Chap 11 Numerical Models

11.1 Introduction

Analytic solutions to the equations of motion are difficult or impossible to obtain for typical oceanic flows.

(1) Non-linear terms are often important;
(2) Frictional terms also.
(3) Together or separate, they make difficult the solutions.
(4) Realistic boundary conditions are often difficult to impose.

The only useful solutions in many instances can be found through numerical integration of the equations.

Two broad types of models are used:

(1) Mechanical models — Simplified models for study of processes; the output is easy to interpret.

(2) Simulation models — Attempt to calculate realistic circulation of oceanic regions. Such models are often very complex because all of important processes are included. Output is difficult to interpret.

Numerical solutions give results on a grid of points — there is no information about the flow between the points. Hence the solutions are approximate, not exact.

(1) A major problem in numerical analysis is to find numerical equations that are good approximation to the original analog equations.

(2) A second problem is to find efficient, accurate solutions to the numerical equations.

11.2 Numerical Methods

The method most commonly used in numerical models, particularly in the simulation models, is that of ***finite differences***. For example, suppose we want to determine $\partial u/\partial x$ at the point x_j. A Taylor series expansion around x_j gives

$$u_{j+1} - u_j = \frac{\partial u}{\partial x}\Delta x + \frac{1}{2}\frac{\partial^2 u}{\partial x^2}(\Delta x)^2 + \frac{1}{6}\frac{\partial^3 u}{\partial x^3}(\Delta x)^3 + \mathrm{O}(\Delta x)^4 \qquad \textbf{\textit{forward}}$$

$$u_j - u_{j-1} = \frac{\partial u}{\partial x}\Delta x - \frac{1}{2}\frac{\partial^2 u}{\partial x^2}(\Delta x)^2 + \frac{1}{6}\frac{\partial^3 u}{\partial x^3}(\Delta x)^3 - \mathrm{O}(\Delta x)^4 \qquad \textbf{\textit{backward}}$$

$$\frac{(u_{j+1} - u_{j-1})}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{1}{6}\frac{\partial^3 u}{\partial x^3}(\Delta x)^2 + \mathrm{O}(\Delta x)^4 \qquad \textbf{\textit{centered}}$$

The schemes where the values at new times are calculated from values at previous times are called ***explicit***.

It is possible to devise schemes in which the values at the new time step depend on spatial gradients of values at the new time step. One then writes down equations for each grid point and must solve the whole set of linear equations simultaneously. Such methods are called ***implicit***.

11.3 General approach to numerical modeling of ocean circulation

The equations of motion, continuity and, where applicable, heat and salt conservation are put in finite difference form. Boundary conditions, for example wind stress, temperature and salinity at the sea surface, and temperature, salinity and velocity on lateral boundaries, are chosen. Initial conditions at time $t = 0$ must also be chosen.

Round-off errors:

–Small errors that grow exponentially with time, especially important in non-linear equations.

Friction and Turbulence:

–Equations of motion are not closed.

–Some scheme must be used for determining the influence of sub- gridscale motion, for example a turbulence-closure scheme for determining the eddy viscosity.

–How to relate friction (turbulent friction) to the large-scale flow?
One solution is to use a variable viscosity, a viscosity that increases as the root-mean-square rate of strain calculated from the shear of the velocity.

Initialization:

–Initial flow field may not be close to geostrophic equilibrium.

–This causes gravitational or sound waves in the solutions, leading to large errors in models that cannot support these waves.

11.4 Descriptions of some models of individual oceans

Historically there have been two types of simulation models used in oceanography:

(1) **Primitive Equation Models**:
- –Based on full equations of motion, including thermodynamic processes.
- –High vertical resolution, low horizontal resolution.
- –High dissipation for numerical stability:
  This inhibits hydrodynamic instability, so no eddies.
  Variability is due only to atmospheric forcing.
- –Used for studies of large-scale hydrographic structure, climate dynamics, and water-mass formation.

