Chap 12 Waves

12.1 Introduction

The main classes of waves and their causes are:

(1) ***ripples, wind waves*** and ***swell*** – due to the effects of the wind on the air/water interface;

(2) ***internal waves*** – which may occur when vertical density variations are present – various causes, e.g. current shear, surface disturbances;

(3) ***tsunamis*** – generated by seismic disturbances of the sea bottom or shore;

(4) ***gyroscopic-gravity waves*** – (surface and internal) of sufficiently long period that the Coriolis effect is important – various causes, e.g. wind stress changes, atmospheric pressure changes;

(5) ***Rossby or planetary waves*** – large-scale and long period, evident as time varying currents – various causes, e.g. time variations in wind stress and perhaps baroclinic and/or barotropic instability;

(6) ***tides*** – due to fluctuating gravitational forces of the moon and sun.

Note: We shall not discuss sound waves here. They have many applications including the determination of depth (echo sounders) and of sub-bottom structure, detecting objects in the body of the ocean (e.g. fish, submarines), transmitting information (e.g. from sub-surface instruments or drifters), and determining ship speed relative to the bottom and current speeds using the Doppler effect from sound reflected from objects in the sea.

Approach:

(1) Consider the fluid dynamics of ideal waves have a sinusoidal shape.

–give information about the relations between the surface shape, the progress of the waves and the motions of the water below the surface.

–ideal regular waves studied bear only a limited resemblance to real waves observed at sea which are characterized by their irregularity in form and period.

(2) Start from observations of the shape of the irregular sea surface, regard it as a composite of a wide range of possible ideal components and carry out a spectral analysis to determine the characteristics of the spectrum of components.

–If we have observations of the surface elevation, η, at a point for a period of time, we can consider this record to be the sum of sine or cosine waves of different amplitudes, phases and frequencies.

–Spectral analysis consists of finding the amplitudes and phases as a function of frequency.

–A plot of amplitude squared (proportional to the wave energy) *versus* frequency is called a ***wave energy spectrum*** (Fig. 12.1).

(3) From the spectrum and the classical theory for each component we should be able to calculate the total effect of the wave field by summing over all the components using appropriate amplitudes and phases.

–The direction of travel should be considered as well and a spectrum including the direction of travel information is called a ***directional spectrum***.

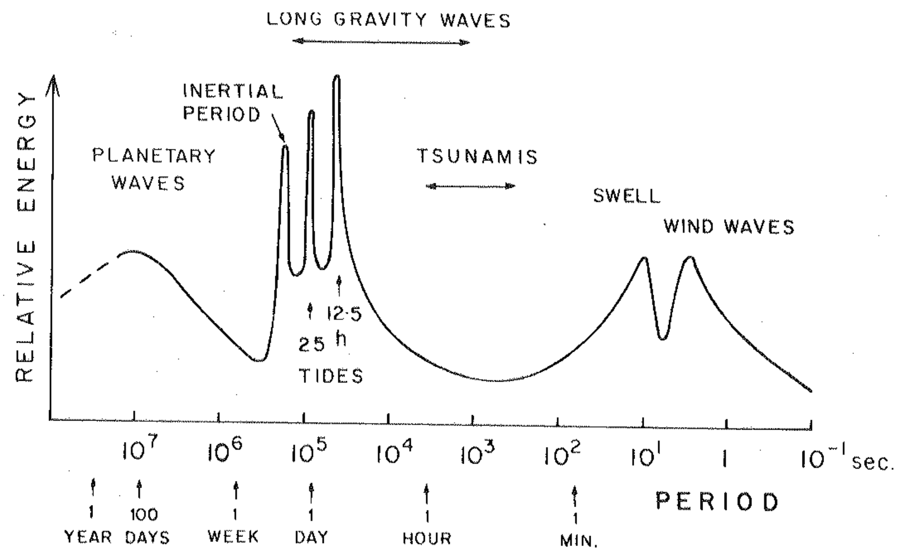

Fig. 12.1 Schematic energy spectrum of oceanic variability, showing approximate relative energy levels. The area under the curve is roughly proportional to the contribution to the total energy of the time-dependent motions. Amplitudes for various parts of the spectrum reflect order of magnitude estimates and are most uncertain for the planetary waves whose average energy levels are least well established.

12.2 Some general characteristics of waves

Assuming that the waves on the sea surface are simple cosine waves (in vertical section) some terms in which we shall use are illustrated in Fig. 12.2.

