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EECS 340 - Algorithms
Assignment 4

22.2-7

Definition:
We have a graph G(V,E) such that each vertex represents a wrestler and each edge represents a rivalry. The graph will contain n vertices because we have n wrestlers, $|V| = n$, and will contain r edges because we have r pairs of wrestlers for which there are rivalries, $|E| = r$.

Algorithm:
1. Continue a series of BFS's until all vertices are visited.
2. We shall arbitrarily assign wrestlers of even distances to be "babyfaces" and those of odd distances to be "heels".
3. Then analyze each edge. Edges between two vertices whose path lengths are even are a rivalry between two "babyfaces". Edges between two vertices whose path lengths are odd are a rivalry between two "heels". Edges between two vertices whose path lengths are one even and one odd are a rivalry between both a "babyface" and a "heel".

Run time Analysis:
A BFS to visit all vertices will take O(V + E) time = O(n + r) time. It will take O(n) time to designate a wrestler as a "babyface" or "heel". It will take O(r) time to analyze each edge. The overall time to perform this algorithm will then be O(n + r) time.

22.3-5

A
The edge representing the connection between u and v is a tree edge if and only if v is a descendant of u in the DFS. The same edge would be a back edge if the opposite were true, where u is a descendant of v, and could potentially be a cross edge if u and v were disjoint. However, as we know from the corollary, u.d < v.d < v.f < u.f, which implies that v is a proper descendant of u as u which has not yet finished. Because v is a descendant of u and a descendant of u in the DFS, the edge (u,v) is a tree edge or forward edge if and only if u.d < v.d < v.f < u.f.

B

If v is an ancestor of u, then edge (u,v) is a back edge. The DFS will need to discover v first, and then discover u before v has finished. Logically, u would then finish before v by the parenthesis theorem. The coordinating behavior that the DFS would need to produce is such that v.d < u.d < u.f < v.f which nearly matches our corollary. The remainder comes from the possibility that u and v are the same vertex. If they are, then u and v are discovered at the same time and finish at the same time. However, the vertex discovered last must be finished first by the parenthesis theorem. Therefore we have the criteria for a back edge to happen if and only if v.d ≤ u.d < u.f ≤ v.f.

C

If v is neither an ancestor or descendant of u, then edge (u, v) is a cross edge. By the parenthesis theorem, the intervals [u.d, u.f] and [v.d, v.f] are entirely disjoint. Thus, the DFS will need to produce a behavior such that u.d < u.f < v.d < v.f or v.d < v.f < u.d < u.f. We can not have the first of these scenarios where u.d < v.d. At the time u is discovered, v will still be white. By the white path theorem, v is then descendant of u, which is not in accordance to being a cross edge. We are only left with the scenario of v.d < v.f < u.d < u.f where, by the parenthesis theorem, neither u nor v is a descendant of the other, which means that edge (u,v) must be a cross edge.

22.3-8

Imagine the following graph: $v \leftarrow S \rightleftharpoons u$

If S is our starting point, then we may very well get the following discovery and finishing times from a depth first search.

Vertex	Discovered	Finish
S	1	6
u	2	3
v	4	5

There is a path in the graph from u to v and u.d = 2 < v.d = 4. However, vertex u finishes before vertex v is discovered, so vertex v is not a descendant of u and we obtain a valid contradiction.