Wind-Generated Waves

The simplest characteristics of a wave are its period and amplitude. Figure 9.1 shows the estimate of the amount of energy in surface waves. Most energy is wind-generated gravity waves that are in the 4 to 12 s range. The horizontal scale is wave frequency ω, where $\omega = 2\pi/T$ and T is the wave period. The vertical scale is the square of the wave amplitude, which is a measure of the wave energy.

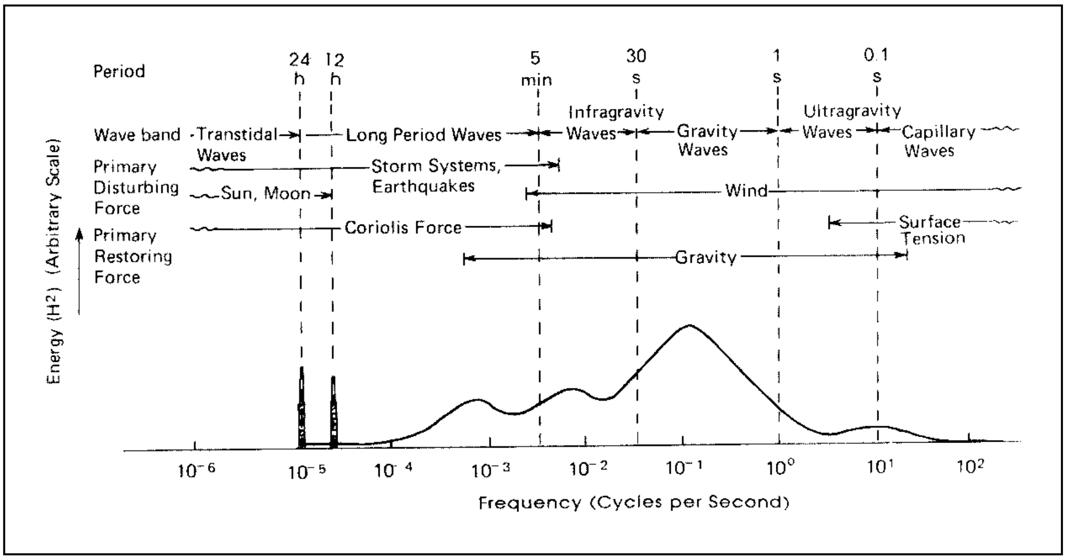


Figure 9.1 Estimate of the amount of energy in surface waves. The 12- and 24-h tides are sharply defined. Most energy is wind-generated gravity waves that are in the 4 to 12 s range. (After Kinsman, Wind Waves. 1965).

Wave Characteristics

Waves can be characterized by their period (T), length (Λ), speed (C), and amplitude (a). In a simple wave, as in Figure 9.2, the length is the distance between two crests. The wave height (H) is the vertical distance from trough to crest and is twice the amplitude. The period is the time between te passage of two successive crests past the same point. Thus, the speed of the wave is

$$C = \frac{\Lambda}{T} \tag{9.1}$$

It is convenient to refer to wavelength in terms of a wave number κ, and period in terms of angular frequency ω:

$$\kappa = \frac{2\pi}{\Lambda} \qquad \omega = \frac{2\pi}{T} \qquad C = \frac{\omega}{\kappa} \tag{9.2}$$

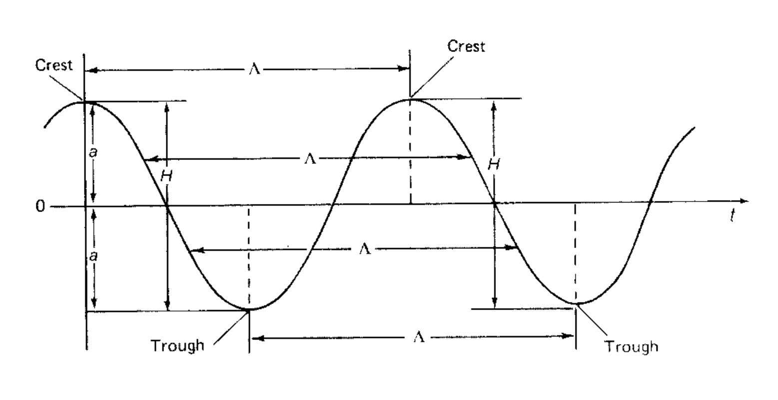


Figure 9.2 Parts of a simple sinusoidal wave.

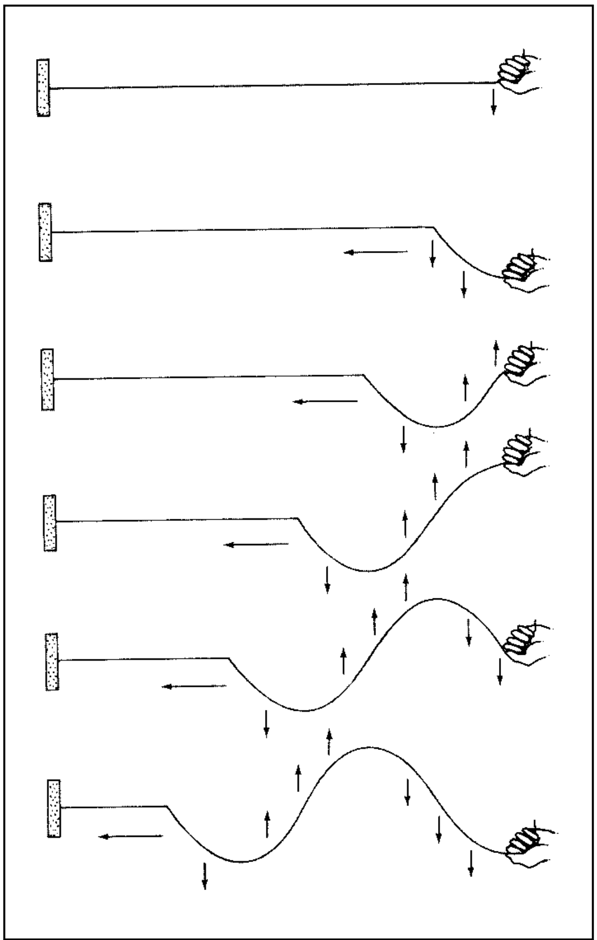


Figure 9.3 Starting with the top panel, when you send a wave down a rope with a flip of your wrist, the wave moves forward, but the rope does not.

Waves motion is of two kinds the movement of (1) the wave form itself and (2) the movement of the water. You can take a length of rope and with a flip of the wrist start a wave moving down its length (Figure 9.3). The wave moves the entire length of the rope, but the rope has not moved any distance. It is similar with ocean waves: They travel along the surface while the water stays put.

For instance, a bird floating on the surface bobs up and down as the waves pass, but it does not travel along the surface with the speed of the wave. It is necessary to distinguish between the movement of the wave (wave motion) and the movement of the water itself (particle motion).

A large portion of the observed characteristics of wind-generated ocean waves can be accounted for by assuming that ocean waves are a combination of simple sinusoidal waves, such as shown in Figure 9.2.

If one assumes that the wavelength is much longer than the wave height (small-amplitude wave, a reasonable assumption for ocean) and that the only external force is gravity, it is possible to derive a relationship between wave speed, wave number, and depth of water (h)

$$C^2 = \frac{g}{\kappa}\tanh \kappa h \tag{9.3}$$

The hyperbolic tangent relationship for these ***surface gravity waves*** can often be further simplified.

(i) When κh is small (< 0.33, shallow water), $\tanh \kappa h \cong \kappa h$

$$C_s^2 = gh \tag{9.4}$$

(ii) When κh is large (> 1.5, deep water), $\tanh \kappa h \cong 1$

$$C_d^2 = \frac{g}{\kappa} \tag{9.5}$$

The significance of the two approximations can be seen in Figure 9.4. Most of the wind-generated surface waves of interest in the ocean can be characterized as shallow or deep water waves, but the larger 6 to 12 s waves that are wo characteristic of the ocean surface change from deep water waves while offshore to shallow water waves before they break on the beach.. It is important to remember that the speed, height and length of a wave may change, but the wave period does not.

