CONTENTS

Chapter 1

BASIC NOTIONS OF LOGIC

1.1 BACKGROUND

This is a text in deductive logic—more specifically, in symbolic deductive logic. Chapters 1–5 are devoted to sentential logic, that branch of symbolic deductive logic that takes sentences as the fundamental units of logical analysis. Chapters 7–10 are devoted to predicate logic, that branch of symbolic deductive logic that takes predicates and individual terms as the fundamental units of logical analysis. Chapter 6 is devoted to the metatheory of sentential logic; Chapter 11, to the metatheory of predicate logic.

In the following chapters we will explore sentential and predicate logic in considerable detail. Here, we try to place that material in a larger context. Historically two overlapping concerns have driven research in deductive logic and the development of specific formal systems of deductive logic: the desire to formulate canons or principles of good reasoning in everyday life, as well as in science and mathematics, and the desire to formalize and systematize existing and emerging work in mathematics and science. Common to these concerns is the view that what distinguishes good reasoning from bad reasoning, and what makes good deductive reasoning "logical" as opposed to "illogical", is truth preservation.

A method or pattern of reasoning is truth-preserving if it never takes one from truths to a falsehood. The hallmark of good deductive reasoning is that it is truth-preserving. If one starts from truths and uses good deductive reasoning, the results one arrives at will also be true. Because we are all interested, as students and scholars, in everyday life and in our careers, in gaining truths and avoiding falsehoods, we all have reason to be interested in reasoning that is truth-preserving.

Most of the deductive systems of reasoning that have been developed for geometry, mathematics, and selected areas of science have been axiomatic systems. And most of us are familiar with at least one axiomatic system—that of Euclidean plane geometry. Euclid, a Greek scholar of the third century B.C., may have been the first person to develop a reasonably complete axiomatic system. Axiomatic systems start with a relatively small number of basic principles, referred to variously as axioms, definitions, postulates, and assumptions, and provide a way of deducing or deriving from them the rest of the claims or assertions of the discipline being axiomatized (in Euclid's case plane geometry). If the starting principles are significantly altered, a new theory may emerge. For example, when Euclid's fifth postulate (the parallel postulate) is modified, theorems of non-Euclidean geometry can be deduced.

Through the centuries scholars have attempted to produce axiomatic systems for a wide variety of disciplines, ranging from plane and solid geometry, to arithmetic (which was successfully axiomatized by Giuseppe Peano in 1889), to parts of the natural and social sciences. Since successful axiomatic systems use only rules of reasoning that are truth-preserving, that never take one from truths to a falsehood, the advantage of successfully axiomatizing a body of knowledge is that it makes all the claims of that body of knowledge as certain as are the starting principles and the rules of reasoning used.

At about the same time that Euclid was developing his axiomatic treatment of plane geometry, another Greek scholar, Aristotle (384–322 B.C.), was developing a general system of logic intended to incorporate the basic principles of good reasoning and to provide a way of evaluating specific cases of reasoning. The system Aristotle produced is variously known as syllogistic, traditional, or Aristotelian logic. Predecessors of Aristotle, in the Greek world and elsewhere, were interested in reasoning well—in offering cogent arguments for their theses and theories and in identifying flaws and fallacies in their own and others' reasoning. But Aristotle was apparently the first person in the Western world to offer at least the outlines of a comprehensive system for codifying and evaluating a wide range of arguments and reasoning.

The following is an argument that has the form of an Aristotelian syllogism:

All mammals are vertebrates.
Some sea creatures are mammals.
___
Some sea creatures are vertebrates.

The horizontal line separates the two premises of this syllogistic argument from the conclusion. This syllogism is an example of good reasoning—it constitutes a good argument—because it is truth-preserving. If the first two sentences (the premises) of the syllogism are true, the third sentence (the conclusion) must also be true. Aristotle's achievement was not in identifying this particular argument about vertebrates, mammals, and sea creatures as a good or truth-preserving argument, but rather in providing an explanation of why this and all reasoning of this form are instances of good reasoning. Aristotle would classify the preceding syllogism as being of the form

All As are Bs.
Some Cs are As.
___
Some Cs are Bs.

And this form or schema produces truth-preserving reasoning whenever 'A', 'B', and 'C' are uniformly replaced by general terms, as in

All cardiologists are wealthy individuals.
Some doctors are cardiologists.
___
Some doctors are wealthy individuals.

Aristotelian logic is a variety of deductive symbolic logic. It is symbolic because it analyzes reasoning by identifying the form or structure of good reasoning, independent of the specific content of particular instances of such reasoning. It is deductive because the requirement it lays down for good reasoning is full truth-preservation. Argument forms all of whose instances are truth-preserving, as well as the arguments that are of those forms, are traditionally termed **valid.** The syllogistic form just displayed is a valid form; that is, no syllogism of this form has true premises and a false conclusion. All actual arguments that can be cast in this syllogistic form are therefore valid arguments.

An example of an **invalid** syllogistic form is

Some As are Bs.
All Cs are As.
___
All Cs are Bs.

There are, to be sure, actual arguments that are of this form and have true premises and a true conclusion—for example,

Some birds are hawks.
All osprey are birds.
___
All osprey are hawks.

But there are also arguments of this form that have true premises and a false conclusion—for example,

> Some positive numbers are even numbers.
> All numbers greater than zero are positive numbers.
> ———
> All numbers greater than zero are even numbers.

The two premises of this syllogism are true, but the conclusion, 'All numbers greater than zero are even', is false. The syllogistic form just displayed is an invalid form precisely because there are instances of it that have true premises and a false conclusion.

Aristotelian logic is very powerful. During the centuries following Aristotle, the rules and techniques associated with syllogistic logic were refined, and various test procedures developed, by Roman, Arabic, medieval, and modern logicians. Until the late nineteenth century Aristotelian logic remained the predominant system for formalizing and evaluating reasoning. It is still taught today in many introductory courses.

Nonetheless, there are important drawbacks to Aristotelian logic. Syllogisms are at the heart of Aristotelian logic, and each syllogism must have exactly two premises and a conclusion. Moreover, every sentence of a syllogism must be of one of the four following forms:

> All As are Bs.
> No As are Bs.
> Some As are Bs.
> Some As are not Bs.

Aristotelian logic is thus best suited to reasoning about relations among groups: 'All members of this group are members of that group', 'Some members of this group are members of that group', and so on. Aristotelian logic thus strains to handle reasoning about individuals. For example, 'Socrates is human' must be recast as something like 'All things that are Socrates [there is, we here assume, only one] are things that are human'.

The Aristotelian requirement that every conclusion be drawn from exactly two premises is unduly restrictive and does not mirror the complexity of actual reasoning and argumentation, a single instance of which may make use of a very large number of premises. Consider, for example, the following reasoning:

> Sarah and Hank are the only finalists for a position with Bowles, Blithers, and Blimy, an accounting firm. Whoever is hired will have a baccalaureate degree in accounting. Hank will get his baccalaureate in accounting only if he passes all the business courses he is taking this semester and completes the general education requirements.

> Sarah will get her baccalaureate only if she passes all her courses and raises her grade point average to 2.5. Hank will fail logic and so will not complete the general education requirements. Sarah will pass all her courses, but her grade point average will not reach 2.5. Therefore Bowles, Blithers, and Blimy will hire neither of the finalists.

The above reasoning is truth-preserving. That is, if the premises are all true, then the conclusion, the last sentence of the paragraph, must also be true. But it would be extremely difficult to recast this chain of reasoning in syllogistic terms.

Finally reasoning that relies on relations[1] cannot readily be accommodated within Aristotelian logic. For example, the reasoning 'Sarah is taller than Tom, and Tom is taller than Betty; therefore Sarah is taller than Betty' presupposes the transitivity of the taller-than relation, that is, presupposes the following truth:

> For any three things, if the first is taller than the second, and the second is taller than the third, then the first is taller than the third.

Principles such as the above and arguments relying on them cannot be incorporated within the Aristotelian framework in any intuitive way.

For these and other reasons, logicians in the mid-to-late 1800s looked for alternatives to Aristotelian logic. This work involved the development of systems of sentential logic, that is, systems based on the way sentences of natural languages can be generated from other sentences by the use of such expressions as 'or', 'and', 'if . . . then . . .', and 'not'. Consider this example:

> Karen is either in Paris or in Nairobi. She is not in Nairobi. So Karen is in Paris.

Simple arguments such as this one are not readily represented within syllogistic logic. Yet the argument is clearly an example of good reasoning. Whenever the first two sentences are true, the last sentence is also true. Reasoning of this sort can readily be symbolized in systems of sentential logic.

On the other hand, sentential logic cannot easily deal with reasoning that rests on claims about all, some, or none of this sort of thing being of that sort—the sort of claims Aristotelian logic can often handle. Predicate logic incorporates sentential logic and is also able to handle all the kinds of sentences that are expressible in Aristotelian logic, as well as many of those that pose difficulties for Aristotelian logic.

[1] See Chapter 7 for an explication of relations.

There are a variety of reasons for studying logic. It is a well-developed discipline that many find interesting in its own right, a discipline that has a rich history and important current research programs and practical applications. Certainly, anyone who plans to major or do graduate work in areas such as philosophy, mathematics, computer science, or linguistics should have a solid grounding in symbolic logic. In general, the study of formal logic also helps develop the skills needed to present and evaluate arguments in any discipline.

Another reason for studying symbolic logic is that, in learning to symbolize natural language sentences (in our case English sentences) in a formal language, students become more aware and more appreciative of the importance of the structure and complexities of natural languages. Precisely what words are used often has a major bearing on whether an argument is valid or invalid, a piece of reasoning convincing or unconvincing. For example, distinguishing between 'Roberta will pass if she completes all the homework' and 'Roberta will pass only if she completes all the homework' is essential to anyone who wants to reason well about the prospects for Roberta's passing.

However, the focus of this text is not primarily on sharpening the critical and evaluative skills readers bring to bear on everyday discourse, newspaper columns, and the rhetoric of politicians. Inculcating these skills is the goal of texts on "critical thinking" or "informal logic", where the primary emphasis is on nonformal techniques for identifying fallacies, figuring out puzzles, and constructing persuasive arguments. Formal or symbolic logic, which is the domain of this book, is a discipline with its own body of theory and results, just as are mathematics and physics. This text is an introduction to that discipline, a discipline whose principles underlie the techniques presented in informal logic texts. This text will help readers not only identify good and bad arguments but also understand why arguments are good arguments or bad arguments. Even though only the most avid devotees of formal systems will be constructing truth-tables, truth-trees, or derivations after completing this text, mastering these formal techniques is a way of coming to understand the principles underlying reasoning and the relations among sentences and sets of sentences.

There is another, quite practical, reason for studying symbolic logic. In most of the chapters that follow, the discussion will center on seven or fewer central concepts. These concepts are related, from chapter to chapter. For example, the concept of truth-functional validity developed in Chapter 3 is one way of refining the concept of logical validity laid out in this chapter. All these concepts are abstract. They cannot be touched or weighed or examined under a microscope. Mastering these concepts and the relations among them is an exercise in abstract thinking. The skills involved are, we think, important and will be useful in a wide variety of theoretical and applied fields. For these reasons the "theory questions" found at the end of most exercise sets are in many ways the most important part of the exercise sets.