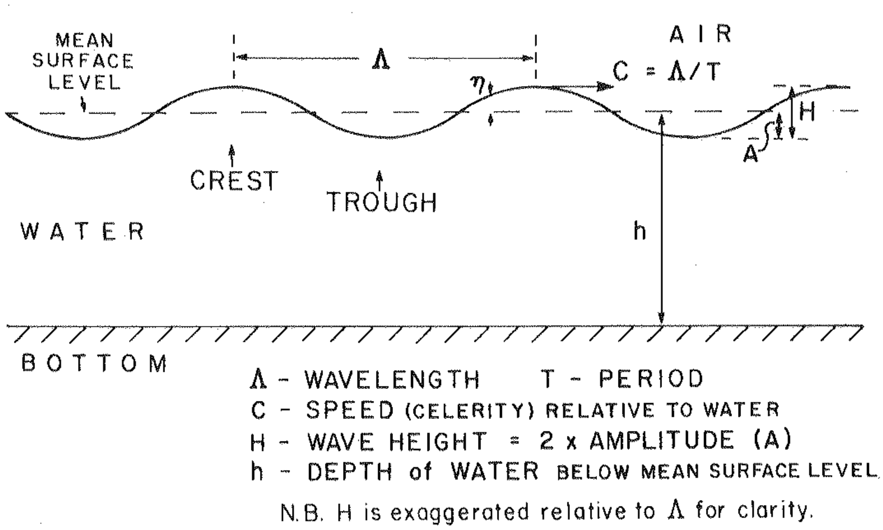

Fig. 12.2 Terms related to ideal (sine or cosine) waves.

–***H*,** the ***height*** of a wave (the vertical distance from trough to crest), is twice what the physicist calls the ***amplitude*** *A* (the maximum displacement) of the vertical oscillatory motion of the surface above or below the mean water level.

–The speed at which a particular part of the wave passes a fixed point is $C = \Lambda/T$ where Λ is the ***wavelength*** (the distance from crest to crest or trough to trough) and T is the ***period*** (the time between two successive crests or troughs passing a fixed point).

–The ***radian frequency*** ($\omega = 2\pi/T$)

–The ***radian wave number*** ($k = 2\pi/\Lambda$)

–The vector wave number $\vec{k}$ has magnitude k and points in the direction of propagation of the waves, e.g. perpendicular to the crests of surface gravity waves.

–$C = \Lambda/T = \omega/k$

For convenience in referring to them, it is common to classify surface waves according to their periods as in Table 12.1.

Table 12.1 Waves classified by period

Period	Wavelength	Name
0–0.2s	centimeters	ripples
0.2–9s	to about 130m	wind waves
9–15s	hundreds of meters	swell
15–30s	many hundreds of meters	long swell or forerunners
0.5min–hours	to thousands of kilometers	long period waves including tsunamis
12.5, 25h, etc.	Thousands of kilometers	tides

–In all these surface waves, gravity is the primary restoring force, allowing oscillations to occur. (If some water lifted up and allowed to fall back under the action of gravity its inertia will cause it to overshoot the equilibrium position; pressure forces will then push it back up and oscillations will ensue.)

–Ripples of wavelength less than about 5 cm are also affected by surface tension; these waves are of very small amplitude and will not be discussed here, although it is considered that they may play a role in determining the drag of wind on water.

–For surface waves of periods of several hours or more, the Coriolis force must also be included in the analysis.

–Wind waves are the locally generated waves. They have a fairly wide range of directions of propagation the sea surface is quite irregular.

–Swell is the term for waves which have been generated elsewhere, it travels in one direction and is much more regular.

–The longer waves travel faster than shorter ones and so at some distance from the source area, at any one time, the swell has a narrow range of frequencies which also makes it more regular than wind waves.

– The ranges of periods of wind waves and swell actually overlap considerably–wind waves may have periods of up to 15 seconds or so if the wind speed is very large, while swell with periods of only a few seconds is possible.

12.3 Small amplitude waves

12.31 Pure waves (single frequency)

The word “small” here is used in a comparative manner and refers to the “relative height” or ***steepness***, H/Λ. For the simple (linear) theory to be correct within a few percent, this ratio should be less than about 1/20 and in many cases for real waves it is 1/50 or less.

For a progressive sine or cosine wave, the vertical displacement η of the free surface from the mean level is given by

$$\eta = A\cos\left[2\pi\left(\frac{x}{\Lambda} - \frac{t}{T}\right)\right] \tag{12.1}$$

Using wave number and frequency, (12.1) can be written more compactly as

$$\eta = A\cos(k\,x - \omega\,t) \tag{12.1′}$$

The argument ($k\,x - \omega\,t$) of the cosine function is termed the ***phase*** of the wave; it goes from 0 to 2π as one goes from one crest to the next (distance Λ) at a fixed time or through one period (T) at a fixed point. Also, since C is the speed at which a point of fixed phase travels it is more completely termed the ***phase speed***.

For such waves it can be shown that the phase speed

$$C = \left[\frac{g\Lambda}{2\pi}\tanh\frac{2\pi h}{\Lambda}\right]^{1/2} = \left[\frac{g}{k}\tanh kh\right]^{1/2} \tag{12.2}$$

where g = acceleration due to gravity and h is the water depth.