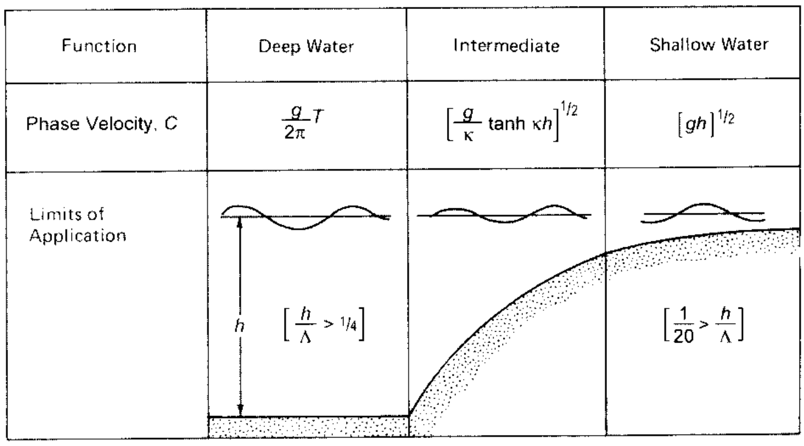


Figure 9.4 The limits of applicability of deep and shallow water waves, as defined by the ratio of water depth to wavelength.

(1) The speed of shallow water waves is independent of wavelength or wave period and is controlled by the depth of water. The deeper the water, the faster the wave. (2) The speed of deep water waves is independent of the depth and is determined by wavelength and period. As long as the water is sufficiently deep (the wavelength is no more than four times the water depth), the wave speed and other parameters are independent of the water depth. Combining Eqs. (9.5) and (9.1) gives

$$C_d = \frac{2\pi}{g}T \cong 1.5T \ \text{ms}^{-1}$$
$$\Lambda = \frac{g}{2\pi}T^2 \cong 1.5T^2 \ \ \text{m} \tag{9.6}$$

One way to visualize the relation of deep water waves to shallow water waves is to imagine what would happen to a deep water wave if its length and period were increased. Let the ocean be 4000 m deep. If the period were 20 s, the wavelength would be about 600 m and the speed about 30 ms^{-1}, a deep water wave. By the time the period was 4 min, the wave would be a shallow water wave. It would be traveling at about 200 ms^{-1} and have a wavelength of nearly 50 km. The shallow water wave speed is the maximum speed a surface gravity can obtain.

Particle Motion

Deep and shallow water waves differ in characteristics other than their wave velocity. Figure 9.5 shows the characteristic particle motion for both.

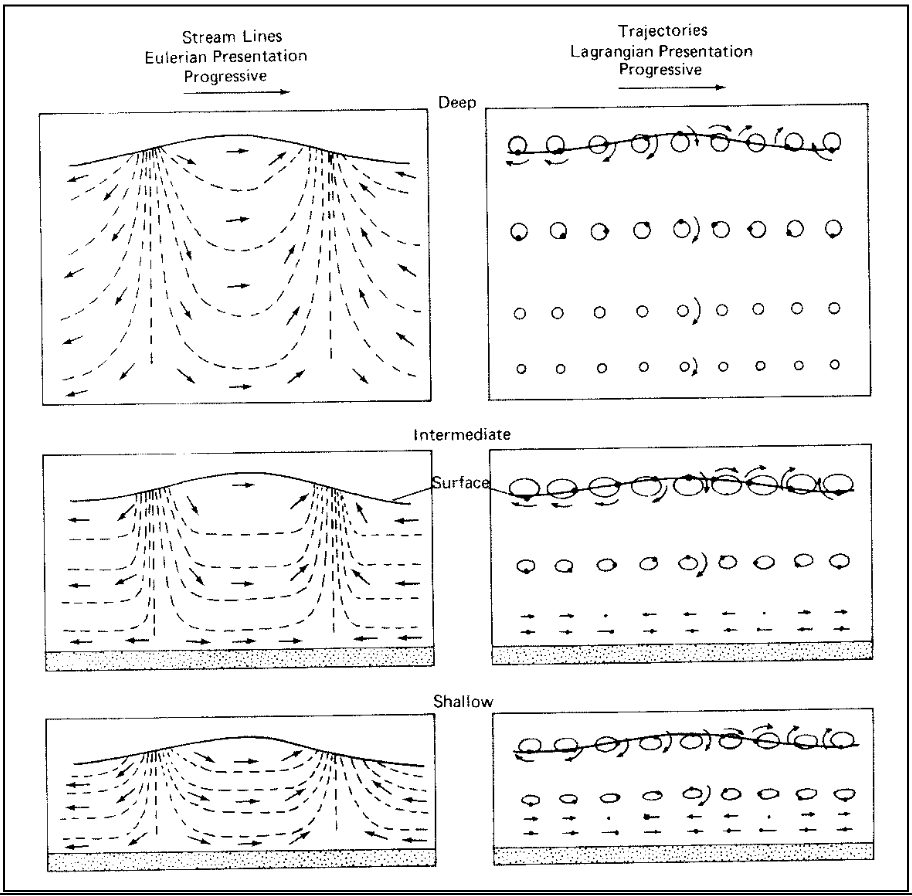


Figure 9.5 Streamlines and trajectories for deep, intermediate, and shallow waves.

(a) Deep water case

The individual water particles describe circles whose radius decreases with depth. At the surface, the radius (r) is the same as the wave amplitude (a), and the particle velocity $\tilde{v} = (u^2 + w^2)^{1/2}$ is simply the circumference of the circle divided by the wave period. A pressure recorder just below the surface would record a pressure change (Δp) equal to the change in hydrostatic pressure as the wave passed by.

$$\begin{aligned} r &= ae^{-kz} \\ \tilde{v} &= \frac{2\pi}{T} ae^{-kz} \\ \Delta p &= a\rho g e^{-kz} \end{aligned} \tag{9.7}$$

The exponential relationship means that these parameters decrease rapidly with depth. Where the depth Z is equal to half the wavelength, the radius, particle velocity, and recorded pressure differences are reduced to 4% of their surface values.

(b) Shallow water case

The water particle motions describe ellipses. The radius along the min or axis is equal to the wave amplitude at the surface and decreases linearly with depth, until at the bottom the minor axis is zero and the motion is horizontal. On the other hand, the radius of the major axis is a function of water depth, wavelength, and amplitude, and does not vary with particle depth.

Table 9.1 gives the exact formulation of the relevant parameters for deep and shallow water waves.

Table 9.1 Comparison of relevant parameters for deep and shallow water waves.

	Deep	Shallow
Surface displacement, η	$a\cos(\kappa x - \omega t)$	$a\cos(\kappa x - \omega t)$
Phase speed, C	$\frac{gT}{2\pi}$	$\sqrt{gh}$
Particles velocity components, u, w	$u = \omega a e^{-\kappa z}\cos(\kappa x - \omega t)$ $w = \omega a e^{-\kappa z}\sin(\kappa x - \omega t)$	$u = \frac{\omega}{\kappa}\frac{a}{h} e^{-\kappa z}\cos(\kappa x - \omega t)$ $w = \omega a\left(1 - \frac{z}{h}\right)\sin(\kappa x - \omega t)$
Pressure differential, Δp	$\rho g a e^{-\kappa z}\cos(\kappa x - \omega t)$	$\rho g a\cos(\kappa x - \omega t)$
Semimajor axes A and semiminor axes B of particle path ellipse	$A = B = ae^{-\kappa z}$	$A = \frac{a}{\kappa h}$ $\quad B = a\frac{h-z}{h}$

Wave energy and Wave Dispersion

The energy in a wave (E) is divided between potential energy (associated with the displacement of water from its equilibrium position) and kinetic energy (associated with particle movement). For both deep and shallow water waves,

$$E = \frac{1}{8}\rho g H^2 = \frac{1}{2}\rho g a^2 \tag{9.8}$$

The units are in terms of energy per unit surface area.

Waves whose speeds are frequency dependent are called *dispersive* waves. Deep water waves are dispersive waves; shallow water waves are *non-dispersive* waves.
(1) Consider two wave trains of slight different periods superimposed on one another, as in Figure 9.6. The resulting envelope shows regions where the waves are in phase (where the wave energy is largest) separated by regions where the waves are out of phase (where the wave height and wave energy are minimum). If both wave trains traveled at the same speed, so would the resulting envelope.

