Beyond-the-standard-model contributions to rare B decays analyzed with variational-Bayes enhanced adaptive importance sampling

Stephan Jahn

March 17, 2015

Overview

Bayes' formula:

$$P(\boldsymbol{\theta}|\mathcal{D},\mathrm{M}) = \frac{P(\mathcal{D}|\boldsymbol{\theta},\mathrm{M})P(\boldsymbol{\theta}|\mathrm{M})}{P(\mathcal{D}|\mathrm{M})} = \frac{P(\mathcal{D}|\boldsymbol{\theta},\mathrm{M})P(\boldsymbol{\theta}|\mathrm{M})}{\int P(\mathcal{D}|\boldsymbol{\theta},\mathrm{M})P(\boldsymbol{\theta}|\mathrm{M})d\boldsymbol{\theta}}$$

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model independent search for new physics (effective theory):

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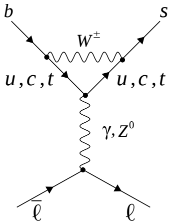
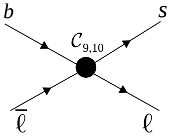


$\boldsymbol{\theta}$ = effective couplings $\mathcal{C}_i$, ...



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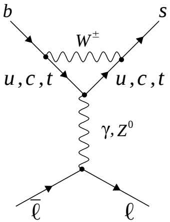

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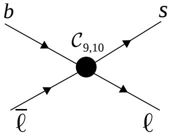

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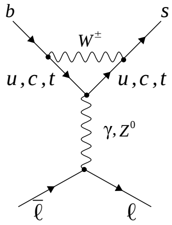

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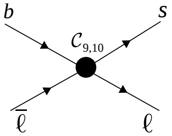
$$\mathrm{M} = \mathrm{EFT}, \mathrm{SM}, \ldots$$



Goals

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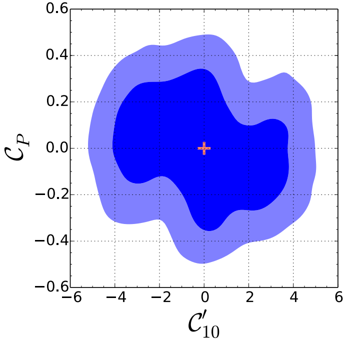

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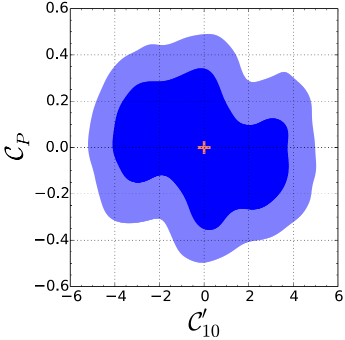

- compare models
  NP $\leftrightarrow$ SM

$$\frac{P(\mathrm{NP}|\mathcal{D})}{P(\mathrm{SM}|\mathcal{D})} = \frac{P(\mathcal{D}|\mathrm{NP})}{P(\mathcal{D}|\mathrm{SM})} \cdot \frac{P(\mathrm{NP})}{P(\mathrm{SM})}$$

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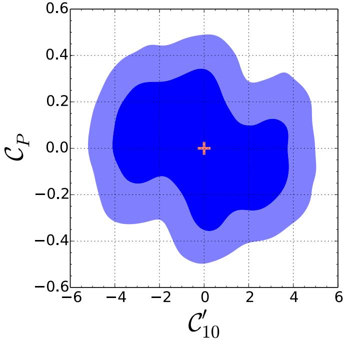

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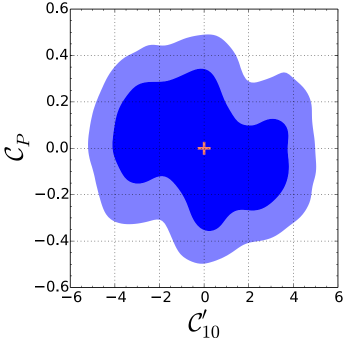

