Solving a River-Crossing similar Problem

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1. The problem

The problem is to bring 8 people: Abel, Boris, Cathy, Damon, Eliza, Flora, George, and Henry from the east side of a river to the west side by a boat in a shortest time. Several constraints below:

(C1) The boat can carry no more than 2 people.

(C2) Only Abel, Boris, Cathy, can operate the boat.

(C3) Boris cannot be with Eliza and Flora without Cathy.

(C4) Cathy cannot be with George and Henry without Boris.

(C5) Damon cannot be with Boris, Cathy, Eliza, Flora, George, or Henry without Abel.

2. Data abstraction

System States The goal of this data abstraction step is to transform the human-readable problem to a machine-readable form. We define the state of this system by a binary number which the i-th bit represents the state of the i-th people (0 is east, 1 is west), the last bit is the state of the boat, and the order is Abel, Boris, Cathy, Damon, Eliza, Flora, George, Henry, and the boat. For example, $145_{10} = 10010001_2$ means Abel, Damon, and the boat is on the West side and the rest on the East.

Nodes and links We model each state by a node and connect a pair of nodes by a link, if the two states can reach to one another directly. As described below, there are two kinds of illegal states which violate C3, C4, and C5:

(1) By C3, states, such as *10*1****, *01*0****, *10**1***, *01**0*** are illegal.

(2) By C4, states, such as *10***0**, *01***1**, *10****0*, *01****1* are illegal.

(3) By C5, states, such as 10*0*****, 01*1*****, 1*00*****, 0*11*****, 1**00****, 0**11****, 1**0*0***, 0**1*1***, 1**0**0**, 0**1**1**, 1**0***0*, 0**1***1* are illegal.

Two legal states can reach each other directly only if C1 and C2 are not violated. For example, 000000000 is not linked to 111111111.

A shortest-path problem The problem involves finding the shortest path between two nodes in the graph. In this case, we can model the legal states as nodes and linking two states which can reach each other directly.

Abstract Data types From the discussions above, we identify the following abstract data types (ADTs) in our data model. These ADTs are abstract, because at this stage we do not concern about the specific implementations of these data types.

(1) A decimal number for the binary-representation state.

(2) A graph for the relationship among all the legal states.

(3) A list of legal states as a solution to the problem