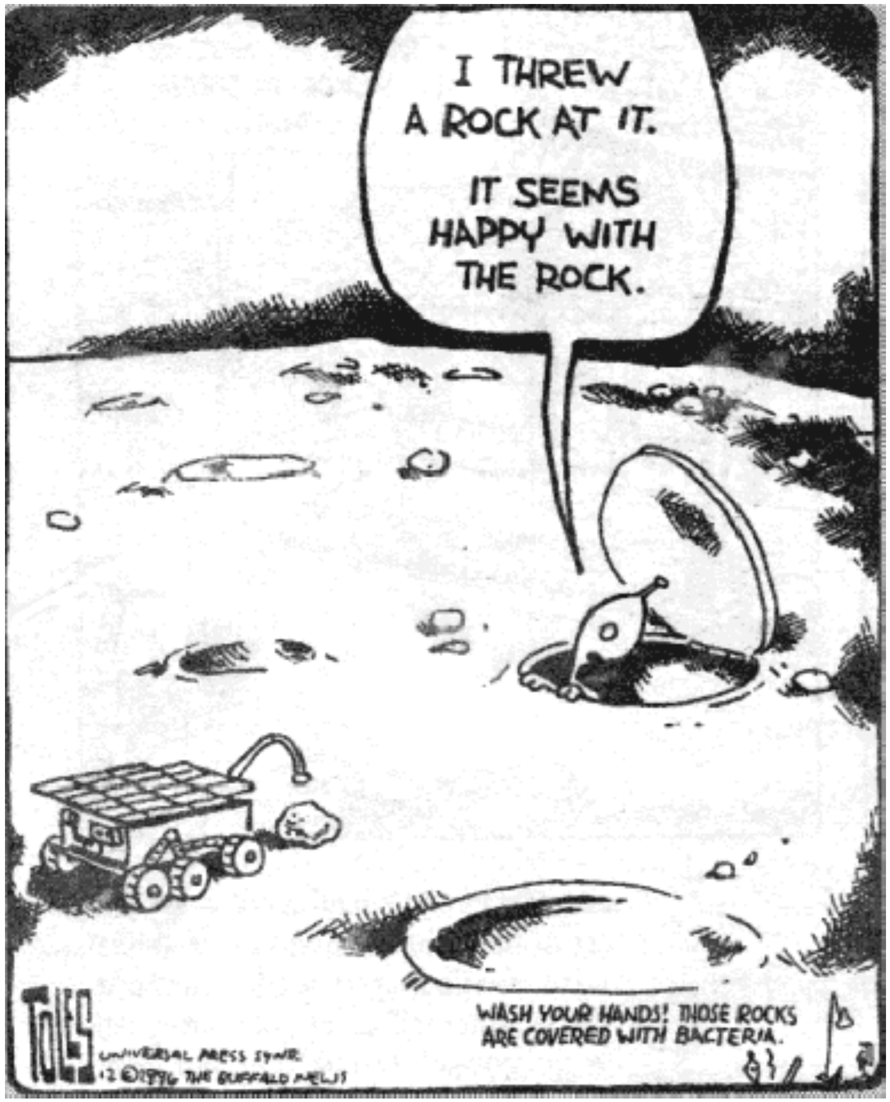
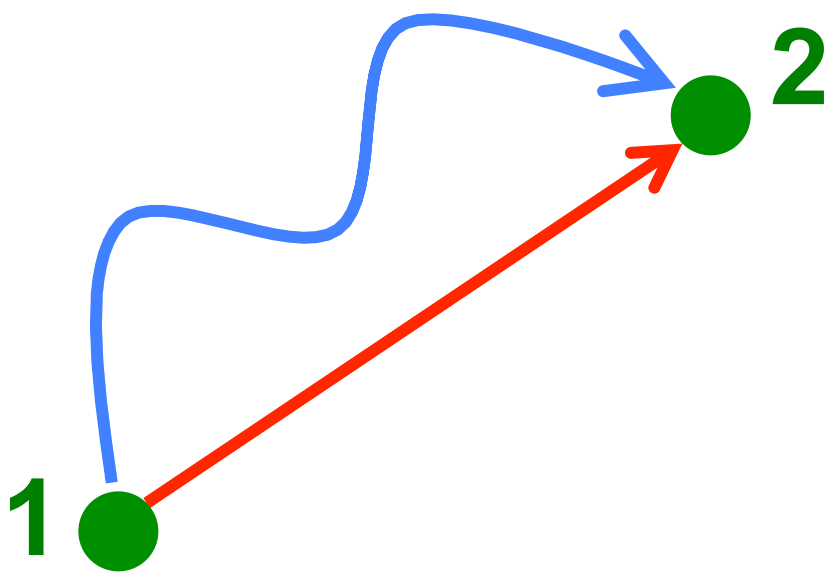
PHYS 121 – SPRING 2014



Chapter 8: Potential Energy & Conservation of Energy

CONSERVATIVE FORCES

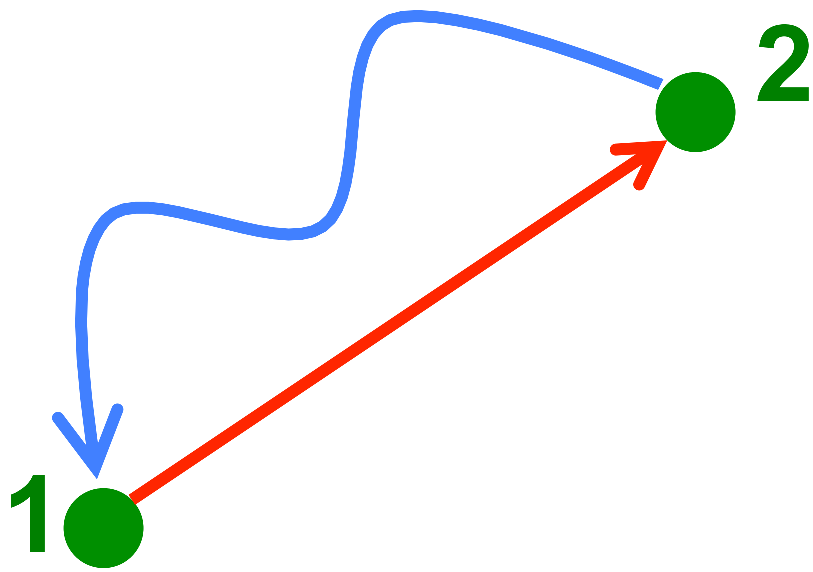
The work done by a conservative force over a path depends only on the start and end points of the path, and not on the path itself.



$$W_{1\to 2} = W_{1\to 2}$$

CONSERVATIVE FORCES

What is the work to go from 1 to 2 along red path, and then back to 1 along blue path?



$$W_{tot} = W_{1\to 2} + W_{2\to 1}$$

$$= W_{1\to 2} - W_{1\to 2}$$

$$= 0$$

Work done over a closed path (round trip) is zero

$$W_{1\to 1} = \oint \vec{F} \cdot d\vec{r} = 0$$

Is Gravity Conservative?

What is the work done by gravity as a mass moves around a closed path?

$$W_{tot} = W_1 + W_2 + W_3 + W_4$$

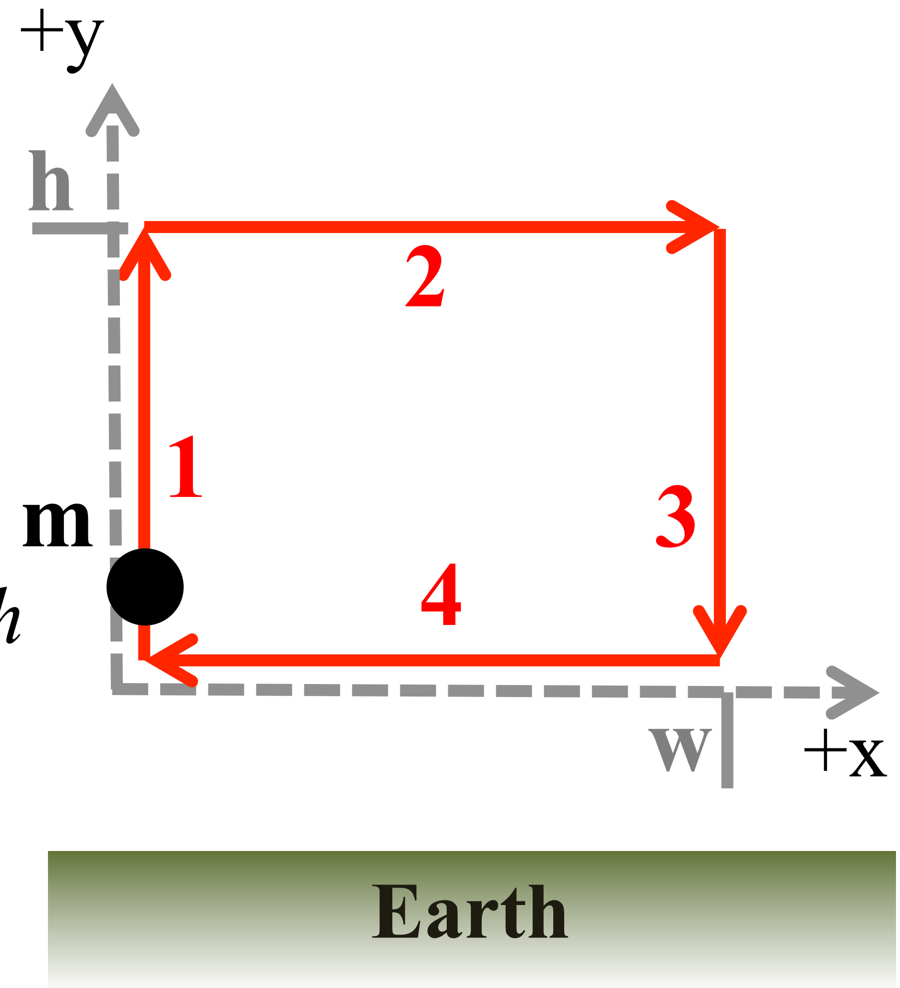
$$W_1 = \vec{F}\cdot\vec{d}_1 = (-mg\,\hat{y})\cdot(h\,\hat{y}) = -mgh$$

$$W_2 = \vec{F}\cdot\vec{d}_2 = (-mg\,\hat{y})\cdot(w\,\hat{x}) = 0$$

$$W_3 = \vec{F}\cdot\vec{d}_3 = (-mg\,\hat{y})\cdot(-h\,\hat{y}) = +mgh$$

$$W_4 = \vec{F}\cdot\vec{d}_4 = (-mg\,\hat{y})\cdot(-w\,\hat{x}) = 0$$

$$W_{tot} = -mgh + 0 + mgh + 0 \boxed{= 0}$$



Gravity is Conservative !

Is Friction Conservative?

What is the work done by friction as a mass moves around a closed path?

$$W_{tot} = W_1 + W_2 + W_3 + W_4$$

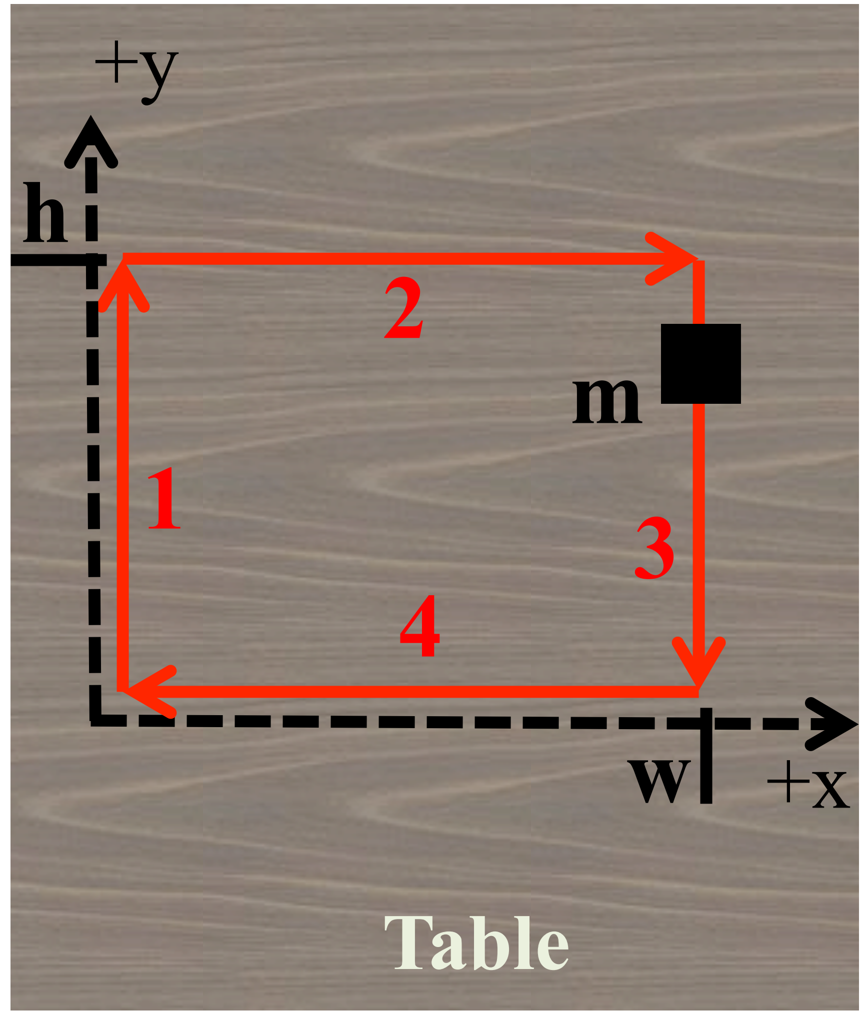
$$W_1 = \vec{F} \cdot \vec{d}_1 = (-F_f\,\hat{y}) \cdot (h\,\hat{y}) = -F_f h$$

$$W_2 = \vec{F} \cdot \vec{d}_2 = (-F_f\,\hat{x}) \cdot (w\,\hat{x}) = -F_f w$$

$$W_3 = \vec{F} \cdot \vec{d}_3 = (+F_f\,\hat{y}) \cdot (-h\,\hat{y}) = -F_f h$$

$$W_4 = \vec{F} \cdot \vec{d}_4 = (+F_f\,\hat{x}) \cdot (-w\,\hat{x}) = -F_f w$$

$$W_{tot} = -2F_f h - 2F_f w \quad \boxed{\neq 0}$$



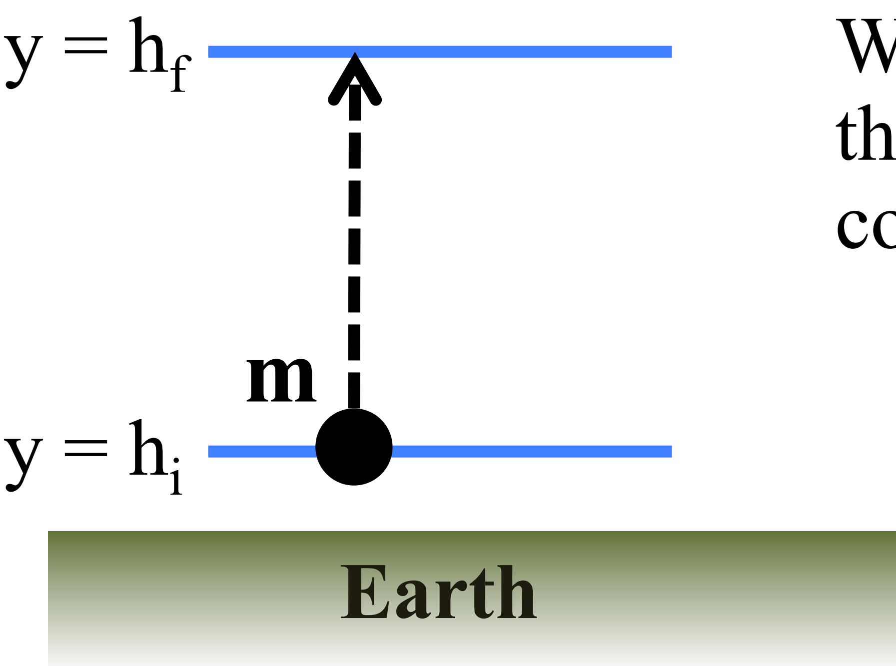
Friction is NOT Conservative !

POTENTIAL ENERGY

If a force is conservative, you can associate a potential energy with that force

- If I do work on an object *against* gravity, I give that object potential energy.
- Potential energy can be recovered when gravity does positive work to pull the object back down.
- This energy is recovered in the form of the object's velocity, or *kinetic energy*.
- Potential energy due to gravity means gravity has the *potential* to do work on the object.

GRAVITATIONAL PE



When I lift on object to a height *h*, the work I do against gravity is converted into PE:

$$W_{me} = \vec{F}_{me} \cdot \vec{d} = +mg\,\hat{y} \cdot (h_f - h_i)\,\hat{y}$$

$$= mgh_f - mgh_i$$

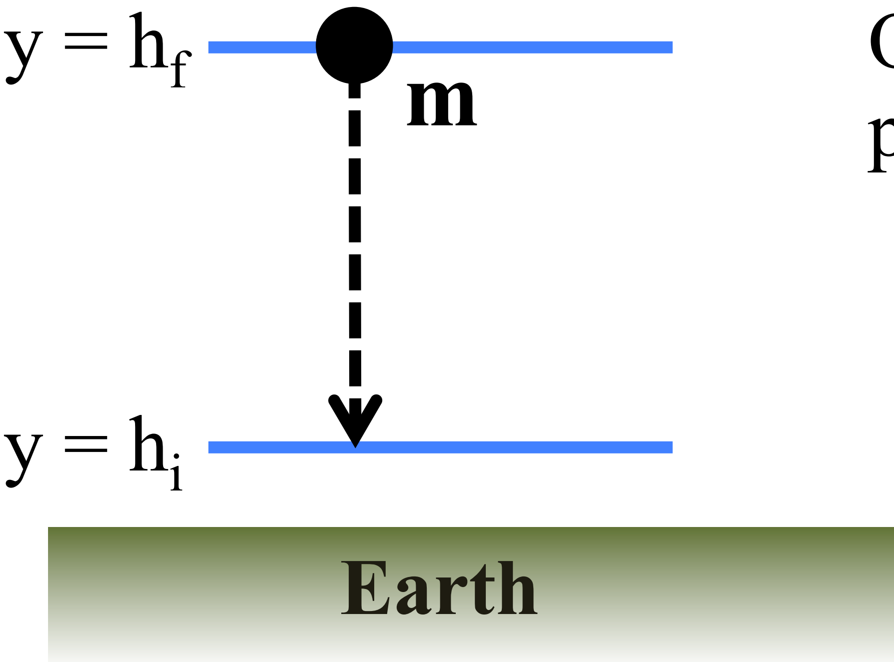
If the object had some PE to start, then the work I do changes the PE.

$$U_f - U_i = mgh_f - mgh_i$$

Thus, we say the Gravitation PE of an object at height *h* is

$$\boxed{U_g = mgh}$$

GRAVITATIONAL PE



On the way down, gravity does positive work on it:

$$W_g = \vec{F}_g \cdot \vec{d} \; = -mg\,\hat{y} \cdot -(h_f - h_i)\,\hat{y}$$

$$= mgh_f - mgh_i$$

If $U_g = mgh$, then the change in PE on the way down is:

$$\Delta U_g = mgh_i - mgh_f$$

Note that: $\Delta U_g = -W_g$

In fact, for all conservative forces: $\boxed{\Delta U_F = -W_{By\,F}}$

CALCULATING PE

If we want to find ΔU for an conservative force $\vec{F}$, then we calculate the work done by that force over some path:

$$\Delta U = U_f - U_i = -\int_{\substack{initial\\pos.}}^{\substack{final\\pos.}} \vec{F}\cdot d\vec{r} \underset{\text{1D}}{=} -\int_{x_i}^{x_f} F\,dx$$

We can reverse the process to find F from U:

$$F_X = -\frac{dU}{dx}$$

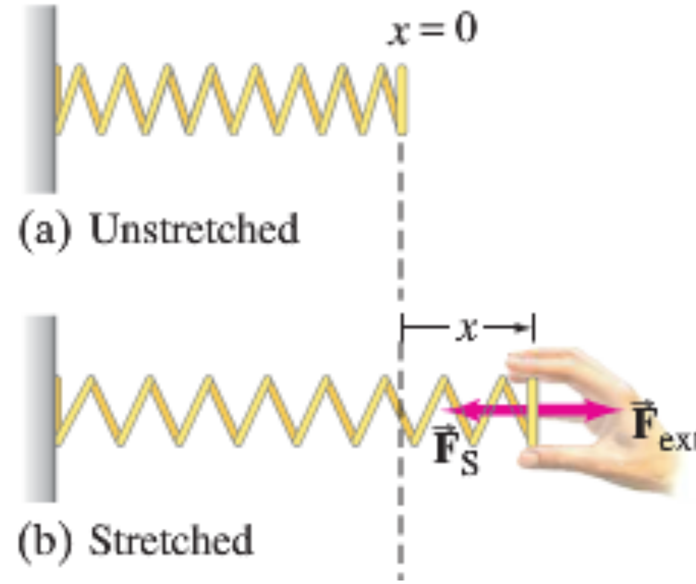
If $\mathbf{F_y = -mg}$, then integral gives $\mathbf{\Delta U = mgy_f - mgy_i}$

If we move our origin to $y = y_i$, then $\mathbf{U(y) = mgy}$

Conversely, if $\mathbf{U(y) = mgy}$, then derivative gives $\mathbf{F_y = - mg}$

ELASTIC PE

What is the PE of a spring?



