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CSCE A365
HW #3 Writeup
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STRING MATCHING

Code Implementation

The provided test.c code was crudely modified to run all three algorithms in succession. Testing was done using a Python script that created n and m then executed test.exe several times with the created parameters. The C code implemented is a direct translation of the pseudo-code in the textbook (Cormen). Corresponding code lines are as follows:

Naive Algorithm

C Implementation	Pseudo-code	Notes
	$n = T.length$	Brought in through args
	$m = P.length$	Brought in through args
`for (int s = 0; s < n - m + 1; s++) {`	for $s = 0$ to $n - m$	Add +1 since we're using <.
`res[s] = 1;`		Setting this to *1* makes it easy to tell if a match is still possible.
`for (int t = 0; t < m && res[s] == 1; t++) {` `if (haystack[s + t] != needle[t]) {`	if $P[1 \ldots m] == T[s+1 \ldots s+m]$	Reversed the logic. Use *res[s]* to see if we should continue check.
`res[s] = 0;`	*Pattern Occurs @ s*	Determined pattern does NOT occur. Breaks current *for* loop. *res[s]* remains *1* if string matched.

Rabin-Karp Algorithm

C Implementation	Pseudo-code	Notes
	$n = T.length$	Brought in through args
	$m = P.length$	
`uint64_t p = 0, t = 0;`	$p = 0$	
	$t = 0$	

`uint64_t h = 1;`
`for (uint64_t i = 0; i < m - 1; i++) {`
`h = mulModuloBase(h);`
`}`
	$h = d^{m-1} \bmod q$	Use multiplication to get power.

`for (uint64_t i = 0; i < m; i++) {`	for $i = 1$ to m	Offset by 1 since we start at 0
`p = (addModuloChar(mulModuloBase(p), needle[i]));`	$p = (dp + P[i]) \bmod q$	
`t = (addModuloChar(mulModuloBase(t), haystack[i]));` `}`	$t = (dt + T[i]) \bmod q$	
`for (uint64_t s = 0; s < n - m + 1; s++) {`	for $s = 0$ to $n - m$	Add +1 since we're using <.
`res[s] = 0;`		Ensures we're initialized properly.
`if (p == t) {`	if $p == t$	If our hashes match…
`}` `//Naive algorithm described above`	if $P[1 \ldots m] == T[s+1 \ldots s+m]$	…double check with Naive
	Pattern Occurs @ s	
`if (s < n - m) {`	if $s < n - m$	

`t = addModuloChar(mulModuloBase(subModulo(t, mulModuloChar(h, haystack[s]))), haystack[s + m]);`
`}`
`}`
	$t = (d(t - T[s+1]h) + T[s+m+1] \bmod q$	Shift to the next hash in the haystack.

Knuth-Morris-Pratt Algorithm

Matcher

C Implementation	Pseudo-code	Notes
	$n = T.length$	Brought in through args
	$m = P.length$	
`int pi[m]`		Create *pi*
`computePrefixFunction(needle, m, pi);`	$\pi = \text{ComputePrefix}(P)$	
`int q = 0;`	$q = 0$	
`for (int i = 0; i < n; i++) {`	for $i = 1$ to n	There are several *±1* offsets due to our arrays starting at *0* instead of *1*
`res[i] = 0;`		Ensures we're initialized properly.
`while (q > 0 && needle[q] != haystack[i]) {`	while $q > 0$ and $P[q+1] \neq T[i]$	
`q = pi[q - 1];` `}`	$q = \pi[q]$	
`if (needle[q] == haystack[i]) {`	if $P[q+1] == T[i]$	
`q++;` `}`	$q = q + 1$	
`if (q == m) {`	if $q == m$	
`res[i - (m - 1)] = 1;`	$Pattern\ Occurs\ @\ i - m$	

