MATH 310 Spring 2017 **Homework #2** **Due Wednesday, January 25**

Please indicate how much time you spent on this assignment and give the name(s) of anyone else you worked with.

1. Write the best proof that you can for the proposition for Bad Proof #2 (see Supplement 1: Bad Proofs).

2. Find examples of a, b, c, $d \in \mathbb{N}$, where a, b, c, $d \neq 0$, such that dividing a into b gives the same quotient and remainder as dividing and d into c, but where $\frac{a}{b} \neq \frac{c}{d}$.

3. Let a, $b \in \mathbb{N}$ and suppose that dividing b into a results in the quotient q with remainder r.

   (a) Let c be a positive integer. Make a conjecture about how to use a, b, c, q, and r to find the quotient and remainder when you divide bc into ac. Support your conjecture with three examples.

   (b) Prove your claim. Communicate your proof in claim/proof format. (Label and state the claim, label the proof.)

4. Prove that if A, B, and C are sets, then
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

5. Use the properties of $\mathbb{Z}$ that we discussed in class to prove that if $a \in \mathbb{Z}$, then $(-1) \cdot a = -a$.

6. Let a and b be integers and assume that $b \geq 1$. By the division algorithm, there are unique integers q and r such that $a = bq + r$ and $0 \leq r < b$.

   Prove that b divides a if and only if $r = 0$.

   Note: To prove a statement of the form "P holds if and only if Q holds," you need to do two things: First, assume that P is true and deduce Q. Second, assume that Q is true and deduce P. So, in this example, first assume that b divides a and deduce that $r = 0$. Second, assume that $r = 0$ and deduce that b divides a.

7. Section 1.1, Problem 7

8. Section 1.2, Problem 8

Challenge Problem

9. Section 1.2, Problem 33