Variational Bayes Applied to Multimodal Distributions in B Physics

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Effective theory

effective Lagrangian for $b \to s\ell^+\ell^-$ (SM):

$$\mathcal{L}_{int} = \frac{4G_F}{\sqrt{2}} \frac{\alpha_e}{4\pi} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i \mathcal{O}_i + ... + \text{h.c.}$$

$$\mathcal{O}_9 = \left[\bar{s}\gamma_\mu P_L \quad b\right]\left[\bar{\ell}\gamma^\mu \ell\right] \qquad \mathcal{O}_{10} = \left[\bar{s}\gamma_\mu P_L \quad b\right]\left[\bar{\ell}\gamma^\mu \gamma_5 \ell\right]$$

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effective Lagrangian for $b \to s\ell^+\ell^-$ (beyond-SM):

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$$\mathcal{O}_9^{(\prime)} = \left[\bar{s}\gamma_\mu P_{L(R)}b\right]\left[\bar{\ell}\gamma^\mu\ell\right] \qquad \mathcal{O}_{10}^{(\prime)} = \left[\bar{s}\gamma_\mu P_{L(R)}b\right]\left[\bar{\ell}\gamma^\mu\gamma_5\ell\right]$$

$$\mathcal{O}_S^{(\prime)} = \left[\bar{s}P_{R(L)}b\right]\left[\bar{\ell}\ell\right] \qquad \mathcal{O}_P^{(\prime)} = \left[\bar{s}P_{R(L)}b\right]\left[\bar{\ell}\gamma_5\ell\right]$$

$$\mathcal{O}_T = \left[\bar{s}\sigma_{\mu\nu}b\right]\left[\bar{\ell}\sigma^{\mu\nu}\ell\right] \qquad \mathcal{O}_{T5} = \left[\bar{s}\sigma_{\mu\nu}b\right]\left[\bar{\ell}\sigma^{\mu\nu}\gamma_5\ell\right]$$

Sensitive observables

- **$B \to K\mu^+\mu^-$ angular distribution**

- **$B_s \to \mu^+\mu^-$ branching fraction**

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$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \frac{3}{4}(1 - F_H)\sin^2\theta + \frac{1}{2}F_H + A_{FB}\cos\theta$$

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$$A_{FB} \propto \mathrm{Re}\Big[(\mathcal{C}_P + \mathcal{C}'_P)\,\mathcal{C}^*_{T5} + (\mathcal{C}_S + \mathcal{C}'_S)\,\mathcal{C}^*_T + \mathcal{O}(m_\ell/\sqrt{q^2})\Big]\Big/\Gamma$$

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- **$\mathbf{B_s \to \mu^+\mu^-}$ branching fraction**

$$\mathcal{B}(B_s \to \mu^+\mu^-) \propto |\mathcal{C}_S - \mathcal{C}'_S|^2 + |(\mathcal{C}_P - \mathcal{C}'_P) + \frac{2m_\ell}{M_{B_s}}(\mathcal{C}_{10} - \mathcal{C}'_{10})|^2$$

Methodology

we want:

$$P(\boldsymbol{\theta}|\mathcal{D},\mathrm{M}) \overset{\text{Bayes}}{\propto} P(\mathcal{D}|\boldsymbol{\theta},\mathrm{M})P(\boldsymbol{\theta}|\mathrm{M})$$

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experiment

measure observables

$P\left(\mathcal{D}|\mathbf{O},\cancel{\boldsymbol{\theta},\mathrm{M}}\right)$

Measurements $P(\mathcal{D}|\mathbf{O})$

- $B \to K\mu^+\mu^-$: $\mathcal{B}, A_{FB}, F_H$
    - LHCb 2014 (arXiv:1403.8044 , arXiv:1403.8045)
    - CDF 2012

      (http://www-cdf.fnal.gov/physics/new/bottom/120628.blessed-b2smumu_96)
- $B_s \to \mu^+\mu^-$: $\mathcal{B}$
    - LHCb+CMS 2014 (arXiv:1411.4413)
- $B \to K^*\mu^+\mu^-$: $\mathcal{B}$
    - LHCb 2013 (arXiv:1304.6325)
    - CMS 2013 (arXiv:1308.3409)
    - CDF 2012