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$\frac{P(\mathrm{NP}|\mathcal{D})}{P(\mathrm{SM}|\mathcal{D})} < 1$ confirm SM ☹

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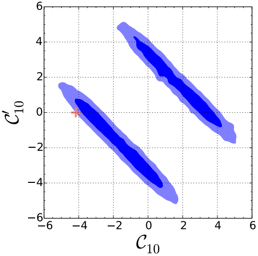

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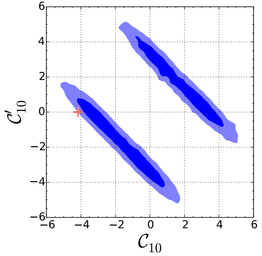

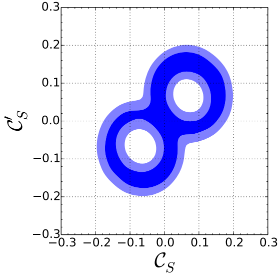

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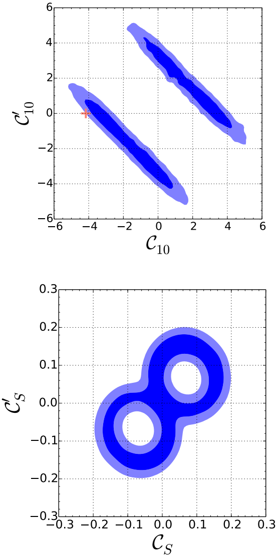
no standard algorithm so far



Contents

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  - known as the *Kullback-Leibler divergence*

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minimize $KL(F\|p)$ and hope to approach the unique global minimum $F = p$

restrict p to Gaussian mixtures

$$p(x_n|\boldsymbol{\theta}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(x_n|\mu_k, \Sigma_k), \quad \boldsymbol{\theta} = \{\pi_k, \mu_k, \Sigma_k\}$$

Variational Bayes (Gaussian mixture model)

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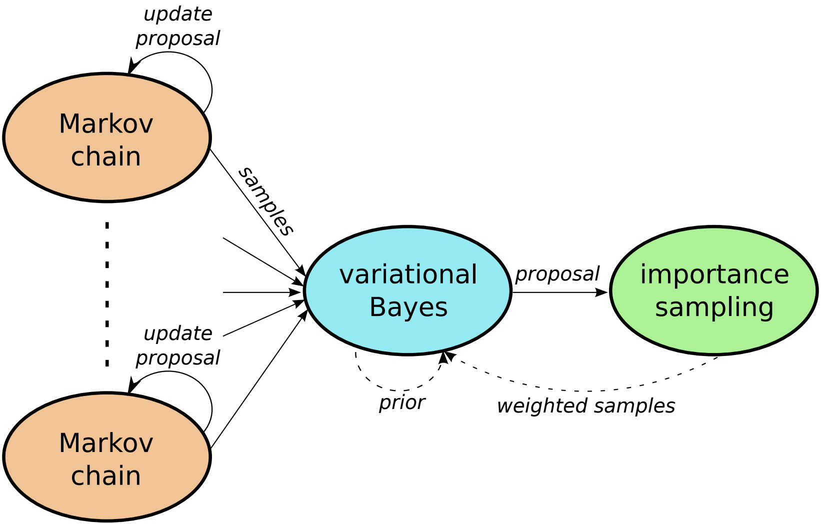
assumption $q(\mathbf{Z}, \boldsymbol{\theta}) = q(\mathbf{Z})q(\boldsymbol{\theta})$

$\Rightarrow$ iterative solution

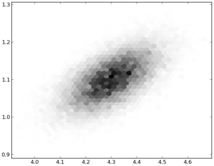
further reading:

Christopher M. Bishop, *Pattern Recognition and Machine Learning*, Springer 2006, chapter 10

