JA.Parsing

An expression parser for C# with easy compilation into delegate functions for fast evaluations, some ability for simplifications and automatic differentialtion. Inspired by https://github.com/toptensoftware/SimpleExpressionEngine.

Code Examples

```
VariableExpr x = "x", y="y";

var f_input = "(x^2-1)/(x^2+1)";
Console.WriteLine($"input={f_input}");
var f = Expr.Parse(f_input);
Console.WriteLine($"f={f}");
Console.WriteLine($"f(0.5)={f.Eval(("x", 0.5))}");

var df = f.Partial(x);
Console.WriteLine($"df={df}");

var fp = f.TotalDerivative();
Console.WriteLine($"fp={fp}");

var w_input = "x^2 + 2*x*y + x/y";
var w = Expr.Parse(w_input);
Console.WriteLine($"w={w}");
var wx = w.Partial(x);
```

```
    Console.WriteLine($"wx={wx}");
    var wy = w.Partial(y);
    Console.WriteLine($"wy={wy}");
    var wp = w.Derivative(x, y);
    Console.WriteLine($"wp={wp}");
```

Highlights

The main type is `Expr` that holds expression trees. An auxilary type is `VariableExpr` which holds variables. A string input is parsed using `Expr.Parse()` and a variable is declared using implicit conversion `Expr x = "x";` or by calling `Expr x = Expr.Variable("x");`.

An expression can either be build by parsing a string such as `1/ln(x+1)` or by using the static functions defined under `Expr`, such as `Expr.Inv(1/Expr.Log(x))`

Several build-in constants are defined similarly to variables. They are accessible with the following function call `Expr.Const("pi")` or by a property `Expr.Pi`.

Each expression has a `.Substiute(symbol,expression)` function that can perform variable substitutions. Numerical evaluation is done via compilation into `Func<double,double>` via the `.Compile<T>( <list of symbols> )` functions. Shortcuts to `.Compile<T>()` are the `GetFunction()` and `GetArray()` findexer`f[]`.

Calculus

Besides standard trig functions such as `Expr.Sin(x)` each `Expr` can evaluate both the partial derivative with respect to a variable with `f.Partial(x)` or the total derivative with `f.TotalDerivative()`.

The **total derivative** needs the derivative of each variable (rate) which is automatically defined by appending the character `p` to the name of the variable. Then it performs the chain rule, such as

```
fp = f.Partial(x)*xp + f.Partial(y)*yp + ...
```

You can define the names of variable rates by either a list of `varaible-rate` tuples, or 2nd argument with an array of rates. The rates can be just variables names, or can be any expression. For example, if the rate of time is `1` then you can write

```
var q = new VariableExpr[] { "t", "x", "y" }
var qp = new Expr[] { 1, "xp", "yp" }
fp = f.TotalDerivative(q, qp);
```

Each variable has `.Rate()` function that declares a new variables denoting the time derivative of the variable. This is done by appending a `p` character after the main symbol and before any underscores _ in the name. For example `"x".Rate()` => `"xp"` and `"x_1".Rate()`=>`"xp_1"`

Lightweight Code Generation

Each expression can be converted into a function for extremely fast evaluation of the expressions. The result might be a scalar `double` value for simple expressions, or an array `double[]` for vector expressions. The user must choose the correct result type and number of arguments by selecting the appropriate `GetFunction()` or `GetArray()` function overload.

Each expression is going to emit IL code via `generator.Emit(OpCodes)` calls and all functions are inlined for very fast evaluations.

Simple Function Example

```
Expr x = "x";
Expr expr = 0.5 + 0.5*Expr.Sin(x/2);
var f = expr.GetFunction("x");
double y = f(Math.PI); // y = 1.0
```

The equivalent IL code to the function evaluation looks something like this

```
ldarg.0         // load 1st argument
ldc_R8.(2)      // load value 2
div             // divide values
call Math.Sin()  // call sine function
ldc_R8.(0.5)    // load 0.5
mul             // multiply values
ldc_R8.(0.5)    // load 0.5
add             // add values
ret             // return result
```

As you can see each node in the expression tree only contributes a minimal amount of IL code and the overall compiled function produces optimized calculations.