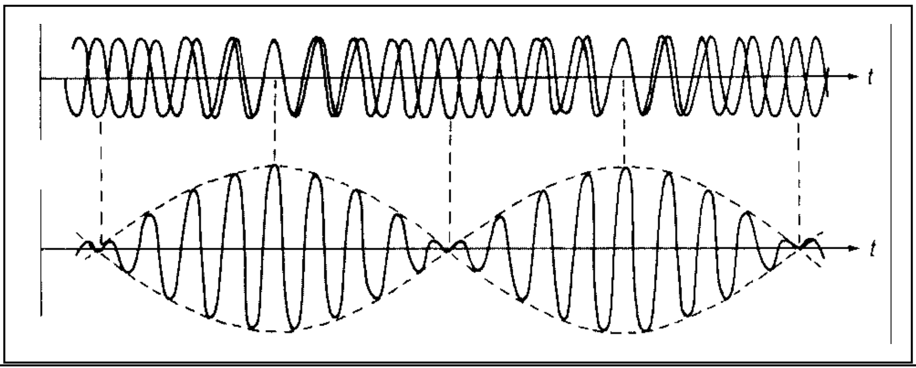


Figure 9.6 Two wave trains of similar amplitude but different periods cause "beats" as they move in and out of phase. Only if the two wave trains travel at identical speeds will the beats progress at the same speed.

(2) If the wave trains move at different speeds, the regions where the two waves are in phase and out of phase will change shown in Figure 9.7. The regions of maximum wave height in the envelope travel at a speed different from either of the individual wave trains. Thus, the wave energy is propagated at a different speed than the *phase velocity* of the individual wave trains. The speed of the envelope is called the ***group velocity***.

$$V = \frac{\Delta\omega}{\Delta\kappa} = \frac{\partial\omega}{\partial\kappa} \tag{9.9}$$

(i) Deep water surface gravity waves (dispersive)

$$V = \frac{1}{2}C_d \tag{9.10}$$

(ii) Shallow water surface gravity waves (non-dispersive)

$$V = C_s \tag{9.11}$$

Consider the superposition of two sets of sinusoidal waves of the same amplitude (a) but slightly different wavelength. The equation of the free surface will be the sum of these waves:

$$\eta = a\sin(\kappa x - \omega\, t) + a\sin(\kappa' x - \omega' t)$$

which can be written as

$$\eta = a\cos\left[(\kappa-\kappa')\frac{x}{2} - (\omega-\omega')\frac{t}{2}\right]\sin\left[(\kappa-\kappa')\frac{x}{2} - (\omega-\omega')\frac{t}{2}\right]$$

If ($\kappa - \kappa'$) is small, then the cosine term varies slowly with x and the resulting free surface is a series of sine waves whose amplitude varies gradually from 0 to 2π. The distance between two successive maxima and minima is $2\pi/(\kappa - \kappa')$ and the time between the passage of two successive troughs is $2\pi/(\omega - \omega')$. The phase velocity of this new newly defined wave on the free surface is the group velocity of the combined waves. The velocity is

$$V = \frac{\omega - \omega'}{\kappa - \kappa'}$$

which is the limit becomes

$$V = \frac{d\omega}{d\kappa}, \qquad V = \frac{d}{d\kappa}(\kappa C)$$

For deep and shallow water surface gravity waves,

$$V_d = \frac{1}{2}\left(\frac{g}{\kappa}\right)^{1/2} = \frac{1}{2}C_d, \qquad V_s = \left(gh\right)^{1/2} = C_s$$

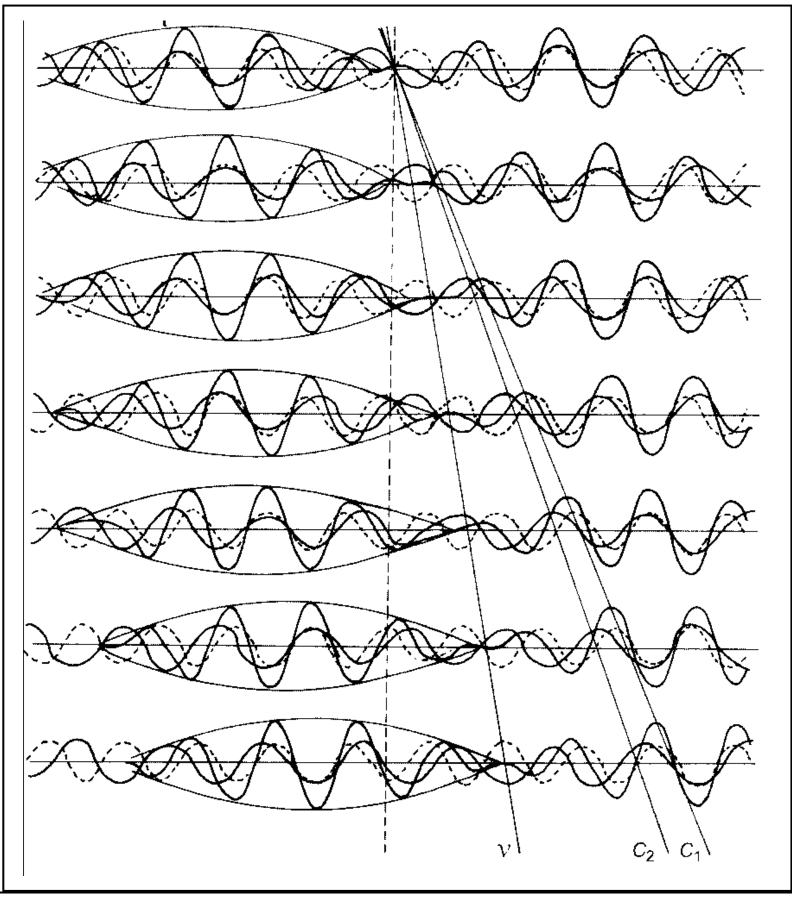


Figure 9.7 Two progressive wave trains of similar amplitude, but different wavelength and wave speed, move in and out of phase. Individual wave nodal points can be tracked at wave speed C_1 and C_2. The nodal point of the wave envelope progresses at the group velocity (V), which is half the phase velocity.

Stokes Waves

For waves of somewhat larger height, the Stokes wave has been used to model the ocean surface and is particularly useful in considering the problem of breaking waves. The trochoidal Stokes wave, with its long troughs and sharp crests, sometimes provides a more realistic profile of the sea surface (Figure 9.8).

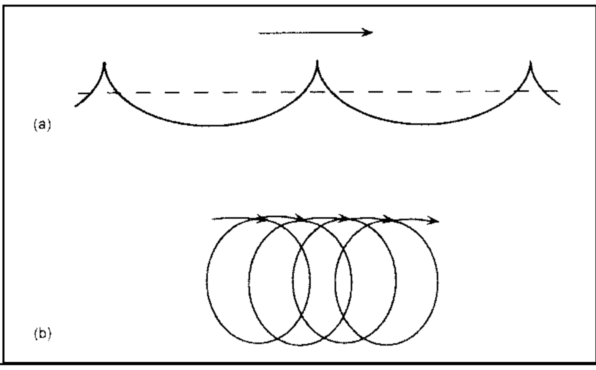


Figure 9.8 (a) The trochoidal Stokes wave is probably a better representation of the sea surface than a sinusoidal. (b) With a Stokes wave, there is a small translation of mass at the surface.

(i) The phase speed of a deep water Stokes wave

$$C^2 = \frac{g}{\kappa}\left(1+\pi^2\delta^2\right) \tag{9.12}$$

where $\delta = H/\Lambda$. For a typical ocean wave $\delta = 1/20$, so a high approximation the phase speed of a Stokes wave is identical to that of a sinusoidal wave of Eq. (9.5).

A more significant difference between the Stokes wave and the sine wave is found in the particle motion (Figure 9.8b). There is a limited translation, u^*, given by

$$u^* = \pi^2\delta^2 C e^{-2\kappa z} \tag{9.13}$$

For a typical ocean wave of period 6 s and a value of $\delta = 0.05$, $u^* \cong 0.25\, e^{-2\kappa z}$ ms^{-1}. At the surface z = 0, there is a significant surface current that drops off rapidly with depth. It is only 4×10^{-4} ms^{-1} at a half wavelength depth of about 28 m.

