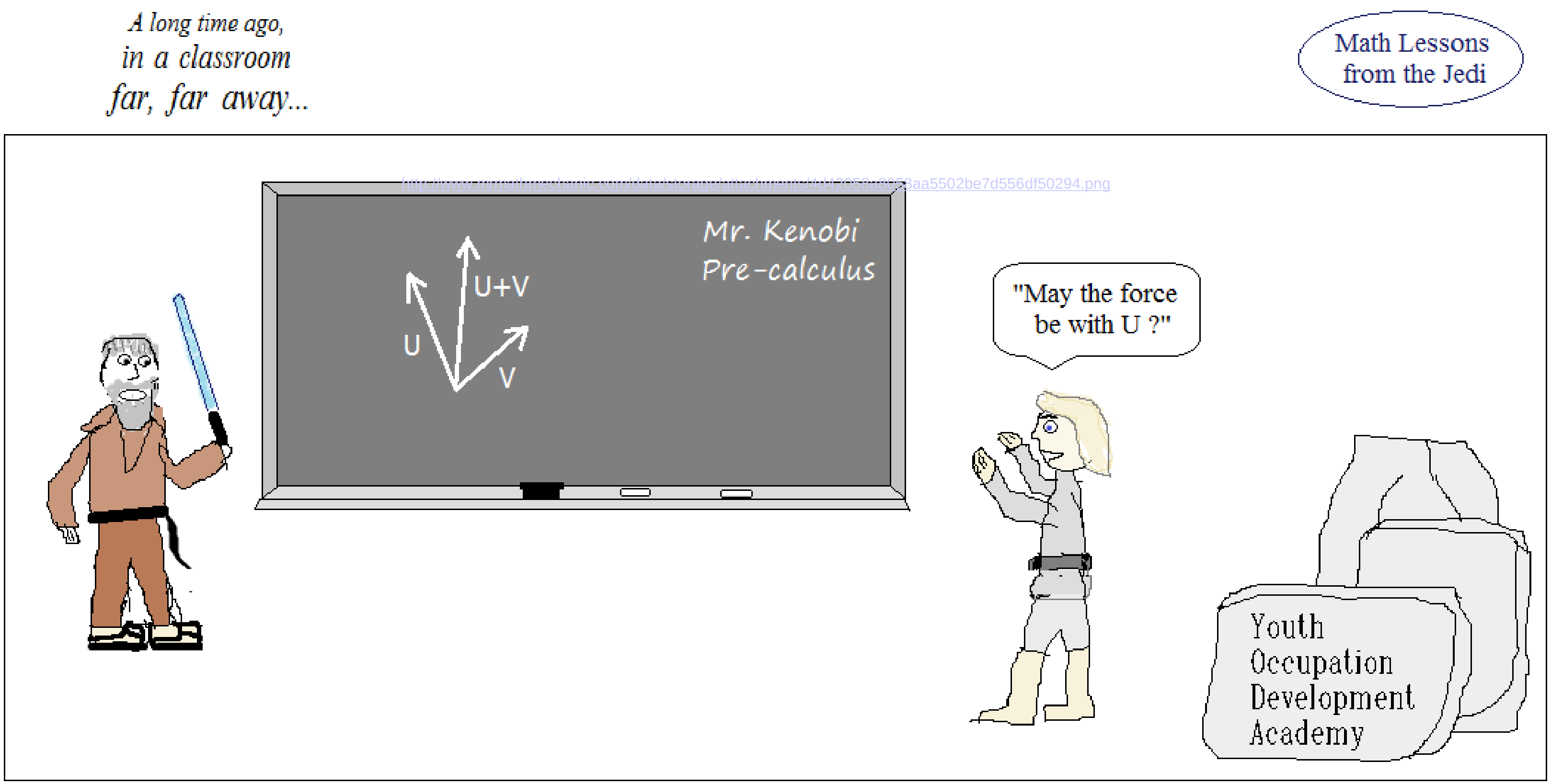
PHYS 121 – SPRING 2015



LanceAF #72 2-17-13
www.mathplane.com

Obi-Wan teaches Luke about
resultant vectors and (the) force

CHAPTER 3: VECTORS

version 01/23/2015, ~ 41 slides

We made it to slide #8 on Wednesday, 1/21/2015.

2D AND 3D – THE NEED FOR VECTORS

Vectors are needed to describe any physical quantity which '*points*' in some specific spatial direction.

It's not sufficient to say that a particle moves 2 m,

we need to know the direction in which it moves.

- We'll assume that our universe contains three dimensions.
  - Quantities like displacement, velocity, acceleration and force may each have three **INDEPENDENT** components.
  - Pay no attention to physics string theorists who say there are ~ 11 dimensions/components. They aren't talking about *classical physics*.
- Most of the examples we'll use this semester will be 2D.
  - There aren't any fundamentally new concepts necessary for working in 3D rather than 2D.
  - **But the math is a lot more annoying in 3D!**

DISPLACEMENT AS A VECTOR

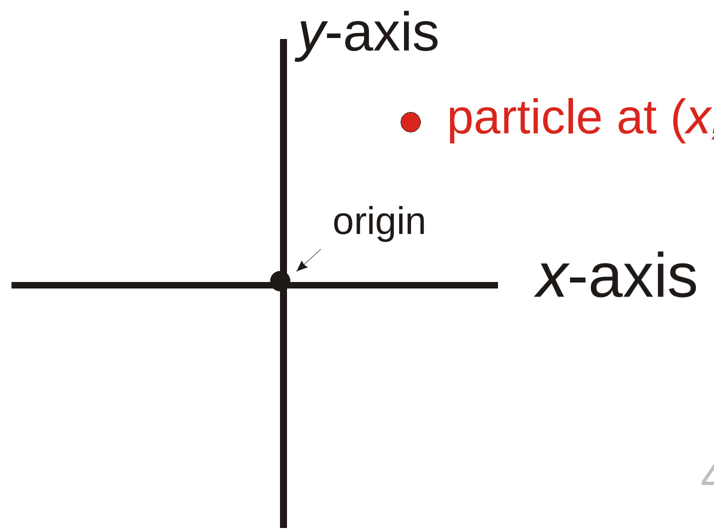
- A '*scalar*' quantity has a numerical value but no *direction* in space.
  - A scalar is a *zero dimensional*, 0D, quantity
  - *Temperature & mass are scalars.*
- Physical quantities that have a direction in space are called '*vectors*'
  - Vectors have *magnitude* and *direction.*
  - The *magnitude* of a vector ~ its *size* and is always positive.
  - A vector is represented in a drawing by an arrow whose length represents the magnitude of the vector.
  - *Displacement* is a vector
    - In 1D its direction can only be *plus* or *minus.*
    - The *magnitude* of a 1D displacement is it's absolute value $|\Delta x|$
    - All vectors behave mathematically (*addition et al*) like displacements.

TWO DIMENSIONS

A two dimensional (2D) universe ~ flat surface.

- 2 values are needed to specify position or displacement.
- A *conventional* coordinate system for 2D consists of axes labeled x & y that are mutually perpendicular to each other and which intersect at the *origin* where $x \equiv 0$ & $y \equiv 0$.
- In empty space, the behavior of x & y components are *independent* from each other

2-D Coordinate System



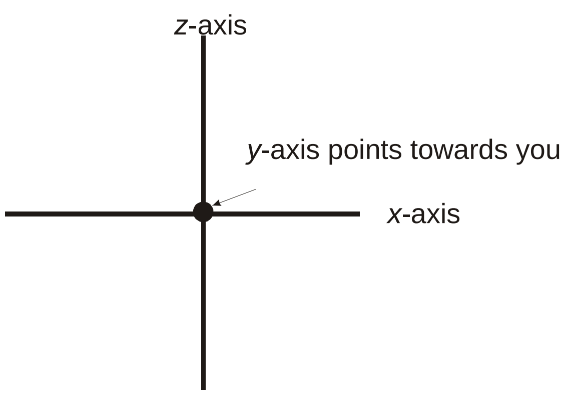
THREE DIMENSIONS

Our universe is three dimensional or 3D (*at least*).

- It takes 3 values to completely specify the position or motion of an object.
- A *Cartesian* 3D coordinate system consists of axes labeled *x*, *y* and *z* that are mutually perpendicular to each other and which intersect at the origin.
    - *y* and *z* are sometimes interchanged
    - You'll encounter non-Cartesian coordinate systems ~ *cylindrical & spherical* .