(1) for $\Lambda < 2h$, called short or deep–water waves ($\because h > \Lambda/2$)

$$\text{then } C_s = (g\Lambda/2\pi)^{1/2} = (g/k)^{1/2} \tag{12.2′a}$$

(2) for $\Lambda > 20h$, called long or shallow–water waves ($\because h < \Lambda/20$)

$$\text{then } C_l = (gh)^{1/2} \tag{12.2′b}$$

Fig. 12.3 illustrates these nomenclatures.

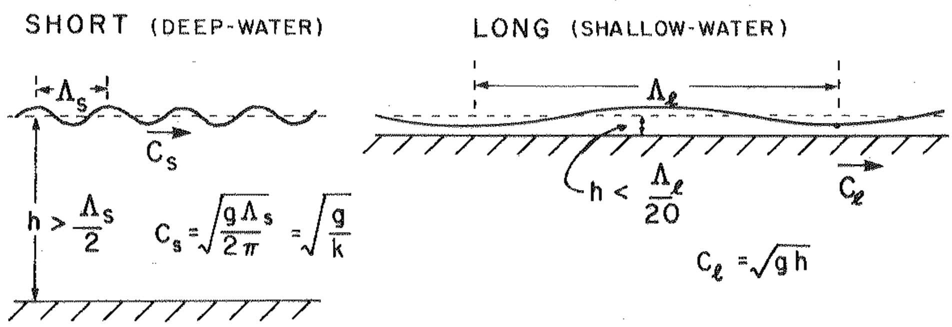

Fig. 12.3 Properties of short and long waves.

The relationship between ω and k is termed the ***dispersion relation***. It is given as

$$\omega^2 = gk\tanh kh \tag{12.3}$$

Fig. 12.4 shows plots of (12.2) as speed C against water depth h for a selection of wavelengths from 10 m to 1 km.

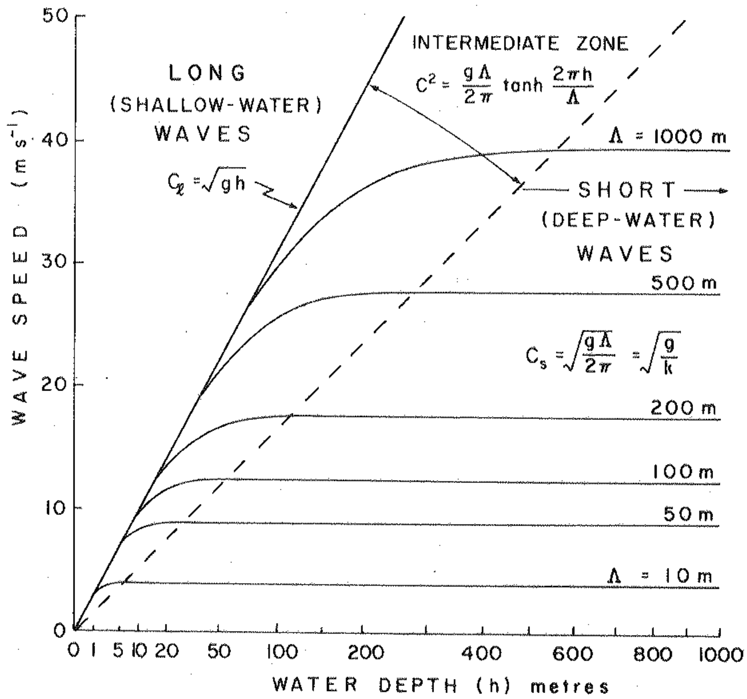

Fig. 12.4 Wave speed versus water depth for various wavelengths.

(1) The left-hand (straight) line is the plot of $C_l = (gh)^{1/2}$ (long-wave speed).

(2) The line for Λ = 200 m shows that the speed follows the long-wave line up to about 10 m water depth ($h = \Lambda/20$) where it commences to curve to the right on the figure eventually reaching its constant value of C_s = 17.7 ms^{-1} at about 100 m water depth ($h = \Lambda/2$).

(3) The zone in the figure to the right of the dashed line is where the short-wave speed approximation holds.

(4) The intermediate zone between the long-wave speed line and the dashed line is where the full expression of (12.2) must be used to calculate the speed.

(5) In practice, the long-wave and short-wave approximations find most use; the intermediate zone applies chiefly in the study of the surf zone near the beach.

If we introduce the values for the constants in the two expressions (12.2′) for the wave speeds we obtain the expressions in Table 12.2 in which are also included a few numerical values for wave properties.

