PHYS 121 – SPRING 2015

Chapter 4: Motion in 2 & 3 Dimensions

version 01/23/2015 , ~ 80 slides

We made it to slide #52 on Monday, January 26, 2015.

VELOCITY as a VECTOR

In 1D, position *wrt* some origin is given by *x*.

In 3D, position is given by $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

The $\hat{x}$, $\hat{y}$, & $\hat{z}$ components can generally be treated independently* of each other.

In 1D, $v = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$

In 3D, $v = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt}$

* Unless there is a *constraint*, like being confined to a circular track.

AVERAGE & INSTANTANEOUS VELOCITY

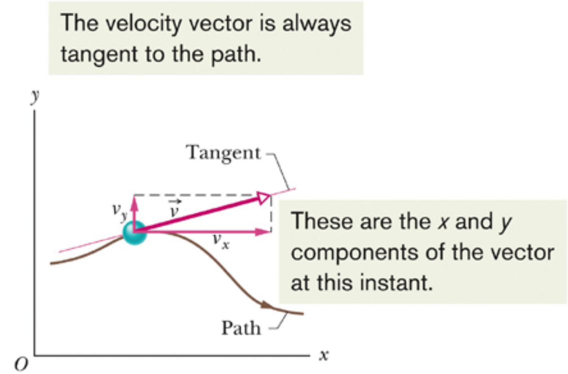
$$\vec{v}_{avg} = \frac{\text{total displacement}}{\text{time}} = \frac{\Delta\vec{r}}{\Delta t} = \left(\frac{\Delta x}{\Delta t}\right)\hat{x} + \left(\frac{\Delta y}{\Delta t}\right)\hat{y} + \left(\frac{\Delta z}{\Delta t}\right)\hat{z}$$

$$\vec{v}_{\text{instantaneous}} = \vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt}$$

$$\vec{v} = (v_x)\hat{x} + (v_y)\hat{y} + (v_z)\hat{z} = \left(\frac{dx}{dt}\right)\hat{x} + \left(\frac{dy}{dt}\right)\hat{y} + \left(\frac{dz}{dt}\right)\hat{z}$$

v is *tangent* to the path of a particle at any given time, since

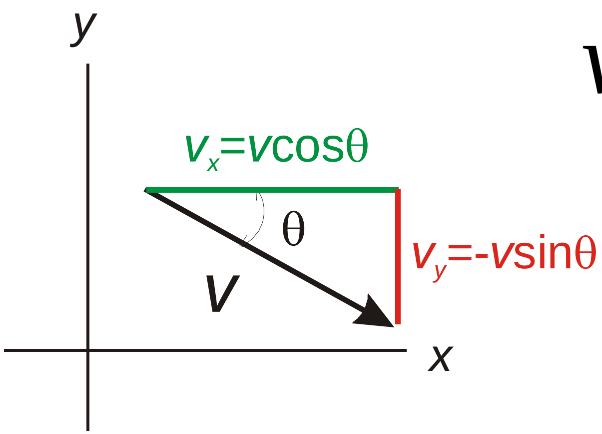
v must point to the position of the particle an instant later.



VELOCITY in 2-D

magnitude & direction

Speed (*magnitude of* **v**) and *direction* $\leftrightarrow$ v_x & v_y



$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1}\left(\frac{|v_y|}{|v_x|}\right)$$

Notice that I'm giving θ CW, <u>down</u> from the positive x-axis.

This is not the conventional CCW choice but it's okay because I am clearly showing you what I've done.

I could instead give θ a negative value or use 360° - θ.

AVERAGE & INSTANTANEOUS VELOCITY EXAMPLE

$$v_{avg} = \frac{\Delta \vec{r}}{\Delta t} \qquad v_{avg} = \frac{\text{total displacement}}{time} = (\Delta x)\hat{x} + (\Delta y)\hat{y} + (\Delta z)\hat{z}$$

$$\vec{v}_{\text{instantaneous}} = \vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt} \qquad \vec{v} = (v_x)\hat{x} + (v_y)\hat{y} + (v_z)\hat{z} = \left(\frac{dx}{dt}\right)\hat{x} + \left(\frac{dy}{dt}\right)\hat{y} + \left(\frac{dz}{dt}\right)\hat{z}$$

A particle's position is described by ***R***; find ***v****instantaneous*

(THEN plug in specific t's if necessary)

$$\vec{R} = \left(2 + 3t + 4t^2\right)\hat{x} + \left(5 - 3t\right)\hat{y}$$

$$\vec{v}(t) = \frac{d\vec{R}}{dt} = (3 + 8t)\hat{x} - 3\hat{y}$$

$$\left|\vec{v}(t)\right| = \sqrt{v_x^2 + v_y^2} = \sqrt{(3+8t)^2 + 9}$$

$$\theta(t) = \tan^{-1}\left(\frac{v_y}{v_x}\right)\tan^{-1}\left(\frac{-3}{3+8t}\right)$$