(ii) The shallow water form of the Stokes wave

$$C^2 = gh\left(1+\frac{H}{2h}\right) \tag{9.14}$$

which reaches to Eq. (9.4) in all cases except when the wave is in very shallow water. Waves will break when the particle speed of the wave outruns the phase speed, when

$$\frac{u}{C} > 1 \tag{9.15}$$

Waves generally break when the ration of wave height to water depth exceeds 0.7.

Capillary waves

The first effect of the wind blowing on a lake or sea surface is to form small surface tension or capillary waves. These wave are present, superimposed on the larger waves and swells, whenever the wind blows. They die out almost immediately when the wind stops. For a wave whose length is less than 0.005 m

$$C_d^2 = \frac{g}{\kappa} + \frac{\kappa}{\rho}\varsigma \tag{9.16}$$

where ς is the surface tension. Surface tension waves, or capillary waves, have very different characteristics than surface gravity wave. In contrast to gravity waves, the shorter the capillary wave, the faster it goes (Figure 9.9). Both gravity waves and capillary waves are dispersive waves, but in the case of capillary waves, the group velocity is greater than the phase velocity. Thus, it appears that new waves are continually building in front of the wave train while those in the rear die out. The minimum speed for a wave from Eq. (9.16) is 0.22 ms^{-1}. Its wavelength is 0.017m.

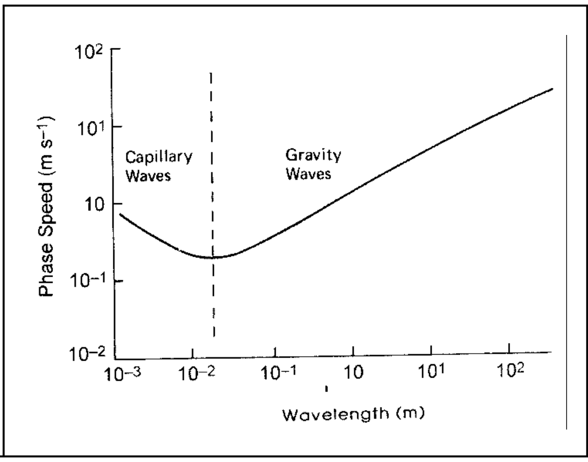


Figure 9.9 For surface waves of wavelength less than 0.01 m, the speed is determined by the surface tension. Surface waves longer than 0.1 m are essentially gravity waves. Those of intermediate wavelength are sometimes called capillary-gravity waves.

Tsunami

The tsunami is one of the most spectacular and devastating of the long-period, shallow water waves. The face that tsunamis are caused by earthquakes has been recognized for several hundred years. The average depth of the ocean is about 4000 m, which means that a tsunami traveling at the speed of $(gh)^{1/2}$ is moving at a speed of about 200 ms^{-1}.

Seiches

The slow oscillation of the water level of a lake or harbor is called a *seiche*. A seiche is a shallow water standing wave, and perhaps the best-known example of a sche is the sloshing wave one can generate in a bathtub.

(1) To understand the physics of a sciche, first consider the motion of water particles in a standing wave. At the nodes, the water motion is entirely horizontal. At the antinodes it is vertical. Elsewhere, it has both horizontal and vertical components

(Figure 9.10).

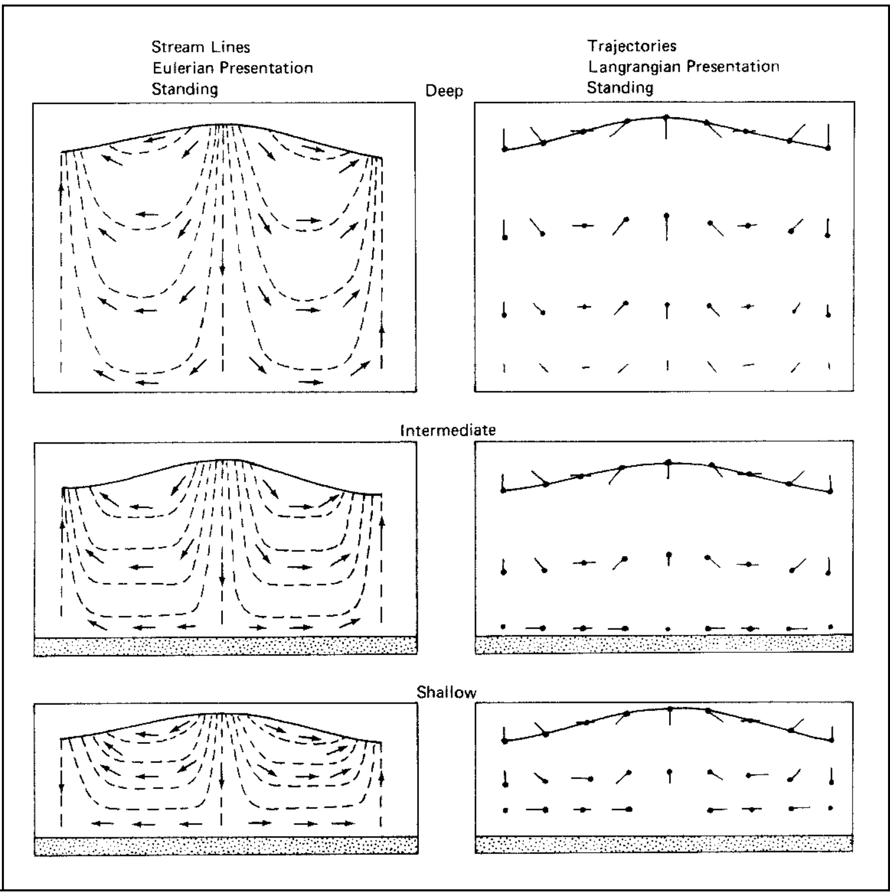


Figure 9.10 In a standing wave, the particle motion is horizontal at the nodal points and vertical at the antinodes. This figure shows a crest in the middle and a trough at the ends. A half period later, a water will crest at both ends and there will be a trough in the middle. At the nodes, which are halfway between crest and trough, there is no vertical movement, only horizontal movement.

(2) Consider next a simple channel closed at both ends in which the water surface can be made to oscillate. The simplest mode is that shown in Figure 9.11. Note that the oscillation has the characteristic of a wave, whose wavelength is twice the length of the channel. The boundary conditions require a wave antinode (vertical movement only at the channel ends), and the wave is clearly a shallow water wave.

$$C = \frac{\Lambda}{T} = \frac{2l}{T} = \sqrt{gh}, \quad T = \frac{2l}{\sqrt{gh}} \tag{9.17}$$

Thus, if one knows the depth (h) and length (l) of the channel, one can determine the period of oscillations. Eq. (9.17) and Figure 9.11 are based on a single node midway in the channel. It is possible to have additional nodes in general

$$T = \frac{2l}{n\sqrt{gh}} \tag{9.18}$$

where n = 1, 2, 3, … is the number of nodes.

(3) Consider a channel open at one end to a level ocean (Figure 9.12). Here the

boundaries require a node at the channel opening end and an antinode at the end. Thus the channel length is a quarter of a wavelength:

$$C = \frac{\Lambda}{T} = \frac{4l}{T} = \sqrt{gh}, \quad T = \frac{4l}{\sqrt{gh}} \tag{9.19}$$

For an open channel with more than one node,

$$T = \frac{4l}{n\sqrt{gh}} \tag{9.20}$$

where n = 1, 3, 5, … is the number of nodes.

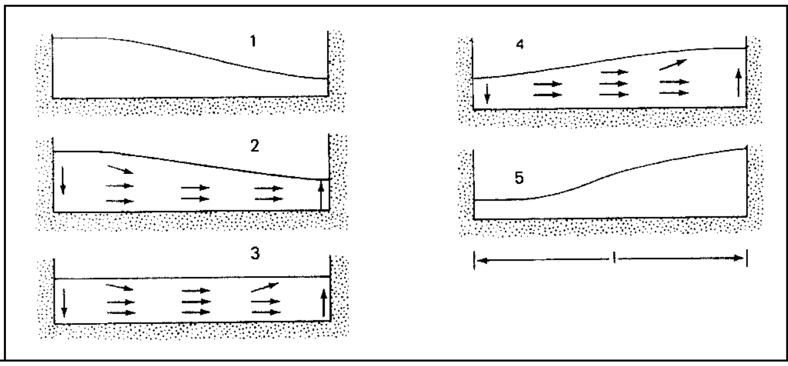


Figure 9.11 Since there can be no flow through the boundary, a seiche in a closed basin requires an antinode at each end of the basin. This means that basin must be a half wavelength for fundamental mode. The seiche progresses through a half a period in the five-step example, with the positions of the crest and trough of panel1 being reversed by panel5.

