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JA.Parsing

An expression parser for **C#** with easy compilation into delegate functions for fast evaluations, some ability for simplifications and automatic differentiation. Inspired by <https://github.com/toptensoftware/SimpleExpressionEngine>.

Code Examples

```
VariableExpr x = "x", y="y";

var f_input = "(x^2-1)/(x^2+1)";
Console.WriteLine($"input={f_input}");
var f = Expr.Parse(f_input);
Console.WriteLine($"f={f}");
Console.WriteLine($"f(0.5)={f.Eval(("x", 0.5))}");

var df = f.Partial(x);
Console.WriteLine($"df={df}");

var fp = f.TotalDerivative();
Console.WriteLine($"fp={fp}");

var w_input = "x^2 + 2*x*y + x/y";
var w = Expr.Parse(w_input);
Console.WriteLine($"w={w}");
var wx = w.Partial(x);
```

```
Console.WriteLine($"wx={wx}");  
var wy = w.Partial(y);  
Console.WriteLine($"wy={wy}");  
var wp = w.Derivative(x, y);  
Console.WriteLine($"wp={wp}");
```

Highlights

The main type is `Expr` that holds expression trees. An auxiliary type is `VariableExpr` which holds variables. A string input is parsed using `Expr.Parse()` and a variable is declared using implicit conversion `Expr x = "x";` or by calling `Expr x = Expr.Variable("x");`.

An expression can either be build by parsing a string such as `1/ln(x+1)` or by using the static functions defined under `Expr`, such as `Expr.Inv(1/Expr.Log(x))`

Several build-in constants are defined similarly to variables. They are accessible with the following function call `Expr.Const("pi")` or by a property `Expr.Pi`.

Each expression has a `.Substiute(symbol,expression)` function that can perform variable substitutions. Numerical evaluation is done via compilation into `Func<double,double>` via the `.Compile<T>(<list of symbols>)` functions. Shortcuts to `.Compile<T>()` are the `GetFunction()` and `GetArray()` findexerf[].

Calculus

Besides standard trig functions such as `Expr.Sin(x)` each `Expr` can evaluate both the partial derivative with respect to a variable with `f.Partial(x)` or the total derivative with `f.TotalDerivative()`.

The **total derivative** needs the derivative of each variable (rate) which is automatically defined by appending the character `p` to the name of the variable. Then it performs the chain rule, such as

```
fp = f.Partial(x)*xp + f.Partial(y)*yp + ...
```

You can define the names of variable rates by either a list of `variable-rate` tuples, or 2nd argument with an array of rates. The rates can be just variables names, or can be any expression. For example, if the rate of time is `1` then you can write

```
var q = new VariableExpr[] { "t", "x", "y" }  
var qp = new Expr[] { 1, "xp", "yp" }  
fp = f.TotalDerivative(q, qp);
```

Each variable has `.Rate()` function that declares a new variables denoting the time derivative of the variable. This is done by appending a `p` character after the main symbol and before any underscores `_` in the name. For example `"x".Rate() => "xp"` and `"x_1".Rate()=>"xp_1"`

Lightweight Code Generation

Each expression can be converted into a function for extremely fast evaluation of the expressions. The result might be a scalar `double` value for simple expressions, or an array `double[]` for vector expressions. The user must choose the correct result type and number of arguments by selecting the appropriate `GetFunction()` or `GetArray()` function overload.

Each expression is going to emit IL code via `generator.Emit(OpCodes)` calls and all functions are inlined for very fast evaluations.

Simple Function Example

```
Expr x = "x";  
Expr expr = 0.5 + 0.5*Expr.Sin(x/2);  
var f = expr.GetFunction("x");  
double y = f(Math.PI); // y = 1.0
```

The equivalent IL code to the function evaluation looks something like this

```
ldarg.0          // load 1st argument  
ldc_R8.(2)       // load value 2  
div  
call Math.Sin()  
ldc_R8.(0.5)  
mul  
ldc_R8.(0.5)  
add  
ret
```

As you can see each node in the expression tree only contributes a minimal amount of IL code and the overall compiled function produces optimized calculations.

Multivariate Example

```
Expr x = "x", y = "y";  
Expr expr = "exp(-(x^2)-(y^2))";  
var f = expr.GetFunction("x","y");  
double z = f(0.2, 0.4); // z = Exp(-0.2)
```

The indexer function `f[]` can be used in place of `GetFunction()` also. For example

```
var f = expr["x","y"];  
double z = f(0.2, 0.4);
```

Arrays of Expressions

Arrays are declared as comma separated values between square brackets, such as `[x,2*x,1/x]` or with the function `Expr.FromArray()`. There is also an implicit conversion between `Expr[]` and an array expression.

