

Beyond-the-standard-model contributions to rare B
decays analyzed with variational-Bayes enhanced
adaptive importance sampling

Stephan Jahn

March 16, 2015

Bayes' formula:

$$P(\boldsymbol{\theta}|\mathcal{D}, M) = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{P(\mathcal{D}|M)} = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{\int P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)d\boldsymbol{\theta}}$$

Bayes' formula:

$$P(\boldsymbol{\theta}|\mathcal{D}, M) = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{P(\mathcal{D}|M)} = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{\int P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)d\boldsymbol{\theta}}$$

our application:

Bayes' formula:

$$P(\boldsymbol{\theta}|\mathcal{D}, M) = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{P(\mathcal{D}|M)} = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{\int P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)d\boldsymbol{\theta}}$$

our application:

$$\boldsymbol{\theta} = \{\mathcal{C}_{7,9,10,S,P,T,T5}^{(i)}, \dots\}$$

Bayes' formula:

$$P(\boldsymbol{\theta}|\mathcal{D}, M) = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{P(\mathcal{D}|M)} = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{\int P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)d\boldsymbol{\theta}}$$

our application:

$$\boldsymbol{\theta} = \{\mathcal{C}_{7,9,10,S,P,T,T5}^{(i)}, \dots\}$$

\mathcal{D} = detector events

Bayes' formula:

$$P(\boldsymbol{\theta}|\mathcal{D}, M) = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{P(\mathcal{D}|M)} = \frac{P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)}{\int P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)d\boldsymbol{\theta}}$$

our application:

$$\boldsymbol{\theta} = \{\mathcal{C}_{7,9,10,S,P,T,T5}^{(i)}, \dots\}$$

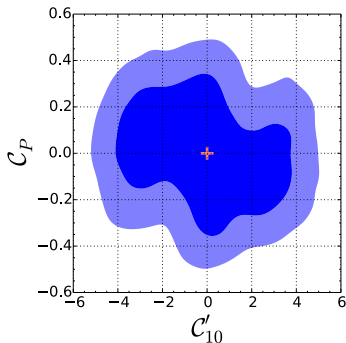
\mathcal{D} = detector events

M = EFT, SM, ...

Goals

Goals

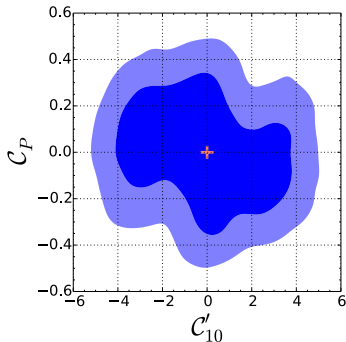
- draw marginal plots of the posterior



Goals

- draw marginal plots of the posterior

- compare models
 $\text{NP} \leftrightarrow \text{SM}$



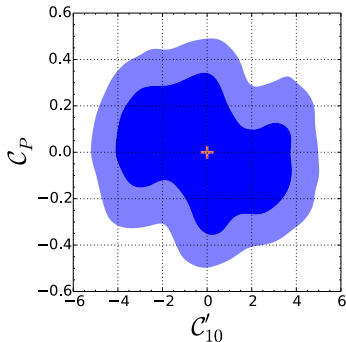
$$\frac{P(\text{NP}|\mathcal{D})}{P(\text{SM}|\mathcal{D})} = \frac{P(\mathcal{D}|\text{NP})}{P(\mathcal{D}|\text{SM})} \cdot \frac{P(\text{NP})}{P(\text{SM})}$$

$$P(\text{M}|\mathcal{D}) = \frac{P(\mathcal{D}|\text{M})P(\text{M})}{P(\mathcal{D})}$$

Goals

- draw marginal plots of the posterior

- compare models
 $\text{NP} \leftrightarrow \text{SM}$



$$\frac{P(\text{NP}|\mathcal{D})}{P(\text{SM}|\mathcal{D})} = \frac{P(\mathcal{D}|\text{NP})}{P(\mathcal{D}|\text{SM})} \cdot \frac{P(\text{NP})}{P(\text{SM})}$$