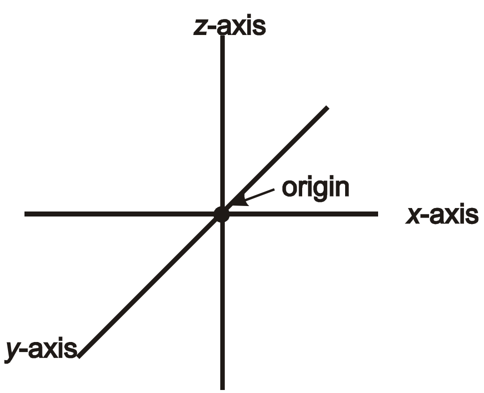
3-D Coordinate System

the y-axis is perpendicular to this page



3-D Coordinate System

in perspective - the y-axis is actually perpendicular to this page



CULTURAL INTERLUDE:
HOW MANY DIMENSIONS ARE THERE?

0D, 1D and 2D systems really do exist, using reasonable definitions of dimensionality, and one can perform experiments that prove this.

Playing around with ‘dimensions’ is not just a game for physicists.

- Any surface or interface between two materials is a 2D system.
  - Computer hard disks rely on a ‘2D electron gas’. http://en.wikipedia.org/wiki/2DEG
  - The thickness of an atom in one direction is negligible if the other dimensions are 10^8 times as large – and experiments prove this.
  - My research field of surface science and thin (~ *1 atom thick*) films focuses on 2D physics
- A long chain molecule (*polymer*) or a step in a 2D surface is a 1D system.
- A small cluster of atoms has 0D-like properties = “*Quantum Dot*”.

You can buy quantum dots online! http://www.nn-labs.com/index.php?option=com_content&view=article&id=50&Itemid=94&gclid=CLK3pM_BgLwCFQPNOgodtSoA4w

Some fundamental principles that you will study this spring, like *conservation of momentum*, are different in lower dimensional systems.

But PHYS 121 deals with conventional 3D space, even if the motion we examine is confined to 1D or 2D within that space.

Lower dimensions are simply useful approximations on the way to 3D physics.

CULTURAL INTERLUDE

Dimensions > 3 (*not counting time as a dimension*) may or may not exist. See, for example

http://en.wikipedia.org/wiki/Dimension_(mathematics_and_physics)

http://mathworld.wolfram.com/Dimension.html.

http://www.pbs.org/wgbh/nova/physics/imagining-other-dimensions.html

- This is not our concern this semester.
- If you want to learn about higher dimensions, up to 11, that might or might not describe our universe (*but perhaps not other universes that exist in parallel with ours*), declare a major in physics.
- If you want to learn about dimensions that DON'T exist, declare a MATH major!
- Engineers probably don't have to worry about extra dimensions,

  unless Aerospace Engineering starts a degree program in **Warp Drive**.

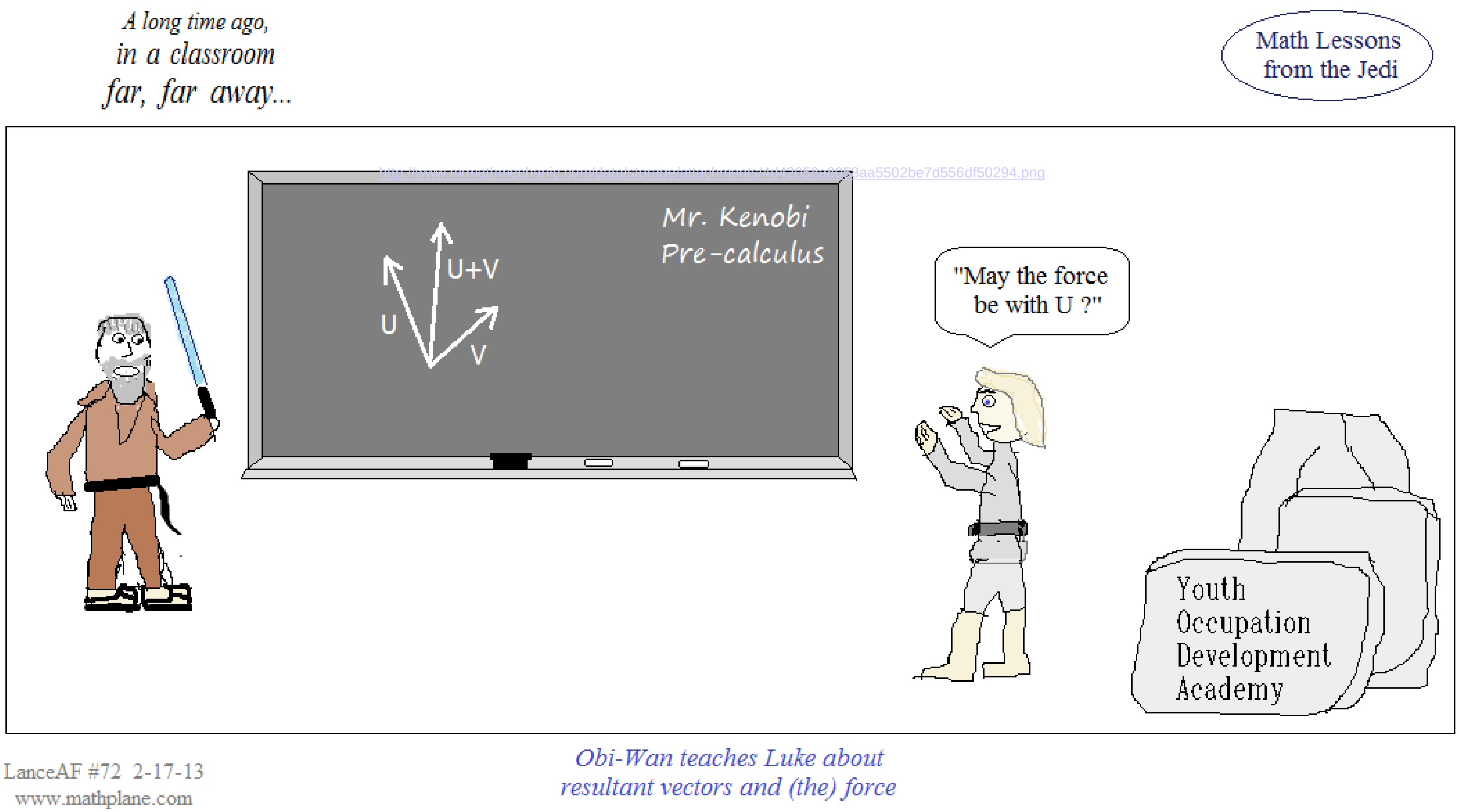
  - But many engineering majors *will* have to worry about things more complicated than *vectors*: the dreaded **TENSORS!**

CULTURAL INTERLUDE

- A scalar is a 0^{th} rank *tensor*.
- A vector is a 1^{st} rank *tensor*.
- *A tensor* of rank n has 3^n components.
  - $3^0 = 1$ component for a scalar
  - $3^1 = 3$ components for a vector
  - $3^2 = 9$ components for a 2^{nd} rank tensor ~ 3 x 3 array or matrix
- The relation between stress and strain (*which are each 2^{nd} rank tensors*) in materials is a 4^{th} rank tensor with 81 parameters, 21 of which are independent. (*Symmetry eliminates the other 60 parameters.*)
  - *A force applied to the xz face of a cube can point in the x, y or z direction.*
  - BE HAPPY that you only see 1^{st} rank tensors $\equiv$ vectors in PHYS 121!
  - Life might be harder in ENGR 200, Statics and Strength of Materials.

We made it to slide #8 on Wednesday, January 21, 2015

PHYS 121 – SPRING 2015



CHAPTER 3: VECTORS

version 01/23/2015, ~ 41 slides

We made it to slide #8 on Wednesday, 1/21/2015.

EXAM #2 & #3 DATES

Answer	Response	%
Wednesday, March 18 & April 15	14	10%
Friday, March 20 & April 17	115	81%
Wednesday, March 25 & April 22	8	6%
Friday, March 27 & April 24	5	4%
Total	142	100%

The winner is Friday, March 20 & April 17!
These dates are posted on Blackboard.

VECTOR SYMBOLS

Displacement, velocity, acceleration and force are vectors.

