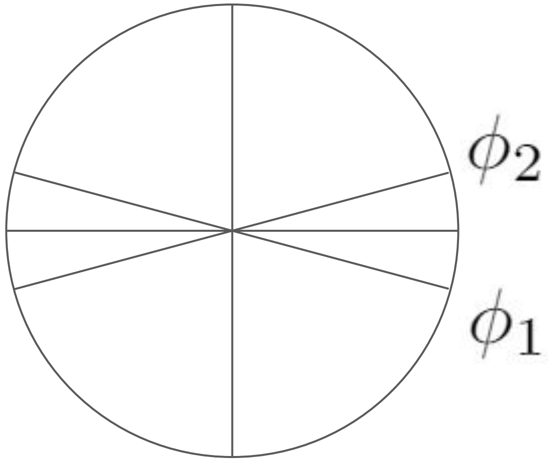
$$\phi_1^2 + \phi_2^2 = 1 \implies 2c \,\mathrm{Re}(h_0) = 1$$

$$\implies -2ct(1 + 2\cos\theta_1 \cos\theta_2) = 1$$

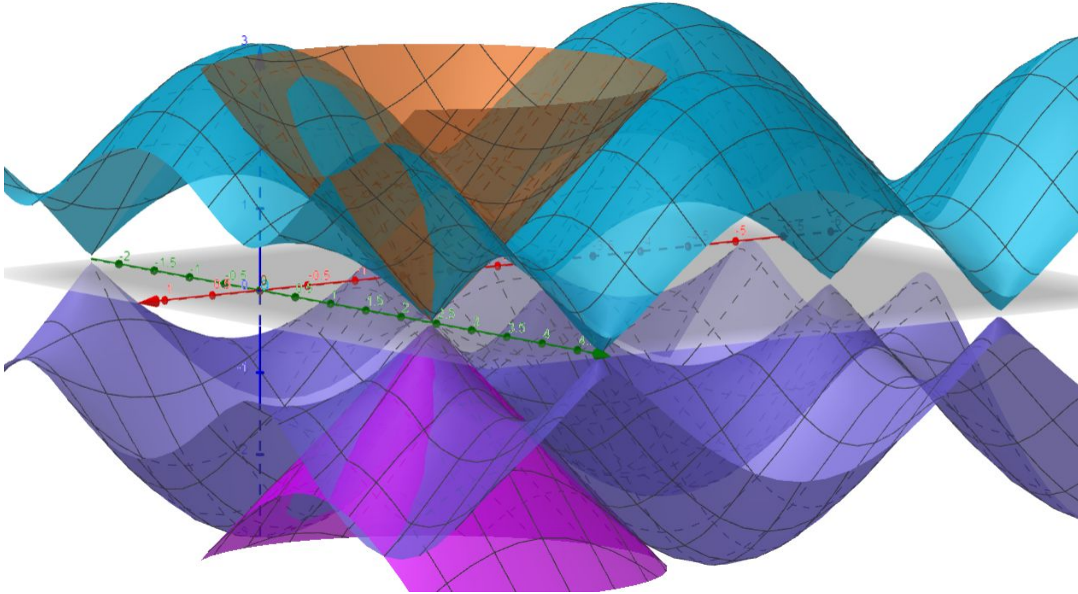
$$\implies c = -\frac{1}{2t(1 + 2\cos\theta_1 \cos\theta_2)}$$