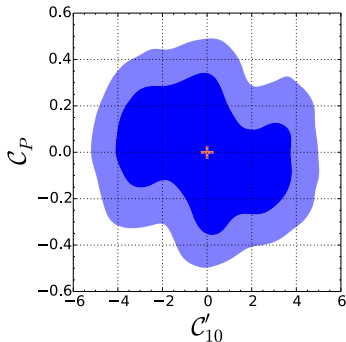
$$P(\text{M}|\mathcal{D}) = \frac{P(\mathcal{D}|\text{M})P(\text{M})}{P(\mathcal{D})}$$

$$\frac{P(\text{NP}|\mathcal{D})}{P(\text{SM}|\mathcal{D})} > 1 \text{ new physics } \odot$$

Goals

- draw marginal plots of the posterior

- compare models
NP \leftrightarrow SM



$$\frac{P(\text{NP}|\mathcal{D})}{P(\text{SM}|\mathcal{D})} = \frac{P(\mathcal{D}|\text{NP})}{P(\mathcal{D}|\text{SM})} \cdot \frac{P(\text{NP})}{P(\text{SM})}$$

$$P(\text{M}|\mathcal{D}) = \frac{P(\mathcal{D}|\text{M})P(\text{M})}{P(\mathcal{D})}$$

$$\frac{P(\text{NP}|\mathcal{D})}{P(\text{SM}|\mathcal{D})} > 1 \text{ new physics } \odot$$

$$\frac{P(\text{NP}|\mathcal{D})}{P(\text{SM}|\mathcal{D})} < 1 \text{ confirm SM } \odot$$

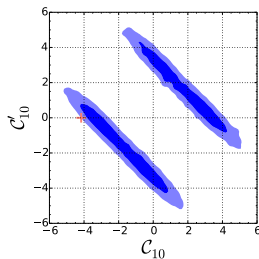
Difficulties

Difficulties

- curse of dimensionality

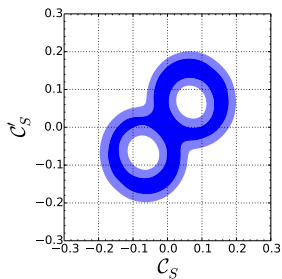
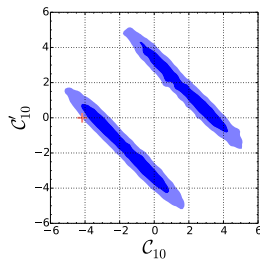
Difficulties

- curse of dimensionality
- multimodality



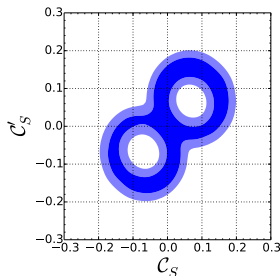
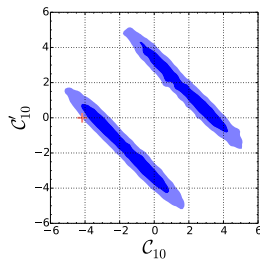
Difficulties

- curse of dimensionality
- multimodality
- degeneracies



Difficulties

- curse of dimensionality
- multimodality
- degeneracies



no standard
algorithm so
far

- 1 Overview
- 2 Adaptive importance sampling with the variational-Bayes approach
- 3 Scalar and tensor contributions to $b \rightarrow s\mu^+\mu^-$
- 4 Summary

