~\OneDrive\Source\GitHub\ja72\Featherstone3D\README.md

FeatherstoneConsoleApp1

A C# implementation of the Featherstone recursvie algorithm for solving the dynamics of serial chains of rigid bodies and/or open tree structures of rigid bodies.

Basic Equations in Spatial Notation

Joint Axis

$$\boldsymbol{s}_i = \begin{bmatrix} \vec{r}_i \times \hat{z}_i \\ \hat{z}_i \end{bmatrix} \text{ or } \begin{bmatrix} \hat{z}_i \\ \end{bmatrix}$$

Spatial Inertia

$$\mathbf{I}_i = \begin{bmatrix} m_i & -m_i[\vec{r}_i^C \times] \\ m_i[\vec{r}_i^C \times] & \mathcal{I}_i - m_i[\vec{r}_i^C \times][\vec{r}_i^C \times] \end{bmatrix}$$

Rigid Body Mechanics

$$\boldsymbol{f}_i - \boldsymbol{f}_{i+1} + \boldsymbol{w}_i = \mathbf{I}_i \boldsymbol{a}_i + \boldsymbol{v}_i \times \mathbf{I}_i \boldsymbol{v}_i$$

Joint Conditions

$$Q_i = \boldsymbol{s}_i^\top \boldsymbol{f}_i$$

Kinematics

Loop up the chain `i=1 to N`

Velocity Kinematics $\boldsymbol{v}_i = \boldsymbol{v}_{i-1} + \boldsymbol{s}_i \dot{q}_i$

Start with $\boldsymbol{v}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Bias Acceleration

$$\boldsymbol{\kappa}_i = \begin{bmatrix} \vec{\omega}_i \times & \vec{v}_i \times \\ 0 & \vec{\omega}_i \times \end{bmatrix} \boldsymbol{s}_i \dot{q}_i$$

Bias Foces

$$\boldsymbol{p}_i = \begin{bmatrix} \vec{\omega}_i \times & 0 \\ \vec{v}_i \times & \vec{\omega}_i \times \end{bmatrix} \mathbf{I}_i \boldsymbol{v}_i$$

Weight

$$\boldsymbol{w}_i = \begin{bmatrix} m_i \vec{g} \\ \vec{r}_i^C \times m_i \vec{g} \end{bmatrix}$$

Articulated Inertia

Loop down the chain `i=N to 1`

Articulated Inertia $\mathbf{I}_i^A = \mathbf{I}_i + \sum_n^{\text{children}} \left(1 - \boldsymbol{T}_n \boldsymbol{s}_n^\top\right) \mathbf{I}_n^A$

Articulated Bias Force $\boldsymbol{p}_i^A = \boldsymbol{p}_i - \boldsymbol{w}_i + \sum_n^{\text{children}} \left(\boldsymbol{T}_n Q_n + \left(1 - \boldsymbol{T}_n \boldsymbol{s}_n^\top\right) \left(\mathbf{I}_n^A \boldsymbol{\kappa}_n + \boldsymbol{p}_n^A\right)\right)$

Start with

$$\mathbf{I}_N^A = \mathbf{I}_N$$
$$\boldsymbol{p}_N^A = \boldsymbol{p}_N - \boldsymbol{w}_N$$

Dynamics Solution

Loop up the chain `i=1 to N`

Joint Acceleration Solution $\ddot{q}_i = \left(\boldsymbol{s}_i^\top \mathbf{I}_n^A \boldsymbol{s}_i\right)^{-1} \left(Q_i - \boldsymbol{s}_i^\top \left(\mathbf{I}_n^A \left(\boldsymbol{a}_{i-1} + \boldsymbol{\kappa}_i\right) + \boldsymbol{p}_i^A\right)\right)$

Acceleration Kinematics $\boldsymbol{a}_i = \boldsymbol{a}_{i-1} + \boldsymbol{s}_i \ddot{q}_i + \boldsymbol{v}_i \times \boldsymbol{s}_i \dot{q}_i$

Joint Forces $\boldsymbol{f}_i = \mathbf{I}_n^A \boldsymbol{a}_i + \boldsymbol{p}_i^A$