A vector is indicated by putting an arrow above a symbol.

$$\vec{\mathrm{r}}, \vec{\mathrm{v}}, \vec{\mathrm{a}} \ \& \ \vec{\mathrm{F}}$$

It's difficult to put arrows above symbols in most software; a simpler alternative is to type the vector quantity in **boldface,** as in

$$\mathbf{r}, \mathbf{v}, \mathbf{a} \ \& \ \mathbf{F}$$

The *magnitude* of a vector is the symbol in italics w/o bolding.

$$r, v, a \ \& \ F \quad \text{with } r = |\mathbf{r}|$$

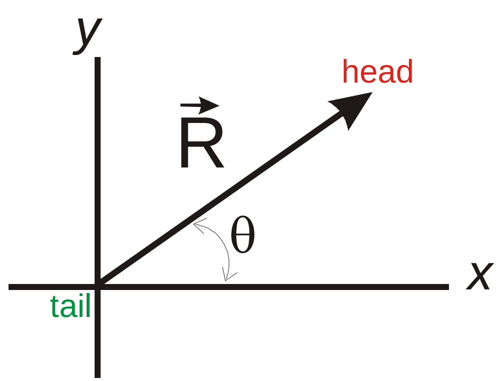
But there are variations in style & carefulness.

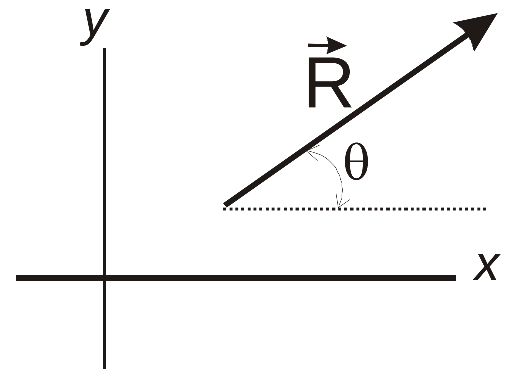
MAGNITUDE & DIRECTION

A vector quantity such as displacement $\vec{R}$ (*physicists often use the symbol R or **r** for displacement and position*) can be described in terms of the vector's

magnitude & direction

with the *convention* that the direction be measured as an angle θ *wrt* the positive *x*-axis CCW from that axis, unless you are told otherwise.





- The vector **R** above is drawn with its **tail** at the origin.
- The length of the arrow is proportional to the magnitude of **R**.
- A negative sign reverses the direction of **R**
- A constant in front can change its magnitude, as in α**R**
- Vectors have magnitude & direction but NOT position.

You can shift vectors parallel to your x, y or z axis for convenience.

ADDING VECTORS GRAPHICALLY/GEOMETRICALLY

Similar vector quantities, such as two displacements (*or two velocities or two accelerations*) can be added, as in

$$\vec{A} + \vec{B} = \vec{C}$$

Displacement **C** describes where you end up if you first do displacement **A** followed by displacement **B**.

A vector *sum* is often called a *resultant*.

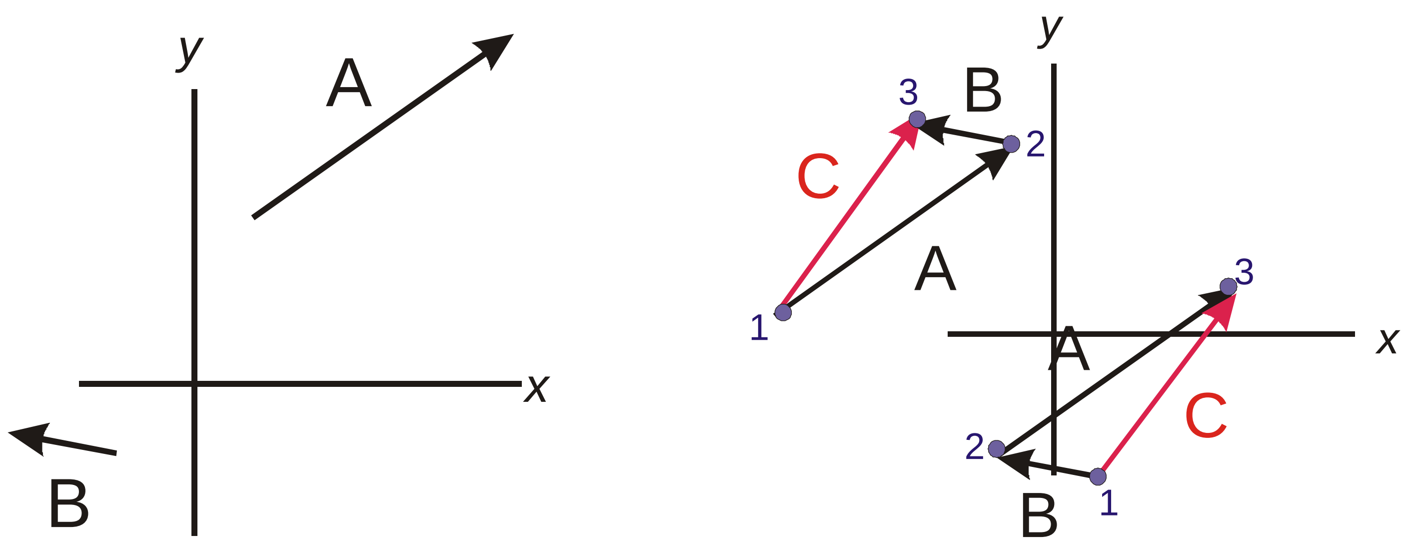
Subtracting vectors:

$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

- To add vectors graphically, position them <u>tail to head</u>.
  - Connect the tail of **B** to the head of **A** to get **A + B = C.**
  - **C**’s tail is the tail of **A** while **C’s** head is the head of **B**.
  - Displacement $1 \rightarrow 2 \rightarrow 3$ = displacement $1 \rightarrow 3$

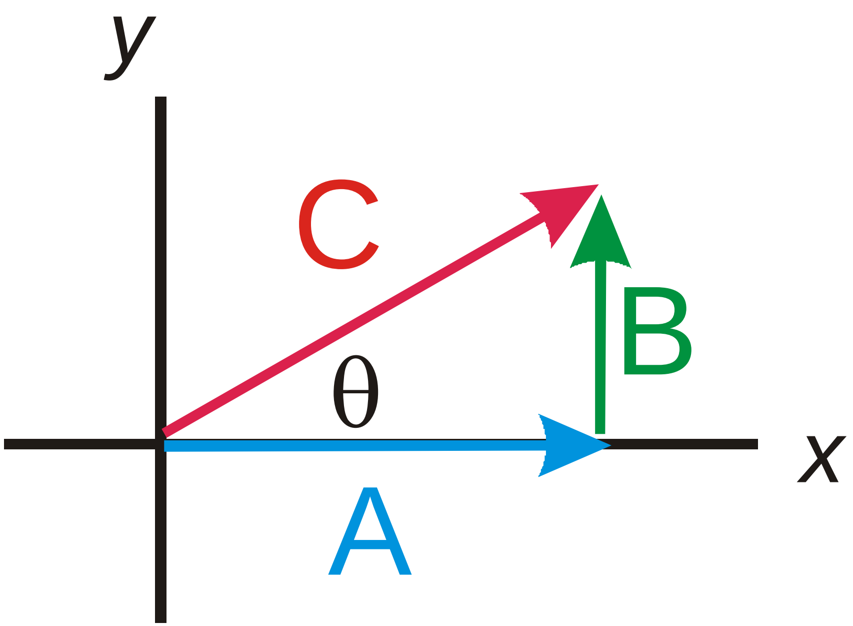
B* + *A *gives the same result as* ***A* + *B***.

Vector addition is commutative (*and associative*)



- You need some high school geometry and/or trig to obtain a numerical description of **C** if you add vectors graphically.

If you're lucky, there's a right angle between **A** & **B** and you can use Pythagorean's Theorem to calculate **C.**



$$\left|\vec{C}\right| = \sqrt{\left|\vec{A}\right|^2 + \left|\vec{B}\right|^2}$$

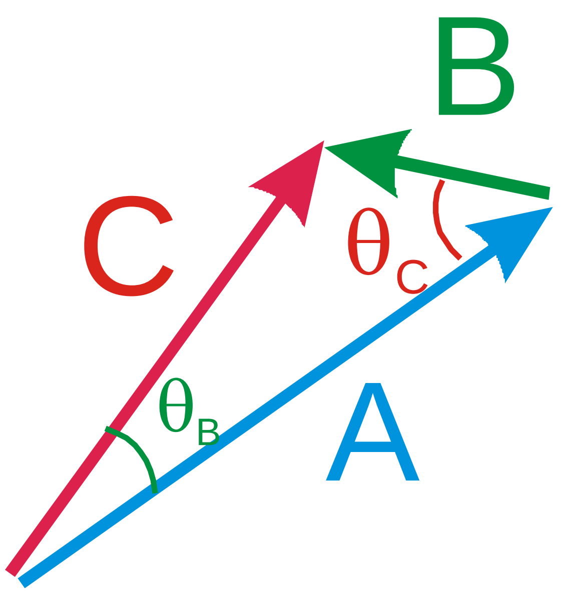
$$\theta = \tan^{-1}\left(\frac{opposite}{adjacent}\right) = \tan^{-1}\left(\frac{\left|\vec{B}\right|}{\left|\vec{A}\right|}\right)$$

If you're not lucky, you can use the Law of Sines and/or the Law of Cosines to find C.

