Monetary Economics

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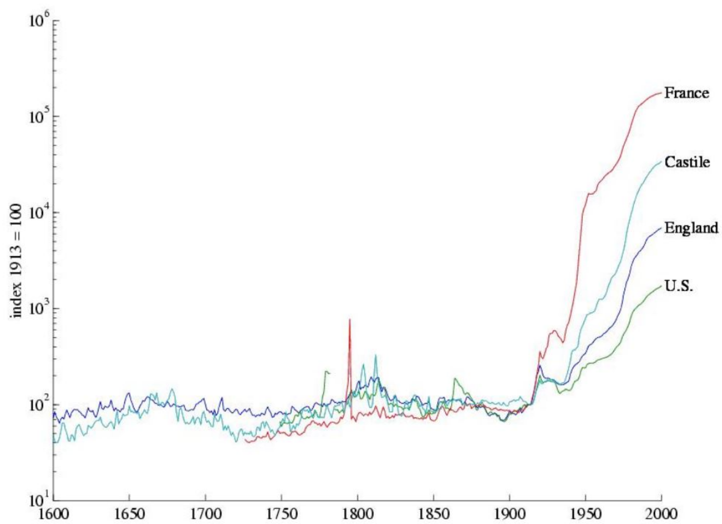

The latest version of this file can be obtained from My Github repository. The figure comes from Sargent and Velde [2014].

Contents

Chapter 1

Introduction

Monetary economics studies the behaviour of monetary phenomena (money, prices) as well as their interactions with the real economy. It can be thought of as branch of macroeconomics and is particularly relevant for those who want to understand the behaviour of central banks.

These notes aim to provide a general overview of key topics in monetary economics. The content is borrowed from other notes found on the internet, textbooks, etc. and is therefore not meant to be original. It is aimed at third year undergraduate students in economics and only assumes a standard background in mathematics: differentiation, equation manipulation and little else. Many of the chapters are self-contained but in order to fully understand the topics you will need to derive the equations with pen and paper.

Chapter 2

Imperfect Information and the Choice of Monetary Policy Instrument

2.1 Introduction

The standard IS-LM model can be used to analyse the consequences of alternative policies, the effects of shocks to the goods or financial sectors, etc. It used to be at the core of any macroeconomic analysis conducted by governments and the private sector for policy simulations and forecasting. Of course, the actual models would have been much more elaborate than the ones you have encountered so far, but their core elements were the same.

Nevertheless, one crucial feature of the model is not a characteristic of actual policy, even at the time when the IS-LM model was dominant. That is the assumption that the central bank chooses the value of the money supply as its policy instrument. In reality it is more accurate to argue that central bankers choose the interest rate. Of course, the two approaches are closely related: we know from the IS-LM model that increases in the money supply result in lower interest rates so whenever we hear that the Bank of England has decided to increase interest rates we can interpret this as a contraction in the money supply. Nevertheless, from a modelling point of view, they are different: in one the interest rate is chosen exogenously, by the the policy maker, and in the other, it is the money supply.

Crucially, does it matter which variable the central bank chooses as its policy instrument? If policy changes in one variable have direct implications for the other one may think that this does not matter. As we shall we, it does and there may be settings where one policy delivers outcomes that are, from an economic perspective, superior.

Before we proceed to analyse this with the use of a model it should be borne in mind that central banks cannot choose both interest rates *and* the money supply. The easiest allegory is with a monopolist: she cannot set both the price (the interest rate) and the quantity produced of her good; rather, she will choose one and this will give the value of the other. Mathematically, this is equivalent to arguing that if the money supply is exogenous – as we have assumed so far – then the model solves for the interest rates (it is endogenous) and vice versa.

The analysis in this chapter follows closely Poole [1970] and I would strongly recommend that you read it.

2.2 The model

Assume that the central bank chooses its instrument (money supply/interest rate) before observing the shocks. The choice of the objective is crucial, and in our example we will assume that the central bank wants to stabilise output (minimise its variance). So the answer can be obtained by solving the model for one choice, obtaining the variance of output and compare the variance of output that one obtains by using an alternative instrument. The instrument that yields the smallest variance is the optimal instrument. Since we will using short-run analysis, the price level will be assumed fixed. A basic ISLM model (in logs) can be written as:

$$y_t = -\alpha i_t + u_t \tag{2.1}$$

$$m_t = -c i_t + y_t + v_t \tag{2.2}$$

The first equation is an aggregate demand relationship where output depends negatively on the interest rate (since there is no inflation in this model, real and nominal interest rates are the same); equivalently, it is simply an IS curve. The second equation represents money demand, or the LM, once the money supply has been substituted in. The variables u and v represent shocks to the economy (these would shift the curves) that are random, mean zero and importantly, the two are uncorrelated with each other. It is also important to note that the money supply, output and interest rates in this model have to be interpreted as deviations from their equilibrium values, so that y represents the output gap (actual output minus potential). Since we are assuming that the central bank wants to stabilise output this would imply making y equal to zero.

Finally, the policy objective is to minimise:

$$E[y_t]^2 \tag{2.3}$$

that is, to minimise the variance of output. In the absence of shocks, monetary policy (via a money supply or interest rate rule) would give $y = 0$.

For our purposes, the central bank first chooses its instrument; then the shocks occur and one can calculate the variance of output, the objective to be minimised.

2.3 A money supply rule

Here m is the central bank's choice. Solving the first two equations and solving for output yields (eliminate the interest rate):

$$y = \frac{\alpha m + cu - \alpha v}{\alpha + c} \tag{2.4}$$

(the time subscripts can be ignore since we are not considering dynamics).

If the central possessed full information (so that she could observe the actual values of the shocks) she would set m such that it yields $y = 0$:

$$m = v - \frac{c}{\alpha} u$$

But under imperfect information the shocks are unobserved. Therefore, choosing m to yield $E[y] = 0$ simply gives $m = 0$ so using this in the equation above leads to

$$y = \frac{cu - \alpha v}{\alpha + c} \tag{2.5}$$

From this equation, the variance of output which arises under a money supply rule, is given by:

$$E_m[y]^2 = \frac{c^2 \sigma_u^2 + \alpha^2 \sigma_v^2}{(\alpha + c)^2} \tag{2.6}$$

2.4 Interest rate rule

The analysis in this case is simpler. Since here we assume that the central bank can control the interest rate directly, one can solve equation (10.4) for output. (the second equation can be ignored).

If the shocks could be observed, then the policy maker can always, as above with the money supply, achieve its target value of output equal to zero via

$$i_t = \frac{1}{\alpha} u_t$$

But with imperfect information the best she can hope for is to achieve $y = 0$ on average. Setting i to give $E[y_t] = 0$ we have

$$E_i[y]^2 = \sigma_u^2 \tag{2.7}$$

Now the two policies can be compared by comparing the alternative variances. Therefore, the interest rate rule is preferred to the money supply rule if and only if:

$$E_i[y^2] < E_m[y^2] \tag{2.8}$$

From the two measures for the variances we can summarise this as:

$$\sigma_u^2 < \frac{c^2\sigma_u^2 + \alpha^2\sigma_v^2}{(\alpha + c)^2} \tag{2.9}$$

Or

$$\sigma_u^2 < \frac{\alpha^2}{(\alpha + c)^2 - c^2}\sigma_v^2 \tag{2.10}$$

The result is as follows: an interest rate rule is preferred when the shocks to the LM are larger, when the LM is steeper (given by $1/c$) and the IS flatter (its slope is $-1/\alpha$).

In the same vein, a money supply rule is preferred when the volatility of the IS shocks is larger, when the LM is flat or the IS steep.

2.5 The monetary base as the instrument

The analysis above assumed that the central bank could choose the money supply as one of its instruments. We know, however, that the central bank can do control this variable indirectly via the monetary base and the money multiplier. Returning to the equation linking the three variables:

$$M = H(i)MB \tag{2.11}$$

where H denotes the money multiplier and is assumed to depend positively on the interest rate. In log form (so that it is linear) and allowing for a shock to this relationship (given by w) we now have a third equation/relationship:

$$m_t = b_t + hi_t + w_t \tag{2.12}$$

and b is the log of the monetary base. Under an interest rule this equation makes no difference (remember that in this case we didn't consider the second equation either). Under a monetary base rule, substitute (2.12) into (2.2), and substitute for the interest rate in this equation, yielding:

$$b_t = y_t - \frac{(c+h)(u_t - y_t)}{\alpha} + v_t - w_t \tag{2.13}$$

$$b_t = \left[\frac{\alpha + c + h}{\alpha}\right] y_t - \frac{(c+h)}{\alpha} u_t + v_t - w_t \tag{2.14}$$

Since all the shocks are mean-zero, this implies setting $b = 0$, so that the solution for output is:

$$y_t = \frac{(c+h)u_t - \alpha v_t + \alpha w_t}{\alpha + c + h} \tag{2.15}$$

The implication now is:

$$E_b \left[y_t\right]^2 = \left[\frac{1}{\alpha + c + h}\right]^2 \left[(c+h)^2 \sigma_u^2 + \alpha^2 \left(\sigma_v^2 + \sigma_w^2\right)\right] \tag{2.16}$$

The fact that there are now money-multiplier shocks makes the money supply rule (via monetary base) less desirable than before, and increases the attractiveness of the interest rate rule, emphasising the main result in Poole (1970): the more shocks to the financial sector (money demand-money supply), the greater the attractiveness of the interest rate as the policy instrument.

2.6 Including endogenous prices into the Poole model

The model above just considered the IS-LM model with fixed prices. A minor extension is to include prices by adding another equation to describe aggregate supply. If output (via labour demand) depends negatively on the real wage we can write aggregate supply (again, de-meaned) as

$$y_t = -\xi \left(w_t - p_t\right)$$

Where w is the real wage and p is the price level. The only problem is that we have one additional equation but two additional variables so to maintain simplicity we shall assume that $w_t = 0$. In other words, nominal wages equal their mean values. Hence we have

$$y_t = \xi p_t \tag{2.17}$$

In addition, we have to modify our LM equation as money demand depends on real money balances $(m - p)$ and now prices are flexible. The new LM is therefore

$$m_t - p_t = -c i_t + y_t + v_t \tag{2.18}$$

Our model therefore consists of (10.4), (2.18) and (4.7).

2.6.1 Interest rates as the policy instrument

This is unchanged on the previous case as we still have a direct relationship between i and y in the IS.

2.6.2 Money supply as the policy instrument

As before, we want output in terms of m and the shocks. This involves eliminating prices and interest rates from the LM equation. Doing so yields

$$y_t = \frac{\alpha\xi}{(1+\xi)\alpha + \xi c}(m_t - v_t) + \frac{c\xi}{(1+\xi)\alpha + \xi c}u_t$$

With full information the central bank could ensure complete stability in output by setting

$$m_t = v_t - \frac{c}{\alpha}u_t$$

And in the presence of imperfect information the volatility of output is given by

$$\sigma_y^2 = \frac{\xi^2\left(\alpha^2\sigma_v^2 + c^2\sigma_u^2\right)}{\left((1+\xi)\alpha + \xi c\right)^2}$$

2.7 Limitations and extensions

In the model above we have so far assumed that the central bank could use one instrument or the other and derived each instrument's performance. However, we could consider a hybrid policy by noting that both interest rate and money supply policies can be thought of as taking the form

$$m_t = \chi i_t \tag{2.19}$$

and that the money supply as instrument involves setting $\chi = 0$ while the interest rate as instrument implies $\chi = \infty$. Now, rather than solving for output in terms of the shocks without m or i; note that we have three equations, IS-LM and (2.19), and three unknowns, output interest rates and the money supply. So using (10.4), (2.2) and (2.19) we have

$$y_t = \frac{(\chi + c)\,u_t - \alpha v_t}{\chi + c + \alpha}$$

And setting $\chi = 0$ or $\chi = \infty$ we can obtain the results derived earlier. One benefit of this approach is that we can move away from a dichotomous choice between one policy instrument and the other. Instead, we could consider what hybrid policy – what value of χ – will give us the least variance in output.

From the equation above, the variance of output is given by

$$\sigma_y^2 = \frac{(\chi + c)^2 \sigma_u^2 + \alpha^2 \sigma_v^2}{(\chi + c + \alpha)^2}$$

The optimal χ is the one that minimises the variance so all we have to do is differentiate the equation above with respect to χ and set that equal to zero. Therefore

$$\chi^* = \frac{\alpha \sigma_v^2}{\sigma_u^2} - c$$

So for example, having derived the optimal χ for given values of variances and model parameters, assume that there is an increase in σ_v^2/σ_u^2 – in other words, more volatility in the LM relative to the IS. Our model suggests that the optimal χ will increase, moving us farther away from the value of zero (interest rate as the instrument). This is exactly what we derived earlier. We can also use this to consider how the slopes of the IS and LM schedules will therefore tilt the hybrid policy more towards one instrument or the other.

We have presented a very simple model and discussed the results that these implied. However, the model itself is basic and ignores inflation, expectations and aggregate supply. More importantly, we have defined the objective as the volatility of output, but one may also want to include other variables, such as the volatility of inflation and interest rates. A recent paper that uses a more realistic setup can be found in Collard and Dellas [2005]. This uses a New Keynesian model and a model-consistent policy objective to assess the relative merits of either policy instrument.

Chapter 3

The Phillips Curve

3.1 Introduction

The basic idea underlying the Phillips curve (PC) is that nominal changes have real consequences. Phillips found a strong and stable negative relationship between unemployment and wage inflation in the UK (1861-1957). Other researchers extended the analysis by considering goods-price inflation, resulting in the Phillips curve (PC): a negative association between higher unemployment and lower inflation. This was obviously just an empirical finding. However, theoretical support could be found in Keynesian models. In fact, the standard Keynesian model is a short run model where aggregate supply (the Phillips curve) is horizontal. Phillips' paper provided empirical support for such an assumption.

In the standard Keynesian model with rigidity in nominal wages, higher prices result in a lower real wage, so that firms would employ more labour (implicitly, workers only care about the nominal wage). Therefore, the PC could be written as:

$$\pi = \beta - \delta u \tag{3.1}$$

The original PC provided a convenient tool for policy makers and for policy analysis. It was easy to understand and it showed that there was a clear (and permanent) trade-off between the unemployment rate and inflation. The policy maker, given her preferences, just had to choose where on the PC to be.

The Phillips curve can also be easily be interpreted as an aggregate supply schedule by using Okun's Law. For now we shall interpret it as postulating that as output rises unemployment falls. We can represent this as:

$$u = -\alpha y \tag{3.2}$$

In other words, there is a negative relationship between the unemployment rate and output. Now we can have the Phillips curve in terms of output:

$$\pi = \beta + \alpha\delta y \tag{3.3}$$

However, this kind of analysis was criticised by Phelps [1968] and Friedman [1968] (1968) for being theoretically flawed: the standard PC did not satisfy the Classical dichotomy in the long run, as it implied that nominal variables (inflation) could determine real variables (unemployment). In other words, fiscal and monetary policy could not affect the unemployment in the long run.

This meant that unemployment would have a tendency to move towards its long run value, called the natural rate (of unemployment). This is the value of unemployment that would be reached in the Classical model. Note that there is nothing desirable in the natural unemployment rate, despite its name. Some of it will be an unavoidable fact of life but it will also depend on policies and institutions.1 In this model one derives the (vertical) aggregate supply curve by assuming that both workers and firms care about real wages. This yields the equilibrium employment and real wage that clears the labour market. Once you have the equilibrium employment rate, then the unemployment rate consistent with this (=(labour force-employment)/labour force) would give the natural rate of unemployment ($\overline{u}$).

Consequently, the PC presented above was theoretically flawed, as the PC had to be vertical in the long run. But then, how do we model the PC in the short run? We need to include the natural rate in the standard PC, but the other factor hitherto ignored is expectations. We could write the PC as:

$$\pi = \beta - \delta(u - \overline{u}) \tag{3.4}$$

So that in the long run $u = \overline{u}$, implying $\pi = \beta$. So what does β now represent? It would be the long run value of inflation. This is often called core inflation. However, we can also call it expected inflation. When expectations of inflation coincide with their actual values economic agents have no incentive to revise their expectations, and unemployment equals its natural rate. This PC incorporating expectations is generally called the Expectations Augmented Phillips Curve (EAPC):

$$\pi = \pi^e - \delta(u - \overline{u}) \tag{3.5}$$

If we re-label the PC in terms of output, then we should modify Okun's law as

$u - \overline{u} = -\alpha x$

Where

$$x = \left(\frac{Y - \overline{Y}}{\overline{Y}}\right) = \log Y - \log \overline{Y} = y - \overline{y} \tag{3.6}$$

and x is the output gap. This states that when output is higher than potential (so that the output gap is positive) the unemployment rate will be falling. Consequently, one can have the PC in terms of inflation and unemployment (a negative relationship) or in terms of inflation and output (a positive relationship). It does not really matter how one writes it. However, as in most of our models we consider output but do not include unemployment, it is easier to get the PC in terms of output and inflation, a supply curve.[1]

From (3.1) above and using $\beta = \pi^e$, the PC in terms of output and inflation is:

$$\pi = \pi^e + \alpha\delta x \tag{3.7}$$

Which we will call an expectations augmented supply curve. Remember: (3.4) and (3.7) represent virtually the same thing, so we can carry out the analysis of the model with only one of the two. We shall focus on (7) as it is the most common practice.

The last element to add to (3.4) or (3.7) is to allow for shocks that affect the relationship between the two variables. This will mean that (3.7) becomes:

$$\pi = \pi^e + \alpha\delta x + \epsilon \tag{3.8}$$

So that ϵ can be interpreted as a cost-push shock. For example, an increase in the oil price (for a net oil importer) would result in higher costs and therefore inflation. The EAPC written in (3.8) performs much better at explaining the data than (3.1). In fact, it is often argued that the breakdown in the Phillips curve occurred during the oil crisis of the early 1970s (that is, there were large cost push shocks). However, the Phillips curve was already shifting by the late 1960s. Why? A straightforward interpretation is that as inflation had been rising (policy makers were exploiting the PC) it was only a matter of time until private sector expectations began to reflect this. In other words, in (3.8) began to rise shifting the PC to the left.

3.2 The Phillips curve: some theories

The Phillips curve models the link between inflation and output in the short run. To the extent that monetary policy determines inflation, then we also

[1] Then, to know what is happening to unemployment just remember that it moves in the opposite direction to output.

have a (short run) link between money and output.[2]

What is the role of money, and monetary policy in the positive short run link between inflation and output? There are several possibilities. First, as in Classical, New Classical and RBC (we shall be covering the latter later on), money and inflation do not affect output (or if it does, the effect is negligible). In other words, we do have a relationship, but it is not a causal one (from money to output). The alternative is that changes in the quantity of money and inflation do cause temporary changes in output. If that is the case, what is the channel? There are two scenarios:

- First, the economy is characterised by flexible prices. As mentioned above, unless one add modifications to the standard model there won't be a causal link. The change comes via introducing imperfect information. The underlying theory originated from Friedman but it was Lucas who provided the theoretical underpinnings. It has subsequently become known as Lucas' islands model.

- Second, there are nominal rigidities in the economy. That is, (some) prices and/or nominal wages are rigid. Then changes in the quantity of money will affect relative prices and hence output.

What model is driving the Phillips curve matters greatly as it is only in the last group where one can argue in favour of activist policy.

3.2.1 Lucas' islands model

The idea is fairly intuitive. Households (who are also firms) start each period on a small island (these are numerous). They can decide how much output to produce and labour is the only input. All they observe is the price of their own good but not the general price level. However, what matters for decisions is the relative price. If their relative price has increased then they will produce more. However, if all prices have gone up (including their individual price) then their relative price has remained unchanged and it is not profitable to increase output.

In addition, they know the volatilities of shocks to individual and aggregate prices. As a result, when they observe an increase in the price of their own good they have to work out whether this represents an increase in their own good. Given that they know how volatile their individual and aggregate prices are, it turns out that the optimal response will be to respond positively to this 'price signal'.

However, how much more the islander will work will depend on how predominant the island-specific shocks are. If for example all the volatility

[2]Recall from McCandless and Weber [1995] that there is little or no long run relationship between real and nominal variables.

came from aggregate shocks, islanders would then always know that higher prices are just higher aggregate prices: they would not modify their labour supply decisions.

There are several other key implications. Only unexpected shocks matter. Turning this into an economic model implies that only unanticipated changes in the money supply will have real effects. Secondly, the slope of the PC will depend on the variance of the demand shocks. This is an implication of the model and Lucas found empirical support for this. Lastly, as only unanticipated policy matters, if policy makers aim to stabilise output, they should keep the money supply fixed. Lastly, in the benchmark model, shocks have no persistence.

Is this a realistic model? One crucial flaw is that it relies on agents' inability to observe aggregate data (prices, the money supply, ...). However, in practice these are widely available with shortish lags.

3.2.2 Overlapping contracts

In this subsection we are going to see that with the presence of overlapping wage contracts shocks will have longer lasting effects on output and that prices will display persistence. The model here is based on Taylor [1979] and we shall keep it as simple as possible. Assume that all wage contracts are set in nominal terms and last for two periods, with half of the workforce re-negotiating its contract in any given period.[3] Moreover, the contract wage is set to reach a desired real wage, w^*:

$$w_{a,t} = \overline{W}_{a,t} - p_t = w^*$$

Note that the model is in logs. While I shall normally denote logged variables using lower case W here represents the real wage. $w_{a,t}$ represents the desired real wage and given the price level the nominal wage is adjusted to ensure that the real wage is w^*, a constant, while p is the aggregate price level. As the actual value of w^* will not affect the results we can simply normalise it to zero.

We can therefore re-write the above as

$$\overline{W}_{a,t} = p_t$$

However, we need to modify this equation in two ways: first, is the contract negotiated before or once period t begins? If it is the former then the wage would be based on expected p_t rather than its actual value. For simplicity, we shall assume that it is once t has begun so that it will be a function of the actual price level (it simplifies the mathematics slightly). Secondly, as the contract is going to last for two periods, the contract (nominal) wage

[3] Think of a union doing it on the workers' behalf.

will take not only p_t into account, but also (expected) p_{+1}, the yield the real wage during the second period of the contract.

Therefore, if we define x_t as the contract wage negotiated in period t, it is defined as

$$x_t = \frac{p_t + E_t p_{t+1}}{2}$$

In any given period the average nominal wage W_t is given by the average of the contract wage set in periods $t-1$ and t:

$$W_t = \frac{1}{2}\left(x_t + x_{t-1}\right)$$

To complete the supply side, if labour is the only input into production and labour demand is a negative function of the real wage, we can write output supply as

$$y_t = y^* - \alpha\left(W_t - p_t\right)$$

Next, to derive our aggregate supply-Phillips curve we combine the above, yielding

$$y_t = y^* - \alpha\left(\frac{1}{2}\left(x_t + x_{t-1}\right) - p_t\right)$$

$$y_t = y^* - \frac{\alpha}{2}\left(\frac{p_t + E_t p_{t+1}}{2} + \frac{p_{t-1} + E_{t-1} p_t}{2} - 2p_t\right)$$

$$y_t = y^* - \frac{\alpha}{4}\left(E_t p_{t+1} + p_{t-1} + E_{t-1} p_t - 3p_t\right) \tag{3.9}$$

If we re-write in terms of prices we have

$$p_t = \frac{4}{3\alpha}(y_t - y^*) + \frac{1}{3}\left(E_t p_{t+1} + p_{t-1} + E_{t-1} p_t\right) \tag{3.10}$$

The key thing to note here is that prices depends on their past values: we have persistence. As a result, one-off shocks will last more than one period. Had we had contracts lasting the more than just two periods then persistence would have been even greater.

Despite this, if we re-write the model in terms of inflation we have

$$\pi_t = \frac{4}{\alpha}(y_t - y^*) + \left[E_t \pi_{t+1} - \eta_t\right] \tag{3.11}$$

Where $\eta \equiv \pi_t - E_{t-1}\pi_t$ and under rational expectations this would be an i.i.d error term. The key thing to note is that this Phillips curve exhibits no persistence in inflation (although as we saw above, there is persistence in prices) as current inflation does not depend on past inflation. This will be important later when we discuss sacrifice ratios.

3.3 What determines the natural rate of unemployment?

The natural rate of unemployment is a variable of crucial importance for forecasting inflation and for policy analysis. Nevertheless, two facts are important to keep in mind: the natural rate is not constant and it is very imprecisely estimated. Note that one can easily get data on unemployment rates. However, the natural rate, just as expectations of inflation, are theoretical concepts for which we have no actual measurement. One way of calculating it is to get a trend for the actual unemployment rate and call that trend the natural rate. The one can carry out regressions with the natural rate on the left hand side and a selection of explanatory variables on the right hand side. To these we turn next:

3.3.1 Demographics

It is well known that unemployment rates vary by age groups. The unemployment rate of the young (e.g., 16-24) is higher than that of older workers (25-45). As this is a long run phenomenon, the implication is that younger workers have a higher natural rate of unemployment. This part of the unemployment rate is likely to evolve very slowly. Educational levels also affect the natural unemployment rates. Low skilled workers have a higher rate of unemployment than higher skilled individuals.

3.3.2 Institutions

This component includes many different factors, such as regulation, taxation, etc. Strong unions are often put forward as the cause of a high natural rate of unemployment (the UK in the 1970s and 1980s). However, this depends on what the union's objectives are. Is their main objective a high wage for their members or to ensure high employment? If it is the former, higher unemployment will result. As this will have a permanent effect, it will be part of the natural rate of unemployment. In contrast, if unions want to ensure high employment, they may negotiate pay restraint in order to ensure few layoffs (Germany in the early noughties). Then unions can bring about a lower natural rate of unemployment. Additional factors include high income taxes (and NI contributions) as well as employer taxes. The former reduce labour supply, the latter labour demand. Either way both result in less employment and higher unemployment. Similarly, minimum wages end up reducing labour demand. Another example with strong relevance to the UK was council housing that workers would lose of they relocated. The Thatcher policy of enabling tenants to purchase their properties aimed to eliminate this factor.

3.3.3 Productivity growth

This factor is not well understood . Normally, periods of slow productivity growth (1970s) have been associated with periods of high unemployment, whilst the reverse (mid-1990s) has also been the case. Why is this the case? Several stories have been put forward. The underlying idea is that during periods of high productivity growth firms can afford to pay higher wages which will induce more workers to accept them. However, in the long run wages cannot grow faster than productivity (remember the profit maximising condition: marginal product of labour = real wage).

3.3.4 Past natural rate

The natural rate does exhibit a lot of inertia (dependence on previous values). Some of it is obvious, think about the demographic causes. However, in addition to this, one could argue that high unemployment in the past would cause those who lost their jobs to de-skill, making them less likely to find another job. Similarly, if some sectors are in decline, a previously high unemployment rate may reflect this, and workers who lost their jobs had sector specific skills, meaning that they will remain unemployed until re-skilling.

There are other factors that affect the natural rate of unemployment, but the crucial thing to remember is that it is independent of monetary policy. Governments playing around with demand (by increasing spending or reducing interest rates) will reduce the unemployment rate. However, this effect will be short-lived as the natural rate of unemployment remains unchanged: expectations of inflation will adjust bringing the unemployment rate back to its long run level. Nevertheless, the unemployment rate can be reduced permanently by policies that target the natural rate of unemployment. Consequently, policy makers no longer face a permanent trade-off between inflation and unemployment, but a short-run one. In the long run, the only thing that the monetary authority can affect is the inflation rate.