Adaptive importance sampling with the variational-Bayes approach



```
'''This example illustrates how to run a Markov Chain using pypmc'''

import numpy as np
import pypmc

# define a proposal
prop_dof   = 50.
prop_sigma = np.array([[0.1 , 0.  ]
                      ,[0.  , 0.02]])
prop = pypmc.density.student_t.LocalStud

# define the target; i.e., the function
# In this case, it is a Gaussian with me
# covariance "target_sigma".
#
# Note that the target function "log_tar
# unnormalized gaussian density.
target_sigma = np.array([[0.01 , 0.003 ]
                        ,[0.003, 0.0025]])
inv_target_sigma = np.linalg.inv(target_sigma)
target_mean  = np.array([4.3, 1.1])

def unnormalized_log_pdf_gauss(x, mu, inv_sigma):
    diff = x - mu
    return -0.5 * diff.dot(inv_sigma).dot(diff)

log_target = lambda x: unnormalized_log_pdf_gauss(x, target_mean, i

# choose a bad initialization
start = np.array([-2., 10.])

# define the markov chain object
mc = pypmc.sampler.markov_chain.AdaptiveMarkovChain(log_target, pro

# run burn-in
mc.run(10**4)

# delete burn-in from history
mc.history.clear()
```



(effective samples).

- **rel_tol** –
  Relative tolerance ϵ. If two consecutive values of the log likelihood bound, L_t, L_{t-1}, are close, declare convergence. More precisely, check that
  $$\left\| \frac{L_t - L_{t-1}}{L_t} \right\| < \epsilon.$$
- **abs_tol** –
  Absolute tolerance ϵ_a. If the current bound L_t is close to zero, $(L_t < \epsilon_a)$, declare convergence if
  $$\| L_t - L_{t-1} \| < \epsilon_a.$$
- **verbose** – Output status information after each update.

`set_variational_parameters()`

Reset the parameters to the submitted values or default.

Use this function to set the prior value (indicated by the subscript 0 as in α_0) or the initial value (e.g., α) used in the iterative procedure to find the posterior value of the variational distribution.

Every parameter can be set in two ways:

1. It is specified for only one component, then it is copied to all other components.
2. It is specified separately for each component as a K vector.

The prior and posterior variational distributions of $\boldsymbol{\mu}$ and $\boldsymbol{\Lambda}$ for each component are given by

$$q(\boldsymbol{\mu}, \boldsymbol{\Lambda}) = q(\boldsymbol{\mu}|\boldsymbol{\Lambda})q(\boldsymbol{\Lambda}) = \prod_{k=1}^{K} \mathcal{N}(\boldsymbol{\mu}_k|\boldsymbol{m}_k, (\beta_k \boldsymbol{\Lambda}_k)^{-1})\mathcal{W}(\boldsymbol{\Lambda}_k|\boldsymbol{W}_k, \nu_k),$$

where $\mathcal{N}$ denotes a Gaussian and $\mathcal{W}$ a Wishart distribution. The weights $\boldsymbol{\pi}$ follow a Dirichlet distribution

$$q(\boldsymbol{\pi}) = Dir(\boldsymbol{\pi}|\boldsymbol{\alpha}).$$

Warning

This function may delete results obtained by `update()`.

https://pypi.python.org/pypi/pypmc

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- neutrino masses
- hierarchy problem
- strong CP problem
- ...

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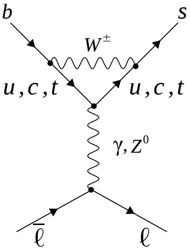
exact structure unknown $\Rightarrow$ model independent analysis

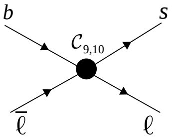
Effective theory

effective Lagrangian for $b \to s\ell^+\ell^-$ (SM):

$$\mathcal{L}_{int} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i \mathcal{O}_i + \ldots + \text{h.c.}$$

$$\mathcal{O}_9 = \left[\bar{s}\gamma_\mu P_L \quad b\right]\left[\bar{\ell}\gamma^\mu \ell\right] \qquad \mathcal{O}_{10} = \left[\bar{s}\gamma_\mu P_L \quad b\right]\left[\bar{\ell}\gamma^\mu \gamma_5 \ell\right]$$