'True' and 'false' are properties of sentences. That is, it is sentences that are either true or false.[2] Throughout this text we will use the notion of a *truth-value.* We will say that true sentences have the truth-value **T**, and false sentences the truth-value **F**. 'Washington, DC, is the capital of the United States' and 'The volume of a gas is directly proportional to its temperature and inversely proportional to its pressure' are both true, and so both have the truth-value **T**. The truth of the first derives from the political organization of the United States, and the truth of the second from the fundamentals of physics and chemistry. 'Toronto is the capital of Canada' and 'Atoms are indivisible' are both false, so both have the truth-value **F**—the first for reasons having to do with the political organization of Canada, and the second for reasons having to do with the existence and behavior of subatomic particles. Although it is only sentences that are either true or false, not every sentence of English is one or the other. Sentences that are obviously neither true nor false include questions ('Where is Kansas City?'), requests ('Please shut the door when you leave'), commands ('Don't darken my door again'), and exclamations ('Ouch!'). The formal systems we develop in this text are intended to deal only with sentences that are either true or false as asserted on a particular occasion in a particular context. To say that the sentences we will be dealing with are those that are either true or false is not, of course, to say that for any given sentence we know which it is, true or false, but only that it is one or the other.

Much of this text is devoted to the study of arguments. Previously, in discussing syllogistic arguments, we presented them by listing the premises followed by the conclusion, with a horizontal line separating the premises from the conclusion. Arguments so displayed are presented in **standard form.** Of course, in natural languages, whether in spoken discourse or in writing, arguments are rarely presented in standard form. Indeed, in English and other natural languages, arguments, or bits of reasoning that can be reconstructed as one or more arguments, generally neither occur in what we call standard form nor are set off from preceding and following discourse. Moreover, the premises are not always given first and the conclusion last. Consider

> Michael will not get the job, for whoever gets the job will have strong references, and Michael's references are not strong.

[2]Many philosophical disputes arise about such basic concepts as sentences, meaning, context, and truth. For example, some philosophers argue that propositions are the kinds of entities that are really true or false, propositions being taken to be the meanings of sentences and to exist independently of any particular language. Other philosophers eschew propositions as unnecessary metaphysical baggage. Because we have to adopt some terminology and because all philosophers agree that sentences play some role in language, we shall talk of sentences as being true or false. But we consider such talk to be shorthand for talk of sentences that have certain meanings as used in particular contexts.

This single sentence can be recast as the following explicit argument in standard form:

> Whoever gets the job will have strong references.
>
> Michael's references are not strong.
>
> ---
>
> Michael will not get the job.

As we just saw, in everyday discourse the conclusion is sometimes given before the premises. The conclusion can also come between premises, with the whole argument being buried in an ongoing text:

> I've got more relatives than I know what to do with. I've got relatives in Idaho and in New Jersey, in Ireland and in Israel. Among them are a couple of cousins, Tom and Fred Culverson. Both Tom and Fred are hard working, and Tom is as tenacious as a bulldog. So Tom is sure to be a success, for if there is one thing I have learned in life, it is that everyone who is both hard working and tenacious succeeds. But I'm sure success won't change Tom. He'll work just as hard after he makes his first million as he does now. He is, after all, a Culverson. And no one is as predictable as a Culverson, unless it's a Hutchings. There are lots of Hutchings on my mother's side, but I haven't had much to do with them. . . .

The following explicit argument can be extracted from this passage and placed in standard form:

> Tom and Fred are hard working.
>
> Tom is tenacious.
>
> Everyone who is both hard working and tenacious succeeds.
>
> ---
>
> Tom will succeed.

There is a lot of information in this passage that is not relevant to the specific argument we have extracted. This is frequently the case.

The first step in analyzing arguments is to extract them from the discourse within which they are embedded and present them in standard form. Doing so requires practice. In natural language the presence of an argument is often signaled by the use of premise and/or conclusion indicator words. *Conclusion indicator words*—that is, words indicating that what follows is intended as the conclusion of an argument—include

> therefore
> thus
> it follows that
> so
> hence
> consequently
> as a result

Premise indicator words—that is, words whose use signals that what follows is intended as a premise of an argument—include

> since
> for
> because
> on account of
> inasmuch as
> for the reason that

Not every piece of discourse is intended as an argument. Consider

> Our galaxy is made up of our sun and billions of other stars. The galaxy is a huge, flat spiral system that rotates like a wheel, and the myriad of stars move around its center somewhat as the planets revolve around our sun. There are millions of other galaxies in addition to our galaxy.

Each of the sentences in this passage is either true or false, but there is no good reason to treat one of the sentences as a conclusion and the others as premises.

So far we have given examples of arguments, talked about them, and presented several of them in standard form. But we have not defined 'argument', and it is time do so. In our definition we make use, as we frequently will throughout the rest of this text, of the notion of a set of sentences. Sets are abstract objects that have members (no members, one member, two members, . . . an infinite number of members). The identity of a set is determined by its members. That is, if set A and set B have exactly the same members, then they are the same set; if they do not, they are different sets.

An *argument* is a set of two or more sentences, one of which is designated as the conclusion and the others as the premises.

This is a very broad notion of an argument. For example, it allows us to count the following as an argument:

> Herbert is four years old.
>
> ---
>
> The sun will shine tomorrow.

This is, of course, a paradigm of a bad argument. The one premise supplies no support whatsoever for the conclusion. The advantage of this broad definition is that it sidesteps the problem of having to give an account of how plausible a line of reasoning has to be to count as an argument or of how likely it is that a given group of sentences will be taken to support a designated sentence in order for those sentences together to count as an argument.

Given our broad definition, we could recast the passage about our galaxy as an argument for one or another of the constituent claims. But there is no reason to do so except to demonstrate that, by our account, any set of two or more sentences can be taken as an argument and evaluated as such. It is, in fact, our contention that it is the job of the logician, not to set limits on what counts as an argument, but rather to provide means of distinguishing good arguments from bad ones. And this we will do. That said, most (but not all) of the examples of arguments we use in this text will be ones in which someone might think that the premises do support the conclusion.

Sometimes an argument turns out to be a good argument even when the premises do not appear, on first review, to support the conclusion. This is another reason for not appealing to some level of apparent support in the definition of an argument. Consider, for example, this passage:

> Everyone loves a lover. Tom loves Alice. Everyone loves everyone.

If by 'lover' we mean 'someone who loves someone', and if we take the last sentence as the conclusion of an argument of which the first two sentences are premises, we have a valid argument. The conclusion does follow, though not obviously, from the premises. The missing reasoning is this: If Tom loves Alice, then Tom is a lover. It follows from the first premise, 'Everyone loves a lover', that everyone loves Tom. And if a lover is someone who loves someone, it further follows that everyone is a lover (because everyone loves Tom). And if everyone is a lover and everyone loves a lover, it follows, finally, that everyone loves everyone. Of course, this reasoning does not work if 'loves' is not being used in the same way in all its occurrences in the original argument, and it may not be ('Everyone loves a lover' may be being used, for example, in the sense of 'Everyone is fond of a person who is in love').

1.3E EXERCISES

(*Note:* The accompanying *Solutions Manual* contains answers to all unstarred exercises.)

1. For each of the following, indicate whether it is the kind of sentence that falls within the scope of this text—that is, is either true or false. If it is not, explain why not.

a. George Washington was the second president of the United States.
*b. The next president of the United States will be a Republican.
c. Turn in your homework on time or not at all.
*d. Would that John Kennedy had not been assassinated.
e. Two is the smallest prime number.

*f. One is the smallest prime number.
g. George Bush senior was the immediate predecessor to George W. as president.
*h. On January 15, 1134, there was a snowstorm in what is now Manhattan, at 3:00 p.m. EST.
i. Sentence m below is true.
*j. May you live long and prosper.
k. Never look a gift horse in the mouth.
*l. Who created these screwy examples?
m. This sentence is false.
*n. Beware of Greeks bearing gifts.

2. For each of the following passages, specify what argument, if any, is being advanced. Where the intent is probably not to express an argument, explain why this is so. Where an argument is probably being expressed, restate the argument in standard form.

a. When Mike, Sharon, Sandy, and Vicky are all out of the office, no important decisions get made. Mike is off skiing, Sharon is in Spokane, Vicky is in Olympia, and Sandy is in Seattle. So no decisions will be made today.
*b. Our press releases are always crisp and upbeat. That's because, though Jack doesn't like sound bytes, Mike does. And Mike is the press officer.
c. Shelby and Noreen are wonderful in dealing with irate students and faculty. Stephanie is wonderful at managing the chancellor's very demanding schedule, and Tina keeps everything moving and cheers everyone up.
*d. This is a great office to work in. Shelby and Noreen are wonderful in dealing with irate students and faculty. Stephanie is wonderful at managing the chancellor's very demanding schedule, and Tina keeps everything moving and cheers everyone up.
e. The galvanized nails, both common and finishing, are in the first drawer. The plain nails are in the second drawer. The third drawer contains Sheetrock screws of various sizes, and the fourth drawer contains wood screws. The bottom drawer contains miscellaneous hardware.
*f. The galvanized nails, both common and finishing, are in the first drawer. The plain nails are in the second drawer. The third drawer contains Sheetrock screws of various sizes, and the fourth drawer contains wood screws. The bottom drawer contains miscellaneous hardware. So we should have everything we need to repair the broken deck chair.
g. The weather is perfect; the view is wonderful; and we're on vacation. So why are you unhappy?
*h. The new kitchen cabinets are done, and the installers are scheduled to come Monday. But there will probably be a delay of at least a week, for the old cabinets haven't been removed, and the carpenter who is to do the removal is off for a week of duck hunting in North Dakota.
i. Wood boats are beautiful, but they require too much maintenance. Fiberglass boats require far less maintenance, but they tend to be more floating bathtubs than real sailing craft. Steel boats are hard to find, and concrete boats never caught on. So there's no boat that will please me.
*j. Sarah, John, Rita, and Bob have all worked hard and all deserve promotion. But the company is having a cash flow problem and is offering those over 55 a $50,000 bonus if they will retire at the end of this year. Sarah, John, and Bob are all over 55 and will take early retirement. So Rita will be promoted.

k. Everyone from anywhere who's anyone knows Barrett. All those who know her respect her and like her. Friedman is from Minneapolis and Barrett is from Duluth. Friedman doesn't like anyone from Duluth. Therefore, either Friedman is a nobody or Minneapolis is a nowhere.

*l. I'm not going to die today. I didn't die yesterday, and I didn't die the day before that, or the day before that, and so on back some fifty years.

m. Having cancer is a good, for whatever is required by something that is a good is itself a good. Being cured of cancer is a good, and being cured of cancer requires having cancer.

*n. The Soviet Union disintegrated because the perceived need for the military security offered by the union disappeared with the end of the cold war and because over 70 years of union had produced few economic benefits. Moreover the Soviet Union never successfully addressed the problem of how to inspire loyalty to a single state by peoples with vastly different cultures and histories.

o. Only the two-party system is compatible both with effective governance and with the presenting and contesting of dissenting views, for when there are more than two political parties, support tends to split among the parties, with no party receiving the support of a majority of voters. And no party can govern effectively without majority support. When there is only one political party, dissenting views are neither presented nor contested. When there are two or more viable parties, dissenting views are presented and contested.

