

Example big-O theorem: Let $f(n) = O(g(n))$ and $t(n) = O(s(n))$, where all functions are positive monotonically growing functions. Prove that $f(n) * t(n) = O(g(n) * s(n))$.

Let c_1 and n_1 be the constants in the big-O definition for $f(n) = O(g(n))$, and c_2 and n_2 be these constants for $t(n) = O(s(n))$. In other words, for any $n > n_1$, $f(n) \leq c_1 * g(n)$ and for any $n > n_2$, $t(n) \leq c_2 * s(n)$. Consider a constant $n_m = \max(n_1, n_2)$. For any $n > n_m$, both above inequalities hold. Therefore, since all functions are positive, for any $n > n_m$ $f(n) * t(n) \leq c_1 * c_2 * g(n) * s(n)$. In other words, we found constants $c_m = c_1 * c_2$ and $n_m = \max(n_1, n_2)$ such that for all $n > n_m$, $f(n) * t(n) \leq c_m * g(n) * s(n)$. Then by the definition of big-O, $f(n) * t(n) = O(g(n) * s(n))$.