`q = pi[q - 1];`
`}`
`}`
	$q = \pi[q]$	

Compute Prefix

C Implementation	Pseudo-code	Notes
	$m = P.length$	Brought in through args.
	let $\pi[1 \dots m]$ be new array	Created in main function and brought in through args.
`pi[0] = 0;`	$\pi[1] = 0$	
`int k = 0;`	$k = 0$	
`for (int q = 1; q < m; q++) {`	for $q = 2$ to m	There are several *±1* offsets due to our arrays starting at *0* instead of *1*
`while (k > 0 && needle[k] != needle[q]) {`	while $k > 0$ and $P[k+1] \neq P[q]$	
`k = pi[k - 1];` `}`	k$= \pi[k]$	
`if (needle[k] == needle[q]) {`	if $P[k+1] == P[q]$	
`k++;` `}`	$k = k + 1$	
`pi[q] = k;` `}`	$\pi[q] = k$	
	return π	Our function modifies *pi* directly, so no need to return it.

Testing

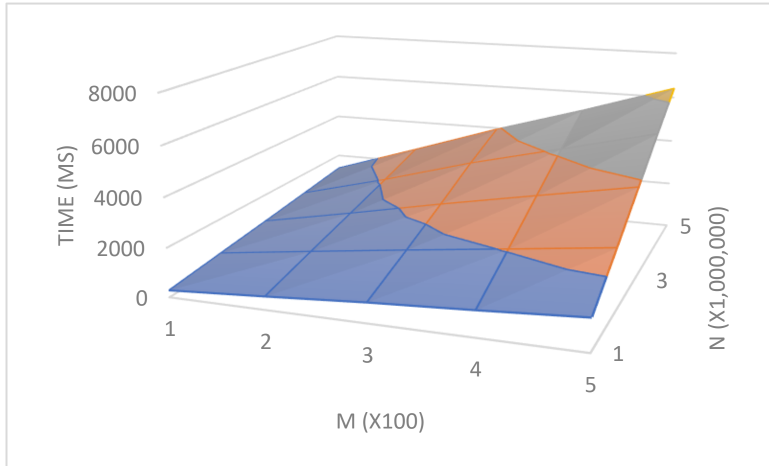
Test #1

This test consists of a haystack $a^n b$ with a needle $a^m b$. It provides a worst-case scenario for the Naive Algorithm.

Naive Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	282	535	809	1034	1288
	2	546	1061	1566	2052	2546
	3	803	1566	2362	3109	3839
	4	1073	2083	3205	4170	5152
	5	1346	2629	3940	5129	6406

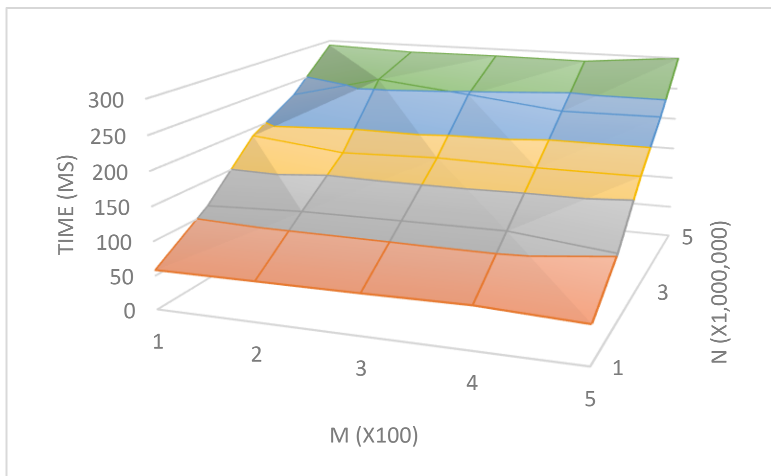
Worst Case



This shows a clear worst-case scenario as the algorithm must check $m - 1$ character for every character in n. Processing time doubles as m doubles and doubles as n doubles. This reflects the theoretical worst-case.