Table 12.2 Short and long-wave formulae and sample values

Short (deep-water) waves				Long (shallow-water) waves
$C_s = (g\Lambda_s / 2\pi)^{1/2} = (g/k)^{1/2} = g/\omega$				$C_l = (gh)^{1/2}$
$C_s = 1.56T = 1.25(\Lambda_s)^{1/2}$				$= 3.13(h)^{1/2}$
$\Lambda_s = 1.56T^2$				$\Lambda_l = 3.13(h)^{1/2}T$
(With Λ and h in m, T in s, C in ms^{-1}, g = 9.8 ms^{-2})				
			Examples	
T	=	5	15 s	h = 5 20 4000m
		Wind-wave	swell	(tsunami)
C_s	=	7.8	23 ms^{-1}	C_l = 7 14 200 ms^{-1}
or C_s	=	28	84 km h^{-1}	or C_l = 25 50 710 kmh^{-1}
and Λ_s	=	39	350 m	and for the tsunami:
				if Λ_l = 200 km
				then h/Λ_l = 1/50
				and T = 17 min

(1) The values for the short waves give an idea of the properties of wind waves and swell, while the first two examples for long waves show the retarding effect of shoaling water on such waves.

(2) The last example, for h = 4000 m, may seem out-of-line for “shallow” water, but it is included to emphasize that the term “shallow” is only relative to the wavelength. The example is typical for the tsunami waves generated by undersea seismic disturbances.

(3) The speed of short waves depends on their wavelength and so on their period, i.e. they are ***dispersive waves***.

(4) For short waves, the speed of the longer waves is greater than that of the shorter ones. Therefore, if a number of waves of different wavelengths are generated simultaneously, the longer ones will move ahead of the shorter ones and be observed first at a distant point.

(5) Shorter waves tend to lose their energy by frictional effects somewhat faster and die out sooner than longer ones, and so do not travel so far.

12.32 Groups of waves; group speed; dispersion

(1) Real ocean waves, even swell from a distant storm, are not pure sine waves but are a sum of sine waves with a range of wavelengths, corresponding periods and amplitudes.

(2) Consider the simplest possible case of two cosine waves, η_1 and η_2, of the same amplitude but of slightly different wave number and frequency, proceeding simultaneous over the same ocean area, i.e.

$$\eta_1 = \cos(k_1 x - \omega_1 t) \quad \text{and} \quad \eta_2 = \cos(k_2 x - \omega_2 t)$$

with $\quad k_1 = k + \Delta k,\ k_2 = k - \Delta k,$ and $\omega_1 = \omega + \Delta\omega,\ \omega_2 = \omega - \Delta\omega$

where $\quad \Delta k = (k_1 - k_2)/2 \ll k = (k_1 + k_2)/2,$

and $\quad \Delta\omega = (\omega_1 - \omega_2)/2 \ll \omega = (\omega_1 + \omega_2)/2.$

Then $\quad \eta_1 = \cos[(kx - \omega t) + (\Delta kx - \Delta\omega t)],$

$$\eta_2 = \cos[(kx - \omega t) - (\Delta kx - \Delta\omega t)],$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B,$

Then $$\eta = \eta_1 + \eta_2 = 2\cos(kx - \omega t)\cos(\Delta kx - \Delta\omega t). \qquad (12.4)$$

–(12.4) describes a higher frequency wave cos(kx–ω t) of wavelength Λ=2π/k whose amplitude is modulated by a lower frequency term cos(Δkx–$\Delta\omega t$).

–Fig. 12.5 shows what η looks like as a function of x at a fixed time t = 0 for $\Delta k/k$ = 1/20, and presents one complete cycle of the envelope cos(Δkx–$\Delta\omega$ t) whose amplitude goes from zero through the sum of the amplitudes of η_1 and η_2 back to zero in a "group length" = $\pi/\Delta k$.

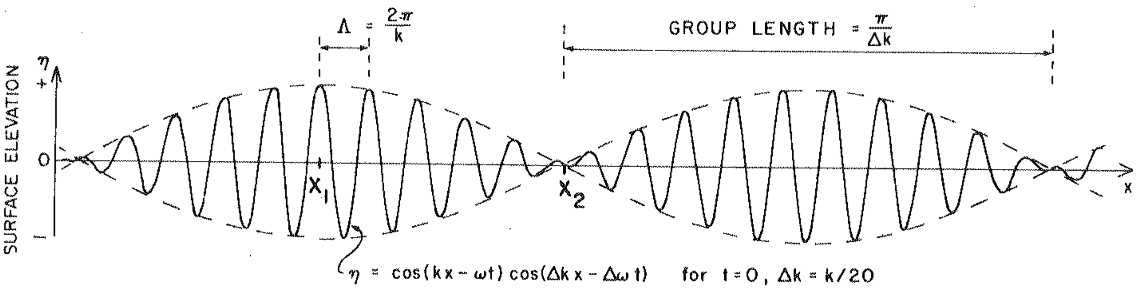

Fig. 12.5 Surface elevations for a "group" made up of two pure cosine waves.

(3) At a fixed time, as we go from one zero to the next of the envelope, (Δkx) changes by π so the "group length" = $\pi/\Delta k$.; likewise at a fixed point ($\Delta\omega t$) changes by π as the successive envelope zeros pass by, so the "group period" = $\pi/\Delta\omega$.