Adaptive importance sampling with the variational-Bayes approach

- 1 Overview
- 2 Adaptive importance sampling with the variational-Bayes approach
- 3 Scalar and tensor contributions to $b \rightarrow s\mu^+\mu^-$
- 4 Summary

$$\int P(x)dx = \int \frac{P(x)}{p(x)} p(x)dx \approx \frac{1}{N} \sum_{n=1}^N \frac{P(x_n)}{p(x_n)} \equiv \mu^N \quad \text{where } x_n \sim p$$

$$\int P(x)dx = \int \frac{P(x)}{p(x)} p(x)dx \approx \frac{1}{N} \sum_{n=1}^N \frac{P(x_n)}{p(x_n)} \equiv \mu^N \quad \text{where } x_n \sim p$$

squared uncertainty (variance):

$$\text{var}(\mu^N) = \frac{1}{N} \left[\int \frac{P(x)}{p(x)} P(x)dx - \left(\int P(x)dx \right)^2 \right]$$

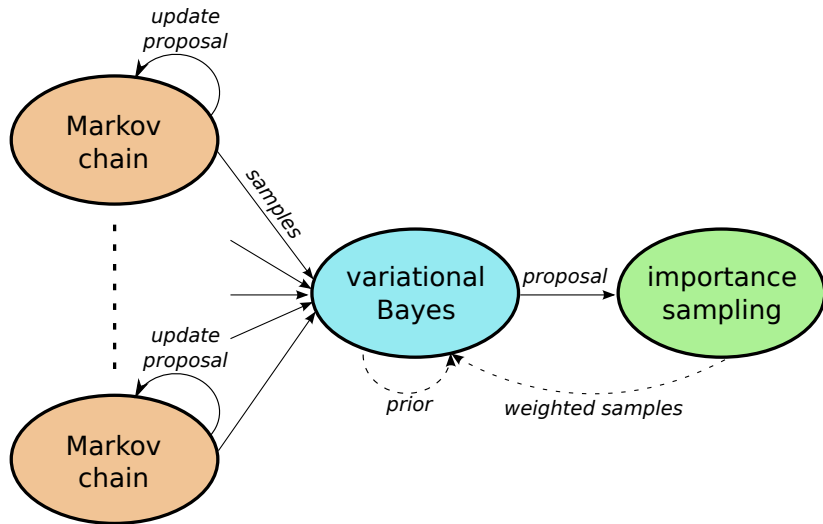
$$\int P(x)dx = \int \frac{P(x)}{p(x)} p(x)dx \approx \frac{1}{N} \sum_{n=1}^N \frac{P(x_n)}{p(x_n)} \equiv \mu^N \quad \text{where } x_n \sim p$$

squared uncertainty (variance):

$$\text{var}(\mu^N) = \frac{1}{N} \left[\int \frac{P(x)}{p(x)} P(x)dx - \left(\int P(x)dx \right)^2 \right]$$

minimize the uncertainty
 $\text{var}(\mu^N)$ with respect to p

Adaptive importance sampling with the variational-Bayes approach



'''This example illustrates how to run a Markov Chain using pypmc'''

```
import numpy as np
import pypmc

# define a proposal
prop_dof = 50.
prop_sigma = np.array([[0.1, 0. ]
                        , [0. , 0.02]])
prop = pypmc.density.student_t.LocalStud

# define the target; i.e., the function
# In this case, it is a Gaussian with me
# covariance "target_sigma".
#
# Note that the target function "log_tar
# unnormalized gaussian density.
target_sigma = np.array([[0.01, 0.003 ]
                        , [0.003, 0.0025]])
inv_target_sigma = np.linalg.inv(target_sigma)
target_mean = np.array([4.3, 1.1])

def unnormalized_log_pdf_gauss(x, mu, inv_sigma):
    diff = x - mu
    return -0.5 * diff.dot(inv_sigma).dot(diff)

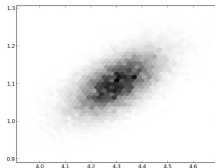
log_target = lambda x: unnormalized_log_pdf_gauss(x, target_mean, i

# choose a bad initialization
start = np.array([-2., 10.])

# define the markov chain object
mc = pypmc.sampler.markov_chain.AdaptiveMarkovChain(log_target, prc

# run burn-in
mc.run(10**4)

# delete burn-in from history
mc.history.clear()
```



(effective samples).