Rabin-Karp Algorithm

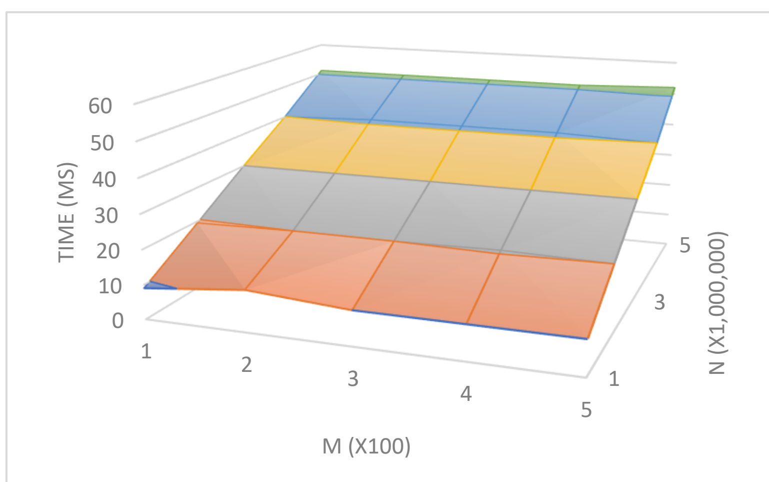
		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	58	59	60	62	56
	2	111	117	118	118	102
	3	188	172	176	173	173
	4	228	263	247	230	231
	5	292	287	288	287	300



This shows a best-case scenario, unaffected by the length of m. Because our hash p never changes and never equals t (except for the very end), we only hit the Naive Algorithm once.

Knuth-Morris-Pratt Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	9	12	10	10	10
	2	19	20	20	21	20
	3	30	30	30	30	30
	4	40	41	41	41	40
	5	51	51	51	51	52



The results show the expected complexity of $O(n)$. Completion time increases linearly with n and is unaffected by n.

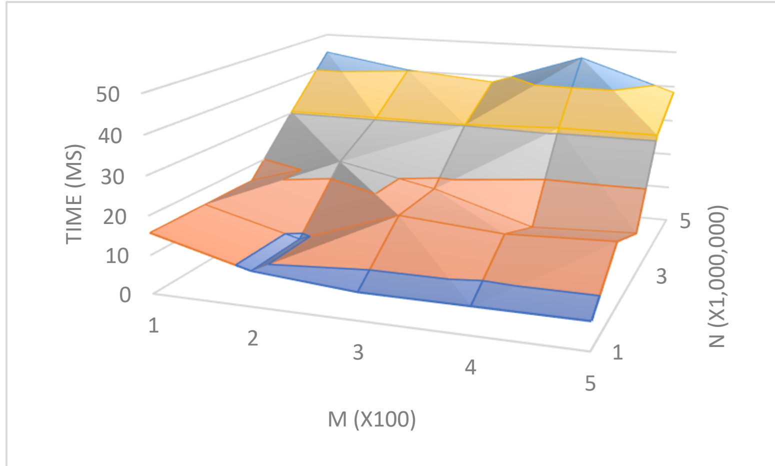
Test #2

This test consists of a haystack of n random characters with a needle equal to the last m characters of n.

Naive Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	16	9	7	7	7
	2	16	9	18	16	17
	3	16	23	18	10	11
	4	30	30	30	31	31
	5	44	40	38	47	38

Average Case

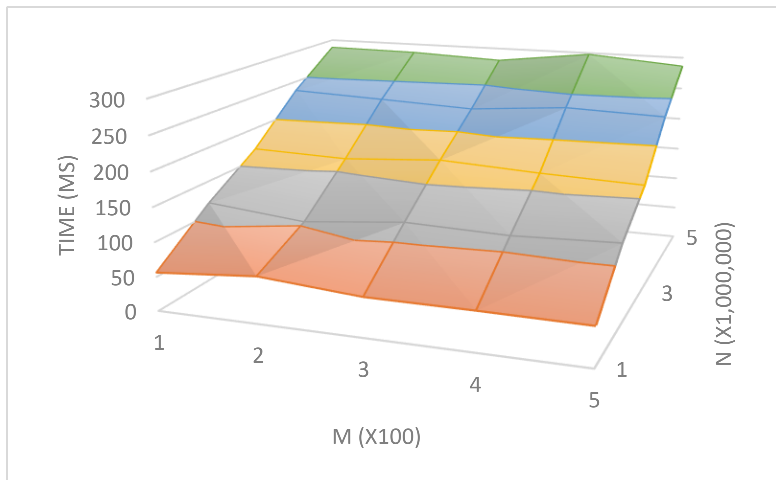


This shows an average scenario (close to best-case). Because the algorithm will likely terminate its inner loop relatively early, the entire length of m will not need to be calculated for each value of n.