(4) The ***group speed***, C_g, at which the group travels is the group length/group period, i.e. $C_g = \Delta\omega/\Delta k$. In the limit as $\Delta\omega$ and $\Delta k \to 0$, the group speed C_g = $d\omega/dk$ which can be evaluated from the dispersion relation to give

$$C_g = \frac{C}{2}\left[1 + \frac{2kh}{\sinh 2kh}\right] \qquad (12.5)$$

–For long waves, $kh \ll 1$ and $\sinh 2kh \rightarrow 2kh$ and $C_g = C_l$.

–For short waves, $kh \gg 1$, $\sinh 2kh \gg 2kh$ and $C_g = C_s/2$.

(5) Wave energy travels at the group speed. If one considers that (in Fig. 12.5) initially at $x = x_2$ there is a zero wave amplitude and therefore zero wave energy, while $x = x_1$ there is maximum amplitude and maximum energy.

(6) As time increases, the group proceeds to the right and the maximum energy region of the group will reach $x = x_2$ after an interval corresponding to the travel time from x_1 to x_2 at the group speed. So energy travels at the group speed.

(7) The ***group velocity***, $\vec{C}_g$, where the vector indicates the direction of travel, is actually a more important characteristic than the phase velocity, $\vec{C}$. The group velocity indicates where the waves go. In the case of surface waves $\vec{C}_g$ and $\vec{C}$ are in the same direction but for other types of waves, $\vec{C}_g$ and $\vec{C}$ may be in different directions.

(8) For short surface gravity waves, the longer waves travel faster than the shorter ones and $C_g < C$. This behavior is termed "normal dispersion." As the individual waves in the group travel at different speeds, the shape of the group changes as it travels and it becomes more spread out with longer waves leading. If one observes the waves while traveling with the group, the individual waves appear near the back of the group, travel forward through it, and disappear at the front.

(9) For very short waves, when surface tension dominates (i.e. when $\Lambda \leq 0.5$ cm), the shorter waves travel faster than the longer ones and $C_g > C$. This situation is termed "anomalous dispersion." The group spreads out as it travels but with the shorter waves leading. If one travels with the group, the individual waves appear near the front, fall behind and disappear at the back.

12.33 Estimating the distance to a wave generation region

Let us determine the distance to a storm which has produced the swell which we see at a wave-observing station. The distance will be $d = C_g(t-t_0)$ where t is the time of arrival and t_0 is the time of generation. For short waves, $\omega C_s = g$ and $C_g = C_s/2 = g/2\omega$, so that $\omega = g(t-t_0)/2d$ at the observing station. Therefore the observed frequency increases linearly with time and from a plot of ω versus t one can determine the generation time t_0 from the intercept $t = t_0$ when $\omega = 0$, and the distance d from the slope which equals $(g/2d)$.

Note that in practice the wave records will provide information about more complex groups than the simple ones of Fig. 12.5, and the calculation is, in fact, a little more complicated than stated above.

12.34 Orbital motion of water particles and wave pressure

For the case of a surface wave with elevation $\eta = A\cos(kx-\omega t)$, the water velocity components at depth z for deep water are

$$u = \omega A \exp(kz)\cos(kx-\omega t)$$
$$w = \omega A \exp(kz)\sin(kx-\omega t)$$

The pressure in the water is p (***hydrostatic***) + p (***wave***) where p (hydrostatic) = $-\rho g z$ as usual and the component due to the wave elevation is

$$p_w = \rho g \eta \exp(k z) = \rho g A \exp(k z)\cos(k x - \omega t)$$

Derivations of these relations may be found in Neumann and Pierson (1966), Wiegel (1964) or LeBlond and Mysak (1978). Fig. 12.6(a) shows a wave profile with the surface velocities indicated by arrows at various points and the wave pressure indicated below them.

