# MATH20142 Cheat Sheet

### 1 Construction and Basic Properties of Complex Numbers

An expression $a + ib (a, b \in \mathbb{R})$ is called a **complex number**. We denote the set of complex numbers by $\mathbb{C}$. For $z = x + iy$, we use $x = \mathrm{Re}z$ and $y = \mathrm{Im}z$ and say that $z$ is real if $\mathrm{Im}z = 0$ and that $z$ is imaginary if $\mathrm{Re}z = 0$.

- $\mathrm{Re}(z \pm w) = \mathrm{Re}z \pm \mathrm{Re}w$
- $\mathrm{Im}(z \pm w) = \mathrm{Im}z \pm \mathrm{Re}w$
- $\overline{(z \pm w)} = \overline{z} \pm \overline{w}$
- $\overline{zw} = \overline{z}\,\overline{w}$
- $\overline{(z/w)} = \overline{z}/\overline{w}$ if $w \neq 0$
- $z + \overline{z} = 2\mathrm{Re}z$
- $z - \overline{z} = 2\mathrm{Im}z$
- $|z| = 0 \iff z = 0$
- $|zw| = |z||w|$
- $|z/w| = |z|/|w|$ if $w \neq 0$
- $|z + w| \leq |z| + |w|$
- $|z - w| \geq ||z| - |w||$