Chap 5 Stability and Double Diffusion

5.1 Static stability

We consider whether or not the variation of density with depth in the ocean is likely to cause the water to move vertically.

(1) If there is light fluid on top of heavy fluid ($\partial\rho/\partial z < 0$) then there is will be no tendency for motion to occur, i.e. **stable**.

(2) If there is heavy fluid above light fluid ($\partial\rho/\partial z > 0$) there will be a tendency for the heavy fluid to sink and the light to rise – the density distribution is **unstable**.

(3) **neutral stable –** if a fluid parcel is moved up or down adiabatically (with no heat exchange) and without salt exchange with its surroundings and then brought to a stop it will not tend to move further because wherever it is moved to it will have the same density as its surroundings ($\partial\rho/\partial z = 0$).

(4) Because of the non-linear equation of state for seawater, using potential density to determine the static stability does not always work in the ocean. For example, North Atlantic Deep Water (NADW) has a slightly larger potential density than Antarctic Bottom Water (ABW). NADW is found above ABW, in which the *in situ* density of ABW is slightly greater than that of the NABW.

5.11 Criterion for static stability (*E*)

Consider a parcel of water moved a short distance vertically from level 1 to level 2 without exchanging heat or salt with its surroundings (Fig. 5.1).

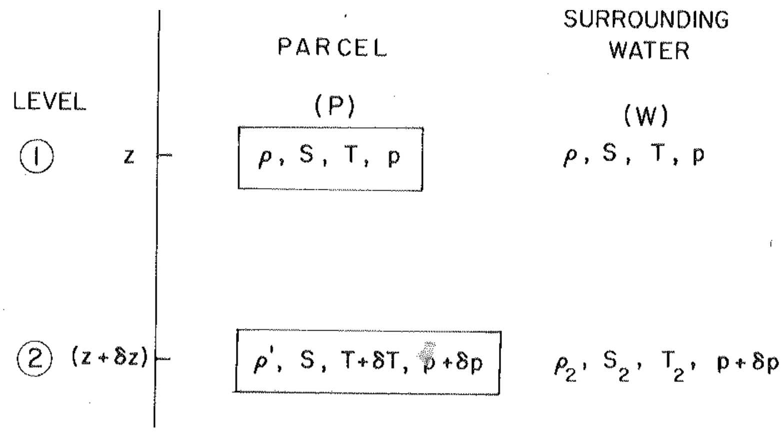


Fig. 5.1 Water properties for calculations of stability.

(i) at level 1 — depth = – z, pressure = – p, *in situ* water properties = (ρ, S, T)

(ii) at level 2 — depth = –(z+δz), pressure = –(p+δp), surrounding water properties = (ρ_2, S_2, T_2), the parcel = (ρ', S, T+δT). Here δT is the adiabatic change of temp- erature due to change of pressure, i.e.

$$\delta T = (dT/dp)_{\text{ad}}\,\delta p = -(dT/dp)_{\text{ad}}\,\rho g \delta z = -\Gamma \delta z$$

where Γ stands for the adiabatic temperature gradient. It is the change of temperature with depth caused by pressure change and is positive, i.e. compression causes the temperature to increase.

The restoring force on the parcel of volume δV will be

$$F = (\text{buoyant upthrust} - \text{weight})$$

$$F = \delta V_2 \rho_2 g - \delta V_2 \rho' g = \delta V_2 g(\rho_2 - \rho') \qquad (5.1)$$

Acceleration: $\quad a_z = \dfrac{F}{M} = \dfrac{\delta V_2 g(\rho_2 - \rho')}{\delta V_2 \rho'} = g\left(\dfrac{\rho_2 - \rho'}{\rho'}\right)$

surrounding water $\quad \rho_2 = \rho + \left(\dfrac{\partial \rho}{\partial z} dz\right)_w = \rho + \left[\dfrac{\partial \rho}{\partial S}\dfrac{\partial S}{\partial z} + \dfrac{\partial \rho}{\partial T}\dfrac{\partial T}{\partial z} + \dfrac{\partial \rho}{\partial p}\dfrac{\partial p}{\partial z}\right]_w \delta z$

the water parcel $\quad \rho' = \rho + \left[\dfrac{\partial \rho}{\partial z}\delta z\right]_p = \rho + \left[-\dfrac{\partial \rho}{\partial T}\Gamma + \dfrac{\partial \rho}{\partial p}\dfrac{\partial p}{\partial z}\right]_p \delta z$