      (http://www-cdf.fnal.gov/physics/new/bottom/120628.blessed-b2smumu_96)

Parameters $\boldsymbol{\theta}$

scan parameters

- Wilson coefficients $\mathcal{C}_{10}^{(\prime)}, \mathcal{C}_S^{(\prime)}, \mathcal{C}_P^{(\prime)}, \mathcal{C}_T$, and $\mathcal{C}_{T5}$

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nuisance parameters

- CKM matrix (4 parameters)
- charm and bottom quark mass (2 parameters)
- form factors
  - $B \to K$ (5 parameters)
  - $B \to K^*$ (6 parameters)
- B_s decay constant f_{B_s}
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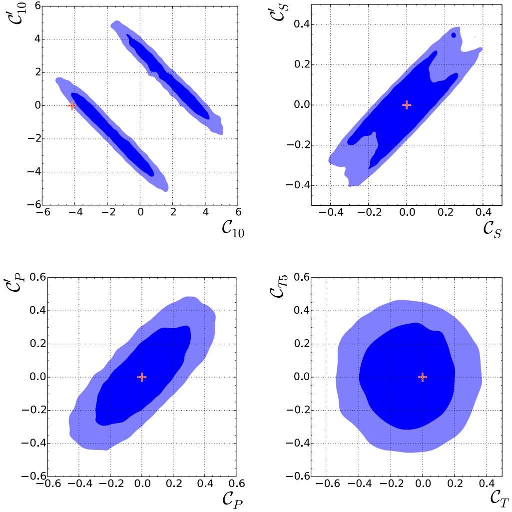
theory calculation O(θ, M):

open-source implementation: EOS-package
`http://project.het.physik.tu-dortmund.de/eos/`

Joint fit of $\mathcal{C}_{10}^{(\prime)}, \mathcal{C}_S^{(\prime)}, \mathcal{C}_P^{(\prime)}, \mathcal{C}_T$, and $\mathcal{C}_{T5}$



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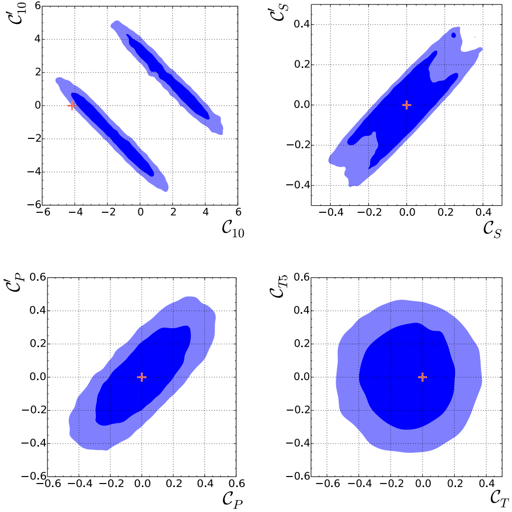

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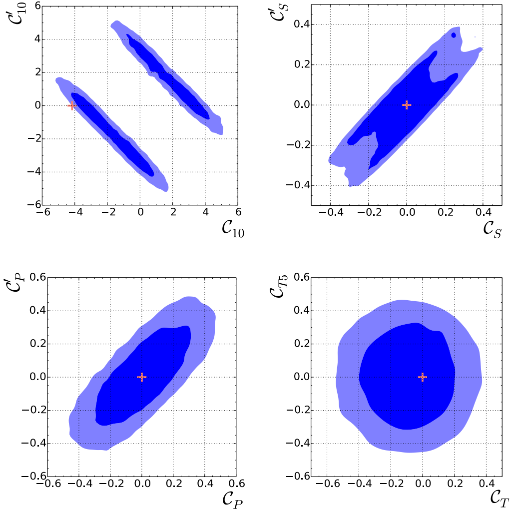