Something that has been ignored so far is what determines the inflation rate. We have been assuming that the policy maker can choose it. Obviously this is unrealistic (just ask Mervyn King!). However (ignoring shocks) monetary policy, either via the money supply or interest rates, affects inflation. We can greatly simplify the analysis and keep the same insights by assuming that the central bank can directly control the inflation rate. Mathematically speaking this just means we do away with one equation.

So far, we have modified the basic PC model to conclude that:

- The unemployment rate will converge towards its natural rate
- The long run Phillips curve is vertical and the intersection between the short and long run PCs occurs when:

  - Unemployment equals its natural rate
  - Expectations of inflation equal actual inflation

- When the unemployment rate differs from its natural rate (we are away from the long run PC) expectations of inflation will be changing

Therefore, expectations of inflation are crucial in reaching equilibrium. Equally, if a cost push shock hits the economy, inflation will rise. What will the consequences be for expected inflation and unemployment? As the PC is just an aggregate supply schedule, how agents form expectations has important consequences on how:

- the central bank should react to events
- how long it will take inflation to return to its long run value (assuming that there is one)

3.4 Policy analysis with the Phillips curve

One cannot do much with the Phillips curve on its own unless we make assumptions about monetary policy such as, for example, the assumption that the central bank is able to control inflation (or output) directly. That being the case, some of the applications one generally encounters are

- Whether central banks should be independent
- If the central bank wants to implement a disinflationary policy,[4] should it do so slowly or should it adopt a cold turkey approach?
- Should central banks stick to a rigid rule or should they be able to adopt the best policy at any given time?

3.4.1 Disinflations

Here we consider the output effects of policy-induced reductions in inflation. For simplicity, assume that central banks – via monetary policy – can directly control the rate of inflation so that we need only consider the Phillips curve. If the monetary authority decides to change the steady state of inflation to a new, lower level, the effects on output will depend not only on the specific form that the PC takes, but also on the timing of the policy announcement as well as its credibility.

For now, let us consider the simple expectations augmented PC:

$$\pi_t = \pi_t^e + \alpha x_t$$

[4] That is, the inflation rate is deemed to be too high and the central bank is going to reduce it.

Where $x = y - y^*$ is the output gap and π^e is expected inflation. Under adaptive expectations disinflations will always be costly, as they will involve losses in output. Moreover, these losses will be independent of any prior announcement or degree of policy credibility, as expectations are backward looking. By contrast, if expectations are rational and the policy is announced prior to its implementation disinflations will be entirely costless (as long as the policy is believed). Note that the key factor affecting the output cost of disinflations is the presence of past inflation in the PC, in other words, inflation persistence. Under rational expectations in the equation above, there will only be inflation persistence if credibility is imperfect, so that expected inflation will partly depend on the level as past inflation as agents believe that the monetary authority will not fully implement the disinflation.

Returning now to the Taylor model of overlapping wage contracts, we can see that even though we have nominal rigidities – in that nominal wages cannot adjust intra-temporally to clear the labour market – disinflations are still costless as there is no inflation persistence.

3.4.2 Evidence on sacrifice ratios

One way of considering the actual real effects of disinflations is to calculate the sacrifice ratio. A large amount of the recent literature on this topic stems from the work of Ball [1994]. First, a definition. Disinflation is the process of bringing inflation down to a new, lower level. It is often believed that bringing down inflation entails (temporary) losses in output. Hence, one definition Ball proposed was to define the sacrifice ratio (sr) as

sr is 'the sum of output losses divided by the change in trend inflation over an episode'

The numerator is calculated by considering the deviations of output from its potential (or trend) level. The assumption he made was that at the beginning of the disinflationary episode output was at its trend level (as well as a year after the new lower level).

Hence sr can be defined as the output losses per percentage point decrease in inflation.

Much of the empirical analysis on the sacrifice ratio is of a reduced form nature. To understand this, note that if we knew the form of the Phillips curve we could calculate the sacrifice ratio as simply the slope of the Phillips curve: lower the inflation rate by one percentage point and find the implied contraction in output. However, this requires knowledge of the Phillips curve (or an assumption of its specific form). The alternative approach we are going to consider here is to calculate sacrifice ratios from the data and then to try and determine what variables provide a good explanation for its magnitude. Typically, the variables of interest have been the speed of disinflation, the original level of inflation, the degree of central bank independence and whether inflation targeting is being implemented.

Recommended reading:

- Sargent [1981]
- Ball [1994]
- Jordan [1997]
- Gonçalves and Carvalho [2009]
- Brito [2010]
- Diana and Sidiropoulos [2004]
- Chevapatrakul and Paez-Farrell [2013]

As mentioned above, having obtained data on sacrifice ratios, the regression to be estimated is something like

$$sr = c + \alpha\pi_0 + \beta SP + \epsilon$$

(In Table 5.10 Ball also shows that for the quarterly sample the initial rate of inflation matters, negatively, as one would expect from the theory).

Where SP is the speed of disinflation and π_0 is the original rate of inflation (at the beginning of the disinflationary episode).

Crucially, as also argued by Sargent, Ball finds that the more rapid the disinflationary episode (a more 'cold turkey' approach) the lower the sacrifice ratio. The obvious conclusion is then that disinflations should be conducted quickly. The rationale for such a result may be attributed to several factors. One of them is reputation. A rapid disinflation can be interpreted as a signal of seriousness and commitment to low inflation, whereas if it is implemented gradually there is always the possibility of a policy reversal in the near future. Also noteworthy is the result that the coefficient on the initial level of inflation is negative. This is consistent with the theory (see Ball, Mankiw and Romer, 1988). The idea is as follows: when inflation is very low (nominal) contracts are signed for longer time periods. This can be interpreted as a greater degree of nominal rigidities (and a flatter Phillips curve), whereas at high levels of inflation contracts are of short duration (due to the costs that inflation imposes) and hence changes in inflation have small effects since firms and workers will be renegotiating contracts soon. In other words, the Phillips curve is steeper. As a result, if at the time of the disinflation the starting level of inflation is high, the steeper Phillips curve implies that a decrease in inflation will result in a lower contraction in output, a lower sacrifice ratio. Regarding the relationship between the degree of central bank independence and the sacrifice ratio, see Jordan [1997] and Daniels et al. [2005].

3.4.3 Inflation targeting and the sacrifice ratio

Ball's paper on sacrifice ratio (SR) gave rise to a small literature on assessing the performance of inflation targeting (whether it is superior to non-IT policies) by focusing on sacrifice ratios. Noteworthy among these are Gonçalves and Carvalho [2009] and Brito [2010]. In essence, Gonçalves and Carvalho [2009] re-did Ball's study but added a dummy variable for the adoption of IT in their sample (they also added a few other variables but that is not important for now). Their main result was that IT resulted in a lower sacrifice ratio (the coefficient on IT was negative). Hence, the policy prescription is that IT can be thought of as 'best practice'. However, Brito [2010] presents strong evidence that this conclusion is not robust as it did not take into account additional factors (such as the fact that some countries were trying to fulfil the Maastricht criteria); the results were highly sensitive to the timing of the disinflations as well as to the presence of particular countries in the sample. Overall the, the conclusion that Brito [2010] reaches is that as yet there is no persuasive evidence that IT reduces sacrifice ratios.

A further related study is Chevapatrakul and Paez-Farrell [2013], who used the same dataset as in Gonçalves and Carvalho [2009] but their focus was on the asymmetric properties of the sacrifice ratio. They employed quantile regressions to determine what factors affect the sacrifice ratio.[5]. The find strong asymmetries: the effect of initial inflation on the SR is positive when the latter is high but then negative for high values of the sacrifice ratio. At the same time, the speed of disinflation only matters when the SR is already low (but not at its lowest values in the distribution), elsewhere it is insignificant. Importantly, the level of debt – which was insignificant under OLS – turned out to positively affect SR only when the latter was high, while inflation targeting was insignificant across all quantiles.

Results: So what does the empirical evidence say on sacrifice ratios?

1. The sacrifice ratio depends negatively on the speed of the disinflationary process: 'cold turkey' is best.

2. There is no significant evidence that inflation targeting lowers the sacrifice ratio.

3. The higher the starting level of inflation the lower the sacrifice ratio.

4. There is some evidence that a greater degree of central bank independence (CBI) increases the sacrifice ratio.

[5] OLS just gives us the response at the mean. With quantile regressions we can determine what the response of Y to X is when the former is low, medium, high

3.5 Recommended further reading

Galí [2008], Chapter 1. This is downloadable from Assaf Razin's webpage:

(www.tau.ac.il/~razin/)

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3.A Taylor Contracts

3.A.1 Introduction

The key factor here is the introduction of overlapping contracts. In the original Taylor and Fischer papers these were in the form of staggered wages but similar insights apply if one is implementing via overlapping price setting.

Some useful references

- Taylor [1979].
- Fischer [1977].
- Romer [2018].
- Walsh [2017].

Each period a fraction (in our case, a half) of all wage contracts are re-negotiated for the following two periods. Contracts are negotiated to achieve a target real wage w^*.[6] So in any given quarter (period), the real wage would ideally be (in logs)

$$w_t = W_t - p_t = w^*$$

We shall use x to denote the contracted nominal wage so we can re-write the above as

$$w_t = x_t - p_t = w^*$$

In the above, x is chosen to deliver (given prices) the target level of the real wage w^*. As set above, this would be a trivial exercise as given the price level and the target real wage the contract wage would always deliver the target wage as we are assuming that we can observe the current price level and the contract wage is set at for one period.

As the precise value of w^* will not play a role in our results, we can set it to zero without loss of generality. This means that for the set up above, contracts wages are set so that

[6] We can think of this target wage as the one that ensures labour market clearing.

$$x_t = p_t$$

In other words, the contract (nominal) wage moves one for one with prices. If we modify this slightly so that contracts are negotiated before the period begins, then they would be designed so as to move one for one with expected prices:

$$x_t = E_{t-1}p_t$$

Now let us introduce overlapping contracts (for two periods). We therefore have that the contracts will be devised such that

$$x_t = \frac{1}{2}\left(E_{t-1}p_t + E_{t-1}p_{t+1}\right) \tag{3.12}$$

Actual real wages will be the average of the contract wage over the current and previous periods.[7] Note that from the above we are assuming that the contract is set in period $t-1$ to apply for periods t and $t+1$, hence the timing of the expectations. Hence, the real wages are

$$w_t = \frac{x_t + x_{t-1}}{2} - p_t \tag{3.13}$$

To keep things as simple as possible, next we shall assume that output (supply) is a negative function of the real wage (think of labour demand and firms' production):[8]

$$y_t = -w_t \tag{3.14}$$

We have all the components we need to derive the Phillips curve. The only thing that remains is to combine the equations above – (3.12), (3.13) and (3.14) – to have a simple relationship between output and prices (or inflation).

Combining, we have

$$y_t = p_t - \left[\frac{x_t + x_{t-1}}{2}\right]$$

$$y_t = p_t - \frac{1}{4}\Big(\left(E_{t-1}p_t + E_{t-1}p_{t+1}\right) + \left(E_{t-2}p_{t-1} + E_{t-2}p_t\right)\Big)$$

We could leave it like this but it doesn't look particularly informative. The equation above can be re-written as:

[7] Recall that at time t half the workforce is bound by x_t and the other half by the contract x_{t-1}.

[8] I could have added a shock to the right hand side representing technology shocks so that we could then analyse its effects in the model but I want to focus on demand shocks.

$$y_t = \frac{1}{4}\Big[(p_t - E_{t-1}p_t) - (E_{t-1}p_{t+1} - p_t) + (p_t - E_{t-2}p_{t-1}) + (p_t - E_{t-2}p_t)\Big]$$

The right hand side is now made up of four blocks. The first and last represent surprises. For the first one, any increase in prices at time t that was not anticipated at time $t-1$ will have a positive effect on y. The same applies to the last element except that now shocks that occurred in $t-1$ (unanticipated in $t-2$) will also affect y. Also, the second block shows that expected future inflation have a negative effect on output. This is because the expected future inflation will lead to an increase in the contract wage today, pushing up current real wages.

An alternative way of re-writing the above is in terms of inflation. Recall that $\pi_t = p_t - p_{t-1}$ and $E_{t-1}\pi_t = E_{t-1}p_t - p_{t-1}$, etc. We then have:

$$y_t = \frac{1}{4}\Big[\pi_t - E_{t-1}\pi_{t+1} + 2(\pi_t - E_{t-1}\pi_t) + (\pi_t - E_{t-2}\pi_t) + (\pi_{t-1} - E_{t-2}\pi_{t-1})\Big] \tag{3.15}$$

The terms in parentheses represent expectational errors ('surprises') and under rational expectations have a mean of zero. The third term represents changes in current inflation not expected at time $t-2$ so any shock that occurred in $t-1$ will be here, hence a somewhat persistent effect. Likewise, the last term represents the surprise element of inflation in $t-1$ not anticipated in period $t-2$. It is worth noting that there are no $t-3$, $t-4$, etc. terms in the Phillips curve. In other words, we have assumed that contracts would last two periods and as a result the output effects on our model of shocks that affect inflation will also last only two periods. Put differently, this model does not embody endogenous persistence. This will be clearer when we have a complete model.

To assess this Phillips curve with overlapping contracts we need to add more equations to close the model. As the PC represents the supply side of the model, let the demand side be given by

$$m_t = p_t + y_t + \epsilon_t$$

and monetary policy is

$$m_t = \mu_t$$

ϵ and μ both represent white noise processes.

The figures show the response to each of the two shocks. Unlike some of the earlier models, now the shocks have protracted effects on output

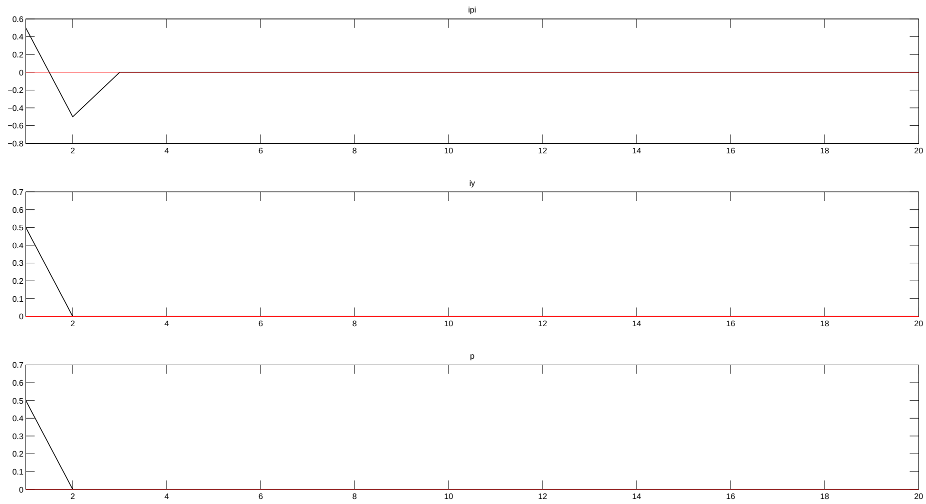

Figure 3.1: Impulse response to a demand shock for inflation, output and prices

3.A.2 Consequences of small modifications

Returning to equation (3.12), recall that we assumed that contracts were devised on the basis of information up to period $t-1$. If instead we had assumed that these were done using period t information equation (3.15) would then become

$$y_t = -\frac{1}{4}\Big[E_t\pi_{t+1} - \pi_t - (\pi_t - E_{t-1}\pi_t)\Big] \tag{3.16}$$

3.B The Lucas Islands Model

3.B.1 Introduction

One of the key contributions in Lucas [1973] is to explain why a Phillips curve may arise when agents have rational expectations and without having to make use of rigid/sticky prices or wages. Recall that depending on the factors that give rise to the same relationship – here it is the Phillips curve – the implications can be markedly different.

The notes here draw heavily on Martin Ellison's notes. The solution is very subtle and when deriving the supply equation note that it will depend on the demand side. Much of the model will make sense upon second reading.

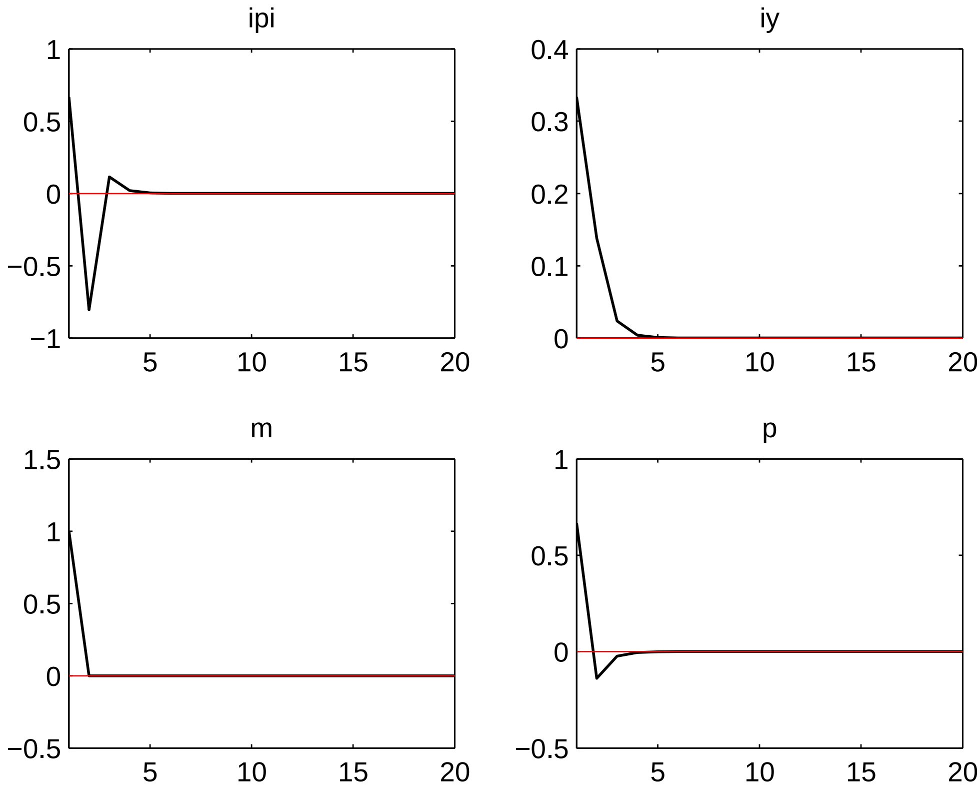

Figure 3.2: Impulse responses to a money supply shock

3.B.2 The supply side

There is a large number of producers-consumers (think of them as yeomen-farmers) and each lives on a separate island. Their work directly translates into output and their supply (output or labour as they are both the same) is a function of the relative price of their own good:

$$y_t(z) = \gamma \left(p_t(z) - p_t\right) \tag{3.17}$$

Where z denotes island z so that this equation describes the behaviour for each islander and $p_t(z)$ will vary from island to island, while p_t is the aggregate price level, hence it is the same for all islands.[9] This equation states that the agent supplies more labour as the price of her good increases relative to the aggregate price level and $y_t(z)$ is the level of output relative to its average value.

Therefore, it is not $p_t(i)$ or p_t in isolation but the gap between the two. Consequently, if they both change by the same amount the producer's output will remain unchanged. One way of understanding this is to think about what you have covered in microeconomics. There you normally abstract from many macroeconomic phenomena and you will rarely hear mention of the aggregate price level so that it can be thought of as constant. In a standard model then, an increase in the price of a firm's good leads the producer to increase her output, all else being the same. This last part is crucial and in the macroeconomic context this means the aggregate price level. It is therefore only *relative* and not absolute prices that matter. If the price on a specific island goes up relative to that in other islands, the islander will increase her output. Another way of seeing this is to consider a worker. Her labour supply is a an increasing function of the real wage only so that changes in nominal wages (with a constant real wage unchanged) will not affect her behaviour. Given that $w_t = W_t - p_t$ (in logarithms and I am assuming you can work out what each variable is) it is the gap between the two right hand side variable that determines labour supply.

While all islands are identical in structure they are subject to idiosyncratic shocks so that $p_t(z)$ will vary across islands (but recall that the average of all of these is just p_t; likewise, aggregate output y_t is the sum of all the $y_t(z)$).

If the agents could observe $p_t(z)$ and p_t then we would just solve for output using the equation above. However, a key insight of Lucas' is to assume that although agents can observe the price of their product, $p_t(z)$, they have to try and estimate/guess the aggregate price level p_t as this cannot be observed contemporaneously. That is the reason it is called the islands model: they can observe the price of the good on their own island but not that in other islands (otherwise they would just calculate the aggregate

[9] Think of all the variables as being in logarithms.

price level). As a result, in the presence of imperfect information their output will be a function of what they expect p_t to be. This means that (3.17) is replaced with

$$y_t(z) = \gamma \left(p_t(z) - E\left(p_t|I_{t-1}(z), p_t(z)\right)\right) \tag{3.18}$$

where $E(p_t|I_{t-1}(z), p_t(z))$ simply means the expectation of the aggregate price level conditional (given) the information available to the agent, to which we shall return shortly. Much of will follow is focused on solving for these expectations.

Under rational expectations agents' forecasts (their rational expectation) of future variables will on average be correct. That is not to say that they will ever be right but that their forecast will not be biased meaning that the one-step ahead forecast error will be a random mean zero i.i.d variable

$$p_t - E(p_t|I_{t-1}) = \epsilon_t \tag{3.19}$$

Where $E(\epsilon^2) = \sigma^2$. Bear in mind that ϵ_t won't necessarily be a single specific shock. The point is that under rational expectations the forecasting error that agents make will be a white noise process, made up of all the shocks in the model; thus we can think of ϵ_t as a composite of these. Later on we shall determine what it is composed of.

The second key equation in the model relates island-specific to aggregate prices:

$$p_t(z) = p_t + z_t \tag{3.20}$$

Where z_t is again a random, mean-zero i.i.d. variable, $z_t \sim N(0, \tau^2)$, representing shocks to island-specific prices (if z_t is positive we can think of this as an increase in the demand for the good produced by island z so that its relative price increases). If we sum across all the islands in any particular period the $z_t(i)$ sum to zero.

Under full information we would just combine (3.17) and (3.20) to solve

$$y_t(z) = \gamma z_t$$

If the agent could observe everything that is affecting her then her output would only be a function of z_t. We can therefore think of z_t as real (as opposed to nominal) shocks. If consumers demand more of your good relative to other goods it becomes relatively more valuable and hence the producer of that particular good will increase production.

However, with imperfect information we have

$$y_t(z) = \gamma \left(p_t + z_t - E(p_t|I_t(z))\right)$$

or

$$y_t(z) = \gamma\left(z_t + \epsilon_t\right)$$

The islander can observe the sum of the two shocks, but not the value of each of them. Ideally, she would respond only to z and not ϵ but this is not part of her information set. However, she has access to past information so given past data on the values of z_t and ϵ_t, she can run the regression

$$z_t = \widehat{\alpha}_1(z_t + \epsilon_t) + u_t$$

Via OLS we obtain

$$\widehat{\alpha}_1 = \frac{Cov\left((z+\epsilon), z\right)}{Var(z+\epsilon)} = \frac{\tau^2}{\tau^2+\sigma^2}$$

So the best guess of z_t is $\widehat{z}_t = \widehat{\alpha}_1(z_t + \epsilon_t)$.
Since we had

$$p_t(z) = p_t + z_t$$

The best guess of p_t given that $p_t(z)$ is observed is

$$E\left(p_t|I_{t-1}(z), p_t(z)\right) = p_t(z) - E\left(z_t|I_{t-1}(z), p_t(z)\right) = p_t(z) - \widehat{\alpha}_1\left(z_t + \epsilon_t\right)$$

Next, use $\epsilon_t = p_t - E(p_t|I_{t-1})$ and $z_t = p_t(z) - p_t$:

$$(z_t + \epsilon_t) = (p_t(z) - p_t) - (p_t - E(p_t|I_{t-1}))$$

$$(z_t + \epsilon_t) = p_t(z) + E(p_t|I_{t-1})$$

Therefore,

$$E\left(p_t|I_{t-1}(z), p_t(z)\right) = p_t(z) - \widehat{\alpha}_1\left(p_t(z) - E(p_t|I_{t-1})\right)$$

$$E\left(p_t|I_{t-1}(z), p_t(z)\right) = (1-\widehat{\alpha}_1)p_t(z) + \widehat{\alpha}_1 E(p_t|I_{t-1})$$

So we now have that

$$E(p_t|I_t(z)) = E(p_t|I_{t-1}(z), p_t(z)) \tag{3.21}$$

Use this in our supply curve:

$$y_t(z) = \gamma\left(p_t(z) - E(p_t|I_t(z))\right)$$

$$y_t(z) = \gamma\left[p_t(z) - (1-\widehat{\alpha}_1)p_t(z) - \widehat{\alpha}_1 E(p_t|I_{t-1})\right]$$

Thus

$$y_t(z) = \widehat{\alpha}_1 \gamma \left[p_t(z) - E(p_t|I_{t-1})\right]$$

If we sum across all islands we have

$$y_t = \widehat{\alpha}_1 \gamma \left[p_t - E(p_t|I_{t-1})\right]$$

We have derived our Lucas surprise supply equation (or Phillips curve): output depends on unanticipated changes in prices and its slope is given by $\widehat{\alpha}_1 \gamma$. Key to this is to also note that the slope depends on the relative contribution of real shocks (z_t) to the overall volatility in price surprises.[10]

3.B.3 The demand side

Assume that the demand for each good is given by

$$y_t^d = m_t(z) - p_t(z)$$

Where m_t is the quantity of money. Assume that

$$m_t(z) = m_t + \eta_t(z)$$

with $\eta_t \sim N(0, \delta^2)$ and that the money supply follows

$$m_t = m_{t-1} + \mu + \xi_t$$

This implies that the growth rate of the money supply is on average equal to μ, and again we shall assume that $\xi \sim N(0, \lambda^2)$.

3.B.4 Equilibrium

In equilibrium the demand and supply in each market (island) are equal to each other. Thus we have

$$\widehat{\alpha}_1 \gamma \left[p_t(z) - E(p_t|I_{t-1})\right] = m_t(z) - p_t(z) \tag{3.22}$$

If we aggregate across all islands the equation above becomes

$$\widehat{\alpha}_1 \gamma \left[p_t - E(p_t|I_{t-1})\right] = m_t - p_t \tag{3.23}$$

Take expectations at $t-1$ of both sides:

$$0 = E_{t-1} m_t - E_{t-1} p_t$$

This gives[11]

[10] Of course, we do not yet have a full solution since $\widehat{\alpha}_1$ is not quite solved for (this will become clearer below) as it depends on the demand shocks and we have not yet considered the demand side of the model. Its value is determined at the end of this document.

[11] Note that we have $E_{t-1} p_t \equiv E(p_t|I_{t-1})$ as they mean the same thing.

$$E_{t-1}p_t = E_{t-1}m_t = m_{t-1} + \mu$$

Thus

$$E(p_t|I_{t-1}) = m_{t-1} + \mu$$

This equation is key: we have now been able to solve for the expectation of the aggregate price level. Now that we have determined what the expected price level is we can return to some of our previous equations that depended on this to fully determine their behaviour.

Returning to the market equilibrium equation we had

$$p_t = E(p_t|I_{t-1}) + \frac{1}{\widehat{\alpha}_1\gamma}(m_t - p_t)$$

Solve for the price level

$$(1 + \widehat{\alpha}_1\gamma)p_t = (m_{t-1} + \mu + \xi_t) + \widehat{\alpha}_1\gamma(m_{t-1} + \mu)$$

Which yields

$$p_t = m_{t-1} + \mu + \frac{1}{1 + \widehat{\alpha}_1\gamma}\xi_t \tag{3.24}$$

From this and noting that earlier we had $E(p_t|I_{t-1}) = m_{t-1} + \mu$, we can combine them to obtain

$$\epsilon_t = p_t - E(p_t|I_{t-1}) = \frac{1}{1 + \widehat{\alpha}_1\gamma}\xi_t$$

Therefore

$$\epsilon_t = \frac{1}{1 + \widehat{\alpha}_1\gamma}\xi_t \tag{3.25}$$

So now we know that the shock ϵ is a multiple of the money growth shock.