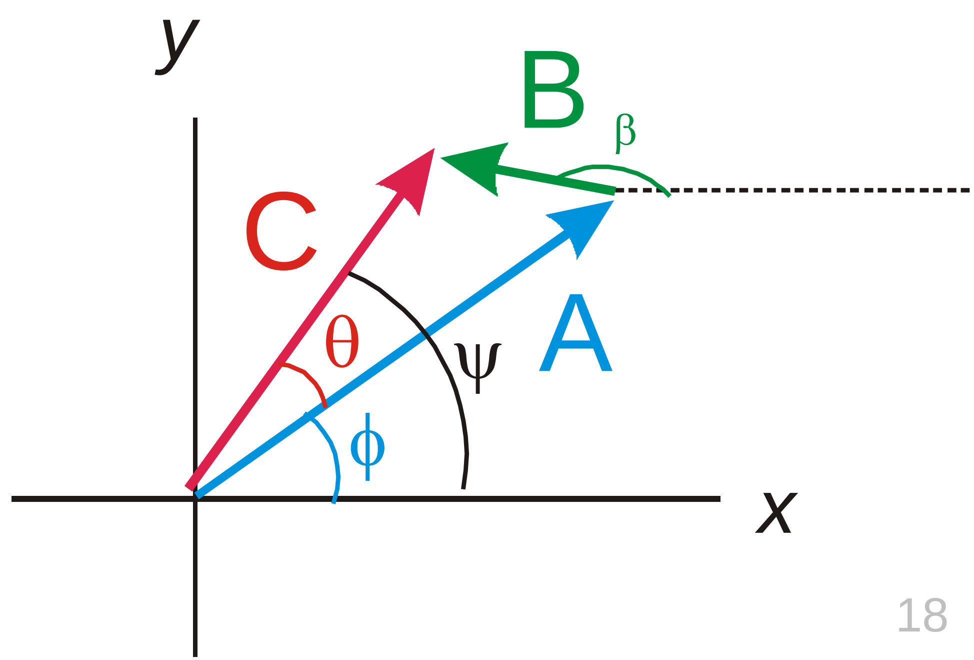
$$\frac{\sin\theta_A}{A} = \frac{\sin\theta_B}{B} = \frac{\sin\theta_C}{C}$$

$$\left|\vec{C}\right|^2 = \left|\vec{A}\right|^2 + \left|\vec{B}\right|^2 - 2\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{AB}$$

θ_{AB} is the smallest angle between **A** & **B** when they are placed tail-to-tail.



Calculating the angle ψ
is annoying!



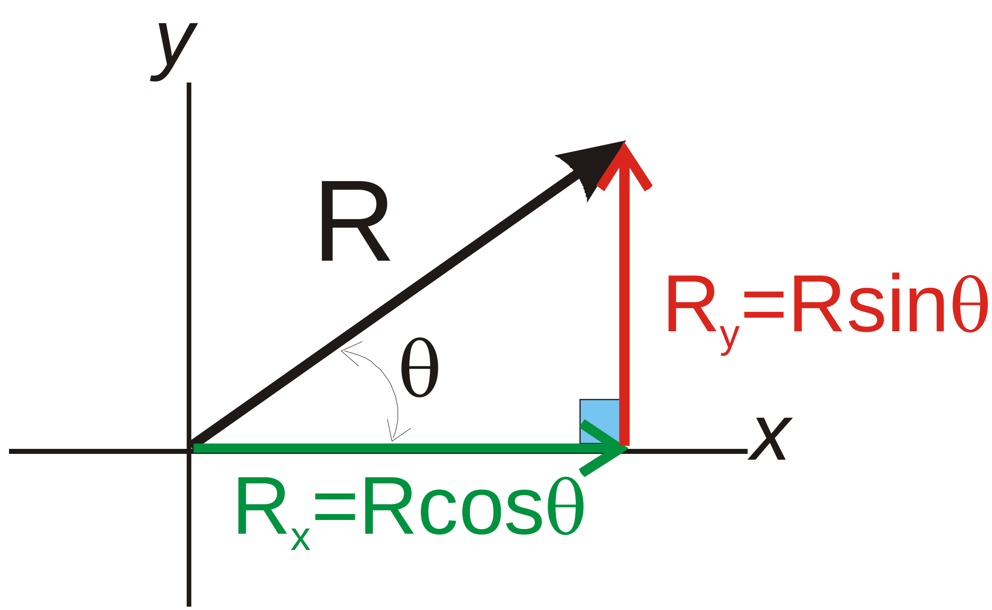
VECTOR COMPONENTS

It's often convenient to think of a vector **R** in terms of "*component*" vectors parallel to each axis that *add* to form **R**.

≡ *resolving it into its components*.

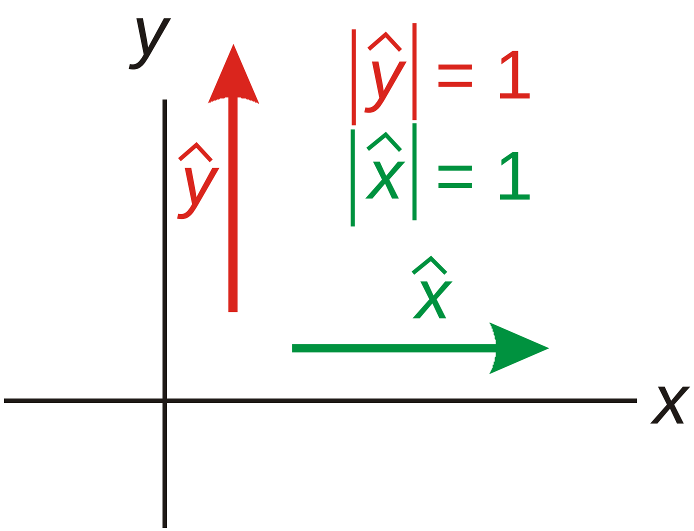
R's components $\mathbf{R_x}$ and $\mathbf{R_y}$ for orthogonal *xy* coordinate system axes are shown below.

ADD = a displacement $\mathbf{R_x}$ followed by a displacement $\mathbf{R_y}$ is equivalent to a displacement **R.**



UNIT VECTORS & COMPONENTS

- A ***unit vector*** is a vector whose magnitude ≡ 1 with no units.
- Unit vectors generally come in sets, with one for each axis of your coordinate system pointing parallel to each axis.
- A unit vector is identified with a small '*hat*' over it instead of a normal vector arrow symbol.
- Most people use $\hat{i}, \hat{j}\ \&\ \hat{k}$ in an *xyz* coordinate system.
- Dr. C. prefers $\hat{x}, \hat{y}\ \&\ \hat{z}$.



UNIT VECTORS & COMPONENTS

You can multiply a unit vector by a number to give it a value other than 1, plus the units associated with that number.

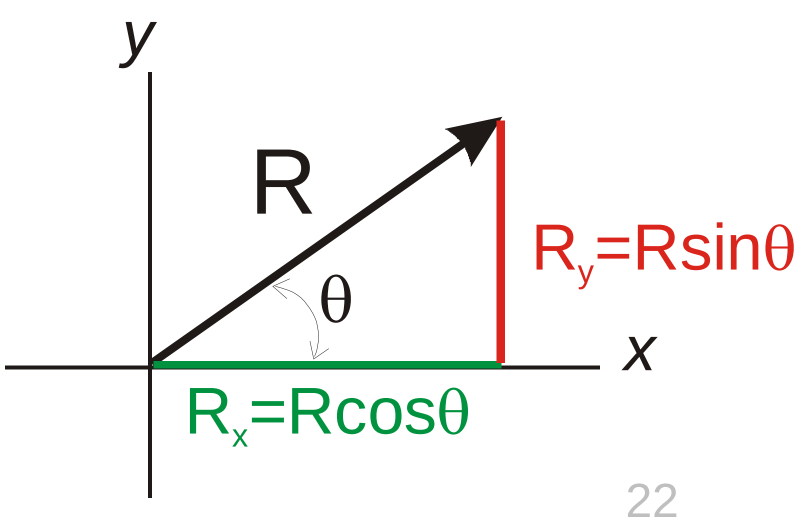
The values associated with each unit vector are the ***COMPONENTS*** of the overall vector.

