Introductory Dynamical Oceanography
Final Exam (1/12/2021)

1. (15%)
   (a) What is the physical meaning of Ekman's theory? (explain what kind of forces balance).

$$f v + A_z \frac{\partial^2 u}{\partial z^2} = \alpha \frac{\partial p}{\partial x}$$

$$-f u + A_z \frac{\partial^2 v}{\partial z^2} = \alpha \frac{\partial p}{\partial y}$$

   (b) Based on what concepts, Ekman simplified the above governing equations as the balance between two forces? Please show the detail how to obtain the simplified Ekman's equations.
   (c) What is the Ekman depth?
   (d) What is the Ekman Spiral?
   (e) What is the empirical formula to calculate the wind stress magnitude, τ_η? Please explain each term.

2. (6%)
   (a) Please explain what is the barotropic and baroclinic condition, respectively?
   (b) Show the "thermal wind" equations from the geostrophic equations and the hydrostatic equation.

3. (12%) Stommel assumes a two-layer model. The upper layer has density ρ_1 and is moving; the lower layer has density ρ_2 and is at rest. If η is the surface elevation and d is the level of the interface between the layers, please answer the following questions:
   (a)What are the pressures in the upper and lower layers, respectively?
   (b)From (a), what is the horizontal pressure gradient in the upper and lower layers, respectively? You can just show the x–component for example.
   (c)What is "reduced gravity"?
   Note: The hydrostatic equation is $dp = -\rho g dz$.

4. (12%) From the governing equations

$$\alpha \frac{\partial p}{\partial x} = f v + \alpha \frac{\partial \tau_x}{\partial z}$$

$$\alpha \frac{\partial p}{\partial y} = -f u + \alpha \frac{\partial \tau_y}{\partial z}$$

   Please show how to obtain the Sverdrup equation, $\beta M_y = \mathrm{curl}_z \vec{\tau}_\eta$?

5. (10%) Please describe the equatorial current systems in the Pacific, Atlantic and Indian Oceans, respectively. (Note: surface and sub-surface if possible)

6. (9%) (a) Please explain what is the Rossby radius of deformation and its physical meaning? Note: external and internal, respectively.
(b) If in coastal waters the scales are h = 10 m, please estimate roughly what is the scale of the internal Rossby radius of deformation?

7. (10%) (a) Stommel was the first one to present an explanation for the *westward intensification*. Please describe what physical mechanisms are responsible for the westward intensification? (b) Please use the conservation of vorticity to interpret the western intensification of the ocean currents.

8. (10%) (a) Please write down the equation the conservation of the potential vorticity and explain its physical meaning. (b) Please discuss the following situations, we can make some predictions about vorticity changes when a parcel of water moves from one place to another.

(1) if D remains constant
(a) If a column of water moves zonally.
(b) If a column of water moves meridionally toward the north pole.

(2) If D increases,
(a) If the water moves zonally.
(b) If the water moves meridionally toward the south pole.