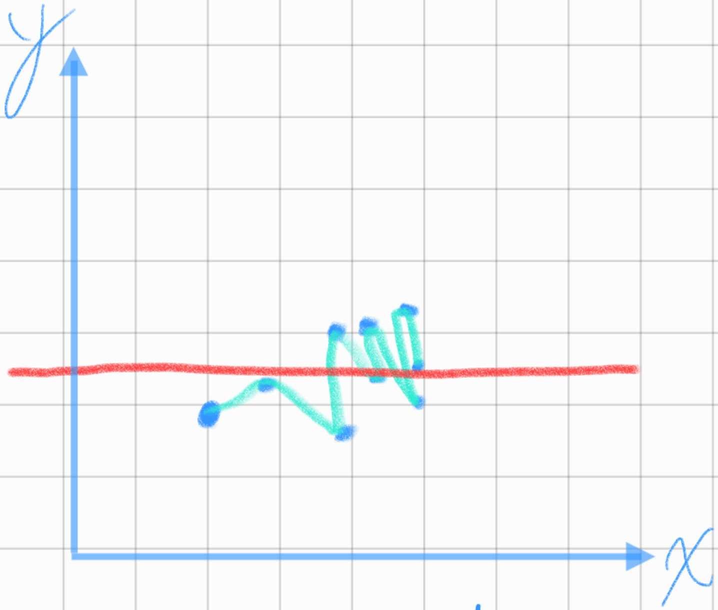

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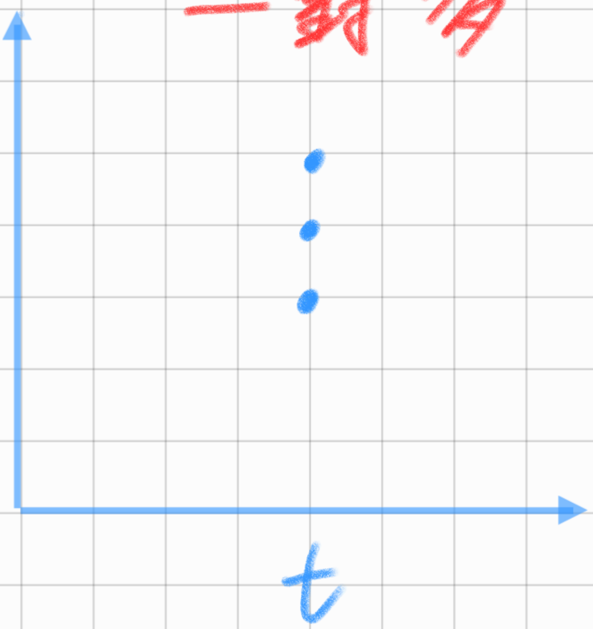
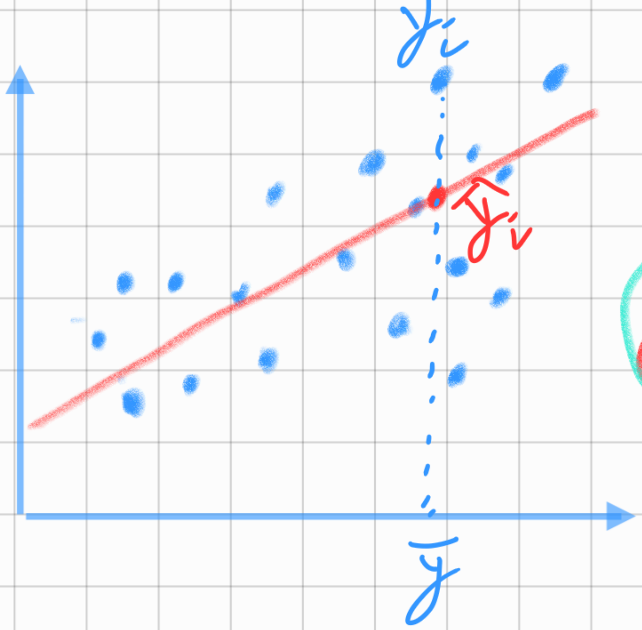




$$y = a_0 + a_1 x_1 + a_2 x_2$$

雨量 氣溫

紅茶產量

迴歸 ⇒ 找最佳化

$$S_r = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

regression

$$\boxed{S_r = \sum_{i=1}^{N} \left[ y_i - (a_0 + a_1 x_1 + a_2 x_2) \right]^2}$$

$$\frac{\partial S_r}{\partial a_0} = 2\cdot(-1)\sum(y_i - a_0 - a_1x_1 - a_2x_2)$$
$$= 0$$

$$\frac{\partial S_r}{\partial a_1} = 2\cdot(-x_1)\sum(y_i - a_0 - a_1x_1 - a_2x_2)$$
$$= 0$$