When the spring is unstretched, we set the end of the spring at $x=0$.

$$U_{S,f} - U_{S,i} = -\int_{\substack{initial\\pos.}}^{\substack{final\\pos.}} \vec{F}_S \cdot d\vec{r} = -\int_0^x F_s\, dx$$

$$= -\int_0^x -kx\, dx$$

$$U_S(x) - U_S(0) = \frac{1}{2}kx^2$$

We define $U_s(0) = 0$, so:

$$\boxed{U_S(x) = \frac{1}{2}kx^2}$$

Also check:

$$F_X = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

SUMMARY

Conservative Forces

1. The work done over a path depends only on the start and end points of the path, and not on the path itself.

   –The work done over a closed path is zero.

2. There is a potential energy U associated with a conservative force.

$$\Delta U = -W_{by\,F} = -\int_{x_i}^{x_f} F\,dx \qquad F_X = -\frac{dU}{dx}$$

- Gravitational PE: $U_g = mgh$
- Elastic PE: $U_s = \frac{1}{2}kx^2$

- For Gravitation PE, only changes have any physical significance. This means you must always define an origin!

SUMMARY

Conservative Forces

1. The work done over a path depends only on the start and end points of the path, and not on the path itself.

   –The work done over a closed path is zero.

2. There is a potential energy U associated with a conservative force.

$$\Delta U = -W_{by\,F} = -\int_{x_i}^{x_f} F\,dx \qquad F_X = -\frac{dU}{dx}$$

- Gravitational PE: $U_g = mgh$
- Elastic PE: $U_s = \frac{1}{2}kx^2$

- Only changes in PE are important – we always need to define were U=0. For springs, U=0 when it is unstretched. For gravity, U=0 wherever we put the origin.

5 options

You are pulling a refrigerator up a rough, inclined plane at a constant speed. Which of the following statements about this situation is *false*?

A. the GPE of the refrigerator is increasing
B. the total work done by all the forces acting on the fridge is zero.
C. you do positive work on the fridge to pull it up the incline
D. the work done on the fridge by gravity is zero
E. the work done on the fridge by the normal force is zero

5 options

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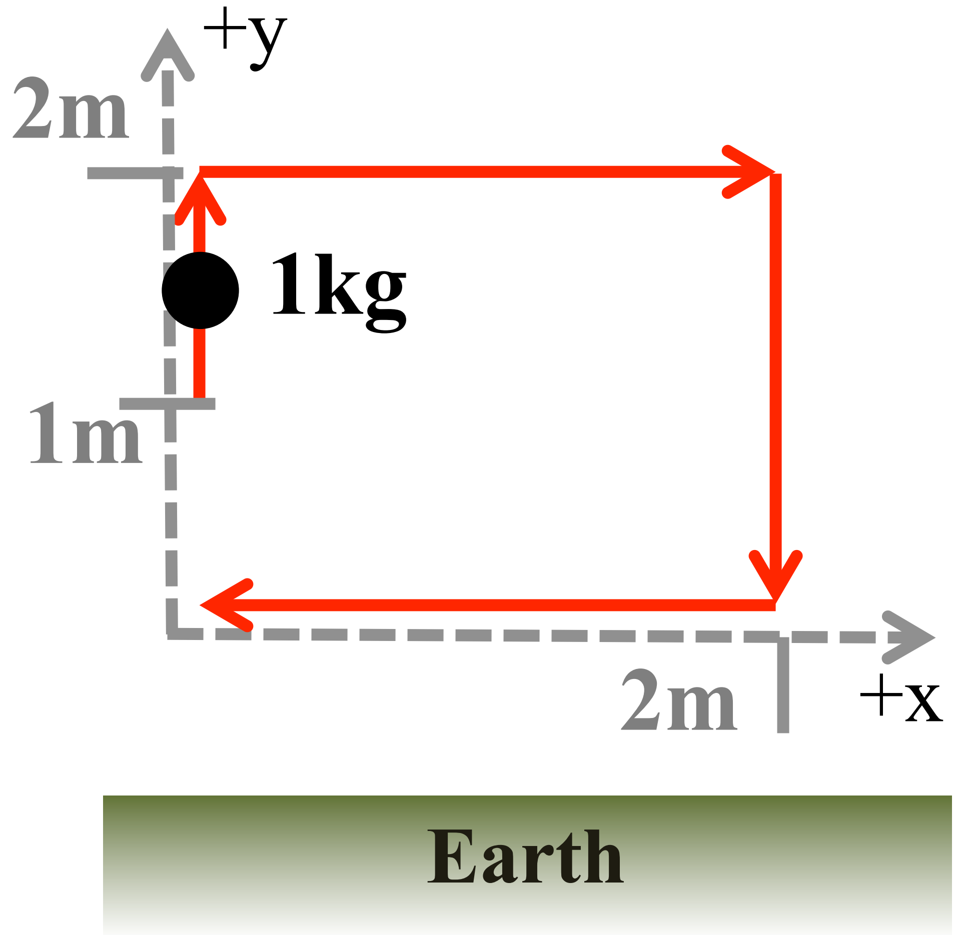

A 1kg mass is moved along the path shown in red, near the surface of the earth. The stopping point is 1m below the starting point. What is the work done on the mass by gravity as it moves around the loop?

(use $g = 10\ m/s^2$)

A. -40 J
B. -20 J
C. 0 J
D. +20 J
E. 40 J
F. None of these

5 options

A 1kg mass is moved along the path shown in red, near the surface of the earth. The stopping point is 1m below the starting point. What is the work done on the mass by gravity as it moves around the loop?

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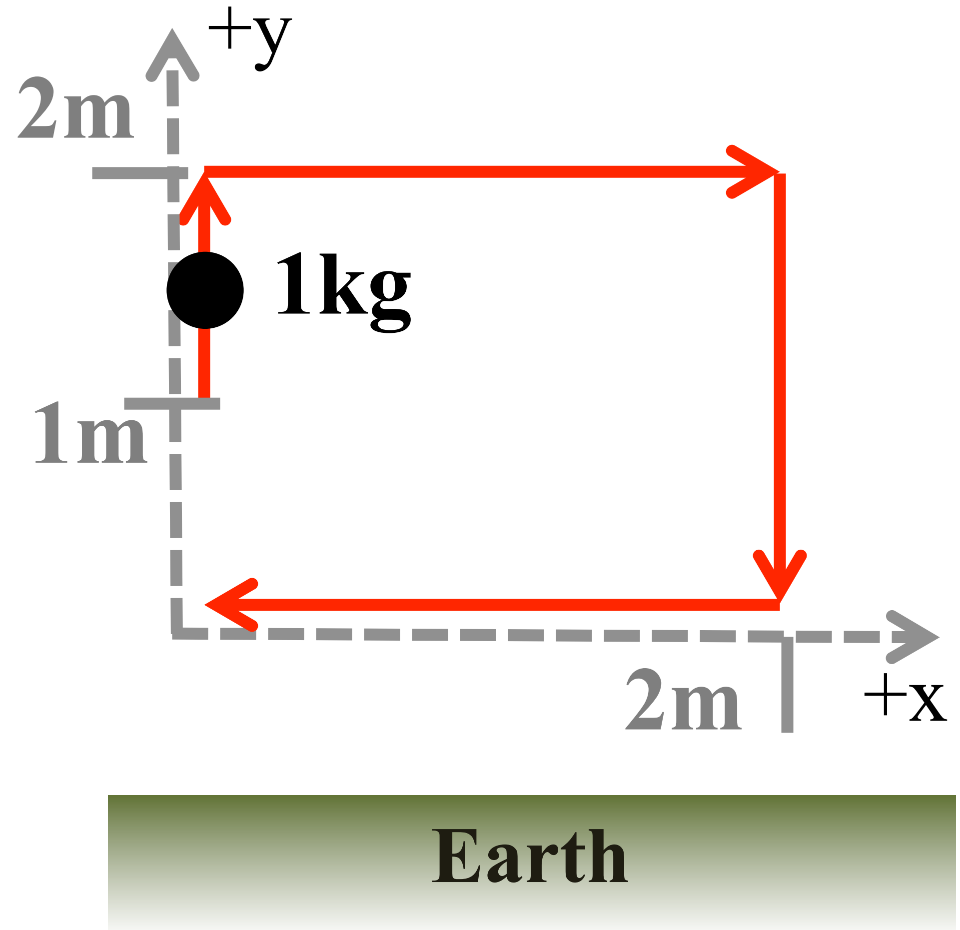


A. -40 J

B. -20 J

C. 0 J

D. +20 J

E. 40 J

F. None of these

$$W_{by\,g} = -\Delta U = -(U_f - U_i)$$

$$= -(mgh_f - mgh_i)$$

$$= +mgh_i$$

$$= 1kg \cdot 10m/s^2 \cdot 1m = 20J$$

LAW OF CONSERVATION OF MECHANICAL ENERGY

If there is a conservative force doing work on a system, then:

$$\Delta U = -W_{by\,F}$$

But the Work-Energy theorem says that $W_{tot} = \Delta K = W_{by\,F}$

$$\Delta U = -\Delta K$$

$$\Delta U + \Delta K = 0$$

$$U_f - U_i + K_f - K_i = 0$$

$$U_i + K_i = U_f + K_f$$

LAW OF CONSERVATION OF MECHANICAL ENERGY

$$U_i + K_i = U_f + K_f$$

We call $U + K$ the **total mechanical energy** E

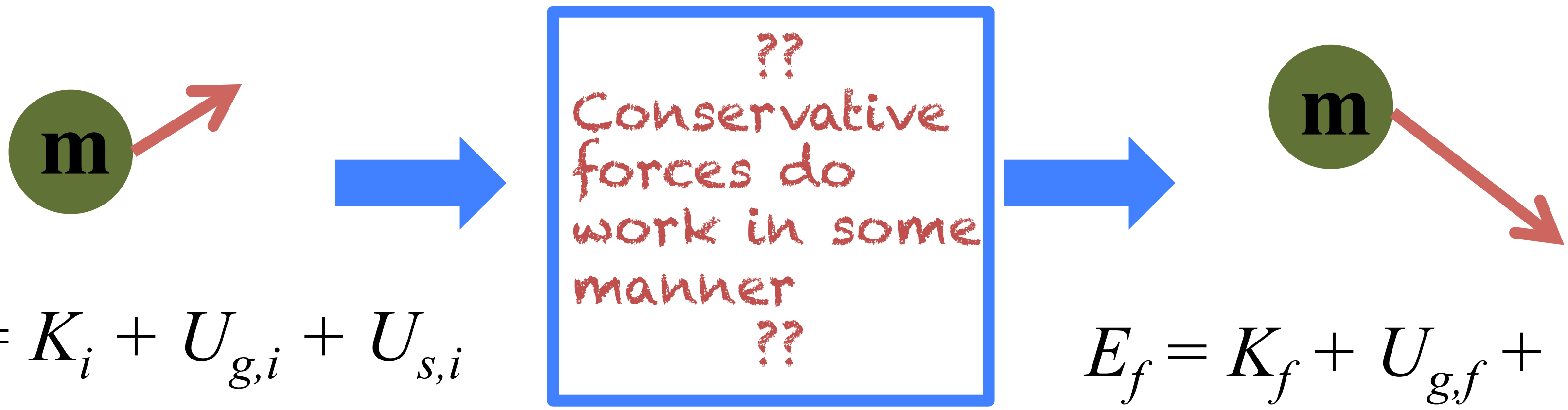
Thus, if there are only conservative forces acting on a system, or any non-conservative forces do no work, then:

$$\mathbf{U + K = constant = E}$$

$$\mathbf{E_i = E_f}$$

THIS IS ONE OF THE MOST IMPORTANT PRINCIPLES IN PHYSICS!

LAW OF CONSERVATION OF MECHANICAL ENERGY



$$E_i = K_i + U_{g,i} + U_{s,i}$$

$$E_f = K_f + U_{g,f} + U_{s,f}$$

The total energy is constant before and after, and we can write

$$\boldsymbol{E_i = E_f}$$

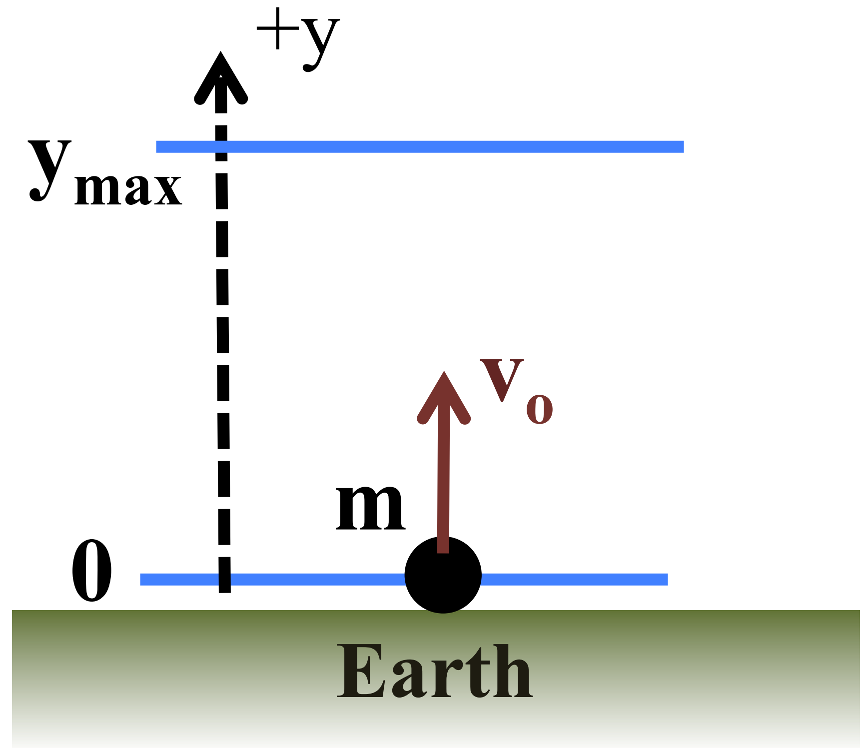
1. Energy is conserved if there are multiple conservative forces (just need to consider PE from each force).

2. If non-conservative are present and do work, we will discuss how to handle that later.

1D MOTION

A mass is launched from the ground at some initial velocity v_o. What is its maximum height?