*p. Humpty Dumpty sat on a wall. Humpty Dumpty had a great fall. All the king's horses and all the king's men couldn't put Humpty together again. So they made him into an omelet and had a great lunch.[3]

1.4 DEDUCTIVE VALIDITY AND SOUNDNESS

We have already noted that truth-preservation is what distinguishes good reasoning from bad reasoning. A deductively valid argument is one whose form or structure is fully truth-preserving—that is, whose form or structure is such that instances of it never proceed from true premises to a false conclusion.[4] A deductively invalid argument is one whose form or structure is such that instances of it do, on occasion, proceed from true premises to a false conclusion.

An example of a **valid deductive argument** is

> There are three, and only three, people in the room: Juarez, Sloan, and Wang.
>
> Juarez is left-handed.
>
> Sloan is left-handed.
>
> Wang is left-handed.
>
> ---
>
> All the people in the room are left-handed.

[3]With apologies to Lewis Carroll.

[4]There are good arguments that are not valid deductive arguments. See Section 1.5.

This argument is truth-preserving. That is, if the premises ('There are three, and only three, people in the room: Juarez, Sloan, and Wang', 'Juarez is left-handed', 'Sloan is left-handed', and 'Wang is left-handed') are all true, then the conclusion ('All the people in the room are left-handed') must also be true. Arguments that are truth-preserving in this strong sense, where it is not possible at the same time for all the premises to be true and the conclusion false, are said to be deductively valid. Such arguments never have true premises and a false conclusion.

> An argument is *deductively valid* if and only if it is not possible for the premises to be true and the conclusion false. An argument is *deductively invalid* if and only if it is not deductively valid.

Consider this example of an invalid deductive argument:

> Sloan is left-handed.
>
> Wang is left-handed.
>
> ---
>
> Everyone is left-handed.

It is invalid because, whereas the premises may well be true, the conclusion is false. Not everyone is left-handed.

Logic is about the relations among sentences and groups of sentences. For example, if we are told that a given argument is deductively valid, we can conclude that if the premises are true the conclusion must also be true. But we cannot conclude, given only that the argument is valid, that the premises are true or that the conclusion is true. Consider, for example, the following argument:

> The corner grocery store was burglarized, and whoever did it both knew the combination to the safe and was in town the night of the burglary.
>
> Carolyn, Albert, and Barbara are the only ones who knew the combination to the safe.
>
> Albert and Barbara were out of town the night of the burglary.
>
> ---
>
> Carolyn committed the burglary.

This argument is deductively valid; that is, if the premises are true, the conclusion must also be true. But it does not follow that the premises are true, and hence it does not follow that the conclusion is true. If we have good reason to believe each of the premises, then we also have good reason to believe the conclusion. But note that it is also the case that, if we have good reason to doubt the conclusion (suppose, for example, that we know Carolyn and also know that burglary is just not her style), then we have good reason to believe that at least one of the premises is false.

The important point to note here is that, given only that an argument is deductively valid, it may still be reasonable to doubt the conclusion or to doubt one or more premises. But what is not reasonable is to accept the premises and doubt or reject the conclusion. One who accepts the premises of a deductively valid argument ought, on pain of irrationality, also accept the conclusion. Correspondingly one who denies the conclusion of a deductively valid argument ought, again on pain of irrationality, reject at least one of the premises of that argument. (In the previous case, no police detective would be impressed by a "defense" of Carolyn that consisted of accepting the premises of the argument but steadfastly denying that Carolyn committed the burglary. If Carolyn did not commit the burglary, then either there was no burglary, or the burglar was not someone who knew the combination, or more people than those mentioned knew the combination, or Carolyn did not know the combination, or Albert and Barbara were not both out of town the night of the burglary, or Carolyn was out of town the night of the burglary, or the person who committed the burglary was not in town the night of the burglary.)

It follows from the definition of deductive validity that if an argument is deductively valid then it does not have all true premises and a false conclusion. But every other combination is possible. For example, a deductively valid argument may have all true premises and a true conclusion. The following is such as argument:

> In 2000 Bush and Gore were the only major party candidates in the presidential election.
>
> A major party candidate won.
>
> Gore did not win.
>
> ---
>
> Bush won the presidential election in 2000.

Deductively valid arguments all of whose premises are true are said to be **deductively sound.**

An argument is *deductively sound* if and only if it is deductively valid and its premises are true. An argument is *deductively unsound* if and only if it is not deductively sound.

The foregoing argument concerning Bush and Gore is both deductively valid and deductively sound.

A deductively valid argument may also have one or more false premises and a conclusion that is false. Here is such an argument:

> France and Great Britain were the major powers in the Napoleonic Wars.
>
> France had the largest army, Great Britain the largest navy.

> The power with the largest army won in the end.
>
> ---
>
> France won in the end.

The third premise, 'The power with largest army won in the end', is false (and the argument is, for this reason, deductively unsound). The conclusion is also false. (Great Britain won the Napoleonic Wars when Wellington defeated Napoleon at the Battle of Waterloo in 1815.)

Finally a deductively valid argument may have a true conclusion and one or more false premises. An example is

> Chicago is the capital of the United States.
>
> The capital of the United States is in Illinois.
>
> ---
>
> Chicago is in Illinois.

Both premises of this argument are false (and the argument is therefore deductively unsound); the conclusion is true. This illustrates that good reasoning can move from one or more false premises to a true conclusion.

A deductively invalid argument may have any combination of truths and falsehoods as premises and conclusion. That is, such an argument may have all true premises and a true conclusion, or all true premises and a false conclusion, or one or more false premises and a true conclusion, or one or more false premises and a false conclusion. Here are some examples:

> Albany is the capital of New York State.
>
> Annapolis is the capital of Maryland.
>
> Columbus is the capital of Ohio.
>
> ---
>
> Denver is the capital of Colorado.

The three premises and the conclusion are all true. But the argument is obviously deductively invalid. Were the legislature of Colorado to vote to move the capital to Boulder, the three premises of this argument would be true and the conclusion false. So it is possible for the premises to be true and the conclusion false. Consider

> Albany is the capital of New York State.
>
> Annapolis is the capital of Maryland.
>
> Columbus is the capital of Ohio.
>
> ---
>
> Minneapolis is the capital of Minnesota.

This deductively invalid argument has true premises and a false conclusion. (St. Paul, not Minneapolis, is the capital of Minnesota.) It is also easy to produce a deductively invalid argument with at least one false premise (two in the following case) and a true conclusion:

> Albany is the capital of New York State.
> Minneapolis is the capital of Minnesota.
> Annapolis is the capital of Maryland.
> Boulder is the capital of Colorado.
> ---
> Columbus is the capital of Ohio.

The conclusion of this argument is true; the second and fourth premises are false. By changing the conclusion to 'Dayton is the capital of Ohio', we produce a deductively invalid argument with at least one (here two) false premise and a false conclusion.

The point to remember is that the only time we can determine whether an argument is deductively valid, given only the truth-values of the premises and conclusion, is when the premises are all true and the conclusion false. We know, again, that such an argument is deductively invalid. In all other cases, to determine whether an argument is deductively valid, we have to consider not what the actual truth-values of the premises and conclusion are, but whether it is possible for the premises all to be true and the conclusion false. For example, consider this argument:

> No sea creatures are mammals.
> Dolphins are sea creatures.
> ---
> Dolphins are not mammals.

It has one false premise (the first), one true premise (the second), and a false conclusion. This information does not determine whether the argument is deductively valid or deductively invalid. Rather, we come to see that the argument is deductively valid when we realize that if both premises were true then the conclusion would have to be true as well—that is, that dolphins, being sea creatures, would have to be nonmammals. This argument is valid because it is not possible for the premises to be true and the conclusion false.

1.4E EXERCISES

1. Which of the following are true and which are false? Explain your answers, giving examples as appropriate.

a. If an argument is valid, all the premises of that argument are true.
*b. If all the premises of an argument are true, the argument is valid.
c. All sound arguments are valid.
*d. All valid arguments are sound.

e. No argument with a false conclusion is valid.
*f. Every argument with a true conclusion is valid.
g. If all the premises of an argument are true and the conclusion is true, then the argument is valid.
*h. If all the premises of an argument are true and the conclusion is false, the argument is invalid.
i. There are sound arguments with false conclusions.
*j. There are sound arguments with at least one false premise.

2. Give arguments with the following characteristics:

a. A valid argument with true premises and a true conclusion.
*b. A valid argument with at least one false premise and a true conclusion.
c. A valid argument with a false conclusion.
*d. An invalid argument all of whose premises are true and whose conclusion is true.
e. An invalid argument all of whose premises are true and whose conclusion is false.
*f. An invalid argument with at least one false premise and a false conclusion.

1.5 INDUCTIVE ARGUMENTS

There are good arguments that are not deductively valid—that is, whose use involves some acceptable risk of proceeding from true premises to a false conclusion. Consider the following example:

> Juarez, Sloan, and Wang are all left-handed.
> Juarez and Sloan both have trouble using can openers made for right-handed people.
> ---
> Wang also has trouble using can openers made for right-handed people.

This argument is not deductively valid. But the conclusion, 'Wang also has trouble using can openers made for right-handed people', is to some extent probable given the fact that Wang is left-handed and that Juarez and Sloan, who are also left-handed, have trouble using can openers made for right-handed people. Nonetheless, the premises could be true and the conclusion false. This might be the case if, for example, Wang is especially adroit with kitchen implements or if the trouble the other two have derives from their having arthritis rather than from their being left-handed.

An argument that is not deductively valid can still be a useful argument—the premises can, as in the prior case, make the conclusion likely even though not certain. Such arguments are said to have **inductive strength,** the strength being proportional to the degree of probability the premises lend to the conclusion.

An argument has *inductive strength* to the extent that the conclusion is probable given the premises.

Inductive strength is thus a matter of degree.

Inductive reasoning is extremely common both in science and in everyday life. Walter Reed's hypothesizing that mosquitoes spread yellow fever is an example of inductive reasoning. While serving in Cuba after the Spanish-American War, Reed, a physician, noticed that those stricken with yellow fever always had recent mosquito bites and that those not stricken tended to work in areas not infested by mosquitoes. Based on these observations, Reed hypothesized that mosquitoes spread yellow fever. He then asked volunteers not infected with yellow fever to allow themselves to be bitten by mosquitoes that had recently bitten yellow-fever-infected patients. The volunteers quickly contracted yellow fever. Reed concluded that mosquitoes transmit yellow fever. His reasoning can be represented as follows:

> Individuals stricken with yellow fever have recent mosquito bites.
>
> Individuals not stricken with yellow fever tend to work in areas not infested with mosquitoes.
>
> Most individuals bitten by mosquitoes that have recently bitten yellow fever patients soon contract yellow fever.
>
> ---
>
> Mosquitoes transmit yellow fever.

Reed's observations made his conclusion probable but not certain. The mechanism of transmission might have turned out to be an airborne bacterium that survives only under conditions that also encourage a high density of mosquitoes. It might have been a coincidence that the volunteers contracted yellow fever soon after being bitten by the test mosquitoes. So we can say of this argument that it is inductively strong but deductively invalid.