• **rel_tol** -

Relative tolerance ϵ . If two consecutive values of the log likelihood bound, L_t, L_{t-1} , are close, declare convergence. More precisely, check that

$$\left| \frac{L_t - L_{t-1}}{L_t} \right| < \epsilon.$$

• **abs_tol** -

Absolute tolerance ϵ_a . If the current bound L_t is close to zero, ($L_t < \epsilon_a$), declare convergence if

$$\|L_t - L_{t-1}\| < \epsilon_a.$$

• **verbose** - Output status information after each update.

set_variational_parameters()

Reset the parameters to the submitted values or default.

Use this function to set the prior value (indicated by the subscript θ as in α_θ) or the initial value (e.g., α) used in the iterative procedure to find the posterior value of the variational distribution.

Every parameter can be set in two ways:

1. It is specified for only one component, then it is copied to all other components.
2. It is specified separately for each component as a K vector.

The prior and posterior variational distributions of μ and Λ for each component are given by

$$q(\mu, \Lambda) = q(\mu|\Lambda)q(\Lambda) = \prod_{k=1}^K \mathcal{N}(\mu_k | m_k, (\beta_k \Lambda_k)^{-1}) \mathcal{W}(\Lambda_k | W_k, \nu_k),$$

where \mathcal{N} denotes a Gaussian and \mathcal{W} a Wishart distribution. The weights π follow a Dirichlet distribution

$$q(\pi) = \text{Dir}(\pi | \alpha).$$

Warning

This function may delete results obtained by [update\(\)](#).

<https://pypi.python.org/pypi/pypmc>

Scalar and tensor contributions to $b \rightarrow s\mu^+\mu^-$

- 1 Overview
- 2 Adaptive importance sampling with the variational-Bayes approach
- 3 Scalar and tensor contributions to $b \rightarrow s\mu^+\mu^-$
- 4 Summary

effective Lagrangian for $b \rightarrow s\ell^+\ell^-$ (SM):

$$\mathcal{L}_{int} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i \mathcal{O}_i + \dots + \text{h.c.}$$

$$\mathcal{O}_9 = [\bar{s}\gamma_\mu P_L \quad b] [\bar{\ell}\gamma^\mu \ell] \qquad \mathcal{O}_{10} = [\bar{s}\gamma_\mu P_L \quad b] [\bar{\ell}\gamma^\mu \gamma_5 \ell]$$

effective Lagrangian for $b \rightarrow s \ell^+ \ell^-$ (**beyond**-SM):

$$\mathcal{L}_{int} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i \mathcal{O}_i + \dots + \text{h.c.}$$

$$\mathcal{O}_9^{(\prime)} = [\bar{s} \gamma_\mu P_{L(R)} b] [\bar{\ell} \gamma^\mu \ell]$$

$$\mathcal{O}_{10}^{(\prime)} = [\bar{s} \gamma_\mu P_{L(R)} b] [\bar{\ell} \gamma^\mu \gamma_5 \ell]$$

$$\mathcal{O}_S^{(\prime)} = [\bar{s} P_{R(L)} b] [\bar{\ell} \ell]$$

$$\mathcal{O}_P^{(\prime)} = [\bar{s} P_{R(L)} b] [\bar{\ell} \gamma_5 \ell]$$

$$\mathcal{O}_T = [\bar{s} \sigma_{\mu\nu} b] [\bar{\ell} \sigma^{\mu\nu} \ell]$$

$$\mathcal{O}_{T5} = [\bar{s} \sigma_{\mu\nu} b] [\bar{\ell} \sigma^{\mu\nu} \gamma_5 \ell]$$