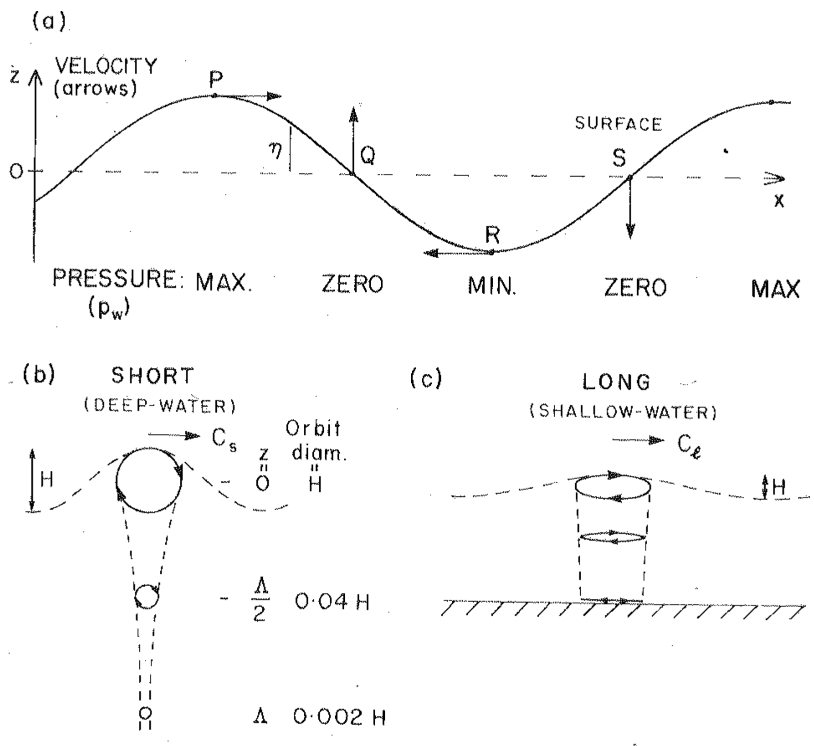

Fig. 12.6 (a) Velocity and wave pressure for surface gravity waves; orbits of water particle motion for (b) short (deep-water) waves, (c) long (shallow-water) waves (for which the water depth is exaggerated by about five times relative to wavelength for clarity).

(1) At the first crest (P), u is a maximum and in the direction of wave travel, while p_w is a maximum in the column below the crest.

(2) At the first zero crossing (Q), w is a maximum (upward), u and p_w are zero.

(3) At the trough (R), $w = 0$ and u and p_w are at their minimal values (u being opposite to the direction of wave travel).

(4) At the second zero crossing (S), u and are p_w zero again and w is a minimal (downward).

Note that it is only the shape of the wave which moves forward continuously at the speed C_s or C_l; the water particles do not travel across the ocean but circulate in ***orbits,*** circular for short (deep-water) waves and elliptical for long (shallow-water) waves. These orbits decrease in size with increase in depth (Fig. 12.6(b,c)).

–For short waves, the diameter of the (circular) orbit is $D_z = H\ \exp(kz) = H\ \exp(2\pi/\Lambda)$ where H is the wave height at the surface and z is the level. For example, at $z = -\Lambda$, the orbit diameter will be only 0.002 of that at the surface (Fig. 12.6(b)).

–For long (shallow-water) waves, the orbits are elliptical at the surface (Fig. 12.6(c)) where the vertical dimension of the orbit is H just as for the short waves. The horizontal dimension is $(\mathrm{H}\Lambda)/(2\pi h)$ and is considerably larger than H since Λ/H » 1; however, since $H/h \le 0.8$ at breaking, the horizontal dimension is always small compared with Λ, i.e. it is $\le \Lambda/8$. The horizontal dimension decreases only slightly while the vertical dimension decreases linearly with increasing depth, until near bottom (if it is flat) the motion will be simply back and forth.

–If we consider higher order corrections, the orbits are quite closed; for short (deep water) waves there is a net flow in the direction of travel of the wave of magnitude $(\pi^2H^2/\Lambda^2)C_s\exp(4\pi z/\Lambda) = A^2k^2C_s\exp(2kz)$. The net transport is called the ***Stokes drift***.

–For Λ = 100 m, H = 3 m, then C_s = 12.5 ms^{-1} while the Stokes drift at the surface is only 0.1 ms^{-1}. The speed of the orbital motion for short waves is $AkC_s\exp(kz)$, so the net flow is only a small fraction $Ak\exp(kz)$ (≤ 0.09 for this case) of the orbital speed.

–One can see intuitively how the Stokes drift comes about from the decrease of the horizontal velocity with depth; a water particle will have a slightly larger forward velocity at the top of its orbit than the backward velocity at the bottom of its orbit.

–For long (shallow water) waves there is also a Stokes drift and additional effects due to bottom friction.

12.35 Wave energy and momentum

(1) Surface gravity waves have energy associated with them, kinetic energy of the water particle motions and gravitational potential energy associated with vertical displacement.

(2) Average over one wavelength or one wave period.

(3) The average potential and kinetic energies are equal and the total energy per unit area of sea surface is $E = (\rho g A^2)/2 = (\rho g H^2)/8$ (joules m^{-2}).

(4) Averaged over a wave, the momentum per unit area = E/C. Thus energy and momentum are related by the phase speed. Both energy and momentum propagate at the group speed C_g.

12.4 Finite amplitude effects

For short (deep water) waves one can work out corrections for the terms neglected in the linear theory. The effect is to add higher harmonics to a wave of a given frequency, e.g. for η, terms involving cos $2(k\,x - \omega\,t)$, cos $3(k\,x - \omega\,t)$, etc. For example, to the first order in AK

$$\eta = A\left[\cos(kx - \omega\,t) + \frac{1}{2}AK\cos 2(kx - \omega\,t)\right] \qquad (12.5)$$

The effect of adding these higher harmonics is to make the crests sharper and the troughs flatter, e.g. compare the second-order Stokers wave (Fig. 12.7(b)) with the simple cosine (linear) wave (Fig. 12.7(a)).