Multivariate Example

```
Expr x = "x", y = "y";
Expr expr = "exp(-(x^2)-(y^2))";
var f = expr.GetFunction("x","y");
double z = f(0.2, 0.4); // z = Exp(-0.2)
```

The indexer function `f[]` can be used in place of `GetFunction()` also. For example

```
var f = expr["x","y"];
double z = f(0.2, 0.4);
```

Arrays of Expressions

Arrays are declared as comma separated values beween square brackers, such as `[x,2*x,1/x]` or with the function `Expr.FromArray()`. There is also an implicit conversion between `Expr[]` and an array expression.

All operators and functions auto-convert to vector functions when defined. For example `2*[x,y] = [2*x,2*y]` , `1+[2*x,2] = [2*x+1,3]` and even for unequal length arrays, the smaller array is copied over until it matches the size of the larger array. An example of this is `[x,y]+[1,2,3,4,5] = [x+1,y+2,x+3,y+4,x+5]`

Evaluation functions need the `GetArray(<list-of-symbols>)` function to return the proper delegate type that returns an array `double[]`

Array Function Example

```
Expr expr = "[1,x,sqr(x),cub(x)]";
var f = expr.GetArray("x");
double[] v = f(0.3);
// v[0] = 1.0
// v[1] = 0.09
// v[2] = 0.027
```

More Details

A list of functions already defined are shown below, but there migth be a few missing

Named Constants

`Expr` function	String syntax	Description & Value
`Zero`	`"0"`	The value zero
`One`	`"1"`	The value one
`Inf`	`"inf"`	Infinity ∞
`Nan`	`"nan"`	Not A Number (IEEE 754 constant)
`PI`	`"pi"`	The number π
`π`	`"π"`	Also the number π
`DivPI`	`"divpi"`	The number $1/\pi$
`E`	`"e"`	The natural log base e
`Φ`	`"Φ"`	The golden ratio $(1+\sqrt{5})/2$
`Deg`	`"deg"`	Degree to radian conversion factor $\pi/180$
`Rad`	`"rad"`	Radian to degree conversion factor $180/\pi$
`Rpm`	`"rpm"`	Revolutions per minute to radians per second conversion $2\pi/60$