Rabin-Karp Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	58	59	60	62	56
	2	111	117	118	118	102
	3	188	172	176	173	173
	4	228	263	247	230	231
	5	292	287	288	287	300

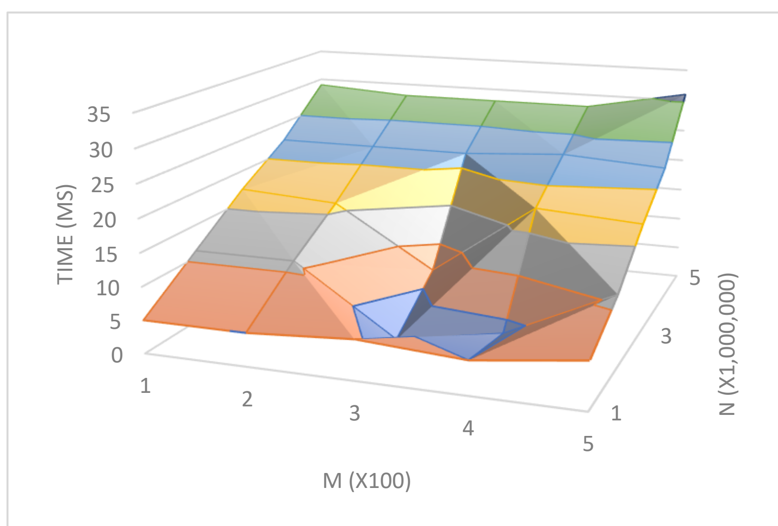
Average Case



This also shows an average scenario (very close to the best-case). With a large enough prime number used for hashing, it's very unlikely we'll receive a false positive.

Knuth-Morris-Pratt Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	5	5	6	5	7
	2	11	11	1	3	11
	3	17	16	7	18	17
	4	22	22	22	23	22
	5	29	28	28	28	31



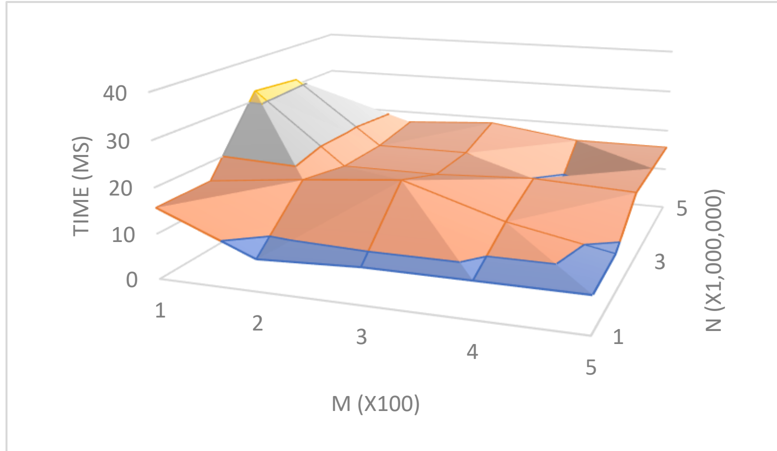
The results again show the expected complexity of $O(n)$. Completion time increases linearly with n and is unaffected by n.

Test #3

This test consists of a haystack of the text from several textbooks with a needle equal to the last m characters of n.