(1) This sharpening of the crests is easily observed with real waves. There is also a correction to the phase speed as

$$C_s = (g/k)^{1/2}\left(1 + \frac{1}{2}A^2K^2\right), \qquad (12.6)$$

i.e. the larger amplitude waves travels a little faster than smaller amplitude ones.

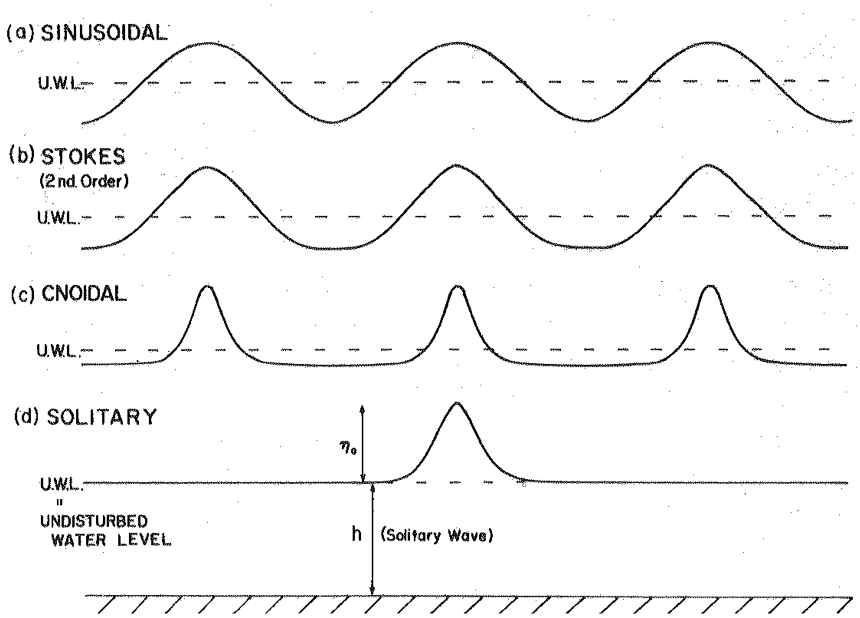

Fig. 12.7 Shapes of (a) pure sine wave; finite amplitude waves, (b) Stokes (2nd order), (c) cnoidal and (d) solitary. (Wave heights are exaggerated relative to lengths by about seven times in order to show differences between shapes more clearly.)

(2) At breaking, $AK \approx 1/4$ (when $H/\Lambda \approx 1/12$), so at most C_s is increased by 3% which is why the linear theory is very good for calculating wave propagation.

(3) In fairly shallow water there will be variations of C_l with both wavelength and amplitude.

(4) It is possible to find solutions where the two types of dispersion balance to permit finite amplitude waves of permanent form, i.e. the shape does not change as they travel. The solutions are shown in Fig. 12.7 for (c) ***cnoidal waves***–a periodic wave train, and (d) for the ***solitary wave***, an isolated traveling disturbance.

(5) For the solitary wave, $C_{sol} = (gh)^{1/2}(1+\eta_0/2h)$ and $\eta_0/h \leq 0.7$ so that the phase and group speeds can be considerably larger than the value $(gh)^{1/2}$ from linear theory.

(6) In very shallow water, as $H/\Lambda \ll 1$ there will be no dispersion of waves of different lengths but the larger amplitude waves will travel faster than smaller amplitude ones. The crest, where the local $(gh)^{1/2}$ is largest, travels faster than the trough, the forward face steepens, and a "shock wave" may form. Tidal bores in river estuaries are an example of this phenomenon as is the formation of surf when waves approach a beach over a shoaling bottom.

12.5 Refraction and breaking in shallow water; diffraction

12.51 Refraction

(1) Small amplitude long (shallow-water) waves all travel at the same speed in water of a given depth h but where the bottom depth is changing their direction of travel may change.

(2) More generally, as waves move into shallow water their period remains constant but C decreases and therefore Λ decreases. As an example, Table 12.3 shows the decrease of speed and wavelength for waves of period 8 seconds on entering shoaling water.

Table 12.3 Decrease of speed and wavelength in shoaling water for waves of period 8 s and length 100 m in deep water

h = 50+	10	5	2	m
C = 12.5	8.9	6.6	4.3	ms^{-1}
Λ = 100	71	53	35	m

(3) If a series of parallel-crested waves approaches at an angle to a straight shoreline (Fig. 12.8(a)) over a smooth sea bottom which shoals gradually, they pro- gressively change direction as the end of the wave nearer to the shore (P in Fig. 12.8(a)) slows down earlier than that farther away (Q in Fig. 12.8(a)). As a result, the waves become more parallel to the shore by the time that they pile up as surf. The change in direction (e.g. PP′, QQ′) associated with the change of speed is called ***refraction***.