Only gravity acts on it, and it is conservative, so we can use Cons. of Energy.



$$E_i = E_f \qquad K_i + U_i = K_f + U_f$$

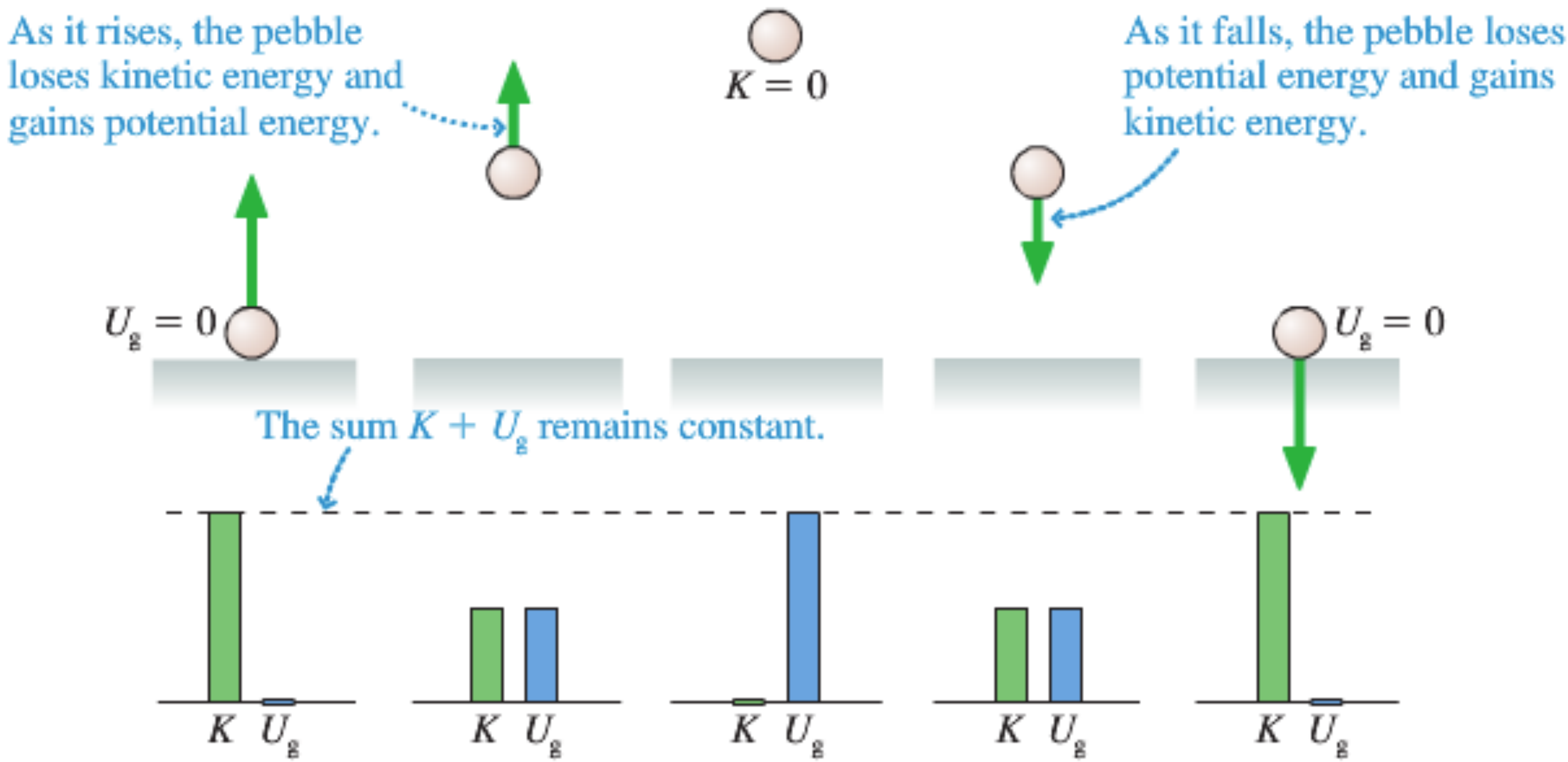
$$\frac{1}{2}mv_0^2 + 0 = 0 + mgy_{max}$$

$$\Rightarrow \boxed{y_{max} = \frac{v_0^2}{2g}}$$

Using Kinematics: $v^2 = v_0^2 + 2a\Delta y \quad 0 = v_0^2 - 2gy_{max} \quad y_{max} = \frac{v_0^2}{2g}$

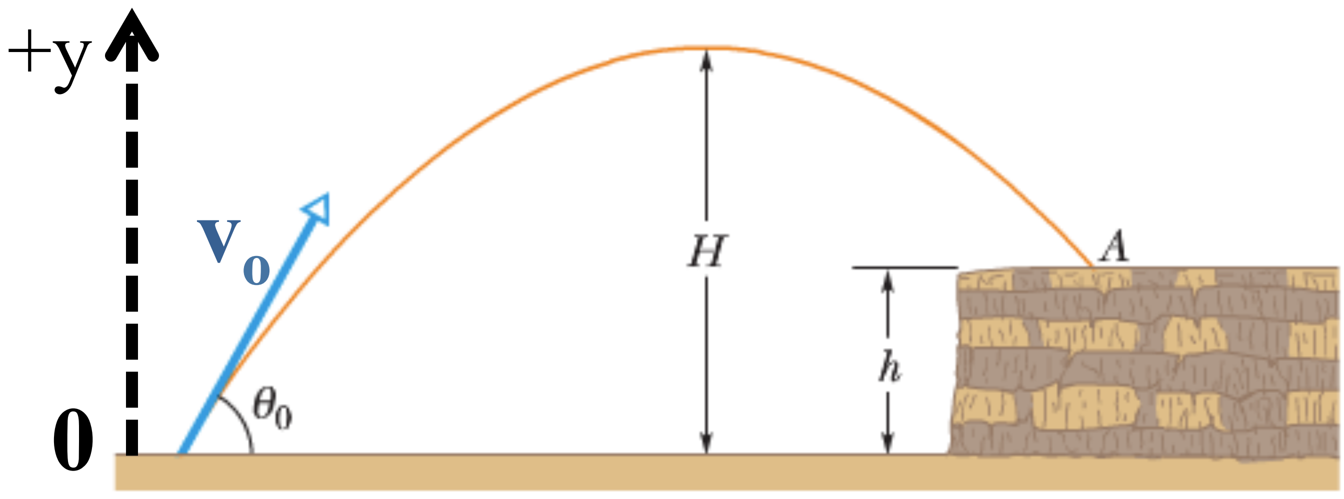
1D MOTION

A mass is launched from the ground at some initial velocity v_o. How what is its maximum height?



2D MOTION

A mass is launched with initial speed v_0 at a wall? What is the speed at point A, when it lands on the wall?



Only gravity acts on it, and it is conservative, so we can use Cons. of Energy.

$$E_i = E_f \qquad K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv_f^2 + mgh$$

Re-arranging: $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_0^2 - mgh$ Solving for v_f: $\boxed{v_f = \sqrt{v_0^2 - 2gh}}$

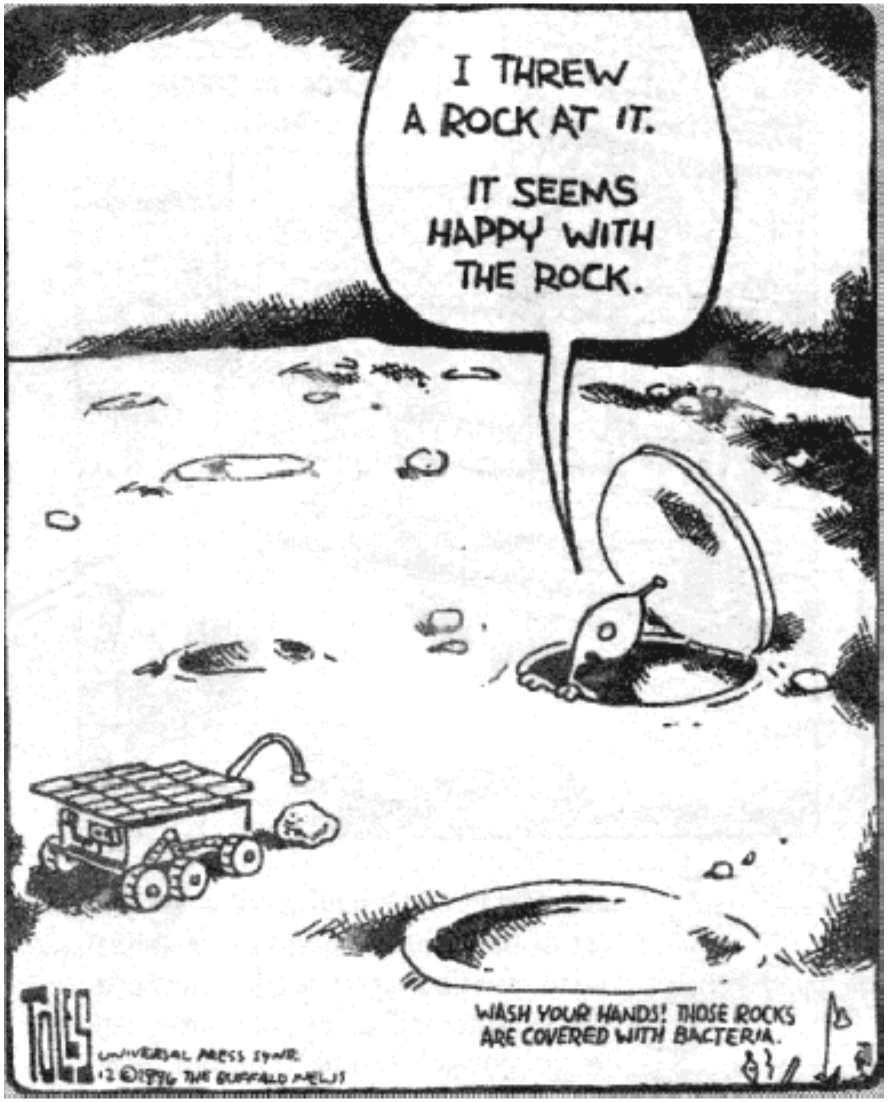
2D MOTION

$$\boxed{v_f = \sqrt{v_0^2 - 2gh}}$$

- ✓ If $h=0$, then $v_f = v_0$, as you would expect of an object that begins and ends at the same height.
- ✓ If v_0 is too small, then you have the square root of a negative number, telling us that it does not have enough energy to reach the top of the building.
- ✓ There is no mass term, which is typical of problems involving only gravity.
- ✓ If we fire the object straight up, at the top $v_f=0$, and this equation tells us the max height $h = \dfrac{v_0^2}{2g}$

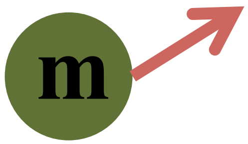
We made it to here on Fri. Feb. 27nd

PHYS 121 – SPRING 2014



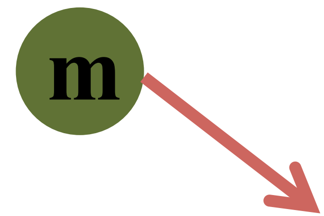
Chapter 8: Potential Energy & Conservation of Energy

CONSERVATION OF ENERGY



$$E_i = K_i + U_{g,i} + U_{s,i}$$

??
Conservative forces do work in some manner
??



$$E_f = K_f + U_{g,f} + U_{s,f}$$

$$\boldsymbol{E_i = E_f}$$

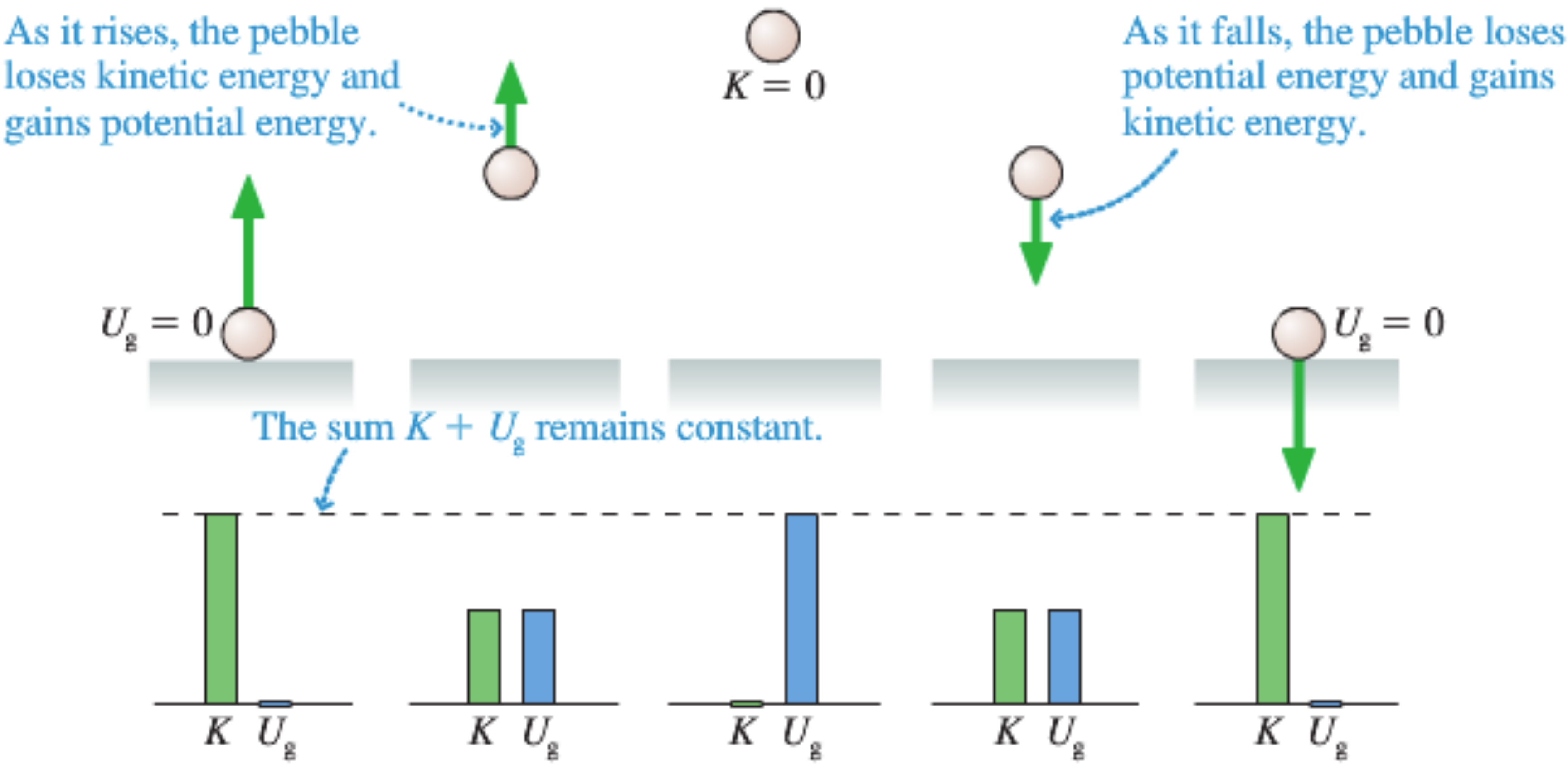
Kinetic Energy $\quad K = \frac{1}{2}mv^2$

Gravitational PE $\quad U_g = mgh$

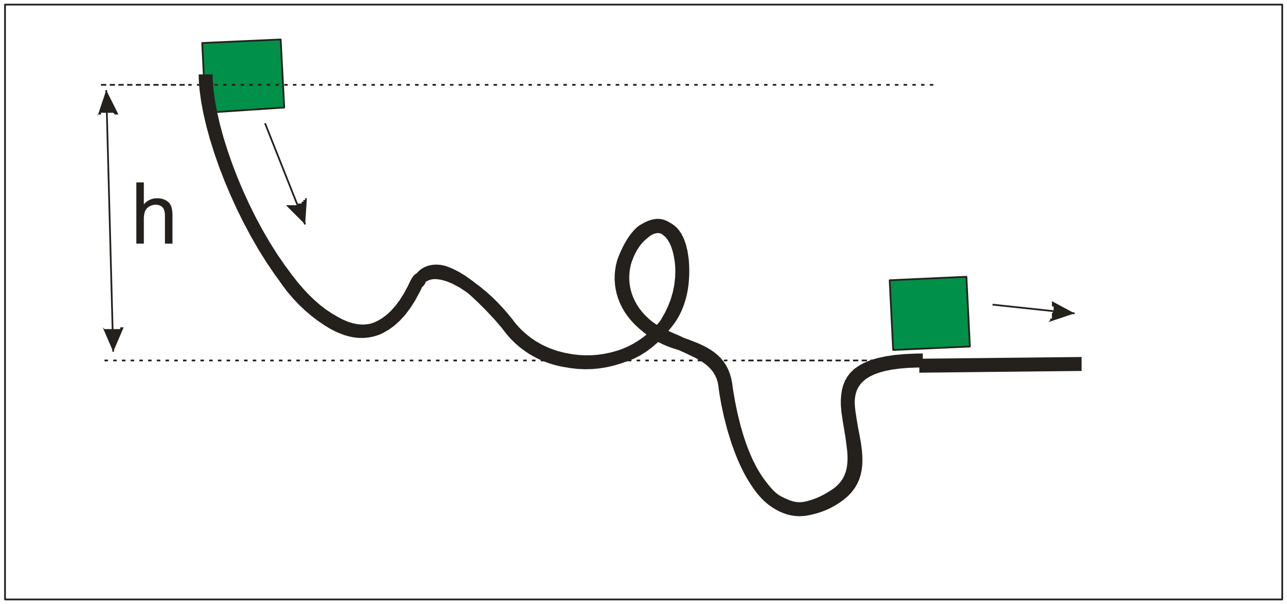
Spring PE $\quad U_s = \frac{1}{2}kx^2$

1D MOTION

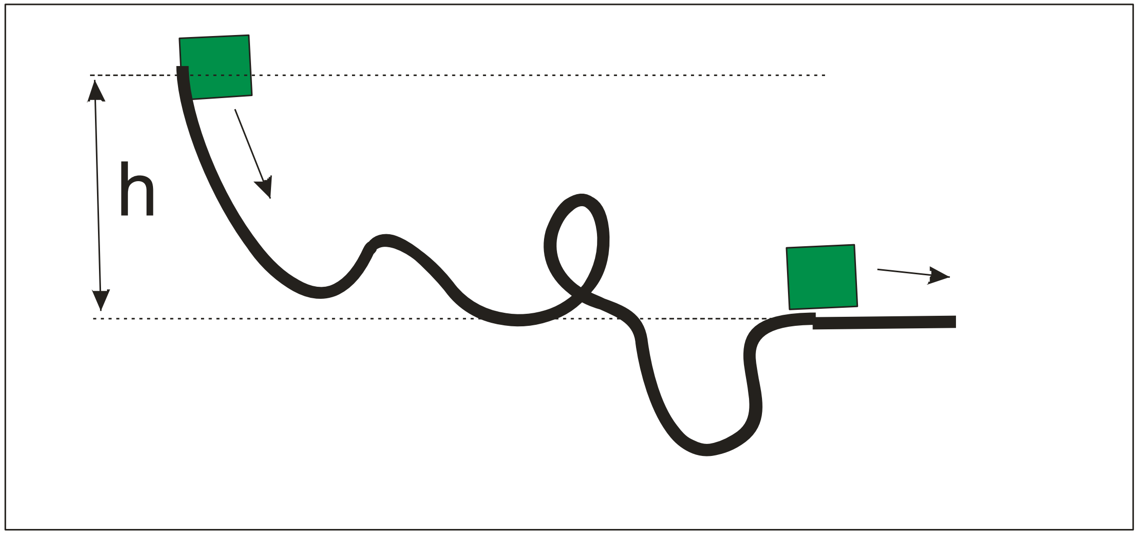
A mass is launched from the ground at some initial velocity v_o. How what is its maximum height?



USING CONSERVATION OF ENERGY



- Consider a frictionless ramp with a complex curved shape but an overall height change of *h* from top to bottom.
- A block of mass M is released from rest at the top. Calculate its speed at the end.



Using Newton's 2nd Law would be VERY HARD, since the x- and y- components of acceleration change as the angle of the ramp changes.

