CPSC 314
Assignment 2

due: Wednesday, October 7, 2015, in class

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: ______________________________

Student Number: ______________________________

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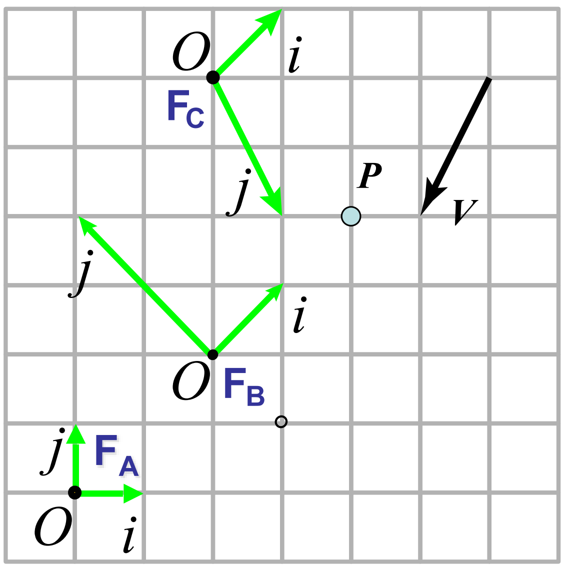
1. Transformations as a change of coordinate frame



(a) (1 point) Express the coordinates of point P with respect to coordinate frames A, B, and C.

(b) (1 point) Express the coordinates of vector V with respect to coordinate frames A, B, and C.

$$\begin{bmatrix} \\ \\ 1 \end{bmatrix}_A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ 1 \end{bmatrix}_C$$

$$\begin{bmatrix} \\ \\ 1 \end{bmatrix}_B = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ 1 \end{bmatrix}_A$$

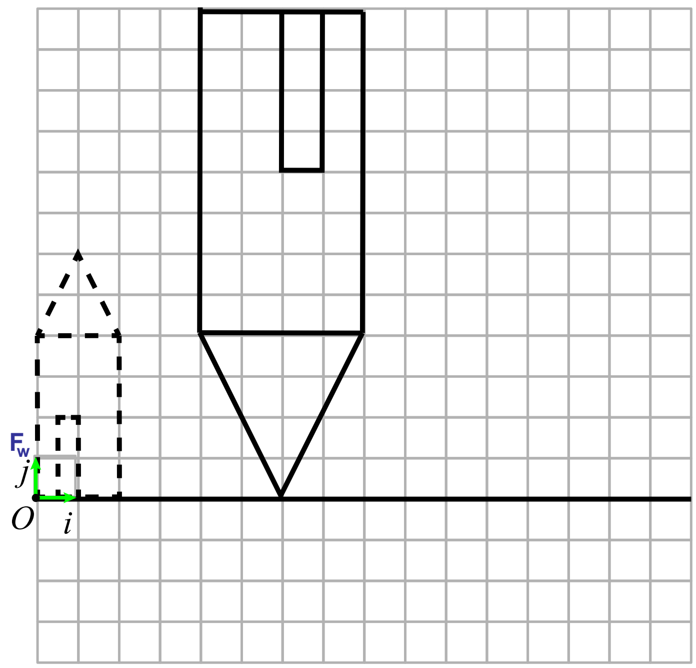
(c) (2 points) Fill in the 2D transformation matrix that takes points from F_C to F_A, as given to the right of the above figure.

(d) (2 points) Fill in the 2D transformation matrix that takes points from F_A to F_B, as given to the right of the above figure.

(e) (2 points) Using the above two matrices, develop a 2D transformation matrix that takes points from F_C to F_B. Test your solution using point P.

2. Composing Transformations

(a) (2 points) Oh no! After a rowdy party, your roommates the decepticons have made a mess of the house! It's up to the transformers to defuse the situation with your landlord. You must help them put house on the ground where it was and make it the correct size (the initial house is drawn in dashed line). What sequence of primitive affine transformations (translate, rotate, scale, shear) should you do?



(b) (2 points) Give the resulting 4×4 transformation matrix. Assume that the transformation leaves z to be unaltered.

$$\begin{bmatrix} \\ \\ \\ 1 \end{bmatrix}_W = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} \\ \\ \\ 1 \end{bmatrix}_{obj}$$

(c) (2 points) What values would need to be assigned to `theta`, `a`, `b`, `c`, `i`, `j`, `k`, `x`, `y`, `z` in order for the following transformations to yield an identical final transformation? Note, `THREE.Matrix4()` constructs an identity matrix and `p,q,r`$\in$ $\{$`m1`, `m2`, `m3`$\}$

```
var m = new THREE.Matrix4();
    var m1 = new THREE.Matrix4();
    var m2 = new THREE.Matrix4();
    var m3 = new THREE.Matrix4();
m1.makeRotateAxis(new THREE.Vector3(i,j,k), theta);
m2.makeScale(a,b,c);
m3.makeTranslate(x,y,z);
m = m3*m2*m1;
house.geometry.applyMatrix(m);
```

3. Decompose the following complex transformations in homogeneous coordinates into a product of simple transformations (scaling, rotation, translation, shear). Pay attention to the order of transformations.

   (a) (1 point)

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (b) (1 point)

$$\begin{bmatrix} 4 & 0 & 0 & -2 \\ 0 & 0.2 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (c) (2 points) What are the inverses of the matrices of parts (a) and (b) above? Hint: if $M = AB$, then $M^{-1} = B^{-1}A^{-1}$

   (d) (2 points) Give the sequence of THREE.js transformations that would produce the same transformation matrix as in part (a) of this question.

4. Rotation Matrices

   (a) (2 points) The columns of a rotation matrix have unit magnitude and they should all be orthogonal to each other, i.e., have a zero dot product. Show that the inverse of a rotation matrix is given by its transpose.

   (b) (4 points) In order for a rotation matrix to represent a rigid body rotation, there is one more constraint that it should satisfy, in addition to those listed above. Which of the following 4×4 matrices represent a valid rotation matrix? Why or why not? What is the extra constraint that rotation matrices should satisfy?

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$