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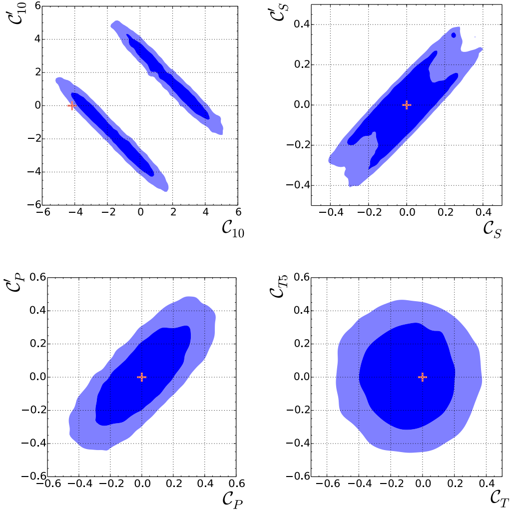
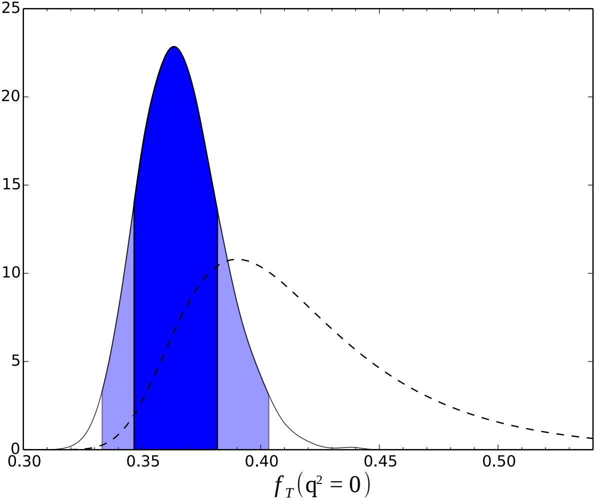
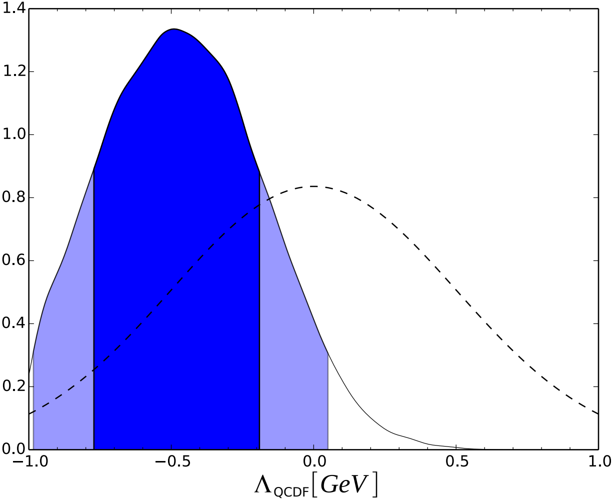


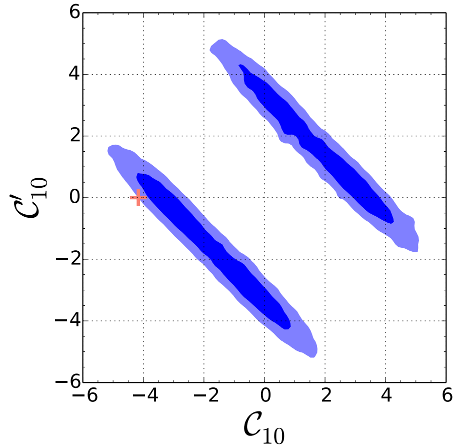
- first *simultaneous* fit
- interference $\mathcal{C}_{10}^{(\prime)} \leftrightarrow \mathcal{C}_{S,P}^{(\prime)}$ in $\mathcal{B}(B_s \to \mu^+\mu^-)$
  - $\Rightarrow$ larger uncertainty than obtained for fixed $\mathcal{C}_{10}^{(\prime)} = \mathcal{C}_{10}^{(\prime)\mathrm{SM}}$
    arXiv:1205.5811,
    arXiv:1206.0273,
    arXiv:1407.7044

Nuisance parameters



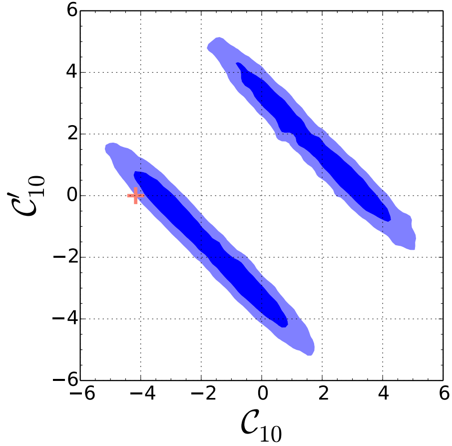

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1.0
−0.5
0.0
0.5
1.0
$\Lambda_{\mathrm{QCDF}}[GeV]$