Turning to the equilibrium equation for each island (supply equals demand) (3.22) re-write it as

$$(1 + \widehat{\alpha}_1\gamma)p_t(z) = m_t(z) + \widehat{\alpha}_1\gamma E(p_t|I_{t-1})$$

Solve this

$$(1 + \widehat{\alpha}_1)p_t(z) = m_t + \eta_t(z) + \widehat{\alpha}_1\gamma(m_{t-1} + \mu)$$

$$(1 + \widehat{\alpha}_1)p_t(z) = (m_{t-1} + \mu + xi_t) + \eta_t(z) + \widehat{\alpha}_1\gamma(m_{t-1} + \mu)$$

$$p_t(z) = m_{t-1} + \mu + \frac{\xi_t + \eta_t(z)}{1 + \widehat{\alpha}_1\gamma} \tag{3.26}$$

This implies that in equilibrium the aggregate price level follows

$$p_t = m_{t-1} + \mu + \frac{\xi_t}{1 + \widehat{\alpha}_1 \gamma} \tag{3.27}$$

Recall that each island's output depended on relative prices
Now combine equations (3.24) and (3.26) and we have

$$z_t = p_t(z) - p_t = \frac{\eta_t(z)}{1 + \widehat{\alpha}_1 \gamma} \tag{3.28}$$

We can now find what $\widehat{\alpha}_1$ is:[12]

$$\widehat{\alpha}_1 \equiv \theta = \frac{Cov[(z_t + \epsilon_t), z_t]}{Var(z_t + \epsilon_t)}$$

Note that

$$Cov[(z_t + \epsilon_t), z_t] = Cov\left[\frac{\eta_t + \xi_t}{1 + \theta\gamma}, \frac{\eta_t}{1 + \theta\gamma}\right] = \frac{\delta^2}{(1 + \theta\gamma)^2}$$

$$Var(z_t + \epsilon_t) = Var[\frac{\eta_t + \xi_t}{1 + \theta\gamma}] = \frac{\delta^2 + \lambda^2}{(1 + \theta\gamma)^2}$$

Therefore,

$$\widehat{\alpha}_1 = \frac{\delta^2}{\lambda^2 + \delta^2}$$

[12] I'll call it θ from now on to be consistent with Ellison's notation.

Chapter 4

Rational Expectations

4.1 Introduction

The solution of models with rational expectations can be quite complex for rich models so the main emphasis will be on understanding the idea behind the rational expectations hypothesis (REH) and how to use it in solving small models.

Useful sources of reading for this are:

- Atfield, Demery and Duck, Rational Expectations in Macroeconomics, Blackwell.
- Hoover, K., The New Classical Macroeconomics. Blackwell.
- McCallum, B. T. Monetary Theory, Ch. 8 (up to 8.4).

Most macroeconomic models contain expectations. For example, the Phillips Curve/supply curve often contains the expected price level, becoming the Expectations Augmented PC. The Fisher (not Fischer) equation contains the expected rate of inflation and so on.

It is important to note that when a model contains expectations, unless you are given additional information regarding expectations formation – for example, an additional equation representing the adaptive expectations hypothesis (AEH) – then you can only solve the model conditional on expectations, whatever they may be. Or put differently, you may be able to simplify the model, but you will not be able to really solve it.

This would require eliminating all expectations from the solution for the endogenous variables, typically output and prices in our exercises. One way of achieving this is to add an additional equation, say AEH, which defines how expectations evolve over time and this allows us to get rid of them. One problem of formulating expectations using the procedure above is that you end up with a very mechanical way as to how expectations are formed, and the typical result will be that it is easy to see how economic agents (households and firms) can make continuous errors in their expectations. Consider the case of a disinflationary episode. When the inflation rate is continuously falling, under AEH agents will constantly overestimate the inflation rate, and even though agents see they are making a systematic mistake they will never converge on the actual inflation rate (catching up never takes place).

Such a formulation for how people form expectations is undesirable. For a start, it is costly for agents to make mistakes: households could have achieved higher utility and firms larger profits if mistakes had not been made. Moreover, AEH in a way treats people as stupid, mechanical automatons that are not aware of their actions. As a result of this, it is necessary to change the way people form expectations in such a way that it overcomes the problems stated above.

So what we want is a model of expectations such that the expectational mistakes people make will be completely random. Otherwise, they could be forecasted and therefore, if people are rational, could not occur. To see this, imagine that when people make a positive mistake this period (the actual value was larger than your expected value), it is normally the case that they will also make a positive mistake in the following period (the same applies when it is negative). Then, under rational expectations you would realise that if you made a positive mistake this period, your mistake is likely to be positive again next period; you consequently revise your expectations upwards (mistakes cost money!) in order to eliminate the bias until, on average, the expected value of the mistake is zero.[1] Hence the expectational mistakes cannot follow any recognisable pattern and have to be completely random. The way to write this is (in an example of inflation):

$$\pi_{t+1} - E_t\pi_{t+1} = \epsilon_{t+1} \tag{4.1}$$

where ϵ is a white noise process. The way we have written this equation, $E_t\pi_{t+1}$ represents the expectation (E), of next period's inflation. Note that this is done with knowledge of all the values of the variables in the economy up to time t (this is the subscript of E). Because we are analysing the expectation of a future variable, we do not know what actual value it will take as we form the expectation at time t of a something that has not yet taken place. To the extent that there is uncertainty in the model – in the presence of shocks that may occur – there will be a discrepancy between actual and expected values. Consequently, its actual value minus its expected value will, on average, equal zero, but normally it will simply be a random error (again, with a mean of zero in order to be consistent with the earlier sentence).

[1] I mention expected because 'events', shocks, may happen in the next period which you could not have foreseen.

An additional implication from the assumption regarding expectations is:

$$E_t \pi_t = \pi_t \tag{4.2}$$

That is, your expectation at time t of a variable at time t, will be its actual value. You cannot make a mistake because under REH, we assume that you have information on all the relevant variables up to the time subscript in the expectations, and inflation at time t is a relevant variable. This is equivalent as asking you today what the weather is like today; because you can observe it (you have information up to the present), you will not make a mistake. The same trick applies if you have to make an expectation of say, past inflation.

We shall in general assume that the shocks hitting the economy are white noise processes. This simply means that they have a constant variance, a mean of zero and they are not correlated with either other shocks or with their own past values. Therefore, if we have that a particular shock took a value of unity in the present period, our best forecast of this shock for the next period is still zero.[2]

Similarly, if we want to make the expectation of past inflation (inflation at t minus something), we are forming the expectation after the event has occurred, such as the present. Then because at the time you make the expectations you are considering a past value, you will choose the actual value. As it is already known you plug in its value.

Using this on the shocks (as long as we assume they are white noise):

$$E_t \epsilon_{t+1} = 0 \tag{4.3}$$

$$E_t \epsilon_t = \epsilon_t \tag{4.4}$$

Lastly, we may occasionally make use of the law of iterated expectations. The intuition is not very hard. Imagine we are considering what we'll expect/forecast next year that inflation will be two years from now. If each period is a year, this is just $E_{t+1}\pi t + 2$. This expectation will differ from the actual value to the extent that unexpected shocks occur between $t+1$ and $t+2$. Everything else is included in the expectation. But what do we expect now that our expectation next year will be of inflation two years' hence? This is just our expectation today of inflation in two years' time.

Mathematically,

$$E_t \left[E_{t+1} \pi_{t+2} \right] = E_t \pi_{t+2} \tag{4.5}$$

[2]If the shock were positively autocorrelated then a high value today implies a high likelihood of a high value tomorrow; solving models with these shocks is not too hard but we'll ignore this aspect for simplicity.

Once we understand the concepts above, it is time to solve for the model's actual values (prices, inflation, output). We are going to solve REH models using a method called the 'minimum state variable' (msv) criterion.

4.2 Solving REH models: the msv method

Recall that in general when we solve a system of equations we solve for the endogenous variables (y) as functions of the exogenous variables (x). In other words for each y we have an equation with the y on the left hand side and only x variables on the right hand side; if we have any ys on the right then we have not yet fully solved the model.

Our approach will be similar in that the solution of the model under rational expectations will give us expressions for the endogenous variables as functions of all the pre-determined variables.

Therefore, the idea is to say that the solution, whatever that may be, will depend on (will be a function of)

- If there are any constants present in the model, the solution will also contain one
- If there are any lagged values in the equation (that is, variables at t-something), then the solution will also depend on each of these
- If there are any shocks in the model, the model will also depend on each of these

Something you should bear in mind is that quite often in mathematical problems there may be more of one way of writing the solution, but they are all equally valid. For example, given points stated above, if we have $E_{t-1}x_t$ in the model we could think of the solution as being a function of this variable as it is pre-determined at time t. We shall not proceed in such a way if only because we want to write the solution, if only because we want to have the solution in terms of 'observables' and expectations cannot be observed.

4.2.1 An example

The easiest way to understand the msv method is to practise it (to death). Consider the following model log-linear, where all coefficients are positive

$$y_t = \gamma_1 m_t - \gamma_2 p_t \tag{4.6}$$

$$y_t = y^* + \phi \left(p_t - E_{t-1}p_t\right) + \epsilon_{yt} \tag{4.7}$$

$$m_t = -\beta y_{t-1} + \epsilon_{mt} \tag{4.8}$$

The first equation is just the LM, or aggregate demand. The reason the two are equivalent is that if money demand is insensitive to interest rates the LM is vertical. In that case the IS is irrelevant.[3] Equation (4.7) is just the Lucas surprise supply function/Phillips curve. Output equals its potential level and also depends positively on price surprises. Lastly, equation (4.8) describes the behaviour of the money supply. Rather than assuming that it is constant, which is quite unrealistic, the monetary authority attempts to stabilise output by contracting the money supply as output increases, hence it is quite activist, the more so the higher the value of β. Presumably it would react to deviations of output from potential $(y - y^*)$, but I'm ignoring the last constant for simplicity, not for realism. Note that monetary policy also contains a white noise process, ϵ_m; this is the random/unpredictable part of monetary policy. The idea is that the central bank cannot control the money supply perfectly (the money multiplier suffers from shocks) or that the central bank reacts to things other than output but agents cannot observe what that is.

If supply contained p_t^e rather than $E_{t-1}p_t$, where the superscript e denotes expectations (not necessarily rational) this would be a fairly Keynesian-monetarist model. In that case, the assumption that expectations are static (they're constant) would give us a very conventional Keynesian model. Similarly, if expectations are adaptive, such as $p_t^e = p_{t-1}$ the implications of the model would also be very Keynesian. Crucially, note that under AEH you are given an equation for expectations (you could think as expected variables as a completely additional variable, hence you need another equation).

But unlike under AEH, with REH we do not need a specific equation for expectations formation. To solve the model, combine the three equations to obtain

$$y_t = \gamma_1 \left(-\beta y_{t-1} + \epsilon_{mt}\right) - \gamma_2 p_t \tag{4.9}$$

$$y^* + \phi \left(p_t - E_{t-1}p_t\right) + \epsilon_{yt} = \gamma_1 \left(-\beta y_{t-1} + \epsilon_{mt}\right) - \gamma_2 p_t \tag{4.10}$$

Now we have only one equation with prices (we can leave past output as it is going to be constant in our model). One could also simplify the equation to have all terms multiplying current prices together, but for our purposes it will be best to leave it as it is, you'll soon see why. Notice that the only endogenous variable at time t is prices. We want to solve for prices first because that is the variable for which we have expectations in the model (there is not expected output or money supply); this way is far easier.

[3] Recall that aggregate demand is obtained by combining the IS and the LM curves.

To solve the model, notice that the equation contains: constants (potential output), lagged variables (past output) and two shocks. As we are only going to consider linear models, our solutions will always be linear models.

Then the solution for prices will take the form:

$$p_t = \lambda_0 + \lambda_1 y_{t-1} + \lambda_2 \epsilon_{mt} + \lambda_3 \epsilon_{yt} \tag{4.11}$$

The reason the solution looks like this is because: the model is linear, so the solution will also be linear. Secondly, prices will be a function, in principle, of everything that is pre-determined so that includes the constants, shocks, and past values of variables. However, for the latter we only include those that are present in the model. As there was no p_{t-1} in the model, for example, then the solution is not going to contain it. That is why the method is called the msv: we use the minimum number of 'state variables' which in our case just means pre-determined.

The lambdas are unknown, so that we have to find their values.[4]

The next thing we need is that if the equation above is a solution, then it also implies that if we take expectations at time t-1 on both sides of (4.11) we have

$$E_{t-1} p_t = \lambda_0 + \lambda_1 y_{t-1} + 0 \tag{4.12}$$

Remember that if you take expectations of a constant it remains a constant, and the expectations at time t of a past variable is the past variable itself. For the shocks, since we have not observed them yet and they are white noise, their expected value is zero.

From these two results, then we also have:

$$p_t - E_{t-1} p_t = \lambda_2 \epsilon_{mt} + \lambda_3 \epsilon_{yt} \tag{4.13}$$

That is, in the expectational error, all the known variables cancel out and you end up with the shocks only. In other words, the unexpected part of today's prices is a function of the shocks only; everything else was taken into account by the expectations in the last period.

Using these results, and substituting them into the equation for prices, (4.10):[5]

$$y^* + \phi \left(p_t - E_{t-1} p_t \right) + \epsilon_{yt} = \gamma_1 \left(-\beta y_{t-1} + \epsilon_{mt} \right) - \gamma_2 p_t \tag{4.14}$$

$$y^* + \phi \left(\lambda_2 \epsilon_{mt} + \lambda_3 \epsilon_{yt} \right) + \epsilon_{yt} = \gamma_1 \left(-\beta y_{t-1} + \epsilon_{mt} \right) - \gamma_2 p_t \tag{4.15}$$

[4] This part of the solution is called 'undetermined coefficients' for a reason.

[5] This is the part where not having simplified for prices makes sense.

$$y^*+\phi\left(\lambda_2\epsilon_{mt}+\lambda_3\epsilon_{yt}\right)+\epsilon_{yt}=\gamma_1\left(-\beta y_{t-1}+\epsilon_{mt}\right)-\gamma_2\left(\lambda_0+\lambda_1 y_{t-1}+\lambda_2\epsilon_{mt}+\lambda_3\epsilon_{yt}\right) \tag{4.16}$$

The only unknowns here are the lambdas. From this equation, if the solution we proposed is correct, then the coefficients multiplying, for example, the constants on both sides of the equation will have to be equal to each other The same will apply to the other components of the solution, such as lagged output.

Therefore, if we begin with the constants, collect the constants on the left hand side and equalise them to those on the right hand side

$$y^* = -\gamma_2\lambda_0 \tag{4.17}$$

Since the only unknown is λ_0, we have found the first component of the solution

$$\lambda_0 = -\frac{y^*}{\gamma_2} \tag{4.18}$$

Next, we need to do the same for components multiplying lagged output. Doing so yields:

$$0 = \gamma_1(-\beta) - \gamma_2(\lambda_1) \tag{4.19}$$

Or

$$\lambda_1 = -\frac{\gamma_1\beta}{\gamma_2} \tag{4.20}$$

Next, for the two shocks. Collecting terms for the monetary shock:

$$\phi\lambda_2 = \gamma_1 - \gamma_2\lambda_2 \tag{4.21}$$

Or

$$\lambda_2 = \frac{\gamma_1}{\phi+\gamma_2} \tag{4.22}$$

Finally, for the supply shock:

$$\phi\lambda_3 + 1 = -\gamma_2\lambda_3 \tag{4.23}$$

Giving

$$\lambda_3 = -\frac{1}{\phi+\gamma_2} \tag{4.24}$$

That's it! Now we have the solution we can put the lambdas into the equation for prices:

$$p_t = \lambda_0 + \lambda_1 y_{t-1} + \lambda_2 \epsilon_{mt} + \lambda_3 \epsilon_{yt} \tag{4.25}$$

$$p_t = -\frac{y^*}{\gamma_2} - \frac{\gamma_1 \beta}{\gamma_2} y_{t-1} + \frac{\gamma_1}{\phi + \gamma_2} \epsilon_{mt} - \frac{1}{\phi + \gamma_2} \epsilon_{yt} \tag{4.26}$$

Now you can analyse the solution and see what REH implies. Prices depend negatively on potential output, which is assumed constant, and this negative relationship makes sense. It also depends negatively on past output, and this is a result of the way the model was written. Monetary policy was written in such a way that the money supply would fall whenever past output had risen, and it is this effect that drives prices down. Finally, the monetary policy shock and the supply shock have positive and negative coefficients, respectively, which also conforms to standard macro theory.

Finally, what does REH imply for output? Given the way we wrote the Phillips curve, this is easy to find out:

$$y_t = y^* + \phi \left(p_t - E_{t-1} p_t \right) + \epsilon_{yt} \tag{4.27}$$

$$y_t = y^* + \phi \left(\lambda_2 \epsilon_{mt} + \lambda_3 \epsilon_{yt} \right) + \epsilon_{yt} \tag{4.28}$$

$$y_t = y^* + \phi \left(\frac{\gamma 1}{\phi + \gamma_2} \right) \epsilon_{mt} + \left[1 - \phi \left(\frac{1}{\phi + \gamma_2} \right) \right] \epsilon_{yt} \tag{4.29}$$

The most important result here, and one that features in most RE models with flexible prices, is that the systematic component of monetary policy (given by beta) cannot affect output, only the monetary policy shock can.

4.3 Contrast with AEH

If we had solved the model under AEH, so that we had:

$$y_t = \gamma_1 m_t - \gamma_2 p_t \tag{4.30}$$

$$y_t = y^* + \phi \left(p_t - p_t^e \right) + \epsilon_{yt} \tag{4.31}$$

$$m_t = -\beta y_{t-1} + \epsilon_{mt} \tag{4.32}$$

$$p_t^e = p_{t-1} \tag{4.33}$$

Because expectations are backward looking you would find that β would be in the solution for output. Hence monetary policy stabilisation can be successful.

4.4 Interest rate pegging under rational expectations

When considering Poole's paper we found that depending on the structure of the economy (slopes of the curves and volatilities of the different shocks) it was preferable to either set the interest rate or the money supply as the policy instrument. If it was the former then we found that the policy maker could set the interest rate at some optimal value and in principle, keep it fixed. As we shall see, such a prescription under rational expectations is likely to result in instability.

The model here is taken from **?** and is comprised of three equations:[6]

$$r_t = b_0 + (E_t p_{t+1} - p_t) + b_1 (p_t - E_{t-1} p_t) + v_t \tag{4.34}$$

$$m_t - p_t = c_0 + c_1 r_t + c_2 (p_t - E_{t-1} p_t) + \eta_t \tag{4.35}$$

$$r_t = r \tag{4.36}$$

r represents the nominal interest rate (I'm using the same notation as in his paper). The first equation could be re-written by taking expected inflation to the left hand side so that we would end up with real interest rates – nominal interest rates minus expected future inflation. The remaining part on the right hand side, price surprises, is just output. McCallum uses this approach rather than including y in the model in order to do away with one equation (and one variable), the Phillips curve. Hence, this equation is the IS[7]

The second equation, money demand, relates the demand for real money balances, as a negative function of the nominal interest rate (so c_1 should be negative) and a positive function of output.

As the policy maker is using the interest rate as the policy instrument the money supply is now endogenous: we do not have an equation for it (this was also discussed when analysing Poole's results). To solve for prices we combine the IS equation with the interest rate rule (note that the second equation just solves for m but we're not interested in it). Doing so yields

$$r = b_0 + (E_t p_{t+1} - p_t) + b_1 (p_t - E_{t-1} p_t) + v_t$$

Combining the coefficients on p_t:

$$r - b_0 = E_t p_{t+1} - (1 - b_1) p_t - b_1 E_{t-1} p_t + v_t$$

[6] I'm using the variant on p. 9 in his paper.

[7] Note that if as we have should have a negative relationship between real rates and output b_1 should be negative.

Our full model contains constants and two shocks so our msv solution for prices is given by

$$p_t = \lambda_0 + \lambda_1 v_t + \lambda_2 \eta_t$$

This implies

$$E_{t-1} p_t = \lambda_0$$

And

$$E_t p_{t+1} = \lambda_0$$

Making use of these in the equation above we have:

$$r - b_0 = E_t p_{t+1} - (1 - b_1) p_t - b_1 E_{t-1} p_t + v_t$$

$$r - b_0 = \lambda_0 - (1 - b_1) \left(\lambda_0 + \lambda_1 v_t + \lambda_2 \eta_t \right) - b_1 \lambda_0 + v_t$$

Collecting terms multiplying v_t we have:

$$0 = -(1 - b_1) \lambda_1 + 1$$

Which gives

$$\lambda_1 = \frac{1}{1 - b_1}$$

Collecting terms multiplying η_t,

$$0 = -\left(1 - b_1\right) \lambda_2$$

so

$$\lambda_2 = 0$$

(This is not surprising as η is the shock to the LM but when we have an interest rate rule the LM is passive)

Solving for the constant we have

$$r - b_0 = b_1 \lambda_0 - b_1 \lambda_0$$

Or

$$r = b_0$$

The process above has failed to determine the value of λ_0! (Furthermore, consistency requires that $r = b_0$).

The reason for these difficulties lies in the fact that in our original model above prices only enter as prices surprises and not on their own.

4.5 The role of money in public finance

Here we are going to focus on the role of money and monetary policy from a more long run perspective, so we are going to ignore monetary policy from the stabilisation point of view.

One of the key features of rational expectations is that models are forward looking. In other words, even though we have analysed how to solve models using the msv criterion and this results in a solution that depends on predetermined variables as well as shocks, one could also have written the equations by iterating into the future, highlighting the fact that the values of the endogenous variables depends on current and expected future events. These two ways of writing the model may look very different but they still imply the same solution. Here is a very simple example:

$$y_t = -\alpha p_t + \beta E_t p_{t+1} \tag{4.37}$$

Think of this as a made up demand equation (for a particular good). Output depends negatively on current prices but positively on expected future prices. Now re-write this with current prices on the left hand side.

$$p_t = -\frac{1}{\alpha}\left(y_t - \beta E_t p_{t+1}\right) \tag{4.38}$$

So by implication:

$$p_{t+1} = -\frac{1}{\alpha}\left(y_{t+1} - \beta E_{t+1} p_{t+2}\right) \tag{4.39}$$

And so on. If we keep substituting for future prices into (4.38) we end up with

$$p_t = -\frac{1}{\alpha}\left[y_t + \frac{\beta}{\alpha} y_{t+1} + \left(\frac{\beta}{\alpha}\right)^2 y_{t+2} + ...\right] \tag{4.40}$$

This can be written as:

$$p_t = -\frac{1}{\alpha}\sum_{i=0}^{\infty}\left(\frac{\beta}{\alpha}\right)^i y_{t+i} + \lim_{i\to\infty}\left(\frac{\beta}{\alpha}\right)^i p_{t+i} \tag{4.41}$$

As long as prices are stable (non-explosive) then the last part is zero. The crucial thing to note is that under this formulation current prices depend not only on current output but also its expected future path (but at a discounted rate). That's the crucial insight of working with dynamic rational expectations models.

4.5.1 Seigniorage

Now we turn to the consolidated government's budget constraint and the effects of money creation.[8] Assuming that all debt is issued for one period the government's budget constraint can be written as:

$$G_t + i_{t-1}B_{t-1} = T_t + (B_t - B_{t-1}) + (H_t - H_{t-1}) \tag{4.42}$$

All variables are in nominal terms (pounds). G denotes government spending on goods and services as well as transfer payments. $i_{t-1}B_{t-1}$ denotes the interest payments that must be paid on the debt stock prevailing in the previous period and T are tax receipts. $B_t - B_{t-1}$ represents new issues of one period debt and ΔH is the new issuance of high powered money, the monetary base. Recall that the latter is just currency held by the non-bank private sector as well as bank reserves.

Interpreting this equation is straightforward. On the left hand side we have government expenditure: this goes towards purchases of goods and services and payment of debt. On the right hand side we have the means by which that expenditure is financed. Hence it is a constraint. We have three sources of finance: taxes, new debt and printing money. In other words, expenditure is not free but must come from somewhere.

In order to see how money creation is source of revenue it is best to write (4.42) in real terms. Dividing by the current price level we have:[9]

$$g_t + \frac{i_{t-1}B_{t-1}}{P_t} = t_t + \frac{(B_t - B_{t-1})}{P_t} + \frac{(H_t - H_{t-1})}{P_t} \tag{4.43}$$

As we sometimes have variables in $t-1$ divided by P_t the trick is to multiply and divide by P_{t-1}. Doing so yields:

$$g_t + i_{t-1}\frac{b_{t-1}}{1+\pi_t} = t_t + \left(b_t - \frac{b_{t-1}}{1+\pi_t}\right) + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right) \tag{4.44}$$

Define the ex-post real interest rate as:

$$1 + \overline{r}_{t-1} = \frac{(1+i_{t-1})}{(1+\pi_t)}$$

So we have

$$g_t + \left(1 + \overline{r}_{t-1} - \frac{1}{1+\pi_t}\right) b_{t-1} = t_t + \left(b_t - \frac{b_{t-1}}{1+\pi_t}\right) + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

$$g_t + (1 + \overline{r}_{t-1})\, b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

[8] This section draws on Walsh [2010], Chapter 4.
[9] Small case letters denotes the variables in real terms.

So now we have the budget constraint in real terms and its interpretation is easier. Crucially for our purposes money creation, ΔH, yields real resources by the value to the extent that h increases and higher inflation erodes the last term, which as it is negative implies that it makes more resources available to the government. The logic is that inflation erodes the value of money and as this is a liability of the central bank (and hence the government) we can think of inflation as a tax. More inflation allows more expenditure just like t does.