Since Weight is the only force doing work, we can use Cons. of Energy.

$$E_i = E_f \qquad K_i + U_i = K_f + U_f$$

$$0 \ + mgh \ = \frac{1}{2} m v_f^2 \ + 0$$

$$\boxed{v_f = \sqrt{2gh}}$$

USING CONSERVATION OF ENERGY

Use Newton's Laws if:

1. The question involves the *time* it takes to move, or

2. The question involves different components of the velocity.

Use Energy if:

1. You are asked to find a final height or speed.

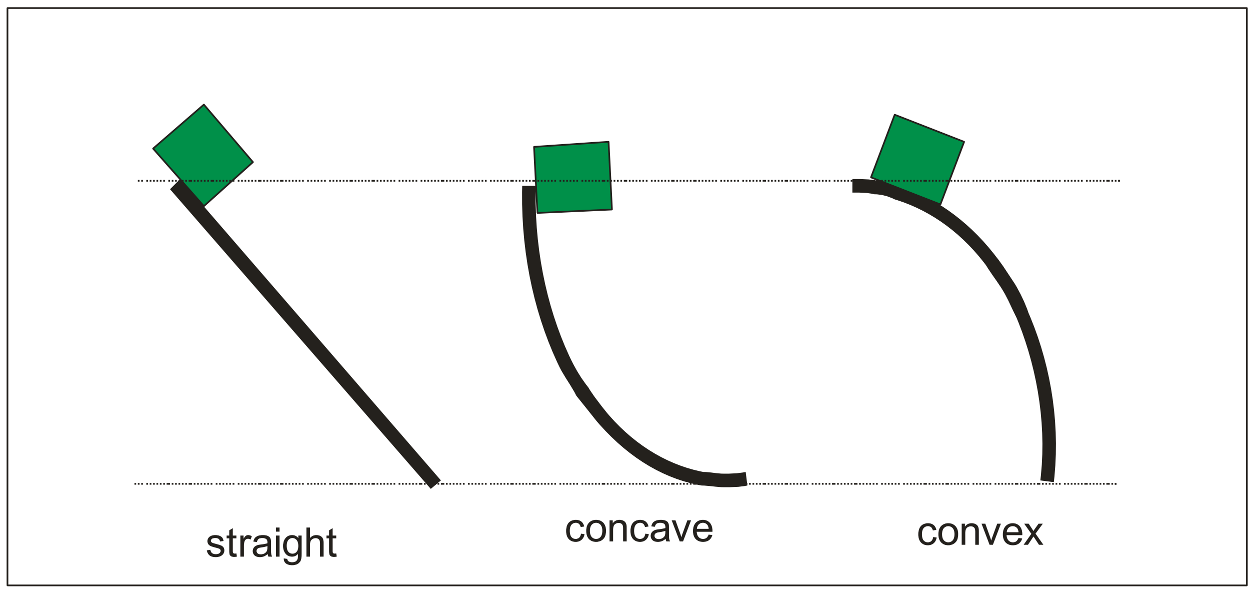
2. The acceleration is not constant or the motion is complicated.

4 options

Three identical blocks are placed at the top of three frictionless ramps, which have different shapes. All blocks start at the same height.

Which block has the largest speed when it reaches the bottom?

A. Straight
B. Concave
C. Convex
D. All the same



4 options

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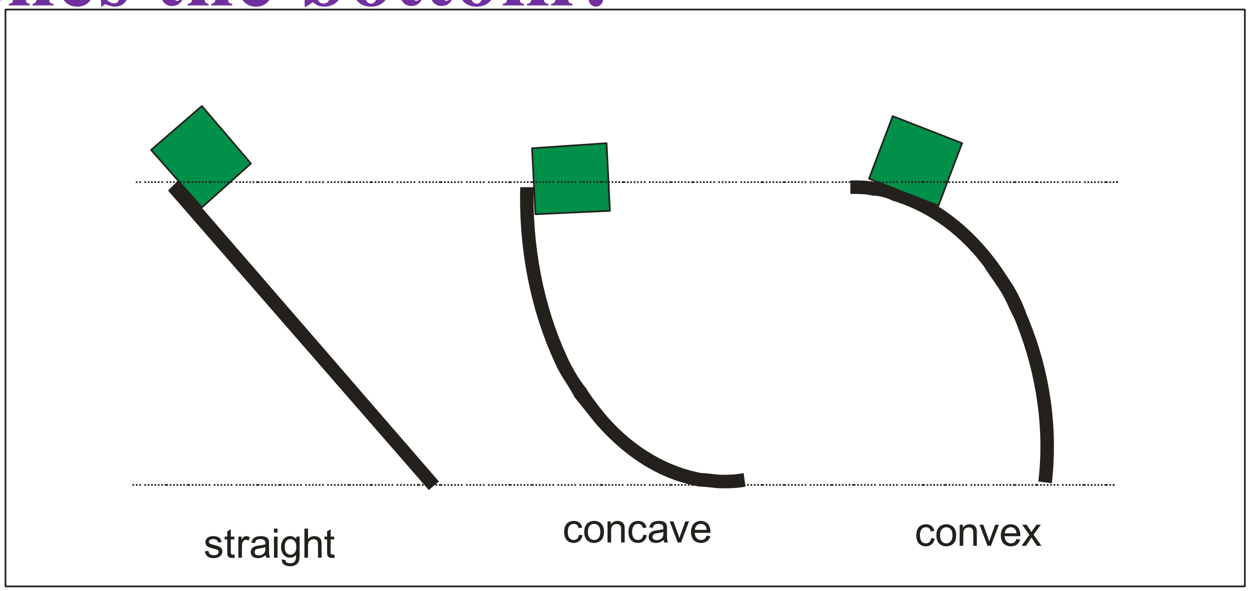


All blocks start with the same PE, and have the same PE at the bottom, regardless of the path down. So they all have the KE at the bottom.

5 options

A block initially at rest slides down a frictionless incline of height *1m*, ending up with a speed *v* at the bottom. To achieve a speed of *2v* at the bottom, what must the height be?

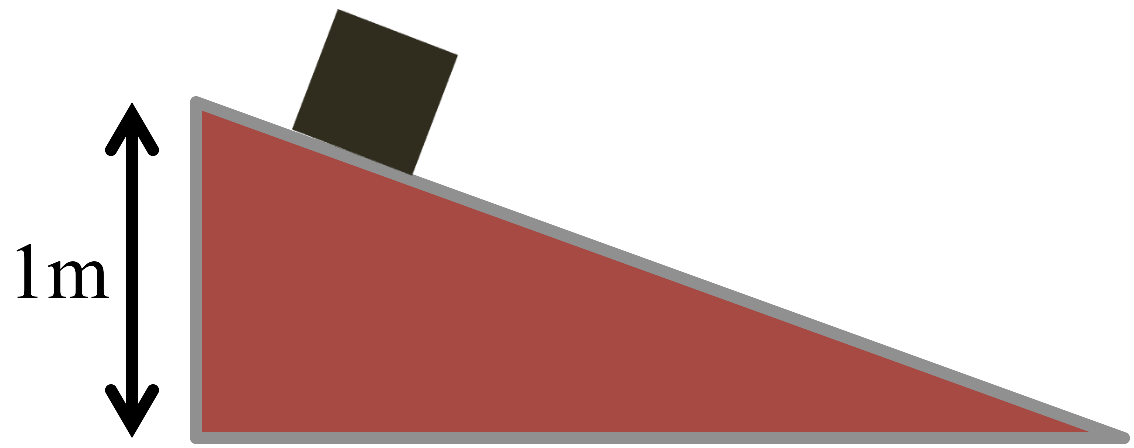
A. 1m

B. 2m

C. 3m

D. 4m

E. 6m



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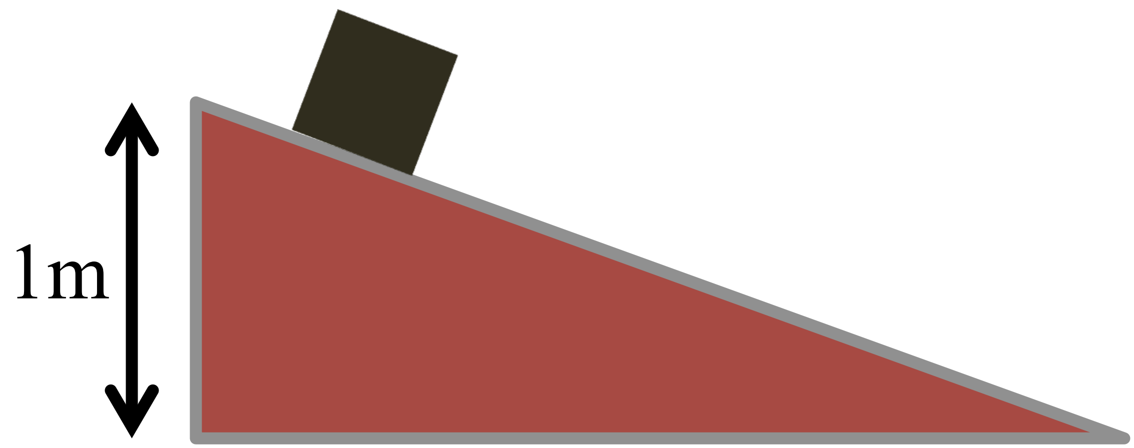


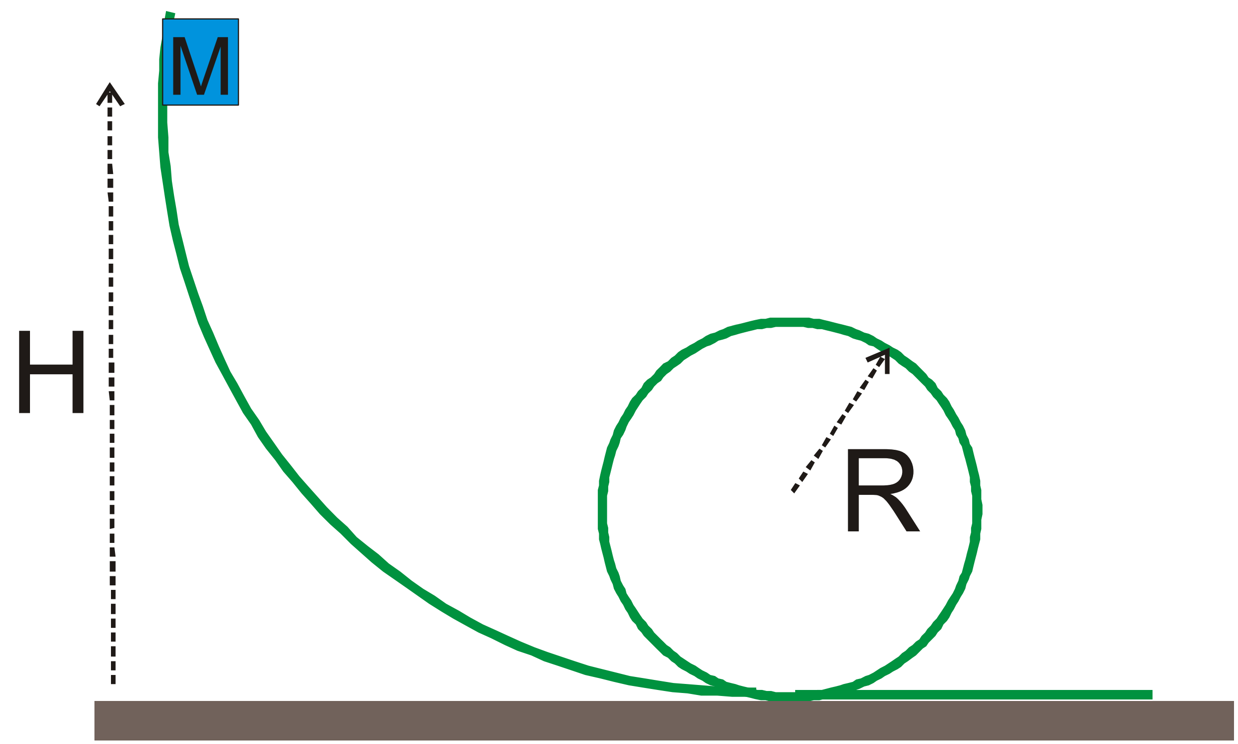
A. 1m
B. 2m
C. 3m
D. 4m
E. 6m

Twice the speed means 4x the KE, so it needs 4x the PE to start. Thus the incline must be 4x as tall.

LOOPS - *a common intro physics problem*

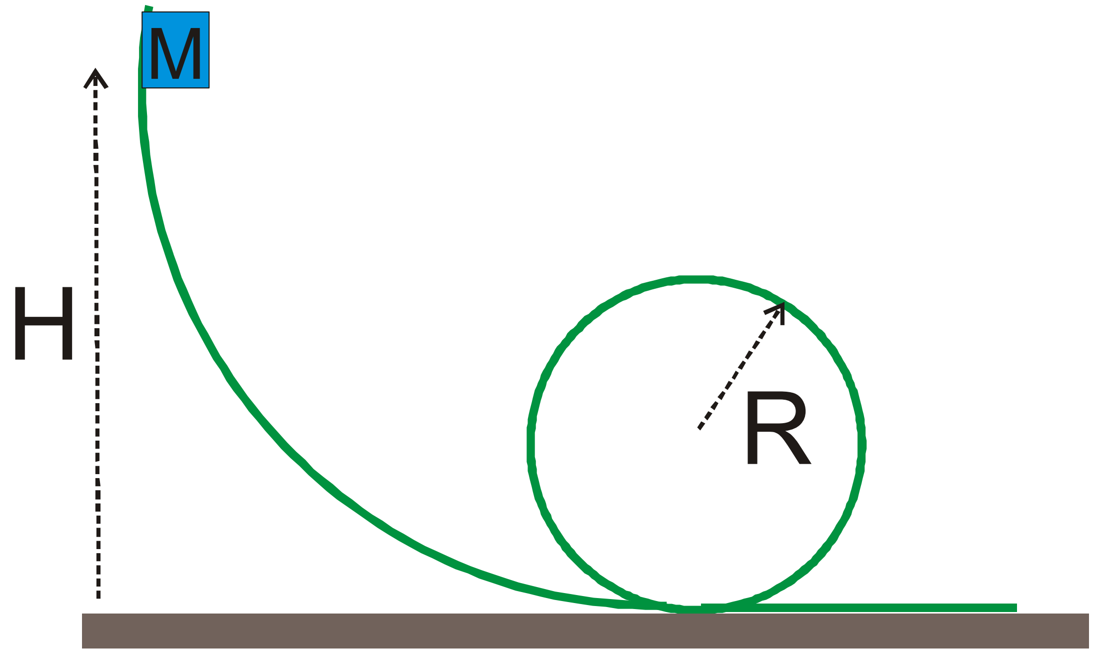
A mass M is traveling around a frictionless vertical circular track of radius R. It starts at a height H.

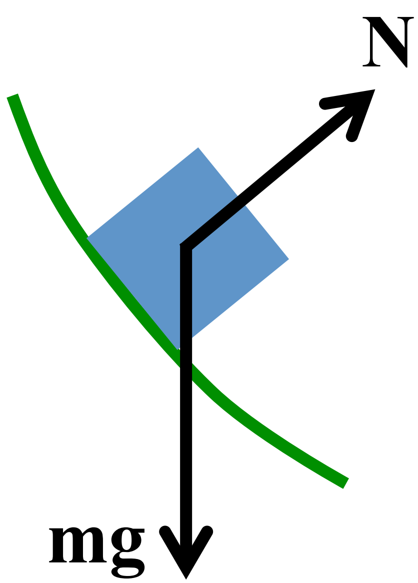
What is the speed of the mass at the top of the loop?



LOOPS *- a common intro physics problem*

What are the forces acting on it, and are they conservative?





Only gravity is doing work, and gravity is conservative, so:

Cons. of Energy ✔

LOOPS - *a common intro physics problem*

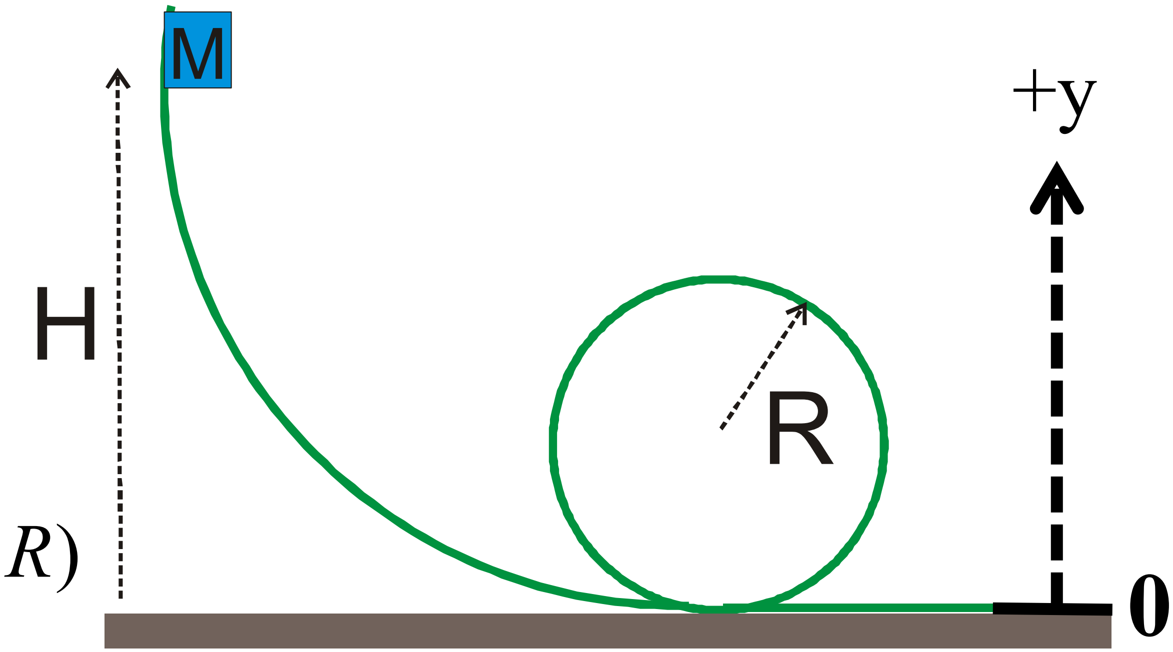
$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$0 + mgH = \frac{1}{2}mv_f^2 + mg(2R)$$

$$\frac{1}{2}mv_f^2 = mg(H - 2R)$$

$$\boxed{v_f = \sqrt{2g(H - 2R)}}$$



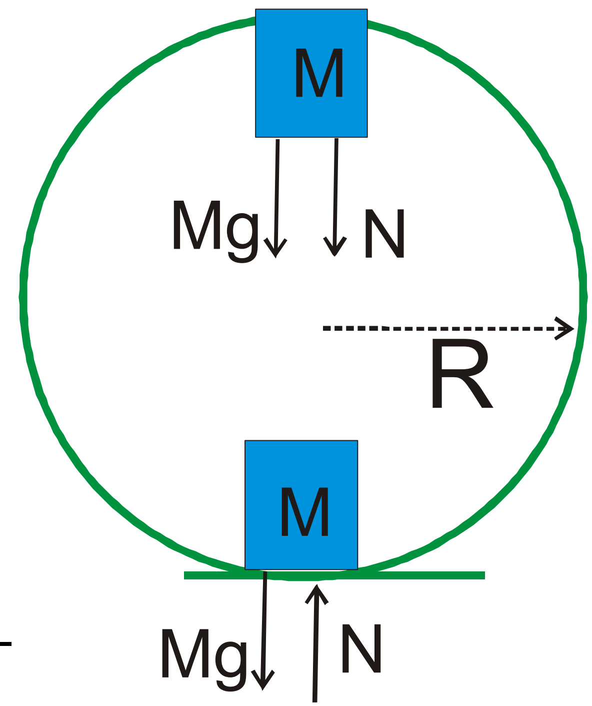
What if H < 2R?

The mass will not make to the top of the loop, it will not have enough energy.

LOOPS - *a common intro physics problem*

$$v_f = \sqrt{2g(H - 2R)}$$

What is the minimum height H such that the mass actually makes it smoothly around the loop?



