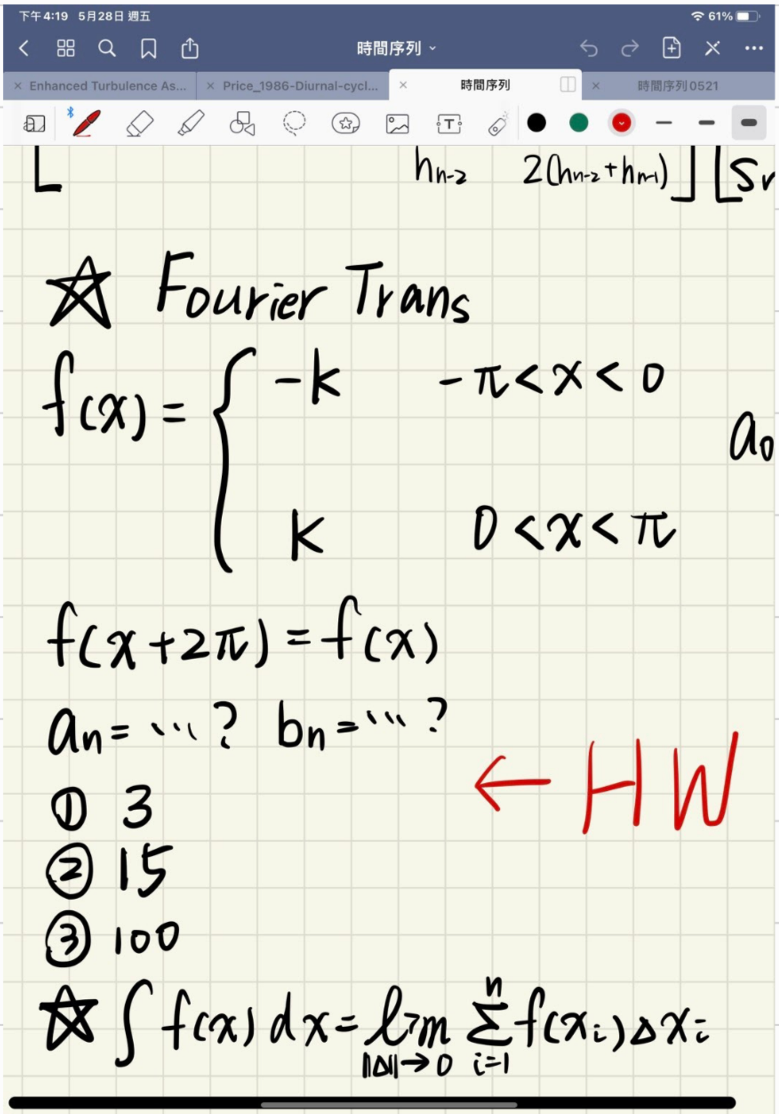


下午4:19 5月28日 週五

時間序列

Enhanced Turbulence As... | Price_1986-Diurnal-cycl... | 時間序列 | 時間序列0521

$L \quad h_{n-2} \quad 2(h_{n-2}+h_{n-1}) \,]\, [S$

☆ Fourier Trans

$$f(x) = \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$$

a_0

$f(x+2\pi) = f(x)$

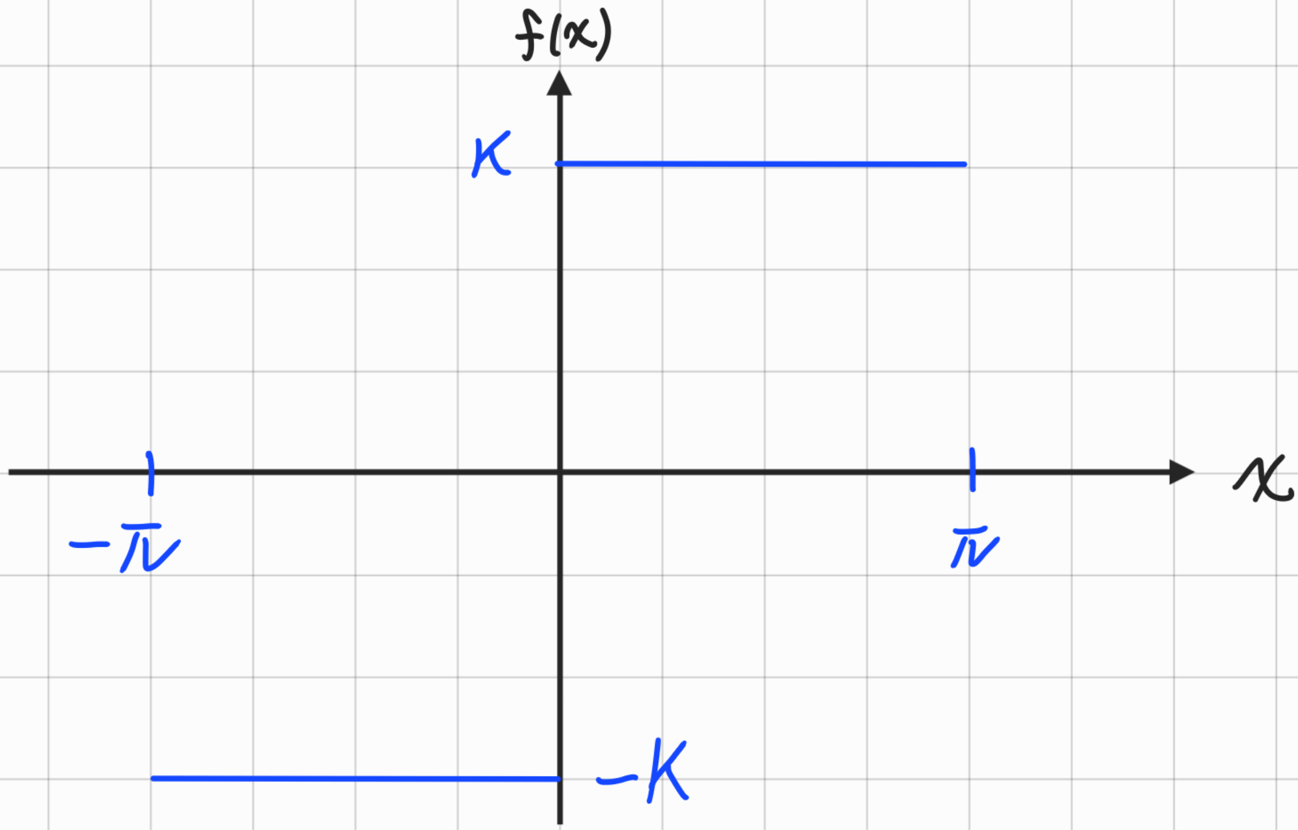
$a_n = \cdots ?$ $b_n = \cdots ?$ ← HW

① 3

② 15

③ 100

☆ $\int f(x)\,dx = \lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} f(x_i)\Delta x_i$



1° $2p = \pi - (-\pi) = 2\pi \Rightarrow p = \pi$

$$2^\circ \quad a_0=\frac{1}{2p}\int_{-p}^{p} f(x)\,dx=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$=\frac{1}{2\pi}\left[\int_{-\pi}^{0} -k\,dx+\int_{0}^{\pi} k\,dx\right]$$

$$=\frac{1}{2\pi}\left[-kx\Big|_{-\pi}^{0}+kx\Big|_{0}^{\pi}\right]$$

$$=\frac{1}{2\pi}\left(-k\pi+k\pi\right)=0$$

$$a_n=\frac{1}{p}\int_{-p}^{p} \overset{\text{odd}}{\boxed{f(x)}}\ \overset{\text{even}}{\boxed{\cos\frac{n\pi}{p}x}}\,dx$$

$$=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$=\frac{1}{\pi}\left[\int_{-\pi}^{0} -k\cos nx\,dx+\int_{0}^{\pi} k\cos nx\,dx\right]$$

$$=\frac{1}{\pi}\left[-\frac{k}{n}\sin nx\Big|_{-\pi}^{0}+\frac{k}{n}\sin nx\Big|_{0}^{\pi}\right]$$

