1 Exercise 2.6

$A = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$

a) Is A symmetric?

$A^T = A = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$

Yes it is.

b) Show that A is positive definite.

R: eigen(A)
$values
10 5

All eigenvalues are positive $\rightarrow$ A is positive definite

2 Exercise 2.7

a) Eigenvalues and Eigenvectors

$$\begin{aligned} \mid A - \lambda I \mid = \begin{vmatrix} 9-\lambda & -2 \\ -2 & 6-\lambda \end{vmatrix} = 0 \\ 54 - 15\lambda + \lambda^2 - 4 = 0 \\ \lambda_1 = 5 \quad \lambda_2 = 10 \end{aligned}$$

$$\begin{aligned} A - 5I = \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} x =& 0 \\ \begin{pmatrix} 1 & -1/2 \\ -2 & 1 \end{pmatrix} x =& 0 \\ \begin{pmatrix} 1 & -1/2 \\ 0 & 0 \end{pmatrix} x =& 0 \end{aligned}$$

$$\begin{aligned} x_1 &= \begin{pmatrix} -1/2 & -1 \end{pmatrix} \\ \frac{x_1}{\mid x_1 \mid} &= \frac{x_1}{\sqrt{(-1)^2 + (-1/2)^2}} \\ &= \frac{x_1}{1.118034} \\ &= \begin{pmatrix} -0.4472136 & -0.8944272 \end{pmatrix} \end{aligned}$$

$$
\begin{aligned}
A - 10I = \begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} x =& 0 \\
\begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix} x =& 0 \\
\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} x =& 0 \\
x_1 =& \begin{pmatrix} 2 & -1 \end{pmatrix} \\
\frac{x_1}{\mid x_1 \mid} =& \frac{x_1}{\sqrt{2^2 + (-1)^2}} \\
=& \frac{x_1}{\sqrt{5}} \\
=& \begin{pmatrix} -0.8944272 & 0.4472136 \end{pmatrix}
\end{aligned}
$$

b) Spectral decomposition

$$
\begin{aligned}
A =& \sum_{i=1}^{n} \lambda_i e_i e_i^T \\
=& 5 \begin{pmatrix} \frac{-1}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \frac{-1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \end{pmatrix} + 10 \begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \frac{-2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{pmatrix} \\
=& 5 \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix} + 10 \begin{pmatrix} \frac{4}{5} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix} \\
=& \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix} \\
=& A
\end{aligned}
$$

c) Inverse matrix

$$
\begin{aligned}
A^{-1} &= \frac{1}{9 \cdot 6 - (-2) \cdot (-2)} \begin{pmatrix} 6 & 2 \\ 2 & 9 \end{pmatrix} \\
&= \begin{pmatrix} 0.12 & 0.04 \\ 0.04 & 0.18 \end{pmatrix}
\end{aligned}
$$