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effective Lagrangian for $b \to s\ell^+\ell^-$ (beyond-SM):

$$\mathcal{L}_{int} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i \mathcal{O}_i + \ldots + \text{h.c.}$$

$$\mathcal{O}_9^{(\prime)} = \left[\bar{s}\gamma_\mu P_{L(R)} b\right] \left[\bar{\ell}\gamma^\mu \ell\right] \qquad \mathcal{O}_{10}^{(\prime)} = \left[\bar{s}\gamma_\mu P_{L(R)} b\right] \left[\bar{\ell}\gamma^\mu \gamma_5 \ell\right]$$

$$\mathcal{O}_S^{(\prime)} = \left[\bar{s} P_{R(L)} b\right] \left[\bar{\ell}\ell\right] \qquad \mathcal{O}_P^{(\prime)} = \left[\bar{s} P_{R(L)} b\right] \left[\bar{\ell}\gamma_5 \ell\right]$$

$$\mathcal{O}_T = \left[\bar{s}\sigma_{\mu\nu} b\right] \left[\bar{\ell}\sigma^{\mu\nu} \ell\right] \qquad \mathcal{O}_{T5} = \left[\bar{s}\sigma_{\mu\nu} b\right] \left[\bar{\ell}\sigma^{\mu\nu}\gamma_5 \ell\right]$$

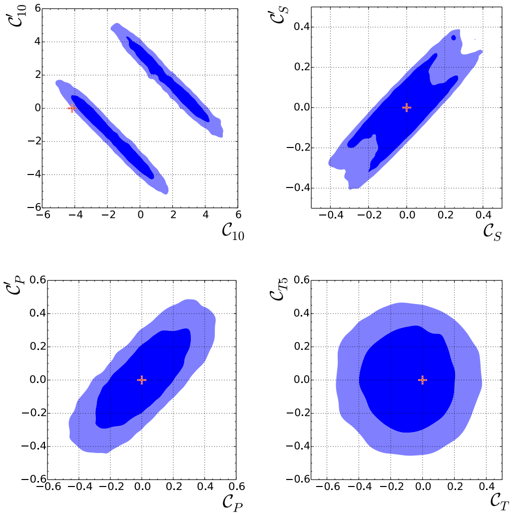
Measurements $P(\mathcal{D}|\mathbf{O})$

- $B \to K\mu^+\mu^-$: $\mathcal{B}, A_{FB}, F_H$
  - LHCb 2014 (arXiv:1403.8044 , arXiv:1403.8045)
  - CDF 2012

    (http://www-cdf.fnal.gov/physics/new/bottom/120628.blessed-b2smumu_96)
- $B_s \to \mu^+\mu^-$: $\mathcal{B}$
  - LHCb+CMS 2014 (arXiv:1411.4413)
- $B \to K^*\mu^+\mu^-$: $\mathcal{B}$
  - LHCb 2013 (arXiv:1304.6325)
  - CMS 2013 (arXiv:1308.3409)
  - CDF 2012

    (http://www-cdf.fnal.gov/physics/new/bottom/120628.blessed-b2smumu_96)

Joint fit of $\mathcal{C}_{10}^{(\prime)}, \mathcal{C}_S^{(\prime)}, \mathcal{C}_P^{(\prime)}, \mathcal{C}_T$, and $\mathcal{C}_{T5}$