Then, $$a_z = \frac{g\left[\rho + \left(\dfrac{\partial \rho}{\partial z}\delta z\right)_w - \rho - \left(\dfrac{\partial \rho}{\partial z}\delta z\right)_p\right]}{\rho\left[1 + \left(\dfrac{1}{\rho}\dfrac{\partial \rho}{\partial z}\delta z\right)_p\right]} \qquad (5.2)$$

As $\delta z \to 0$, $(1/\rho)(\partial\rho/\partial z)\delta z$ in the denominator of (5.2) may be neglected. So,

$$a_z = \frac{g}{\rho}\left[\left(\frac{\partial \rho}{\partial z}\delta z\right)_w - \left(\frac{\partial \rho}{\partial z}\delta z\right)_p\right]$$

Define **stability** $\equiv E \equiv -\dfrac{a_z}{g}$ for δz = unit length

$$E = -\frac{1}{\rho}\left[\left(\frac{\partial \rho}{\partial z}\right)_w - \left(\frac{\partial \rho}{\partial z}\right)_p\right]$$

(I) Simplified approach for evaluating E

$$\rho(S, T, 0) - 1000 = \sigma_t$$

$$E \approx -\frac{1}{\rho}\left[\left(\frac{\partial \sigma_t}{\partial z}\right)_w - \left(\frac{\partial \sigma_t}{\partial z}\right)_p\right]$$

We note that the salinity of the second term is independent of z, and its temperature dependence is small; therefore

$$E \approx -\frac{1}{\rho}\frac{\partial \sigma_t}{\partial z} \qquad \text{For water in the upper km of the ocean}$$

(II) Full accuracy solution

Deep in the ocean, the change in density with depth is so small that we must consider the small change in density of the parcel due to changes in pressure as it is moved vertically. So, (5.2) becomes

$$\frac{a_z}{g} = \frac{1}{\rho}\left[\frac{\partial \rho}{\partial S}\frac{\partial S}{\partial z} + \frac{\partial \rho}{\partial T}\left(\frac{\partial T}{\partial z} + \Gamma\right)\right]\delta z \tag{5.3}$$

where we have used $\left(\frac{\partial p}{\partial z}\right)_w = \left(\frac{\partial p}{\partial z}\right)_p$ and $\left(\frac{\partial \rho}{\partial p}\right)_w = \left(\frac{\partial \rho}{\partial p}\right)_p$ because ρ_w is not much different from ρ_p if T, S vary slowly with depth.

$$E = -\frac{1}{\rho}\left[\frac{\partial \rho}{\partial S}\frac{\partial S}{\partial z} + \frac{\partial \rho}{\partial T}\left(\frac{\partial T}{\partial z} + \Gamma\right)\right] \tag{5.4}$$

5.12 Numerical values for stability

– Stability is defined such that (i) $E > 0$, stable (ii) $E = 0$, neutral stability (iii) $E < 0$, unstable.

– In the upper kilometer of the ocean, $z < 1{,}000$ m, $E = 100\sim1000\times10^{-8}$/m, and in deep trenches where $z > 7{,}000$ m, $E = 1\times10^{-8}$/m.

– For $E > 50\times10^{-8}$/m, $E = -\frac{1}{\rho}\frac{\partial \sigma_t}{\partial z}$ is a good approximation.

If the stability of the water were neutral, the *in situ* gradient must be the same as that for the water parcel.