At top: $\Sigma F = N + mg$

For circular motion, ΣF must equal $\dfrac{mv^2}{r}$

$$N + mg = \frac{mv^2}{R}$$

When it falls off, the ***N*** goes to zero: $g = \dfrac{v^2}{R}$

We know *v* at the top:

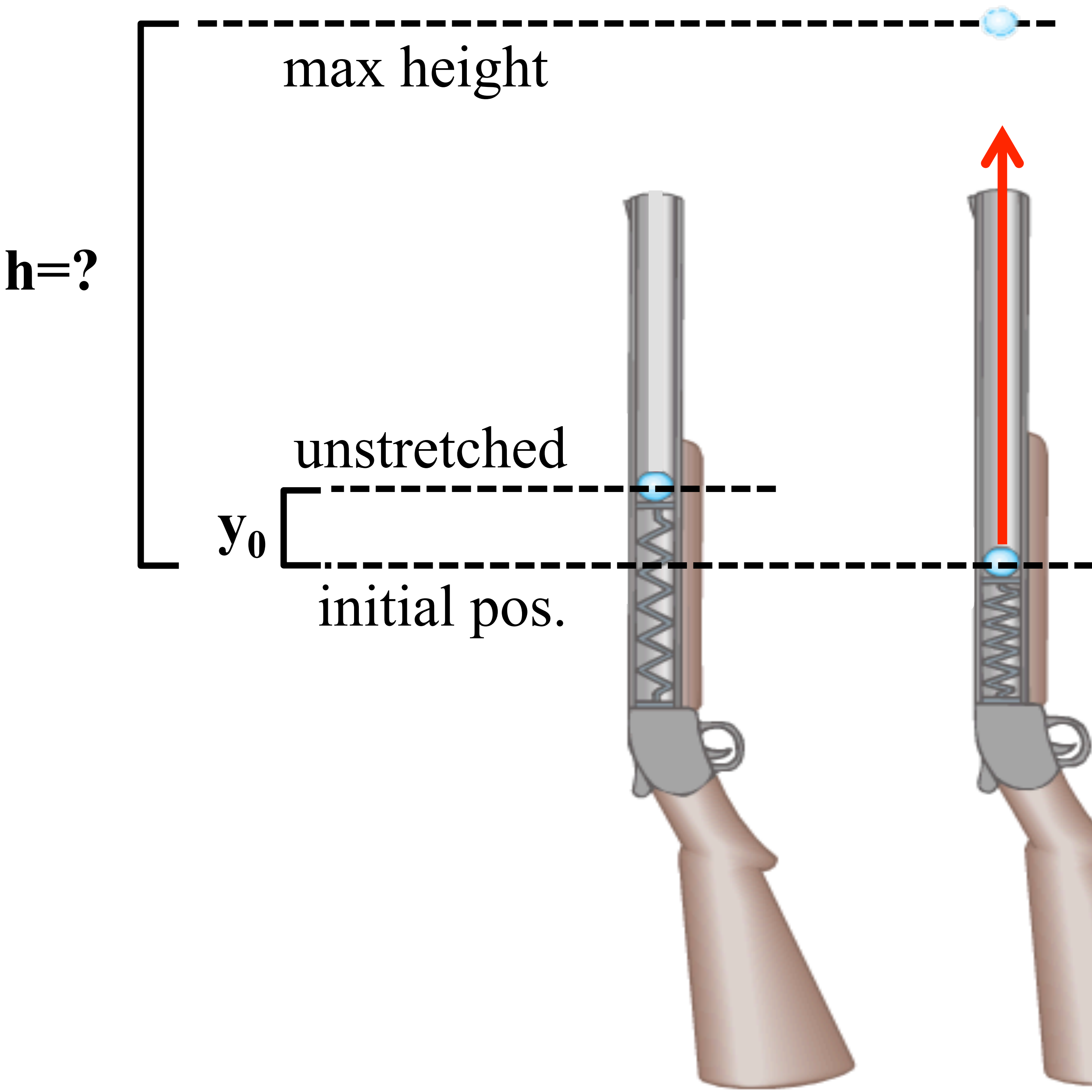
$$g = \frac{2g(H - 2R)}{R} \qquad gR = 2gH - 4gR$$

$$\boxed{H = \frac{5}{2}R}$$

SPRING EXAMPLE #1

A small mass is placed into a spring loaded gun. The mass/spring is then compressed by a distance y_0 from the spring's unstretched position. When released, what is the maximum height h the mass achieves, relative to its initial, compressed, position?

SPRING EXAMPLE #1



From Serway

SPRING EXAMPLE #1

Only forces acting are gravity and spring, which are both conservative.

Cons. of Energy ✔

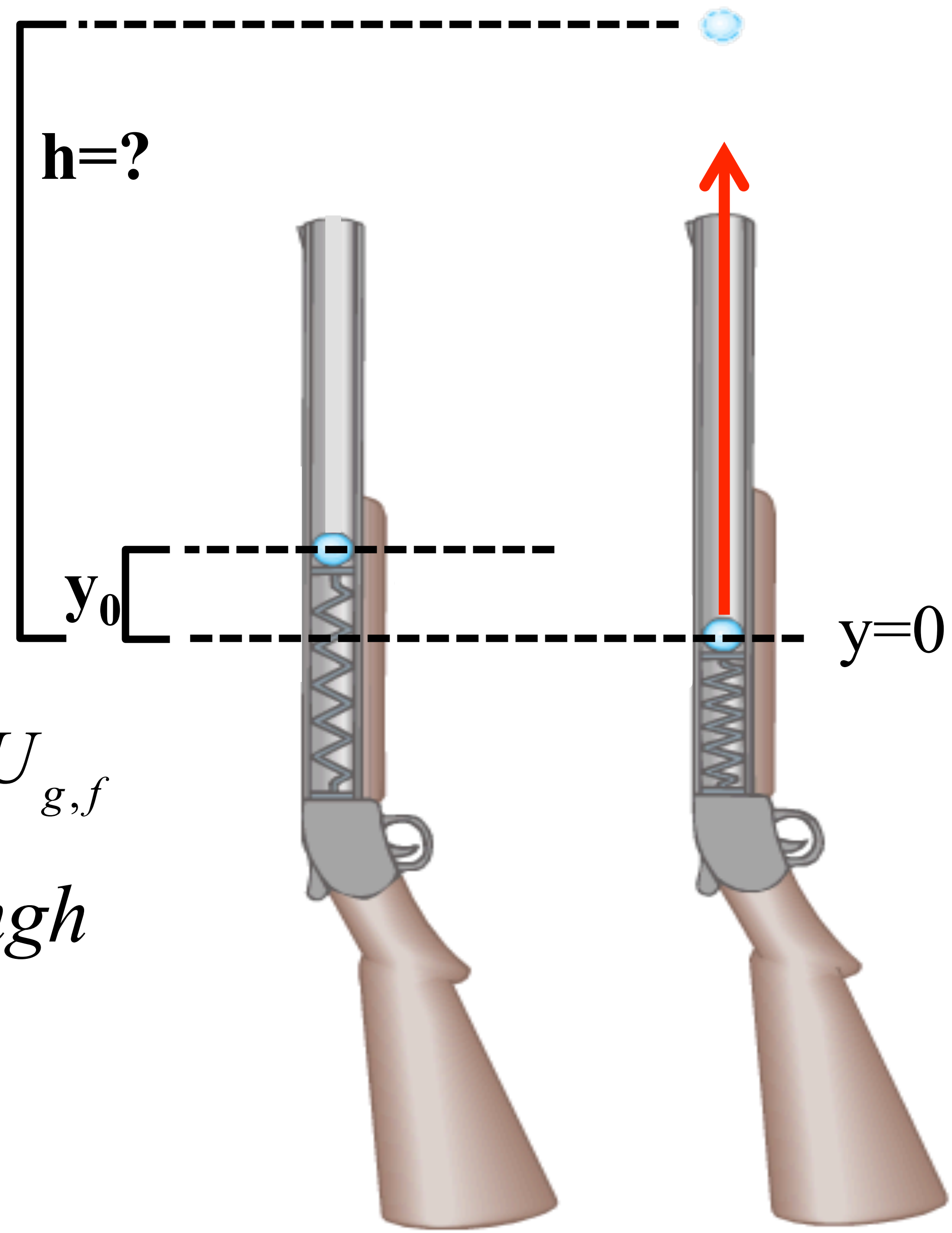
$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$K_i + U_{S,i} + U_{g,i} = K_f + U_{S,f} + U_{g,f}$$

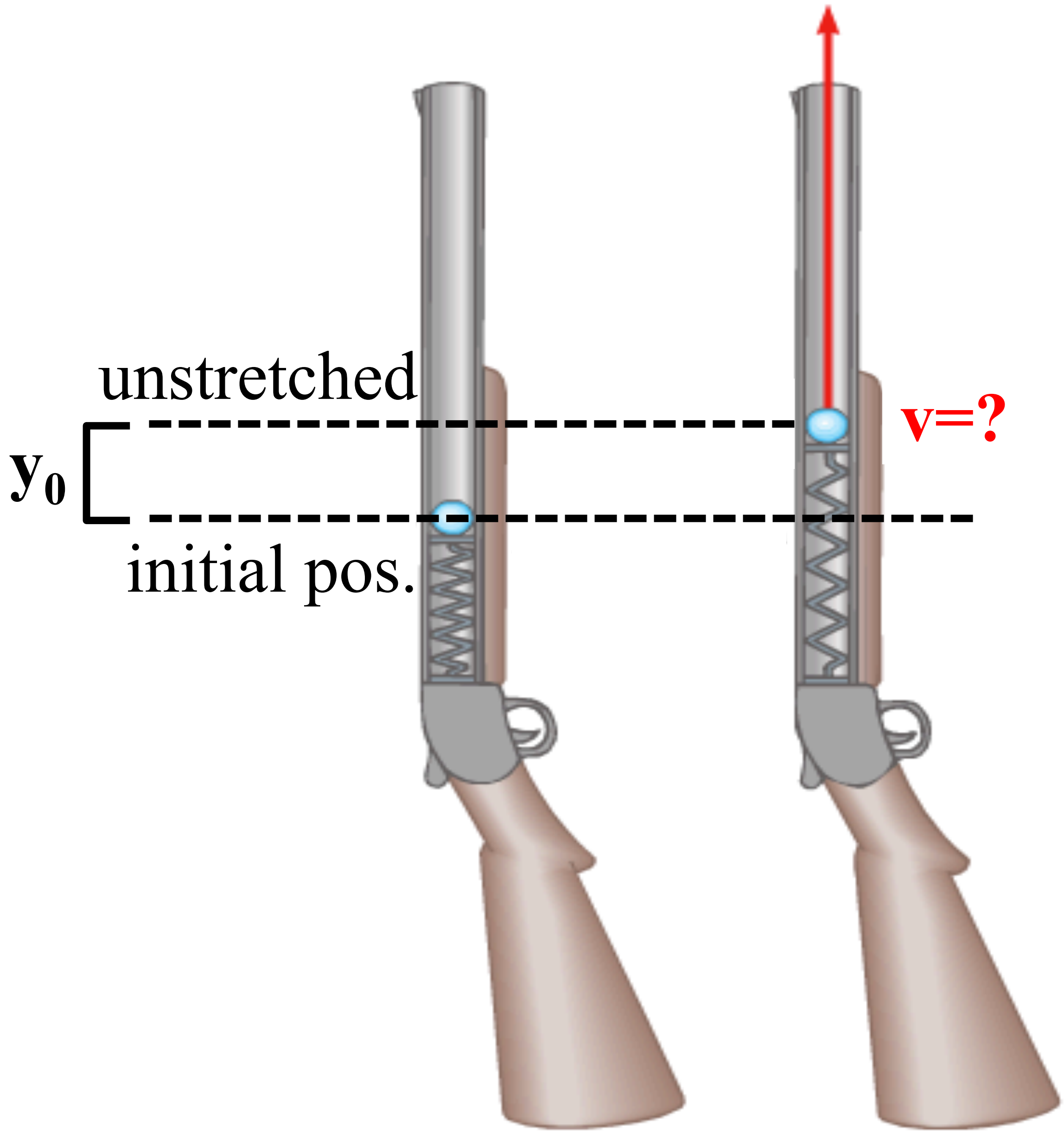
$$0 + \frac{1}{2}k{y_0}^2 + 0 = 0 + 0 + mgh$$

$$\boxed{h = \frac{1}{2g}\frac{k}{m}{y_0}^2}$$



SPRING EXAMPLE #1

For the same initial conditions, what is the velocity of the mass when it passes through the spring's original unstretched position?



SPRING EXAMPLE #1

Only forces acting are gravity and spring, which are both conservative.

Cons. of Energy ✔

$$E_i = E_f$$

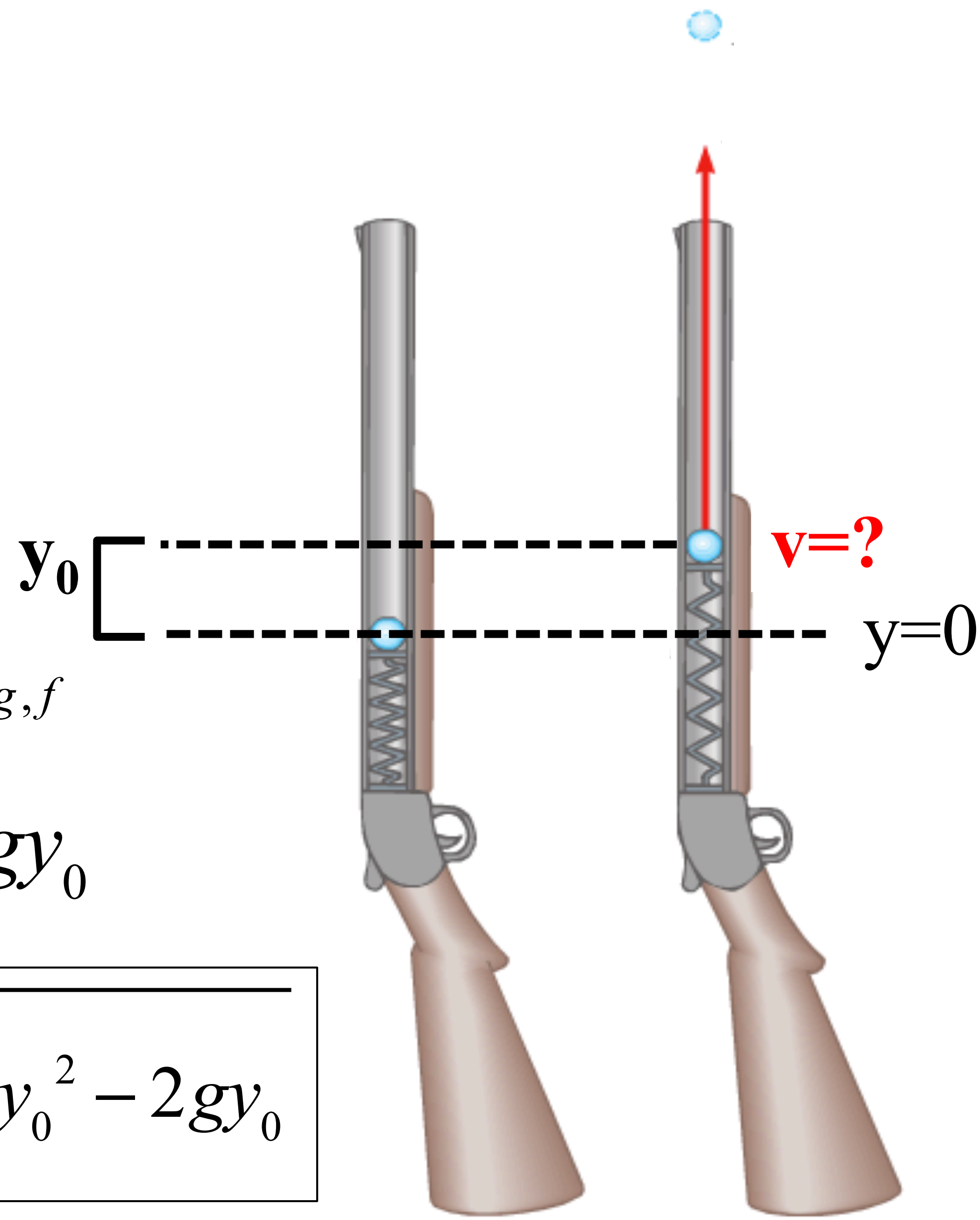
$$K_i + U_i = K_f + U_f$$

$$K_i + U_{S,i} + U_{g,i} = K_f + U_{S,f} + U_{g,f}$$

$$0 + \frac{1}{2}ky_0^{\ 2} + 0 = \frac{1}{2}mv^2 + 0 + mgy_0$$

$$\frac{1}{2}mv^2 = \frac{1}{2}ky_0^{\ 2} - mgy_0$$

$$\boxed{v = \sqrt{\frac{k}{m}y_0^{\ 2} - 2gy_0}}$$



6 options

A spring is used to launch a ball straight up in the air, and the ball reaches a maximum height of 24 m. The same ball is launched a second time but this time the spring is compressed only half as far before firing. How far up does the ball go this time?

A. 96m
B. 48m
C. 24m
D. 12m
E. 6m
F. 3m

6 options

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A. 96m
B. 48m
C. 24m
D. 12m
E. 6m
F. 3m

Spring PE is proportional to the square of its compression, $U = ½kx^2$, and if the compression is reduced by a factor of 2, then the energy is reduced by a factor of 4.

All of this is converted to gravitational PE, *mgh*, at the top, so *h* is reduced by a factor of 4: from 24 m to 6 m.

SPRING EXAMPLE #2

A mass *m* slides down a curved, quarter-circle, frictionless ramp of radius *R*. At the bottom of the ramp, the mass strikes a spring.

- How much will the spring be compressed?
- How far up the ramp will the mass rebound?

SPRING EXAMPLE #2

How much will the spring be compressed?

Only forces acting are gravity and spring, which are both conservative.

Cons. of Energy ✔ $E_i = E_f$

$$K_i + U_{S,i} + U_{g,i} = K_f + U_{S,f} + U_{g,f}$$

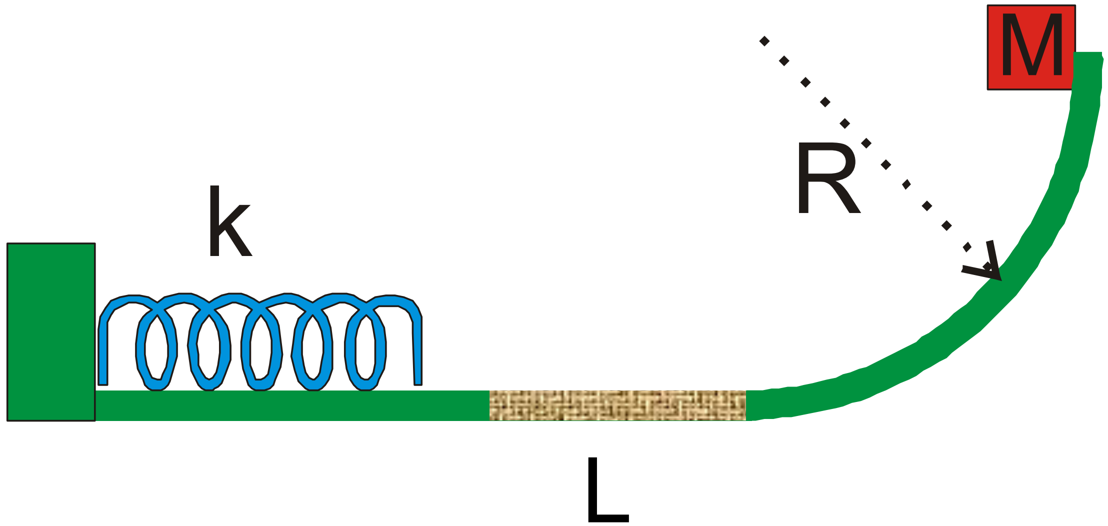
$$0 + 0 + mgR = 0 + \frac{1}{2}kx_0^2 + 0$$

$$\boxed{x_0 = \sqrt{\frac{m}{k}2gR}}$$

How high will the mass rebound?