Unary Functions

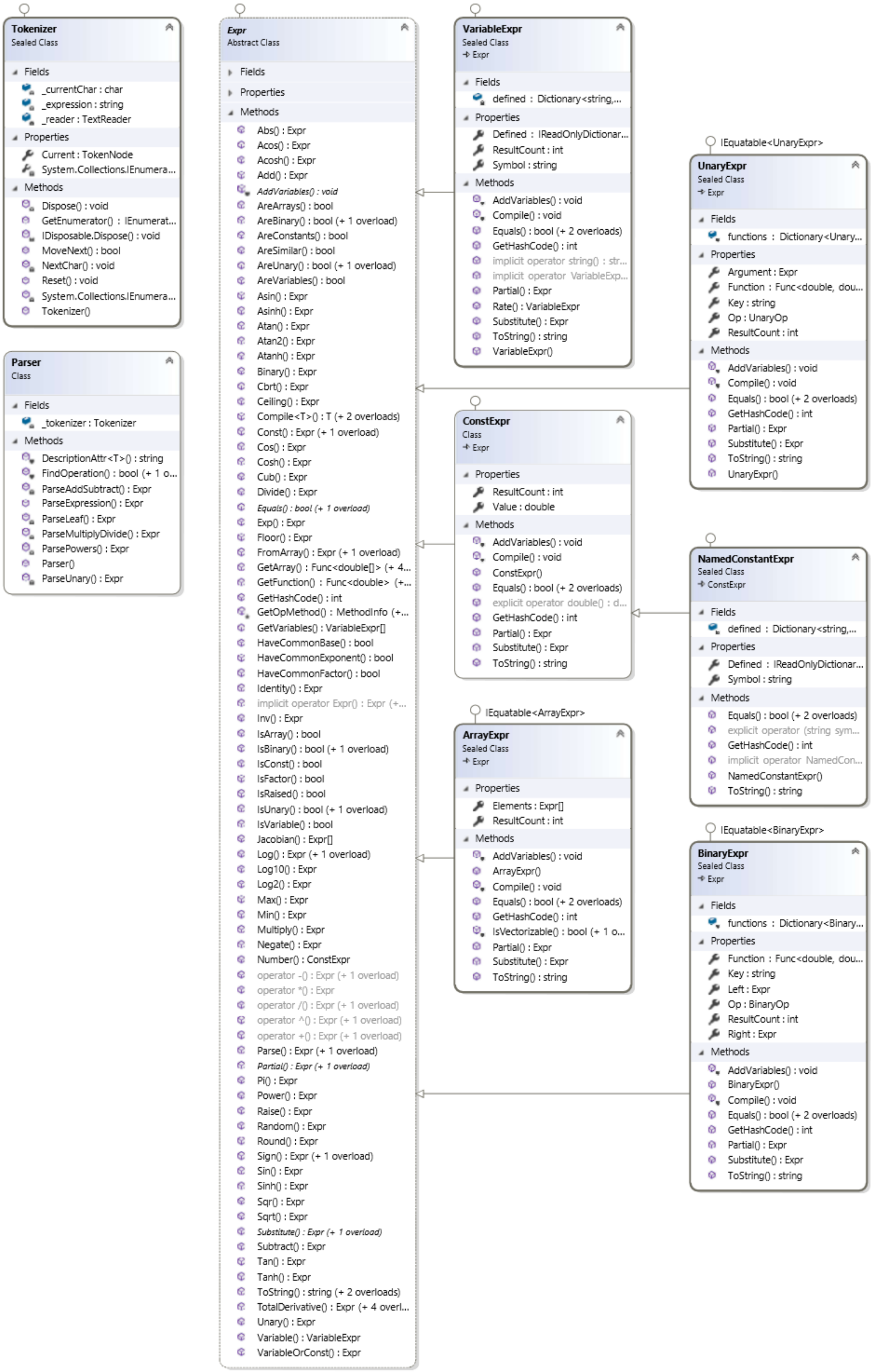
`Expr` function	String syntax	Description
`Identity(x)`	`"+"`	Does nothing
`Negate(x)`	`"-"`	Negative
`Inverse(x)`	`"inv(x)"`	Inverse `1/x`
`Rnd(x)`	`"rnd(x)"`	Random number between `0` and `x`