- $B \rightarrow K\mu^+\mu^-$ angular distribution

- $B_s \rightarrow \mu^+\mu^-$ branching fraction

- $B \rightarrow K\mu^+\mu^-$ angular distribution

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta} = \frac{3}{4}(1 - F_H) \sin^2\theta + \frac{1}{2}F_H + A_{FB} \cos\theta$$

- $B_s \rightarrow \mu^+\mu^-$ branching fraction

- $B \rightarrow K \mu^+ \mu^-$ angular distribution

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta} = \frac{3}{4}(1 - F_H) \sin^2\theta + \frac{1}{2}F_H + A_{FB} \cos\theta$$

$$F_H^{\text{SM}} = \mathcal{O}(m_\ell^2/q^2) \quad A_{FB}^{\text{SM}} = 0$$

- $B_s \rightarrow \mu^+ \mu^-$ branching fraction

- $B \rightarrow K \mu^+ \mu^-$ angular distribution

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta} = \frac{3}{4}(1 - F_H) \sin^2\theta + \frac{1}{2}F_H + A_{FB} \cos\theta$$

$$F_H^{\text{SM}} = \mathcal{O}(m_\ell^2/q^2) \quad A_{FB}^{\text{SM}} = 0$$

- $B_s \rightarrow \mu^+ \mu^-$ branching fraction

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \propto |\mathcal{C}_S - \mathcal{C}'_S|^2 + |(\mathcal{C}_P - \mathcal{C}'_P) + \frac{2m_\ell}{M_{B_s}}(\mathcal{C}_{10} - \mathcal{C}'_{10})|^2$$

we want:

$$P(\boldsymbol{\theta}|\mathcal{D}, M) \stackrel{\text{Bayes}}{\propto} P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)$$

we want:

$$P(\boldsymbol{\theta}|\mathcal{D}, M) \stackrel{\text{Bayes}}{\propto} P(\mathcal{D}|\boldsymbol{\theta}, M)P(\boldsymbol{\theta}|M)$$

split theory and experiment - *observables* **O**:

$$P(\mathcal{D}|\boldsymbol{\theta}, M) = P(\mathcal{D}|\mathbf{O}(\boldsymbol{\theta}, M))$$

we want:

$$P(\boldsymbol{\theta}|\mathcal{D}, \mathbf{M}) \stackrel{\text{Bayes}}{\propto} P(\mathcal{D}|\boldsymbol{\theta}, \mathbf{M})P(\boldsymbol{\theta}|\mathbf{M})$$

split theory and experiment - *observables* \mathbf{O} :

$$P(\mathcal{D}|\boldsymbol{\theta}, \mathbf{M}) = P(\mathcal{D}|\mathbf{O}(\boldsymbol{\theta}, \mathbf{M}))$$

theory

experiment

we want:

$$P(\theta|\mathcal{D}, M) \stackrel{\text{Bayes}}{\propto} P(\mathcal{D}|\theta, M)P(\theta|M)$$

split theory and experiment - *observables* \mathbf{O} :

$$P(\mathcal{D}|\theta, M) = P(\mathcal{D}|\mathbf{O}(\theta, M))$$

theory

calculate observables
 $\mathbf{O}(\theta, M)$

experiment



we want:

$$P(\theta|\mathcal{D}, M) \stackrel{\text{Bayes}}{\propto} P(\mathcal{D}|\theta, M)P(\theta|M)$$

split theory and experiment - *observables* \mathbf{O} :

$$P(\mathcal{D}|\theta, M) = P(\mathcal{D}|\mathbf{O}(\theta, M))$$

theory

calculate observables
 $\mathbf{O}(\theta, M)$



experiment

measure observables
 $P(\mathcal{D}|\mathbf{O})$



we want:

$$P(\theta|\mathcal{D}, M) \stackrel{\text{Bayes}}{\propto} P(\mathcal{D}|\theta, M)P(\theta|M)$$

split theory and experiment - *observables* \mathbf{O} :