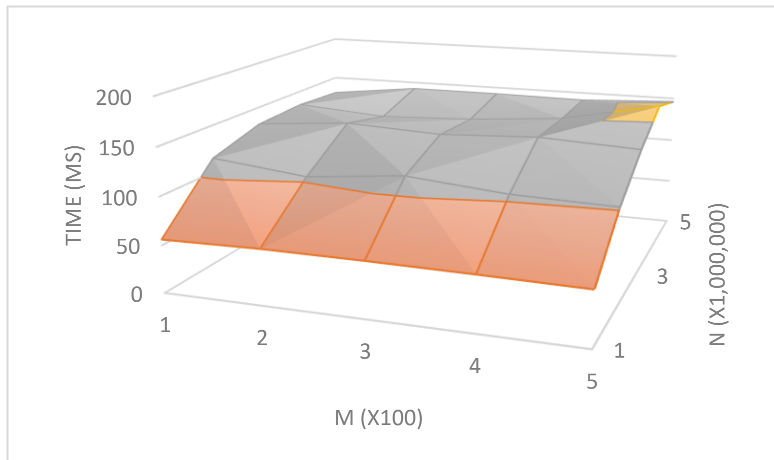
Naive Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	16	7	8	8	8
	2	16	18	20	13	9
	3	32	16	16	17	16
	4	31	16	16	7	16
	5	16	17	19	16	16



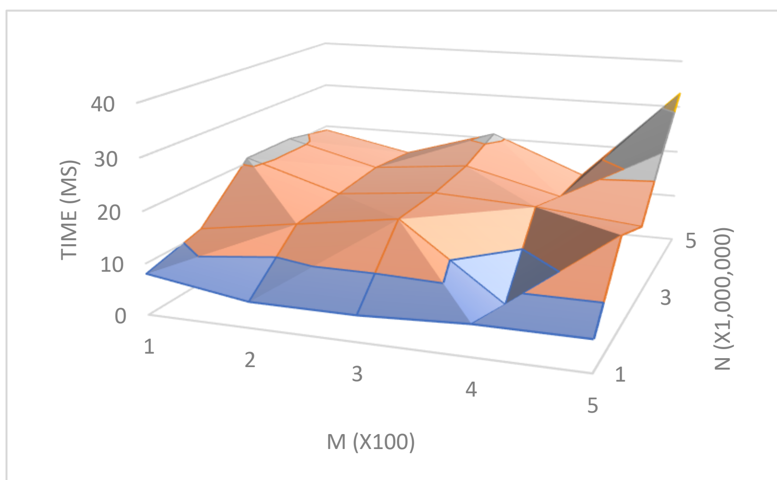
Rabin-Karp Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	56	58	58	57	56
	2	115	104	114	105	102
	3	131	139	134	139	134
	4	134	126	130	139	160
	5	130	142	142	141	146



Knuth-Morris-Pratt Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	8	5	5	6	6
	2	11	14	17	3	18
	3	21	15	17	16	14
	4	21	16	18	13	24
	5	19	15	21	13	33



While I would expect this test to run similarly to Test #2, the results from all three algorithms resemble a complexity closer to $O(\log n)$ than $O(n)$. I could speculate that there is a change in some calculations or inner loop run times due to the characters following patterns found in the English language. It's also possible this test is flawed in a way I'm not seeing, but it would require further examination. I've kept the results in the writeup for informative purposes but will not be attempting to analyze them.

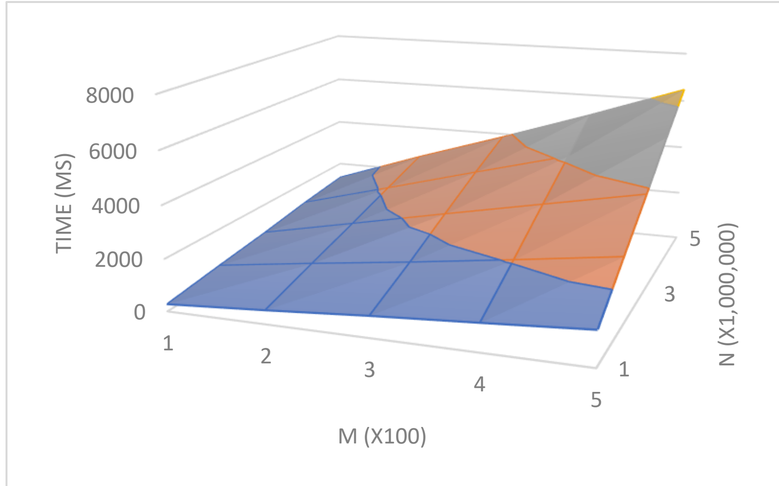
Test #4

This test consists of a haystack a^n with a needle a^m. It provides a worst-case scenario for the Naïve and Rabin-Karp Algorithms.

