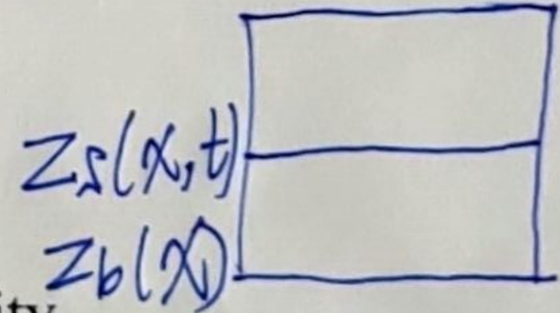
(4) Consider a homogeneous fluid in a channel of constant width but variable depth in which the flow is non-divergent and confined to a vertical plane $\boxed{y\text{=constant}}$ so

Constant width

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{1}$$



where

$u = x$-component velocity

$w = z$-component velocity

Let $z_s(x, t)$ be the free surface elevation and $z_b(x)$ be the bottom elevation and

$$h \equiv z_s(x,t) - z_b(x) = \int_{Z_b}^{Z_s} dZ \tag{2}$$

Using the usual form of Leibnitz' rule show that the vertical integral form of (1) is

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}\int_{z_b}^{z_s} u dz = 0$$

provided that

$$\begin{cases} w = \dfrac{\partial z_s}{\partial t} + u\dfrac{\partial z_s}{\partial x} & \text{at } z = z_s \\ w = u\dfrac{\partial z_b}{\partial x} & \text{at } z = z_b \end{cases} \qquad \frac{u}{\delta x} = \frac{w}{\delta z_b} \tag{3}$$

Note: Leibnitz' rule for differentiation of integrals

$$\frac{d}{dx}\int_{a(x)}^{b(x)} F(x,t)dt = \int_{a(x)}^{b(x)} \frac{\partial F}{\partial x} dt + F(x,b)\frac{db}{dx} - F(x,a)\frac{da}{dx}$$