`Expr` function	String syntax	Description
`Pi(x)`	`"pi(x)"`	Value of π multiplied by `x`
`Abs(x)`	`"abs(x)"`	Absolute value of `x`
`Sign(x)`	`"sign(x)"`	Either `-1` or `+1` depending on sign of `x`
`Exp(x)`	`"exp(x)"`	Exponatiation e^x
`Log(x)`	`"ln(x)"`	Natural Logarithm $\ln(x)$
`Log2(x)`	`"log2(x)"`	Logarithm Base 2 $\ln(x)/\ln(2)$
`Log10(x)`	`"log10(x)"`	Logarithm Base 10 $\ln(x)/\ln(10)$
`Sqr(x)`	`"sqr(x)"`	Square x^2, opposite of `"sqrt(x)"`
`Sqrt(x)`	`"sqrt(x)"`	Square root $\sqrt{x}$, opposite of `"sqr(x)"`
`Cub(x)`	`"cub(x)"`	Cube x^4, opposite of `"cbrt(x)"`
`Cbrt(x)`	`"cbrt(x)"`	Cube root $\sqrt{x}$, opposite of `"cub(x)"`
`Floor(x)`	`"floor(x)"`	Floor function (biggest integer <= `x`)
`Ceiling(x)`	`"ceil(x)"`	Ceiling function (smallest integer >= `x`)
`Round(x)`	`"round(x)"`	Rounding function to nearest integer
`Sin(x)`	`"sin(x)"`	Sine function (argument in radians)
`Cos(x)`	`"cos(x)"`	Cosine function (argument in radians)
`Tan(x)`	`"tan(x)"`	Tangent function (argument in radians)
`Sinh(x)`	`"sinh(x)"`	Sine hyperbolic function
`Cosh(x)`	`"cosh(x)"`	Cosine hyperbolic function
`Tanh(x)`	`"tanh(x)"`	Tangent hyperbolic function
`Asin(x)`	`"asin(x)"`	Arc-sine function (result in radians)
`Acos(x)`	`"acos(x)"`	Arc-cosine function (result in radians)
`Atan(x)`	`"atan(x)"`	Arc-tangent function (result in radians)
`Asinh(x)`	`"asinh(x)"`	Arc-Sine hyperbolic function
`Acosh(x)`	`"acosh(x)"`	Arc-Cosine hyperbolic function
`Atanh(x)`	`"atanh(x)"`	Arc-Tangent hyperbolic function

Binary Functions

`Expr` function	String syntax	Description
`Add(x,y)`	`"+"`	Addition
`Substract(x,y)`	`"-"`	Subtraction
`Multiply(x,y)`	`"*"`	Multiplication
`Divide(x,y)`	`"/"`	Division
`Max(x,y)`	`"max(x,y)"`	Maximum value
`Min(x,y)`	`"min(x,y)"`	Minimum value
`Sign(x,y)`	`"sign(x,y)`	`abs(x)*sign(y)`
`Log(x,y)`	`"log(x,y)"`	Logarith of `x` with base `y`
`Atan2(dy,dx)`	`"atan2(dy,dx)"`	Full quadrant arc tangent function