$$P(\mathcal{D}|\theta, M) = P(\mathcal{D}|\mathbf{O}(\theta, M), \text{ ~~\theta, M~~ })$$

assumption

theory

calculate observables
 $\mathbf{O}(\theta, M)$



experiment

measure observables
 $P(\mathcal{D}|\mathbf{O}, \text{ ~~\theta, M~~ })$



- $B \rightarrow K\mu^+\mu^-$: \mathcal{B} , A_{FB} , F_H
 - LHCb 2014 (arXiv:1403.8044 , arXiv:1403.8045)
 - CDF 2012
(http://www-cdf.fnal.gov/physics/new/bottom/120628.blessed-b2smumu_96)
- $B_s \rightarrow \mu^+\mu^-$: \mathcal{B}
 - LHCb+CMS 2014 (arXiv:1411.4413)
- $B \rightarrow K^*\mu^+\mu^-$: \mathcal{B}
 - LHCb 2013 (arXiv:1304.6325)
 - CMS 2013 (arXiv:1308.3409)
 - CDF 2012
(http://www-cdf.fnal.gov/physics/new/bottom/120628.blessed-b2smumu_96)

scan parameters

- Wilson coefficients $\mathcal{C}_{10}^{(\prime)}, \mathcal{C}_S^{(\prime)}, \mathcal{C}_P^{(\prime)}, \mathcal{C}_T$, and \mathcal{C}_{T5}

scan parameters

- Wilson coefficients $C_{10}^{(\prime)}, C_S^{(\prime)}, C_P^{(\prime)}, C_T$, and C_{T5}

nuisance parameters

- CKM matrix (4 parameters)
- charm and bottom quark mass (2 parameters)
- form factors
 - $B \rightarrow K$ (5 parameters)
 - $B \rightarrow K^*$ (6 parameters)
- B_s decay constant f_{B_s}
- subleading corrections (11 parameters)

scan parameters

- Wilson coefficients $\mathcal{C}_{10}^{(\prime)}, \mathcal{C}_S^{(\prime)}, \mathcal{C}_P^{(\prime)}, \mathcal{C}_T$, and \mathcal{C}_{T5}

nuisance parameters

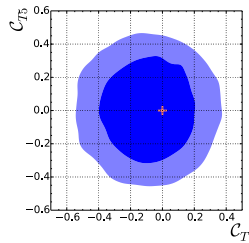
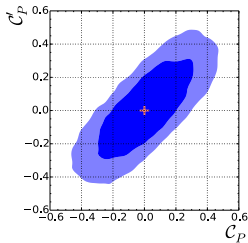
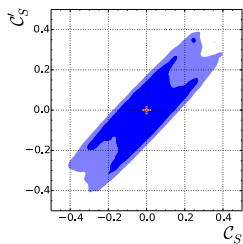
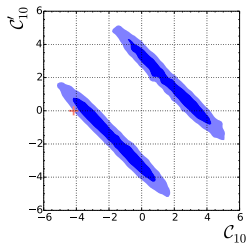
- CKM matrix (4 parameters)
- charm and bottom quark mass (2 parameters)
- form factors
 - $B \rightarrow K$ (5 parameters)
 - $B \rightarrow K^*$ (6 parameters)
- B_s decay constant f_{B_s}
- subleading corrections (11 parameters)

theory calculation $\mathbf{O}(\theta, \mathbf{M})$:

open-source implementation: EOS-package

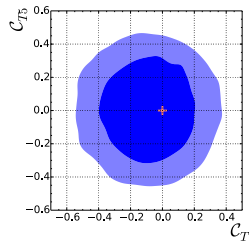
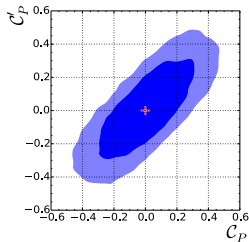
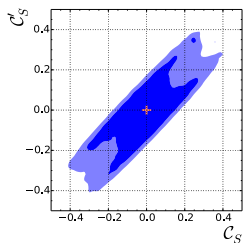
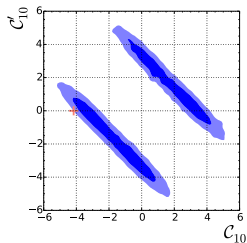
<http://project.het.physik.tu-dortmund.de/eos/>