$$\omega^2 = h_0$$

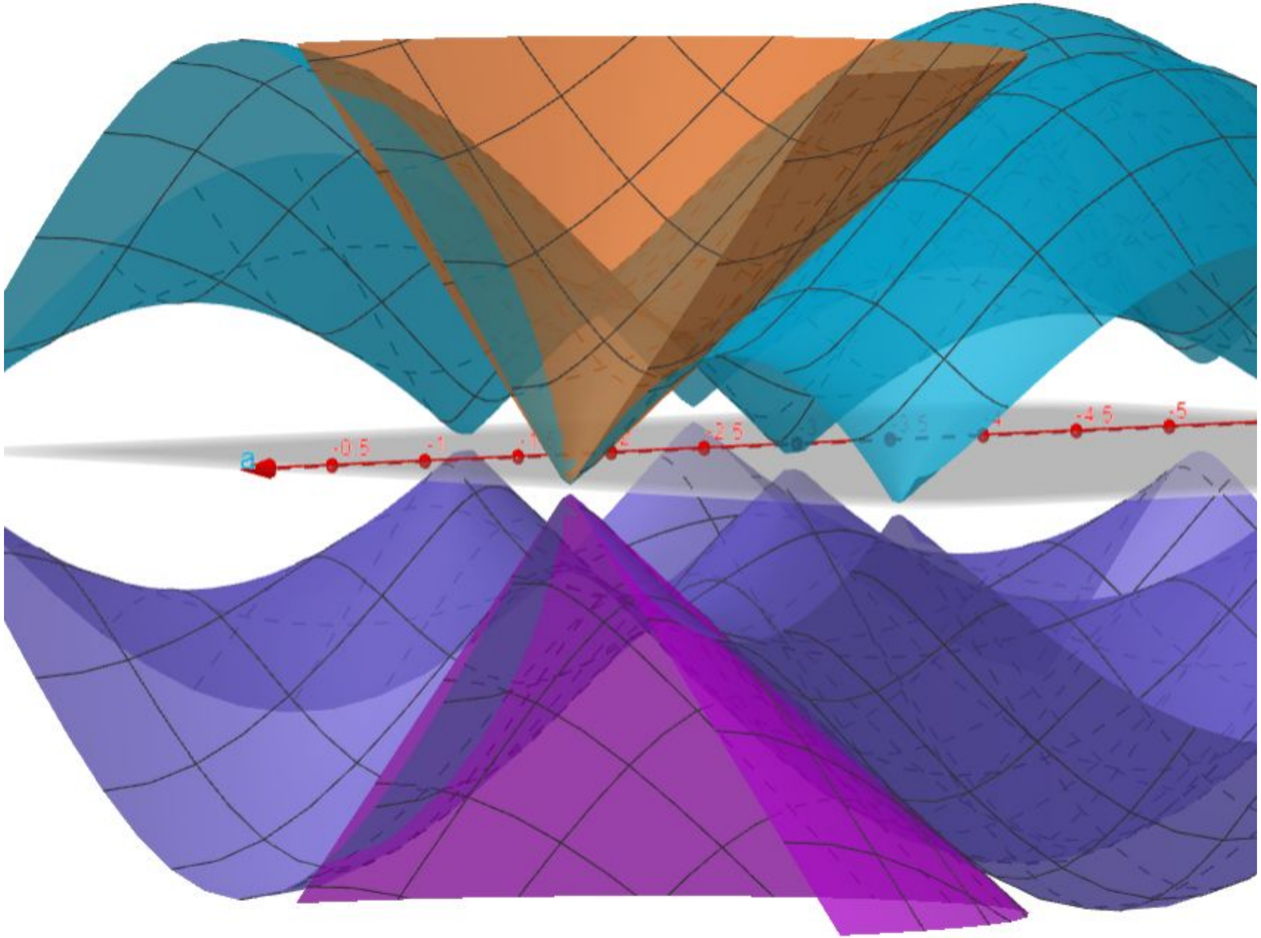
$$\boxed{\psi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \pm\sqrt{c} \begin{bmatrix} \pm\omega^* \\ \omega \end{bmatrix}}.$$



$$
\begin{aligned}
h_0 &= -t\big(1 + 2\exp(i\theta_1)\cos\theta_2\big) \\
&\approx -t\big(1 + 2(1 + id\theta_1)(-\frac{1}{2} - \frac{\sqrt{3}}{2}d\theta_2)\big) \\
&= t(id\theta_1 + \sqrt{3}d\theta_2) \\
&= \frac{3}{2}at(idk_x + dk_y)
\end{aligned}
$$