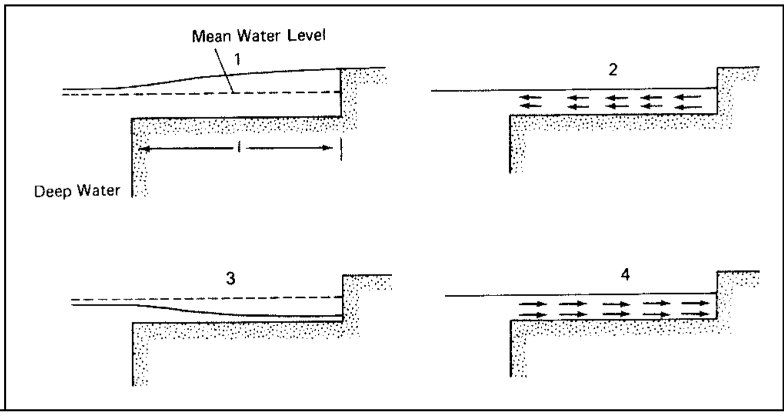


Figure 9.12 Boundary conditions for a seiche in a narrow basin open at one end (for example, a bay opening onto the ocean) require a nodal point where the bay connects with the ocean, and an antinode at the closed end. For the fundamental mode, this means that the basin is a quarter wavelength. Water runs out of the basin as the crest begins to fall and returns as the trough begins to fill.

Edge Waves

There is a class of waves called *edge waves* that run parallel to the shore of the ocean or lake. They can be generated by sharp weather patterns: for example, a passing line squall or a hurricane running up the coast some miles offshore. Their characteristic phase speed is that of a deep water wave modified by the slope of the sea bottom. One can imagine them as trapped waves. The speed at which they travel along the coast is given by

$$C^2 = \frac{g}{\kappa}\sin t \tag{9.21}$$

where t is the slope of the bottom, which is typically 10^{-2} to 10^{-4}. Since slopes are usually small, their phase speeds are much less (1 to 10%) than those of corresponding deep water waves. The amplitude of these edge waves is highest at the shore and falls off rapidly as one proceeds seaward:

$$a = a_0 e^{-\kappa x} \tag{9.22}$$

Where a_0 is the amplitude at shoreline and x is the distance normal to the shore. Essentially all of the energy in edge waves is trapped within one wavelength of the shore.

Very Long Waves: The Coriolis Effect

Waves in which the forces are weak enough for the Coriolis term to be important must have periods of at least several hours. They must be a form of the shallow water waves.

(1) In the absence of the Coriolis force, a simple shallow water wave in a constant density ocean traveling in the x direction satisfies the simplified balance of forces

$$\begin{aligned} \frac{\partial u}{\partial t} &= -g\frac{\partial \eta}{\partial x} \\ \frac{\partial \eta}{\partial t} &= -h\left(\frac{\partial u}{\partial x}\right) \end{aligned} \tag{9.23}$$

Note:

$$\int_\eta^z dp = -\rho g \int_\eta^z dz \;\Rightarrow\; p_z = -\rho g(z-\eta)$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{1}{\rho}\frac{\partial[-\rho g(z-\eta)]}{\partial x} = -g\frac{\partial \eta}{\partial x}$$

From the equation of continuity

$$\int_\eta^{-h}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dz + \int_\eta^{-h} dw = 0 \;\Rightarrow\; -\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)h + w_{-h} - w_\eta = 0$$

Boundary conditions:

At the bottom $w = 0$ at $z = -h$

At the free surface $w = \partial\eta/\partial t + u\partial u/\partial x + v\partial\eta/\partial y$ at $z = \eta$

which for small perturbations, reduced to

$w = \partial\eta/\partial t$ at $z = \eta$

Thus,

$$\frac{\partial \eta}{\partial t}+h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$$

Consider that a water wave travels in the x direction so $\partial v/\partial y = 0$.

By appropriately differentiating Eq. (9.23) to eliminate u, one can form a simple wave equation

$$\frac{\partial^2 \eta}{\partial x^2}=\frac{1}{C^2}\frac{\partial^2 \eta}{\partial t^2} \tag{9.24}$$

where C is the shallow water wave velocity:

$$C=\sqrt{gh} \tag{9.25}$$

(2) Adding the Coriolis force means that one no longer has a one-dimensional set of equations. Eq. (9.23) becomes

$$\begin{aligned}\frac{\partial u}{\partial t}&=+fv-g\frac{\partial \eta}{\partial x}\\ \frac{\partial v}{\partial t}&=-fu-g\frac{\partial \eta}{\partial y}\\ \frac{\partial \eta}{\partial t}&=-h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\end{aligned} \tag{9.26}$$

These are the governing equations for both the Kelvin wave and inertial gravity waves, both of which play important roles in the ocean.

Kelvin Waves

The Kelvin wave is a long period, small amplitude wave that travels parallel to a fixed boundary, such as a wide channel or a coastline. Because the role of the Coriolis force, the equator, where f goes through zero and change sign, also serves as a Kelvin wave boundary. In the direction of the wave, the solution is identical to that of the non-dispersive shallow water wave, with the wave traveling in the north-south direction. In the transverse (x) direction the balance is geostrophic. There is a slope to the sea surface such that the wave amplitude (η0) at the boundary falls off exponentially as one moves away from the channel wall (Figure 9.13):

$$\begin{aligned}\eta&=\eta_0\exp\left(-fx/\sqrt{gh}\right)\\ &=\eta_0\exp\left(-x/R_d\right)\end{aligned} \tag{9.27}$$

where $R_d=\sqrt{gh}/f$ is the *Rossby radius of deformation*. For the open ocean, R_d is on the order of 2000 km; for coastal waters, it is on the order of 300 km. In this example, R_d is a measure of the transverse distance at which one can expect to observe the effect of a Kelvin wave.

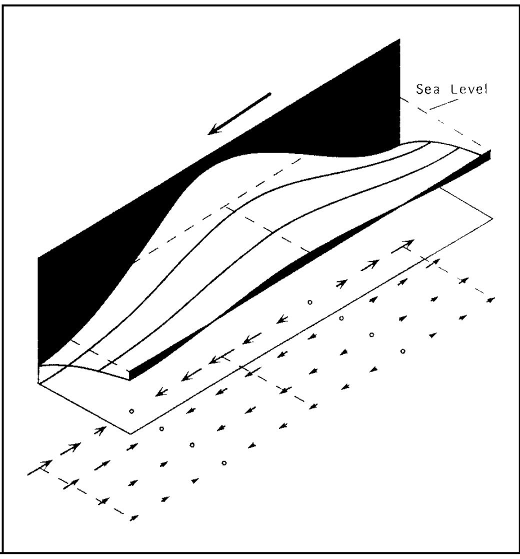


Figure 9.13 For a Kelvin wave running along a channel the balance of forces are (a) those of a shallow water wave in the direction the wave is progressing along the channel, and (b) geostrophic for those perpendicular to the direction of the wave. The maximum wave amplitude is at the channel edge, and the maximum amplitude decreases exponentially away from the channel.

As one faces in the direction of the Kelvin wave's progress, the maximum amplitude is always on the right (left) in the Northern Southern) Hemisphere. In a closed basin, therefore, Kelvin waves travel counterclockwise (clockwise) in the Northern (Southern) Hemisphere. Along the equator, the Kelvin wave travels from west to east in both hemispheres, with maximum amplitude at the equator and the amplitude decreasing exponentially both north and south of the equator.

Kelvin waves are often associated with tides. The height of the semidiurnal tidal wave that runs eastward up the English Channel is several times higher on the right-hand (French) side of the channel than on the left-hand side (English) side. In the North Sea, there is a complex rotary tide system that shows counterclockwise rotation. (Figure 9.14).

