

MATH20142 Cheat Sheet

1 Construction and Basic Properties of Complex Numbers

An expression $a + ib$ ($a, b \in \mathbb{R}$) is called a **complex number**. We denote the set of complex numbers by \mathbb{C} . For $z = x + iy$, we use $x = \operatorname{Re} z$ and $y = \operatorname{Im} z$ and say that z is real if $\operatorname{Im} z = 0$ and that z is imaginary if $\operatorname{Re} z = 0$.

- $\operatorname{Re}(z \pm w) = \operatorname{Re} z \pm \operatorname{Re} w$
- $\operatorname{Im}(z \pm w) = \operatorname{Im} z \pm \operatorname{Im} w$
- $\overline{(z \pm w)} = \bar{z} \pm \bar{w}$
- $\overline{zw} = \bar{z} \bar{w}$
- $\overline{(z/w)} = \bar{z}/\bar{w}$ if $w \neq 0$
- $z + \bar{z} = 2\operatorname{Re} z$
- $z - \bar{z} = 2i\operatorname{Im} z$
- $|z| = 0 \iff z = 0$
- $|zw| = |z||w|$
- $|z/w| = |z|/|w|$ if $w \neq 0$
- $|z + w| \leq |z| + |w|$
- $|z - w| \geq ||z| - |w||$

2 Topology in \mathbb{C}

ε -neighbourhood of z_0 : $N_\varepsilon(z_0) = \{z \in \mathbb{C} : |z - z_0| < \varepsilon\}$ (disc centred at z_0 containing points with distance $< \varepsilon$)

limit point: $z_0 \in \mathbb{C}$ is a limit point of a set $S \subset \mathbb{C}$ if, for every $\varepsilon > 0$, $N_\varepsilon(z_0)$ contains a point in $S \setminus \{z_0\}$

interior point: let $S \subset \mathbb{C}$, z_0 a limit point of S , then z_0 is an interior point of S if $\exists \varepsilon > 0$, $N_\varepsilon(z_0) \subset S$

boundary point: let $S \subset \mathbb{C}$, z_0 a limit point of S , then z_0 is a boundary point of S if it is not an interior point

open: a set $S \subset \mathbb{C}$ is called open if it consists only of interior points

domain: let $S \subset \mathbb{C}$, $S \neq \emptyset$, then S is called a domain if S is open and every pair of points can be connected by a polygonal arc lying entirely in S

function: let $S \subset \mathbb{C}$, $S \neq \emptyset$, a function $f : S \rightarrow \mathbb{C}$ is a rule which assigns to each $z \in S$, an image $f(z) \in \mathbb{C}$

$\lim_{z \rightarrow z_0} f(z)$: let $f : S \rightarrow \mathbb{C}$ be a function. if z_0 is a limit point of S then we say $\lim_{z \rightarrow z_0} f(z) = l$ if, $\forall \varepsilon > 0, \exists \delta > 0, s \in S$ and $0 < |z - z_0| < \delta \implies |f(z) - l| < \varepsilon$

continuity: $f(z)$ is continuous at z_0 if $\lim_{z \rightarrow z_0} f(z) = f(z_0)$

proposition 2.1 a set $S \subset \mathbb{C}$ is closed \iff its complement $\mathbb{C} \setminus S$ is open

proposition 2.2 if $\lim_{z \rightarrow z_0} f(z) = l$ and $\lim_{z \rightarrow z_0} g(z) = k$, then

1. $\lim_{z \rightarrow z_0} (f(z) \pm g(z)) = l \pm k$
2. $\lim_{z \rightarrow z_0} (f(z)g(z)) = lk$
3. $\lim_{z \rightarrow z_0} (f(z)/g(z)) = l/k$ (for $k \neq 0$)

proposition 2.3 $\lim_{z \rightarrow z_0} f(z) = l = \alpha + i\beta$ ($\alpha, \beta \in \mathbb{R}$) $\iff u(x, y) \rightarrow \alpha, v(x, y) \rightarrow \beta$, as $(x, y) \rightarrow (\operatorname{Re} z_0, \operatorname{Im} z_0)$