Add the components to form a normal vector.

A displacement or position vector in 3-D can described as

$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k} = R_x\hat{x} + R_y\hat{y} + R_z\hat{z}$$

R_x , R_y & R_z are the components of **R.**

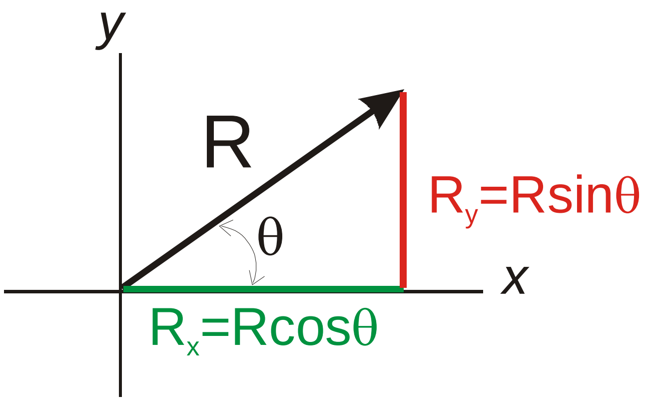


COMPONENTS *vs* MAGNITUDE/DIRECTION

The connection between these descriptions in 2D is

$$R_x = R\cos\theta \qquad R_y = R\sin\theta$$

$$R = |R| = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



You DO have to remember your trig!

sin = opposite/hypotenuse

cos = adjacent/hypotenuse [*'cos' ~ close?*]

tan = sin/cos = opposite/adjacent

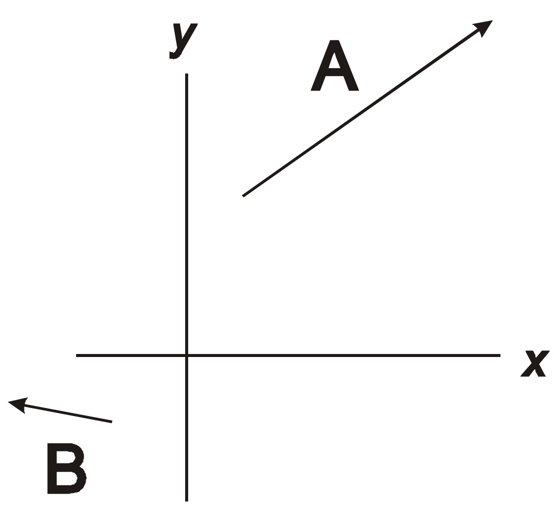
Do I need to include this on exam formula sheets?

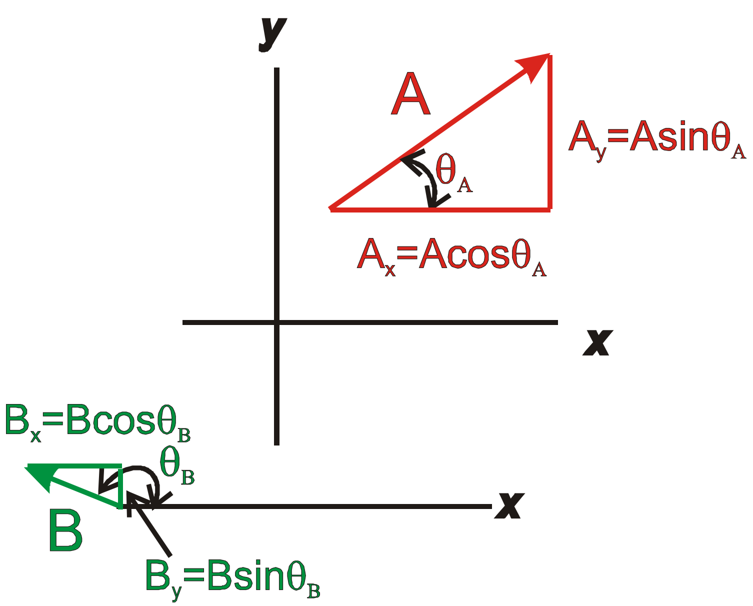
ADDING VECTORS USING COMPONENTS

It's often easier to carry out vector addition with components

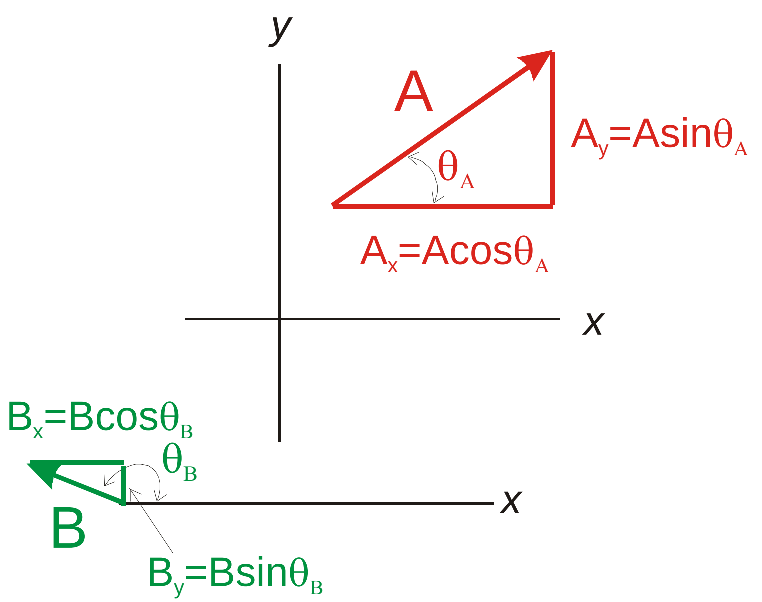
(*rather than the Laws of Sines & Cosines*)

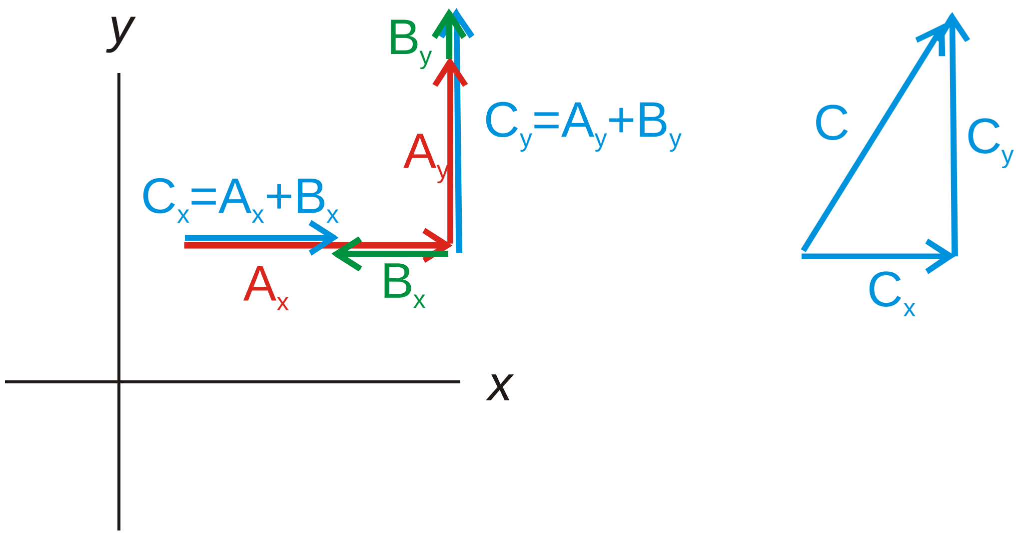
- Resolve each vector into its *x* and *y* components.





- Add each type (x, y, z) of component like normal numbers.