$$=\frac{1}{\pi}\left[-\frac{k}{n}(0-\sin n\pi)+\frac{k}{n}\sin n\pi+0\right]$$

$$=\frac{1}{\pi}\times\frac{2k}{n}\sin n\pi=\frac{2k}{n\pi}\sin n\pi=0$$

$$b_n=\frac{1}{p}\int_{-p}^{p}\boxed{f(x)}\ \boxed{\sin\frac{n\pi}{p}x}\,dx=\frac{2}{p}\int_{0}^{p} f(x)\sin\frac{n\pi}{p}x\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} K\sin nx\,dx = \frac{2}{\pi}\times\frac{-K}{n}\cos nx\Big|_0^{\pi}$$

$$= \frac{2}{\pi}\times\frac{-K}{n}(\cos n\pi - 1) = \frac{-2K}{n\pi}\left[(-1)^n - 1\right]$$

$$= \frac{2K}{n\pi}\left[1-(-1)^n\right]$$

3° $$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi}{p}x + b_n\sin\frac{n\pi}{p}x\right]$$

$$= \sum_{n=1}^{N}\left[b_n\sin nx\right]$$

$$= \frac{2K}{\pi}\sum_{n=1}^{N}\left[1-(-1)^n\right]\frac{\sin nx}{n}$$

4° $N=3 \Rightarrow f(x) = \frac{4K}{\pi}\left(\sin x + \frac{1}{3}\sin 3x\right)$

$$N=15 \Rightarrow f(x) = \frac{4K}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \frac{1}{9}\sin 9x + \frac{1}{11}\sin 11x + \frac{1}{13}\sin 13x + \frac{1}{15}\sin 15x\right)$$

$$N=100 \Rightarrow f(x) = \frac{4K}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \cdots + \frac{1}{99}\sin 99x\right)$$