Difficulties



Difficulties

- curse of dimensionality



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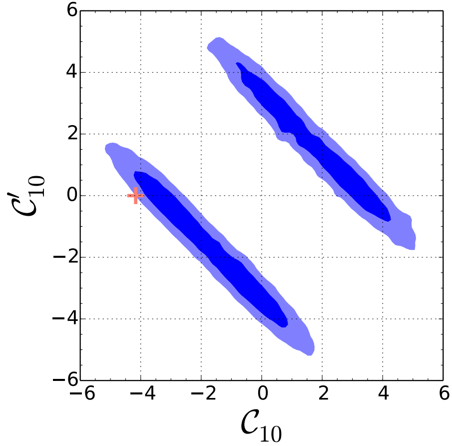

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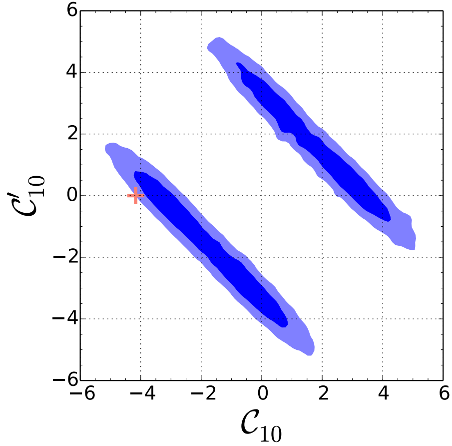

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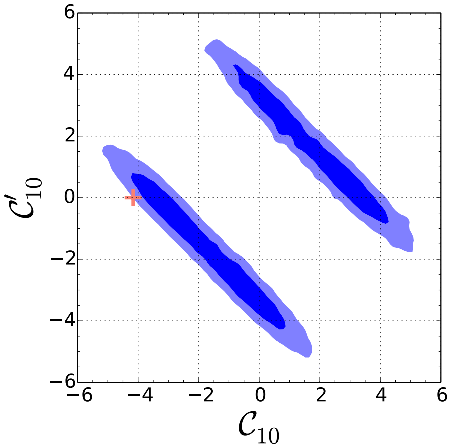
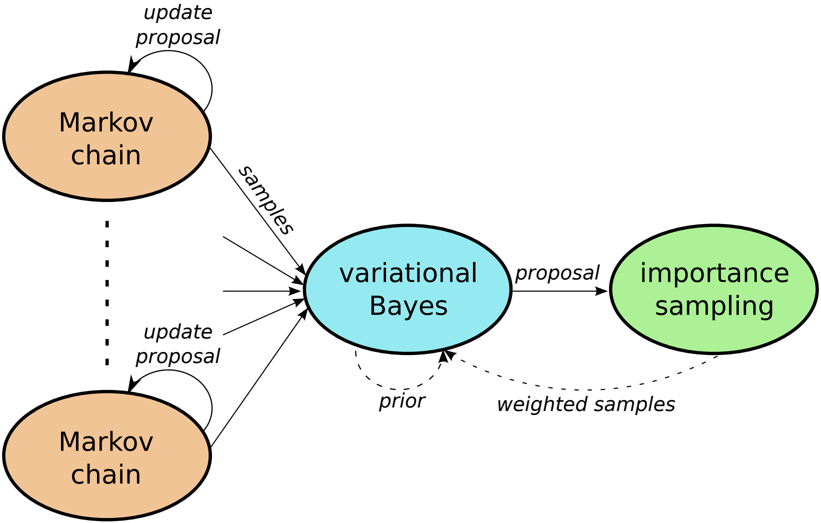
no standard algorithm so far



Sampling algorithm



https://pypi.python.org/pypi/pypmc

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- sampling algorithm to handle multimodality in dim $= \mathcal{O}(40)$

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  http://www.physi.uni-heidelberg.de/Forschung/he/LHCb/documents/WorkshopNeckarzMar14/NeckarzimmernKstmumuExp.pdf

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- paper work in progress