- $C_x = A_x + B_x \qquad C_y = A_y + B_y$
- If you want the answer in terms of magnitude & direction

$$\left|\vec{C}\right| = \sqrt{C_x^2 + C_y^2} \qquad\qquad \theta_{\text{wrt pos x-axis}} = \tan^{-1}\frac{C_y}{C_x}$$

DISPLACEMENT

DISPLACEMENT ≡ **CHANGE** in position

In 1D, displacement $\Delta x = x_{final} - x_{original} = x_f - x_o$

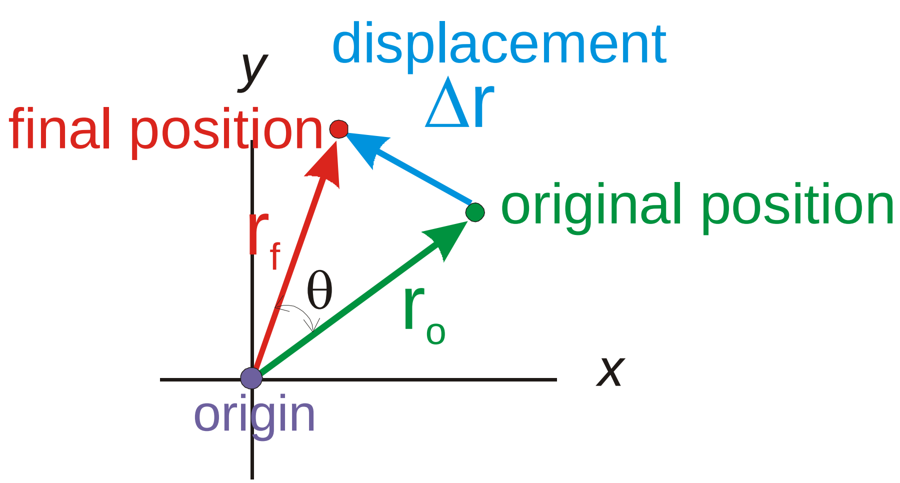
In 2D & 3D, displacement = $\Delta \mathbf{r} = \mathbf{r}_{final} - \mathbf{r}_{original} = \mathbf{r}_{\mathrm{f}} - \mathbf{r}_o$

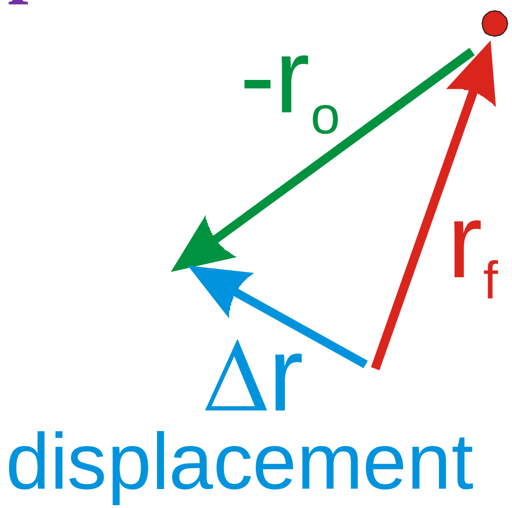
Subtracting a vector such as $\mathbf{r}_o$ is the same as *adding* $-\mathbf{r}_o$

$-\mathbf{r}_o$ is found from $\mathbf{r}_o$ by reversing the vector head-to-tail graphically

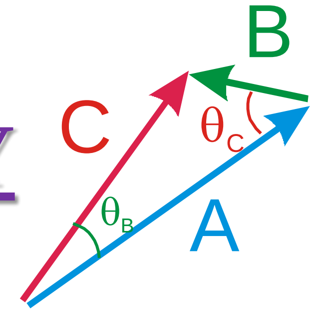
= changing the signs of each of its components

You can calculate a displacement by summing vectors graphically or with components.



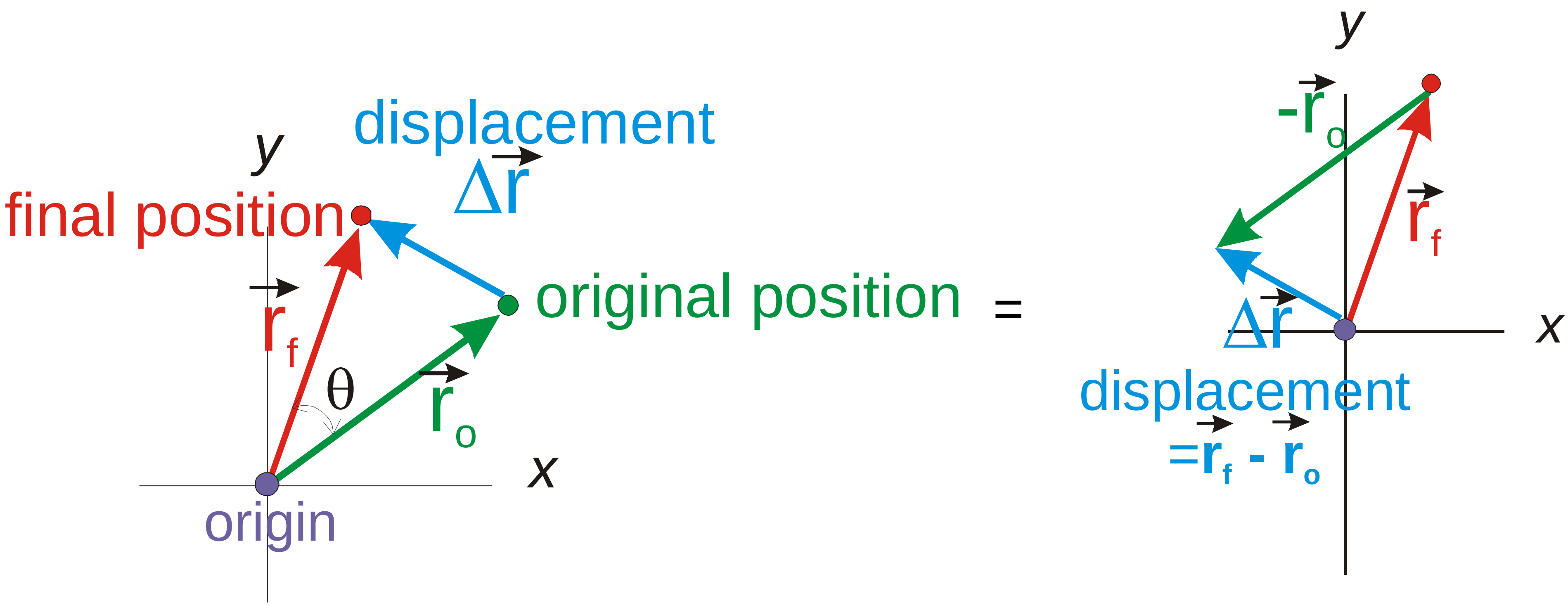


DISPLACEMENT GRAPHICALLY



$\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_o$ is related to $\mathbf{r}_o$ and $\mathbf{r}_f$ by the laws of sines & cosines.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad c^2 = a^2 + b^2 - 2ab\cos C$$



DISPLACEMENT with COMPONENTS

$$\Delta r_x = r_{fx} - r_{ox}$$
$$\Delta r_y = r_{fy} - r_{oy}$$
$$\Delta r_z = r_{fz} - r_{oz}$$

$$\Delta\vec{r} = \Delta r_x\hat{x} + \Delta r_y\hat{y} + \Delta r_z\hat{z}$$

If you want $\Delta\mathbf{r}$ in terms of magnitude & direction

$$\left|\Delta\vec{r}\right| = \sqrt{\left(\Delta r_x\right)^2 + \left(\Delta r_y\right)^2 + \left(\Delta r_z\right)^2}$$

In 2D: $\theta_{\text{wrt pos x-axis}} = \tan^{-1}\dfrac{\Delta r_y}{\Delta r_x}$

Velocity and acceleration vectors and follow the same vector rules as displacement, just replace the ***r*** with ***v*** or ***a***.

POSITION in 3D

3D systems are hard to draw in 2D; you need to use perspective.

