Homework #3
Math 201-003
Dr. Ramesh Garimella
Due: July 17, 2023 (in class)

1. Is $b = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$ a linear combination of the vectors $v_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 8 \\ -2 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$?

2. For what value(s) of h, $b = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$. Justify your answer.

3. Write the linear span of the vectors v_1, v_2 and v_3 given in problem 1.

4. Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $b = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$.

    (a) Write a vector equation corresponding to the matrix equation $Ax = b$.

    (b) Write a linear system of equations corresponding to the matrix equation $Ax = b$.

    (c) Do the system of linear equations in part (b) need to be consistent for **b** to be a linear combination of the columns of A? Justify your answer.

5. The following reaction between potassium permanganate ($KMnO_4$) and manganese sulfate in water produces manganese dioxide, potassium sulphate and sulfuric acid:

$$KMnO_4 + MnSO_4 + H_2O \rightarrow MnO_2 + K_2SO_4 + H_2SO4$$
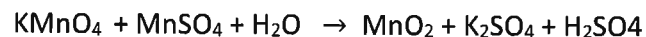
For each compound, construct a vector that lists the number of atoms of potassium (K), manganese (Mn), oxygen (O), sulfur (S) and hydrogen (H) in the above order. Then balance the chemical equation.

6. (a) Find the general traffic patterns in the free network shown in the figure.
    (b) Describe the general traffic pattern when the road whose flow is x_4 is closed?
    (c) when $x_4 = 0$, what is the minimum value of x_1