We can gain further insights if we also define the ex-ante real interest rate (this is the one you will be most familiar with), r_t. This is given by:

$$(1+r_{t-1}) = \frac{1+i_{t-1}}{1+\pi_t^e}$$

So

$$1+i_{t-1} = (1+r_{t-1})(1+\pi_t^e)$$

Using these in the equation above we have:

$$g_t + \frac{(1+i_{t-1})}{(1+\pi_t)}b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

$$g_t + \frac{(1+r_{t-1})(1+\pi_t^e)}{(1+\pi_t)}b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

The only thing we are going to modify now is the coefficient on b_{t-1} (you'll see why at the end):

$$g_t + \frac{(1+r_{t-1})}{(1+\pi_t)}b_{t-1} + \frac{(1+r_{t-1})\pi_t^e}{(1+\pi_t)}b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

$$g_t + \frac{(1+r_{t-1})}{(1+\pi_t)}b_{t-1} + \frac{(1+r_{t-1})\left(\pi_t^e - \pi_t + \pi_t\right)}{(1+\pi_t)}b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

$$g_t + \frac{(1+r_{t-1})}{(1+\pi_t)}b_{t-1} + \frac{(1+r_{t-1})\pi_t}{(1+\pi_t)}b_{t-1} + \frac{(1+r_{t-1})(\pi_t^e - \pi_t)}{(1+\pi_t)}b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

Collecting terms we have:

$$g_t + \frac{(1+r_{t-1})(1+\pi_t)}{(1+\pi_t)}b_{t-1} + \frac{(1+r_{t-1})(\pi_t^e - \pi_t)}{(1+\pi_t)}b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

$$g_t + (1+r_{t-1})b_{t-1} + \frac{(1+r_{t-1})(\pi_t^e - \pi_t)}{(1+\pi_t)}b_{t-1} = t_t + b_t + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right)$$

Simplifying we have:

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right) + \left(\frac{\pi_t - \pi_t^e}{1+\pi_t}\right)(1+r_{t-1})\,b_{t-1} \tag{4.45}$$

We have spent a lot of time playing around with the budget constraint and the reason is that now we can infer additional insights from the constraint as it is represented in (4.45). First, unanticipated inflation is a source of revenue, that is the last term on the right hand side. One way of thinking about this is that it erodes the value of sovereign debt and could therefore be treated as a tax on bond holdings. However, even if all inflation is anticipated the inflation still generates revenue from money creation, seigniorage. That is the third term on the right hand side. Calling seigniorage s we have:

$$s_t = h_t - \frac{h_{t-1}}{(1+\pi_t)}$$

We can manipulate this slightly:

$$s_t \equiv \frac{H_t - H_{t-1}}{P_t} = h_t - \frac{H_{t-1}}{P_t}$$

$$= h_t - \frac{H_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} = h_t - h_{t-1}\left(\frac{1}{1+\pi_t}\right) =$$

$$= \Delta h_t + \left(1 - \frac{1}{1+\pi_t}\right)h_{t-1}$$

$$= \Delta h_t + \left(\frac{\pi_t}{1+\pi_t}\right)h_{t-1}$$

The right hand side tells us that seigniorage is made up of two sources. The first one, Δh is the change in real money balances (money in purchasing power terms). As the government is the monopoly supplier of monetary base its supply is a source of revenue. However, in our models with no long run output growth h will be a constant in steady state.[10]

If in steady state h is constant the first term on the right hand side of the equation above is zero. Hence, in the long run this tells us that the revenue from s is just

$$\frac{\pi}{1+\pi}h$$

[10] Steady state is just an equilibrium concept. It is the long run solution to all the models we consider. In very simple models all the variables are fixed at their equilibrium values, that's what you are used to. However, in dynamic models, say, one with inflation, the idea some variables are allowed to grow at a constant rate. Bear in mind that it would only make sense for some specific variable to grow in the long run, such as output and prices (therefore giving inflation), but we could not have inflation growing over time.

Turning to the creation of the monetary base, this is implemented by monetary policy. If we assume that H grows at some rate θ, then this implies that the growth of h is

$$h_t/h_{t-1} - 1 = \frac{H_t/P_t}{H_{t-1}/P_{t-1}} - 1 = \frac{H_t/H_{t-1}}{P_t/P_{t-1}} - 1 = \frac{1+\theta}{1+\pi_t} - 1 = \frac{\theta_t - \pi_t}{1+\pi_t}$$

Since in the long run we had that h is constant the last expression must equal zero. In other words, $\pi = \theta$. Inflation is caused by money creation.

4.5.2 Unpleasant monetarist arithmetic

Now we consider one implication of the government's budget constraint on fiscal-monetary policy coordination. The idea is this: can a central bank reduce the inflation rate (remember that it is driven by s) by implementing a contractionary monetary policy regardless of what the fiscal authority is doing?

Let us return to equation (4.45), ignoring surprise inflation we have:

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + \left(h_t - \frac{h_{t-1}}{1+\pi_t}\right) \tag{4.46}$$

Or:

$$g_t + r_{t-1}b_{t-1} = t_t + (b_t - b_{t-1}) + s_t \tag{4.47}$$

To highlight the point, assume that interest rates are positive and constant, $r_t = r > 0$. Now we have:

$$(1 + r_{t-1})\, b_{t-1} = b_t + (t_t - g_t) + s_t$$

If that is the equation for b_{t-1}, the same thing would apply to b_t, which could then be substituted into the above. Iterating this way we end up with

$$(1+r)\, b_{t-1} = -\sum_{i=0}^{\infty} \frac{g_{t+i} - t_{t+i}}{(1+r)^i} + \sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} + \lim \frac{b_{t+i}}{(1+r)^i} \tag{4.48}$$

Note that if we rule out explosive paths the last term is just zero, and that the left hand side was chosen in the previous period. If we just re-write this with s on the left hand side we have:

$$\sum_{i=0}^{\infty} \frac{s_{t+i}}{(1+r)^i} = (1+r)\, b_{t-1} + \sum_{i=0}^{\infty} \frac{g_{t+i} - t_{t+i}}{(1+r)^i} \tag{4.49}$$

The next part is the intuition and it will be quite easy. Given that the first term on the right hand side is fixed (at time t, anyway), if the fiscal authorities fix the paths of g and t exogenously then we know the value of the right hand side, no matter what the central bank does. If that is the case, as this is a constraint and both sides have to be equal to each other, it pins down the left hand side. If does not fix a particular value of s at some specific date, but it fixes the sum. So turning to our question: if the central bank contracts the growth rate of the money supply today but the fiscal authorities work independently, then the left hand side of the equation above will be unchanged. A decrease in s_t will have to be matched by a corresponding increase in future s. A monetary contraction today can cause an increase in inflation tomorrow! That's the key insight in Sargent and Wallace [1981].

4.5.3 Solutions to the problems raised by Sargent and Wallace [1981]

The results above highlight how important coordination between the fiscal and monetary authorities is. In principle, a lack of proper coordination between the two sets of policy makers can give rise to a conflict between them. Typically, this analysis is framed in the context of a 'responsible' central bank concerned with low inflation and an irresponsible government that has ambitious plans for fiscal policy – due to, for example, its desire to get re-elected. The interactions between the two agents are often explored using game theory.

We could determine the alternative outcomes that may arise is via changing the institutional assumptions in terms of who goes first. If the central bank acts as the Stackelberg leader we can end up with both policy makers acting responsibly – this is the dominant monetary policy solution analysed by Sargent and Wallace [1981]. By contrast, if the government is the Stackelberg leader then both agents act irresponsibly – we have an expansionary fiscal policy and this is accommodated by the central bank. Clearly, the latter case leads to inferior outcomes and a large literature has emerged discussing mechanisms to ensure that it is the former solution that prevails.

One way of ensuring that the central bank acts as the Stackelberg leader is to provide the central bank with a stronger monetary commitment. This would in principle not only generate better policy on the part of the central bank but also lead the fiscal authorities toward a less expansionary (irresponsible) policy. Some preliminary and supportive analysis on this issue is provided by **?**. As they point out, even an independent central bank with an inflation target may not be sufficient. In these cases we need to ensure fiscal commitment via the legislation of fiscal rules.

A further introductory article that discusses some of these issues is **?**. For example, the cite they case of New Zealand, one of the pioneers of inflation targeting. In their case, a planned fiscal expansion – under an independent and inflation targeting central bank – led the central banker not only to criticise such a policy in public.

However, note that in principle if both policy makers coordinate their policies with the aim of achieving common objectives the outcome will be superior. For example, during a crisis the central bank may implement an accommodating policy towards the fiscal authority but this will later revert back to the case where the central bank is the Stackelberg leader.[11] Such an arrangement would lead to superior outcomes. But if the central bank is playing a passive role during the crisis it is putting its independence at risk and there we be no guarantee that after the crisis is over that it can retain its role as Stackelberg leader during the normal times.

[11] One could argue that this is what has been occurring in both the UK and the US in recent years.

4.6 Application: the Gold Standard

The gold standard as a monetary rule.

Consider the following log-linear model of an economy in which gold is held as an asset:

$$m_t^d = p_t + y_t - a_1 \left(E_t p_{t+1} - p_t\right) - a_2 \left(E_t q_{t+1} - q_t\right) + \eta_t \tag{4.50}$$

$$m_t^s = \lambda + g_t^m + q_t + \theta_t \tag{4.51}$$

$$m_t^s = m_t^d \tag{4.52}$$

Where p_t is the price level and q_t is the price of gold; both are measured in terms of the numeraire (pound). η and θ are white noise processes and m is the money stock. g^m is the stock of monetary gold (the monetary base) and y_t is real output.

The stock demand for non-monetary gold is given by

$$g_t^{nm} = b_1 \left(p_t - q_t\right) - b_2 \left[\left(E_t p_{t+1} - p_t\right) - \left(E_t q_{t+1} - q_t\right)\right] + u_t \tag{4.53}$$

and

$$g_t^m = g - g_t^{nm} \tag{4.54}$$

Where g is the total stock of gold, assumed to be constant, and u is also a white noise process.

Lastly, assume that the supply of output is given by

$$y_t = h \left(p_t - E_{t-1} p_t\right) + v_t \tag{4.55}$$

The logic behind the equations is as follows. Equation (4.50) is our demand for money equation, which depends positively on income, as we would expect (here the income elasticity of money demand is one). The coefficient on the price level is one so that it is an equation for real money balances. Our money demand equation also depends negatively on the inflation rate as we would expect. If the real interest rate is a constant, then the nominal rate moves one-for-one with expected inflation; that is what this equatioin is picking up and recall that the return on money is minus the inflation rate. Lastly, the return on holding gold as an asset is its expected increase in value: this is picked up by the $E_t \Delta q_{t+1}$. As gold and money are substitutes, expected increases in the value of gold reduce the demand for money.

Equation (4.51) limits the money supply: apart from a constant, it equals the amount of gold plus its price (recall that the model is log-linear). This ensures the convertibility of paper money for gold. One of the reasons for the popularity in some circles of the gold standard is that it is a device to ensure commitment by the monetary authority: gold is the nominal anchor. Equation (4.53) can be thought of as the demand equation for any good. It depends negatively on its price relative to other goods $(q-p)$, but current demand is a positive function of expected increases in prices: if we expect something to become more expensive in the next period, we demand more of it in the present. Lastly, (4.54) simply states that the total supply of gold is fixed so that greater use for one role implies less for the other.

4.6.1 A gold standard monetary policy

Let us assume that monetary policy is based on fixing the price of gold: $q_t = 0 \quad \forall t$.

What would the consequences be for output and inflation? To answer this question, we must solve the model using the techniques we are already familiar with. Given that q is fixed our model becomes

$$\lambda + g_t^m + \theta_t = p_t + y_t - a_1\left(E_t p_{t+1} - p_t\right) + \eta_t$$

$$g_t^m = g - \left[b_1 p_t - b_2\left(E_t p_{t+1} - p_t\right) + u_t\right]$$

$$y_t = h\left(p_t - E_{t-1} p_t\right) + v_t$$

Simplifying:

$$\lambda + g - \left[b_1 p_t - b_2\left(E_t p_{t+1} - p_t\right) + u_t\right] + \theta_t = p_t + y_t - a_1\left(E_t p_{t+1} - p_t\right) + \eta_t$$

then

$$\lambda + g - \left[b_1 p_t - b_2\left(E_t p_{t+1} - p_t\right) + u_t\right] + \theta_t = p_t + h\left(p_t - E_{t-1} p_t\right) + v_t - a_1\left(E_t p_{t+1} - p_t\right) + \eta_t$$

This is a linear rational expectations model. Our msv solution is

$$p_t = \bar{p} + \delta_1 u_t + \delta_2 \theta_t + \delta_3 v_t + \delta_4 \eta_t$$

This implies that

$$E_{t-1} p_t = \bar{p}$$

$$E_t p_{t+1} = \bar{p}$$

We can substitute these into our equation above, although I shall simplify it first. We have

$$\lambda + g - (1+b_1)\,p_t + b_2\,(E_t p_{t+1} - p_t) - u_t + \theta_t = h\,(p_t - E_{t-1}p_t) + v_t - a_1\,(E_t p_{t+1} - p_t) + \eta_t$$

$$\lambda + g - (1+b_1)\,(\bar{p} + \delta_1 u_t + \delta_2\theta_t + \delta_3 v_t + \delta_4\eta_t) - b_2\,(\delta_1 u_t + \delta_2\theta_t + \delta_3 v_t + \delta_4\eta_t) - u_t + \theta_t$$
$$= h\,(\delta_1 u_t + \delta_2\theta_t + \delta_3 v_t + \delta_4\eta_t) + v_t + a_1\,(\delta_1 u_t + \delta_2\theta_t + \delta_3 v_t + \delta_4\eta_t) + \eta_t$$

This gives:

$$\bar{p} = \frac{\lambda + g}{1 + b_1}$$

For u_t:

$$-(1+b_1)\delta_1 - b_2\delta_1 - 1 = h\delta_1 + a_1\delta_1$$

so that

$$\delta_1 = -\frac{1}{1 + b_1 + b_2 + h + a_1}$$

Next, for θ_t:

$$-(1+b_1)\delta_2 - b_2\delta_2 + 1 = h\delta_2 + a_1\delta_2$$

Giving:

$$\delta_2 = \frac{1}{h + a_1 + 1 + b_1 + b_2}$$

For v_t:

$$-(1+b_1)\delta_3 - b_2\delta_3 = h\delta_3 + 1 + a_1\delta_3$$

thus

$$\delta_3 = -\frac{1}{1 + b_1 + b_2 + h + a_1}$$

Lastly, for η_t:

$$-(1+b_1)\delta_4 - b_2\delta_4 = h\delta_4 + a_1\delta_4 + 1$$

so that

$$\delta_4 = -\frac{1}{1 + b_1 + b_2 + h + a_1}$$

Our solution for prices is therefore

$$p_t = \frac{\lambda + g}{1 + b_1} + \frac{\theta_t - u_t - v_t - \eta_t}{1 + b_1 + b_2 + h + a_1} \tag{4.56}$$

So for output we have:

$$y_t = h\left(\frac{\theta_t - u_t - v_t - \eta_t}{1 + b_1 + b_2 + h + a_1}\right) + v_t$$

Giving

$$y_t = \frac{h(\theta_t - u_t - \eta_t) + (1 + b_1 + b_2 + a_1)v_t}{1 + b_1 + b_2 + h + a_1} \tag{4.57}$$

For future reference, note that this equation can be re-written as:

$$y_t = \frac{h(\theta_t - u_t - \eta_t)}{1 + b_1 + b_2 + h + a_1} + \frac{1 + b_1 + b_2 + a_1}{1 + b_1 + b_2 + h + a_1}v_t \tag{4.58}$$

These two equations give us the equilibrium behaviour of prices and inflation in our simple model under a gold standard monetary policy. In isolation it is not clear what it might imply or what we can infer from the results. Imagine, therefore, that we compare it to a policy where the central bank uses monetary policy to fully stabilise the price level: it can successfully fix $p_t = \bar{p}\ \forall t$, by allowing q_t to vary. Obviously, this means that prices will be fixed (for comparison purposes, assume it's the same $\bar{p}$ that we obtained above). What are the consequences for output?

This is very simple, from the Phillips curve, if the policy maker is able to fully stabilise prices and is thus expected to do so, we have

$$y_t = v_t \tag{4.59}$$

The variance of output can be calculated from the two equilibrium solutions for output very easily. Crucially, under the gold standard all four shocks affect output; under our price-stability policy only the Phillips curve shock drive the volatility of output. Moreover, the coefficient on v_t equals one under price stability while its value will be less than one under the gold standard. This implies that (they are different ways of saying the same thing) the gold standard will be best only when

- Output movements are primarily driven by v shocks and the others are trivial (very low volatilities)
- Output is hardly responsive to price surprises (almost vertical Phillips curve)

In addition, if society also cares about price stability, then this makes the gold standard even less appealing.

The discussion above was based on the assumption that both policies would deliver the same average price level so that they imply the same

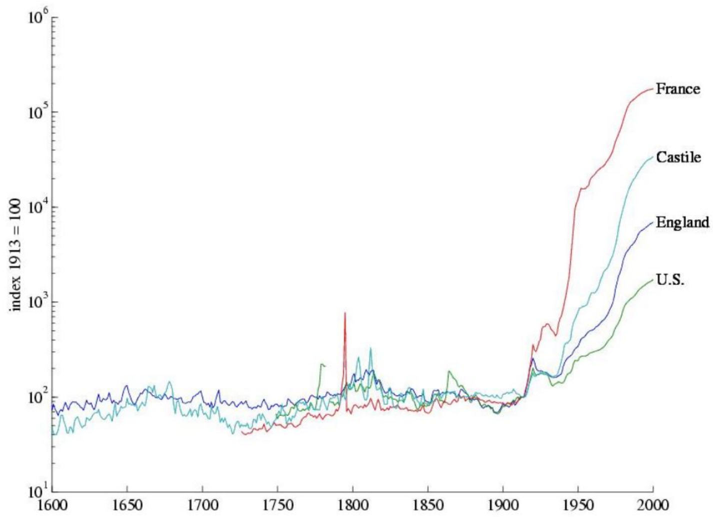

Figure 4.1: Price levels during and after the gold standard (Sargent)

long-run behaviour, but differed in terms of short-run behaviour. One key feature of the period during the gold standard is that prices were remarkably stable but after it was abandoned countries experienced sustained periods of inflation (see Figure (4.1). The reason can be attributed to the gold standard being a commitment device, restricting the actions of monetary policy makers, whereas when it was abandoned monetary policy did not possess a nominal anchor: unlike a fixed money supply/exchange rate policy, or inflation targeting, there was no clear nominal objective for monetary policy.

4.7 Application: Nominal income targeting

The idea of a central bank targeting a level or growth rate for nominal income is decades long and was often discussed during the Great Recession. Here we shall use a simple model to see how it would fare:

$$m_t + v_t = p_t + y_t \tag{4.60}$$

$$v_t = \alpha_1 i_t + \alpha_2 y_t + \epsilon_t \tag{4.61}$$

$$y_t = -\alpha_3 \left[i_t - (E_t p_{t+1} - p_t) \right] \tag{4.62}$$

$$p_t = E_{t-1}p_t + \alpha_4 y_t + \eta_t \tag{4.63}$$

The first equation just defines velocity, while the second equation gives us a theory of what is driving it. Under a fixed nominal income rule we have that $p_t + y_t = A$, a constant. The value of this constant is not that important as we are not really considering whether nominal income targeting (NIT) produces more or less inflation than an alternative policy; rather, we are focusing on the effects on business cycles. Hence, we shall let $A = 0$.

Given this we have

$$p_t + y_t = 0$$

and the Phillips curve.

This implies that $E_{t-1}p_t = 0$ so we can easily solve for prices and output, giving

$$y_t = -\frac{1}{1+\alpha_4}\eta_t$$

$$p_t = \frac{1}{1+\alpha_4}\eta_t$$

Hence, according to this model under the gold standard only supply-side (PC) shocks affect output, while those to the IS-LM (aggregate demand) do not. Moreover, inflation and output share the same volatility. One way of thinking about NIT is to consider a growing economy so that the target is in terms of the growth in nominal income. Then, if there is an economic slowdown so that the growth of output is lower than the target, the central bank aims for a higher inflation target, so that $\pi_t + \Delta y_t = A$, with the right hand side being the NIT growth rate. This is in contrast to inflation targeting (IT), where the primary aim is solely in terms of inflation.

Now, let us compare the performance of NIT in the economy above with one where the central bank sets policy by fixing the money supply. This would have followed Friedman's recommendation and is consistent with the IS-LM model as taught in introductory courses. This policy implies that $m_t = \bar{m}$.[a]

The first equation becomes

$$\bar{m} + \alpha_1 i_t + \alpha_2 y_t + \epsilon_t$$

If $\bar{m} = 0$, our model does not contain lags or constants. Thus

$$E_{t-1}p_t = E_t p_{t+1} = 0$$

Combining the equations we obtain

$$y_t = \frac{\alpha_3 \left[\epsilon_t - (1+\alpha_1)\eta_t\right]}{\alpha_3 \left[1 - \alpha_2 + \alpha_4(1+\alpha_1)\right] + \alpha_1}$$

To solve for prices we use this result in the Phillips curve:

$$p_t = \alpha_4 y_t + \eta_t$$

$$p_t = \frac{\alpha_3 \alpha_4 \epsilon_t + \alpha_3(1+\alpha_2)\eta_t}{\alpha_3 \left[1 - \alpha_2 + \alpha_4(1+\alpha_1)\right] + \alpha_1}$$

The results above indicate that both shocks now lead to business cycles and fluctuations in prices. How do we compare this to NIT? Firstly, it the volatility of ϵ_t is sufficiently large, this will make η_t relatively unimportant as a source of business cycles. In that case NIT is clearly preferable. However, in the reverse case the conclusion will depend on the coefficient in front of η_t under each alternative policy and for both prices and output.

[a] Again, we can set $\bar{m}$ to ensure that the average price level coincides with that under NIT for comparability.

Chapter 5

Determinacy in Rational Expectations Models: the Cagan model

One characteristic of forward-looking models is that we may end up with non-unique solutions.

5.1 Models with expectations of future variables

The Cagan model (Cagan [1956]) is one most influential papers in macroeconomics. It considered the relationship between changes in the quantity of money and the price level during hyperinflations. The countries he considered – Austria, Germany, Greece, Hungary, Poland and Russia – all experienced hyperinflationary episodes after the end of the First World War.[1] Although we think of hyperinflations (and also high inflation) as being driven by monetary policy, at a deeper level such events often have fiscal underlying causes: it is the fiscal authority's need for revenue that results in rapid money creation.

Although **?** used adaptive expectations we are going to use the rational expectations version of the same model, following Sargent and Wallace [1973]. The starting point is a standard money demand equation where the demand for real money balances depends positively on output and negatively on the nominal interest rate. As the objective in the Cagan model was to consider hyperinflations, we can assume that output and the real interest rates are fixed.[2] Making use of the Fisher equation, our money demand equation can be written as

[1]While the Russian Revolution is taught as having taken place in October 1917 this does not imply that after this date the Bolsheviks had full control of the country.

[2]In other words, money and prices are rising so rapidly that any movement in real variables is trivial by comparison.

$$m_t - p_t = -\eta \left(E_t p_{t+1} - p_t \right) \tag{5.1}$$

We can re-write this equation as

$$p_t = \alpha E_t p_{t+1} + (1-\alpha) m_t \tag{5.2}$$

Where $\alpha = \eta/(1+\eta)$.

The results we are going to discuss pertain to the general class of models where we have

$$y_t = \alpha E_t y_{t+1} + \beta x_t$$

The Cagan model therefore belongs to this category. We are going to solve this model in an non-MSV manner to understand the concept of bubbles but also because solving forward yields many insights. Nonetheless, we shall also use the MSV approach at the end of this section.

5.1.1 Solving when $\alpha < 1$

We can iterate equation (5.2) forward until period T:

$$p_t = (1-\alpha) \sum_{j=0}^{T-1} \alpha^j E_t m_{t+j} + \alpha^T E_t p_{t+T} \tag{5.3}$$

Notice on important feature: as the price level is a forward-looking variable, agents need to formulate beliefs regarding the behaviour of monetary policy. In other words, to solve for the price level we need an equation that describes the behaviour of m_t.

Ruling out bubbles means that for the last term on the right hand side we have

$$\lim_{T \to \infty} \alpha^T E_t p_{t+T} = 0$$

so that our bubble-free, or fundamental, solution for prices ($\overline{p}_t$) is given by

$$\overline{p}_t = (1-\alpha) \sum_{j=0}^{T} \alpha^j E_t m_{t+j} \tag{5.4}$$

It is called fundamental as the behaviour of prices is solely determined by economic (model-relevant) variables. This is consistent with the MSV solution. Note that this equation implies (we shall need it later):

$$\overline{p}_t = (1-\alpha) m_t + \alpha E_t \overline{p}_{t+1} \tag{5.5}$$

5.1.2 The solution with bubbles

We can define bubbles as the discrepancy between an actual variable and its fundamental value. Hence, in our case the bubble $-b_t$ – is given by

$$b_t = p_t - \overline{p}_t$$

It turns out that the process

$$b_t = \alpha E_t b_{t+1}$$

Is consistent with the model (hence it is a rational bubble).[3] To see this, note use

$$p_t = b_t + \overline{p}_t$$

Into equation (**??**) and we have[4]

$$p_t = \alpha E_t p_{t+1} + (1-\alpha)m_t$$

$$b_t + \overline{p}_t = \alpha E_t \left(b_{t+1} + \overline{p}_{t+1}\right) + (1-\alpha)m_t$$

Re-write as

$$b_t + \overline{p}_t = \alpha E_t b_{t+1} + \alpha E_t \overline{p}_{t+1} + (1-\alpha)m_t$$

That that this is consistent with everything we have stated above. As long as the process for the bubble follows the equation described earlier, the equation for prices:

$$p_t = \overline{p}_t + b_t$$

Is our bubble solution (after substituting the solutions for fundamental prices and bubbles). An example of b consistent with the above is

$$b_t = \frac{z_t}{\alpha^t}$$

with

$$z_t = z_{t-1} + \epsilon_t$$

z being any exogenous variable that follows the process above. A key result is that the process with the bubbles as described above implies an explosive path for prices.

Solving using the MSV method is the same as imposing the bubble-free solution. Sometimes bubbles can be rules out because they violate other

[3] An example is provided below.

[4] Note that I make use of the law of iterated expectations: $E_{t-k}\left(E_t x_{t+k}\right) = E_{t-k}x_{t+k}$. Or in words: 'what I expected yesterday that I expect today about tomorrow is what I expected yesterday about tomorrow'.

conditions of the model, for example, that agents cannot borrow indefinitely. However, such reasoning is not always possible, such as when the bubbles lead to explosive solutions for prices but not for real variables.

Another way of viewing our results above is that with $\alpha < 1$ all paths but one are explosive. If we can therefore rule out those explosive paths then we have a unique rational expectations equilibrium.

The idea behind the Taylor principle in the Taylor rule is also based on this. The Taylor principle means that only one path for inflation is consistent with the (New Keynesian) model.

5.1.3 Solution when $\alpha < 1$

Now a different, but less common, problem arises. To see what this implies here is a simple example:

$$y_t = \alpha E_t y_{t+1} + \beta x_t$$

$$x_t = \overline{x} + \epsilon_t$$

Our MSV solution is

$$y_t = \lambda_0 + \lambda_1 \epsilon_t$$

giving us

$$\tilde{y}_t = \frac{\beta}{1-\alpha}\overline{x} + \beta \epsilon_t$$

We could think of this as the solution to our model. However, this solution is not unique (hence the tilde to point out that this is 'one' solution). An alternative solution is

$$y_t = \tilde{y}_t + b_t \tag{5.6}$$

with $b_t = \alpha E_t b_{t+1}$. If we substitute this alternative solution into our earlier equation for y we shall see that it holds:

$$y_t = \alpha E_t y_{t+1} + \beta x_t$$

$$\tilde{y}_t + b_t = \alpha \left(\lambda_0 + E_t b_{t+1}\right) + \beta x_t$$

$$\frac{\beta}{1-\alpha}\overline{x} + \beta \epsilon_t \ + b_t = \alpha \left(\lambda_0 + E_t b_{t+1}\right) + \beta x_t$$

$$\frac{\beta}{1-\alpha}\overline{x} + \beta \epsilon_t \ + b_t = \alpha \left(\frac{\beta}{1-\alpha}\overline{x} + E_t b_{t+1}\right) + \beta \left(\overline{x} + \epsilon_t\right)$$

Both sides of the equation are equal to each other so again our solution for y in (5.6) with the process for b as described above is valid. Unlike the previous case where there was a unique non-explosive solution, because the process for b is non-explosive we have an infinite number of stable solutions, each one given by any process b, hence their name of 'sunspot equilibria'.