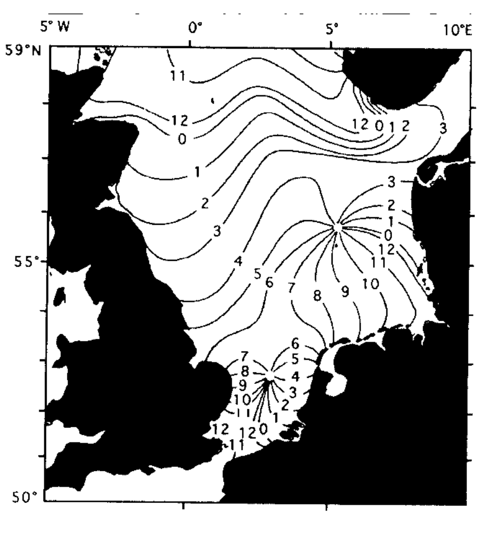


Figure 9.14 The semidiurnal tide in the North Sea is a combination of a progressive tidal wave from the north and two counterclockwise rotary tides in the south. The lines represent the time of high water in hours, in reference to Greenwich.

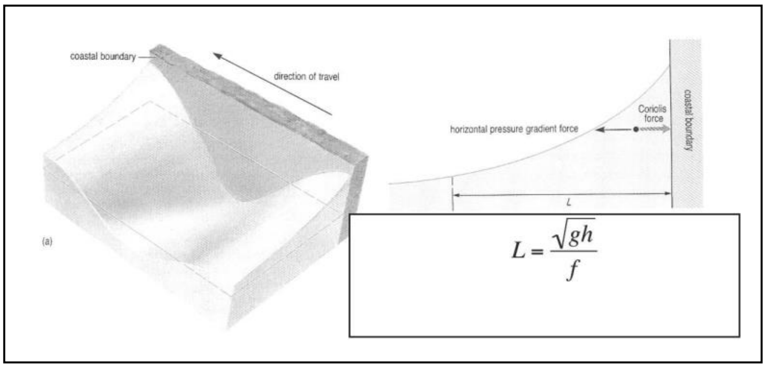
Notes:

Coastal Kelvin waves

(1) Costal Kelvin waves balance the Coriolis force against a topographic boundary (i.e. coastline). They always propagate with the shoreline on the right in the northern and the left in the southern hemisphere.

(2) A coastal Kelvin wave moving northward along is deflected to the right, but the coast prevent the wave from turning right and instead causes water to pile up on the coast. The pile of water creates a pressure gradient directed offshore and a geostrophic current directed northward.

(3) Kelvin wave amplitude is negligible at a distance offshore given by the Rossy radius of deformation. For mid-latitude Kelvin waves traveling in the thermocline, this is about 25 km. Because of the rapidly decay coastal Kelvin waves appear to be trapped close to the coast.

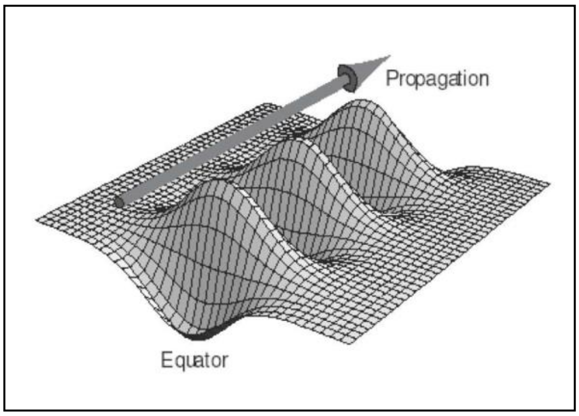


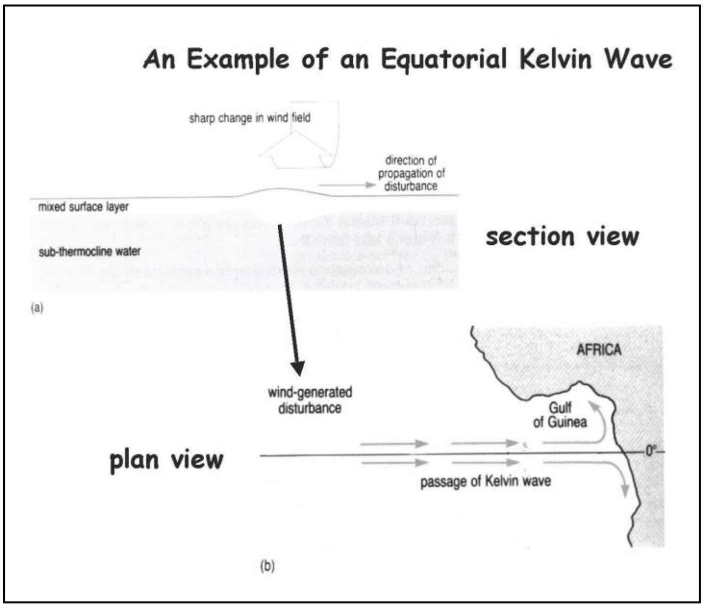
$$L = \frac{\sqrt{gh}}{f}$$

Equatorial Kelvin waves

(1) Equatorial Kelvin waves are a special type of Kelvin wave that balances the Coriolis force in the northern hemisphere against its southern hemisphere counterpart. This wave always propagates eastward and only exists on the equator.

(2) Equatorial Kelvin waves propagating <u>in the thermocline</u> have wave speeds slow enough to give a Rossby Radius of Deformation that is on the order of 250 km and thus they appear to be trapped close to the equator.





Inertia Gravity Waves

The wave velocity is

$$C_i = \frac{\sqrt{gh}}{\sqrt{1-\left(f/\omega\right)^2}} \tag{9.28}$$

The phase velocity is a function of wave frequency, so these are dispersive waves. The group velocity is given by

$$V_i = \sqrt{gh}\sqrt{1-\left(\frac{f}{\omega}\right)^2} \tag{9.29}$$

These are *inertia gravity waves*, often referred to as ***Poincaré waves***. Note that the term under the second square root sign is imaginary unless the wave frequency (ω) is larger than the Coriolis parameter (f), which means that these waves cannot have periods longer than the inertial period.

For a wave traveling in the x direction, the particle velocity u in the x direction is the same as for the classic shallow water wave. However, if the wave frequency is small enough for the Coriolis force to be important, there is transverse particle motion parallel to the wave crest.

$$\frac{v}{u} = \frac{f}{\omega} \tag{9.30}$$

The water particles travel in an ellipse. As the frequency increases, the right-hand side of Eq. (9.30) approaches zero and Eqs. (9.28) and (9.29) approach the pase and group velocity of a classical shallow water wave.

Rossby Waves

Kelvin waves and inertia gravity waves are adequately described by ignoring the change in the Coriolis parameter with latitude. It is precisely that change of the Coriolis parameter with latitude that determines the Rossby wave. The governing equations are given as

$$\begin{aligned}\frac{\partial u}{\partial t} &= +(f_0 + \beta y)v - g\frac{\partial \eta}{\partial x} \\ \frac{\partial v}{\partial t} &= -(f_0 + \beta y)u - g\frac{\partial \eta}{\partial y}\end{aligned} \tag{9.31}$$

For a class of waves in the ocean called *Rossby* or planetary waves, the restoring force is not gravity and the particle motion associated with these waves is in the horizontal, not the vertical. For a westward-moving particle, the (βy) term provides the necessary restoring force, where y is the north-south distance the particle if from its equilibrium position (f_0) latitude.

Another way of conceptualizing Rossby waves is in terms of the conservation of potential vorticity $(\zeta+f)/D$. Imagine a column of water of constant layer depth (D) and with zero relative vorticity (ζ) at latitude with Coriolis parameter f_0. If the column is displaced northward, the Coriolis parameter increases. To conserve potential vorticity, the layer must increase its negative relative vorticity. Likewise, if the water is displaced southward, the relative vorticity must increase to balance the decrease in the Coriolis parameter (Figure 9.15).