Since energy is conserved, it must have the same GPE after rebounding as it did when it started, so it *will reach the same height.*

SPRING EXAMPLE #2



What if there is now a rough section of track of length L and coefficient of kinetic friction μ_k before the spring?

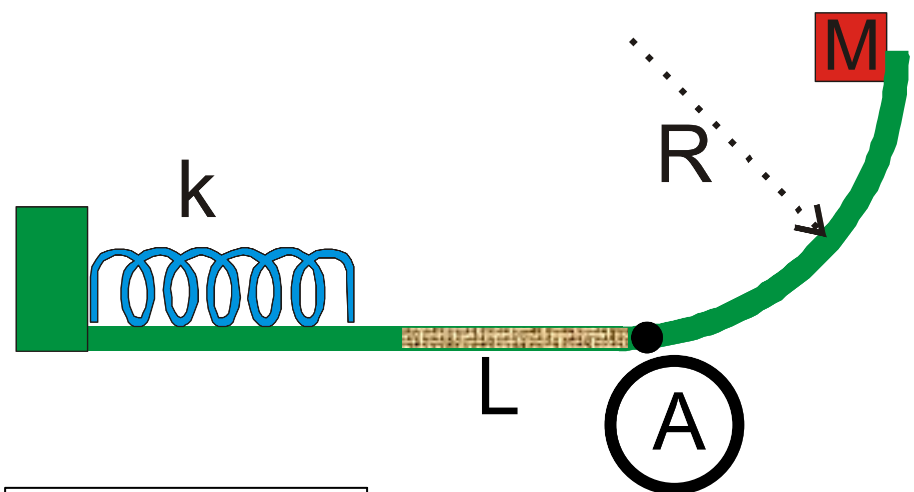
- How much will the spring be compressed?

SPRING EXAMPLE #2

General approach:

1. Use CoE to find speed at bottom, before friction.
2. Calculate Work done by friction and use Work-Energy theorem to find new speed when it hits the spring.
3. Use CoE to find maximum spring compression.

1. Speed at point A.



$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$0 + mgR = \frac{1}{2}mv_A^2 + 0$$

$$\boxed{K_A = mgR}$$

SPRING EXAMPLE #2

2. Change in KE across rough section.

$$\boxed{K_A = mgR}$$

$$W_{tot} = W_f = \Delta K$$

$$W_f = \vec{F}_f \cdot \vec{d} = -\mu_k mg\,\hat{x} \cdot L\hat{x} = -\mu_k mgL$$

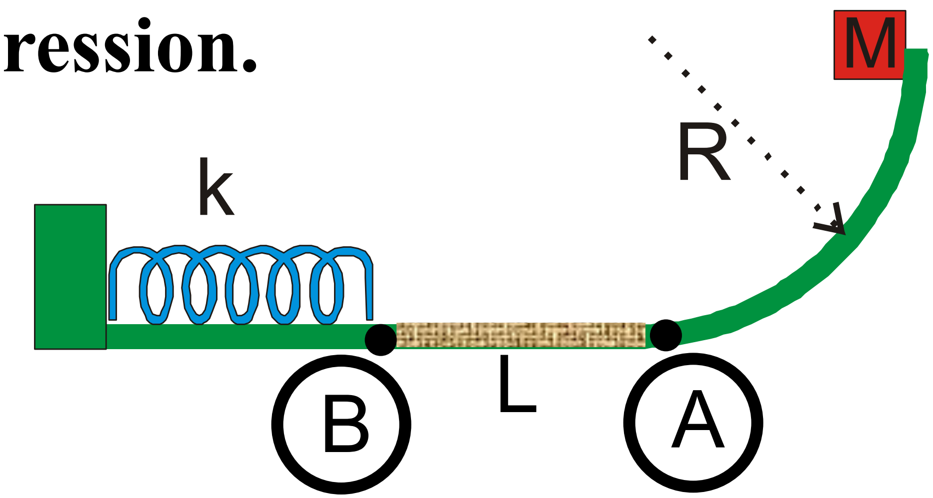
$$K_B = K_A + \Delta K = K_A + W_f = \boxed{mg(R - \mu_k L)}$$

3. Use CoE to find spring compression.

$$E_i = E_f \qquad K_i + U_i = K_f + U_f$$

$$mg(R - \mu_k L) + 0 = 0 + \frac{1}{2}kx_0^2$$

$$\boxed{x_0 = \sqrt{\frac{m}{k}2g(R - \mu_k L)}}$$



SPRING EXAMPLE #2

Alternative Approach:

Say that the change in energy of the system is no longer zero, but equal to whatever is taken out by friction.

With any non-conservative or external force:

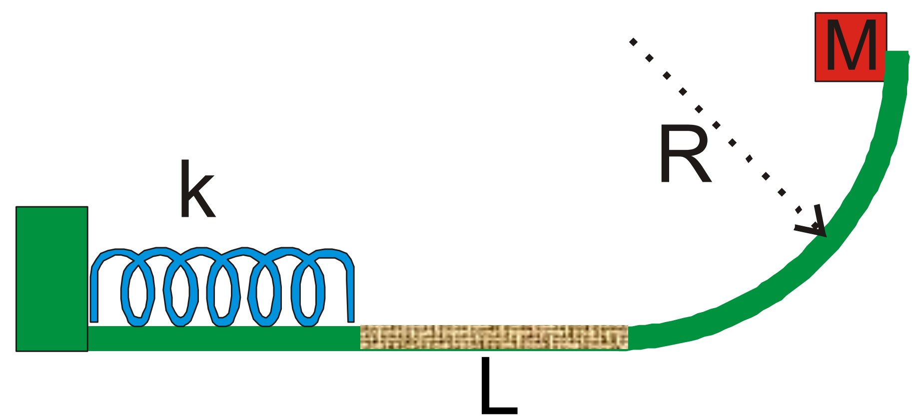
$$\Delta E = W_{by\, non-cons\; force}$$

$$E_f - E_i = W_{by\; friction}$$

$$(K_f + U_f) - (K_i + U_i) = -\mu_k mgL$$

$$(0 + \frac{1}{2}kx_0^2) - (0 + mgR) = -\mu_k mgL$$

$$\frac{1}{2}kx_0^2 = mgR - \mu_k mgL$$

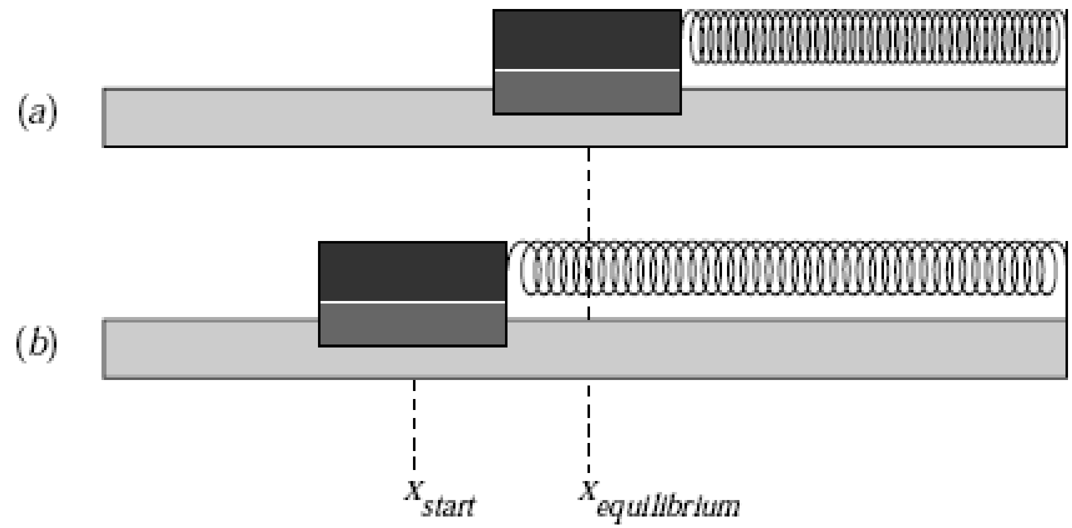


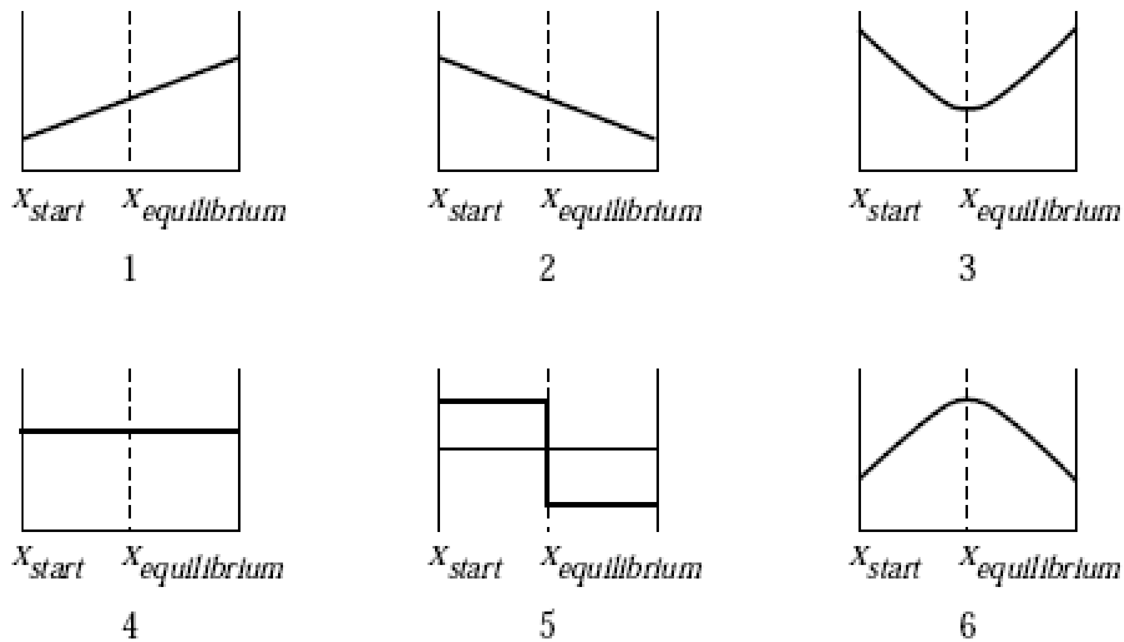
$$\boxed{x_0 = \sqrt{\frac{m}{k}2g(R - \mu_k L)}}$$

6 options

A car attached to a spring on a frictionless track is pulled from the equilibrium position to a ‘start’ position & released. The cart then oscillates about $x_{equilibrium}$.

Which graph most closely represents the potential energy of the spring/mass system as a function of the position of the cart?





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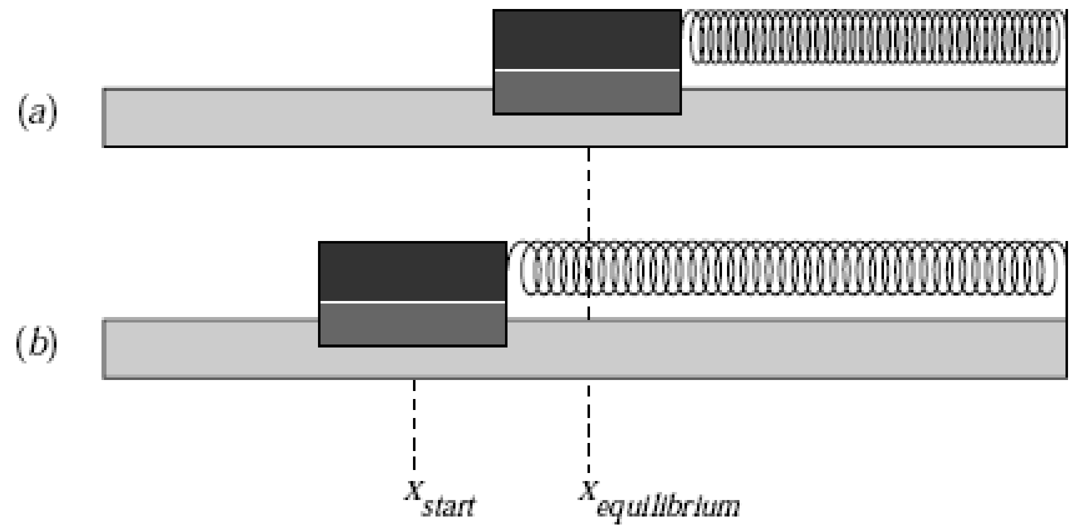
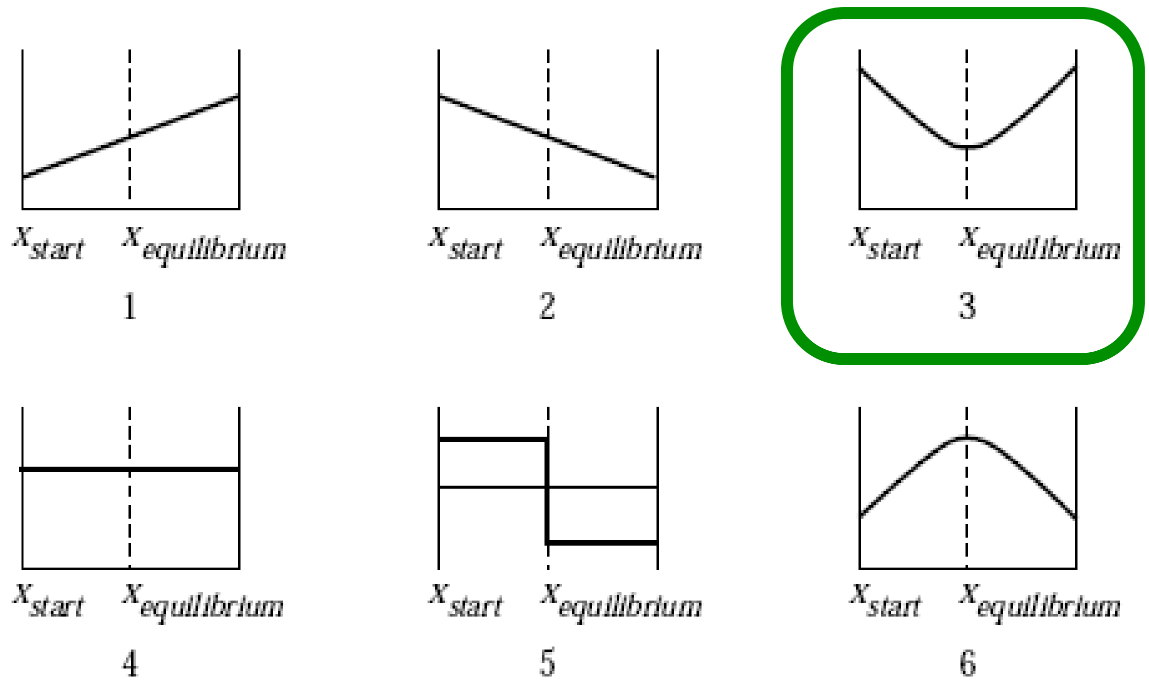




#3: $U = \frac{1}{2}kx^2$

We made it to here on Mon. Mar. 2^{nd}

PHYS 121 – SPRING 2014

Chapter 8: Potential Energy & Conservation of Energy

CONSERVATION OF ENERGY

If only conservative forces are doing work: $\Delta E = E_f - E_i = 0$

Total Energy is sum of kinetic and potential energies

$$E = K + U_g + U_s$$

Kinetic Energy $K = \frac{1}{2}mv^2$

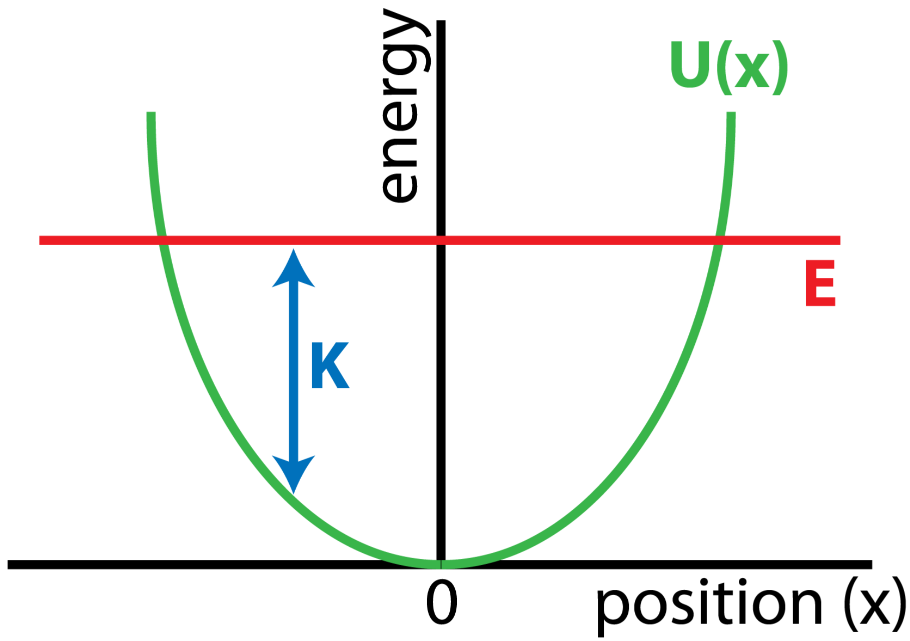
Gravitational PE $U_g = mgh$

Spring PE $U_S = \frac{1}{2}kx^2$

If non-conservative forces are doing work:

$$\Delta E = E_f - E_i = W_{by\,non-cons\ forces}$$

POTENTIAL ENERGY CURVES

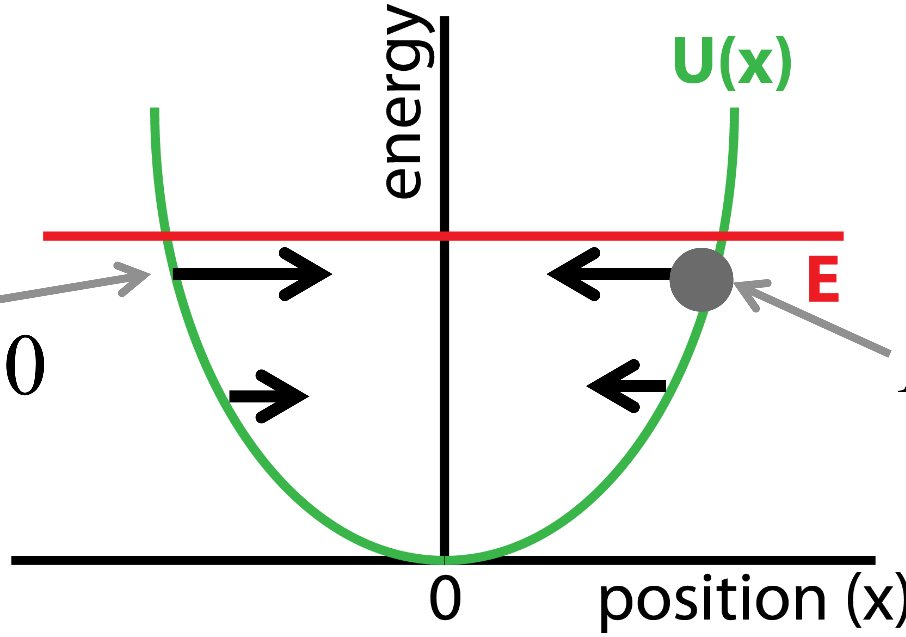


For a mass-spring system

- $U = \frac{1}{2}kx^2$ = a parabola opening upwards
- E = constant, the total energy of the system
- K(x) = E – U(x) = vertical distance between E and U curves at any given position.