5.2 General specification

More generally, if our rational expectations model is written as

$$A \begin{bmatrix} X_{t+1} \\ E_t Y_{t+1} \end{bmatrix} = B \begin{bmatrix} X_t \\ Y_t \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ 0 \end{bmatrix} \tag{5.7}$$

Where X_t represents pre-determined variables (there are n_1 of them) and Y_t are 'jump' variables (there are n_2 of them). We can re-write this as

$$\begin{bmatrix} X_{t+1} \\ E_t Y_{t+1} \end{bmatrix} = C \begin{bmatrix} X_t \\ Y_t \end{bmatrix} + A^{-1} \begin{bmatrix} \epsilon_{t+1} \\ 0 \end{bmatrix} \tag{5.8}$$

If we let $C = Q^{-1} \Lambda Q$, with the eigenvalues in *Lambda* ordered from smallest to largest, for a unique rational expectations equilibrium we require the first n_1 (last n_2) eigenvalues to be inside (outside) the unit circle.

Put differently, we need the number of stable roots to be equal to the number of predetermined variables and the number of unstable (explosive) roots to be equal to the number of jump variables.

5.3 Solving the Cagan model using the MSV approach

The equation we have above was

$$p_t = \alpha E_t p_{t+1} + (1-\alpha) m_t$$

As noted above, in order to solve the model, we need an equation describing the money supply. Assume that it is given by

$$m_t = \bar{m} + \varepsilon_t$$

Where ε_t is a shock.

Under the MSV approach

$$p_t = \lambda_0 + \lambda_1 \varepsilon_t$$

$$E_t p_{t+1} = \lambda_0$$

Combining:

$$(1-\alpha)\lambda_0 + \lambda_1 \varepsilon_t = (1-\alpha)\left(\bar{m} + \varepsilon_t\right)$$

The last step is left as an exercise.

5.4 Summary

We have considered some of the problems that arise when we have rational expectations of future variables. Our MSV solution method is therefore based on the first case and we rule out explosive solutions. If we had backward looking variables then the criteria would be reversed (for uniqueness we then require $\alpha < 1$).

It is worth noting that when we have several endogenous variables in our model then the equivalent criterion is in terms of eigenvalues. If all y the variables are forward looking then the matrix α would have to have all eigenvalues less than one. In many models it turns out that some variables will be forward looking and others backward looking (such as the capital stock). When this is the case, then the requirement is that the number of eigenvalues of α less than one be equal to the number of 'jump' or forward looking variables.

When using Dynare, if the conditions above are not satisfied the programme stops with the error message 'Blanchard-Kahn conditions not satisfied'.

Chapter 6

Time Inconsistency

6.1 Introduction

One of the key results of the rational expectations revolution is that central banks cannot easily fool people. In fact, any systematic policy will always be known by private agents and they will take this into account when forming expectations and only the random (unpredictable) component of policy will not affect real variables.[1]. An important line of research has analysed contexts where central banks aim to maximise social welfare and in order to achieve this they have an incentive to make policy announcements/promises which they will then renege on. This can be applied to a variety of settings, but the most famous application originated in a paper by Kydland and Prescott (1977).

6.2 The model

To understand the workings of the model it will be best to analyse each agent separately. A good textbook treatment of this topic is Walsh [2017].

Private agents form their expectations rationally. In addition, firms each period set the prices of their goods before the period begins. That is, for period t, they will set their prices at the end of period $t-1$. This means that when they do so they will know everything that has happened up to (and including) $t-1$. Remember that under fully flexible prices (think of the Classical model) output is always equal to potential and prices are always equal to expected prices. Here, because prices are fixed for only a very short period, the model will be similar, with the exception that shocks (or surprises) that occur in period t were not taken into account by firms when they set their prices for that period. Consequently, since prices cannot react it will be output (the real side of the 1 Models were later adapted to

[1]Models were later adapted to allow for this result to be watered down, but to some extent it remains valid

allow for this result to be watered down, but to some extent it remains valid. economy) that will absorb the shock. We can write the supply side (Phillips curve) under this formulation as:

$$y_t = y^* + \alpha\left(\pi_t - E_{t-1}\pi_t\right) \tag{6.1}$$

We have used such a Phillips curve (PC) in earlier topics on solving RE models. Note that whenever expectations are correct: $\pi_t = E_{t-1}\pi_t$, so that $y_t = y^*$. In other words, y^* denotes potential output (the level of output when all markets clear and expected prices equal actual prices). In addition, (1) gives a positive relationship between inflation and output, so that plotting the PC in a diagram the slope would be given by $1/\alpha^2$.

Any shock/event that happened in $t-1$ or earlier will not affect current output, as it is included in the expectations. In other words, this is a New Classical Phillips curve. An important element in the model above which is not always obvious is that firms set prices for one period. Remember that under perfect competition all agents, including firms, are price takers. Consequently, they would never have the market power to set the prices of their product. This means that we are dealing with a model with some degree of monopolistic competition. Whilst knowledge of this does not change the results in any way there is one aspect of this which is important: under monopoly/monopolistic competition, prices (output) are higher (lower) than under perfect competition. Since perfect competition delivers the maximum social welfare, we can say that under the model in (6.1), y^* is inefficiently low. This will be important for some of the results below.[2] The policy maker, much as we did when analysing the optimal choice of monetary instrument aims to maximise an objective function. We will assume that the policy maker (central bank) wants to maximise social welfare, which is given by:

$$L = -\frac{1}{2}\left(\pi_t - \pi^*\right)^2 - \frac{1}{2}\left(y_t - ky^*\right)^2 \tag{6.2}$$

Each component has a negative sign in front, indicating that the objective is to minimise it. In other words, the Bliss point, where maximum welfare (and minimum loss) is achieved is when $\pi = \pi^*$ and $y = ky^*$. One can then think of π^* as the optimal level of inflation (e.g. the inflation target); whilst the optimal amount of output is higher than the natural rate y^* . The reason for this latter result is linked to the earlier paragraph. As the natural rate is inefficiently low, society would prefer the level of output that could be achieved under perfect competition (let us call this the perfect competition level of output). In order to keep the model as simple as possible

[2] Remember: in (6.2) we have three variables (expected inflation is the third one), so when plotting we are assuming that the third variable is exogenous, i.e., when it changes it causes shifts in the PC.

we will assume that the central bank (the only policy maker in this setup) is able to control the inflation rate directly. Remember that this is a short cut. If the central bank uses interest rates as its monetary policy instrument we would need an IS equation (and if it cannot control the money supply directly, an additional equation linking it to the monetary base). However, be aware that we could make the set up more realistic but all it would add is complexity. Consequently, each period the central bank chooses the inflation rate and plugging this into the PC (subject to the value of expected inflation) we can find out the value of output. Before we attempt to solve the model it will be best to try to use our intuition in order to understand what the implications are:

- Agents have rational expectations (and remember that expectations will adjust in response to any policy change). Moreover, as long as expectations are correct output will equal the natural level of output, and not its perfect competition level;

- this affects expectations of inflation at t-1. Having announced the inflation rate, the private sector then forms its expectations;

- period t begins with agents having already formed their expectations, and the central bank then chooses the actual inflation rate it will deliver.

Let us assume that the central bank's announced inflation is believed. Given the setup, it makes sense for the announced inflation rate to be $\pi = \pi^*$, as this is the one that minimises the loss function (6.2). As it is believed, expected inflation will also equal this rate. Next, period t begins with $E_{t-1}\pi_t = \pi^*$.

But is it optimal for the central bank to follow through with its announcement? Doing so would deliver the solution that is generally termed 'under commitment'. That is, the central bank makes announcements and fulfils them. In fact it will be best for the central bank to renege on its promise. With expectations already set at a low level, it is optimal to cheat a little in order to bring output a little closer to its perfect competition level. Whilst this increases the loss in (6.2) because inflation is higher than announced, it is on the hand lower because the second part of (6.2), which contains output, is sufficiently lower to compensate. Hence, overall the losses are minimised by cheating. That is the source of the time inconsistency: before period t begins, it is optimal for the central bank to announce a policy and to stick to it. However, after the period has begun, this is no longer optimal, hence it is inconsistent. One can think of many examples where such a setup arises. The government could announce that all capital taxes have been abolished. This will lead to a surge in investment and therefore the capital stock. However, after this capital stock is in place, it will be optimal to tax it! Similarly, let us say you will be tested on this topic a week from today. Having announced this (wishfully on my part) you will have learnt it before then. However, when the day of the test arrives, since I do not enjoy marking scripts and you have already learnt the material, I have no incentive to test you. Again, the gains from cheating outweigh those from commitment. However, the catch is that under rational expectations agents know what the central bank's incentives are. In other words, it knows that the central bank will renege on its promises. This implies that private agents will not expect the inflation rate as announced by the central bank. Instead as the central bank's incentive is to deliver an inflation rate that is higher than the announced one their expectations of inflation will be higher. How much higher? High enough until the central bank has no incentive to cheat. That is, when cheating (by increasing inflation a little than expected to get the gains from higher output) does not pay. This is the solution of the model under discretion. Because the central bank is unable to commit itself (the gains from cheating are too great) it ends up in the discretionary equilibrium. Here inflation is higher than π^* , but as agents have not been fooled, output will equal its natural level. Overall, this solution is worse than if it had committed. We can categorise the results so far as follows:

- the best solution from society's and the central bank's points of view is where the announcements are believed but reneged on. Inflation will be higher than the target, but output will be closer to its perfect competition level;
- the second best solution is the one where the central bank does not renege on its promises. Inflation equals its announced level but output equals its natural level;
- the third best solution is where the central bank has no credibility: private agents know it will cheat. Here inflation will be higher than in a) or b) but output will be equal to its natural level.

6.2.1 Solving the model mathematically

The central bank maximises its objective subject to the PC. Moreover, it takes expectations as exogenous. This means that it does not take into account the fact that its policies affect expectations. Setting up the Lagrangean:

$$L = -\frac{1}{2}(\pi_t - \pi^*)^2 - \frac{1}{2}b(y_t - ky^*)^2 + \lambda(y_t - y^* - \alpha(\pi_t - E_{t-1}\pi_t))$$

first order condition for inflation:

$$(\pi_t - \pi^*) = \alpha\lambda$$

first order condition for output:

$$b\left(y_t - ky^*\right) = -\lambda$$

Combining these two yields a relationship between output and inflation:

$$\left(\pi_t - \pi^*\right) = -\alpha b\left(y_t - ky^*\right) \tag{6.3}$$

We can think of (6.3) as the optimal monetary policy. That is, the central bank will adjust the inflation rate in order to ensure that (6.3) is satisfied. In order to determine what the effects of such a policy will be all we have to do next is to plug in the PC into (6.3). Substituting for output yields:

$$\left(\pi_t - \pi^*\right) = -\alpha b\left[y^* + \alpha\left(\pi_t - E_{t-1}\pi_t\right) - ky^*\right] \tag{6.4}$$

Assuming that the central bank's announcement that inflation will equal its target is believed implies $E_{t-1}\pi_t = \pi^*$. Using this in (6.4):

$$\left(\pi_t - \pi^*\right) = -\alpha b\left[y^* + \alpha\left(\pi_t - \pi^*\right) - ky^*\right]$$

Solve for inflation:

$$\pi_t = \frac{\alpha b(k-1)}{1+\alpha^2 b} y^* + \pi^* \tag{6.5}$$

As argued above, (6.5) clearly shows that once the central bank's announcement of low inflation is believed, actual inflation as implemented by the policy maker will be higher than this. To get the solution for output substitute (6.5) into the Phillips curve:

$$y_t = y^* + \alpha\left(\pi_t - \pi^*\right)$$

$$y_t = y^* + \alpha\left[\frac{\alpha b(k-1)}{1+\alpha^2 b} y^* + \pi^* - \pi^*\right]$$

$$y_t = \frac{1+\alpha^2 bk}{1+\alpha^2 b} y^* \tag{6.6}$$

Recall that $k > 1$ as a result of the distortions arising from monopolistic competition, so that if the policy maker cheats (and is believed) output will be above its natural level, which from a social point of view is too low. However, if agents have rational expectations the solution given by (6.5) and (6.6) will never occur. In fact, agents know that the policy maker will be maximising its objective and will therefore follow rule (**??**). To see the implications of this, substitute the policy maker's rule (**??**) into the Phillips curve in order to solve for inflation:

$$\pi_t - \pi^* = -\alpha b \left(y_t - k y^* \right)$$

$$\pi_t - \pi^* = -\alpha b \left[y^* + \alpha \left(\pi_t - E_{t-1}\pi_t \right) - k y^* \right]$$

$$\pi_t - \pi^* = -\alpha b (k-1) y^* - \alpha^2 b \left(\pi_t - E_{t-1}\pi_t \right)$$

Forming (rational) expectations of this equation gives[3]

$$\left(E_{t-1}\pi_t - \pi^* \right) = \alpha b (k-1) y^* - \alpha^2 b \left(E_{t-1}\pi_t - E_{t-1}\pi_t \right)$$

so

$$E_{t-1}\pi_t = \pi^* + \alpha b (k-1) y^* \tag{6.7}$$

The repercussions of this will become evident once we substitute (6.7) into the Phillips curve and (6.1). For the Phillips curve we have:

$$y_t = y^* + \alpha \left(\pi_t - E_{t-1}\pi_t \right)$$
$$y_t = y^* + \alpha \left(\pi_t - \pi^* + \alpha b (k-1) y^* \right)$$

$$y_t = \left[1 - \alpha^2 b (k-1) \right] y^* + \alpha \left(\pi_t - \pi^* \right)$$

Use this in (6.3) to then solve for the actual inflation rate under discretion:

$$\left(\pi_t - \pi^* \right) = -\alpha b \left(y_t - k y^* \right)$$

$$\pi_t - \pi^* = -\frac{\alpha b}{1 + \alpha^2 b} \left[1 - \alpha^2 b (k-1) - k \right] y^*$$

$$\pi_t = \pi^* + \alpha b (k-1) y^* \tag{6.8}$$

Which is consistent with (6.7), the private sector's expectation of inflation. So far, we have that under discretion inflation and its expectation are equal to each other and given by (6.7). This implies that output is equal to its natural level. Hence discretion yields higher inflation than is optimal without an offsetting gain in output.

Categorising the different equilibria in order of welfare, from best to worst:

The overall conclusion emanating from all this is that because of time inconsistency society ends up in an undesirable equilibrium. Is there anything that can be done to overcome this? I will only briefly discuss some of the potential solutions to the time inconsistency problem in monetary policy.

[3] Recall that the expectation of a constant is the constant itself.

Solutions	y_t	π_t
Cheating	$\frac{1+\alpha^2 bk}{1+\alpha^2 b} y^*$	$\frac{\alpha b(k-1)}{1+\alpha^2 b} y^* + \pi^*$
Commitment	y^*	π^*
Discretion	y^*	$\pi^* + \alpha b(k-1)y^*$

Table 6.1: Losses under alternative scenarios

6.3 The Conservative Central Banker

See Rogoff [1985]. Recall that the monetary authority was trying to maximise social welfare. That is, the agent in charge of choosing the inflation rate had the same welfare function as society at large, and this was where the problem arose. Consequently, a possible solution is to appoint a central banker who does not care about output at all! This would imply that the conservative central banker's preferences are given by:

$$L = -\frac{1}{2} (\pi_t - \pi^*)^2$$

Minimising this loss function is then easy: choose $\pi_t = \pi^*$. Since this is now credible (the central banker has no incentive to cheat), it will be believed and output will equal its natural level. Hence the commitment solution is achieved by appointing a central banker with preferences different from society's.

6.4 Reputation

See Barro and Gordon [1983] The time inconsistency problem above arose partly because the game is played only once. However, if such a problem is repeated for a large number of periods the solution may be more subtle. The government may be able to cheat today and get the gains in output. However, this would imply a loss of credibility that could last for a long period, in which inflation would be high but output would equal its natural level. The solution (what the government will do) will then depend on how much it cares about the present vs the future and how long the punishment will last, among other factors.

6.A Linear Contracts for Central Bankers

6.B Introduction

This brief note covers one potential solution to the problem of time inconsistency that has been widely covered in the literature.[4]. We have already

[4] For a textbook treatment see Walsh [2003]; this note is based on the book.

seen that one potential solution to the inflation bias involves delegating monetary policy to a conservative central banker (the Rogoff model). An alternative approach is to design a contract whereby the 'agent' (the central bank) is punished by the 'principal' (the government) whenever inflation deviates from its target. We can think of this as the central banker receiving an income (or deduction) bonus when she hits the inflation target.

6.C The model

We assume that the central banker's preferences are still the same as society's

$$L' = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + b\,(y_t - ky^*)^2\right]$$

However, she is now subject to an inflation contract meaning that the loss becomes

$$L = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + b\,(y_t - ky^*)^2\right] + t\pi_t$$

and the Phillips curve remains unchanged

$$y_t = y^* + \alpha\,(\pi_t - E_{t-1}\pi_t) + u_t$$

The principal (the government) sets t to alter the central banker's incentives so that her behaviour will not result in the inflation bias. This means that we choose the value of t that eliminates this bias. The implication then would be that the central bank could respond to shocks in order to minimise losses but without the desire to inflate the economy so that the solution for inflation will be of the form

$$\pi_t = \pi^* + \delta u_t$$

So that on average inflation equals its target but it will also respond to shocks (δ is as yet unknown).

Setting up the Lagrangean with the central banker's objective and solving gives

$$\pi_t = \pi^* + \alpha b(k-1)y^* - t - \frac{\alpha b}{1+\alpha^2 b}u_t$$

The government knows that this is the inflation rate that the central banker will set. The government's aim is to choose t to minimise its expected loss $E(L')$ since it appoints the central banker prior to observing the shocks. So what is the optimal inflation contract? It is the one that delivers $\pi_t = \pi^*$ on average. This means that t is chosen to equal

$$t = \alpha b(k-1)y^*$$

and hence the inflation bias is eliminated. This provides an alternative to the scheme where the government delegates monetary policy to a central banker with an inflation target lower than society's.

The resulting equilibrium outcomes for inflation is then

$$\pi_t = \pi^* - \frac{\alpha b}{1 + \alpha^2 b} u_t$$

(you can easily derive the one for output).

Chapter 7

Unpleasant Monetarist Arithmetic

This is a summary of the main insights in Sargent and Wallace [1981], henceforth SW, and I would urge you to read the original paper. The key contribution of the paper is to consider the interaction between monetary and fiscal policy – both branches of government – given that they are not really independent. In other words, when studying monetary policy, can we do this independently of what is occurring with fiscal policy? The answer is a clear no. Moreover, any attempts by a central bank to control inflation will eventually be fruitless if fiscal policy is not consistent with this.

In what follows, the notation will follow the original paper as much as possible. In my version there is no population growth, so we set $n = 0$.

The model is very monetarist:

1. A constant growth rate of output (we set it to zero).
2. A constant return on government bonds ($R_{t-1} = R$, although we shall impose this at the end).
3. A quantity theory demand for money so that there is a strong and direct link between the quantity of money and the price level (it has constant velocity).

A model like this embodies the limitations on monetary policy pointed out by Friedman: don't use monetary policy to control the economy.

7.1 The setup

The consolidated government budget constraint is given by (in real terms)

$$D(t) = \frac{\Delta H(t)}{P(t)} + B(t) - B(t-1)(1 + R(t-1)) \tag{7.1}$$

Where D is the primary deficit, H is the stock of base or high powered money and B is the stock of one-period bonds. R is the real interest rate. What this equation states is that in any given period the deficit can be financed by money creation ($\Delta H(t)$) or by issuing new debt, with $B(t-1)$ measured in units of time $t-1$ goods. To re-iterate: the budget constraint above is all in terms of goods (real).

Re-arranging we can write the budget constraint as:

$$B(t) = (1 + R(t-1))\, B(t-1) - D(t) - \frac{\Delta H(t)}{P(t)} \tag{7.2}$$

Now we're going to use this equation and our three assumptions above – that set the monetarist model – to illustrate that:

- If fiscal policy in the form of the path for $D(t)$ sequence is taken as given, then tighter current monetary policy implies higher future inflation.

We specify alternative monetary policies (tight/loose)in the following way: we take $H(1)$ as given and we assume constant growth rates for H, denoted by θ for periods $t = 2, 3, ..., T$, where T is some date in the future (so $T \geq 2$. However, after that, $t > T$ we assume that the path of $H(t)$ is determined by the condition that the stock of debt is held constant at whatever level was reached in period T, implying that $B(T) = B(T+1) = \dots$.

The last point above is consistent with there being an upper limit on the stock of debt. Therefore, with H_1 given, we assume that the monetary base follows the process:

$$H(t) = (1 + \theta)\, H(t-1) \tag{7.3}$$

and the equation above holds for $t = 2, 3, ..., T$. Therefore θ represents the growth rate of the money supply.

So what we want to do is to consider the consequences of various choices of θ and T and when we discuss a tight(er) monetary policy this will be taken to imply a low value of θ.

It is worth noting that we are also assuming that the paths of $D(t)$, θ and T are announced in period $t = 1$ and believed by all agents.

We can then return to our 'monetarist' assumption (the third one) regarding the link between the price level and the money stock and write this as

$$P(t) = \frac{1}{h} H(t) \tag{7.4}$$

for some positive constant h.[1] From this equation it then follows that

$$1+\pi(t)=1+\theta$$

Thus, in the discussion above when we were setting values for θ and T (this is the monetary policy from period 1 to T), we are also choosing the rates of inflation from 2 to T . The crucial thing is: what does this imply for the inflation rate after period T?

We solve this problem in two steps:

1. First, we determine how inflation after T depends on the stock of debt reached in period T. Let this final stock of debt be written as $B_\theta(T)$.

2. We then show that $B_\theta(T)$ depends on θ.[2]

Step 1:

To show how inflation after T depends on $B_\theta(T)$ we use equation (7.2) for periods $t>T$, noting that debt will from then onwards be constant – by assumption – and using our equation linking prices to base money (7.4). This gives

$$D(t)=\frac{\Delta H(t)}{P(t)}+\cancel{B(t)}-(\cancel{1}+R(t-1))B_{t-1}$$

$$D(t)=\frac{h\Delta P(t)}{P(t)}-B(t-1)R(t-1)$$

$$D(t)=h-h\frac{P(t-1)}{P(t)}-R(t-1)B_\theta(T)$$

$$1-\frac{P(t-1)}{P(t)}=\frac{D_t+R(t-1)B_\theta(T)}{h}$$

(Or equivalently,)

$$\frac{\pi(t)}{1+\pi(t)}=\frac{D(t)+R(t-1)B_\theta(T)}{h}$$

From the equation above, we can see that inflation for $t>T$ is increasing in the stock of debt. Note from this equation that an increase in $B(t-1)$ increases the right hand side. Given the equality, the left hand side must increase by the same amount. From the left hand side we can see that it is

[1] One can obtain this equation from a standard money demand equation but where the interest rate elasticity of money demand is zero and imposing the assumption in SW that output is constant. Note that this equation posits a strong link between money and prices.

[2] That is the reason for having θ as a subscript.

an increasing function of $\pi(t)$.[3] Because we solved this equation for $t > T$ and assumed that the stock of debt would be constant for that period we have just found that inflation will be higher the higher the level of debt $B_\theta(T)$, for all $t > T$.

If we assume a constant deficit and interest rates we have

$$\frac{\pi(t)}{1+\pi(t)} = \frac{D + RB_\theta(t-1)}{h}$$

Step 2: For the second part, we want to show that tighter money now (lower θ) results in greater $B_\theta(T)$.[4] For this we need to solve for the paths of $B(1)$, $B_\theta(2)$, ..., $B_\theta(T)$.

To find $B(1)$, using the period $t = 1$ version of (7.2):

$$B(1) = (1 + R(0))B(0) + D(1) - \frac{H(1) - H(0)}{P(1)}$$

This, plus the equation linking P to H (to eliminate prices above) means we can write $B(1)$ as a function of $D(1), H(1), H(0), B(0)$. Hence, $B(1)$ does not depend on θ.

Next step is to find $B_\theta(2)$, $B_\theta(3)$, ..., $B_\theta(T)$ using equations (7.3) and (7.4)

$$B(t) = (1 + R_{t-1})B_{t-1} + D_t - \frac{h\theta}{1+\theta} \tag{7.5}$$

for $t = 2, 3, ..., T$.

If we now impose constant deficits and interest rates ($D(t) = D$ and $R(t) = R$) as assumed above, we therefore have that for $2 < t \leq T$

$$B(2) = (1 + R)B(1) + D - h\left(\frac{\theta}{1+\theta}\right)$$

$$\begin{aligned} B(3) &= (1 + R)B(2) + D - h\left(\frac{\theta}{1+\theta}\right) \\ &= (1+R)\left[(1 + R)B(1) + D - h\left(\frac{\theta}{1+\theta}\right)\right] + D - h\left(\frac{\theta}{1+\theta}\right) \\ &= (1+R)^2 B(1) + [1 + (1+R)]\left[D - h\left(\frac{\theta}{1+\theta}\right)\right] \end{aligned} \tag{7.6}$$

so for $t \geq 2$ and $t < T$:

$$B_\theta(t) = (1+R)^{t-1}B(1) + \sum_{s=2}^{t}(1+R)^{s-2}\left[D - h\left(\frac{\theta}{1+\theta}\right)\right]$$

[3] If you do not see this, try $y = \frac{x}{1+x}$ and find $\partial y/\partial x$. Here $x \equiv \pi$ and y is the left hand side.

[4] Now we need to show how $B_\theta(T)$ depends on θ.

From this equation we can see that tight money now – lower θ – increases the right hand side, implying a higher level of $B_\theta(t)$ and this applies to $B_\theta(T)$. This completes our proof.

Therefore we have that **less inflation now achieved through monetary policy on its own implies more inflation in the future**. For such a result it is crucial that the real rate of return be greater than the rate of population growth (here set to zero) and that the path of the fiscal deficit be independent of θ.

7.2 Tighter money can mean higher inflation now

To get this result we need to modify our money demand function. Now, let it be of the Cagan type so that it depends on expected inflation (it equals actual future inflation under perfect foresight)

$$\frac{H_t}{P_t} = h_0 - h_1(1 + \pi_{t+1})$$

Recall from the notes on REH and the Cagan model that we can then solve for prices by iterating forward. We can then see that tight money now implies loose money later.

Tight money now will have a negative effect on inflation, but expectations of future loose money will then have the effect of raising current prices so the overall effect is in principle ambiguous (it depends on parameter values).

7.3 Conclusion

We have used a simple model to show that if fiscal policy is exogenous, monetary policy is pinned down by the government's budget constraint. This implies that lower inflation now will have to be matched by higher inflation in the future. The argument is akin to the logic behind Ricardian equivalence and the effects of temporary tax cuts when government spending is exogenous.

The implication from this model is that for central banks to control inflation successfully, fiscal policy has to be consistent with such a mandate. In the model above this was not the case: the path of D was exogenous, rather than given by the government's budget constraint.

Chapter 8

Real Business Cycles

There is no government so that we only have two kinds of agents: firms and households. All agents are price takers as we are assuming perfect competition and there are no externalities (so that the perfectly competitive equilibrium will be the Pareto optimum). One way of thinking about this is to consider that there is a very large number of firms and of households so that no single agent's decision will have an effect on prices, although we shall assume that the number of agents sums to one for simplicity. Lastly, we are going to assume that all agents are identical so that the decisions of a single agent will represent the decisions of all agents (the 'representative agent').

As we are considering an RBC model note that all variables are real and, at least initially, in levels rather than logarithms. To keep the notation simple we are going to ignore the expectations notation for now.

Let us now turn to each agent's decisions in turn.

