Applying Ekman's theory to the bottom friction layer.
The governing equations

$$0 = -\alpha \frac{\partial p}{\partial x} + fv + A_z \frac{\partial^2 u}{\partial z^2}$$

$$0 = -\alpha \frac{\partial p}{\partial x} - fu + A_z \frac{\partial^2 v}{\partial z^2}$$

Boundary conditions:

$$\begin{cases} z = 0, & u = v = 0 \\ z \to \infty, & u = u_g,\ v = 0 \end{cases}$$

This is, assume that the geostrophic current only has the flow in the *x* direction.

Similarly, we can think of the velocity as having two parts (1) one associated with the horizontal pressure gradient and (2) one with vertical friction, i.e.

$$\begin{cases} -\alpha \dfrac{\partial p}{\partial x} + fv_g = 0 \quad \Rightarrow \quad -\alpha \dfrac{\partial p}{\partial x} = 0 \\ -\alpha \dfrac{\partial p}{\partial y} - fu_g = 0 \quad \Rightarrow \quad -\alpha \dfrac{\partial p}{\partial y} = fu_g \end{cases}$$

Thus, the governing equations can be written as

$$fv + A_z \frac{\partial^2 u}{\partial z^2} = 0$$

$$fu_g - fu + A_z \frac{\partial^2 v}{\partial z^2} = 0$$

Let $w = u_1 + iv$, where $u_1 = u - u_g$

$$fv + A_z \frac{\partial^2 u}{\partial z^2} = 0$$

$$-fu_1 + A_z \frac{\partial^2 v}{\partial z^2} = 0$$

$$\frac{\partial^2 w}{\partial z^2} = \frac{\partial^2 u_1}{\partial z^2} + i\frac{\partial^2 v}{\partial z^2} = -\frac{fv}{A_z} + i\frac{fu_1}{A_z} = \frac{if}{Az}\left(u_1 + iv\right) = \frac{if}{A_z} w$$

$$\therefore\ w = c_1 e^{(if/A_z)^{1/2} z} + c_2 e^{-(if/A_z)^{1/2} z}$$

B.Cs. $\begin{cases} z = 0, & w = -u_g \\ z \to \infty, & w = 0 \end{cases}$

$\therefore\ c_1 = 0$ and $c_2 = -u_g$

$$w = -u_g e^{-(if/A_z)^{1/2} z} = -u_g e^{-(f/2A_z)^{1/2}(1+i)z}$$

$$= -u_g e^{-(f/2A_z)^{1/2} z}\left(\cos\sqrt{\frac{f}{2A_z}}z - i\sin\sqrt{\frac{f}{2A_z}}z\right)$$

Let $\dfrac{\pi}{D_E} = \sqrt{\dfrac{f}{2A_z}}$

$$\therefore\ w = u_1 + iv = (u - u_g) + iv = -u_g e^{-\frac{\pi}{D_E}z}\left(\cos\frac{\pi}{D_E}z - i\sin\frac{\pi}{D_E}z\right)$$

Therefore, the solution for the northern hemisphere is

$$u = u_g\left(1 - e^{-\frac{\pi}{D_E}z}\cos\frac{\pi}{D_E}z\right)$$

$$v = u_g e^{-\frac{\pi}{D_E}z}\sin\frac{\pi}{D_E}z$$