<u>POTENTIAL ENERGY CURVES</u>

$$F_X = -\frac{dU}{dx} > 0$$



$$F_X = -\frac{dU}{dx} < 0$$

1. *U=E* positions are called <u>turning points</u>. K=0, object turns around.

picture the object bouncing between the two turning points.

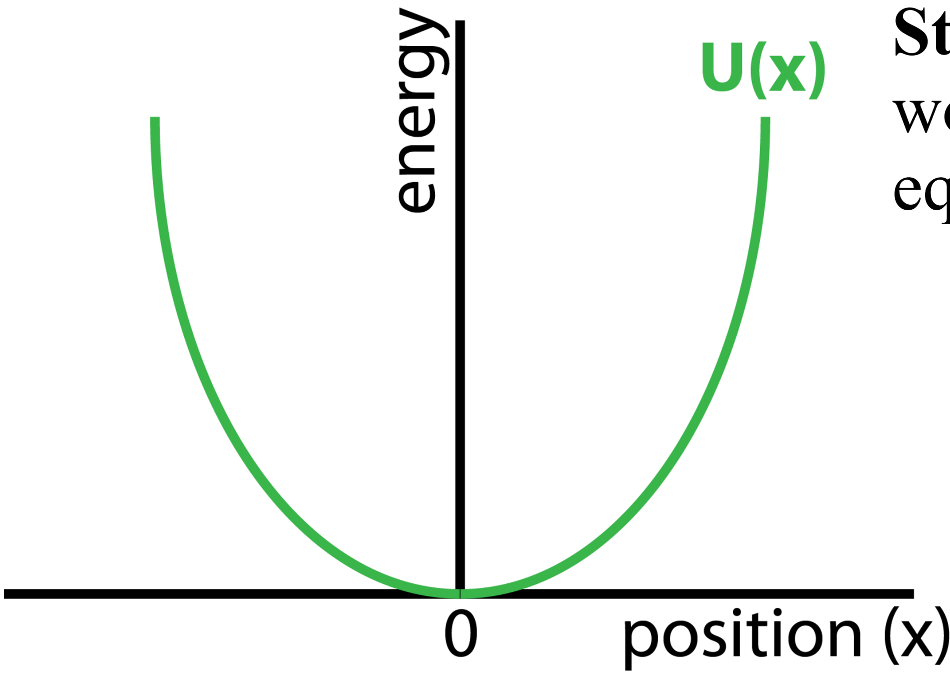
2. *U>E* positions are <u>forbidden regions</u>. Energy is too small.

3. Force is given by the derivative of U(x) curve.

4. If object is placed at the center initially, with no speed (E=0), it will stay there.

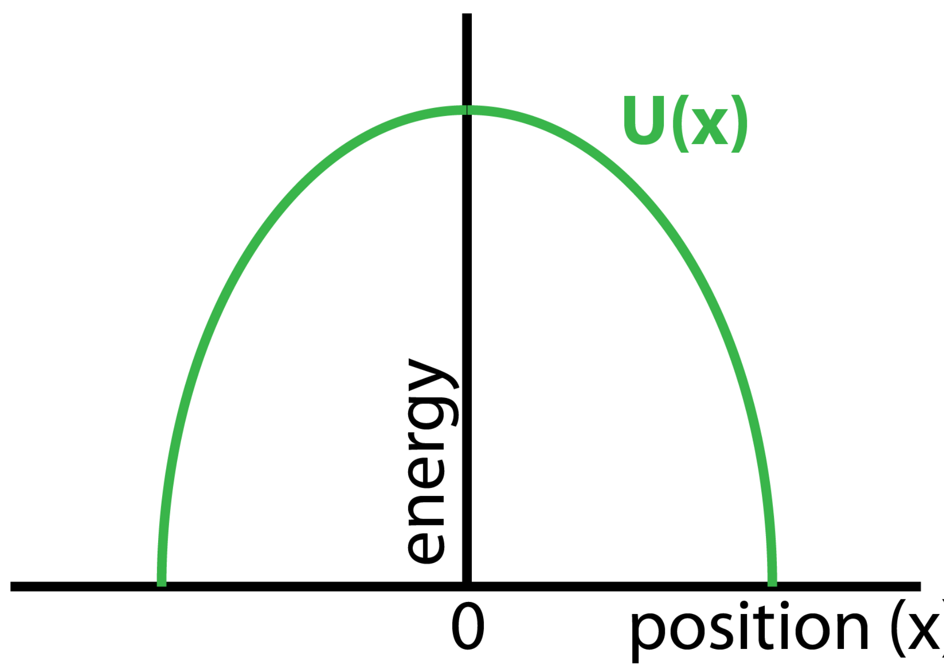
STABLE vs. UNSTABLE

Equilibrium: point on the PE curve where the slope is zero (*F=0*). If placed here, an object would remain here indefinitely.



Stable: if displaced a bit, object would tend to move back to equilibrium, or oscillate about it.

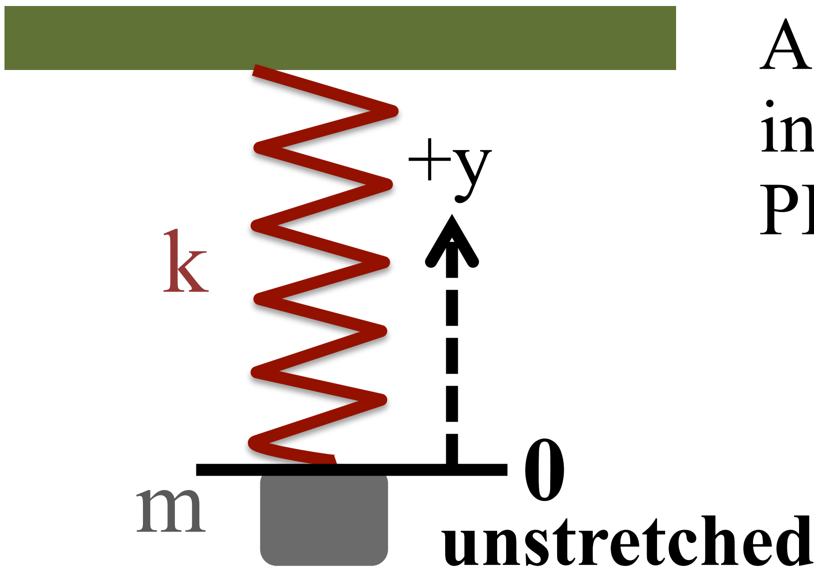
U(x) curves upward



Unstable: if displaced a bit, object would continue moving away from equilibrium.

U(x) curves downward

VERTICAL SPRING

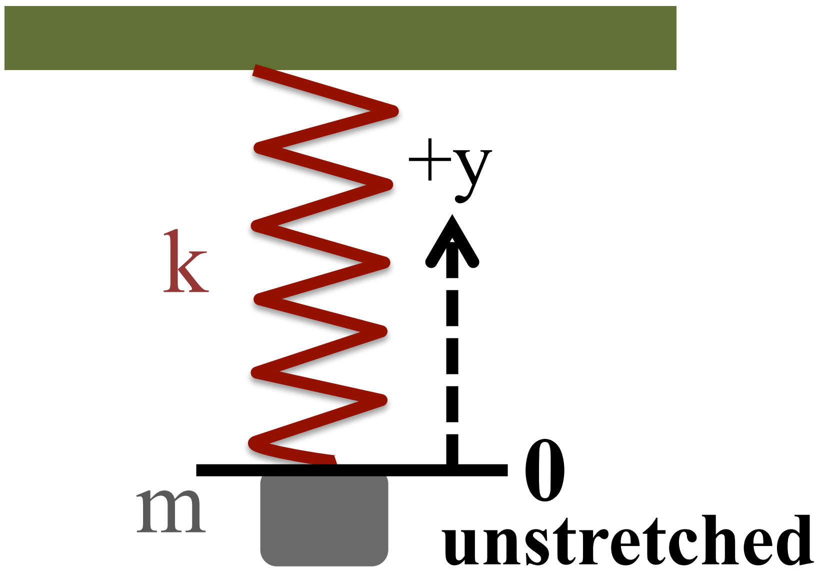


A mass is placed on a spring, originally in its unstretched position. What is the PE curve of the mass-spring system?

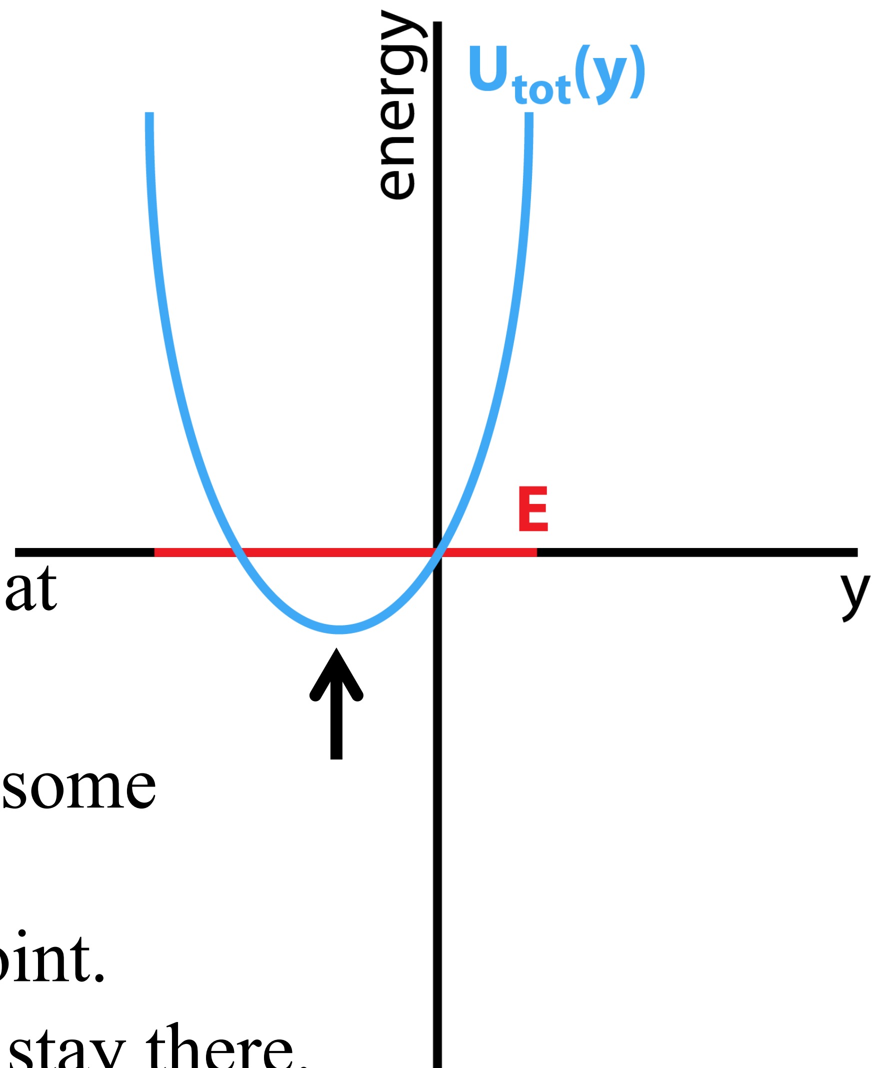
$$U(y) = U_s(y) + U_g(y)$$

$$U(y) = \frac{1}{2}ky^2 + mgy$$

VERTICAL SPRING



$$U(y) = \frac{1}{2}ky^2 + mgy$$



If we place the mass on the spring at y=0, then we start it with E=0.

There is an equilibrium position at some negative y-value.

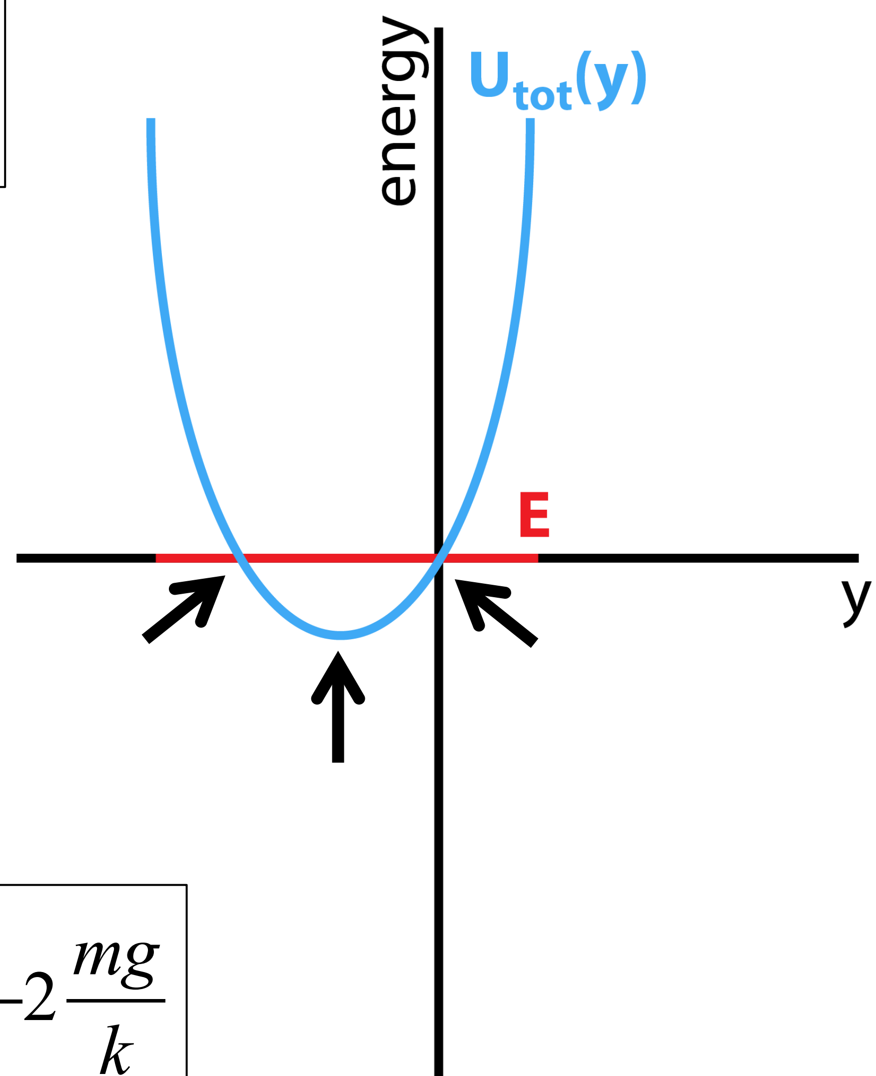
- It will oscillate around this point.
- If placed here initially, it will stay there.

VERTICAL SPRING

Where is equilibrium position?

$$U(y) = \frac{1}{2}ky^2 + mgy$$

$$\frac{dU}{dy} = 0 = ky + mg \qquad \boxed{y = -\frac{mg}{k}}$$



Where are turning points?

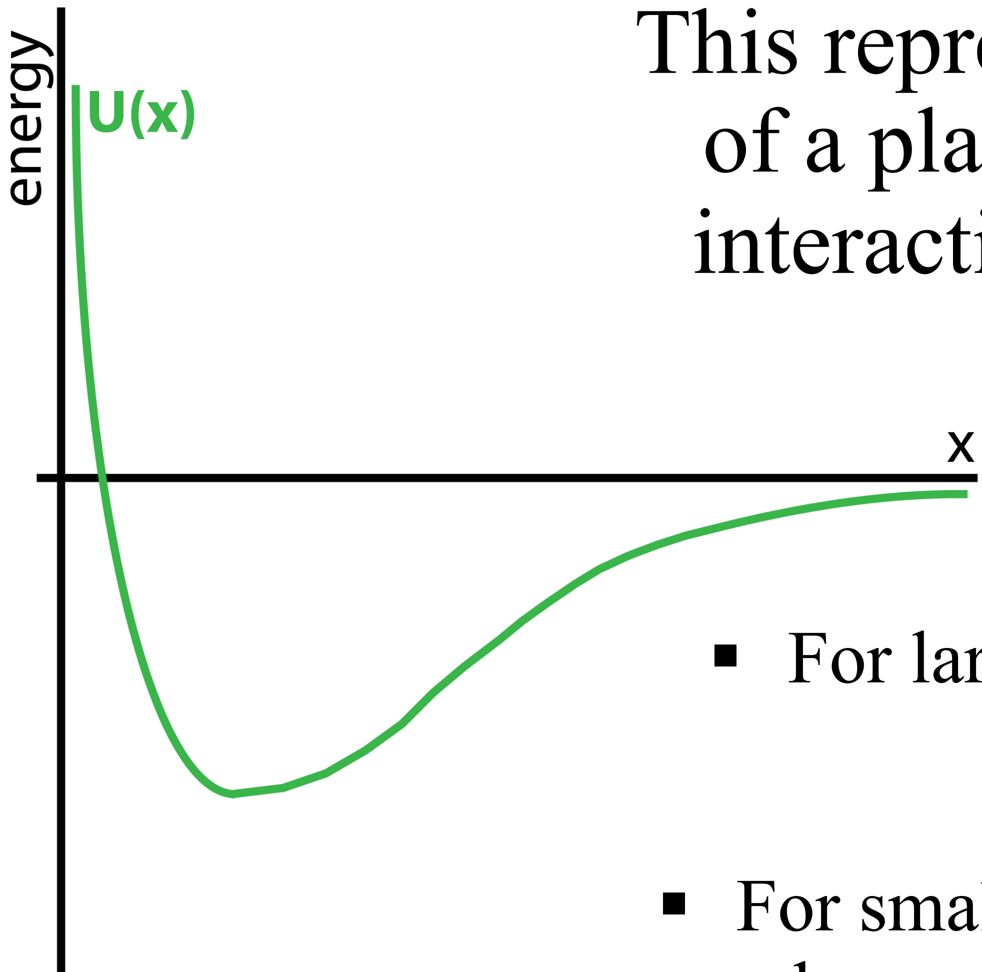
$$U(y) = E = 0 = \frac{1}{2}ky^2 + mgy$$