8.1 Firms

All firms produce output via a production function

$$Y_t = A_t F(K_t, L_t) \tag{8.1}$$

Where A is technology, K is capital and L is labour. The production function is homogeneous of degree one so that we have constant returns to scale. Moreover, Euler's theorem implies that

$$Y_t = \frac{\partial Y}{\partial K} K_t + \frac{\partial Y}{\partial L} L_t = MPK_t K_t + MPL_t L_t$$

Investment and capital are related via the transition equation for capital, which is given by

$$K_{t+1} = (1-\delta)K_t + I_t$$

with $0 < \delta < 1$ being the depreciation rate.

The objective of the firm is to maximise its discounted stream of profits π, given by

$$V_t = \sum_{s=t}^{\infty} \Delta_s \pi_s$$

Where Δ is the discount factor on profits and this is given by

$$\Delta_0 = 1$$

$$\Delta_1 = 1/R_1$$

$$\Delta_2 = \frac{1}{R_1 R_2}$$

And so on, with $R_t = 1 + r_t$ being the gross real rate of interest. This implies that

$$\frac{\Delta_t}{\Delta_{t+1}} = R_{t+1}$$

Profits are given by

$$\pi_t = Y_t - w_t L_t - (K_{t+1} - (1-\delta)K_t)$$

We therefore have

$$\begin{aligned} V_t = &(A_t F(K_t, L_t) - w_t L_t - (K_{t+1} - (1-\delta)K_t)) \\ &+ \frac{(A_{t+1} F(K_{t+1}, L_{t+1}) - w_{t+1} L_{t+1} - (K_{t+2} - (1-\delta)K_{t+1}))}{R_t} + \ldots \end{aligned}$$

The firm chooses $\{K_{t+1}, K_{t+2}, ..., L_t, L_{t+1}, ...\}$ subject to K_t being predetermined and it takes wages, technology and interest rates as exogenous. The first order conditions are

$$R_{t+1} = MPK_{t+1} + 1 - \delta \tag{8.2}$$

$$w_t = MPL_t \tag{8.3}$$

These equations hold for all t although that for capital begins in $t+1$.

8.2 Households

The representative household maximises its discounted expected utility given by

$$\sum_{s=t}^{\infty} \beta^t \left[U(C_t) - \Gamma(L_t)\right] \tag{8.4}$$

where $\Gamma(L_t)$ represents the disutility of labour, with $\Gamma'(L) > 0$ and $\Gamma''(L) > 0$. The representative household maximises its utility subject to its budget constraint:

$$C_t + K_{t+1} + B_{t+1} = w_t L_t + R_t K_t + R_t^b B_t$$

On the left hand side we have the expenditure components: the household can consume, purchase capital today (for the next period) or bonds. Recall that households own the firms. The bonds are traded among agents so they can lend/borrow to each other at the interest rate R_t^b.

To derive the first order conditions, we set up the Lagrangean:

$$L_t = \sum_{s=t}^{\infty} \beta^s \left\{[U(C_s) - \Gamma(L_s)] + \lambda_s[w_s L_s + R_s K_s + R_s^b B_s - C_s - K_{s+1} - B_{s+1}]\right\} \tag{8.5}$$

Where again, K_t and B_t are pre-determined and the household takes interest rates and wages as given. Another way of understanding this equation is to expand it:

$$\begin{aligned} L_t = &\left\{[U(C_t) - \Gamma(L_t)] + \lambda_t[w_t L_t + R_t K_t + R_t^b B_t - C_t - K_{t+1} - B_{t+1}]\right\} \\ &+ \beta\left\{[U(C_{t+1}) - \Gamma(L_{t+1})] + \lambda_{t+1}[w_{t+1} L_{t+1} + R_{t+1} K_{t+1} + R_{t+1}^b B_{t+1} - C_{t+1} - K_{t+2} - B_{t+2}]\right\} \end{aligned}$$

We therefore have

$$U'(C_t) = \lambda_t \tag{8.6}$$

$$\Gamma'(L_t) = w_t \lambda_t \tag{8.7}$$

$$\lambda_t = \beta \lambda_{t+1} R_{t+1} \tag{8.8}$$

$$\lambda_t = \beta \lambda_{t+1} R_{t+1}^b \tag{8.9}$$

The first equation is fairly standard: the Lagrange multiplier equals the marginal utility of consumption. The idea is that if we relax the constraint by a marginal amount utility will increase by $U'(C)$. The second equation can be understood in terms of marginal cost equals marginal benefit. If the household works an additional unit of time its utility will fall by the marginal disutility of labour (the left hand side); this is the marginal cost. The gain from that additional unit of labour will be the additional wage earned, and this is converted into utility by multiplying it by the marginal utility of consumption; this is the marginal benefit. At the optimum, these two values must be equal to each other.

The third equation represents an intertemporal choice and is one of the key elements of the mode where we have dynamics as it pertains to the savings decision. It can again be thought of in terms of marginal cost/marginal benefit. If the household decides to save one unit of consumption this means that it'll consume one unit less in the present: its marginal cost is therefore the present period's marginal utility of consumption. Savings are just postponed consumption so the benefit is the consumption to be gained in the next period.[1] The one unit saved in t therefore pays R_{t+1} in the next period and this is converted into utility by multiplying it by the marginal utility of consumption in the next period. Lastly, as households discount the future, this amount is multiplied by β. This represents the marginal gain.

It is also worth noting that the last two equations are identical and this is not surprising. As we are assuming no default the expected returns on bonds and capital must be identical (otherwise there would be an arbitrage opportunity). Moreover, as all agents are identical to each other the net supply of bonds must be zero in equilibrium.[2] In other words, as they're identical they are either all lending or all borrowing but in that case, who is the counterparty? If we had an open economy this would not be the case as the representative household could be selling/buying bonds to/from foreigners.

Another important result arises when we use the household's budget constraint and the production function (also recall that the net supply of bonds is zero) in equilibrium:

$$C_t + K_{t+1} + B_{t+1} = w_t L_t + R_t K_t + R_t^b B_t$$

$$C_t + K_{t+1} = w_t L_t + R_t K_t$$

$$C_t + K_{t+1} = w_t L_t + (MPK_t + 1 - \delta)K_t$$

$$C_t + K_{t+1} = w_t L_t + (MPK_t + 1 - \delta)K_t$$

[1] Postponing it several periods does not alter the insight, we would just have the interest rates over several periods.

[2] The way this is achieved is via the interest rate. It will be such that agents will not want to lend or borrow to each other.

$$C_t + I_t = w_t L_t + MPK_t K_t$$

$$C_t + I_t = Y_t$$

$$C_t + I_t = Y_t \tag{8.10}$$

So the equation for aggregate demand is an equilibrium condition.

8.3 General Equilibrium

Our full model in general equilibrium is given by (including expectations)

$$Y_t = A_t F(K_t, L_t) \tag{8.11}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{8.12}$$

$$R_t = MPK_t + 1 - \delta \tag{8.13}$$

$$w_t = MPL_t \tag{8.14}$$

$$\Gamma'(L_t) = w_t U'(C_t) \tag{8.15}$$

$$U'(C_t) = \beta E_t U'(C_{t+1}) R_{t+1} \tag{8.16}$$

As MPL and MPK represent the marginal products of labour and capital, respectively, and these are just the derivatives of Y with respect to each input I am not going to consider them additional variables. Our model therefore comprises 6 equations and six endogenous variables: Y, K, L, I, R, w.

8.4 An example

Let us give our model some specific functional forms:

$$Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha}$$
$$U = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{\eta}}{\eta}$$

Our model therefore becomes

$$Y_t = e^{z_t} K_t^{\alpha} L_t^{1-\alpha} \tag{8.17}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{8.18}$$

$$R_t = \alpha e^{z_t} \left(\frac{L_t}{K_t}\right)^{1-\alpha} + 1 - \delta \tag{8.19}$$

$$w_t = (1-\alpha)e^{z_t} \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{8.20}$$

$$L_t^{\eta-1} = w_t C_t^{-\sigma} \tag{8.21}$$

with $z_t = \rho_z z_{t-1} + \epsilon_t$. This is a rational expectations model with one endogenous state variable, K_t and one shock, technology (z_t). However, as it stands this is not something you can solve by hand because the model is non-linear (the production function, the equations for interest rates and wages). The production function could easily be made linear by taking logarithms with the aim of solving for the logs of the variables but this is not enough because the logarithm of the remaining equations would be non-linear. Consequently, the general approach in the literature has been to conduct log-linear approximations to these equations.

Chapter 9

The Cash in Advance Model

The Cash in Advance (CIA) model

This approach to introducing money in a representative agent model focus on the transactions role of money (as opposed to viewing money as an asset, for example). The basic idea is that money is essential for carrying out transactions and this is modelled in the form of a constraint. The model is in other respects identical to its money in the utility function (MIU) counterpart or just an RBC without money.

9.1 Model setup

As with the other models, the economy is populated by a large number of identical households and firms, all of which are price takers. As we have introduced money into an otherwise RBC model this implies the existence of a government, with the consequence that we shall have to formulate a government budget constraint. Be aware that all variables are in levels (not logarithms) unless otherwise stated.

9.1.1 Households

Household utility is given by

$$\sum_{s=t}^{\infty} \beta^{s-t} \left\{ U(C_t) - \Gamma(L_t) \right\} \tag{9.1}$$

The representative agent is subject to an intertemporal budget constraint. Timing is very important in CIA models and different timing assumptions will yield somewhat different results. At the beginning of the period the agent has an amount Ω_t of financial assets that she can re-allocate between money and bonds

$$\Omega_t = M_t + B_t$$

After this, the agent is able t carry out transactions subject to the CIA constraint:

$$M_t \geq P_t C_t \tag{9.2}$$

so that her previous choice of M will limit the purchases she will be able to undertake. Next, the agent supplies her labour, receives lump-sum transfers (or pays lump-sum taxes) so that her wealth at the beginning of the next period (Ω_{t+1}) will be

$$\Omega_{t+1} = B_{t+1} + M_{t+1} = W_t L_t - P_t C_t + (1+i_t) B_t + M_t + W_t L_t + \Pi_t + T_t \tag{9.3}$$

The agent's problem then is to maximise her utility subject to the CIA constraint and (9.3) by choosing the optimal values of C_t, L_t, M_t and Ω_{t+1}.[1] The process discussed above implies that the bond market opens first and that the goods market follows so that the agent is able to adjust her financial portfolio to have enough money as required for transactions.

Setting up the Lagrangean we have

$$L = \sum_{s=t}^{\infty} \beta^{s-t} \Big\{ U(C_s) - \Gamma(L_s) + \lambda_s \Big[W_s L_s + \Pi_s - P_s C_s + (1+i_s)\Omega_s - i_s M_s + W_s L_s + T_s - \Omega_{s+1} \Big] \Big\} + \sum_{s=t}^{\infty} \beta^{s-t} \mu_s \left(M_s - P_s C_s \right) \tag{9.4}$$

The way it is written the optimisation is implemented in period t for all periods $s \geq t$. You can think of this as a contingent plan that the agents makes and expectations have been included. As the optimisation begins in period t assets carried over from the previous period are taken as given.

The first order conditions are:

$$U'(C_t) - P_t \left(\lambda_t + \mu_t \right) = 0$$

$$-\Gamma'(L_t) + W_t \lambda_t = 0$$

$$\beta \lambda_{t+1} (1 + i_{t+1}) = \lambda_t$$

$$i_t \lambda_t = \mu_t$$

plus the CIA constraint:

[1] Alternatively, you can differentiate with respect to B_{t+1} but not B_t since Ω_t is fixed at time t.

$$M_t = P_t C_t$$

Combining the above and using $\lambda_t = U'(C_t)/(P_t(1+i_t))$ we have

$$\frac{U'(C_t)}{P_t(1+i_t)} = \beta \frac{U'(C_{t+1})(1+i_{t+1})}{P_{t+1}(1+i_{t+1})}$$

$$\Gamma'(L_t) = \frac{U'(C_t)}{(1+i_{t-1})} \frac{W_t}{P_t}$$

$$M_t = P_t C_t$$

And the first of these three equations can be re-written as

$$U'(C_t) = \beta \frac{U'(C_{t+1})(1+i_t)}{(1+\pi_{t+1})}$$

This is just the nominal (monetary) equivalent of the consumption Euler equation first encountered in the RBC model since the real interest rate is given by $(1+r_t) = (1+i_t)/(1+\pi_{t+1})$.[2] Also, the equation $M_t = P_t C_t$ is not only our CIA constraint but our money demand equation, which implies constant velocity.[3] This is an undesirable feature because it is at odds with the data but relaxing the CIA constraint so that some goods can be purchased on credit, for example, can overcome this.

9.1.2 Firms

Firms seek to maximise the discounted stream of current and future profits which in this model is given by[4]

$$V_0 = \sum_{s=0}^{\infty} \Delta_s \pi_s$$

Where Δ is the discount factor on profits and this is given by

$$\Delta_0 = 1$$

$$\Delta_1 = 1/R_1$$

$$\Delta_2 = \frac{1}{R_1 R_2}$$

[2]However, if we change the formulation or timing of the CIA constraint the equation above may end up looking quite different.

[3]A constant velocity does not arise in the MIU model.

[4]Here π represents profits in real terms, while that in the household's budget constraint, Π is nominal. They are related via $\Pi = P\pi$. As we do shall no longer use real profits in the remainder of the model this should not confuse you as to what variable is used to denote inflation or profits.

As there is no capital profits are given by

$$\pi_t = A_t F(L_t) - w_t L_t$$

with $Y_t = A_t F(L_t)$ being the representative firm's output. This problem straightforward as direct optimisation shows that each period the firm will ensure that

$$A_t F'(L_t) = w_t$$

In other words, as long as the firm is maximising profits it will ensure that the marginal product of labour equals the real wage. This is our labour demand function.

9.1.3 The Government

We want to keep the remainder of the model as simple as possible and that is the reason we assumed no government debt (bonds). Just like households and firms, the government in subject to a constraint. Note that the government sector has only appeared twice in the model thus far: by issuing currency that is then held by households, and by the transfer payments that households receive. If we also assume that there is no government spending then the government's budget constraint is given by:

$$\Delta M_{t+1} = T_t$$

That is, any new money creation is used to finance the lump sum transfers to the household. We can think of this as a balanced budget policy as the transfers are fully financed every single period. While we have considered the government's budget constraint that it must abide by, it contains to elements (money creation and transfers) so it is free to choose one of them, which we shall call the government's policy. We shall assume that the government follows a policy of money growth (in nominal terms) given by

$$M_t = (1 + \theta_t) M_{t-1} \tag{9.5}$$

Or in real terms

$$m_t = (1 + \theta_t) \frac{m_{t-1}}{1 + \pi_t}$$

Where θ_t represents shocks (that may be persistent) to the growth of the money supply. To see this, take logs of (9.5) and you will see that the growth rate of the money supply is given by θ_t. If this were made constant then the money supply would be growing by the same amount every single period. If we assume that shocks to the money supply exhibit some persistence this could be modelled as

$$\theta_t = \overline{\theta} + \rho_\theta \theta_{t-1} + \epsilon_{\theta,t}$$

$\epsilon_{\theta,t}$ represents a while noise process.

9.1.4 General Equilibrium

If we combine the constraints for the firm, the household and the government we have

$$\Omega_{t+1} = W_t L_t - P_t C_t + (1+i_t)B_t + M_t + \Pi_t + T_t$$

$$\Pi_t = P_t A_t F(L_t) - W_t L_t = P_t Y_t - W_t L_t$$

$$\Delta M_{t+1} = T_t$$

Using the second and third equations in the first one we have

$$(M_{t+1} + B_{t+1}) = W_t L_t - P_t C_t + (1+i_t)(M_t + B_t) - i_t M_t + \Pi_t + T_t$$

$$(M_{t+1} + B_{t+1}) = W_t L_t - P_t C_t + (1+i_t)(M_t + B_t) - i_t M_t + \Pi_t + \Delta M_{t+1}$$

$$B_{t+1} = W_t L_t - P_t C_t + (1+i_t)(M_t + B_t) - i_t M_t + \Pi_t + M_t$$

$$B_{t+1} = W_t L_t - P_t C_t + (1+i_t)B_t + W_t L_t + (P_t Y_t - W_t L_t)$$

$$B_{t+1} = -P_t C_t + (1+i_t)B_t + P_t Y_t$$

In equilibrium all households are identical (that is why we use the term 'representative agent') and recall that B denoted the one-period nominal bonds held by households and issued by other households. But as all agents are identical then they would all want to be in the same situation (that is, they would all like to be borrowing/lending exactly the same amount). This means that in equilibrium the supply of bonds (at the aggregate level) is equal to zero.[5] Therefore we have

[5] Another way of seeing this is as follows. As the budget constraint shows you are free to buy or issue debt to smooth your consumption if you wish to do so. All your other 'twins' are in exactly the same situation. What prevents everyone from trying to lend/borrow in equilibrium is the real interest rate: it will be such that you will wish not to issue or buy any bonds. If B had represented government bonds then this would not be the case but the result below would still follow as they would disappear when adding household and government assets.

$$0 = -P_t C_t + W_t L_t + Y_t$$

Or

$$Y_t = C_t$$

This is the economy's aggregate resource constraint, where we can see the equation for aggregate demand is not imposed but emerges by combining the constraints of the different agents in the model. It is therefore an equilibrium condition. In the remainder of the discussion below – as is the general approach – this is the equation we shall be using instead of the separate budget constraints.

9.2 Putting it all together

If we summarise our model equations and using $C = Y$ throughout we have

$$U'(Y_t) = \beta \frac{U'(Y_{t+1})(1 + i_t)}{(1 + \pi_{t+1})} \tag{9.6}$$

$$\Gamma'(L_t) = \frac{U'(Y_t)}{(1 + i_t)} A_t F'(L_t) \tag{9.7}$$

$$m_t = Y_t \tag{9.8}$$

$$Y_t = A_t F(L_t) \tag{9.9}$$

$$m_t = (1 + \theta_t) \frac{m_{t-1}}{1 + \pi_t} \tag{9.10}$$

Where $m = M/P$ denotes real money balance (so it is in levels, not logs) while A and θ are both exogenous processes. We therefore have five endogenous variables (Y, i, π, L and m) and five equations so that if we had used some functional forms we could have solved the model.

9.2.1 Steady State Analysis

To see what the effects of money and inflation are in this model we are going to consider its steady state properties. Steady state is a long run concept where some variables are allowed to grow – but only at their long run values – while others are stationary. Assuming that technology (exogenous) does not grow over time means that in the long run its value is fixed. At the same time we modelled the money supply in terms of growth rates so the steady state implies a constant growth in the money supply. In the short run both the growth in the money supply and the level of technology would be subject to shocks but in steady state we are not focusing on that.

We can therefore conclude that in steady state (the long run) output and labour will be fixed. This implies that m will also be a fixed but as M has steady state growth rate so does P. In other words, in steady state the inflation rate is constant. From the last equation above with θ exogenous and $m_t = m_{t-1} = \overline{m}$ in steady state we have

$$\overline{\pi} = \overline{\theta}$$

That is, the inflation rate is given by the growth rate of the money supply. Using this in the first equation we have[6]

$$(1 + \overline{i}) = \frac{(1 + \overline{\pi})}{\beta}$$

So with a constant β the steady state nominal interest rate is driven by the state state rate of inflation (and hence nominal money growth).

The reason for going through these steps will hopefully now become clearer. Using these results in equation (9.7) we can see that a higher rate of money growth will lead to a greater increase in the nominal rate of interest. The right hand side of this equation has the marginal product of labour (equal to the real wage in equilibrium) divided by $(1 + i)$ so the interest rate acts like a labour income tax. Consequently, higher inflation in this model will have long run effects on output by reducing labour supply so if you like to think in terms of Phillips curves, the CIA model above with endogenous labour implies that the Phillips curve (with output) is downward sloping.[7] The intuition is that household utility depends on both leisure and consumption. In order to consume households supply their labour. Consumption requires money and inflation (via the nominal interest rate) erodes money holdings thereby altering the trade-offs, the marginal rate of substitution between leisure and consumption because leisure does not suffer from this 'inflation tax'.

It is also worth noting that the RBC counterpart of this model would not have contained inflation and the resulting equation would have been

$$\Gamma'(L) = U'(C)AF'(L)$$

So we can see how the cash in advance constraint creates a distortion. Given the above, we can therefore ask, what is the optimal rate of inflation?

[6] Note that since the real interest rate is $(1+r) = (1+i)/(1+\pi)$, equation (9.6) implies $(1 + r) = 1/\beta$ in steady state. The real interest rate is determined by the household discount factor. This is intuitive as the more impatient households are (the lower their β, the higher the real interest rate will have to be in equilibrium.

[7] That said, it may be that a quantitative version of the model will show that the effects of inflation on output are very small.

9.3 The Friedman Rule

To consider the optimal rate of inflation we can think of a benevolent policy maker who maximises social welfare by deciding on the allocations of consumption and leisure – the two arguments in the utility function – subject to ensuring that the economy's constraints are satisfied. Doing so will give us the choices for consumption and leisure that will maximise utility but this may differ from our results above because earlier agents were acting in isolation – they were all price takers – whereas now the social planner internalises the distortions. The approach is straightforward. The Lagrangean is

$$L_t = \sum_{s=0}^{\infty} \beta^s \left[U(C_s - \Gamma(L_s) + \lambda_s \left(A_s F(L_s) - C_s \right) \right]$$

Combining the first order conditions we then have

$$\Gamma'(L) = U'(C) A F'(L)$$

So that the RBC solution – the solution to this model without money – is the Pareto optimum.[8] The allocation under the social planner maximises social welfare while that under the decentralised competitive equilibrium was given by equation (9.7) differs from this. Consequently, the competitive equilibrium solution will be sub-optimal unless we have

$$\bar{i} = 0$$

In other words, the optimal nominal rate of interest is zero as it eliminates the distortions caused by the cash in advance constraint. Given that

$$\bar{i} \approx \bar{r} + \bar{\pi}$$

and r being pinned by β, this implies that the optimal rate of inflation is one of deflation – delivered via the growth rate of the money supply θ – with a value of

$$\bar{\pi} \approx -\bar{r}$$

This result has been a very influential and emerges in several models, rather than being specific to our CIA framework. As we have been working with steady states the Friedman rule should be interpreted as what the optimal inflation target should be, or 'optimal monetary policy' or the optimal growth of the money supply', but in all cases pertaining to the long run, not

[8] The solution under the social planner would therefore be $\Gamma'(L_t) = U'(A_t F(L_t)) A_t F'(L_t)$. After solving for L we can therefore obtain the optimal values for the remaining variables.

what the monetary authority should do on a quarterly basis. That said, no central bank has ever set a negative inflation target.

9.4 A Specific Quantitative Example*

To understand our cash in advance model further we shall need to use specific functional forms as well as parameter values. However, we are also going to enrich the model by including capital. This is not only a more realistic framework but it also enables us to determine the importance of capital as an internal propagation mechanism.[9]

In order to better capture the CIA constraint with uncertainty we are going to model it – in real terms – as[10]

$$\frac{m_{t-1}}{1+\pi_t} + \frac{(1+i_{t-1})}{1+\pi_t} b_{t-1} + \tau_t \geq b_t + c_t \tag{9.11}$$

The agent begins the period with cash balances left over from the previous period as well as the lump-sum transfer (τ) from the government. In addition, at the beginning of the period she can make use of the bond market in order to ensure that she can finance her purchases.

The agent's budget constraint, again in real terms, is given by[11]

$$c_t + k_{t+1} + m_t + b_t = w_t L_t + r_t k_t + \Pi_t + \frac{m_{t-1}}{1+\pi_t} + \frac{(1+i_{t-1})}{1+\pi_t} b_{t-1} + \tau_t + (1-\delta) k_t \tag{9.12}$$

Starting with the right hand side, the agent receives labour income as well as money from the previous period. In addition to this, households own the physical capital (k) which they lend to firms at the rate r_t. Households also decide how much capital to acquire in the following period, hence the k_{t+1} on the left hand side. At the same time, the agents possesses bonds, which yield an interest i.

Setting up the Lagrangean with λ and μ representing the multipliers for the budget and CIA constraints, respectively, the first order conditions with respect to consumption, labour, bonds, money and capital are

$$U'(C_t) = \mu_t + \lambda_t$$

$$\Gamma'(L_t) = \lambda_t w_t$$

[9] The idea is that a shock, say, technology, leads to an increase in capital and as this is a stock variable output will then remain high for many more subsequent periods.

[10] See the notes on the money in the utility model (MIU) if you do not understand how this is obtained.

[11] I am using Π to denote real rather than nominal profits as we shall be using π to represent inflation.

$$\mu_t + \lambda_t = \beta E_t \left(\frac{(1+i_t)}{1+\pi_{t+1}} (\mu_{t+1} + \lambda_{t+1}) \right)$$

$$\lambda_t = \beta E_t \left(\frac{\lambda_{t+1} + \mu_{t+1}}{1+\pi_{t+1}} \right)$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \left(r_{t+1} + 1 - \delta \right) \right]$$

By combining the third and fourth equations one can show that we obtain $\mu_t = i_t \lambda_t$, just as we did with the earlier CIA model.

9.4.1 Firms

In this formulation the firm's problems is straightforward. Firms purchase labour and rent capital from the household – with a given level of technology – to produce output. Hence their objective is to maximise

$$\Pi_t = A_t F(K_t, L_t) - w_t L_t + r_t K_t$$

This gives

$$w_t = A_t F_{L,t}$$

$$r_t = A_t F_{K,t}$$

Where $A_t F_{x,t}$ represents the derivative of the production function with respect to input x.

9.4.2 Government

The money supply follows

$$M_t = (1 + \theta_t) M_{t-1}$$

where θ_t denotes the growth rate of the money supply. New currency enters the economy via lump-sum transfers ($T_t = \theta_t M_{t-1}$) so that the government's budget constraint is given by

$$\Delta M_t = T_t$$

Combining this equation with the household's budget constraint and the fact that firms do not make profits in equilibrium (because of perfectly competitive markets) we then have

$$Y_t = C_t + I_t$$

Lastly, the money supply process above in real terms is given by

$$m_t = \frac{(1+\theta_t)}{1+\pi_t} m_{t-1}$$

And we shall assume

$$\theta_t = (1-\rho_\theta)\overline{\theta} + \rho_\theta\theta_{t-1} + \epsilon_{\theta,t}$$

9.5 Functional Forms

In solving the model below, we shall assume that the production function is Cobb-Douglas:

$$Y_t = e^{z_t}K_t^\alpha + L_t^{1-\alpha}$$

Where z is the level of technology, assumed to follow

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}$$

This implies that the marginal products of capital and labour are given

$$MPK_t = \alpha Y_t/K_t$$

$$MPL_t = (1-\alpha)Y_t/L_t$$

The utility function exhibits constant relative risk aversion (CRRA):

$$U(C_t) - \Gamma(L_t) = \frac{C_t^{1-\phi}-1}{1-\phi} + \Psi\frac{(1-L_t)^{1-\eta}}{1-\eta}$$

so that $1-L$ denotes the proportion of time devoted to leisure activities, while ϕ, Ψ and η are all positive.[12]

This yields

$$U'(C_t) = C_t^{-\phi}$$

$$\Gamma'(L_t) = \Psi(1-L_t)^{-\eta}$$

9.6 The Full Model

Our model equations are therefore

$$C_t^{-\phi} = \mu_t + \lambda_t$$

$$\Psi(1-L_t)^{-\eta} = \lambda_t(1-\alpha)Y_t/L_t$$

$$\mu_t = i_t\lambda_t$$

[12] In the limiting case where $\phi = 1$ we get a logarithmic utility function. To see this use L'Hopital's rule.

$$\lambda_t = \beta E_t \left(\frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}} \right)$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \left(r_{t+1} + 1 - \delta \right) \right]$$

$$r_t = \alpha Y_t / K_t$$

$$Y_t = e^{z_t} K_t^{\alpha} + L_t^{1-\alpha}$$

$$\theta_t = (1 - \rho_\theta)\overline{\theta} + \rho_\theta \theta_{t-1} + \epsilon_{\theta,t}$$

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}$$

9.6.1 The linear version

The log-linear approximation to these equations (see Walsh, Chapter 3) is[13]

$$-\phi c_t = i_t + \lambda_t$$

$$y_t + \lambda_t = \left(1 + \eta \frac{L}{1 - L} \right) l_t$$

$$\lambda_t = -\phi E_t c_{t+1} - E_t \pi_{t+1}$$

$$\lambda_t = E_t \lambda_{t+1} + E_t r_{t+1}$$

$$r_t = \alpha \frac{Y}{K} \left(y_t - k_t \right)$$

$$y_t = \alpha k_t + (1 - \alpha) l_t + z_t$$

$$(y_t = \frac{C}{Y} c_t + \frac{K}{Y} (k_{t+1} - k_t)$$

$$c_t = m_t$$

$$m_t = m_{t-1} - \pi_t + u_t$$

[13] I am recycling variables here as now all variables are in percentage deviations from their steady states.