All operators and functions auto-convert to vector functions when defined. For example `2*[x,y] = [2*x,2*y]`, `1+[2*x,2] = [2*x+1,3]` and even for unequal length arrays, the smaller array is copied over until it matches the size of the larger array. An example of this is `[x,y]+[1,2,3,4,5] = [x+1,y+2,x+3,y+4,x+5]`

Evaluation functions need the `GetArray(<list-of-symbols>)` function to return the proper delegate type that returns an array `double[]`

Array Function Example

```
Expr expr = "[1,x,sqr(x),cub(x)]";  
var f = expr.GetArray("x");  
double[] v = f(0.3);  
// v[0] = 1.0  
// v[1] = 0.09  
// v[2] = 0.027
```

More Details

A list of functions already defined are shown below, but there might be a few missing

Named Constants

Expr function	String syntax	Description & Value
Zero	"0"	The value zero
One	"1"	The value one
Inf	"inf"	Infinity ∞
Nan	"nan"	Not A Number (IEEE 754 constant)
PI	"pi"	The number π
π	" π "	Also the number π
DivPI	"divpi"	The number $1/\pi$
E	"e"	The natural log base e
Φ	" Φ "	The golden ratio $(1 + \sqrt{5})/2$
Deg	"deg"	Degree to radian conversion factor $\pi/180$
Rad	"rad"	Radian to degree conversion factor $180/\pi$
Rpm	"rpm"	Revolutions per minute to radians per second conversion $2\pi/60$

Unary Functions

Expr function	String syntax	Description
Identity(x)	"+"	Does nothing
Negate(x)	"-"	Negative
Inverse(x)	"inv(x)"	Inverse $1/x$
Rnd(x)	"rnd(x)"	Random number between 0 and x

Expr function	String syntax	Description
Pi(x)	"pi(x)"	Value of π multiplied by x
Abs(x)	"abs(x)"	Absolute value of x
Sign(x)	"sign(x)"	Either -1 or +1 depending on sign of x
Exp(x)	"exp(x)"	Exponatiation e^x
Log(x)	"ln(x)"	Natural Logarithm $\ln(x)$
Log2(x)	"log2(x)"	Logarithm Base 2 $\ln(x)/\ln(2)$
Log10(x)	"log10(x)"	Logarithm Base 10 $\ln(x)/\ln(10)$
Sqr(x)	"sqr(x)"	Square x^2 , opposite of "sqrt(x)"
Sqrt(x)	"sqrt(x)"	Square root \sqrt{x} , opposite of "sqr(x)"
Cub(x)	"cub(x)"	Cube x^4 , opposite of "cbrt(x)"
Cbrt(x)	"cbrt(x)"	Cube root $\sqrt[3]{x}$, opposite of "cub(x)"
Floor(x)	"floor(x)"	Floor function (biggest integer \leq x)
Ceiling(x)	"ceil(x)"	Ceiling function (smallest integer \geq x)
Round(x)	"round(x)"	Rounding function to nearest integer
Sin(x)	"sin(x)"	Sine function (argument in radians)
Cos(x)	"cos(x)"	Cosine function (argument in radians)
Tan(x)	"tan(x)"	Tangent function (argument in radians)
Sinh(x)	"sinh(x)"	Sine hyperbolic function
Cosh(x)	"cosh(x)"	Cosine hyperbolic function
Tanh(x)	"tanh(x)"	Tangent hyperbolic function
Asin(x)	"asin(x)"	Arc-sine function (result in radians)
Acos(x)	"acos(x)"	Arc-cosine function (result in radians)
Atan(x)	"atan(x)"	Arc-tangent function (result in radians)
Asinh(x)	"asinh(x)"	Arc-Sine hyperbolic function
Acosh(x)	"acosh(x)"	Arc-Cosine hyperbolic function
Atanh(x)	"atanh(x)"	Arc-Tangent hyperbolic function

Binary Functions

Expr	function	String syntax	Description
Add	(x,y)	"+"	Addition
Substract	(x,y)	"-"	Subtraction
Multiply	(x,y)	"*"	Multiplication
Divide	(x,y)	"/"	Division
Max	(x,y)	"max(x,y)"	Maximum value
Min	(x,y)	"min(x,y)"	Minimum value
Sign	(x,y)	"sign(x,y)"	abs(x)*sign(y)
Log	(x,y)	"log(x,y)"	Logarith of x with base y
Atan2	(dy,dx)	"atan2(dy,dx)"	Full quadrant arc tangent function

A class diagram is added below