Naive Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	276	527	810	1057	1318
	2	552	1062	1545	2063	2603
	3	804	1576	2357	3114	3866
	4	1104	2115	3142	4143	5215
	5	1337	2645	3861	5133	6473

Worst Case

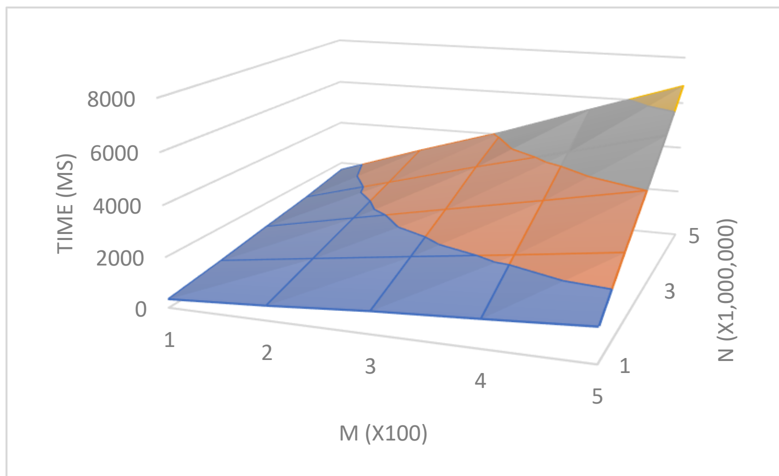


Same results as Test #1, which is to be expected.

Rabin-Karp Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	341	577	872	1095	1337
	2	633	1156	1673	2178	2705
	3	963	1730	2490	3235	4002
	4	1270	2318	3361	4340	5347
	5	1661	2934	4122	5451	6760

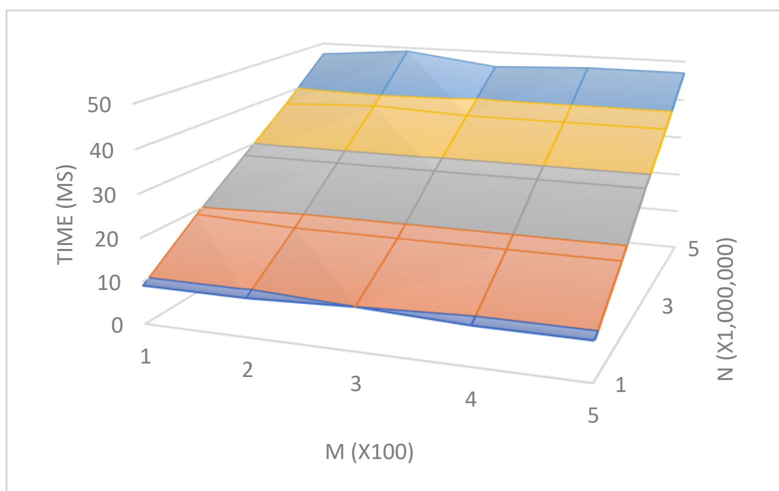
Worst Case



This shows slightly worse results than the Naive Algorithm. This is expected as this algorithm is essentially running the entire Naive Algorithm plus it's own preprocessing and modulo overhead.

Knuth-Morris-Pratt Algorithm

		m (x100)				
	time (ms)	1	2	3	4	5
n (x1,000,000)	1	9	12	10	10	10
	2	19	20	20	21	20
	3	30	30	30	30	30
	4	40	41	41	41	40
	5	51	51	51	51	52



Same results as Test #1, which is to be expected.

Notable Observations

- The Knuth-Morris-Pratt Algorithm seems to have a worst-case when m is homogenous, a best/better case when m followed a certain pattern (Test #3 with the English language), and an "in-the-middle" case when m and n were randomized.

- The Rabin-Karp Algorithm shows a fair amount of overhead. I would speculate this is due to the modulo arithmetic that needs to be calculated during every loop iteration. It still scales as expected.

- Test #3 was included, but not analyzed. The test material was less controlled, and the results were unexpected.