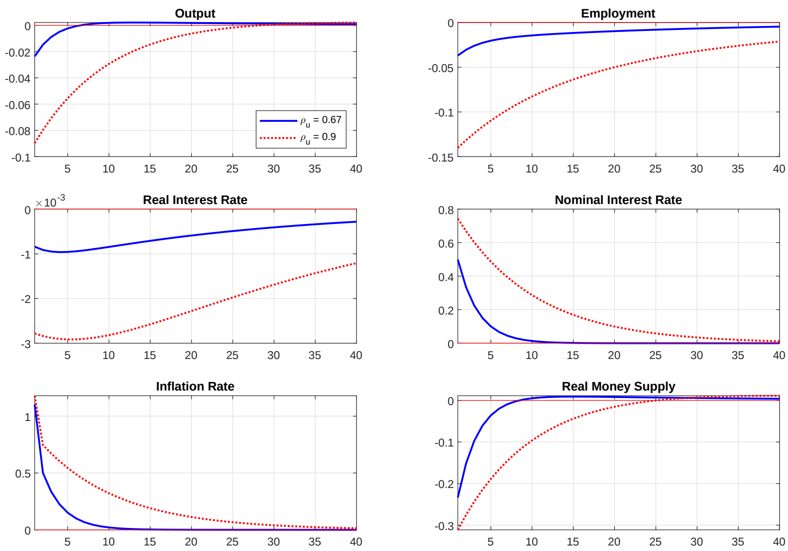

Figure 9.1: Impulse responses to money supply shock (ρ_u denotes the degree of persistence in the shock)

$$u_t = \rho_\theta u_{t-1} + \epsilon_{\theta,t}$$

Where we have defined $u_t = \theta_t - \overline{\theta}$.

The model is solved using Dynare with parameter values as in Walsh. Our focus is on the impact on the endogenous variables of a shock to the money supply and this is can be seen in Figure (9.1). Note that the persistent monetary shock leads to an increase in inflation and most importantly, the nominal interest rate (I in the figure), shown in the bottom left panel. This is contrast to the standard result that increases in the money supply lower the nominal interest rate (think of the IS-LM model) and that also features in the New Keynesian model. This negative relationship between interest rates and monetary injections is called the liquidity effect. This effect is supported by econometric evidence and standard flexible price cannot capture this. Nonetheless, it is also worth noting that even in this flexible-price model money has real effects. The monetary expansion leads to a reduction in labour and therefore output. However, the latter variable does increase later on because the monetary shock leads to a reduction in labour but an increase in investment (think of increases in savings).

9.7 Conclusion

A key issue worth noting is that the model presented above, as well as the standard MIU model, do not feature a Phillips curve. We have perfectly flexible prices and a role for money is introduced (via utility or a transactions constraint) so that we can discuss issues such as the welfare costs of inflation, the optimal growth rate of the money supply, etc. In the CIA model the growth rate of the money supply has real, albeit small, effects – so that we do not have monetary superneutrality – but there is no Phillips mechanism at work. However, with the parameter values in the Dynare file and allowing for both money supply and technology shocks the correlation between output and inflation is 0.46. On observing such a correlation one would have been tempted to interpret many of the model's correlations with some sort of Keynesian model but as we know what model is generating these correlations we can see that this is obviously not the case. Instead, the movements in the model's variables is dominated by the technology shock and θ (by assumption) responds positively to the level of technology so that money, nominal interest rates and inflation are all procyclical.

Chapter 10

The New Keynesian Model

10.1 Introduction

When analysing the RBC model we saw how one could try and understand the causes of business cycles from optimising behaviour in the presence of competitive equilibrium. As a result agents are always optimising and in the standard RBC model stabilisation policy is undesirable. For example, if leisure is a proxy for unemployment, the latter often increases in response to temporary decreases in the level of technology. Whilst such increases in unemployment are always voluntary in these models that is not to say that those who are unemployed are 'happy' about it. Rather, the lower level of technology reduces the real wage and so it is not worthwhile to work as much as before, but agents' welfare (utility) has nevertheless decreased. The would rather work at a higher wage.

The other key result is that monetary policy, in the form of interest rate movements in order to stabilise output, are pretty much ineffective. These effects are non-zero in an MIU model where consumption and m are non-separable: higher inflation causes a decrease in m and affects the marginal utility of consumption, but such a channel is quantitatively very small. The MIU model assumed that prices were flexible so even though we had monetary elements it was, in terms of behaviour, almost identical to an RBC model.

The primary contribution of the New Keynesian (NK) model is to provide a framework that originates in the RBC tradition but by assuming nominal rigidities we now have real effects on the part of monetary policy. The basic idea is as follows: firms' prices are partly rigid – rather, in any given period some firms are unable to reset prices so that changes in aggregate prices cause changes in relative prices. Because firms operate under monopolistic competition their prices are above marginal cost. Given their temporarily fixed prices they will absorb any increase in the demand for their product by increasing their output.

How does monetary policy affect output? Given the existence of sticky prices, any change in nominal interest rates – the policy instrument – will in part be reflected in an increase in real interest rates as inflation doesn't fully adapt. Recall that consumption and investment decisions depend on real rates (think of the RBC model), so changes in R therefore cause changes in output, employment, etc.

We proceed by going over each of the agents in the model before putting them all together to analyse the full model.

10.2 Households

Households in the basic model consume a basket of goods, hold real money balances and supply their labour to firms. As we now have monopolistic firms each firm produces a slightly differentiated good. Hence we no longer have a one-good economy and the mathematics get a lot messier. The only thing to bear in mind is that c in the utility function is an aggregate of the consumption's basket.

The household's consumption and labour supply decisions are the same as in the MIU model as they both make the same assumptions. Recall that the only difference between the NK and RBC models is firms' pricing decisions: that's part of the firm's problem. As a result, households' first order conditions from the MIU model are repeated here:[1]

$$U_{c_t} = \beta E_t U_{c_{t+1}} \left(1 + r_{t+1}\right) \tag{10.1}$$

$$U_{l_t} = w_t U_{c_t} \tag{10.2}$$

So we have a consumption Euler equation and the household's labour supply decision – the marginal disutility of leisure is equal to the real wage times the marginal utility of consumption.

In deriving the IS side of the model we are going to assume that the only input into production is labour (and technology). As there is no capital there will be no investment either.[2] Therefore we have

$$Y = C$$

So we can write our Euler equation as:

$$U_{Y_t} = \beta E_t U_{Y_{t+1}} \left(1 + r_{t+1}\right) \tag{10.3}$$

This equation is non-linear and we want to work with linear models so that we can then use the methods we already know. The general method is

[1] I'm including here a first order condition for labour supply but we shan't focus on this.

[2] We shall initially assume no government spending.

to take logs of the equation and then perform a first order Taylor expansion.[3] For the CRRA utility function we then have the following equation, with σ denoting the coefficient of relative risk aversion ($\sigma > 0$):

$$y_t = E_t y_{t+1} - \sigma^{-1} \widehat{r}_{t+1} \tag{10.4}$$

Note that I have slightly changed the notation of the variables. y and $\widehat{r}$ now represent percentage deviations of output and the real interest rate from their steady state (long run) values. This is now a dynamic IS equation and its format is fairly general. It states that output (in percentage terms) depends on its expected future value with a coefficient of unity, and is also a negative function of the real interest rate. I am still using the notation r_{t+1} to denote the interest rate over period t to the beginning of period $t+1$.

It will be more convenient to re-write the IS in terms of the output gap x_t. This is straightforward as it is defined as the difference between actual and potential GDP.[4]

$$x_t = y_t - y_t^n$$

where I am using the notation y^n to denote the potential or natural, level of output. This is the level of output that would prevail if all prices were fully flexible. In an RBC model with flexible prices the output gap would always be equal to zero; it just wouldn't be useful as a concept.

Using the definition of the output gap and the Fisher equation we have

$$(x_t + y_t^n) = E_t \left(x_{t+1} + y_{t+1}^n\right) - \sigma^{-1} \widehat{r}_{t+1}$$

$$x_t = E_t x_{t+1} - \sigma^{-1} \widehat{r}_{t+1} - \left(y_t^n - E_t y_{t+1}^n\right)$$

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(R_t - E_t \pi_{t+1}\right) - \left(y_t^n - E_t y_{t+1}^n\right)$$

Changes in the natural level of output arise from real shocks (the same ones as in the RBC models), such as technology shocks. We can therefore write simplify the model as:

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(R_t - E_t \pi_{t+1}\right) + g_t \tag{10.5}$$

So that we can think of g as an exogenous shock. This is the only equation where we have the monetary policy instrument, the short term interest rate R.

[3] If you do not know what I am talking about just ignore it.

[4] If we were working in levels it would be the percentage difference but as our model is log-linear then it is just the difference.

10.3 Firms

We now have a continuum of profit maximising firms who, because of monopolistic competition, are able to have to power over pricing decisions. Hence, they set their prices in order to maximise profits. If that were the only assumption this model would have very little that is different compared to the MIU model. Prices would be set higher and output would be lower than under the perfect consumption setting in the MIU model. However, the economy's response to different shocks would be the same as would the ineffectiveness of monetary policy.

Consequently, the next assumption is crucial: price rigidity/stickiness. In any given period only a random proportion of firms is allowed to reset its prices. This proportion is exogenous so that firms, when setting their prices, never know when they will be able to re-optimise. They know the probability but that is all. If prices were set for only one period and firms knew that they would always be able to reset their prices in the following period, all firms would behave the same way – they would set their prices as a markup over marginal costs. If prices were to be fixed by two periods, then they would choose the prices today (that will be binding for two periods) to maximise profits over the two periods. It will be a sort of average of what is best in each period. Likewise if the prices is fixed for many periods. If, by contrast, prices have to be fixed today but there is a positive probability of resetting them in the following period (and so on) then the firm will set prices today taking into account that they may be fixed for quite some time. As a result, the optimal pricing decision will depend on today's real marginal cost, period $t+1$'s real marginal cost (times the probability that prices will be fixed then), and so on.

Once we take into account that all firms are faced by such a framework and that they all behave the same way, the result is that inflation depends on current real marginal costs but also expected future inflation:

$$\pi_t = \beta E_t \pi_{t+1} + \widehat{\kappa} mc_t \tag{10.6}$$

Note that inflation depends on expected future inflation and real marginal costs, therefore future inflation will depends on inflation in $t+2$ and future real marginal costs. If we keep extrapolating we can rewrite the NKPC as

$$\pi_t = E_t \sum_{s=0}^{\infty} \beta^s \kappa mc_{t+s} \tag{10.7}$$

We shall not be using the latter formulation, although you should note that the two expressions are equivalent. The second representation clearly shows that inflation depends not only on current real marginal costs, but also on its infinite discounted future path. As a result, an increase in expected future real marginal costs will have a positive on current inflation. For example, if because of anticipated changes in regulations, future higher oil prices, etc. agents anticipate an increase in mc_{t+2}, then those firms resetting prices today (period t) will note that they may still be stuck with the same prices in period $t+2$. Consequently this will have a positive effect on the prices they set now.

Although the representation in (10.6) is correct – in our model – it is often convenient to work with the NKPC re-written in terms of output, if only to reduce the number of variables in our model. I shan't go into the details, but there is a positive relationship between real marginal costs and the output gap.[5] Therefore, the NKPC we shall use will be the following:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{10.8}$$

10.3.1 An alternative derivation of the New Keynesian Phillips curve

There is an alternative way of formulating the firm's problem that, while not as micro-founded, is very simple to derive.[6] and is based on Rotemberg price adjustment costs.

Assume that the firm's desired price, $p_t^*(j)$ is a positive function of the aggregate price level and of current economic activity:[7]

$$p_t^*(j) = p_t + \alpha x_t$$

Next, assume that the firm's profits are negatively affected by the gap between its current price and its desired price in a quadratic manner:

$$\Pi_t(j) = -\delta \left[p_t(j) - p_t^*(j)\right]^2 = -\delta \left[p_t(j) - p_t(j) - \alpha x_t\right]^2$$

In the absence of any adjustment costs, the desired and actual prices would always equal each other. Consequently, let us assume that the costs of adjusting prices is quadratic, so that small changes are not as costly as large changes:

$$c_t(j) = \phi \left[p_t(j) - p_{t-1}(j)\right]^2$$

Each period, firm j chooses $p_t(j)$ to maximise its present discounted profits net of price-adjustment costs

$$\sum_{i=0}^{\infty} \beta^i E_t \left[\Pi_{t+i}(j) - c_{t+i}(j)\right]$$

[5] The exact relationship between the two depends on the specific assumptions underlying the model.

[6] This follows Walsh [2017], p. 233.

[7] Assume that there is a large number of firms, indexed by j.

The first order condition is

$$-\delta\left[p_t(j)-p_t-\alpha x_t\right]-\phi\left(p_t(j)-p_{t-1}(j)\right)+\beta\phi E_t\left(p_{t+1}(j)-p_t(j)\right)=0$$

Assuming symmetry, we have $p_t(j)=p_t(s)=p_t$, which gives

$$-\alpha\delta x_t-\phi\pi_t+\beta\phi E_t\pi_{t+1}=0$$

or

$$\pi_t=\beta E_t\pi_{t+1}+\frac{\alpha\delta}{\phi}x_t$$

Which is the New Keynesian Phillips curve. Note that the slope of the Phillips curve is a function of α, δ and ϕ, which have an economic meaning. One shortcoming of the Rotemberg formulation of the NKPC is that it implies that all firms will re-set prices at the same time and in every period, contrary to the microeconomic evidence on the frequency of price changes.

10.4 Monetary policy

For now, we shall assume that the central bank sets the nominal interest rate R according to the following interest rate rule

$$R_t=\mu_1\pi_t+\mu_2 x_t \tag{10.9}$$

Where again, all variables are in deviation from steady state (that is the reason there are no constants in the model) and $\mu_1,\mu_2>0$. The equation above is called a Taylor rule, following Taylor [1993]. In essence, it states that central banks will raise interest rates whenever inflation is above its target (that's its steady state value) or when the output gap is positive.[8]

The Taylor rule has been very influential in both academic and policy circles.[9] It seems to describe monetary policy in the US (and other countries) reasonably well. That is not to say that central banks necessarily follow a Taylor rule; rather, you could think of it as a summary of the behaviour of interest rates, and the Taylor rule is 'picking up' whatever the central bank is actually reacting to. Its influence on research stems from the fact that Taylor-type rules perform very well across a wide range of models – they are robust, even when compared to optimal rules.

[8] Also note that, as already previously discussed, if the central bank had set an exogenous interest rule so that it does not respond to endogenous variables, this would result in indeterminacy.

[9] See, for example, Paez-Farrell [2009] and Gali et al. [2001].

10.5 The complete model

Our full New Keynesian model is therefore given by

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(R_t - E_t \pi_{t+1} \right) + g_t \tag{10.10}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{10.11}$$

$$R_t = \mu_1 \pi_t + \mu_2 x_t \tag{10.12}$$

10.5.1 Implications

This is a linear rational expectations model with three equations and three endogenous variables: the output gap, interest rates and inflation, and it could be solved using the methods we have already covered. However, given the dynamic structure of the model it's not as simple as you'd think. Nevertheless, the main insights can be gleaned by studying each of the equations.

This section borrows heavily from Clarida et al. [1999] and I would urge you to read it. First, monetary policy in these models cannot affect the average level of output or any other real variable, so that we have long run monetary neutrality. Whilst we are solving the model assuming a constant long run level of inflation – set by monetary policy so that we can call it the inflation target – we are taking that value as given. Hence our analysis concerns movements around the target. In this sense this is short run analysis, as opposed to the MIU model we have already considered, which focused on the short run. Consequently, monetary policy in our set up determines the volatilities of the output gap and inflation but not their average values. To the extent that policy makers wish to stabilise the economy, we could think of this as saying that they want to minimise the variances of x and π.

However, we can think of the NKPC as a constraint (the supply side of the economy) so that the volatilities of inflation and output cannot be both brought down to zero. Central bankers will therefore have to trade off lower volatility in one variable at the expense of another. Good policies are those that bring us down to the policy frontier (see Figure 1 in [Clarida et al., 1999]).

Secondly, due to price stickiness, the central bank can, temporarily, control the output gap directly. To understand this, ignore the Taylor rule for the moment. We have two equations, the IS and the NKPC. If the central bank has access to real time data it can observe the values in the IS and therefore choose any value of R it wants in order to achieve the value of x it desires. From this we get two results: when it comes to analysing monetary policy we shall assume that the CB controls the output gap directly and we'll ignore the IS (this solves for R but it won't be the focus of our analysis). The other key result is that if the central bank can control x directly, say, by keeping it constant, then as long as the only shocks in our model are g, there is no trade-off between stabilising output and stabilising inflation. The central bank can always ensure that x equals zero (its long run value) and inflation always equals its target.

Such a result is very strong and relies on the assumption of perfect information on the part of the central bank and only IS shocks. If we modify the model by allowing cost-push shocks, so that our NKPC becomes

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \tag{10.13}$$

Now we have a trade-off. Keeping x constant is not sufficient for full stability as then inflation will absorb the cost-push shock.[10]

Another key result concerns monetary policy. If the interest rate rule is modified to

$$R_t = v_t$$

So that in essence it follows an exogenous process so that it is not reacting to economic conditions, we have indeterminacy. We already explored this in the context of the Poole model: exogenous interest rate rules lead to multiple equilibria.

Similarly, if the central bank follows a Taylor rule but $\mu_1 < 1$, the same problem will often (but not always) arise. This is called the **Taylor principle**, the requirement that the coefficient on inflation must be greater than unity.[11] To understand this, think of a shock that causes inflation to increase by one percentage point. If the nominal interest rate does not increase by more than that, the real interest rate will fall, stimulating the economy. This stimulus will then push up inflation, but again, real interest rates will fall, and so on.

10.5.2 Determinacy

A potential problem with interest rate rules (Taylor rules) already alluded to above is the fact that we may not have a unique equilibrium. Figure (10.1) shows the regions of determinacy when the Taylor rule responds contemporaneously to both inflation and output (ϕ_π is the coefficient on inflation). Clearly, a value of ϕ_π is sufficient for determinacy but slightly lower values

[10] Note that one cannot just add shocks here and there as one pleases in micro-founded models; they must originate from somewhere. In the NK model a common way to do this is to assume that firms' markup over marginal costs are subject to shocks. The result will then be the presence of a stochastic element in the NKPC.

[11] This result is often misunderstood by applied macroeconomists. I mentioned that it isn't always necessary because if the response to output μ_2 is strong enough, the response to inflation can be weaker and we still get determinacy.

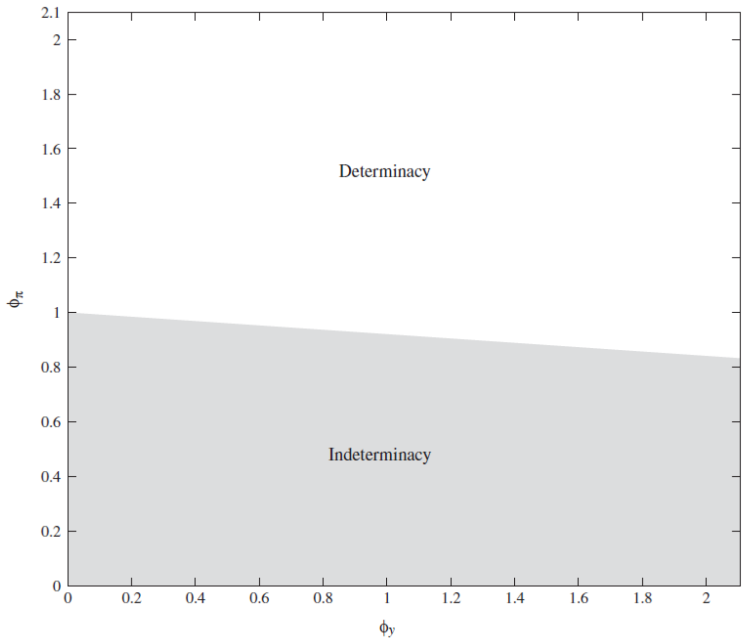

Figure 10.1: Determinacy in the basic NK model (source: Galí [2008]).

are also acceptable is accompanied by sufficiently large responses to output.[12]

By contrast, Figure (10.2) shows that if the TR is implemented using the one period forecasts of inflation and output (so that our TR is forward looking), then too strong a reaction is clearly undesirable, although such high values for the coefficients are unlikely to be observed in practice. It turns out that if one solved the model with a Taylor rule that depends on past values of inflation and output the region of determinacy is in general larger. Moreover, if central banks react to real time data, which is often subject to revisions, using forecasts or current data in implementing policy is going to be affected by errors in the data.[13] but the drawback of implementing such a rule is that the central bank would not be reacting to observed increases in inflationary pressures.

One limitation of the Taylor rule in practice is that the output gap is unobservable. Moreover, something that I have neglected up until now, is that the natural rate of interest r_t^n – the interest rate that would prevail under flex-price conditions – would be affected by real shocks. Such an interest rate should be on the right hand side of the Taylor rule in levels (I had been assuming that it was constant so that in deviations from steady

[12]I have not said much about the values used to solve the models as shown in the figures but they are fairly standard and small modifications would not alter the main conclusions.

[13]For the use of real data in monetary policy see Orphanides [2003].

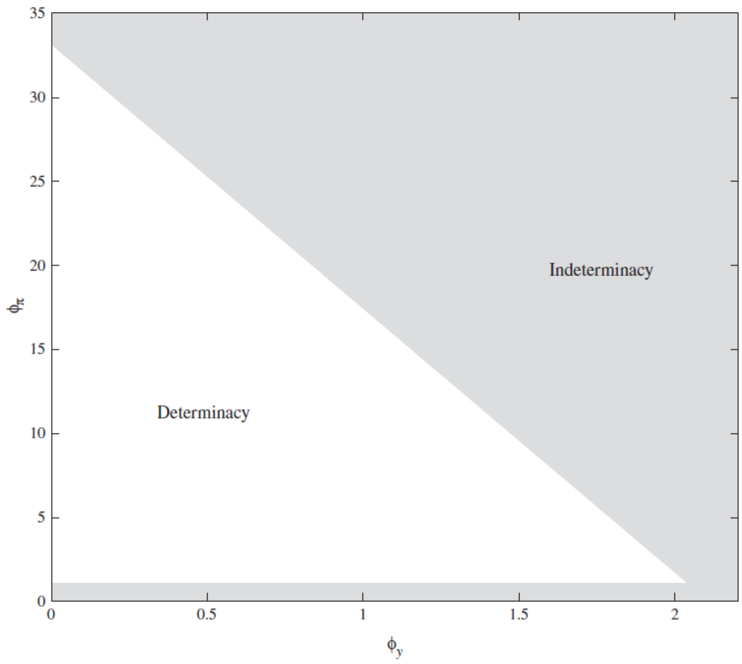

Figure 10.2: Determinacy in the basic NK model with a forward-looking Taylor rule (source: Galí [2008]).

state it was set to zero) so that the standard Taylor rule (in levels) would be written as

$$R_t = r_t^n + \overline{\pi} + 1.5\left(\pi_t - \overline{\pi}\right) + 0.5x_t$$

in steady state this becomes

$$R = r + \overline{\pi}$$

Subtracting this from the Taylor rule we have

$$R_t - R = r_t^n - r + 1.5\left(\pi_t - \overline{\pi}\right) + 0.5x_t$$

$$\widehat{R}_t = \widehat{r}_t^n + 1.5\widehat{\pi}_t + 0.5x_t$$

Or more generally,

$$\widehat{R}_t = \widehat{r}_t^n + 1.5\widehat{\pi}_t + 0.5x_t$$

However, this variable is also unobservable and hence even if we are choosing values of δ_1 and δ_2 to maximise some objective – such as the minimisation in the volatility of inflation and the output gap – in practice we are asking too much of such a simple rule. As a result, these shortcoming in implementing a Taylor rule in practice have led to calls for alternative simple rules, where interest rates respond to observable variables.

10.6 Optimal monetary policy

Assume that the central bank's objective is to minimise

$$Loss_t = E_t \frac{1}{2} \sum_{s=0}^{\infty} \beta^s \left(\pi_{t+s}^2 + \omega x_{t+s}^2 \right)$$

Subject to the NKPC:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

We shall first solve this under commitment (you will see the distinction with discretion later). If at time t the central banker solves the problem above, the Lagrangean is given by

$$\begin{aligned} L = E_t \Big\{ & \frac{1}{2} \left[\left(\pi_t^2 + \omega x_t^2 \right) + \beta \left(\pi_{t+1}^2 + \omega x_{t+1}^2 \right) + \beta^2 \ldots \right] \\ & + \lambda_t \left(\beta \pi_{t+1} + \kappa x_t + u_t - \pi_t \right) + \beta \lambda_{t+1} \left(\beta \pi_{t+2} + \kappa x_{t+1} + u_{t+1} - \pi_{t+1} \right) + \ldots \Big\} \end{aligned} \tag{10.14}$$

The first order conditions are
for π_t $(s = 0)$

$$\pi_t - \lambda_t = 0$$

for π_{t+s} $(s > 0)$

$$\beta^s \pi_{t+s} + \beta^s \lambda_{t+s-1} - \beta^s \lambda_{t+s} = 0$$

for x_{t+s}, $s \geq 0$

$$\omega x_{t+s} + \kappa \lambda_{t+s} = 0$$

Once again, every period the policy maker chooses the output gap to be consistent with the above. As a result, even though this is the plan it makes now – hence the expectations at time t – it is assumed that when choosing x in $t+1$ it uses information in $t+1$, that is the reason we do not have E_t in the second first order condition above.

The crucial result of the equations above stems from the first two. The central bank optimises in period t and it finds that at the optimum, the inflation rate equals its Lagrange multiplier. Then, for all future inflation rates, it also takes into account the next period's Lagrange multipliers. So the rule for $t+1$ inflation is

$$\beta\pi_{t+1} + \beta\lambda_t - \beta\lambda_{t+1} = 0$$

But what if the central bank were to re-do the whole Lagrangean all over again but starting in $t + 1$? Then the first order condition for inflation in period $t + 1$ would be given by

$$\pi_{t+1} - \lambda_{t+1} = 0$$

Hence, the commitment solution is time inconsistent. What is going on? The whole problem arises from the Phillips curve. When optimising the central bank takes the $E_t\pi_{t+1}$ as endogenous: it tries to manipulate expectations in order to improve its inflation-output trade-off. However, once those expectations are set, the optimal behaviour is different. We can therefore think of λ_{t+s-1} as the value of one's past promise. It is optimal to cheat today (so today, the previous period's value of λ has a value of zero) but then to keep one's word. Obviously, that's not credible.