Deductive logic, which is the province of this text, and inductive logic, which lies beyond the scope of this text, both provide methods for evaluating arguments, and methods of both sorts can be applied to the same argument, as above. Which methods are most appropriately applied depends on the context. If an argument is given with the assumption that if the premises are true the conclusion must also be true, then the argument should be evaluated by the standards of deductive logic. However, if an argument is given with the weaker assumption that if the premises are true the conclusion is probable, then the argument should be evaluated by the standards of inductive logic.

1.5E EXERCISES

Evaluate the passages in Exercise 2 in Section 1.3 that contain arguments. In each case say whether deductive or inductive standards are most appropriate. If the former, state whether the argument is deductively valid. If the latter, state to what extent the argument is inductively strong.

1.6 LOGICAL CONSISTENCY, TRUTH, FALSITY, AND EQUIVALENCE

One of the important relations that can hold among a set of sentences is **consistency.**

A set of sentences is *logically consistent* if and only if it is possible for all the members of that set to be true. A set of sentences is *logically inconsistent* if and only if it is not logically consistent.

We will indicate that we are talking about a set of sentences by enclosing the component sentences within curly braces— '{' and '}'. The following set is logically consistent:

> {Texas is larger than Oklahoma. The Phlogiston theory of heat has been disproven. The United States Congress consists of the Senate and the House of Representatives.}[5]

Note that there is no requirement that the members of a set have "something to do with each other". The three sentences listed are largely if not entirely unrelated. Together they constitute a consistent set because it is possible that all three are true at the same time. (In fact, all three are true.) We obtain a different but also consistent set by replacing the second sentence, 'The Phlogiston theory of heat has been disproven', with 'The Phlogiston theory of heat has been proven'.

> {Texas is larger than Oklahoma. The Phlogiston theory of heat has been proven. The United States Congress consists of the Senate and the House of Representatives.}

Someone who believes all the members of this new set has, to be sure, at least one false belief ('The Phlogiston theory of heat has been proven'), but this does not make the set inconsistent. There is nothing in the nature of the three sentences and their relations to one another that keeps all three from being true. What keeps the second listed sentence from being true is the nature of heat, that it does not behave the way the Phlogiston theory says it behaves.

The following set of sentences is inconsistent:

> {Michael and Benjamin both applied for positions at the local fast-food outlet, and at least one of them will be hired. No one who applied for a position will get it.}

[5]Technically what should be listed between the curly braces are the names of the members of the set—here the names of sentences. These are formed by placing single quotation marks around the sentences. See Chapter 2, Section 2.4.

If the first listed sentence is true, then the second sentence, 'No one who applied for a position will get it', is false. In contrast, if the second sentence is true, then it cannot be (as the first sentence claims it is) that Michael and Benjamin both applied and that at least one of them will be hired. That is, if the second sentence is true, then the first sentence is false. So it is not possible for both members of this set to be true. We are able to figure this out without knowing who Michael and Benjamin are. The relationships between the members of the set make it impossible for all the members to be true.

The following set is also inconsistent:

> {Anyone who takes astrology seriously is foolish. Alice is my sister, and no sister of mine has a husband who is foolish. Horace is Alice's husband, and he reads the horoscope column every morning. Anyone who reads the horoscope column every morning takes astrology seriously.}

A little reflection shows that not all the members of the foregoing set can be true. If the first, third, and fourth sentences are true, the second cannot be. Alternatively, if the second, third, and fourth are true, the first cannot be. And so on.

Logic cannot normally tell us whether a given sentence is true or false, but we can use logic to discover whether a set is consistent or inconsistent. And if a set is inconsistent, we know that at least one member of it is false, and hence that believing all the members of the set would involve believing at least one false sentence, something we don't want to do. Establishing that a set is consistent does not establish that all, or even any, of its members are true; but it does establish that it is possible for all the members to be true.

Although logic cannot normally be used to determine whether a particular sentence is true or false, there are two special cases. Some sentences are true because of their form or structure. For example, 'Either Cynthia will get a job or she will not get a job' is true no matter how Cynthia fares vis-à-vis her job-seeking activities. Indeed, every sentence that is of the form 'either . . . or . . .' and is such that what comes after the 'or' is the denial of what comes after the 'either' is true. Sentences such as these do not give us any new information. They do not "tell us anything about the world". Whoever Cynthia is, we all know that she either will or will not get a job, and so being told this does not convey any new information. Other sentences of this type include 'If Henry gets fired, he gets fired', 'If everyone passes, Denise will pass', and 'If Sarah will go mountain climbing if and only if Marjorie does, then if Marjorie does not go, neither will Sarah'. Sentences of this sort are said to be **logically true.**

A sentence is *logically true* if and only if it is not possible for the sentence to be false.

Just as some sentences are true by virtue of their form or structure, so too some sentences are false by virtue of their form or structure. These include 'Sarah is an A student and Sarah is not an A student', 'All lions are ferocious but

there are lions in zoos that are not ferocious', 'I'm here and nobody is here', and 'Some dollar bills are not dollar bills'. Such sentences are said to be **logically false.**

A sentence is *logically false* if and only if it is not possible for the sentence to be true.

Logically false sentences, like logically true sentences, give us no information about the world.[6]

Sentences that purport to give us information about the world—and these constitute most of the sentences we encounter outside logic and mathematics—are neither logically true nor logically false. They include 'Ivan is driving from Boston to New Orleans', 'Anyone who takes astrology seriously is foolish', and 'Perkins advocates the relaxation of air pollution standards because he owns a lot of stock in a company producing coal with a high sulfur content'. Such sentences claim that the world, or some part of it, is a certain way, and to determine whether they are true we have to gather information about the world, and not merely about how those sentences are constructed. Such sentences are said to be **logically indeterminate.**

A sentence is *logically indeterminate* if and only if it is neither logically true nor logically false.

The final concept we introduce in this chapter is that of logical equivalence. Sentences are sometimes related in such a way that, because of their structure or form, if one is true the other is as well, and vice versa. Examples of such pairs of sentences include these:

> Henry loves Sarah.
> Sarah is loved by Henry.
>
> Both Sarah and Henry will pass.
> Both Henry and Sarah will pass.
>
> Not all tumors are cancerous.
> Some tumors are not cancerous.

Of course, the members of this pair, perhaps to Henry's dismay, are not **logically equivalent:**

> Henry loves Sarah.
> Sarah loves Henry.

[6]But logically true sentences are useful in ways that logically false sentences are not. By some accounts mathematics consists exclusively of logical truths.

The members of a pair of sentences are *logically equivalent* if and only if it is not possible for one of the sentences to be true while the other sentence is false.

Note that we allow a sentence to be equivalent to itself, by counting, for example, 'Sarah is very bright' and 'Sarah is very bright' as constituting a pair of (identical) sentences. On this definition of logical equivalence, it also follows that all logically true sentences are logically equivalent and that all logically false sentences are logically equivalent. But it does not follow that all logically indeterminate sentences are logically equivalent. Clearly, logically indeterminate sentences with different truth-values—for example, 'Philadelphia is in Pennsylvania' (true) and 'Denver is in Wyoming' (false)—are not logically equivalent. Moreover, not all logically indeterminate sentences with the same truth-value are logically equivalent. 'California produces red wine' and 'California produces white wine' are both true, but these claims are not logically equivalent. The test for logical equivalence is not sameness of truth-value, but rather whether the sentences in question *must* have the same truth-value—whether it is impossible for them to have different truth-values. Since it is possible for 'California produces white wine' to be true but 'California produces red wine' to be false (California vintners might decide that all the money is to be made in white wine and stop producing red wine), these sentences are not logically equivalent.

1.6E EXERCISES

1. Where possible, give an example of each of the following. Where not possible, explain why no example can be given.
a. A consistent set all of whose members are true.
*b. A consistent set with at least one true member and at least one false member.
c. An inconsistent set all of whose members are true.
*d. A consistent set all of whose members are false.

2. For each of the following sets of sentences, indicate whether the set is consistent or inconsistent, and why.
a. {Good vegetables are hard to find. The Dodgers are no longer in Brooklyn. Today is hotter than yesterday.}
*b. {Henry likes real ice cream. Real ice cream is a dairy product. There isn't a dairy product Henry likes.}
c. {Washington, D.C., is the capital of the United States. Paris is the capital of France. Ottawa is the capital of Canada.}
*d. {Washington, D.C., is the capital of the United States. Paris is the capital of France. Toronto is the capital of Canada.}
e. {The weather is fine. Tomorrow is Tuesday. Two plus two equals four. We're almost out of gas.}
*f. {Sue is taller than Tom. Tom is taller than Henry. Henry is just as tall as Sue.}
g. {Tom, Sue, and Robin are all bright. No one who fails "Poetry for Scientists" is bright. Tom failed "Poetry for Scientists".}

*h. {The United States does not support dictatorships. In the 1980s the United State supported Iraq. Iraq has been a dictatorship since 1979.}
i. {Roosevelt was a better president than Truman, as was Eisenhower. Eisenhower was also a better president than his successor, Kennedy. Kennedy was the best president we ever had.}
*j. {Jones and his relatives own all the land in Gaylord, Minnesota. Smith is no relation to Jones. Smith owns land in Gaylord, Minnesota.}
k. {Everyone who likes film classics likes *Casablanca*. Everyone who likes Humphrey Bogart likes *Casablanca*. Sarah likes *Casablanca*, but she doesn't like most film classics and she doesn't like Humphrey Bogart.}
*l. {Everyone who likes film classics likes *Casablanca*. Everyone who likes Humphrey Bogart likes *Casablanca*. Sarah likes film classics and she likes Humphrey Bogart, but she can't stand *Casablanca*.}

3. Give an example of each of the following. Explain, in each case, why the given example is of the sort requested.
a. A logically true sentence
*b. A logically false sentence
c. A logically indeterminate sentence

4. For each of the following, indicate whether it is logically true, logically false, or logically indeterminate, and why.
a. Sarah passed the bar exam but she never went to law school.
*b. Helen is a doctor but not an MD.
c. Helen is an MD but not a doctor.
*d. Bob is in London but his heart is in Texas.
e. Robin will either make it to class by starting time or she won't.
*f. Robin will either make it to class by starting time or she will be late.
g. Bob knows everyone in the class, which includes Robin, whom he doesn't know.
*h. Sarah likes all kinds of fish but she doesn't like ocean fish.
i. If Sarah likes all kinds of fish, then she likes ocean fish.
*j. Anyone who likes rare beef likes rare emu.
k. Anyone who loves everyone is lacking in discrimination.
*l. Anyone who loves everyone loves a lot of people.

5. Where possible, give examples of the following. Where not possible, explain why not.
a. A pair of sentences, both of which are logically indeterminate and are logically equivalent.
*b. A pair of sentences that are not logically equivalent but that are both true.
c. A pair of sentences that are logically equivalent, one of which is logically true and one of which is not.
*d. A pair of sentences that are logically equivalent and both false.
e. A pair of sentences, at least one of which is logically true, that are logically equivalent.
*f. A pair of sentences that are logically equivalent, one of which is logically false and the other of which is logically true.

6. For each of the following pairs of sentences, indicate whether the sentences are logically equivalent, and explain why.
a. Henry is in love with Sue.
Sue is in love with Henry.

*b. Sue married Barbara
Barbara married Sue.

c. Tom likes all kinds of fish.
Tom claims to like all kinds of fish.