(2) **Quasi-Geostrophic Models**:
- –Based on geostrophic approximations to equations of motion, limited thermodynamics.
- –High horizontal resolution, low vertical resolution.
- –Used for studies of the dynamics of time-dependent circulation, eddies, the interaction of eddies with the mean flow, and western boundary currents.

Now, with the development of fast computers with large memory it is becoming possible to combine the two models.

(3) **Modern Models**:
- –Primitive equation
- –Resolution of 0.3° latitude, 0.4° longitude (37km × 37km at 34° Latitude) which equals the Rossby radius of deformation for the first baroclinic mode.
- –30 vertical levels.
- –Real coasts and bathymetry.
- –Initialized with known distributions of temperature and salinity.
- –50 Cray XMP hours per year of integration time.

Quasi-Geostrophic Models

O'Brien's 2-Dimensional, Wind-driven Model of the North Pacific (1971)
–Realistic extension of Munk's model.
–Constant density.
–One year (barotropic).
–Polar coordinates, 2°×2° resolution, β–plane.
–Bathymetry for $z < 2$ km.
–Variable horizontal eddy viscosity.

Spin up over several months.

Kuroshio leaves the coast at the correct latitude 35°N
–Transport is correct at 60×10^6 m^3/s.
–Increase in transport above Munk's 38 Sv due to non-linear terms.
–1/3 of transport is due to re-circulation.

Drawbacks:
–No thermocline.
–No thermohaline – wind transport interactions.

Eddy Resolving Numerical Models

To understand the role of eddies in numerical models it was necessary to develop regional models with high spatial resolution capable of reproducing eddies.

Eddies in the atmosphere (storms) often have negative viscosity.
–Small features in the flow drive the larger features.
–Eddies can accelerate the flow, an example is the jet stream.

Eddies are now known to be very important for understanding the oceanic circulation.
–Eddies are fed by the potential energy of the main flow.
–This is an example of ***baroclinic instability***.

Models have been possible run over the past few years with the advent of the modern supercomputers.

Eddy-resolving General Circulation Models (EGCM):
–Attempt to model the circulation of an oceanic region.

–To keep within the limitations of even very fast computers, the models must still be limited to ocean basins.
–The models take many days of computer time to produce useful results, and the output is very large because intermediate results are stored for later analysis.

Harvard Open Ocean Model (1988):
–Eddy resolving.
–Quasi-geostrophic.
–Baroclinic, 6-levels in vertical.
–15 km resolution, 1 hour time step.
–High-frequency smoothing to damp grid-scale variability (Shapiro filter).

The model reproduces the important features of the Gulf Stream and meander region, including:
–Meanders,
–Cold and warm core rings,
–Interaction of rings with the stream,
–Baroclinic instability.

Model is now being used for forecasting the dynamics of the Gulf Stream
–Satellite data used to determine the location of features in the region.
–Features are represented by simple analytic functions in the model.
–The model dynamically adjusts the features and interpolates between them, a **nowcast**.
–The model is used for evolving the features in time, a **forecast**.

The model has been used for making successful one week forecasts of the Gulf Stream and Meander Region.

Primitive Equation Models:
–Use momentum equation + continuity equation.
–Use equation of state to relate density to temperature and salinity.
–Models requires very many computations, hence very fast computers.
–There is yet no model that reproduces most of the known features of the circulation, including the thermocline.

Semptner & Chervin's (1988) Global, Eddy-Resolving Model:
–3-dimensional, 20-layer model.

–Wind driven + thermohaline forcing.
–Rigid lid to remove surface gravity waves, this allows larger time steps.
–Started from rest with observed vertical density distribution.
–Bi-harmonic eddy viscosity, which varies with scale.
–No static instability allowed.
–Spun up for 22.5 years with mean annual thermohaline and wind forcing.
–Integrated for 10 years with monthly thermohaline and wind forcing.

The model gives:

–Realistic picture of the global ocean circulation plus the statistics of the variability.
–Realistic heat and mass transports.
–Realistic eddy heat transports.

Semptner, A.J. & R.M. Chervin, 1988. A simulation of the global ocean circulation with resolved eddies. *J. Geophysical Research* **93**: 15,502- 15,522 & 15,767-15,775.