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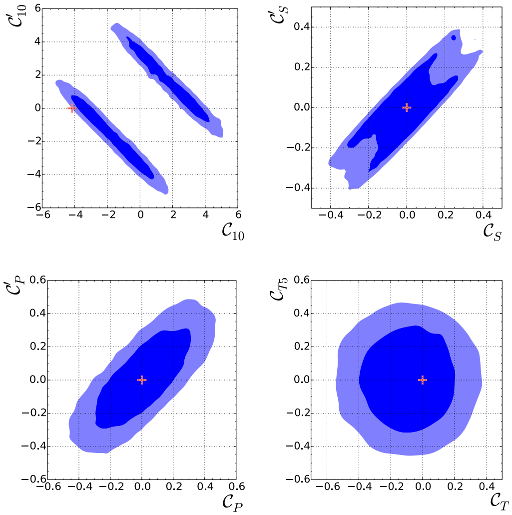
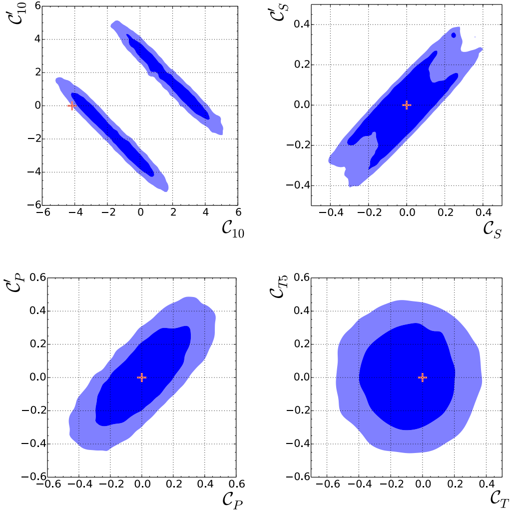


- first *simultaneous* fit



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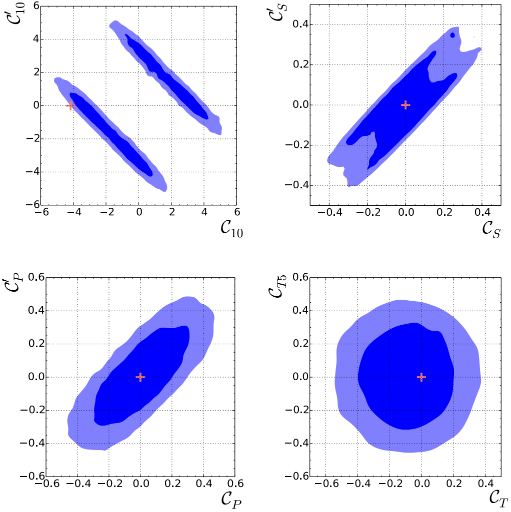

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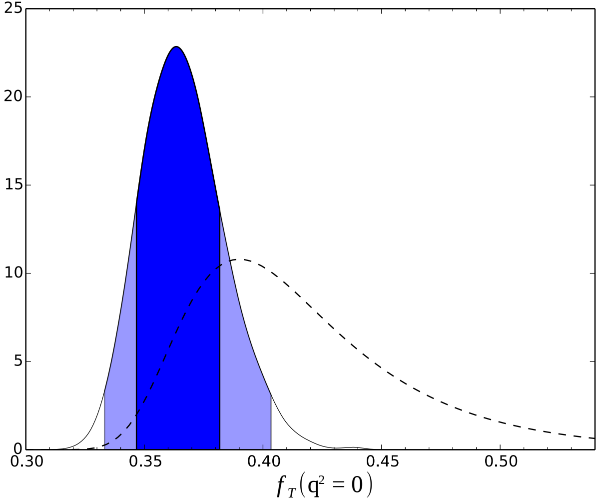
- interference $\mathcal{C}_{10}^{(\prime)} \leftrightarrow \mathcal{C}_{S,P}^{(\prime)}$ in $\mathcal{B}(B_s \to \mu^+\mu^-)$
  - $\Rightarrow$ larger uncertainty than obtained for fixed $\mathcal{C}_{10}^{(\prime)} = \mathcal{C}_{10}^{(\prime)\mathrm{SM}}$

    arXiv:1205.5811,

    arXiv:1206.0273,

    arXiv:1407.7044

Nuisance parameters



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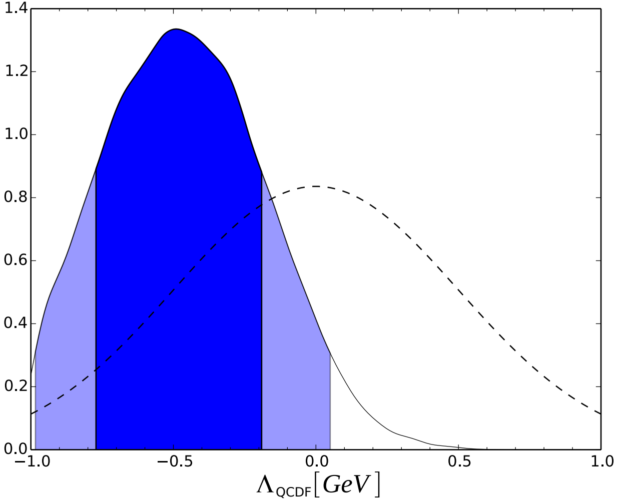

