Math 310 Homework 6

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February 26, 2017

Note: This homework took a total of 6 hours. I initially did it alone, but I did review with Jacob Warner.

Problem 1. Section 2.2 #13

Proposition: Prove or disprove: If $[a] \odot [b] = [a] \odot [c]$ and $[a] \neq [0]$ in $\mathbb{Z}_n$, then $[b] = [c]$.

Problem 2. Section 2.3 #1,2

Proposition: Find all the units and zero divisors in (Do both together):

(a) $\mathbb{Z}_7$ Hello

(b) $\mathbb{Z}_8$ Hello

(c) $\mathbb{Z}_9$ Hello

(d) $\mathbb{Z}_10$

Problem 3. Section 2.3 #8

Proposition:

(a) Give three examples of equations of the form $ax = b$ in $\mathbb{Z}_12$ that have no nonzero solutions.

(b) For each of the equations in part (a), does the equation $ax = 0$ have a nonzero solution?

Problem 4. Section 2.3 #11

Proposition: Without using Exercises 13 and 14, prove: If $a, b \in \mathbb{Z}_n$ and a is a unit, then the equation $ax = b$ has a unique solution in $\mathbb{Z}_n$. [*Note:* You must find a solution for the equation *and* show that this solution is the only one.]

Problem 5. Section 2.3 #13

Proposition: Let a, b, n be integers with $n > 1$. Let $d = (a, n)$ and assume $d|b$. Prove that the equation $[a]x = [b]$ has a solution in Z_n as follows

(a) Explain why there are integers u, v, a_1, b_1, n_1 such that $au_nv = d$,$a = da_1$, $b = db_1$, and $n = dn_1$.

(b) Show that each of

$$[ub_1],[ub_1 + n_1],[ub_1 + 2n_1],[ub_1 + 3n_1],\cdots,[ub_1 + (d-1)n_1]$$

is a solution of $[a]x = [b]$.

Problem 6. Section 2.3 #14

Proposition: Let a, b, n be integers with $n > 1$. Let $d = (a, n)$ and assume $d|b$. Prove that the equation $[a]x = [b]$ has d unique solutions in Z_n as follows

(a) Show that the solutions listed in Exercise 13(b) are all distinct. [*Hint*: $[r] = [s]$ if and only if $n|(r - s)$.]

(b) If $x = [r]$ is any solution of $[a]x = [b]$, show that $[r] = [ub_1 + kn_1]$ for some integer k with $0 \leq k \leq d - 1$. [*Hint*: $[ar] - [aub_1] = [0]$ (Why?), so that $n|(a(r - ub_1))$. Show that $n_1|(a_1(r - ub_1))$ and use Theorem 1.4 to show that $n_1|(r - ub_1)$.]

Problem 7. Section 2.3 #17

Proposition: Prove that the product of two units in $\mathbb{Z}_n$ is also a unit.