Joint fit of $\mathcal{C}_{10}^{(j)}$, $\mathcal{C}_S^{(j)}$, $\mathcal{C}_P^{(j)}$, \mathcal{C}_T , and \mathcal{C}_{T5}

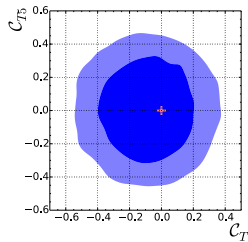
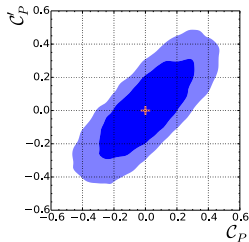
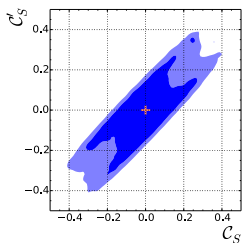
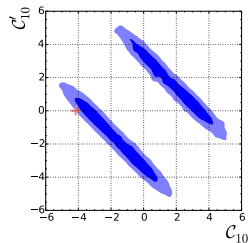


Joint fit of $\mathcal{C}_{10}^{(j)}, \mathcal{C}_S^{(j)}, \mathcal{C}_P^{(j)}, \mathcal{C}_T$, and \mathcal{C}_{T5}

- first *simultaneous* fit

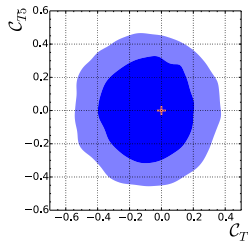
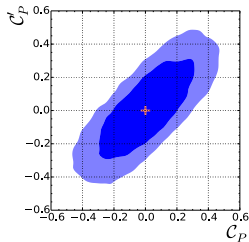
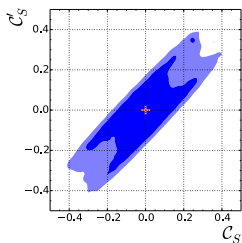
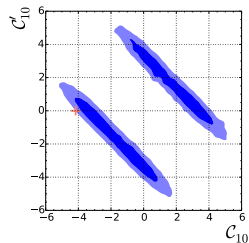


Joint fit of $\mathcal{C}_{10}^{(f)}$, $\mathcal{C}_S^{(f)}$, $\mathcal{C}_P^{(f)}$, \mathcal{C}_T , and \mathcal{C}_{T5}



- first *simultaneous* fit
- interference $\mathcal{C}_{10}^{(f)} \leftrightarrow \mathcal{C}_{S,P}^{(f)}$ in $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$

Joint fit of $\mathcal{C}_{10}^{(\prime)}$, $\mathcal{C}_S^{(\prime)}$, $\mathcal{C}_P^{(\prime)}$, \mathcal{C}_T , and \mathcal{C}_{T5}



- first *simultaneous* fit

- interference $\mathcal{C}_{10}^{(\prime)} \leftrightarrow \mathcal{C}_{S,P}^{(\prime)}$ in $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$