$$\frac{\partial S_r}{\partial a_2} = 2(-x_2)\sum(y_i - a_0 - a_1x_1 - a_2x_2)$$

$$= 0$$

$$\sum y_i - \sum a_0 - \sum a_1x_1 - \sum a_2x_2 = 0$$

$$\rightarrow \sum y_i = \sum a_0 + \sum a_1x_1 + \sum a_2x_2$$

$$\rightarrow \begin{cases} \sum a_0 + \sum a_1x_1 + \sum a_2x_2 = \sum y_i \\ \sum a_0x_1 + \sum a_1x_1^2 + \sum a_2x_1x_2 = \sum x_1y_i \\ \sum a_0x_2 + \sum a_1x_1x_2 + \sum a_2x_2^2 = \sum x_2y_i \end{cases}$$

$$\begin{cases} a_0\boxed{\sum(1)}^{=n} + a_1\sum x_1 + a_2\sum x_2 = \sum y_i \\ a_0\sum x_1 + a_1\sum x_1^2 + a_2\sum x_1x_2 = \sum x_1y_i \\ a_0\sum x_2 + a_1\sum x_1x_2 + a_2\sum x_2^2 = \sum x_2y_i \end{cases}$$

$$\underbrace{\begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 \\ \sum x_2 & \sum x_1 x_2 & \sum x_2^2 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} \sum y_i \\ \sum x_1 y_i \\ \sum x_2 y_i \end{bmatrix}}_{Y}$$

$$A = D \backslash Y$$

↳ 多維

$$\begin{bmatrix} n & \sum x_1 & \sum x_2 & \sum x_3 & \cdots & \sum x_m \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 & \sum x_1 x_3 & \cdots & \sum x_1 x_m \\ \sum x_2 & \sum x_1 x_2 & \sum x_2^2 & \sum x_2 x_3 & \cdots & \sum x_2 x_m \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ \sum x_m & \sum x_m x_1 & \sum x_m x_2 & \sum x_m x_3 & \cdots & \sum x_m^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_1 y_i \\ \sum x_2 y_i \\ \sum x_3 y_i \\ \vdots \\ \sum x_m y_i \end{bmatrix}$$

$$\begin{cases} u = a_0 + a_1 x_1 + a_2 x_2 \\ V = A_0 + A_1 x_1 + A_2 x_2 \end{cases}$$

$$\hat{y}_i = a_0 + a_1 x + a_2 x^2$$

Least Square Method

$$S_r = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$$S_r = \sum_{i=1}^{N} (y_i - a_0 - a_1 x_i - a_2 x_i^2)^2$$

$$\frac{\partial S_r}{\partial a_0} = 2 \sum (y_i - a_0 - a_1 x_i - a_2 x_i^2)(-1)$$

$$= 0$$

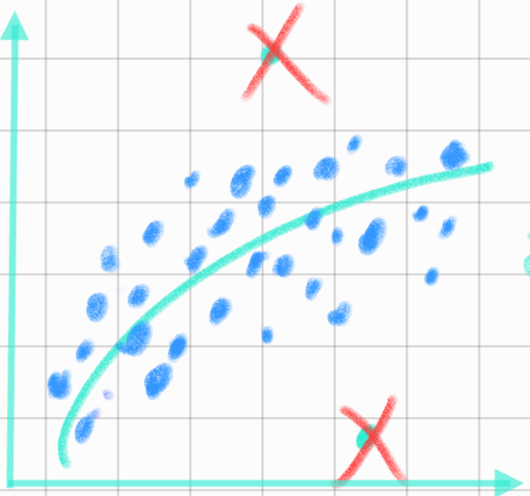
$$\frac{\partial S}{\partial a_1} = 2 \sum (y_i - a_0 - a_1 x - a_2 x^2)(-x)$$

$$= 0$$

$$\frac{\partial S}{\partial a_2} = 2 \sum (y_i - a_0 - a_1 x - a_2 x^2)(-x^2)$$

$$= 0$$

$$\begin{bmatrix} N & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x y_i \\ \sum x^2 y_i \end{bmatrix}$$



拋物線不需要
通過每一點

$$\begin{bmatrix} n & \sum x & \sum x^2 & \cdots & \sum x^m \\ \sum x & \sum x^2 & \sum x^3 & \cdots & \sum x^{m+1} \\ \vdots & \vdots & \vdots & & \vdots \\ \sum x^m & \sum x^{m+1} & \sum x^{m+2} & \cdots & \sum x^{2m} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x y_i \\ \vdots \\ \sum x^m y_i \end{bmatrix}$$

作業：使用以上公式
找出 $\hat{y}_i = a_0 + a_1 x + a_2 x^2$
的 $a_0$、$a_1$、$a_2$

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y | 2.1 | 7.7 | 13.6 | 27.2 | 40.9 | 61.1 |