

Sorting Algorithm

The following algorithm sorts an array by searching the unsorted portion of the array for the lowest value and placing it in the greatest slot of the sorted portion of the array starting with the lowest value at index 1:

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Sort(A)
n = length[A]
tj = # of times a smaller value is found
1  for j ← 1 to n - 1 do
2      key ← j
3      for i ← j + 1 to n do
4          if A[i] < A[key] do
5              key ← i
6      temp ← A[j]
7      A[j] ← A[key]
8      A[key] ← temp

```

line	cost	times
1	C ₁	n
2	C ₂	n - 1
3	C ₃	$\sum_{j=1}^{n-1} (n - j + 1)$
4	C ₄	$\sum_{j=1}^{n-1} (n - j)$
5	C ₅	$\sum_{j=1}^{n-1} (t_j)$
6	C ₆	n - 1
7	C ₇	n - 1
8	C ₈	n - 1

$$C_1 n + (C_2 + C_6 + C_7 + C_8)(n - 1) + C_3 \sum_{j=1}^{n-1} (n - j + 1) + C_4 \sum_{j=1}^{n-1} (n - j) + C_5 \sum_{j=1}^{n-1} (t_j)$$

$$C_1 n + (C_2 + C_6 + C_7 + C_8)(n - 1) + (C_3 + C_4) \left(\sum_{j=1}^{n-1} n - \sum_{j=1}^{n-1} j \right) + C_3 \sum_{j=1}^{n-1} 1 + C_5 \sum_{j=1}^{n-1} t_j$$

$$C_1 n + (C_2 + C_6 + C_7 + C_8)(n - 1) + (C_3 + C_4) \left(n^2 - n - \frac{(n - 1)^2 + n - 1}{2} \right) + C_3 (n - 1) + C_5 \sum_{j=1}^{n-1} t_j$$

$$C_1 n + (C_2 + C_3 + C_6 + C_7 + C_8)(n - 1) + (C_3 + C_4) \left(\frac{2n^2 - 2n - (n^2 - 2n + 1 + n - 1)}{2} \right) + C_5 \sum_{j=1}^{n-1} t_j$$

$$C_1 n + (C_2 + C_3 + C_6 + C_7 + C_8)(n - 1) + (C_3 + C_4) \left(\frac{n^2 - n}{2} \right) + C_5 \sum_{j=1}^{n-1} t_j$$

$$C_1 n + (C_2 + C_3 + C_6 + C_7 + C_8)(n - 1) + (C_3 + C_4) \left(\frac{1}{2} n^2 - \frac{1}{2} n \right) + C_5 \sum_{j=1}^{n-1} t_j$$

$$\left(\frac{1}{2} C_3 + \frac{1}{2} C_4 \right) n^2 + \left(\text{CONST}_2 - \frac{1}{2} C_3 - \frac{1}{2} C_4 \right) n - \text{CONST}_2 + C_5 \sum_{j=1}^{n-1} t_j$$

$$\text{CONST}_1 n^2 + \text{CONST}_3 n - \text{CONST}_2 + C_5 \sum_{j=1}^{n-1} t_j$$

Best Case: $\forall j, t_j = 0$

$$CONST_1 n^2 + CONST_3 n - CONST_2 + C_5 \sum_{j=1}^{n-1} 0$$

$$CONST_1 n^2 + CONST_2 n - CONST_3$$

$$O(n^2)$$

Worst Case: $\forall j, t_j = (n - j)$

$$CONST_1 n^2 + CONST_3 n - CONST_2 + C_5 \sum_{j=1}^{n-1} (n - j)$$

$$CONST_1 n^2 + CONST_3 n - CONST_2 + C_5 \left[\sum_{j=1}^{n-1} n - \sum_{j=1}^{n-1} j \right]$$

$$CONST_1 n^2 + CONST_3 n - CONST_2 + C_5 \left[\frac{1}{2} n^2 - \frac{1}{2} n \right]$$

$$\left(CONST_1 + \frac{1}{2} C_5 \right) n^2 + \left(CONST_3 - \frac{1}{2} C_5 \right) n - CONST_2$$

$$CONST_4 n^2 + CONST_5 n - CONST_2$$

$$O(n^2)$$

The best and worst case scenarios are similar. Regardless of the amount of times a smaller value is found, the algorithm still must check all unsorted array values for every array slot.