One solution is: $\boxed{y = 0}$

A second solution is given by:

$$0 = \frac{1}{2}ky + mg \qquad \boxed{y = -2\frac{mg}{k}}$$

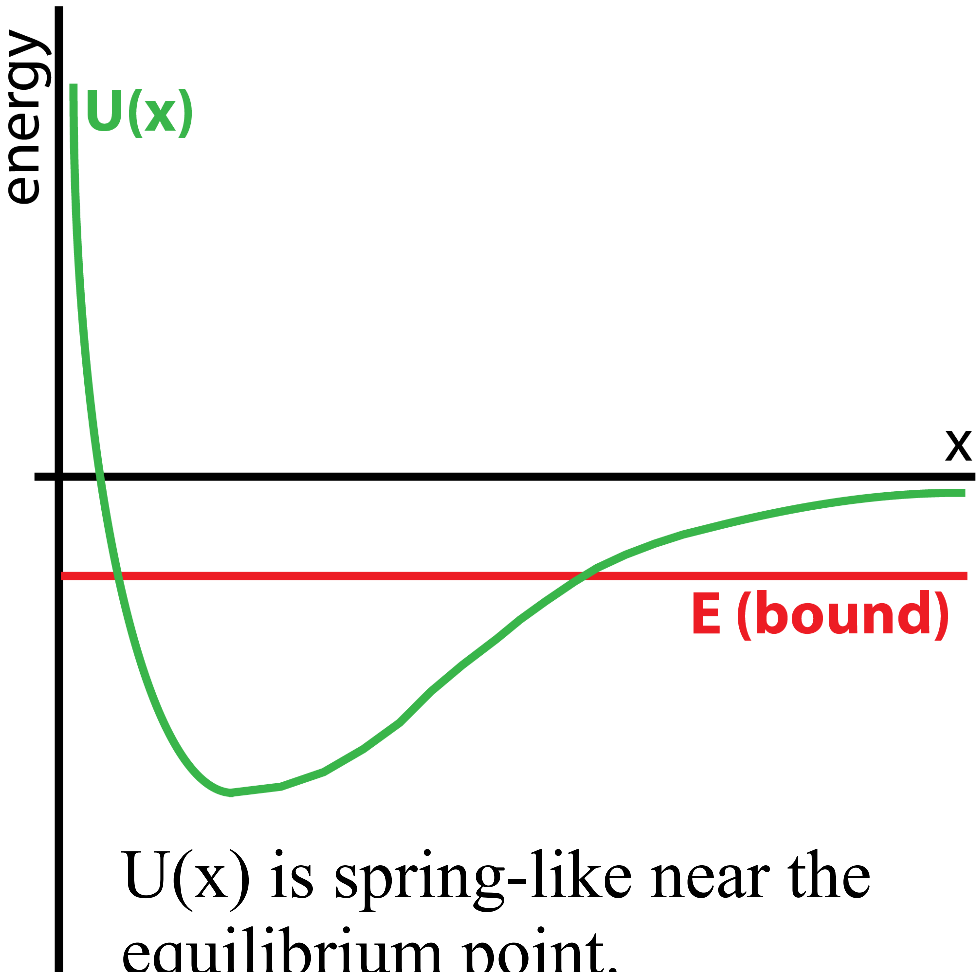
POTENTIAL ENERGY CURVES



This represents the potential energy of a planet orbiting a star, or the interaction of two neutral atoms.

- For large x, there is an attractive force pulling the particle in.
- For small x, there is a repulsive force that keeps the particle from reaching $x=0$.

POTENTIAL ENERGY CURVES



U(x) is spring-like near the equilibrium point.

Almost ALL systems with an equilibrium can be modeled as a spring!

1. Unbound State: E > 0

- Only 1 turning point exists near the origin.
- Particle can have non-zero KE at $x=\infty$.

2. Bound State: E < 0

- 2 turning points exist.
- Particle cannot escape to $x=\infty$.
- Bounces between the two turning points.

CULTURAL: TAYLOR EXPANSION

Any function near near $x=a$ can be written as:

$$f(x) = f(a) + \frac{1}{1!}\frac{df}{dx}(x-a) + \frac{1}{2!}\frac{d^2 f}{dx^2}(x-a)^2 + \frac{1}{3!}\frac{d^3 f}{dx^3}(x-a)^3 + \ldots$$

If $f(x)$ is the PE function, $U(x)$

The constant term corresponds to 0 force

The next term corresponds to a constant, non-zero force.

The third term corresponds to a spring-like force.

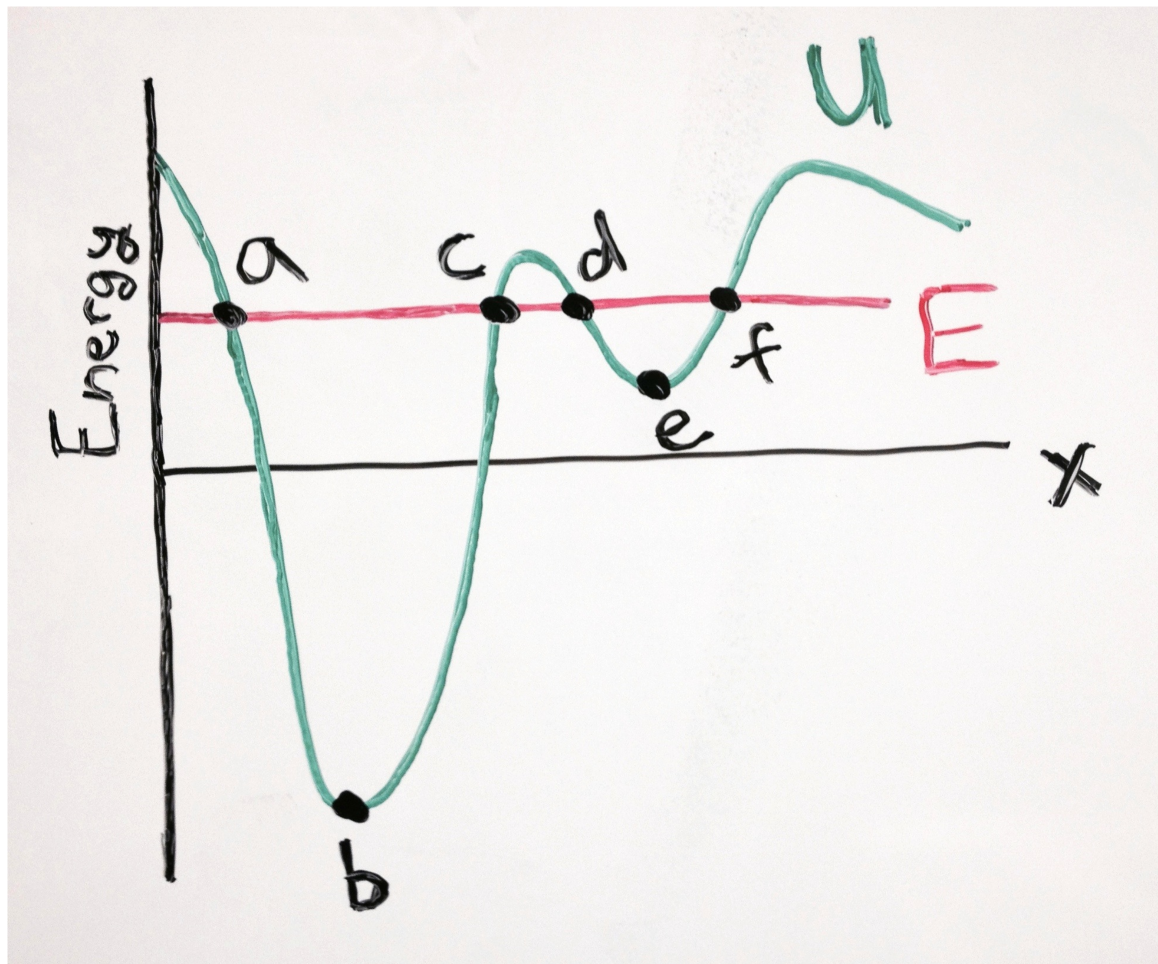
Near an equilibrium point, the first derivative will be approximately zero. So the spring-force dominates.

For more information, see http://mathworld.wolfram.com/TaylorSeries.html or http://en.wikipedia.org/wiki/Taylor_series

6 options

The potential energy diagram for a particle is shown in the figure. It starts at position “a” with zero velocity. At what position(s) in its motion is the kinetic energy zero?

A. b
B. e
C. a & c
D. d & f
E. a, c, d, & f
F. b & e



6 options

The potential energy diagram for a particle is shown in the figure. It starts at position “a” with zero velocity. At what position(s) in its motion is the kinetic energy zero?

A. b

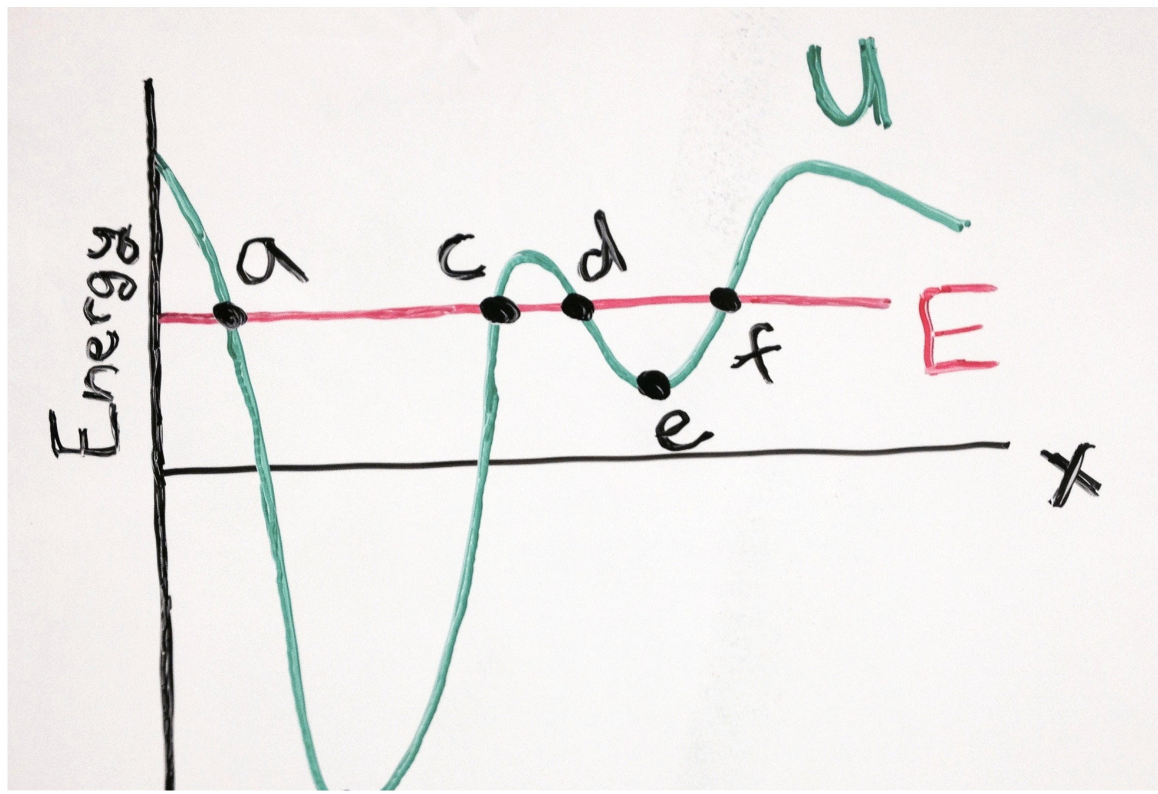
B. e

C. $\boxed{\text{a \& c}}$

D. d & f

E. a, c, d, & f

F. b & e



K=0 when U=E. This happens at “a” and “c”. It would also happen at “d” and “f” but the particle never reaches these positions.

http://mexicotoday.org/article/mexico-wind-power-sets-new-generation-record

POWER

http://heartfeltworkforce.com/wp-content/uploads/2012/11/sun-power.jpg

Power is rate at which work is done.

$$\boxed{P = \frac{dW}{dt}}$$

If the rate does not change over time, then

$$P = \frac{\Delta W}{\Delta t}$$

Units: $\dfrac{\text{Joules}}{\text{second}} = \text{Watt (W)}$

http://mexicotoday.org/article/mexico-wind-power-sets-new-generation-record

POWER

http://heartfeltworkforce.com/wp-content/uploads/2012/11/sun-power.jpg

$$\boxed{P = \frac{dW}{dt}}$$

The work done by a force over a small displacement $d\vec{r}$ is

$$dW = \vec{F} \cdot d\vec{r}$$

$$\Rightarrow P = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \boxed{\vec{F} \cdot \vec{v}}$$

This is the power delivered by the force at any instant in time.

ELEVATOR POWER

A elevator car has a mass *m=1500kg,* and the engineer wants it to move upward at a constant speed of *v=3m/s*. While moving, the elevator experiences a constant friction force of *f=4000N*. What is the power of the motor required?

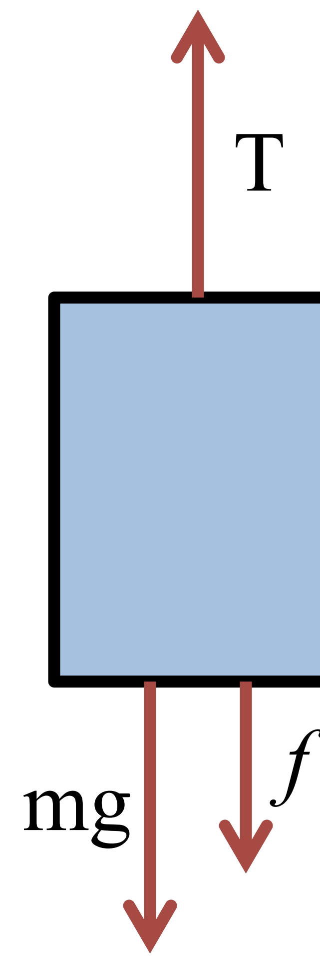
Power supplied by motor: $P = \vec{T} \cdot \vec{v} \; = Tv$

Need to find tension:

$$\Sigma F = T - mg - f \; = 0$$

$$\Rightarrow T = mg + f \; = 18.7 \times 10^3 N$$

$$P = (18.7 \times 10^3 N)(3m/s) = \boxed{56kW}$$



POWER ↔ ENERGY CONFUSION

Energy is measured in *Joules*

Power is measured in *Watts*

Power plants, cars, motors, horses, etc. usually have a specific power, but could supply an any amount of energy (if they always have fuel and never break down).

- Human ~ 100W for many minutes
- Car ~ 100-300 horsepower (1hp = 746W)
- Veale Wind Turbine ~ 100kW maximum
- National Ignition Facility ~ 500 x 10^{12} W (for 10^{-12} sec)

Electrical energy sold in units of kW-hours (a unit of energy), and a single AC outlet delivers this energy at a rate of 1-2kW.

NYT review of 2014 Mazda 6, which stores energy from braking in a capacitor:
"The stored juice, roughly 2,000 watts, then powers the electric system and accessories." (Apr. 21, 2014)

This is impossible! You can't store power, only energy.

6 options

A 1966 AC Cobra 427 can accelerate from 0 to 30 mph in 1.5 s. How long would it take for this car to accelerate from zero to 60 mph, assuming the power of the engine is independent of velocity and friction/air resistance is not a factor?

A. 2 s
B. 3 s
C. 4.5 s
D. 6 s
E. 9 s
F. 12 s

http://upload.wikimedia.org/wikipedia/commons/8/81/AC_Shelby_Cobra_(Auto_classique).JPG

6 options

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A. 2 s

B. 3 s

C. 4.5 s

D. 6 s

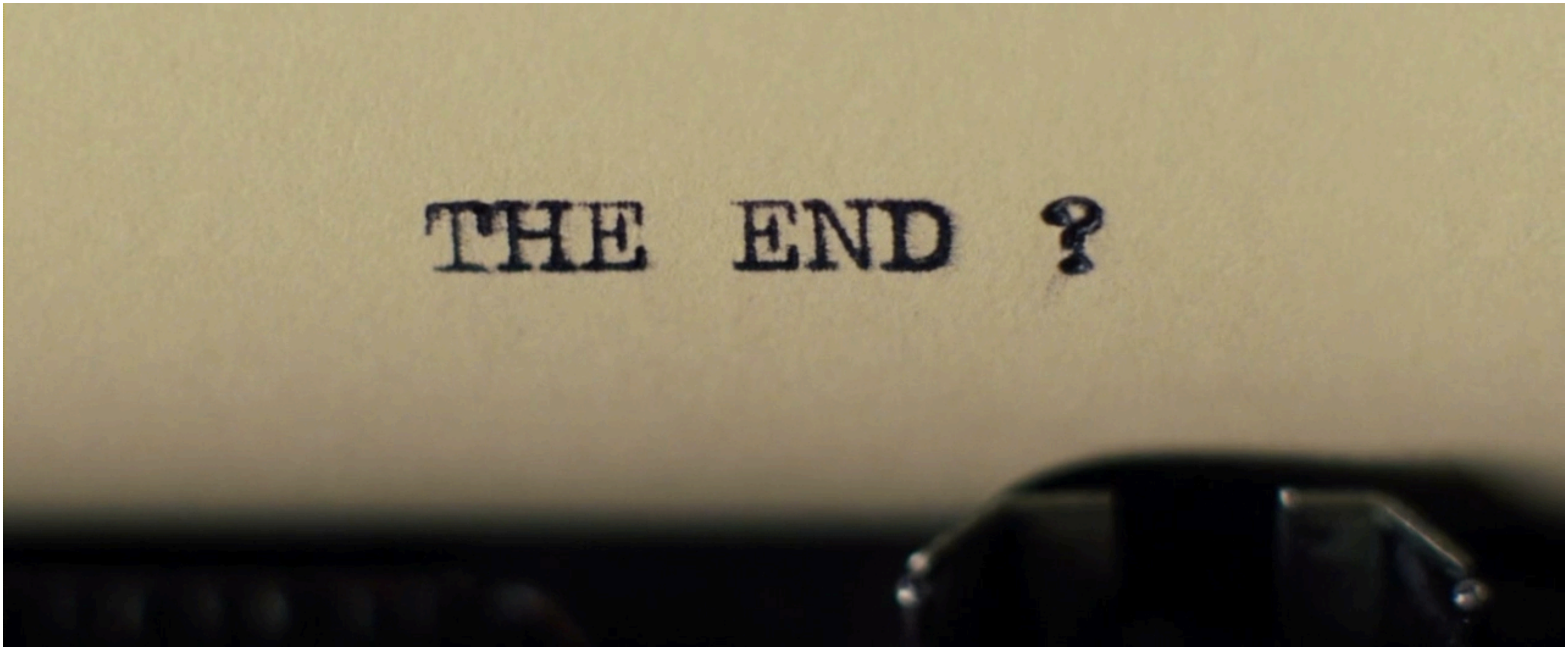
E. 9 s

F. 12 s

Power is constant: $$P = \frac{\Delta W}{\Delta t} \Rightarrow \Delta t = \frac{\Delta W}{P}$$

Work is from change in KE: $$\Delta t = \frac{\Delta KE}{P}$$

If final speed increases 2x, ΔKE increases 4x.
If power is the same, then Δt will increase 4x.
(1.5s x 4 = 6s)



Sherlock Holmes: A Game of Shadowsn