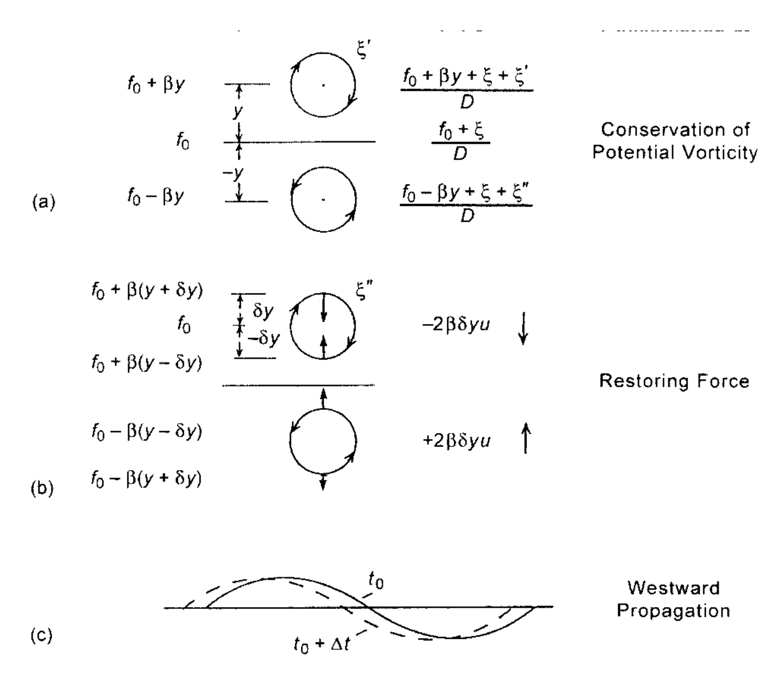


Figure 9.15 (a) To conserve potential vorticity, a layer of water of constant thickness perturbed from its equilibrium position along f0 will gain negative relative vorticity (ζ') north of the line and positive relative vorticity (ζ'') south of the line. (b) The effect of the Coriolis force on the change in relative vorticity is to drive the layer back toward its equilibrium line at f0. The farther from equilibrium, the greater is the restoring force. (c) The periodic motion generated by the effect of the Coriolis force on the relative vorticity drive the wave westward.

The periods of Rossby waves are longer than the inertial period. Their phase velocity always has westward component. It is given by

$$C_R = \frac{\beta}{\kappa^2 + (f^2 / gh)} \tag{9.32}$$

which means that the period is a function of wavelength and the shorter the wave-length, the longer the period. However, although the phase velocity always has a westward component, the group velocity and the energy can travel either west or east. Rossby waves are the principal wave form with period longer than a day. Some Rossby waves travel very slowly (taking months to years to cross an ocean basin).

Notes:

(1) The restoring force for a Rossby wave is the requirement to conserve potential vorticity.

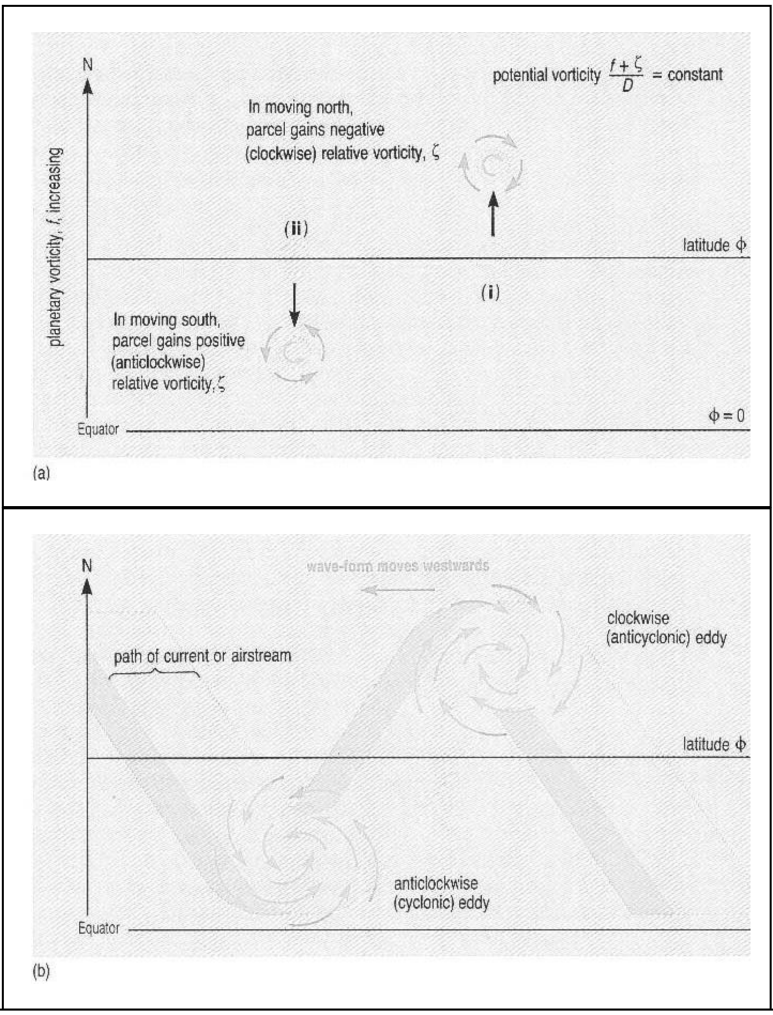
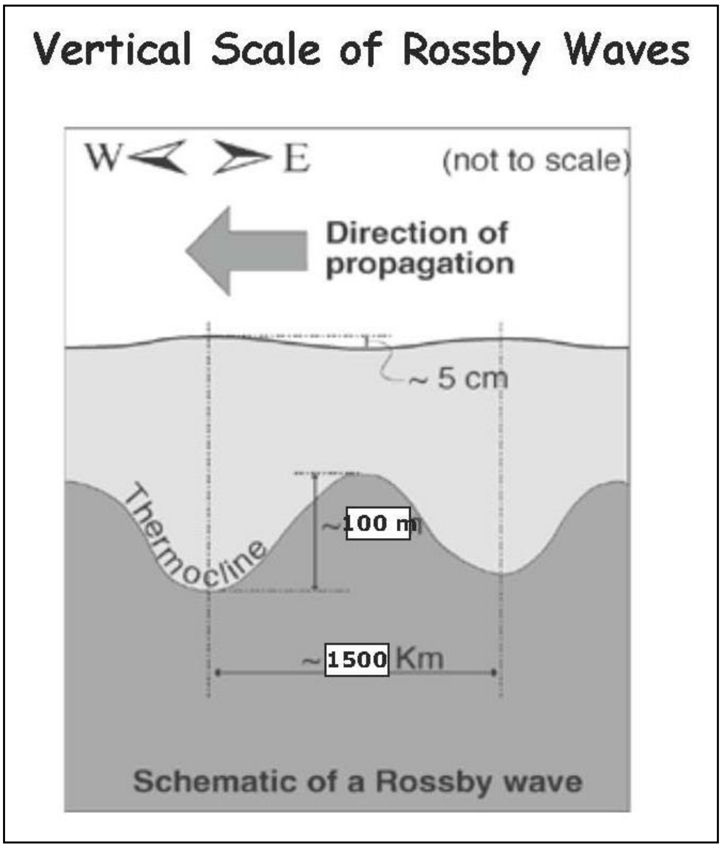
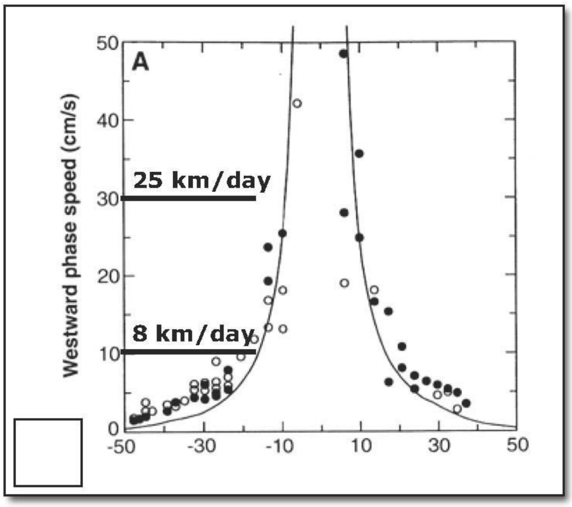


Figure (a) Diagram to show how in a Rossby wave the need to conserve potential vorticity $(f+\zeta)/D$ leads to a parcel of water oscillating about a line of latitude ϕ while alternately gaining and losing relative vorticity ζ. (b) The path taken by a current or airstream affected by a Rossby wave. Note that the flow pattern is characterized by anticyclonic and cyclonic eddies, and that the wave-form moves westwards relative to the current or airstream.