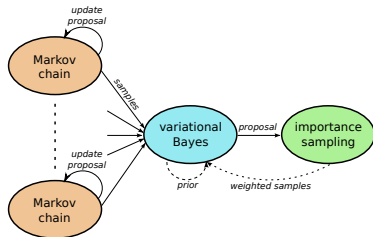
\Rightarrow larger uncertainty than obtained for fixed $\mathcal{C}_{10}^{(\prime)} = \mathcal{C}_{10}^{(\prime)\text{SM}}$

arXiv:1205.5811,

arXiv:1206.0273,

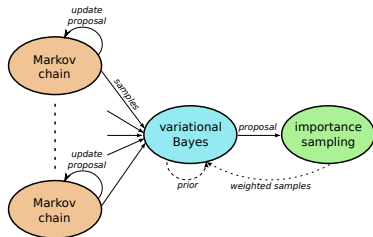
arXiv:1407.7044

algorithm to sample and
integrate in $\text{dim} = \mathcal{O}(40)$

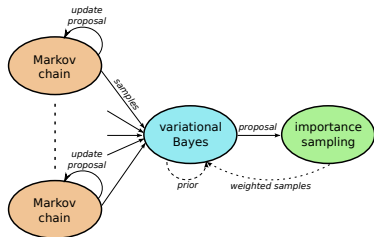


algorithm to sample and
integrate in $\text{dim} = \mathcal{O}(40)$

model-independent search
for new physics



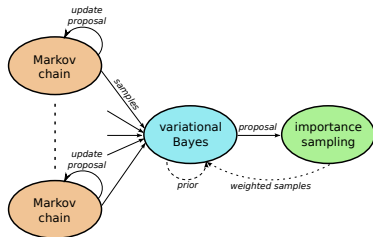
algorithm to sample and
integrate in $\text{dim} = \mathcal{O}(40)$



model-independent search
for new physics

- **simultaneous** fit of $\mathcal{C}_{10}^{(f)}$, $\mathcal{C}_S^{(f)}$, $\mathcal{C}_P^{(f)}$, \mathcal{C}_T , and \mathcal{C}_{T5}
⇒ updated constraints

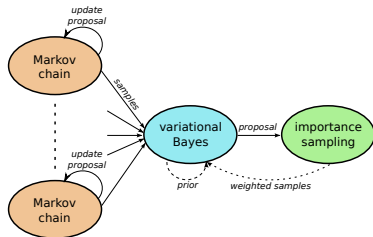
algorithm to sample and
integrate in $\text{dim} = \mathcal{O}(40)$



model-independent search
for new physics

- **simultaneous** fit of $\mathcal{C}_{10}^{(I)}, \mathcal{C}_S^{(I)}, \mathcal{C}_P^{(I)}, \mathcal{C}_T$, and \mathcal{C}_{T5}
⇒ updated constraints
- no significant deviation from the SM

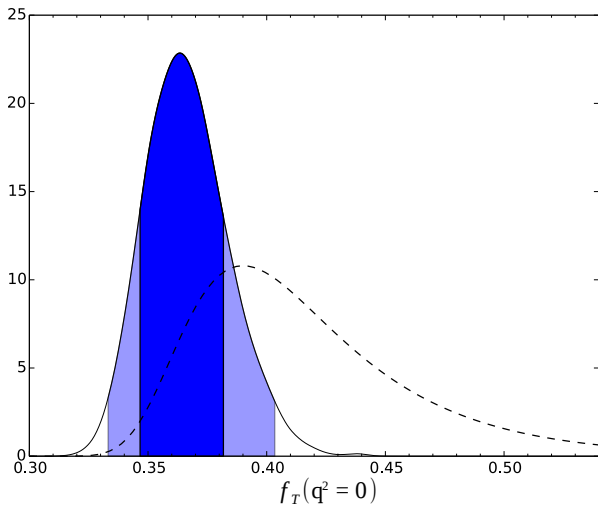
algorithm to sample and
integrate in $\text{dim} = \mathcal{O}(40)$



model-independent search
for new physics

- **simultaneous** fit of $\mathcal{C}_{10}^{(l)}, \mathcal{C}_S^{(l)}, \mathcal{C}_P^{(l)}, \mathcal{C}_T$, and \mathcal{C}_{T5}
⇒ updated constraints
- no significant deviation from the SM
- need better theoretical control

Nuisance parameters



Nuisance parameters