AVERAGE & INSTANTANEOUS VELOCITY EXAMPLE

$$v_{avg} = \frac{\Delta \vec{r}}{\Delta t} \qquad v_{avg} = \frac{\text{total displacement}}{time} = (\Delta x)\hat{x} + (\Delta y)\hat{y} + (\Delta z)\hat{z}$$

$$\vec{v}_{\text{instantaneous}} = \vec{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{r}}{\Delta t}\right) = \frac{d\vec{r}}{dt} \qquad \vec{v} = (v_x)\hat{x} + (v_y)\hat{y} + (v_z)\hat{z} = \left(\frac{dx}{dt}\right)\hat{x} + \left(\frac{dy}{dt}\right)\hat{y} + \left(\frac{dz}{dt}\right)\hat{z}$$

A particle's position is described by ***R***; find ***v**$_{AVG}$*

$$\vec{R} = \left(2 + 3t + 4t^2\right)\hat{x} + \left(5 - 3t\right)\hat{y}$$

$$\vec{v}_{AVG}(t) = \frac{\Delta R}{\Delta t} = \frac{R_f - R_o}{t_f - t_o}$$

$$\vec{v}_{AVG}(t) = \frac{\left(3\left(t_f - t_o\right) + 4\left(t_f^2 - t_o^2\right)\right)\hat{x} - 3\left(t_f - t_o\right)\hat{y}}{t_f - t_o}$$

AVERAGE & INSTANTANEOUS ACCELERATION

$$\vec{a}_{avg} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{\Delta\vec{v}}{\Delta t} = \left(\frac{\Delta v_x}{\Delta t}\right)\hat{x} + \left(\frac{\Delta v_y}{\Delta t}\right)\hat{y} + \left(\frac{\Delta v_z}{\Delta t}\right)\hat{z}$$

$$\vec{a}_{\text{instantaneous}} = \vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{v}}{\Delta t}\right) = \frac{d\vec{v}}{dt}$$

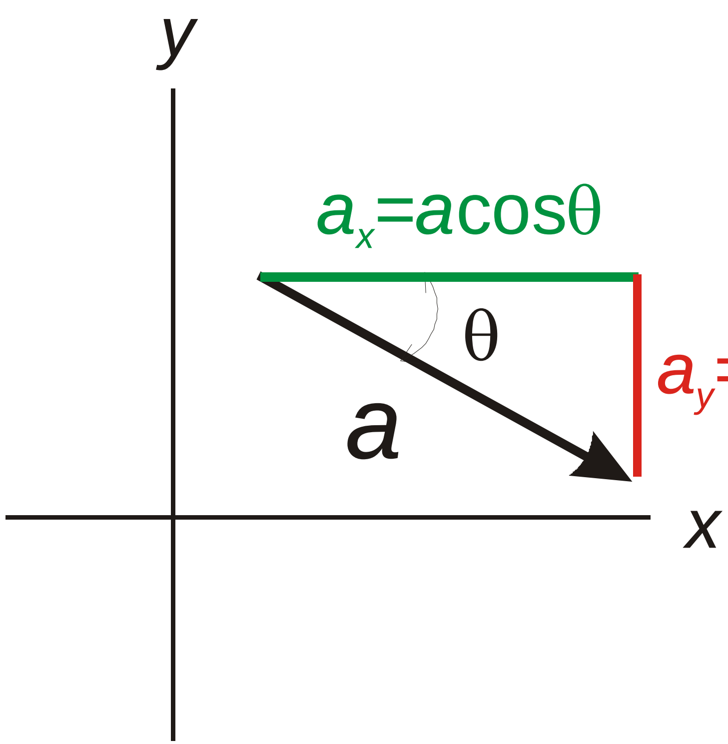
$$\vec{a} = (a_x)\hat{x} + (a_y)\hat{y} + (a_z)\hat{z} = \left(\frac{dv_x}{dt}\right)\hat{x} + \left(\frac{dv_y}{dt}\right)\hat{y} + \left(\frac{dv_z}{dt}\right)\hat{z}$$

- ***a*** is **NOT** *tangent* to the path of a particle at any given time
- ***a*** points in the direction of Δ***v*****,** not Δ***r***

ACCELERATION in 2-D

magnitude & direction

Acceleration in terms of magnitude & direction can be found from a_x and a_y.



$$a = |a| = \sqrt{a_x^2 + a_y^2}$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

ACCELERATION EXAMPLE

$$\vec{a}_{\text{instantaneous}} = \vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{v}}{\Delta t}\right) = \frac{d\vec{v}}{dt}$$

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$$\vec{R} = \left(2 + 3t + 4t^2\right)\hat{x} + \left(5 - 3t\right)\hat{y}$$

$$\vec{v}\left(t\right) = \frac{d\vec{R}}{dt} = \left(3 + 8t\right)\hat{x} - 3\hat{y}$$

$$\vec{a}\left(t\right) = \frac{d\vec{v}}{dt} = 8\hat{x}$$

$$\left|\vec{a}\left(t\right)\right| = 8\ \hat{x}\ \left(\text{m/s}^2\right)$$

$$\theta\left(t\right) = 0$$

PROJECTILE MOTION

Projectile motion (*aka ballistic motion*) is the motion of an object '*launched' near the earth's surface*