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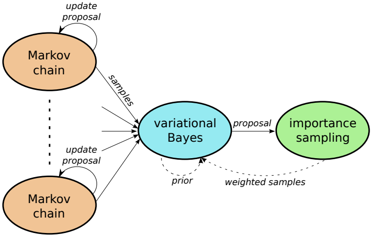
algorithm to sample and integrate in dim = $\mathcal{O}(40)$



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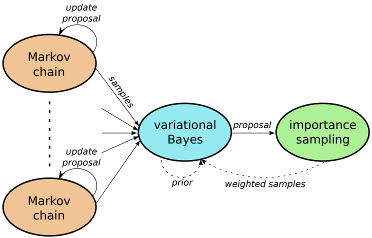

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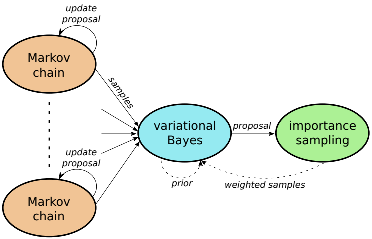

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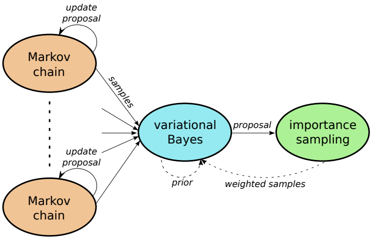

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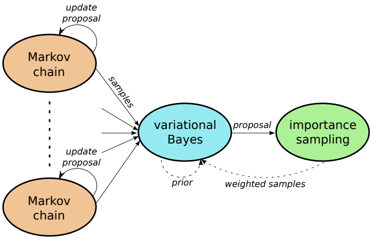

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  $\Rightarrow$ updated constraints
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- need better theoretical control (form factors, QCDF)

Parameters $\boldsymbol{\theta}$

scan parameters

- Wilson coefficients $\mathcal{C}_{10}^{(\prime)}, \mathcal{C}_{S}^{(\prime)}, \mathcal{C}_{P}^{(\prime)}, \mathcal{C}_{T}$, and $\mathcal{C}_{T5}$

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- charm and bottom quark mass (2 parameters)
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theory calculation $\mathbf{O}(\theta, \mathbf{M})$:

open-source implementation: EOS-package

`http://project.het.physik.tu-dortmund.de/eos/`

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we want:

$$P(\boldsymbol{\theta}|\mathcal{D},\mathrm{M}) \overset{\text{Bayes}}{\propto} P(\mathcal{D}|\boldsymbol{\theta},\mathrm{M})P(\boldsymbol{\theta}|\mathrm{M})$$

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Sensitive observables $\mathbf{O}$

- **$\mathbf{B} \to \mathbf{K}\mu^+\mu^-$ angular distribution**

- **$\mathbf{B_s} \to \mu^+\mu^-$ branching fraction**

Sensitive observables O

- **B → K$\mu^+\mu^-$ angular distribution**

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \frac{3}{4}(1 - F_H)\sin^2\theta + \frac{1}{2}F_H + A_{FB}\cos\theta$$

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$$\mathcal{B}(B_s \to \mu^+\mu^-) \propto |\mathcal{C}_S - \mathcal{C}'_S|^2 + |(\mathcal{C}_P - \mathcal{C}'_P) + \frac{2m_\ell}{M_{B_s}}(\mathcal{C}_{10} - \mathcal{C}'_{10})|^2$$