Competitive equilibrium and the optimal allocation: two examples

Juan Paez-Farrell

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1 Introduction

In the standard model with no rigidities or externalities, the competitive equilibrium results in the same allocation as that chosen by the social planner. Here we consider two modifications that such a result can easily be overturned.

2 Income taxes

Consider a representative agent model with only one period. The agent's utility is given by

$$U = c_t + \psi \ln l_t \tag{1}$$

where l_t is leisure and we assume that $1 = n_t + l_t$, with n_t denoting labour input. The budget constraint is given by

$$c_t = w_t(1 - \tau_t)(1 - l_t) + T_t + \pi_t \tag{2}$$

τ_t represents the income tax rate and T_t is a lump-sum tax/transfer (the latter if positive). The equation shows that the household's consumption is equal to its net labour income plus any transfers and profits (π_t) from the firm. The first order conditions are given by

$$\begin{aligned} 1 &= \lambda \\ \frac{\psi}{l_t} &= \lambda w_t \end{aligned} \tag{3}$$

Firms operate under perfect competition. Their objective is to maximise

$$\pi_t = z_t(1 - l_t) + w_t(1 - l_t) \tag{4}$$

Given perfect competition, we have $\pi_t = 0$ and from the first order condition we have

$$w_t = z_t \tag{5}$$

Lastly, the government's budget constraint is given by

$$T_t = w_t \tau_t (1 - l_t) \tag{6}$$

In other words, we are assuming that all the income tax revenue is returned to households as a lump-sum tranfer. In other words, if the agent has to pay 10 goods, given her chosen labour supply, as income taxes she also receives 10 goods as a lump-sum transfer from the government. You would think that these two cancel out so that there is no effect. In solving the model, we are going to assume that the government chooses τ_t exogenously (so that T_t is endogenous; reversing this does not alter the conclusions but the key element is that one must be endogenous and the other exogeneous).

2.1 Equilibrium

To solve our model, that is, to express every single endogenous variable as a function of exogenous variables only, we need to combine some of the equations above. First, combining the household's first order conditions and using the fact that wages equal the value of technology z_t, we have

$$\begin{aligned}\frac{\psi}{l_t} &= w_t(1 - \tau_t)\lambda \\ l_t &= \frac{\psi}{z_t(1 - \tau_t)}\end{aligned}$$

Given the solution for leisure, finding the values of the remaining variables is straightforward

$$
\begin{aligned}
y_t &= z_t - \frac{\psi}{1-\tau_t} \\
T_t &= \tau_t \left[z_t - \frac{\psi}{1-\tau_t} \right] \\
c_t &= w_t(1-\tau_t) + T_t = z_t - \frac{\psi}{1-\tau_t}
\end{aligned}
\tag{7}
$$

Note that the introduction of a distortionary income tax results in a reduction in labour supply, output and consumption whereas leisure increases. Why does this happen, when the income tax is fully rebated? The reason is straightforward: from the point of view of a single agent, the transfer she receives is independent of her income but the tax she has to be is not. Consequently, she will reduce her labour supply while at the same time receiving the same transfer. However, in equilibrium this will never occur as T is dependent on the chosen l.

What about the social planner? The SP maximises social welfare (household utility) subject to the feasibility constraint, which is given by

$$
y_t = z_t n_t = z_t(1-l_t) = c_t
$$

To see this, combine the household and government's budget constraint along with the firm's production function. The resulting first order conditions are

$$
1 = \lambda \frac{\psi}{l_t} = z_t \lambda
$$

Using this along with the feasibility constraint we have

$$
\begin{aligned}
l_t &= \frac{\psi}{z_t} \\
y_t = c_t &= z_t \left(1 - \frac{\psi}{z_t} \right) = z_t - \psi
\end{aligned}
$$

The income tax does not alter the allocation under the social planner as it internalises the fact that the two cancel each other out. Moreever, recall that the allocation under the SP results in the welfare-maximising equilibrium.

Given that welfare is higher under the SP, you might be wondering why agents do not choose the same values under the competitive equilibrium. The reason is as follows:

- Assume that under the competitive equilibrium all agents choose the allocation selected by the SP.
- An individual agent will then find that she would be better off reducing her labour supply as the transfer she receives remains unchanged (each individual agent's decision has no effects on aggregate values).
- However, as all agents are identical, they all find that they would be better off individually by reducing their labour supply.
- As this means that total labour supply falls, the resulting income tax revenue that the government receives then also falls.

3 An example with externalities

The agent's utility is given by

$$U = c_t + \psi \ln l_t - \phi y_t$$

The only modification compared to the previous case is that welfare depends negatively on aggregate output (think of pollution). Crucially, the value of y_t is exogenous to the household so it can only choose consumption and leisure while taking firms' outputs as given.

The budget constraint is the same as before so combining first order conditions we have

$$\frac{\psi}{l_t} = w_t \Rightarrow l_t = \frac{\psi}{w_t} \tag{8}$$

Again, the problem for the firm has not changed so labour demand is chosen so that the marginal product of labour z_t equals the real wage

$$w_t = z_t$$

Combining and then using the household' budget constraint (with $\tau = T = 0$)

$$\begin{aligned} l_t &= \frac{\psi}{z_t} \\ y_t = c_t &= z_t - \psi \end{aligned} \tag{9}$$

This is the same allocation that we obtained under the social planner earlier when $\tau = T = 0$. Is this allocation optimal? To determine this, we must compare our results to those obtained by solving the social planner's problem.

The SP's objective is to maximise

$$U = c_t + \psi \ln l_t - \phi y_t$$

subject to

$$c_t = y_t$$

and

$$y_t = z_t(1 - l_t)$$

Substituting the production function into the objective and the constraint, the Lagrangean becomes

$$\mathcal{L} = c_t + \psi \ln l_t - \phi \left[z_t(1 - l_t) \right] + \lambda \left[z_t(1 - l_t) - c_t \right.$$

The first order conditions are

$$
\begin{aligned}
1 &= \lambda \\
\frac{\psi}{l_t} + \phi &= \lambda z_t
\end{aligned}
$$

Simplifying, we obtain

$$
\begin{aligned}
l_t &= \frac{\psi}{z_t - \phi} \\
y_t = c_t &= z_t \left(1 - \frac{\psi}{z_t - \phi}\right)
\end{aligned}
$$

Notice that in the present of a negative externality the optimal level of output is lower than in the competitive equilibrium. In the case of the later, individual agents recognise that their individual actions in isolation have a negligible effect on the externalities. As all agents collectively behave in the same way, the result is that the externality is not taken into account when choosing how much to consume and work. In contrast, the social planner does recognise that the externality arises from producing output and the result is that the optimal level of output is then lower.