$$
\boxed{E(\vec{k}) = E_0 \pm \frac{3}{2}at|\vec{k} - K|.}
$$


a
-0.5
-1
-2.5
-4.5
-5

Similarities to the Dirac Equation

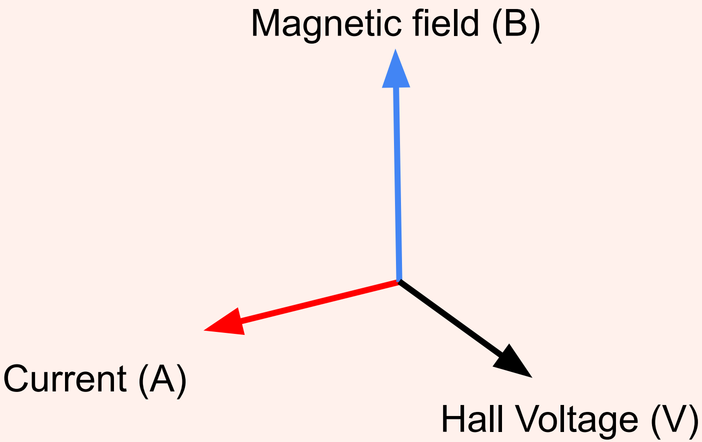
As you can see the equation of a electron in graphene at a Dirac point is similar to that of a relativistic particle.

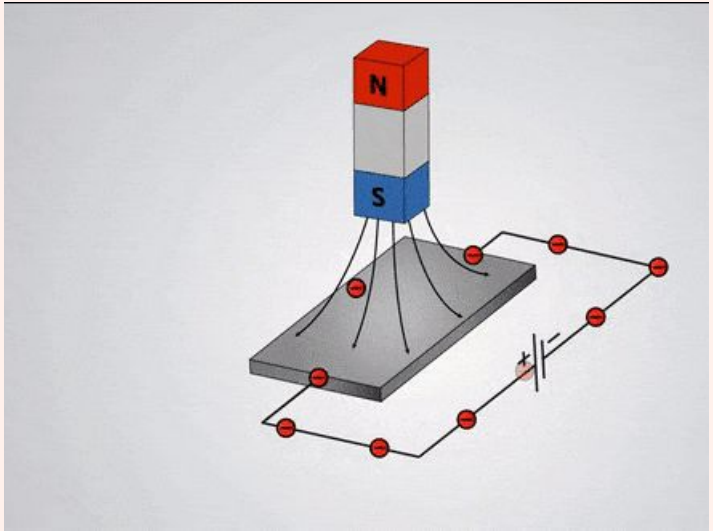
$E^2 = m^2c^4 + c^2p^2$

$m = 0 \rightarrow E = cp$

Classical Hall Effect

- Circuit with thin metal plate - current
- Magnetic field perpendicular to the metal plate
- potential difference (voltage)