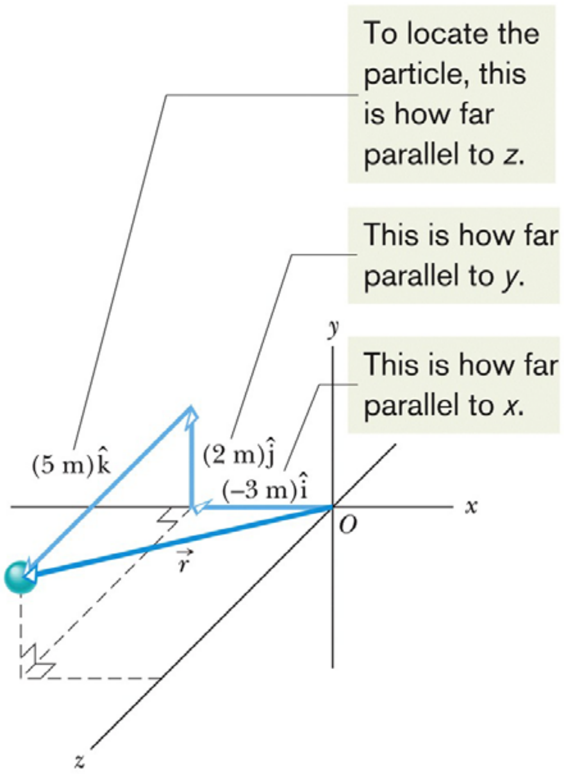
It's also tricky to describe direction in 3D; 2 angles are necessary.

like latitude & longitude or *angles with respect to two axes*

It's straight-forward to use components in 3D.

$$\vec{r} = -3\hat{x} + 2\hat{y} + 5\hat{z} \quad \text{(in meters)}$$

$$|r| = \sqrt{r_x^2 + r_y^2 + r_z^2}$$



3D POSITION ANALYTICALLY

Position in 1D might be given in the form of an equation like

$$x(t) = x_o + v_o t + ½at^2$$

Position in 2D might be given as a pair of *parametric* equations (*in terms of the 'parameter' t*) like: $x(t) = x_o + v_{xo}t + ½a_x t^2$ & $y(t) = y_o + v_{yo}t + ½a_y t^2$

EXAMPLE: $x(t) = 2 + 3t$ $y(t) = 10 - 4.9t^2$ describes an object that ???

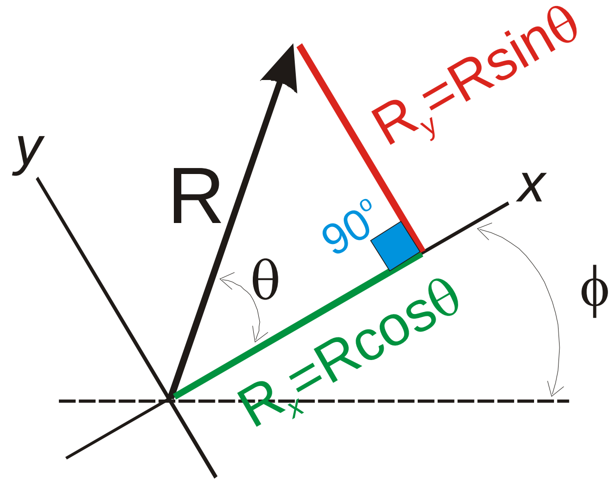
- starts at x,y = 2, 10 m
- the horizontal component of velocity v_x is a constant 3 m/s
- accelerates downwards at $a = -g = -9.8$ m/s^2
- starting with an initial $v_y = 0$

$y(x)$ or $x(y)$ might also be useful

- These describe the path more directly on an *xy* plot
- Time does not appear ⇒ velocity & acceleration are uncertain.
- In this example, solve for $t(x)$ & plug into $y(t)$.

TILTED COORDINATE SYSTEM

We don't have to stick with *x* = horizontal and *y* = vertical.

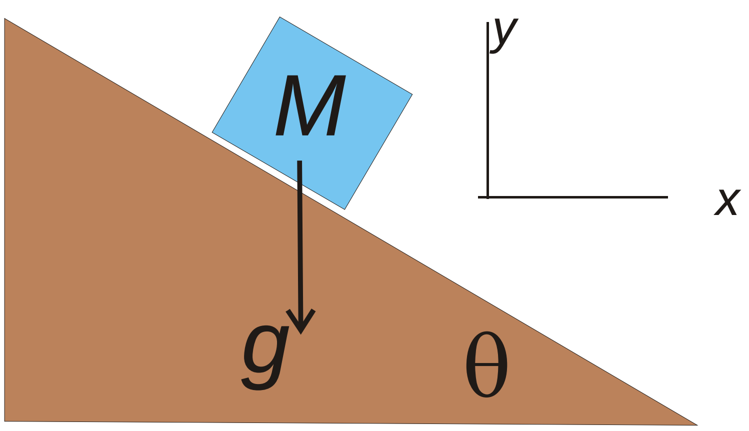


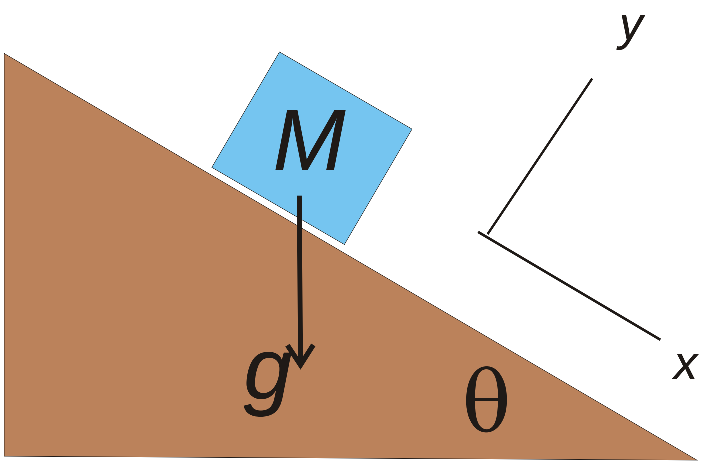
- Break the vector into components that are parallel to the axes of your coordinate system.
- Your axes, and therefore your components, are perpendicular to each other
- Think in terms of right triangles with a 90° angle between the components.
- Former PHYS 121 student: "*Components can just be simply thought of as triangle problems*."

TILTED COORDINATE SYSTEM

We'll examine systems this semester where horizontal and vertical are NOT the best options for the x and y axes of our coordinate system.

- One example is an inclined plane that makes an angle θ with respect to horizontal.
- What is the component of *g along the incline?*
- What is the component of *g perpendicular to the incline?*



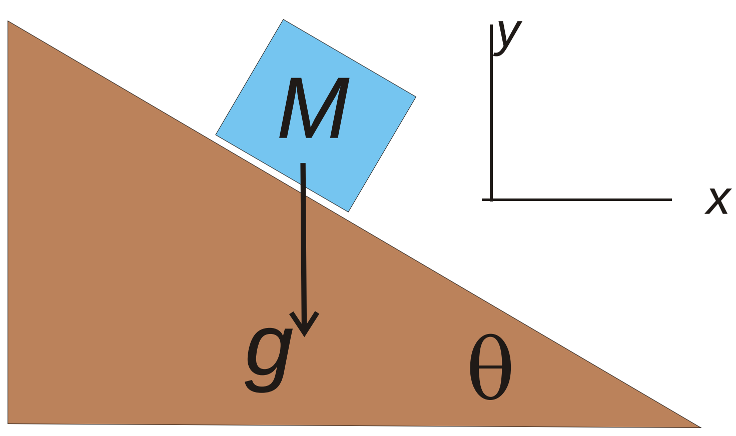


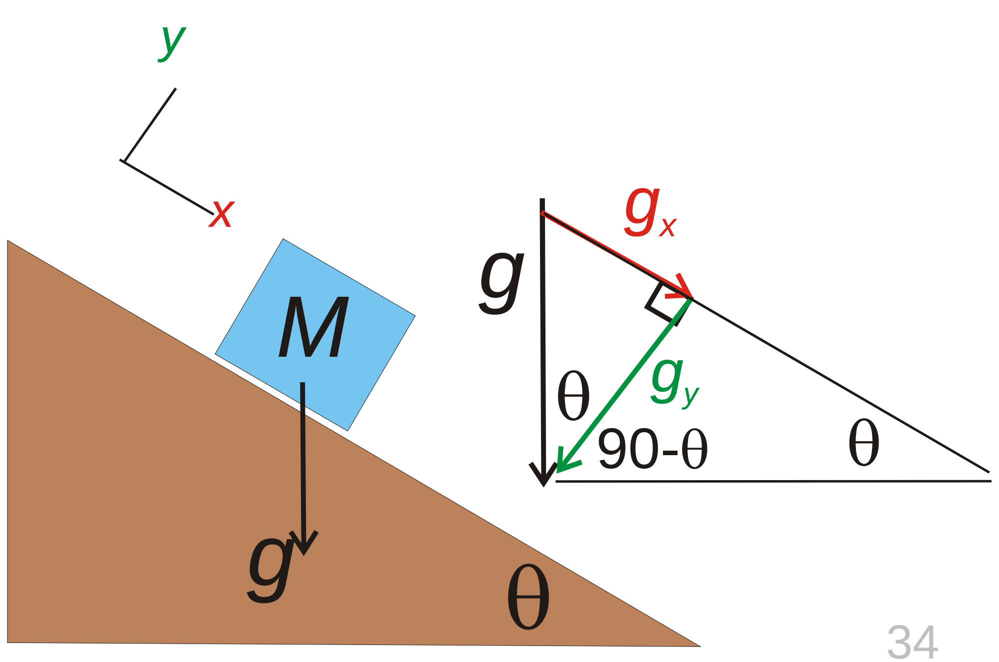
TILTED COORDINATE SYSTEM

In the figure on the left, $\vec{g} = -g\hat{y}$

In the figure on the right,

$$\vec{g} = g\sin\theta\,\hat{x} - g\cos\theta\,\hat{y}$$





VECTOR MULTIPLICATION

There are two different methods for multiplying one vector by another vector, depending on the physics involved.

