CPSC 314
Assignment 4

due: Wednesday, Dec, 2nd 2014, in class

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: SOLUTION

Student Number: ______________________

Question 1	/ 18
Question 2	/ 12
TOTAL	/ 30

1. (18 points) Ray Intersections with an Ellipse Given a ray originating at $C = (1, -1)$ with a direction $v = (1, 1)$, determine whether it will intersect an ellipsoid given by the implicit equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$. If it does intersect the ellipse, find the coordinates of the intersection and normal to the ellipse at that point. Reminder: use parametric ray equation: $\mathbf{P}(t) = \mathbf{C} + t\mathbf{v}, t \geq 0$

   Solution. Let's start by finding intersection points. Writing out the parametric ray equation for our point and direction, and the ellipsoid equation, we get a system of equations and an inequality:

$$\begin{cases} x = 1 + t \\ y = -1 + t \\ t > 0 \\ \frac{x^2}{9} + \frac{y^2}{4} = 1 \end{cases}$$

   By substituting first two equations into the third one, we get:

$$(1 + t)^2/9 + (-1 + t)^2/4 = 1,$$

   which can be simplified as

$$13t^2 - 10t - 23 = 0.$$

   The two roots of this equation are

$$t_1 = -1, t_2 = 23/13$$

   Those two roots correspond to the intersections of a **line** passing through the ellipse; in our case it's a ray, so we take the positive root only. Since one t is negative and one is positive, this proves that the point C is inside the ellipse. Thus, the intersection point is $(36/13, 10/13)$.

   To find the normal at that point, let's compute gradient of the ellipse equation:

$$\begin{pmatrix} n_x \\ n_y \end{pmatrix} = \begin{pmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{pmatrix} = \begin{pmatrix} 2x/9 \\ y/2 \end{pmatrix} = \begin{pmatrix} 8/13 \\ 5/13 \end{pmatrix}$$

   Optionally you can normalize the normal, but that was not required.

2. (12 points) Light and Color Please answer the following questions regarding light and color. Explain your answer.

   - Is color a physical property of light? *No, it's a property of perception. Even though color depends on the physical light properties, such as frequency and intensity, color is defined by what we perceive.*
   - Imagine you are in 19th century. How would you test how many different types of cones do people have? *Well first we have to assume we know everything about light that we know now. We just have no idea what's happening in the eye. Then the logical thing to do was to try mixing lights of different frequency (say, spectral green and spectral red) and comparing that to pure spectral light. In 19th century it was probably not trivial to get spectral colors, but we could potentially use a rainbow as a 'dirty approximation'. Colors of the rainbow are not pure, but close enough for our experiment. Now all we have to create a rainbow using a prism (or find one, if you're up to it), and now using some sequence of lenses mix red and green and let the viewer decide if it's the same as yellow part of the rainbow. This way we'll figure out if we have cones sensitive to yellow light (which we don't have). And so on. It's certainly not a clean and bullet-proof experiment though.*
   - Can modern displays show all the colors we can see? If yes, how? If no, why? *No. Any modern display can mix a few colors, therefore forming a convex polygon on the chromaticity diagram (XYZ) - gamut. Since we have to start with some color, all the vertices of that polygon must lie on the horseshoe diagram. And since the horseshoe diagram is not a convex polygon, we can't cover it this way.*
   - If we mix cyan ink with yellow ink in equal proportions, what color will we see? What color will get absorbed? *Cyan ink has blue-green color, so it absorbs red. Yellow is a mix of red and green light, so yellow ink absorbs blue. When mixed, the final mixture will absorb red and blue, therefore reflecting green light.*