*d. Bill and Mary were both admitted to the Golden Key Honor Society.
Bill was admitted to the Golden Key Honor Society and Mary was admitted to the Golden Key Honor Society.

e. Neither Bill nor Mary will get into law school.
Bill will not get into law school or Mary will get into law school.

*f. The judge pronounced Bill and Mary husband and wife.
Bill and Mary got married.

g. Only Mariner fans came to the rally.
All Mariner fans came to the rally.

*h. I know there are people who are starving in every large city in America.
In every large city in America I know people who are starving.

i. Every newscast reported that a strike is imminent.
A strike is imminent.

*j. A bad day of sailing is better than a good day at work.
A good day at work isn't as good as a bad day of sailing.

k. Sarah and Anna won't both be elected president of the senior class.
Either Sarah will be elected president of the senior class or Anna will be elected president of the senior class.

*l. Sarah and Anna won't both be elected president of the senior class.
Either Sarah will not be elected president of the senior class or Anna will not be elected president of the senior class.

m. Everyone dislikes someone.
There is someone whom everyone dislikes.

*n. Everyone dislikes someone.
There is no universally liked person.

o. Everyone likes someone.
Someone is liked by everyone.

*p. Not everyone likes someone.
There is someone who doesn't like anyone.

1.7 SPECIAL CASES OF VALIDITY

Having introduced the notions of logical consistency, logical truth, and logical falsity, we are now in a position to consider some special and rather counterintuitive cases of validity. We have defined a deductively valid argument to be one in which it is impossible for the premises to be true and the conclusion false. Such an argument, we have said, is truth-preserving in that it never takes us from true premises to a false conclusion. Here we consider two special cases of validity. Consider first an argument whose conclusion is logically true. An example is

> The Philadelphia Phillies are the best team in the National League.
>
> ---
>
> Either the next president will be a woman or the next president will not be a woman.

The conclusion of this argument is clearly logically true. No matter who wins the next presidential election, that person either will or will not be a woman. The premise is, at the moment of this writing, anything but true. Note that the premise is utterly unconnected with the conclusion. For the latter reason one might very well be tempted to say that this is an invalid argument, for surely the premises of a valid argument must be relevant to (have some connection with) the conclusion. But recall our definition of validity: An argument is deductively valid if and only if it is not possible for the premises to be true and the conclusion false. The above argument does satisfy this requirement. It is not possible for the conclusion, a logical truth, to be false. Therefore it is not possible for the premises to be true and the conclusion false—again, because the conclusion cannot be false.[7]

To put the point another way, this argument is truth-preserving. It will never lead us from truths to a falsehood because it will never lead us to a falsehood—because the conclusion is logically true. There is no risk of reaching a false conclusion here precisely because there is no risk that the conclusion is false. All arguments whose conclusions are logically true are deductively valid for this reason.[8]

Consider next arguments whose premises form logically inconsistent sets. This may be because one or more of an argument's premises are logically false (in which case it is impossible for those premises to be true, and hence impossible for all the premises to be true), or it may be because, while no single premise is logically false, the premises taken together are nonetheless logically inconsistent. The following is a case of the latter sort:

> Albert is brighter than all his sisters.
>
> Albert and Sally are brother and sister.
>
> Sally is brighter than all her brothers.
>
> ---
>
> *Tyrannosaurus rex* was the fiercest of all dinosaurs.

In this case, if the first and second premises are both true, the third premise cannot be true. And, if the second and third premises are both true, the first premise cannot be true. So not all the premises can be true. The set consisting of the premises is logically inconsistent. Here, as in the preceding case, there is no obvious connection between the premises and the conclusion. Yet the argument does satisfy our definition of deductive validity because it is impossible for all the premises of this argument to be true and therefore

[7]Arguments whose conclusions are logically true are deductively valid whether or not their conclusions are related to their premises. For example,

> The Philadelphia Phillies are the best team in the National League; therefore the Phillies either will or will not win the National League pennant

is a deductively valid argument, not because the premise and conclusion both concern the Phillies but because the conclusion is logically true and it is therefore impossible for the premise to be true *and* the conclusion false.

[8]One way to think of such an argument is that, since the conclusion is logically true, it requires *no* support. Hence, whatever support the premises provide (even if it is none at all—even if the premises are utterly unrelated to the conclusion) is enough.

impossible for all the premises to be true and the conclusion false. The argument is truth-preserving because it will never take us from truths to a falsehood. It will not do so because the premises cannot all be true, and hence there is no possibility of going from truths to a falsehood. Arguments whose premises are inconsistent, while valid, are of course never sound.

Every argument whose premises constitute a logically inconsistent set is thus deductively valid. As a further example, consider

> Sandra will get an A in the course and Sandra will not get an A in the course.
>
> ---
>
> Sandra will graduate.

The one premise of this argument is logically false. Therefore that premise cannot be true. And so it is impossible for every premise of this argument (there is only one) to be true and the conclusion false. The conclusion may be false, but not while the premise is true.

Arguments of the sort we are discussing here are sometimes dismissed as not being arguments at all, precisely because their validity does not depend on a relation between the premises and conclusion. There are, however, systematic reasons for allowing these cases to constitute arguments and thus for recognizing them as valid deductive arguments. It is important to remember that such arguments are valid because they meet the requirement of truth preservation—they will never take us from truths to a falsehood—not because the premises support the conclusion in any intuitive way.

1.7E EXERCISES

1. Which of the following are true, and which are false? Explain your answers giving examples where appropriate.

a. If at least one member of a set of sentences is logically false, then the set is logically inconsistent.
*b. No two false sentences are logically equivalent.
c. Every argument whose conclusion is logically equivalent to one of its premises is valid.
*d. Any argument that includes among its premises 'Everyone is a scoundrel' and 'I'm no scoundrel' is deductively valid.
e. Every argument that has 'Whatever will be, will be' as a conclusion is deductively valid.
*f. Every argument that has 'Everyone is a scoundrel and I'm no scoundrel' as a conclusion is deductively invalid.
g. Every argument all of whose premises are logically true is deductively valid.

2. Answer each of the following:

a. Does every person who believes that New York City is the capital of the United States have inconsistent beliefs?
*b. Need one be engaged in a disagreement or dispute to have use for an argument as we have been using the term 'argument'? Explain.

c. Explain why logic cannot normally tell us whether a valid argument is sound. Under what conditions could we decide, on logical grounds alone, that a valid argument is sound?
*d. Suppose an argument is valid but has a false conclusion. What can we conclude about the premises? Explain.
e. Explain why an argument with at least one logically false premise must be valid no matter what the other premises are and no matter what the conclusion is.
*f. Suppose an argument has a premise that is logically equivalent to a logical falsehood. Must the argument be valid? Explain.
g. Suppose an argument has a logical truth as its conclusion. Explain why the argument must be valid no matter what its premises are. Explain why some such arguments are sound and some are not.
*h. Suppose the premises of an argument form an inconsistent set of sentences. Explain why the argument must be valid but unsound.
i. Suppose a set of a million sentences is consistent. Now suppose a new set of sentences is constructed so that every sentence in the new set is logically equivalent to at least one of the sentences in the old set. Must the new set be consistent? Explain.

GLOSSARY

ARGUMENT: An argument is a set of two or more sentences, one of which is designated as the conclusion and the others as the premises.

DEDUCTIVE VALIDITY: An argument is *deductively valid* if and only if it is not possible for the premises to be true and the conclusion false. An argument is *deductively invalid* if and only if it is not deductively valid.

DEDUCTIVE SOUNDNESS: An argument is *deductively sound* if and only if it is deductively valid and all its premises are true. An argument is *deductively unsound* if and only if it is not deductively sound.

INDUCTIVE STRENGTH: An argument has *inductive strength* to the extent that the conclusion is probable given the premises.

LOGICAL CONSISTENCY: A set of sentences is *logically consistent* if and only if it is possible for all the members of that set to be true. A set of sentences is *logically inconsistent* if and only if it is not logically consistent.

LOGICAL TRUTH: A sentence is *logically true* if and only if it is not possible for the sentence to be false.

LOGICAL FALSITY: A sentence is *logically false* if and only if it is not possible for the sentence to be true.

LOGICAL INDETERMINACY: A sentence is *logically indeterminate* if and only if it is neither logically true nor logically false.

LOGICAL EQUIVALENCE: The members of a pair of sentences are *logically equivalent* if and only if it is not possible for one of the sentences to be true while the other sentence is false.

Chapter 2

SENTENTIAL LOGIC: SYMBOLIZATION AND SYNTAX

2.1 SYMBOLIZATION AND TRUTH-FUNCTIONAL CONNECTIVES

Sentential logic, as the name suggests, is a branch of formal logic in which sentences are the basic units. In this chapter we shall introduce *SL*, a symbolic language for sentential logic, which will facilitate our development of formal techniques for assessing the logical relations among sentences and groups of sentences. The sentences of English that can be symbolized in *SL* are those that are either true or false, that is, have truth-values.

In English there are various ways of generating sentences from other sentences. One way is to place a linking term such as 'and' between them. The result, allowing for appropriate adjustments in capitalization and punctuation, will itself be a sentence of English. In this way we can generate

Socrates is wise and Aristotle is crafty

by writing 'and' between 'Socrates is wise' and 'Aristotle is crafty'. Some other linking terms of English are 'or', 'although', 'unless', 'before', and 'if and only if'. As used to generate sentences from other sentences, these terms are called

sentential connectives (they connect or join sentences to produce further sentences).

Some sentence-generating words and expressions do not join two sentences together but rather work on a single sentence. Examples are 'it is not the case that' and 'it is alleged that'. Prefacing a sentence with either of these expressions generates a further sentence. Since these expressions do not literally connect two sentences, the term "sentential connective" is perhaps a little misleading. Nonetheless, such sentence-generating devices as these are commonly classified as sentential connectives, and we shall follow this usage.

Sentences generated from other sentences by means of sentential connectives are **compound sentences.** All other sentences are **simple sentences.** In developing sentential logic we shall be especially interested in the **truth-functional** use of sentential connectives. Intuitively a compound sentence generated by a truth-functional connective is one in which the truth-value of the compound is a function of, or is fixed by, the truth-values of its components.

> A sentential connective is used *truth-functionally* if and only if it is used to generate a compound sentence from one or more sentences in such a way that the truth-value of the generated compound is wholly determined by the truth-values of those one or more sentences from which the compound is generated, no matter what those truth-values may be.

Few, if any, connectives of English are always used truth-functionally. However, many connectives of English are often so used. We shall call these connectives, as so used, **truth-functional connectives.** A **truth-functionally compound sentence** is a compound sentence generated by a truth-functional connective.

In English 'and' is often used truth-functionally. Consider the compound sentence

Alice is in England and Bertram is in France.

Suppose that Alice is in Belgium, not England. Then 'Alice is in England' is false. The compound sentence is then clearly also false. Similarly, if 'Bertram is in France' is false, the compound 'Alice is in England and Bertram is in France' is false as well. In fact, this compound will be true if and only if both of the sentences from which it is generated are true. Hence the truth-value of this compound is wholly determined by the truth-values of the component sentences from which it is generated. Given their truth-values, whatever they may be, we can always compute the truth-value of the compound in question. This is just what we mean when we say that 'and' functions as a truth-functional connective.