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial z}\right)_p = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_{\text{ad}}\frac{\partial p}{\partial z} = -g\left(\frac{\partial \rho}{\partial p}\right)_{\text{ad}}$$

but $$\left(\frac{\partial \rho}{\partial p}\right)_{\text{ad}} = \frac{1}{C^2}$$ where C is the speed of sound,

so $$-\frac{1}{\rho}\left(\frac{\partial \rho}{\partial z}\right)_{w} = \frac{g}{C^2} \cong 400 \times 10^{-8}\,\text{m}^{-1}$$

If one wishes to use *in situ* density, $\rho(s, t, p)$, then to correct for compressibility the stability is given by

$$E = -\frac{1}{\rho}\frac{\partial \rho}{\partial z} - \frac{g}{C^2} \quad (5.10)$$

5.13 Buoyancy frequency (*N*)

The ***Brunt-Väisälä*** (or ***buoyancy***) ***frequency N*** is given by

$$N^2 = (gE) = g\left[-\frac{1}{\rho}\frac{\partial \rho(s,t,p)}{\partial z} - \frac{g}{C^2}\right] \cong g\left[-\frac{1}{\rho}\frac{\partial \sigma_t}{\partial z}\right] \text{ (radians s}^{-1})^2 \quad (5.11)$$

– The frequency in cycle s–1 (hertz) is $N/2\pi = (gE)^{1/2}/2\pi$.

– The maximum frequency of ***internal waves*** in water of stability E.

– High values of N are usually found in the main pycnocline zone, i.e. where the vertical density gradient is greatest.

– This is usually in the ***thermocline*** in oceanic waters (where density variations are determined chiefly by temperature variations) or in the ***halocline*** in coastal waters (where density variations may be determined chiefly by salinity variations).

– Typical values of N are a few cycles per hour (Fig. 5.2).

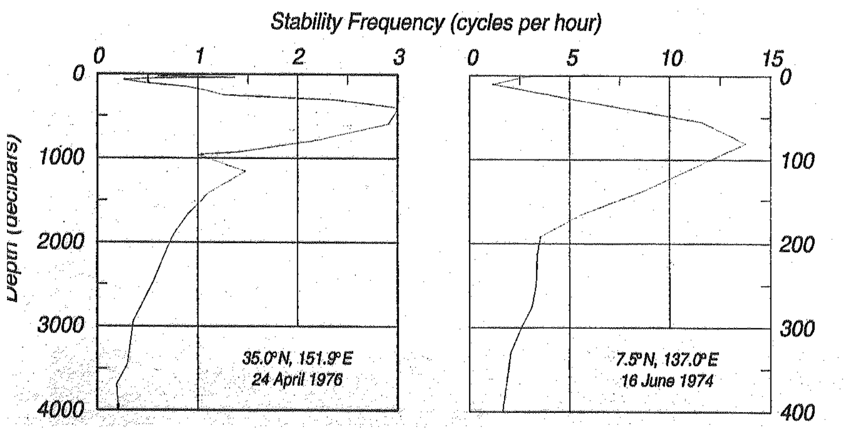


Fig. 5.2 Observed stratification frequency in the Pacific. Left: Stability of the deep thermocline east of the Kuroshio. Right: Stability of a shallow thermocline typical of the tropics.

5.2 Double Diffusion

In some regions of the ocean, the water column is statically stable, but instability may develop because the rates at which heat and salt diffuse molecularly are dif- ferent, diffusion of heat ≈ 100 times diffusion of salt A result is that if two water masses of the same density but different combinations of temperature and salinity are in contact, the “double” diffusion of these two properties may give rise to density changes which render the layers unstable.

(I) ***salt fingering***

Suppose that there is a layer of warmer, saltier water above cooler, fresher water, such that the upper layer is of the same density as or is less dense than the lower layer (Fig. 5.3).

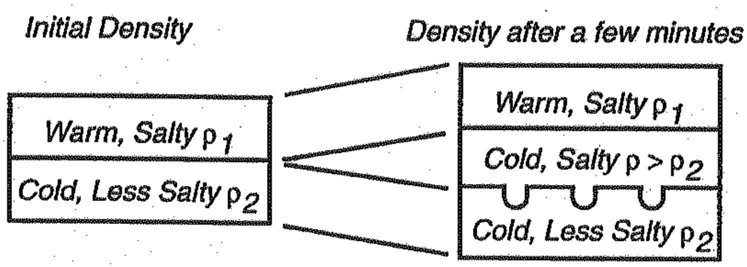


Fig. 5.3 Left: Initial distribution of density in the vertical. Right: After some time, the diffusion of heat leads to a thin unstable layer between the two initially stable layers. The thin unstable layer sinks into the lower layer as salty fingers. The vertical scale in the figures is a few cm.

