EECS 281, January 29, 2015

Idempotency:

T12: $X + X + X + \dots + X = X$

T12': $X \cdot X \cdot \dots \cdot X = X$

De Morgan's Theorem:

T13: $(X_1 \cdot X_2 \cdot \dots X_n)' =$

$X_1' + X_2' + \dots + X_n'$

T13': $(X_1 + X_2 + \dots + X_n)' =$

$X_1' \cdot X_2' \cdot \dots \cdot X_n'$

T14: $[F(X_1, X_2, \dots, X_n, +, \cdot)]' =$

$F(X_1', X_2', \dots, X_n', \cdot, +)$

Shannon's expansion theorems:

T15: $F(X_1, \ldots, X_n) =$

$$X_1 \cdot F(1, X_2, \ldots, X_n) +$$
$$X_1' \cdot F(0, X_2, \ldots, X_n)$$

T15': $F(X_1, \ldots, X_n) = [X_1 + F(0, X_2, \ldots X_n)] \cdot$
$$[X_1' + F(1, X_2, \ldots X_n)]$$

Proof using finite induction:

T12: $\underbrace{X + X + \ldots + X}_{n} = X$

First prove for $n = 2$.

Then prove: if theorem is true for $n = i$

then prove that it is true for $n = i + 1$

$n=2$ $\quad X + X = X$ (From T3: $X+X=X$)

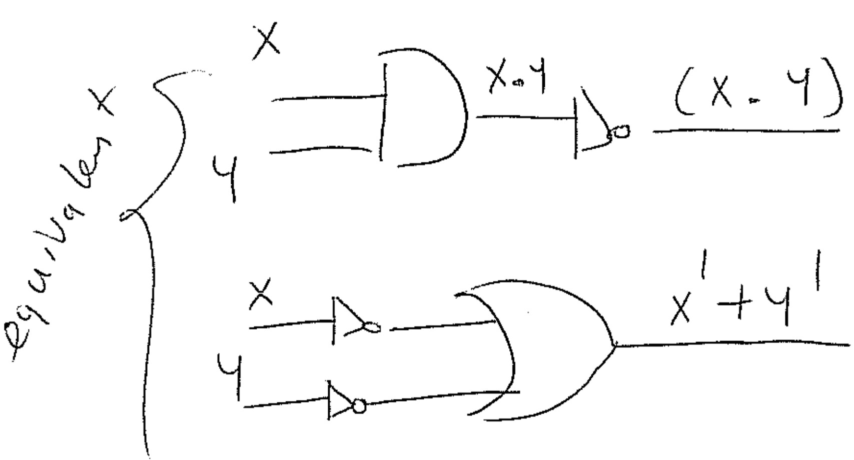
$n = i+1$

$$\underbrace{X + X + \ldots + X}_{i+1} = X \ ?$$

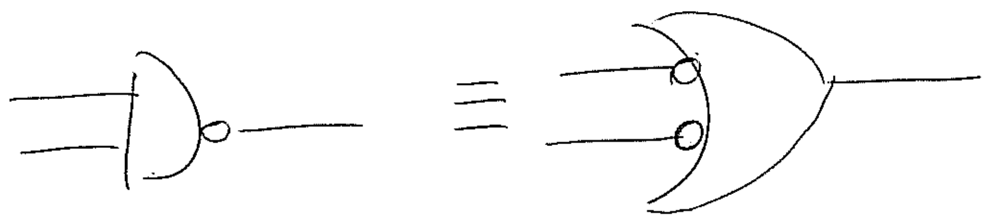
$$X + \underbrace{\underbrace{(X \ldots + X)}_{i}}_{=X} = X + X = X$$

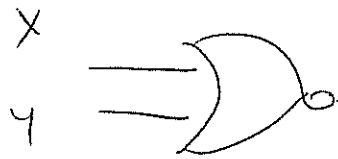
From T3

$\Rightarrow$ T12 true for all finite values of n.



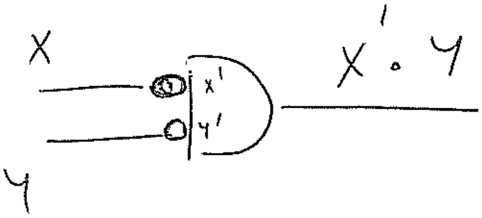
$(x \cdot y)' = x' + y'$





$(X+Y)' = X' \cdot Y'$

$\equiv$



$X' \cdot Y'$

Duality:

Any theorem or identity in switching algebra remains true if 0 and 1 are swapped and $\cdot$ and $+$ are swapped throughout.