SENTENCES OF SENTENTIAL LOGIC

In *SL* capital Roman letters are used to abbreviate individual sentences of English. Thus

Socrates is wise

can be abbreviated as

W

Of course, we could have chosen any capital letter for the abbreviation, but it is common practice to select a letter that reminds us of the sentence being abbreviated. In this case 'W' reminds us of the word 'wise'. But it is essential to remember that the capital letters of *SL* abbreviate entire sentences and *not* individual words within sentences.

To ensure that we have enough sentences in our symbolic language to represent any number of English sentences, we shall also count capital Roman letters with positive-integer subscripts as sentences of *SL*. Thus all the following are sentences of *SL*:

A, B, Z, T_{25}, Q_6

In *SL* capital letters with or without subscripts are **atomic sentences.** Sentences of *SL* that are made up of one or more atomic sentences and one or more sentential connectives of *SL* are *molecular sentences.*

CONJUNCTION

We could abbreviate

Socrates is wise and Aristotle is crafty

in our symbolic language as 'A', but in doing so we would bury important information about this English sentence. This sentence is a compound made up of two simple sentences: 'Socrates is wise' and 'Aristotle is crafty'. Furthermore, in this case the word 'and', which connects the two sentences, is serving as a truth-functional connective. This compound sentence is true if both of its component sentences are true and is false otherwise. We shall use '&' (ampersand) as the sentential connective of *SL* that captures the force of this truth-functional use of 'and' in English. Instead of symbolizing 'Socrates is wise and Aristotle is crafty' as 'A', we can now symbolize it as

W & C

where 'W' abbreviates 'Socrates is wise' and 'C' abbreviates 'Aristotle is crafty'. Remember that the letters abbreviate entire sentences, not merely specific words like the words 'wise' and 'crafty'. The compound sentence 'W & C' is an example of a molecular sentence of *SL*.

A sentence of the form

P & Q

where **P** and **Q** are sentences of *SL*, is a **conjunction.**[1] **P** and **Q** are the **conjuncts** of the conjunction. Informally we shall use the terms "conjunction" and "conjunct" in talking of English sentences that can be symbolized as conjunctions of *SL*. The relation between the truth or falsity of a conjunction and the truth or falsity of its conjuncts can be simply put: A conjunction is true if and only if both of its conjuncts are true. This is summarized by the following table:

P	Q	P & Q
T	T	T
T	F	F
F	T	F
F	F	F

Such a table is called a *characteristic truth-table* because it defines the use of '&' in *SL*. The table is read horizontally, row by row. The first row contains three T's. The first two indicate that we are considering the case in which **P** has the truth-value **T** and **Q** has the truth-value **T**. The last item in the first row is a **T**, indicating that the conjunction has the truth-value **T** under these conditions. The second row indicates that, when **P** has the truth-value **T** and **Q** has the truth-value **F**, the conjunction has the truth-value **F**. The third row shows that, when **P** has the truth-value **F** and **Q** has the truth-value **T**, the conjunction has the truth-value **F**. The last row indicates that when both **P** and **Q** have the truth-value **F**, the conjunction has the truth-value **F** as well.

Sometimes an English sentence that is not itself a compound sentence can be paraphrased as a compound sentence. The sentence

Fred and Nancy passed their driving examinations

can be paraphrased as

<u>Both</u> Fred passed his driving examination <u>and</u> Nancy passed her driving examination.

We underscore the connectives in paraphrases to emphasize that we are using those connectives truth-functionally. We use '<u>both</u> . . . <u>and</u> . . .',

[1] Our use of boldface letters to talk generally about the sentences of *SL* is explained in Section 2.4.

rather than just '*and*', to mark off the conjuncts unambiguously. Where the example being paraphrased is complex, we shall sometimes also use parentheses—'(' and ')'—and brackets—'[' and ']'—to indicate grouping. The foregoing paraphrase is an adequate paraphrase of the original sentence inasmuch as both the original sentence and the paraphrase are true if and only if 'Fred passed his driving examination' and 'Nancy passed her driving examination' are both true. The paraphrase is a conjunction and can be symbolized as

F & N

where 'F' abbreviates 'Fred passed his driving examination' and 'N' abbreviates 'Nancy passed her driving examination'.

Symbolizing English sentences in *SL* should be thought of as a two-step process. First, we construct in English a truth-functional paraphrase of the original English sentence; next, we symbolize that paraphrase in *SL*. The paraphrase stage serves to remind us that the compounds symbolized as molecular sentences of *SL* are always truth-functional compounds.

The preceding example illustrates that the grammatical structure of an English sentence is not a completely reliable indication of its logical structure. Key words like 'and' serve as clues but are not infallible guides to symbolization. The sentence

Two jiggers of gin and a few drops of dry vermouth make a great martini

cannot be fairly paraphrased as

Both two jiggers of gin make a great martini *and* a few drops of dry vermouth make a great martini.

Together these ingredients may make a great martini, but separately they make no martini at all. Such a paraphrase completely distorts the sense of the original sentence. Thus the original sentence must be regarded as a simple sentence and symbolized in *SL* as an atomic sentence, say

M

Many sentences generated by such other connectives of English as 'but', 'however', 'although', 'nevertheless', 'nonetheless', and 'moreover' can be closely paraphrased using 'and' in its truth-functional sense. Consider some examples:

Susan loves country music, but she hates opera

can be paraphrased as

Both Susan loves country music *and* Susan hates opera.

The paraphrase can be symbolized as 'L & H', where 'L' abbreviates 'Susan loves country music' and 'H' abbreviates 'Susan hates opera'.

The members came today; however, the meeting is tomorrow

can be paraphrased as

Both the members came today *and* the meeting is tomorrow

which can be symbolized as 'C & M', where 'C' abbreviates 'The members came today' and 'M' abbreviates 'The meeting is tomorrow'.

Although George purchased a thousand raffle tickets, he lost

can be paraphrased as

Both George purchased a thousand raffle tickets *and* George lost

which can be symbolized as 'P & L', where 'P' abbreviates 'George purchased a thousand raffle tickets' and 'L' abbreviates 'George lost'.

In each of these cases, the paraphrase perhaps misses part of the sense of the original English sentence. In the last example, for instance, there is the suggestion that it is surprising that George could have purchased a thousand raffle tickets and still have lost the raffle. Truth-functional paraphrases often fail to capture all the nuances present in the sentences of which they are paraphrases. This loss is usually not important for the purposes of logical analysis.

In symbolizing sentences of a natural language—in our case English—grammatical structure and key words provide important clues, but they are not infallible guides to correct symbolizations. Ultimately we have to ask ourselves, as speakers of English, whether the sentence can be reasonably paraphrased as a truth-functional compound. If so, we can symbolize it as a molecular sentence of *SL*. If not, we have to symbolize it as an atomic sentence of *SL*.

DISJUNCTION

Another sentential connective of English is 'or', used in such sentences as

Henry James was a psychologist or William James was a psychologist.

This English sentence contains two simple sentences as components: 'Henry James was a psychologist' and 'William James was a psychologist'. The truth-value of the compound wholly depends upon the truth-values of the component sentences. As long as at least one of the component sentences is true, the compound is true; but if both the components are false, then the compound is false. When used in this way, 'or' serves as a truth-functional connective of English. In *SL* '∨' (wedge) is the symbol that expresses this truth-functional relation. Thus the sentence about Henry and William James can be symbolized as

$$H \vee W$$

where 'H' abbreviates 'Henry James was a psychologist' and 'W' abbreviates 'William James was a psychologist'. 'H ∨ W' is true if 'H' is true or 'W' is true, and it is false only when both 'H' and 'W' are false.

A sentence of the form

$$\mathbf{P} \vee \mathbf{Q}$$

where **P** and **Q** are sentences of *SL*, is a **disjunction. P** and **Q** are the **disjuncts** of the sentence. Informally we shall use the terms "disjunction" and "disjunct" in talking of English sentences that can be symbolized as disjunctions of *SL*. A disjunction is true if and only if at least one of its disjuncts is true. This is summarized by the following characteristic truth-table:

P	**Q**	**P ∨ Q**
T	T	T
T	F	T
F	T	T
F	F	F

The only case in which a disjunction has the truth-value **F** is when both disjuncts have the truth-value **F**.

Some sentences of English that do not contain the word 'or' can be paraphrased as a disjunction. For instance,

> At least one of the two hikers, Jerry and Amy, will get to the top of the mountain

can adequately be paraphrased as

> Either Jerry will get to the top of the mountain or Amy will get to the top of the mountain.

This paraphrase can be symbolized as 'J ∨ A', where 'J' abbreviates 'Jerry will get to the top of the mountain' and 'A' abbreviates 'Amy will get to the top of the mountain'. Remember, the letters abbreviate the entire sentences, not just the words 'Jerry' and 'Amy'. In paraphrasing English sentences as disjunctions of *SL*, we use the 'either . . . or . . .' construction to mark off the two disjuncts unambiguously.

In English sentences that can be paraphrased as disjunctions, 'or' does not always occur between full sentences. For example,

> Nietzsche is either a philosopher or a mathematician

can be paraphrased as

> Either Nietzsche is a philosopher or Nietzsche is a mathematician.

This truth-functional paraphrase can be symbolized as 'P ∨ M', where 'P' abbreviates 'Nietzsche is a philosopher' and 'M' abbreviates 'Nietzsche is a mathematician'.

We use the wedge to symbolize disjunctions in the *inclusive* sense. Suppose the following appears on a menu:

> With your meal you get apple pie or chocolate cake.

We might try to paraphrase this as

> Either with your meal you get apple pie or with your meal you get chocolate cake.

Since we use 'or' only in the inclusive sense in paraphrases, this paraphrase is true if either or both of the disjuncts are true. In ordinary English, on the other hand, 'or' is sometimes used in a more restrictive sense. In the present example, if someone orders both pie and cake, the waiter is likely to point out that either cake or pie, but *not* both, comes with the dinner. This is the exclusive sense of 'or'—either one or the other but not both. Although this sense of 'or' cannot be captured by '∨' alone, there is, as we shall soon see, a combination of connectives of *SL* that will allow us to express the exclusive sense of 'or'.

NEGATION

'It is not the case that' is a sentential connective of English. Consider the following compound generated by this connective:

> It is not the case that Franklin Pierce was president.

This sentence is true if its component sentence, 'Franklin Pierce was president', is false, and it is false if that component sentence is true. 'It is not the case that' is a truth-functional connective because the truth-value of the generated sentence is wholly determined by the truth-value of the component sentence. In *SL* '~ ' (tilde) is the sentential connective that captures this truth-functional relationship. Thus the sentence in question can be symbolized as

~ F

where 'F' abbreviates 'Franklin Pierce was president'. The tilde is a **unary connective,** because it "connects" only one sentence. On the other hand, '&' and '∨' are **binary connectives** since each connects two sentences. When '~ ' is placed in front of a sentence, the truth-value of the generated sentence is the opposite of the truth-value of the original sentence. So the characteristic truth-table for negation is this:

P	~ P
T	F
F	T

Notice that, because '~ ' is a unary connective, we need a truth-table of only two rows to represent all the possible "combinations" of truth-values that a single sentence to which '~ ' is attached might have.