(4) If the sea bottom does not have a uniform slope along the full length of the shore, the refraction may be more complicated. Two simple examples are where there is an underwater ridge running out at right angles to the shore or where there is an underwater valley. The fraction pattern for waves coming straight towards the shore would then be as in Fig. 12.8(b).

(5) In this figure are shown not the wave crests but the wave ***orthogonals*** which are perpendiculars to the wave crests, i.e. parallels to $\vec{k}, \vec{C}$ and $\vec{C}_g$, which indicate the direction of travel of the waves, as do the arrows in Fig. 12.8(a).

(6) Refraction of waves round a headland, for instance, occurs if the water deepens gradually to seaward from the land but not if the water is of relatively uniform depth off the headland. Waves are often observed to be refracted round islands and one can sometimes see an interference pattern set up where the waves which are refracted around the two sides of a small

island meet behind it.

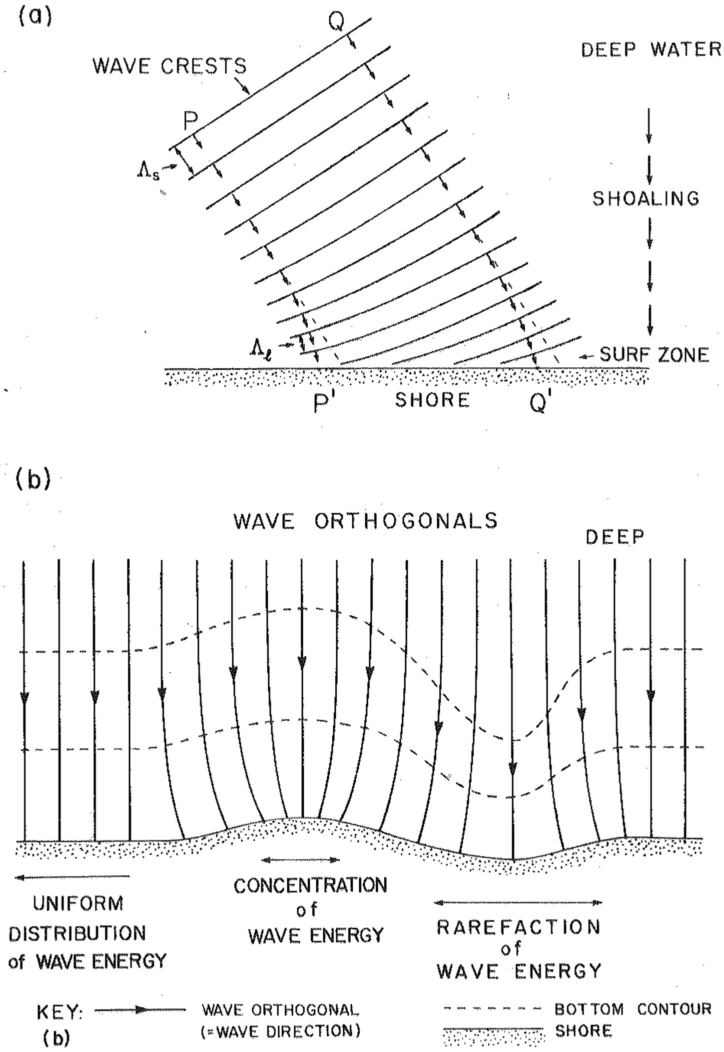

Fig. 12.8 (a) Refraction of waves approaching a beach over a smoothly shoaling bottom, (b) wave refraction on approaching an underwater ridge (left) and a valley (right). Dashed lines represent depth contours, full lines represent wave orthogonals (directions of travel of waves), in (b).

12.52 Wave breaking

(1) As the waves move inshore and slow down, not only does the wavelength decrease but also the wave height changes.

(2) In a steady state, then energy flux is constant as the waves move inshore (until they breaks), with a value averaged between two successive crests of ($\rho g H^2 C_g/8$) joules per meter width of crest per second. If the flux is not constant, the divergence of flux, $\partial(EC_g)/\partial x$, would cause the energy level to change locally, i.e. $\partial E/\partial t$ would not vanish.

(3) If the waves are initially long, e.g. tsunamis, $C_g = C_l$, both speed decrease, and H increases.

(4) If the waves are initially short, at first C_g increases, reaching a maximum value of 1.2 times the deep-water value when $h/\Lambda \approx 0.19$, where Λ is the local value not the deep-water value ($h/\Lambda_d = 0.16$). In this zone, H decreases to a minimum of about 90% of the deep-water value when C_g is a maximum. H/Λ is nearly constant at first but begins to increase before $h/\Lambda \approx 0.19$ because Λ decreases faster than H. At $h/\Lambda = 0.19$, the steepness H/Λ is about 10% greater than the deep-water value. As the waves move further inshore, C_g decreases and H must increase. However, the decrease in Λ dominates the H/Λ changes.