1 Exercise 2.6

$$A = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$$

a) Is A symmetric?

$$A^T = A = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$$

Yes it is.

b) Show that A is positive definite.

R: eigen(A)

$values

10 5

All eigenvalues are positive $\rightarrow$ A is positive definite

2 Exercise 2.7

a) Eigenvalues and Eigenvectors

$$| A - \lambda I | = \begin{vmatrix} 9 - \lambda & -2 \\ -2 & 6 - \lambda \end{vmatrix} = 0$$

$$54 - 15\lambda + \lambda^2 - 4 = 0$$

$$\lambda_1 = 5 \quad \lambda_2 = 10$$

$$A - 5I = \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 1 & -1/2 \\ -2 & 1 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 1 & -1/2 \\ 0 & 0 \end{pmatrix} x = 0$$

$$x_1 = \begin{pmatrix} -1/2 & -1 \end{pmatrix}$$

$$\frac{x_1}{| x_1 |} = \frac{x_1}{\sqrt{(-1)^2 + (-1/2)^2}}$$

$$= \frac{x_1}{1.118034}$$

$$= \begin{pmatrix} -0.4472136 & -0.8944272 \end{pmatrix}$$

$$A - 10I = \begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} x = 0$$

$$x_1 = \begin{pmatrix} 2 & -1 \end{pmatrix}$$

$$\frac{x_1}{\mid x_1 \mid} = \frac{x_1}{\sqrt{2^2 + (-1)^2}}$$

$$= \frac{x_1}{\sqrt{5}}$$

$$= \begin{pmatrix} -0.8944272 & 0.4472136 \end{pmatrix}$$

b) Spectral decomposition

$$A = \sum_{i=1}^{n} \lambda_i e_i e_i^T$$

$$= 5 \begin{pmatrix} \frac{-1}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \frac{-1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \end{pmatrix} + 10 \begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \frac{-2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \end{pmatrix}$$

$$= 5 \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix} + 10 \begin{pmatrix} \frac{4}{5} & \frac{-2}{5} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$$

$$= \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$$

$$= A$$

c) Inverse matrix

$$A^{-1} = \frac{1}{9 \cdot 6 - (-2) \cdot (-2)} \begin{pmatrix} 6 & 2 \\ 2 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0.12 & 0.04 \\ 0.04 & 0.18 \end{pmatrix}$$

d) Eigenvalues and eigenvectors of A^{-1}

$$\lambda_1 = 1/10$$

$$\lambda_2 = 1/5$$

$$e_1 = \begin{pmatrix} -0.8944272 & 0.4472136 \end{pmatrix}$$

$$e_2 = \begin{pmatrix} -0.4472136 & -0.8944272 \end{pmatrix}$$