Putting a '~ ' in front of a sentence forms the negation of that sentence. Hence '~ A' is the negation of 'A' (though 'A' is *not* the negation of '~ A'), '~ ~ A' is the negation of '~ A' (though '~ A' is not the negation of '~ ~ A'), and so forth. Informally we shall use the term "negation" in talking about sentences of English that can be symbolized as negations in *SL*. Thus

It is not the case that Franklin Pierce was president

is the negation of

Franklin Pierce was president.

Whether an English sentence should be symbolized as a negation depends on the context. As before, grammar and key words give us clues. Consider some examples:

Not all sailors are good swimmers

is readily paraphrased as

It is not the case that all sailors are good swimmers.

This paraphrase can be symbolized as '~ G', where 'G' abbreviates 'All sailors are good swimmers'. But the following example is not as straightforward:

No doctors are rich.

One might be tempted to paraphrase this sentence as 'It is not the case that all doctors are rich', but to do so is to treat 'No doctors are rich' as the negation of 'All doctors are rich'. This is a mistake because a sentence and its negation are so related that, if one is true, the other is false, and vice versa. In fact, since some doctors are rich and some doctors are not rich, both 'All doctors are rich' and 'No doctors are rich' are false. Hence the latter cannot be the negation of the former. Rather, 'No doctors are rich' is the negation of 'Some doctors are rich'. 'No doctors are rich' is true if and only if 'Some doctors are rich' is false, so the former sentence can be paraphrased as

It is not the case that some doctors are rich.

This can be symbolized as '~ D', where 'D' abbreviates 'Some doctors are rich'.

Some further examples will be helpful:

Chlorine is not a metal

can plausibly be understood as

It is not the case that chlorine is a metal.

This paraphrase can be symbolized as '~ C', where 'C' abbreviates 'Chlorine is a metal'. Notice that 'Chlorine is a metal' and 'Chlorine is not a metal' are such that if either is true the other is false, which must be the case if the latter is to be the negation of the former. But now consider an apparently similar case:

Some humans are not male.

This sentence should not be paraphrased as 'It is not the case that some humans are male'. The latter sentence is true if and only if *no* humans are male, which is not the claim made by the original sentence. The proper paraphrase is

It is not the case that all humans are male

which can be symbolized as '~ H', where 'H' abbreviates 'All humans are male'. Often sentences containing words with such prefixes as 'un-', 'in-', and 'non-' are best paraphrased as negations. But we must be careful here.

Kant was unmarried

can be understood as

It is not the case that Kant was married and then symbolized as '~ K', where 'K' abbreviates 'Kant was married'. 'Kant was unmarried' is the negation of 'Kant was married'. But

Some people are unmarried

should not be paraphrased as 'It is not the case that some people are married'. 'Some people are married' and 'Some people are unmarried' are both true. A proper paraphrase in this case is

It is not the case that all people are married

which can be symbolized as '~ M', where 'M' abbreviates 'All people are married'.

COMBINATIONS OF SENTENTIAL CONNECTIVES

So far we have discussed three types of truth-functional compounds—conjunctions, disjunctions, and negations—and the corresponding sentential connectives of *SL*— '&', '∨', and '~ '. These connectives can be used in combination to symbolize complex passages. Suppose we wish to symbolize the following:

> Either the steam engine or the computer was the greatest modern invention, but the zipper, although not the greatest modern invention, has made life much easier.

The main connective in this sentence is 'but', and the sentence can be paraphrased as a conjunction. The left conjunct can be paraphrased as a disjunction, and the right can be paraphrased as a conjunction making the claim that the zipper was not the greatest modern invention and the claim that the zipper has made life much easier. Finally the claim that the zipper was not the greatest modern invention can be paraphrased as a negation. The resulting truth-functional paraphrase is

> Both (either the steam engine was the greatest modern invention or the computer was the greatest modern invention) and (both it is not the case that the zipper was the greatest modern invention and the zipper has made life much easier).

For clarity we have inserted some parentheses in the paraphrase to emphasize the grouping of the components. The order of placement of 'both' and 'either' is important. In this case 'both' occurring before 'either' at the beginning shows that the overall sentence is a conjunction, not a disjunction. The paraphrase can be symbolized as

$(S \vee C) \ \& \ (\sim Z \ \& \ E)$

where 'S' abbreviates 'The steam engine was the greatest modern invention', 'C' abbreviates 'The computer was the greatest modern invention', 'Z' abbreviates 'The zipper was the greatest modern invention', and 'E' abbreviates 'The zipper has made life much easier'.

The connectives '&', '∨', and '~ ' can be used in combination to symbolize English sentential connectives such as 'neither . . . nor . . .'. The sentence

Neither Sherlock Holmes nor Watson is fond of criminals

can be paraphrased as

> Both it is not the case that Sherlock Holmes is fond of criminals and it is not the case that Watson is fond of criminals.

This can be symbolized as

$\sim H \ \& \sim W$

where 'H' abbreviates 'Sherlock Holmes is fond of criminals' and 'W' abbreviates 'Watson is fond of criminals'.

Another equally good paraphrase of the original sentence is

> It is not the case that either Sherlock Holmes is fond of criminals or Watson is fond of criminals.

This paraphrase can be symbolized using the above abbreviations as

$\sim (H \vee W)$

Note that the original sentence, the paraphrases, and the symbolic sentences are all true if Sherlock Holmes is not fond of criminals and Watson is not fond of criminals, and they are all false otherwise.

A similar, but nonequivalent, connective is 'not both . . . and . . .'. Consider this claim:

A Republican and a Democrat will not both become president.

Truth-functionally paraphrased this becomes

> It is not the case that both a Republican will become president and a Democrat will become president

which is symbolized as

$\sim (R \ \& \ D)$

This sentence does not maintain that neither a Republican nor a Democrat will become president but only that not both of them will become president. '~ (R ∨ D)' is not an acceptable symbolization, but '~ (R & D)' is. Another possible and acceptable paraphrase of this particular 'not both . . . and . . .' claim is

> Either it is not the case that a Republican will become president or it is not the case that a Democrat will become president

which when symbolized becomes

> ~ R ∨ ~ D

Here is a table summarizing the truth conditions for 'neither . . . nor . . .'. Notice that a 'neither . . . nor . . .' expression is true only when both of its components, **P** and **Q**, are false.

Truth Conditions for
'Neither . . . nor . . .'

P	**Q**	**~ P & ~ Q**	**~ (P ∨ Q)**
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Compare this table with the next table, which shows the truth conditions for 'not both . . . and . . .':

Truth Conditions for
'Not both . . . and . . .'

P	**Q**	**~ (P & Q)**	**~ P ∨ ~ Q**
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

A 'not both . . . and . . .' expression is false only when both of its components, **P** and **Q**, are true.

A combination of the sentential connectives of *SL* can also be used to capture the exclusive sense of 'or' discussed earlier. Recall that the sentence

> With your meal you get apple pie or chocolate cake

is true in the exclusive sense of 'or' if with your meal you get apple pie or chocolate cake but not both apple pie and chocolate cake. We now know how to paraphrase the 'not both . . . and . . .' portion of the sentence. The paraphrase of the whole sentence is

> Both (either with your meal you get apple pie or with your meal you get chocolate cake) and it is not the case that (both with your meal you get apple pie and with your meal you get chocolate cake).

This can be symbolized as

> (A ∨ C) & ~ (A & C)

where 'A' abbreviates 'With your meal you get apple pie' and 'C' abbreviates 'With your meal you get chocolate cake'. Here is a table showing the truth conditions for exclusive 'or':

Truth Conditions for Exclusive 'Or'

P	**Q**	**(P ∨ Q) & ~ (P & Q)**
T	T	F
T	F	T
F	T	T
F	F	F

MATERIAL CONDITIONAL

One of the most common sentential connectives of English is 'if . . . then . . .'. A simple example is

> If Jones got the job then he applied for it.

This can be paraphrased as

> Either it is not the case that Jones got the job or Jones applied for the job

which can be symbolized as

> ~ G ∨ A

where 'G' abbreviates 'Jones got the job' and 'A' abbreviates 'Jones applied for the job'. It will be convenient to have a symbol in *SL* that expresses the truth-functional sense of 'if . . . then . . .'; we introduce '⊃' (horseshoe) for this purpose. The sentence 'If Jones got the job then Jones applied for the job' can then be symbolized as

$G \supset A$

A sentence of the form $\mathbf{P} \supset \mathbf{Q}$, where **P** and **Q** are sentences of *SL*, is a **material conditional. P**, the sentence on the left of the '$\supset$', is the **antecedent,** and **Q**, the sentence on the right of the '$\supset$', is the **consequent** of the conditional. It is important to remember that, whenever we write a sentence of the form $\mathbf{P} \supset \mathbf{Q}$, we could express it as $\sim \mathbf{P} \vee \mathbf{Q}$. A sentence of the form $\sim \mathbf{P} \vee \mathbf{Q}$ is a disjunction, and a disjunction is false in only one case—when both disjuncts are false. Thus a sentence of the form $\sim \mathbf{P} \vee \mathbf{Q}$ is false when $\sim \mathbf{P}$ is false and **Q** is false, that is, when **P** is true and **Q** is false. This is also the only case in which a sentence of the form $\mathbf{P} \supset \mathbf{Q}$ is false, that is, when the antecedent is true and the consequent is false. The characteristic truth-table is shown here:

P	**Q**	$\mathbf{P} \supset \mathbf{Q}$
T	T	T
T	F	F
F	T	T
F	F	T

Informally we can regard the 'if' clause of an English conditional as the antecedent of that conditional and the 'then' clause as the consequent. Here is an example of an English conditional converted to a truth-functional paraphrase that is symbolized by the material conditional:

If Michelle is in Paris then she is in France.

Expressed in a truth-functional paraphrase this becomes

If Michelle is in Paris then Michelle is in France.

The truth-functional paraphrase can be symbolized as a material conditional

$P \supset F$

Notice that the truth-functional paraphrase is false if Michelle is in Paris but is not in France—that is, if the antecedent is true and the consequent is false. But the truth-functional paraphrase is true under all other conditions. Thus, if Michelle is in Paris and in France, the paraphrase is true. If Michelle is not in Paris but is somewhere else in France, the paraphrase is true. If Michelle is not in Paris and not in France, the paraphrase is true.

However, the material conditional is not adequate as a complete treatment of conditional sentences in English. Material conditionals are truth-functional, but conditionals in English frequently convey information that exceeds

a truth-functional analysis. For instance, 'if . . . then . . .' constructions sometimes have a causal force that is lost in a truth-functional paraphrase. Consider:

1. If this rod is made of metal then it will expand when heated.
2. If this rod is made of metal then it will contract when heated.

Each of these sentences can be used to make a causal claim, to assert a causal relation between the substance of which the rod in question is composed and the reaction of the rod to heat. But sentence 1 is in accord with the laws of nature, and sentence 2 is not. So, as used to make causal claims, sentence 1 is true and sentence 2 is false, even if it is false that the rod is made of metal.

Now suppose we paraphrase these two sentences as material conditionals:

1a. If this rod is made of metal then this rod will expand when heated.
2a. If this rod is made of metal then this rod will contract when heated.