An alternative solution method is discretion. Here, the policy maker minimises losses but takes the expectations of future inflation as exogenous. In this case, the problem is no longer dynamic. We could think of this as

$$\max -\frac{1}{2}\left(\pi_t^2 + \omega x_t^2\right)$$

Subject to

$$\pi_t = F_t + \kappa x_t + u_t$$

And this is done every single period. In this case, the first order conditions are always given by:

$$\pi_t - \lambda_t = 0$$

$$\omega x_t + \kappa\lambda_t = 0$$

The losses under discretion will be higher than under commitment, as shown in Dennis [2008], but the latter policy suffers from time inconsistency. By contrast, discretion is time consistent.

In terms of how the differ in performance, the logic is as follows. Assume that there is a cost-push shock. By combining the first order conditions you will see that under commitment the central bank will cause a persistent, but small, reduction in the output gap. By making is persistent it affects expectations of future inflation and makes stabilisation easier. Under discretion we also get a contraction in the output gap, but it is not persistent as the central bank is not attempting to manipulate expectations. The end result is that under discretion the output gap (inflation) is stabilised too much (little) relative to commitment. This is called the **stabilisation bias** of discretion.

An alternative policy that delivers some of the gains from commitment without suffering from time inconsistency is called the 'timeless perspective'. The idea is to assume that the optimisation at some point in the past so that the effects of the first period policy have worn off. Hence, policy under the timeless perspective is just given by combining the first order condition for inflation derived above (for $s > 0$) and that for output so that combining them we have

$$\pi_t = -\frac{\omega}{\kappa}(x_t - x_{t-1}) \tag{10.15}$$

This is identical to the commitment policy described earlier except that now we assume that the policy in (10.15) is delivered every single period, including the present.

10.7 Inflation persistence

Some authors have criticised the NKPC because inflation is purely forward looking and possesses no persistence (dependence on past inflation), whereas empirically inflation has often been highly persistent. There are several extension of our framework that can result in a hybrid NKPC of the form:[14]

$$\pi_t = \beta\theta E_t\pi_{t+1} + (1-\theta)\pi_{t-1} + \kappa x_t + u_t$$

Why does it matter? Here is one example. Assume that $\beta = \kappa = 1$ and $\pi_{t-1} = 1$. If $\theta = 1$ so that inflation is purely forward looking, we have

$$\pi_t = E_t\pi_{t+1} + x_t + u_t$$

If $u_t = 1$ the central bank could let inflation absorb all the shock today $\pi_t = 1$ so that expected inflation and current output equal zero, or alternatively, keep both inflation rates at zero and let $x_t = -1$, so that we have a recession. If by contrast we have $\theta = 0$, then either inflation rises to 2 or the output gap is -2. So we have either a larger increase in inflation or a sharper recession.[15]

Hence, the more persistent inflation is, the costlier it is to stabilise the economy.

We can glean further insights by considering the timeless perspective in this model. Setting up the Lagrangean and differentiating with respect to inflation and output we have[16]

[14] This model is discussed in detail in McCallum and Nelson [2004].

[15] The actual outcome would be the solution of the first order conditions we had earlier. Which variable absorbs most of the shock depends on the value of ω.

[16] See McCallum and Nelson [2004] for details. The key thing to note is that the first order condition for π_{t+1} is also used for π_t, hence we only derive the former.

$$\pi_t + \beta^2(1-\theta)E_t\lambda_{t+1} + \theta\lambda_{t-1} - \lambda_t = 0 \tag{10.16}$$

$$\omega y_t = -\alpha\lambda_t \tag{10.17}$$

So that combining we have

$$\pi_t = -\frac{\omega}{\alpha}\left[\theta y_{t-1} - y_t + \beta^2(1-\theta)E_t y_{t+1}\right] \tag{10.18}$$

10.8 Empirical evidence

For this section I shall just provide some reading as the papers make the main points very clearly. Some useful papers are:

- Paez-Farrell [2009].
- Clarida et al. [1999].
- Clarida et al. [1998].

On the performance of the NKPC:

- Gali et al. [2005].
- Rudd and Whelan [2005b].
- Rudd and Whelan [2005a].

10.8.1 What are the impacts of monetary policy shocks? (Short run)

In the early lectures I mentioned that one way of assessing the short run relationships between monetary and real variables is to look at their dynamic cross correlations (correlations between any two variables at different leads and lags). An alternative framework is done via Vector Autoregressions (VARs). If Y is a vector of endogenous variables (such as money, output and prices), then the VAR can be written as

$$Y_t = AY_{t-1} + \epsilon_t$$

(The presentation here is intentionally very basic). The above is a VAR(1) as it only covers a one period lag but adding more does not alter the main intuition. From the above equation and subject to identification restrictions one can model the impact of shocks to, say, the money supply on output. The impact of these shocks is called impulse response functions

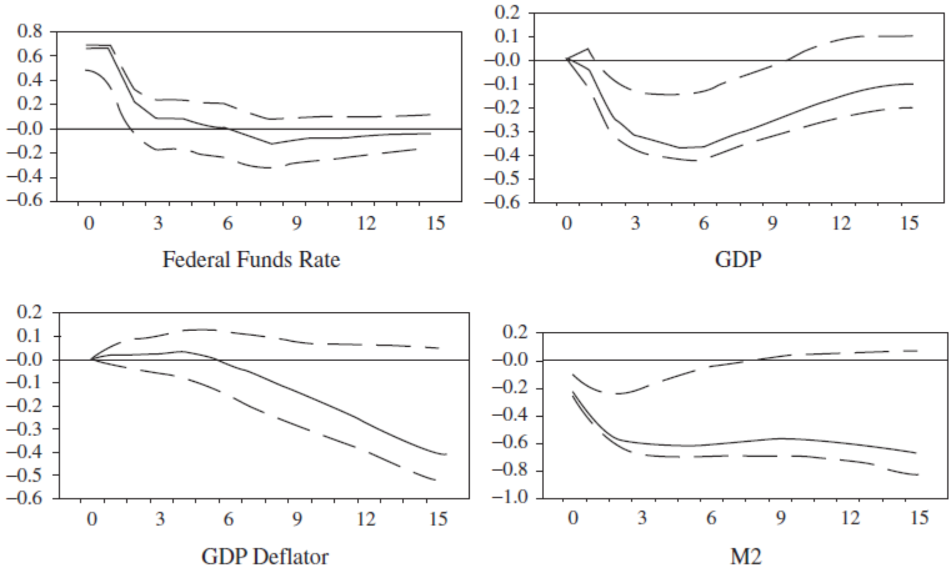

Figure 10.3: VAR impulse response from CEE 1999

and an example is given in Figure (10.3) for the US, taken from Christiano et al. [1999].

The crucial thing to note is that the data show that a monetary policy shock has a protracted effect on output and that it returns to its pre-shock level after approximately 10 quarters. Can the standard NK model replicate these effects of a monetary policy shock? It cannot; our standard model just embodying price stickiness is insufficient to generate such a persistence in output. We also need real rigidities.

10.9 Real rigidities and the persistence of output

In standard models with endogenous labour supply households maximise their utility subject to the budget constraint. Consider the very simple example of maximising (adding more periods would not make a difference)

$$u(c_t, n_t) = \ln c_t + \psi \ln (1 - n_t) \tag{10.19}$$

Subject to

$$c_t = w_t n_t \tag{10.20}$$

So that the household's only source of income is labour income and all of it is consumed. Combining the first order conditions we have

$$\frac{\psi}{1 - n_t} = \frac{w_t}{c_t}$$

Using the budget constraint and if labour is the only input (so that there is no investment and $c = y$) we have an implicit labour supply equation:

$$w_t = (1 + \psi)\, y_t$$

The equation above describes the behaviour of real wages in perfectly competitive markets. As we shall see below, this equation combined with the rest of the model will imply that the NK/DSGE model is unable to capture the persistence of output following a monetary shock. We can alternatively re-write the above equation as

$$w_t = \frac{1}{\alpha} y_t \tag{10.21}$$

Where if $1/\alpha = 1 + \psi$ we end up with the previous equation and the assumption of perfectly competitive labour. However, later we shall introduce real wage rigidities by allowing α to vary.

10.9.1 A simple NK model with real rigidities (Jeanne, 1998)

This model is due to Jeanne (1998) and I use his notation for consistency, although I have simplified some elements.

The starting setup is standard. We have a large number of households and firms. Households maximise their utility function, which depends on consumption and leisure. The consumption Euler equation is given by

$$u'(c_t) = \beta E_t \left(\frac{1 + R_t}{1 + \pi_{t+1}} u'(c_{t+1}) \right)$$

Where R is the nominal interest rate. If utility is separable in consumption and leisure and the former is of the CRRA form, we can write the above as:[17]

$$\widehat{c}_t = -\sigma \left[\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right] + E_t \widehat{c}_{t+1}$$

(Note that the variables are in deviation from steady state).

The model further assumes that the only production input is labour. As a result, there is neither investment nor capital so $c = y$

$$\widehat{y}_t = -\sigma \left[\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right] + E_t \widehat{y}_{t+1} \tag{10.22}$$

In other words, we have a dynamic IS.

Next, the model introduces monetary non-neutralities by imposing a cash in advance constraint (as opposed to MIU) so that money is required in order to carry out (nominal) expenditures:

[17] CRRA stands for constant relative risk aversion. This form of utility function is often used in macro models. The log utility function arises when σ approaches unity.

$$P_t c_t = M_t$$

Or in log-deviation:

$$\widehat{c}_t = \widehat{m}_t$$

Where m represents real money balances ((M/P)) and the 'hat' simply represents percentage deviation from steady state. In our model there is not capital (and hence no investment) so output equals consumption, giving

$$\widehat{y}_t = \widehat{m}_t \tag{10.23}$$

10.9.2 The supply side

Next, we assume Calvo pricing, where the fraction of firms that are unable to re-set prices is equal to ϕ. This will be our degree of nominal rigidities. Recall that when firms are able to re-set prices the respond to marginal costs. In our model there is a direct relationship between real marginal costs and the real wage (as labour is the only input) so we have

$$\widehat{w}_t = \widehat{mc}_t$$

And using the previous equation for the real wage

$$\widehat{w}_t = \frac{1}{\omega}\widehat{y}_t \tag{10.24}$$

The NKPC is given by

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi}\widehat{mc}_t \tag{10.25}$$

In our model there is a direct relationship between real marginal costs and the real wage (as labour is the only input) so we have

$$\widehat{w}_t = \widehat{mc}_t$$

And using the previous equation for the real wage

$$\widehat{w}_t = \frac{1}{\alpha}\widehat{y}_t$$

We can re-write the NKPC in terms of output:

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi\alpha}\widehat{y}_t \tag{10.26}$$

So far, we have the NKPC with two variables (output and inflation) plus our CIA constraint with output and money, making it two equations and three variables. If we used the interest rate as the policy instrument we would make use of the IS equation (linking y and R) plus an equation for the behaviour of R. In that case, the CIA constraint would be of little interest as it would just explain the behaviour of m. Instead, let us assume that that the central bank uses the money supply. The growth of the money supply is given by

$$\mu_t \equiv \frac{M_t}{M_{t-1}} - 1$$

and let us assume that it follows

$$\widehat{\mu}_t = \rho_m \widehat{\mu}_{t-1} + \epsilon_t \tag{10.27}$$

Where ϵ is a white noise process. In other words, whenever growth rate of the money supply is hit by a (money supply) shock ϵ, it takes some time – determined by ρ_m until it returns to its long run growth rate.

To get the equation for real money balances ($m = M/P$) we make use of the one above but we divide and multiply by the same number:

$$1 + \mu_t = \frac{M_t}{M_{t-1}} \frac{P_t}{P_t} \frac{P_{t-1}}{P_{t-1}}$$

$$1 + \mu_t = \frac{m_t}{m_{t-1}} \frac{P_t}{P_{t-1}}$$

$$1 + \mu_t = \frac{m_t}{m_{t-1}} (1 + \pi_t)$$

And in deviations from steady state:

$$\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t \tag{10.28}$$

10.9.3 Solving the model

Now we all of the equations we need. They are repeated below (I'm ignoring the 'hats' for simplicity):

$$y_t = m_t$$

$$\mu_t = m_t - m_{t-1} + \pi_t$$

$$\mu_t = \rho_m \mu_{t-1} + \epsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi\omega} y_t$$

We have four equations and four unknowns. Recall that we are here discussing the persistence of output so that is the variable we want to solve for. If we eliminate m we have:

$$\mu_t = y_t - y_{t-1} + \pi_t$$

$$\mu_t = \rho_m \mu_{t-1} + \epsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi\alpha} y_t$$

Eliminate π:

$$\mu_t = \rho_m \mu_{t-1} + \epsilon_t$$

$$(\mu_t - y_t + y_{t-1}) = \beta E_t \left(\mu_{t+1} - y_{t+1} + y_t\right) + \frac{(1-\phi)(1-\beta\phi)}{\phi\alpha} y_t$$

Combining these two equations we have

$$(1-\beta\rho_m)\,\mu_t - \left(1+\beta+\frac{(1-\phi)(1-\beta\phi)}{\phi\alpha}\right) y_t + y_{t-1} = -\beta E_t y_{t+1}$$

Our MSV solution is of the form:

$$y_t = \delta_1 y_{t-1} + \delta_2 \mu_t$$

And this implies

$$E_t y_{t+1} = \delta_1 y_t + \delta_2 \rho_m \mu_t$$

Substituting into the equation above we have

$$\mu_t - \left(1+\beta+\frac{(1-\phi)(1-\beta\phi)}{\phi\alpha}\right) \delta_1 y_{t-1} + \delta_2 \mu_t + y_{t-1} = \beta\rho_m \mu_t - \beta\left(\delta_1\left(\delta_1 y_{t-1} + \delta_2 \mu_t\right) + \delta_2 \rho_m \mu_t\right)$$

Collecting coefficients we have
For y_{t-1}:

$$-\left(1+\beta+\frac{(1-\phi)(1-\beta\phi)}{\phi\alpha}\right)\delta_1 + 1 = -\beta\delta_1^2 \tag{10.29}$$

If we solve the above for δ_1 we have two possible solutions but only one provides is with a stable solution for output.[18] The coefficient for δ_2 can be obtained from:

[18] The stable root satisfies $|\delta_1| \leq 1$.

$$1+\delta_2 = \beta\rho_m - \beta\delta_1\delta_2 + \beta\delta_2\rho_m$$

$$\delta_2 = \frac{\beta\rho_m - 1}{1+\beta\left(\delta_1 - \rho_m\right)}$$

The parameter δ_1 captures the persistence and amplitude of the response of output to a monetary shock and from the solution we note that δ_1 is increasing in

- ϕ
- and α

The intuition with respect to α is as follows: recall that at any point in time we have to groups of firms: flex-price and fixed-price producers. The latter cannot change their prices by assumption and the only reason why flex-price producers raise their price is that the increase in labour demand by fix-price producers bis up the real wage above its natural level. Therefore, the more rigid the real wage the less flex-price producers have to adjust their price and the more they adjust their quantities.

Given the above, we can note that for a given value of δ_1, ϕ and α are substitutes: we can vary their values whilst ensure that δ_1 is constant. Jeanne considers the range $\alpha \in [5, 15]$ as plausible for OECD economies. However, note that under the assumption of perfectly competitive markets α should be around 1/2, suggesting that hours worked and the real wage should be strongly correlated, although Christiano and Eichenbaum [1992] show that this is at odds with the data. Some of these parameter combinations are shown in Figure (10.4), where some important non-linearities are evident. More specifically, note that around $\phi = 1$ the iso-persistence curves are flat; this implies that only a small amount of wage rigidity is necessary to offset a large degree of nominal rigidity.[19] To see this in a different way, Figure (10.5) shows the impulse response functions of the model with values $\phi = 0.5$, $\alpha = 3$ and $\rho_m = 0.5$ and this is compared with the impulse response estimated on US data.

The other aspect is seeing how much rigidity we need in order to replicate impulse responses of output to monetary shocks. Figure (10.5) shows the IRF from the model with that for US output. The dotted line shows the impulse response function of output in our model to a monetary policy shock when prices are fixed for two quarters on average (this is a low degree of price rigidity) and $\alpha = 3$ and $\rho_m = 0.5$. As Jeanne points out, the model is able to capture the 'hump' response of output. Hence the model is able to mimic the persistence of output observed in the data with only small degrees of real and nominal rigidities.

[19] In other words, implausibly large amounts of real rigidities are not necessary.

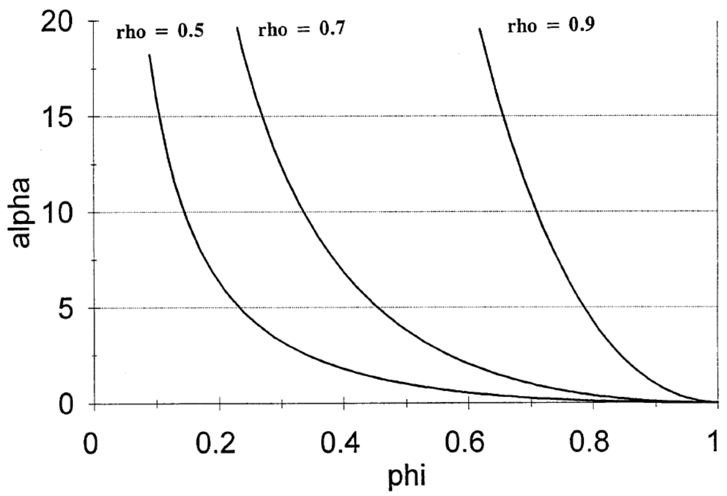

Figure 10.4: Iso-persistence curves in Jeanne's 1998 model ($\delta_1 = \rho$)

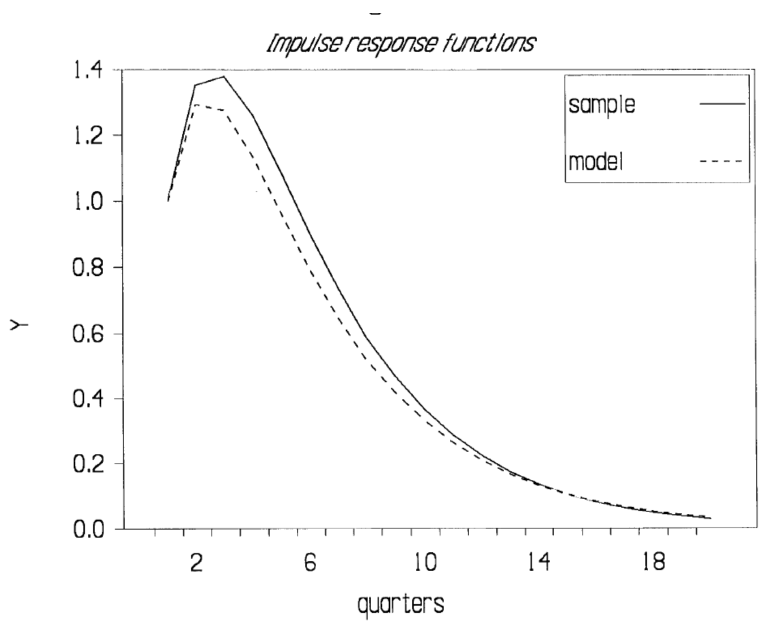

Figure 10.5: Impulse responses for the US

10.10 Final thoughts

The models analysed in this topic form the building blocks of the richer models used at central banks, think tanks, etc. for policy purposes and increasingly, for forecasting.

Returning to some of the key results: in the absence of shocks to the Phillips curve, there is no output (gap)- inflation stabilisation trade-off. In such a scenario a central bank that focuses solely on stabilising inflation will also succeed at fully stabilising the output gap. However, once shocks to the Phillips curve are introduced this is no longer the case.

One could similarly think about recent calls for central banks to stabilise the financial sector. However, for such proposals to make sense

1. We must have a clear definition of what we mean by financial stability
2. Monetary policy must be able to deliver financial stability
3. Having financial stability as an additional policy objective must pose a trade-off with the other goals

The third point just extends the basic NK model and the output-inflation trade-off. If no trade-off exists then asking including output in the policy objective is completely unnecessary; inf.ation stabilisation will deliver that. Likewise, if stabilising inflation and output also delivers financial stability, then it is not necessary to include it as an additional objective.

Recent promising research that considers financial stability in terms of build ups in leverage and interest rate spreads shows that central bank actions may affect the composition of banks' balance sheets and hence the risks that they take, posing risks for overall financial stability.[20]

[20] See Gertler and Kiyotaki [2010] and Curdia and Woodford [2009].

Chapter 11

Summary of different models

11.1 A roughly chronological sequence of macroeconomic models

The models presented here are highly stylised. There are also other models (or extensions of these models) not considered here.

11.1.1 IS-LM

$$y_t = -\sigma r_t + u_{1,t}$$
$$m_t = \alpha y_t - \beta r_t + u_{2,t}$$

Plus an equation describing monetary policy.

- $u_{x,t}$ is a shocks, for $x = \{1, 2\}$.
- Prices are assumed fixed so real and nominal interest rates are the same.
- Ignores expectations
- Any normative discussion will inherently use ad hoc objectives.

11.1.2 Neoclassical synthesis

$$y_t = -\sigma r_t + u_{1,t}$$
$$m_t = \alpha y_t - \beta R_t + u_{2,t}$$
$$R_t = r_t + \pi^e_{t+1}$$
$$\pi^e_t = \pi_{t-1}$$
$$\pi_t = pi^e_t + \delta\left(y_t - y^*\right)$$

Plus a description of monetary policy

- Model now allows for inflation.
- Although the model includes expectations, these are adaptive.
- The supply side is included (PC).
- Any normative discussion will inherently use ad hoc objectives.
- Model is subject to the Lucas critique.

11.1.3 Models with rational expectations

$$y_t = -\sigma r_t + u_{1,t}$$

$$m_t = \alpha y_t - \beta R_t + u_{2,t}$$

$$R_t = r_t + E_t \pi_{t+1}$$

$$\pi_t = \pi_t^e + \delta (y_t - y^*) + u_{3,t}$$

Plus a description of monetary policy.

- Now with rational expectations agents do not make systematic mistakes.
- Policy announcements matter.
- Model allows for inflation.
- Any normative discussion will inherently use ad hoc objectives.
- But still subject to Lucas critique.

Two further points:

1. In all the models considered above the shocks do not always have a structural interpretation. For example, we can call the IS shocks, demand shocks, etc. But what exactly are they? Where do they come from?
2. The models above can all be interpreted in terms of supply and demand, IS-LM etc.

11.1.4 Micro-founded models

(This specific model is based on Uhlig, Section 4.4)
Households: given K_0, market wages w_t and returns R_t, the representative agent chooses $\{c_t, k_t\}_{t=0}^{\infty}$ to maximise

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\eta} - 1}{1 - \eta} \right]$$

Subject to $n_t^s = 1$ (inelastic labour supply) and

$$C_t + K_{t+1} = W_t N_t^s + R_t K_t$$

Given wages and returns, the representative firm chooses $\{k_t, n_t\}_{t=0}^{\infty}$ to maximise

$$Z_t K_t^{\alpha} N_t^{1-\alpha} + (1 - \delta) K_t - W_t N t - R_t K_t$$

where $Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha}$.

Markets clear:

$$N_t = 1$$

$$C_t + K_t - (1 - \delta) K_t = Z_t K_t^{\alpha}$$

- The above give rise to a (typically non-linear) rational expectations model.
- If correctly specified, the model is not subject to the Lucas critique.
- Can discuss welfare and policy objectives by focusing on the welfare of the representative agent.

The non-linear rational expectations model is given by (from first order conditions and constraints):

$$C_t = Z_t K_t^{\alpha} + (1 - \delta) K_t - K_{t+1}$$

$$R_t = \alpha Z_t K_t^{\alpha - 1} + (1 - \delta)$$

$$1 = E_t \left[\beta \left(\frac{C_t}{C_{t+1}} \right)^{\eta} R_{t+1} \right]$$

$$\log Z_t = (1 - \rho) \log \overline{Z} + \rho \log Z_{t-1} + \epsilon_t$$

Log-linearising the system above we have (small case denote log-deviations from steady state and capitals are the steady state values):

$$c_t = \frac{Y}{C} z_t + \frac{K}{C} R k_t - \frac{K}{C} k_t$$

$$r_t = (1 - \beta(1 - \delta)) (z_t + (\alpha - 1) k_t)$$

$$c_t = E_t c_{t+1} - \eta^{-1} r_{t+1}$$
$$z_t = \rho z_{t-1} + \epsilon_t$$
$$y_t = z_t + \alpha k_t$$

This is a linear rational expectations model that could be solved using the msv method. As there are no constants (because the variables are all in deviations from steady state) the only state variables are k_t and z_t (or alternatively, k_t, z_{t-1} and ϵ_t). See Uhlig for further details.

Note that you can add shocks to the first set of equations (preferences, the production function, ...) but not suddenly in the linear system. Kydland and Prescott emphasised the role of technology shocks (z_t) as a driver of business cycles.

Here shocks have a clear structural interpretation because we can see exactly where they originate.

New Keynesian models

$$x_t = E_t x_{t+1} - \sigma \left(R_t - E_t \pi_{t+1} \right) + g_t$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \mu_t$$

Plus some description of monetary policy (normally an interest rate rule)

- It is important to note that this is based on the RBC model but with the addition of nominal rigidities.
- In this model monetary does have real effects

Note that for both the RBC and NK models (and DSGE models in general) thinking in terms of supply and demand etc. will often be unhelpful.

For example, assume government spending (and hence also taxes due to the budget constraint; we shall assume lump-sum taxes) in the RBC model. Assuming a given process for g_t (that for taxes is unnecessary as their present value has to be consistent with the present value of spending) we have:

1. A positive government spending shock will imply that at some point, taxes will have to increase by the same amount in present value terms;
2. as a result, households will be poorer;
3. if both consumption and leisure are normal goods then they will decrease their choice of leisure;
4. this implies more labour supply;
5. output would therefore rise;
6. real wages would fall.

See the file `UhligHansen.mod` on Mole.

Chapter 12

CIA Model (Walsh)

12.1 Model equations

$$z_t = \rho_z z_{t-1} + \epsilon_{z,t}$$

$$u_t = \rho_u u_{t-1} + \phi z_{t-1} + \epsilon_{u,t}$$

$$y_t = \alpha k_{t-1} + (1-\alpha) n_t + z_t$$

$$\frac{Y}{K} y_t = \frac{C}{K} c_t + \delta i_t$$

$$k_t = (1-\delta) k_{t-1} + \delta i_t$$

$$R_t = \alpha \frac{Y}{K} E_t \left(y_{t+1} - K_t \right)$$

$$y_t = \left(1 + \left(\frac{\eta N}{1-N} \right) \right) n_t - U_{c,t}$$

$$U_{c,t} = E_t U_{c,t+1} + R_t$$

$$U_{c,t} = -\Phi c_t - i_t$$

$$m_t = c_t$$

$$m_t = m_{t-1} + u_t - \Pi_t$$

$$I_t = R_t + E_t \Pi_{t+1}$$

Variables are: z_t is technology shock, u_t is the money growth shock; y_t is output; k_{t-1} is capital; n_t is labour, i_t is investment; R_t is the real interest rate; $U_{c,t}$ is the marginal utility of consumption; m_t is real money balances, Π_t is inflation, I_t is the nominal interest rate.

List of Figures

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Appendix A

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