The DOT product is useful when discussing WORK.

$$\vec{A} \bullet \vec{B} = \vec{C}$$

The CROSS product is useful for ROTATIONAL DYNAMICS.

$$\vec{A} \times \vec{B} = \vec{C}$$

We'll delay discussing vector multiplication methods until we need them in chapters 7 & 13.

SIX CLICKER QUESTIONS FOLLOW

to try on your own

- 5 options

Which of the following expressions is *false* concerning the vectors shown in the sketch?

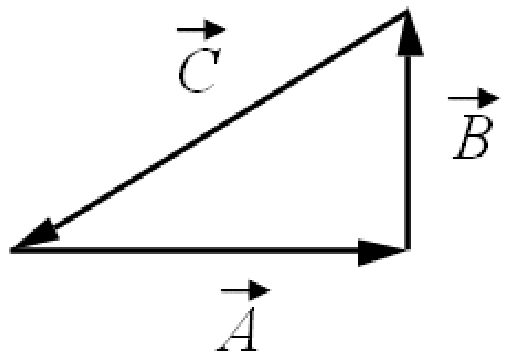
a) $\vec{C} + \vec{A} = -\vec{B}$

b) $\vec{C} = \vec{A} + \vec{B}$

c) $\vec{A} + \vec{B} + \vec{C} = 0$

d) $C < A + B$

e) $A^2 + B^2 = C^2$



- 5 options

Which of the following expressions is *false* concerning the vectors shown in the sketch?

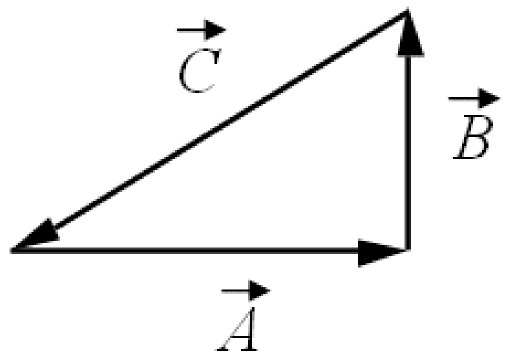
a) $\vec{C} + \vec{A} = -\vec{B}$

b) $\vec{C} = \vec{A} + \vec{B}$

c) $\vec{A} + \vec{B} + \vec{C} = 0$

d) $C < A + B$

e) $A^2 + B^2 = C^2$



b is false: **A** + **B** = **-C**

- 5 options

Two vectors $\vec{a}$ and $\vec{b}$ are added together to form a vector $\vec{c}$.
The relationship between the magnitudes of the vectors is given by $a + b = c$.
Which one of the following statements concerning these vectors is true?

a) $\vec{a}$ and $\vec{b}$ must point in the same direction.

b) $\vec{a}$ and $\vec{b}$ must be displacements.

c) $\vec{a}$ and $\vec{b}$ must be at right angles to each other.

d) $\vec{a}$ and $\vec{b}$ must point in opposite directions.

e) $\vec{a}$ and $\vec{b}$ must have equal lengths.

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The relationship between the magnitudes of the vectors is $a^2 + b^2 = c^2$.
Which one of the following statements concerning these vectors is true?

a) $\vec{a}$ and $\vec{b}$ must be parallel.

b) $\vec{a}$ and $\vec{b}$ could have any orientation relative to each other.

c) $\vec{a}$ and $\vec{b}$ must be at right angles to each other.

d) $\vec{a}$ and $\vec{b}$ must point in opposite directions.

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- 4 options

What is the minimum number of vectors with <u>*unequal*</u> magnitudes whose vector sum can be zero?

a) 2
b) 3
c) 4
d) 5
e) 6

 - 4 options

What is the minimum number of vectors with *unequal* magnitudes whose vector sum can be zero?

a) 2
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c) 4
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e) 6

- 5 options

A PHYS 121 student adds two displacement vectors with magnitudes of 8.0 km and 6.0 km.

Which one of the following statements is TRUE concerning the magnitude of the resultant displacement?

a) The magnitude must be 14.0 km.

b) The magnitude must be 10.0 km.

c) The magnitude could be equal to zero kilometers, depending on how the vectors are oriented.

d) The magnitude could have any value between 2.0 km and 14.0 km, depending on how the vectors are oriented.

e) No conclusion can be reached without knowing the directions of the vectors.

- 5 options

A PHYS 121 student adds two displacement vectors with magnitudes of 8.0 km and 6.0 km. Which one of the following statements is true concerning the magnitude of the resultant displacement?

a) The magnitude must be 14.0 km.

b) The magnitude must be 10.0 km.

c) The magnitude could be equal to zero kilometers, depending on how the vectors are oriented.

d) The magnitude could have any value between 2.0 km and 14.0 km, depending on how the vectors are oriented.

e) No conclusion can be reached without knowing the directions of the vectors.

To picture the possible magnitudes, draw a circle of radius 6 km around the head of the 8 km vector. The resultant displacement can be anything that starts at the tail of the 8 km vector and ends on this circle.

- 4 options

Two displacement vectors of magnitudes 21 cm and 79 cm are added. Which one of the following is the only possible option for the magnitude of the resultant?

a) 0 cm

b) 28 cm

c) 37 cm

d) 82 cm

e) 114 cm

- 4 options

Two displacement vectors of magnitudes 21 cm and 79 cm are added. Which one of the following is the only possible option for the magnitude of the resultant?

a) 0 cm

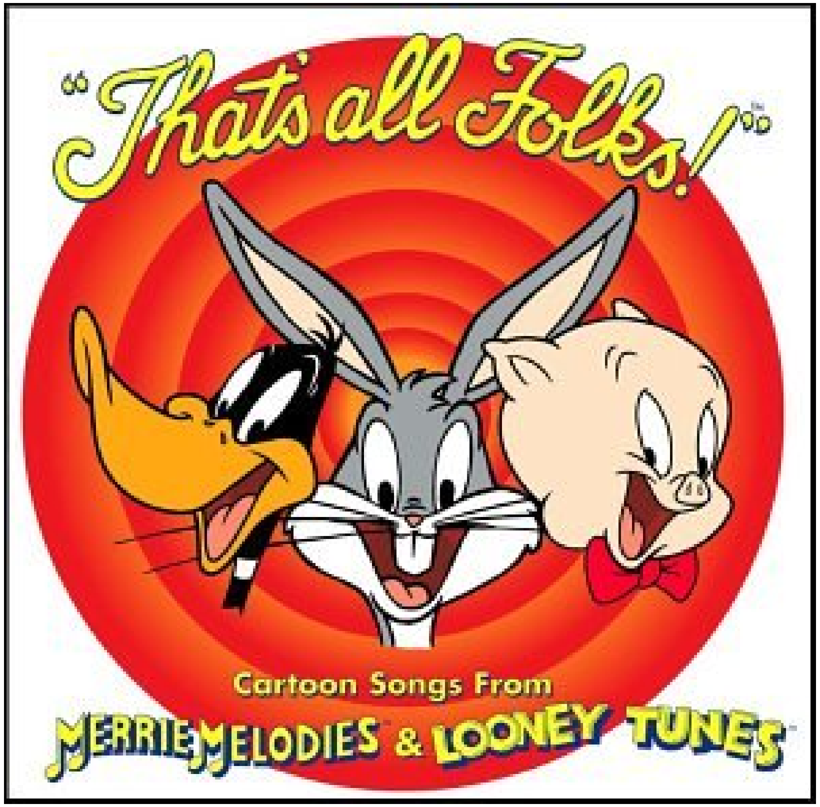
b) 28 cm

c) 37 cm

d) 82 cm

e) 114 cm

THE END!



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