These paraphrases can be symbolized as

1b. $M \supset E$
2b. $M \supset C$

where 'M' abbreviates 'The rod is made of metal', 'E' abbreviates 'This rod will expand when heated', and 'C' abbreviates 'This rod will contract when heated'. Remember that a material conditional is true if the antecedent is false. If the rod in the example is not made of metal, then both sentences 1a and 2a, and consequently their symbolizations 1b and 2b, are true. Sentence 1 says more than either 1a or 1b, and sentence 2 says more than either 2a or 2b. The fact that sentence 2 is false, whereas 2a and 2b are both true, shows this. It follows that when they are used to assert a causal relation, sentences 1 and 2, like many other English conditionals, are not truth-functional compounds. When it is and when it is not appropriate to paraphrase such sentences as material conditionals will be discussed further in Section 2.3.

Here are further examples of English sentences that can be paraphrased by using 'if . . . then . . .', but here and elsewhere we must keep in mind that sometimes information contained in the English conditionals will be lost in truth-functional paraphrasing.

Larry will become wealthy provided that he inherits the family fortune

can be paraphrased as

If Larry inherits the family fortune then Larry will become wealthy

which can be symbolized as

$F \supset W$

where 'F' abbreviates 'Larry inherits the family fortune' and 'W' abbreviates 'Larry will become wealthy'.

> The Democratic candidate will win the election if he wins in the big cities

can be paraphrased as

> If the Democratic candidate wins in the big cities then the Democratic candidate will win the election

which can be symbolized as '$C \supset E$', where 'C' abbreviates 'The Democratic candidate wins in the big cities' and 'E' abbreviates 'The Democratic candidate will win the election'.

> Betty is in London only if Betty is in England

can be paraphrased as

> If Betty is in London then Betty is in England

which can be symbolized as '$L \supset E$', where 'L' abbreviates 'Betty is in London' and 'E' abbreviates 'Betty is in England'. In this case be sure to notice the order in which the sentences are paraphrased. A common mistake in paraphrasing the sentential connective 'only if' is to ignore the word 'only' and reverse the order of the sentences. It is *incorrect* to paraphrase the original as 'If Betty is in England then Betty is in London'.

A connective that can be paraphrased either as a disjunction or as a conditional is 'unless'. Consider the sentence

> This plant will die unless it is watered.

The only circumstance under which this sentence is false is the situation in which this plant does not die and is not watered. If either of the sentences that 'unless' connects is true, then the whole sentence is true. The simplest paraphrase is to treat the sentence as the disjunction

> Either this plant will die or it is watered

which can be symbolized as

> $D \vee W$

We can also understand the sentence 'This plant will die unless it is watered' as expressing a conditional:

> If it is not the case that it is watered, then this plant will die

which can be symbolized as

> $\sim W \supset D$

Equally well, we can understand the sentence as expressing the equivalent conditional:

> If it is not the case that this plant will die, then it is watered

which when symbolized is

> $\sim D \supset W$

The two conditional paraphrases look different from each other and from the disjunction, but they make identical truth-functional claims. The disjunction claims that at least one of its component sentences is true. Each of the conditionals claims that, if one of two component sentences is not true, the other one is true. Here is a table that shows the truth-functional equivalence of the symbolizations for 'unless':

Truth Conditions for 'Unless'

P	Q	$P \vee Q$	$\sim P \supset Q$	$\sim Q \supset P$
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	F

MATERIAL BICONDITIONAL

In English the connective 'if and only if' is used to express more than either the connective 'if' or the connective 'only if'. For example

> John will get an A in the course if and only if he does well on the final examination

can be paraphrased as

> Both (if John will get an A in the course then John does well on the final examination) and (if John does well on the final examination then John will get an A in the course).

We can symbolize the paraphrase as

> $(C \supset E) \& (E \supset C)$

where 'C' abbreviates 'John will get an A in the course' and 'E' abbreviates 'John does well on the final examination'. The original sentence can also be paraphrased as

> Either (both John will get an A in the course and John does well on the final examination) or (both it is not the case that John will get an A in the course and it is not the case that John does well on the final examination).

Using the same abbreviations, this paraphrase is symbolized as

$$(C \& E) \vee (\sim C \& \sim E)$$

Both of these paraphrases and their corresponding symbolizations are truth-functional compounds. Each is true just in case either both atomic sentences are true or both atomic sentences are false. We introduce the connective '≡' (triple bar) to capture the truth-functional use of the connective 'if and only if'. The original English sentence can be symbolized as

$$C \equiv E$$

A sentence of the form

$$\mathbf{P} \equiv \mathbf{Q}$$

where **P** and **Q** are sentences of *SL*, is a **material biconditional.** Informally we shall use the term "material biconditional" when describing English sentences that can be symbolized as material biconditionals in *SL*. Here is the characteristic truth-table for '≡':

P	Q	P ≡ Q
T	T	T
T	F	F
F	T	F
F	F	T

The connective 'just in case' is sometimes used in English as an equivalent to 'if and only if'.

> Andy will win the lottery just in case Andy has the winning ticket

can be properly paraphrased as

> Andy will win the lottery if and only if Andy has the winning ticket

and symbolized as

$$W \equiv T$$

However, care must be taken when paraphrasing 'just in case' because this connective sometimes is used in ways *not* equivalent to 'if and only if'. Consider

> Marty takes her umbrella to work just in case it rains.

This does not mean 'Marty takes her umbrella to work if and only if it rains'. Rather, the sentence means

> Marty takes her umbrella to work because it may rain.

SUMMARY OF SOME COMMON CONNECTIVES

Note that we use lowercase boldface '**p**' and '**q**' to designate sentences of English and uppercase boldface '**P**' and '**Q**' to designate sentences of *SL*.

English Connectives	*Paraphrases*	*Symbolizations*
not **p**	it is not the case that **p**	~ **P**
p and **q** **p** but **q** **p** however **q** **p** although **q** **p** nevertheless **q** **p** nonetheless **q** **p** moreover **q**	both **p** and **q**	**P** & **Q**
p or **q**	either **p** or **q**	**P** ∨ **Q**
p or **q** [*exclusive*]	both either **p** or **q** and it is not the case that both **p** and **q**	(**P** ∨ **Q**) & ~ (**P** & **Q**)
if **p** then **q** **p** only if **q** **q** if **p** **q** provided that **p** **q** given **p**	if **p** then **q**	**P** ⊃ **Q**
p if and only if **q** **p** if but only if **q** **p** just in case **q**	**p** if and only if **q**	**P** ≡ **Q**
neither **p** nor **q**	both it is not the case that **p** and it is not the case that **q** it is not the case that either **p** or **q**	~ **P** & ~ **Q** ~ (**P** ∨ **Q**)
not both **p** and **q**	it is not the case that both **p** and **q** either it is not the case that **p** or it is not the case that **q**	~ (**P** & **Q**) ~ **P** ∨ ~ **Q**
p unless **q**	either **p** or **q** if it is not the case that **p** then **q** if it is not the case that **q** then **p**	**P** ∨ **Q** ~ **P** ⊃ **Q** ~ **Q** ⊃ **P**

The connective 'because' is not truth-functional. ('Because' can join two true sentences resulting in a true sentence and 'because' can join two true sentences resulting in a false sentence.) Hence 'Marty takes her umbrella to work just in case it rains' should be symbolized by a single sentence letter such as 'M'.

In our discussion of the material conditional and the material biconditional, we have been careful to distinguish among connectives such as 'if', 'only if', and 'if and only if'. These distinctions are very important in logic, philosophy, and mathematics. However, in everyday discourse people speak casually. For example, people may use 'if' or 'only if' when they mean 'if and only if'. Our general policy in this book is to take disjunctions and conditionals in their weaker rather than their stronger senses. That is, normally 'or' will be read in the inclusive sense, and 'if . . . then . . .' (and other conditional connectives) will be taken in the material conditional sense (not the biconditional sense). When stronger readings are intended, we will indicate that by explicitly using expressions such as 'either . . . or . . . but not both' and 'if and only if'.

2.1E EXERCISES

1. For each of the following sentences, construct a truth-functional paraphrase and symbolize the paraphrase in *SL*. Use these abbreviations:

A: Albert jogs regularly.
B: Bob jogs regularly.
C: Carol jogs regularly.

a. Bob and Carol jog regularly.
*b. Bob does not jog regularly, but Carol does.
c. Either Bob jogs regularly or Carol jogs regularly.
*d. Albert jogs regularly and so does Carol.
e. Neither Bob nor Carol jogs regularly.
*f. Bob does jog regularly; however, Albert doesn't.
g. Bob doesn't jog regularly unless Carol jogs regularly.
*h. Albert and Bob and also Carol do not jog regularly.
i. Either Bob jogs regularly or Albert jogs regularly, but they don't both jog regularly.
*j. Although Carol doesn't jog regularly, either Bob or Albert does.
k. It is not the case that Carol or Bob jogs regularly; moreover Albert doesn't jog regularly either.
*l. It is not the case that Albert, Bob, or Carol jogs regularly.
m. Either Albert jogs regularly or he doesn't.
*n. Neither Albert nor Carol nor Bob jogs regularly.

2. Using the abbreviations given in Exercise 1, construct idiomatic English sentences from the following sentences of *SL*:

a. $A \& B$
*b. $A \vee \sim A$

c. $A \vee C$
*d. $\sim (A \vee C)$
e. $\sim A \& \sim C$
*f. $\sim \sim B$
g. $B \& (A \vee C)$
*h. $(A \vee C) \& \sim (A \& C)$
i. $(A \& C) \& B$
*j. $\sim A \vee (\sim B \vee \sim C)$
k. $(B \vee C) \vee \sim (B \vee C)$

3. Assuming that 'Albert jogs regularly' is true, 'Bob jogs regularly' is false, and 'Carol jogs regularly' is true, which of the symbolic sentences in Exercise 2 are true and which are false? Use your knowledge of the characteristic truth-tables in answering.

4. Paraphrase each of the following using the phrase 'it is not the case that'. Symbolize the results, indicating what your abbreviations are.

a. Some joggers are not marathon runners.
*b. Bob is not a marathon runner.
c. Each and every marathon runner is not lazy.
*d. Some joggers are unhealthy.
e. Nobody is perfect.

5. For each of the following sentences, construct a truth-functional paraphrase and symbolize the paraphrase in *SL*. Use these abbreviations:

A: Albert jogs regularly.
B: Bob jogs regularly.
C: Carol jogs regularly.
L: Bob is lazy.
M: Carol is a marathon runner.
H: Albert is healthy.

a. If Bob jogs regularly he is not lazy.
*b. If Bob is not lazy he jogs regularly.
c. Bob jogs regularly if and only if he is not lazy.
*d. Carol is a marathon runner only if she jogs regularly.
e. Carol is a marathon runner if and only if she jogs regularly.
*f. If Carol jogs regularly, then if Bob is not lazy he jogs regularly.
g. If both Carol and Bob jog regularly, then Albert does too.
*h. If either Carol or Bob jogs regularly, then Albert does too.
i. If either Carol or Bob does not jog regularly, then Albert doesn't either.
*j. If neither Carol nor Bob jogs regularly, then Albert doesn't either.
k. If Albert is healthy and Bob is not lazy, then both jog regularly.
*l. If Albert is healthy, he jogs regularly if and only if Bob does.
m. Assuming Carol is not a marathon runner, she jogs regularly if and only if Albert and Bob both jog regularly.
*n. Although Albert is healthy he does not jog regularly, but Carol does jog regularly if Bob does.
o. If Carol is a marathon runner and Bob is not lazy and Albert is healthy, then they all jog regularly.