A class diagram is added below



Tokenizer
Sealed Class
Fields
_currentChar : char
_expression : string
_reader : TextReader
Properties
Current : TokenNode
System.Collections.IEnumera...
Methods
Dispose() : void
GetEnumerator() : IEnumerat...
IDisposable.Dispose() : void
MoveNext() : bool
NextChar() : void
Reset() : void
System.Collections.IEnumera...
Tokenizer()
Parser
Class
Fields
_tokenizer : Tokenizer
Methods
DescriptionAttr<T>() : string
FindOperation() : bool (+ 1 o...
ParseAddSubtract() : Expr
ParseExpression() : Expr
ParseLeaf() : Expr
ParseMultiplyDivide() : Expr
ParsePowers() : Expr
Parser()
ParseUnary() : Expr
Expr
Abstract Class
Fields
Properties
Methods
Abs() : Expr
Acos() : Expr
Acosh() : Expr
Add() : Expr
AddVariables() : void
AreArrays() : bool
AreBinary() : bool (+ 1 overload)
AreConstants() : bool
AreSimilar() : bool
AreUnary() : bool (+ 1 overload)
AreVariables() : bool
Asin() : Expr
Asinh() : Expr
Atan() : Expr
Atan2() : Expr
Atanh() : Expr
Binary() : Expr
Cbrt() : Expr
Ceiling() : Expr
Compile<T>() : T (+ 2 overloads)
Const() : Expr (+ 1 overload)
Cos() : Expr
Cosh() : Expr
Cub() : Expr
Divide() : Expr
Equals() : bool (+ 1 overload)
Exp() : Expr
Floor() : Expr
FromArray() : Expr (+ 1 overload)
GetArray() : Func<double[]> (+ 4...
GetFunction() : Func<double> (+...
GetHashCode() : int
GetOpMethod() : MethodInfo (+...
GetVariables() : VariableExpr[]
HaveCommonBase() : bool
HaveCommonExponent() : bool
HaveCommonFactor() : bool
Identity() : Expr
implicit operator Expr() : Expr (+...
Inv() : Expr
IsArray() : bool
IsBinary() : bool (+ 1 overload)
IsConst() : bool
IsFactor() : bool
IsRaised() : bool
IsUnary() : bool (+ 1 overload)
IsVariable() : bool
Jacobian() : Expr[]
Log() : Expr (+ 1 overload)
Log10() : Expr
Log2() : Expr
Max() : Expr
Min() : Expr
Multiply() : Expr
Negate() : Expr
Number() : ConstExpr
operator -() : Expr (+ 1 overload)
operator *() : Expr
operator /() : Expr (+ 1 overload)
operator ^() : Expr (+ 1 overload)
operator +() : Expr (+ 1 overload)
Parse() : Expr (+ 1 overload)
Partial() : Expr (+ 1 overload)
Pi() : Expr
Power() : Expr
Raise() : Expr
Random() : Expr
Round() : Expr
Sign() : Expr (+ 1 overload)
Sin() : Expr
Sinh() : Expr
Sqr() : Expr
Sqrt() : Expr
Substitute() : Expr (+ 1 overload)
Subtract() : Expr
Tan() : Expr
Tanh() : Expr
ToString() : string (+ 2 overloads)
TotalDerivative() : Expr (+ 4 overl...
Unary() : Expr
Variable() : VariableExpr
VariableOrConst() : Expr
VariableExpr
Sealed Class
→ Expr
Fields
defined : Dictionary<string,...
Properties
Defined : IReadOnlyDictionar...
ResultCount : int
Symbol : string
Methods
AddVariables() : void
Compile() : void
Equals() : bool (+ 2 overloads)
GetHashCode() : int
implicit operator string() : str...
implicit operator VariableExp...
Partial() : Expr
Rate() : VariableExpr
Substitute() : Expr
ToString() : string
VariableExpr()
ConstExpr
Class
→ Expr
Properties
ResultCount : int
Value : double
Methods
AddVariables() : void
Compile() : void
ConstExpr()
Equals() : bool (+ 2 overloads)
explicit operator double() : d...
GetHashCode() : int
Partial() : Expr
Substitute() : Expr
ToString() : string
IEquatable<ArrayExpr>
ArrayExpr
Sealed Class
→ Expr
Properties
Elements : Expr[]
ResultCount : int
Methods
AddVariables() : void
ArrayExpr()
Compile() : void
Equals() : bool (+ 2 overloads)
GetHashCode() : int
IsVectorizable() : bool (+ 1 o...
Partial() : Expr
Substitute() : Expr
ToString() : string
IEquatable<UnaryExpr>
UnaryExpr
Sealed Class
→ Expr
Fields
functions : Dictionary<Unary...
Properties
Argument : Expr
Function : Func<double, dou...
Key : string
Op : UnaryOp
ResultCount : int
Methods
AddVariables() : void
Compile() : void
Equals() : bool (+ 2 overloads)
GetHashCode() : int
Partial() : Expr
Substitute() : Expr
ToString() : string
UnaryExpr()
NamedConstantExpr
Sealed Class
→ ConstExpr
Fields
defined : Dictionary<string,...
Properties
Defined : IReadOnlyDictionar...
Symbol : string
Methods
Equals() : bool (+ 2 overloads)
explicit operator (string sym...
GetHashCode() : int
implicit operator NamedCon...
NamedConstantExpr()
ToString() : string
IEquatable<BinaryExpr>
BinaryExpr
Sealed Class
→ Expr
Fields
functions : Dictionary<Binary...
Properties
Function : Func<double, dou...
Key : string
Left : Expr
Op : BinaryOp
ResultCount : int
Right : Expr
Methods
AddVariables() : void
BinaryExpr()
Compile() : void
Equals() : bool (+ 2 overloads)
GetHashCode() : int
Partial() : Expr
Substitute() : Expr
ToString() : string