DT\_proof

Josh Soboil

13/09/2020

# An incoherent, Dutch Book series of bets:

p1 <- .3  
p2 <- .1  
p3 <- .4  
c(p1, p2, p3)

## [1] 0.3 0.1 0.4

The set of probabilities do not meet the criterion P() = 1:

sum(p1 + p2 + p3)

## [1] 0.8

In matrix form:

R <- matrix(c(1 - p1, - p2, - p3,  
 - p1, 1 - p2, - p3,  
 - p1, - p2, 1 - p3),   
 nrow = 3)  
R

## [,1] [,2] [,3]  
## [1,] 0.7 -0.3 -0.3  
## [2,] -0.1 0.9 -0.1  
## [3,] -0.4 -0.4 0.6

and that we can invert the matrix because it is not singular:

matrixcalc::is.singular.matrix(R)

## [1] FALSE

matlib::inv(R)

## [,1] [,2] [,3]  
## [1,] 2.5 1.5 1.5  
## [2,] 0.5 1.5 0.5  
## [3,] 2.0 2.0 3.0

Notice matrix determinant > 0:

det(R)

## [1] 0.2

Here we give some arbitrary numerical stakes to each event with probability:

s <- c(5, 2, 1)  
s

## [1] 5 2 1

Accordingly, if we solve for the system of linear equations, the gains for each bet equal:

G <- solve(a = inv(R), b = s)  
G

## [1] 2.6 1.2 -2.2

Notice the sum have positive gains of 2:

round(sum(G))

## [1] 2

Because this series of bets is inconsistent, we can solve for s again:

s\_solved <- solve(a = R, b = G)  
s\_solved

## [1] 5 2 1

and this will work for any arbitrary size of stakes (bets):

s <- c(20291, 213, 13512)  
s

## [1] 20291 213 13512

Accordingly, if we solve for the system of linear equations, the gains for each bet equal:

G <- solve(a = inv(R), b = s)  
G

## [1] 10086.2 -3188.6 -94.4

Again, notice the sum have positive gains:

round(sum(G))

## [1] 6803

# A coherent series of bets:

p1 <- .3  
p2 <- .3  
p3 <- .4  
c(p1, p2, p3)

## [1] 0.3 0.3 0.4

The set of probabilities do meet the criterion P() = 1:

sum(p1 + p2 + p3)

## [1] 1

In matrix form:

R <- matrix(c(1 - p1, - p2, - p3,  
 - p1, 1 - p2, - p3,  
 - p1, - p2, 1 - p3),   
 nrow = 3)  
R

## [,1] [,2] [,3]  
## [1,] 0.7 -0.3 -0.3  
## [2,] -0.3 0.7 -0.3  
## [3,] -0.4 -0.4 0.6

and we cannot invert the matrix, because it is a singular matrix with a determinant of zero:

matrixcalc::is.singular.matrix(R)

## [1] TRUE

det(R)

## [1] -6.661338e-17

Here we give assign numerical stakes to each event with probability pn:

s <- c(5, 2, 1)  
s

## [1] 5 2 1

Accordingly, if we solve for the system of linear equations, the gains for each bet equal (we even have to revert to the %\*% matrix operator)

G <- R%\*%s  
G

## [,1]  
## [1,] 2.6  
## [2,] -0.4  
## [3,] -2.2

Notice the net overall gains are 0:

round(sum(G))

## [1] 0

… and theefore we cannot solve “for s = R^-1 \* g”

This will always result in a net gain of 0 for any arbitrary size of stakes (bets):

s <- c(20291, 213, 13512)  
s

## [1] 20291 213 13512

Accordingly, if we solve for the system of linear equations, the gains for each bet equal:

G <- R%\*%s  
G

## [,1]  
## [1,] 10086.2  
## [2,] -9991.8  
## [3,] -94.4

Again, notice the net overall gains are 0:

round(sum(G))

## [1] 0