

Question 1

A) Keys: $\{C, D, M, P, S, T\}$

Candidate Keys: CM and DM

M only appears in the left side of our FD's, therefore it must be a part of our candidate key. However, $M^+ = \{M\}$, so we need to pair M with C or D. Doing so we get the entire set of keys $\{C, D, M, P, S, T\}$.

B) Use DM as candidate key and put into canonical form

$C \rightarrow D$: Redundant

BCNF

$C \rightarrow S$: Not BCNF

$R1 = \{\underline{C}, S\}$

$C \rightarrow S$

$DM \rightarrow P$: BCNF

$R2 = \{\underline{D}, \underline{M}, P, C\}$

$DM \rightarrow PC$

$DM \rightarrow C$: BCNF

$R3 = \{\underline{D}, T\}$

$D \rightarrow T$

$D \rightarrow T$: not BCNF

This does not preserve all dependencies because $C \rightarrow D$ is lost.

C) Use DM as candidate key and put into canonical form

$C \rightarrow D$

3NF

$C \rightarrow S$

$R1 = \{\underline{C}, S, D\}$

$DM \rightarrow P$

$R2 = \{\underline{D}, \underline{M}, P, C\}$

$DM \rightarrow C$

$R3 = \{\underline{D}, T\}$

$D \rightarrow T$

Question 2

Aa) $\{ACE\}^+ = \{A, B, C, D, E\}$

Ab) R_1 is NOT in 3NF. The FD $A \rightarrow B$ isn't trivial. A isn't a superkey of R_1 , and B isn't part of a candidate key for R_1 .

Ac) R_1 is NOT in BCNF because it isn't in 3NF.

A BCNF decomposition:

R1a: $\{\underline{A}, B\}$ $A \rightarrow B$

R1b: $\{\underline{C}, D\}$ $C \rightarrow D$

R1c: $\{A, C, E\}$ Included to ensure lossless join and avoid losing E entirely.

Ba) $\{AB\}^+ = \{A, B, F\}$

Bb) R_2 is NOT in 3NF. The FD $B \rightarrow F$ isn't trivial. B isn't a superkey of R_2 , and F isn't part of a candidate key for R_2 .

Bc) R_2 is NOT in BCNF because it isn't in 3NF.

A BCNF decomposition:

R2a: $\{\underline{B}, F\}$ $B \rightarrow F$

R2b: $\{A, B\}$ Included to ensure lossless join and avoid losing A entirely.

Note that $AB \rightarrow F$ isn't specifically included because it is trivially derived from $B \rightarrow F$.

Ca) For R_3 $\{A\}^+ = \{A, B\}$; For R_4 $\{C\}^+ = \{C, D, E, F\}$; $\{AC\}^+ = \{A, B, C, D, E, F\}$

Cb) R_3 is in 3NF. For the FD $A \rightarrow B$, A is a superkey of R_3 . R_4 is NOT in 3NF. The FD $D \rightarrow EF$ isn't a trivial FD, and D isn't a superkey of R_4 , and neither E nor F are part of a superkey in R_4 .

Cc) R_3 is in BCNF since A is a R_3 superkey. R_4 is NOT in BCNF because it isn't in 3NF. A BCNF decomposition:

R_3 : $\{\underline{A}, B\}$ $A \rightarrow B$

R_{4a} : $\{\underline{C}, D\}$ $C \rightarrow D$

R_{4b} : $\{D, E, F\}$ $D \rightarrow EF$

Da) $\{CDE\}^+ = \{A, B, C, D, E\}$

Db) R_5 is NOT in 3NF. The FD $D \rightarrow B$ is not trivial and D isn't a superkey of R_5 , and B isn't part of a key in R_5 .

Dc) R_5 is NOT in BCNF because it isn't in 3NF.

A BCNF decomposition:

R_{5a} : $\{B, \underline{D}\}$ $D \rightarrow B$

R_{5b} : $\{A, \underline{C}, \underline{E}\}$ $CE \rightarrow A$

Ea) $\{ACD\}^+ = \{A, B, C, D, E\}$; $\{BCD\}^+ = \{A, B, C, D, E\}$; $\{CDE\}^+ = \{A, B, C, D, E\}$

Eb) R_6 is in 3NF. For every non-trivial FD, there is a candidate key that includes the "Y" part of $X \rightarrow Y$. For example, in $A \rightarrow E$, CDE is a candidate key, for $BC \rightarrow A$, ACD is candidate key, and for $DE \rightarrow B$, BCD is a candidate key).

Ec) R_6 is NOT in BCNF. Example, $A \rightarrow E$ isn't a trivial FD, and A isn't a superkey of R_6 .

A BCNF decomposition:

R_{6a} : $\{\underline{A}, E\}$ $A \rightarrow E$

R_{6b} : $\{A, \underline{B}, \underline{C}\}$ $BC \rightarrow A$

R_{6c} : $\{B, \underline{D}, \underline{E}\}$ $DE \rightarrow B$

Question 3

A) Attribute closure of $AB^+ = \{ABCDEF\}$

$AB \rightarrow A$

$A \rightarrow D$

$D \rightarrow C$

$D \rightarrow F$

$A \rightarrow E$

$AB \rightarrow B$

B) Lossless: After doing table of decomposition (Chase Algorithm), relation ABC has a row filled with distinguished variables, therefore lossless. Also, $R_1 \cap R_3 \rightarrow R_3$.

Dependency Preserving: Not dependency preserving, for example lost $A \rightarrow D$.

Answer: This relation is lossless, not dependency preserving.

C) Lossless: After doing table of decomposition (Chase Algorithm), relation ABCE has a row filled with distinguished variables, therefore lossless. Also, $R_1 \cap R_2 \rightarrow R_2$.

Dependency Preserving: All FD's can be found in the given relations.

Answer: This relation is lossless and dependency preserving.

Question 4

A) The given set of FD's do not provide a minimum cover because there are a number of redundant variables on the RHS of the FD's. One minimal cover is:

$A \rightarrow B$ $B \rightarrow C$ $AD \rightarrow E$ $BE \rightarrow F$ $D \rightarrow G$

B) F^+ is the set of all FD's entailed by F. So $AD \rightarrow BCEFG$ and all of the FD's implied by this FD is the collection of FD's that make up F^+ . The implied FD's include $AD \rightarrow A$, $AD \rightarrow D$, $AD \rightarrow AD$, $AD \rightarrow B$, $AD \rightarrow BC$, and so on. This results in over 2400 FD's.

C) $\{B\}^+ = \{B \text{ (trivial), } C \text{ (FD 2)}\}$

$\{G\}^+ = \{G \text{ (trivial)}\}$

$\{AD\}^+ = \{A \text{ (trivial), } D \text{ (trivial), } B \text{ (FD 1), } C \text{ (FD 3), } E \text{ (FD 3), } G \text{ (FD 3)}\}$