Math 325-001 - Fall 2016. Homework 5.

Due: Friday, Oct 28, in class.

- *STAPLE!*
- *NO fuzzy edges on pages improperly torn from spiral binder notebooks!*
- *Typing is not mandatory, but highly encouraged if you have a suitable typesetting system at your disposal.*
- *If it's not legible, it's not there.*

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1. Page 64. Problem #8: The "Squeeze theorem".
2. This problem consists of two related parts:
   (a) Page 64. Problem #9.
   (b) Page 70. Problem #6.
3. Prove (from the "ε-δ definition") that $\lim_{x\to 3}\sqrt{3-x}$ does not exist, but $\lim_{x\to 3^-}\sqrt{3-x} = 0$.
4. Consider the set of numbers $\{a_n : n \in \mathbb{N}\}$ where each a_n is determined via this recursive definition:
$$a_1 = 2 \quad \text{and} \quad a_n = 2 - \frac{1}{a_{n-1}} \quad \text{for} \quad n \geq 2\,.$$
   For example,
$$a_1 = 2, \quad a_2 = 2 - \frac{1}{a_1} = 2 - \frac{1}{2} = \frac{3}{2}, \quad a_3 = 2 - \frac{1}{(3/2)} = \frac{4}{3}\,, \qquad \text{etc.}$$
   Use induction to prove that for all $n \in \mathbb{N}$ we have $a_n = \dfrac{n+1}{n}$.
5. **Problem to be done with a partner. Same instructions as before. Remember, that you submit it SEPARATELY from the rest of the questions: I should be able to easily tell where the original draft, the feedback and the revision are.**

   Recall that a function $f \subset \mathbb{R}^2$ is called a polynomial if there is a non-negative integer n (called "the degree" of f) and real numbers $c_0, c_1, \ldots, c_n$, such that for every $x \in \mathbb{R}$
$$f(x) = \sum_{i=0}^{n} c_i x^i$$
   Use induction and the theorem about combining the limits to prove that every polynomial is continuous on $\mathbb{R}$.
   (Note that the results about combining limits apply to at most two functions at a time. Also don't forget that you must check the case when $n = 0$, specifically, when f is a constant function).