(2) Latitudinal variation of the phase speeds of nondispersive Rossby waves obtained from historical hydrographic data based on the classical theory (solid line) and from T/P observations in the Pacific (solid circles) and the Atlantic and Indian oceans (open circles).



$$C_R = -\frac{\beta}{\kappa^2 + \frac{f^2}{gh}}$$

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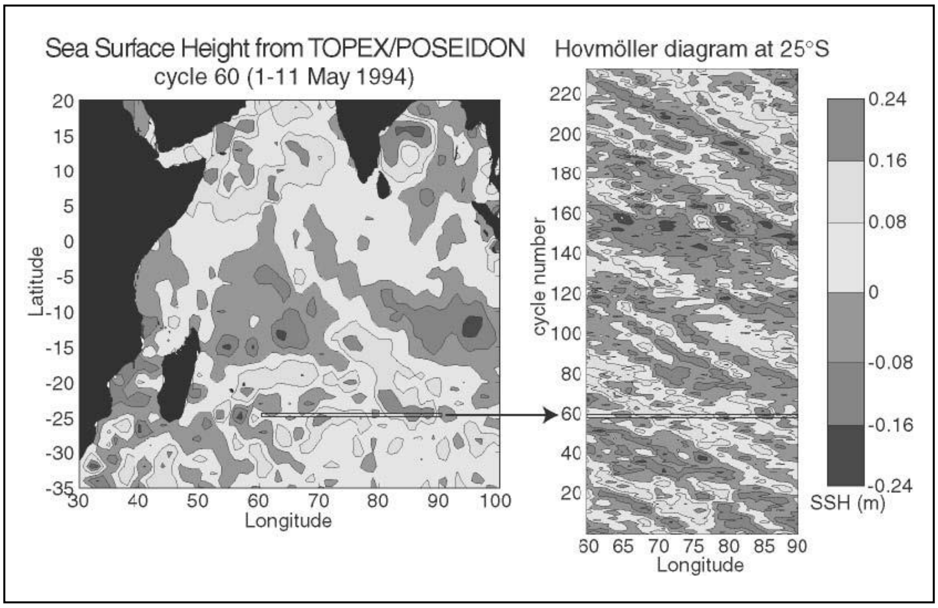
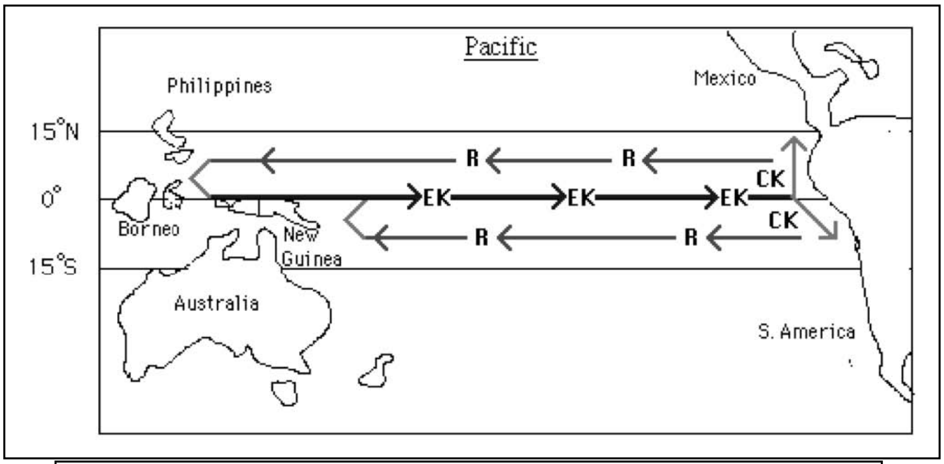
(3) Time-distance or "Hovmoller" diagrams are commonly used to depict wave propagation in the ocean.



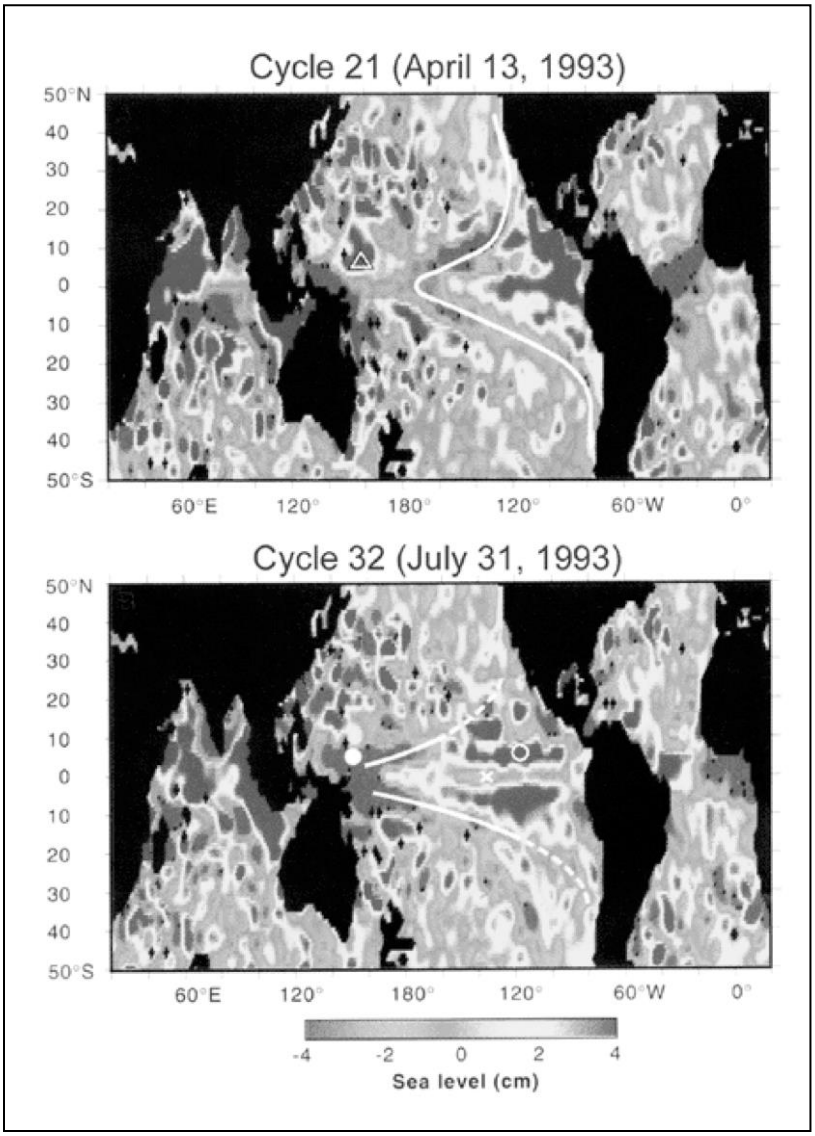
Kelvin-Rossby wave interactions



EK: easterly Kelvin waves
R: Rossby wave
CK: coastal Kelvin waves

(1) A westward-propagating Rossby wave trough centered on the equator and extending to mid-latitudes in both hemispheres can be seen in the Pacific Ocean in the April 13, 1993 frame. The refracted shape that is characteristic of Rossby waves is due to the latitudinal variation of phase speed.

(2) In the July 31, 1993 frame, this Rossby wave trough has impinged on the western boundary of the Pacific and an equatorial Kelvin wave trough centered at about 140 W has propagated rapidly eastward more than half way across the Pacific, splitting a newly formed Rossby wave crest that has propagated westward from South America.



(3) A westward-propagating Rossby wave trough centered on the equator and extending to mid-latitudes in both hemispheres can be seen in the Pacific Ocean in the April 13, 1993 frame. The refracted shape that is characteristic of Rossby waves is due to the latitudinal variation of phase. In the July 31, 1993 frame, this Rossby wave trough has impinged on the western boundary of the Pacific and an equatorial Kelvin wave trough cantered at 140^{o}W has propagated rapidly eastward more than half way across the Pacific, splitting a newly formed Rossby wave crest that has propagated westward from South America.