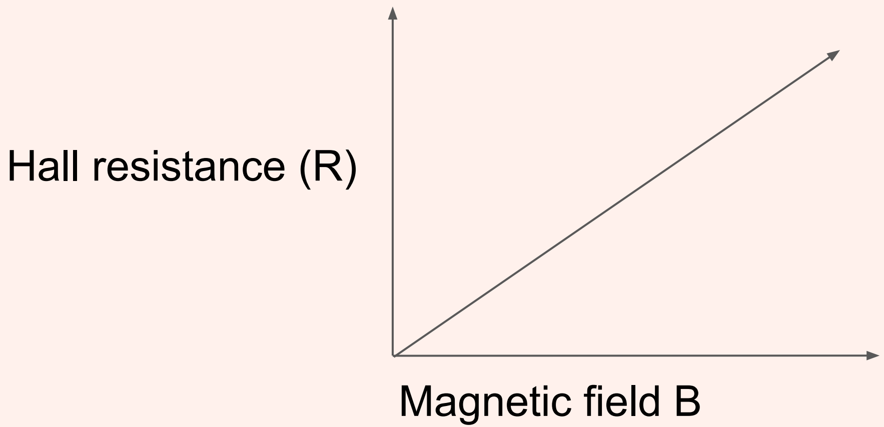


Classical Hall Effect

- Magnetic Field B to Hall Resistance
  - Denoted as R_{hall} or R_{xy}

$$R_{hall} = \frac{B_z}{e\rho}$$

ρ (rho): electron density

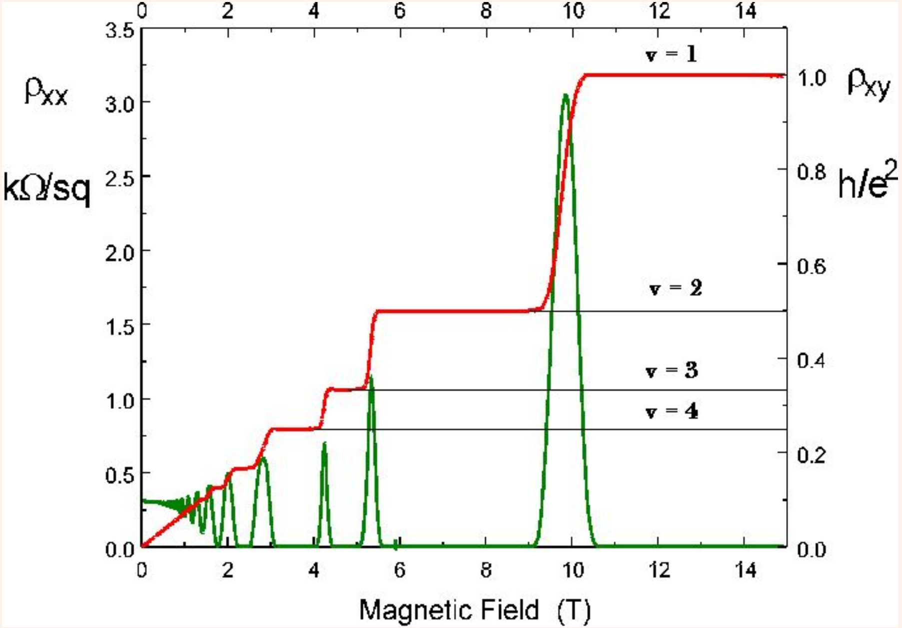


Quantum Hall Effect

$$R_{xy} = \frac{V_{\text{Hall}}}{I_{\text{channel}}} = \frac{h}{e^2 \nu}$$

- Hall Effect at low temperatures, high magnetic fields, 2D
  - Hall Resistances are discrete
  - Regular resistance is effectively 0

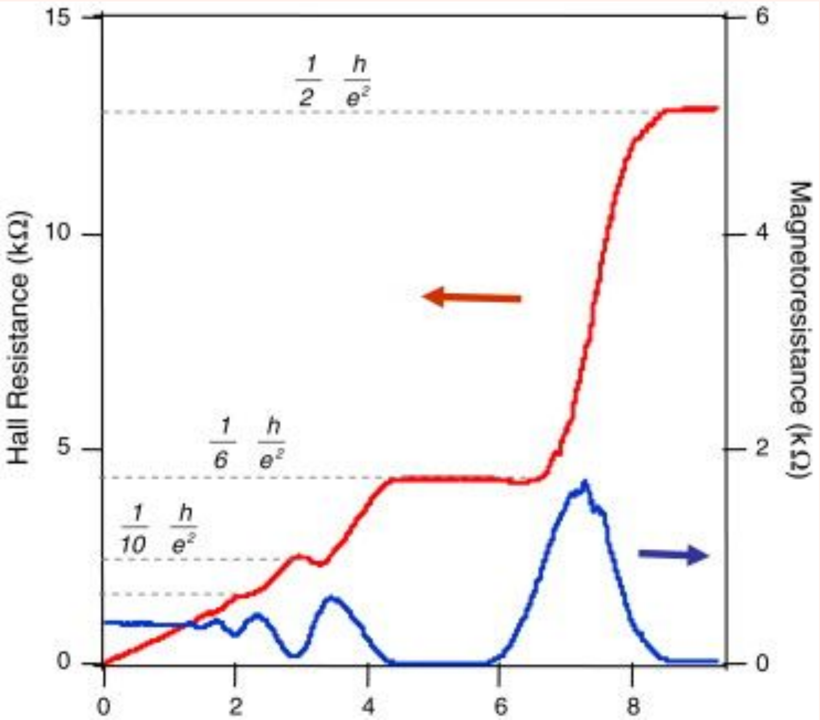
- Electrons confined 2D with B field
  - Landau level



Quantum Hall Effect in Graphene

$$R_{xy} = \frac{V_{\text{Hall}}}{I_{\text{channel}}} = \frac{h}{e^2 \nu}$$

- Graphene instead of metal plate
  - Consistent, not many impurities
  - v_F is close to speed of light at Dirac points
- Fractional QHE
  - No confirmed explanation



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