– If the density difference between the layers is small, the saltier water above may become heavier than the cooler, fresher layer below. So, the interface becomes unstable.

– Because the layer is thin, the fluid sinks 1–5 cm in diameter and 100 cm long, not much different in size and shape from our fingers.

– Because two constituents diffuse across the interface, the process is called ***double-diffusive instability***.

– There is evidence for its occurrence in the ocean at the lower surface of the outflow of warm, saline Mediterranean water from the Strait of Gibraltar into the cooler, less saline Atlantic water.

– Salt fingering eventually leads to density increasing with depth in a series of steps. That is, layers of constant-density are separated by thin layers with large changes in density, and the profile of density as a function of depth looks like stair steps.

– Schmitt et al (1987) observed 5–30 m thick steps in the western, tropical North Atlantic that were coherent over 200–400 km and that lasted for at least 8 months.

(II) ***layering***

If a layer of colder, fresher water is above a layer of warmer, saltier water (Fig. 5.4).

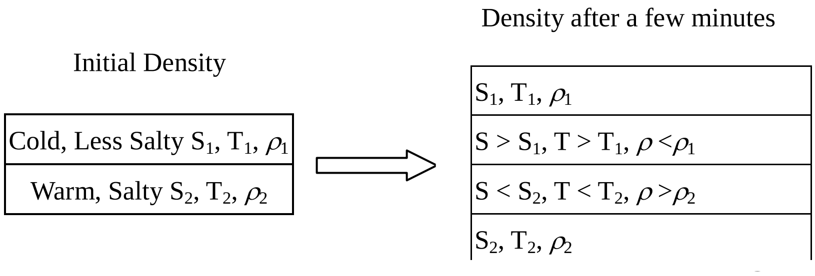


Fig. 5.4 The sketch of the occurrence of layering.

– The water just above the interface becomes lighter than that above it and tends to rise while water below gets heavier and tends to sinks.

– This phenomenon is called ***layering or diffusive convection*** and may lead to fairly homogeneous layers separated by thinner regions of high gradients of temperature and salinity.

– It is much less common than salt fingering, and it is mostly found at high latitudes.

– Layering also leads to a stair step of density as a function of depth.

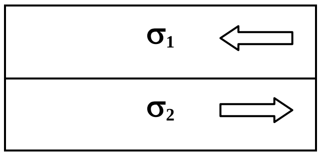
– There is evidence for its occurrence in the Arctic Ocean among other locations.

– Neal et al (1969) observed 2–10 m thick layers in the sea beneath the Arctic ice.

5.3 Dynamic stability

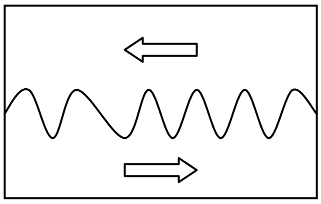
If motion is initiated it may be dynamically unstable and it may break down into smaller-sized irregular turbulent motions.

(i) Consider two stable layers of water moving relative to each other.



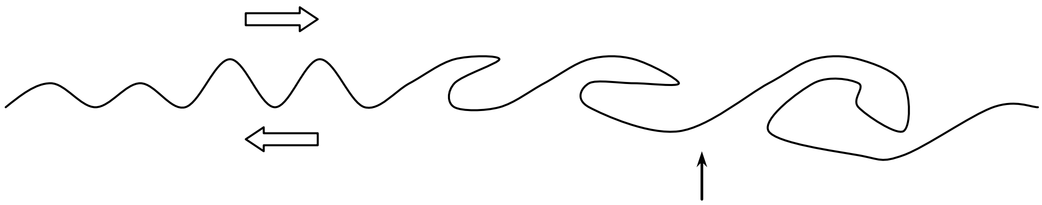
$\sigma_1 < \sigma_2$ stable flow

The relative motion can lead to dynamic instability through the formation of waves on the interface.



Dynamic Instability

Waves can grow until they break.



As waves grow, they eventually become so large as they are unstable.

unstable wave