When taking the dual be careful about the precedence.

e.g. $X + X \cdot Y = X$ $\neq$

$\cancel{X \cdot X + Y = X + Y}$

Row	X	Y	Z	F
0	0	0	0	F(0,0,0)
1	0	0	1	F(0,0,1)
2	0	1	0	F(0,1,0)
3	0	1	1	F(0,1,1)
4	1	0	0	F(1,0,0)
5	1	0	1	F(1,0,1)
6	1	1	0	F(1,1,0)
7	1	1	1	F(1,1,1)

Table 4-4

General truth table structure for a 3-variable logic function, F(X,Y, Z).

$$X + (X \cdot Y) = X$$

$$X \cdot (X + Y) = X \checkmark$$

Standard Representation of logic Functions:

Truth Table:

for n-variables $\longrightarrow$ 2^n rows in truth table (all possible input combinations)

we can turn the information in a truth table into algebraic expressions.

Definitions:

Literal: variable or complement of a variable: e.g. x, x', y'

Product term: single literal or logical product of two or more literals.
e.g. X', $W \cdot Y'$, $X \cdot Y \cdot Z \cdot W'$

Sum-of-products expression: logical sum of product terms.

e.g. $z' + w' \cdot x \cdot y + y' \cdot w$

Sum term: single literal or logical sum of two or more literals.

e.g. w, $x + y' + w$

Product-of-sums expression: logical product of sum terms.

e.g. $x' \cdot (w + x + y)$

Normal term: product or sum term in which no variable appears more than once.

e.g. $w \cdot x \cdot x \cdot y' \longrightarrow w \cdot x \cdot y'$

nonnormal

n-variable minterm: normal product term with n literals. There are 2^n such product terms.

e.g. $w' \cdot x \cdot y \cdot z'$.

n-variable maxterm: normal sum term with <u>n literals</u>. There are 2^n such terms.

e.g. $w' + x' + y + z'$

Correspondence between truth table and minterms/maxterms:

minterm: product term that is 1 in exactly one row.

maxterm: sum term that is 0 in exactly one row.

Row	X	Y	Z	F	minterms	maxterms
0	0	0	0	F(0,0,0)	$X' \cdot Y' \cdot Z'$	$X + Y + Z$
1	0	0	1	'	$X' \cdot Y' \cdot Z$	$X + Y + Z'$
2	0	1	0	'	$X' \cdot Y \cdot Z'$	$X + Y' + Z$
3	0	1	1	'	$X' \cdot Y \cdot Z$	$X + Y' + Z'$
4	1	0	0	'	$X \cdot Y' \cdot Z'$	$X' + Y + Z$
5	1	0	1	(	$X \cdot Y' \cdot Z$	$X' + Y + Z'$
6	1	1	0	(	$X \cdot Y \cdot Z'$	$X' + Y' + Z$
7	1	1	1	'	$X \cdot Y \cdot Z$	$X' + Y' + Z'$

Canonical sum: of a logic function is the sum of minterms corresponding to truth-table rows for which the function produces a 1 output.

Example:

Rows	X	Y	Z	F	minterms
0	0	0	0	1	$X'.Y'.Z'$
1	0	0	1	0	
2	0	1	0	0	
3	0	1	1	1	$X'\cdot Y\cdot Z$
4	1	0	0	1	$X\cdot Y'\cdot Z'$
5	1	0	1	0	
6	1	1	0	1	$X\cdot Y\cdot Z'$
7	1	1	1	1	$X\cdot Y\cdot Z$

$$F = \sum_{X,Y,Z}^{\text{Row}} (0, 3, \underline{4}, 6, 7)$$

$X = 1$
$Y = 0$
$Z = 0$

$$F = X'\cdot Y'\cdot Z' + X'\cdot Y\cdot Z + X\cdot Y'\cdot Z' + X\cdot Y\cdot Z' + X\cdot Y\cdot Z$$

Example: simplify to $B' + A \cdot C'$

$$F = \underbrace{A' \cdot B' \cdot C' + A' \cdot B' \cdot C} + A \cdot B \cdot C' + \underbrace{A \cdot B' \cdot C' + A \cdot B' \cdot C} + A \cdot B' \cdot C'$$

$$= A' \cdot B' \cdot \underbrace{(C' + C)}_{1} + A \cdot B' \underbrace{(C' + C)}_{1} + \underbrace{A \cdot B \cdot C' + A \cdot B' \cdot C'}_{A \cdot C' \underbrace{(B + B')}_{1}}$$

$$= \underbrace{A' \cdot B' + A \cdot B'}_{B' \underbrace{(A' + A)}_{1}} + \underbrace{A \cdot C'}_{+ A \cdot C'} = B' + A \cdot C'$$

Example:

$B'C' + AB'$

$$F = A'B'C' + AB'C' + AB'C + AB'$$

$$= \underbrace{(A' + A)}_{1} \cdot B'C' + \cancel{AB'C} AB'\underbrace{(C + C')}_{1}$$

$$= B'C' + AB'$$