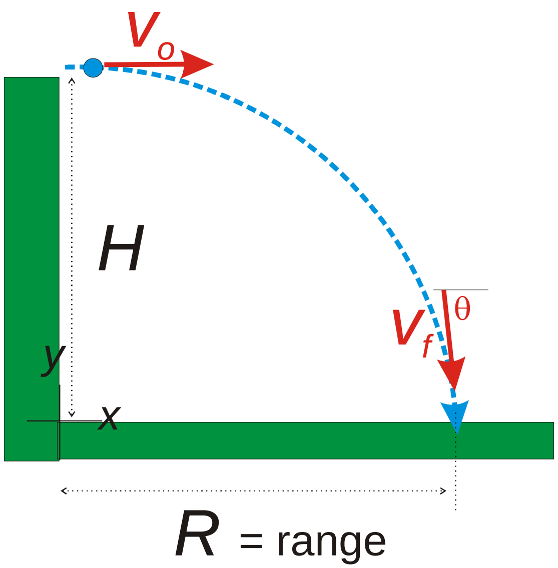
= subject to the earth's gravitational acceleration g.

- Projectile motion is confined to a *vertical plane*.

  This plane is defined by two lines,

  - one derived from the original velocity vector
  - the other from the direction of $\boldsymbol{g}$

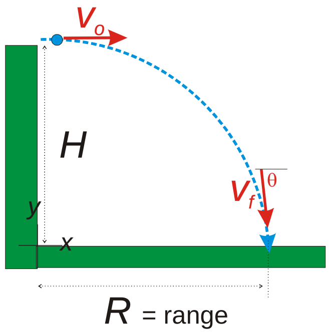
PROJECTILE MOTION



A normal projectile motion problem involves a particle *projected* into the air at some angle.

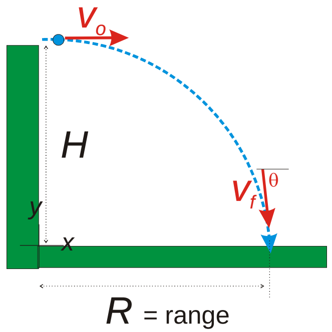
- We'll start with a simplified version where the particle starts (*or ends*) with pure horizontal motion.
- You kick a ball *horizontally* off of a cliff of height H at speed v_o.
  - Where does it land, *R*, and how long does it take to hit the ground?
- Treat the x & y components **separately**; combine them as needed.
  - **The time the ball spends in the air depends only on the vertical component of motion!**
  - **The horizontal displacement depends on the (*constant*) horizontal velocity and the time the ball spends in the air.**

y (*vertical*) COMPONENT



- The y-component of projectile motion is handled just like 1D free fall.
  The acceleration is a constant , $a = -g$ in the y direction (*if +y is up*)
- The time in the air depends solely on the distance it falls.
  - Use the constant acceleration formula with $a = -g$.
    $$y = y_0 + v_{oy}t - \tfrac{1}{2}gt^2$$
  - With $v_{oy} = 0$, $y - y_o = -H$ and
    $$t = \sqrt{\frac{2H}{g}}$$
  - The vertical component of velocity when it hits the ground is
    $$v_y = v_{oy} + at = -gt$$
  - These equations for $y(t)$ and $v_y(t)$ give a complete description of the vertical component of the motion.

x (horizontal) COMPONENT



There is no acceleration in the x direction ⇒ $\boldsymbol{a_x = 0}$.

$$\Rightarrow \quad v_x = v_{ox} + \cancel{a}_x t = v_{ox}$$

$$\boldsymbol{v_x = v_{ox}}$$

The component of velocity in the x direction is constant.

$$\Rightarrow \quad x = x_o + v_{ox}t + \tfrac{1}{2}\cancel{a}_x t^2$$

$$\boldsymbol{x = x_o + v_{ox}t}$$

The object travels at a constant velocity in the *x*-direction; gravity has no effect on this aspect of its motion, as long as the object is still in the air.

The distance it lands from the end of the table depends only on the initial horizontal velocity and the time it spends in the air.

$$t = \sqrt{\frac{2H}{g}} \qquad x = x_o + v_{ox}t \;\Rightarrow\; \left(x - x_o\right) = R = v_{ox}\sqrt{\frac{2H}{g}}$$

PARABOLIC PATH

The (*parametric*) equations for $x(t)$ & $y(t)$

$x(t) = x_o + v_{xo}t$ & $y(t) = y_o + v_{yo}t - \frac{1}{2} gt^2$

describe the projectile's path in terms of the time t

but it's also useful to know $y(x)$

which requires eliminating t from the equations for x & y

How do you eliminate t from $x(t)$ & $y(t)$?

PARABOLIC PATH *cont'd*

Starting with

$$x(t) \equiv x = x_o + v_{xo}t$$

we have

$$t = (x - x_o)/v_{xo}$$

Plugging this into the equation for $y(t)$

$$y(t) \equiv y = y_o + v_{yo}t - \tfrac{1}{2}\, gt^2$$

becomes

$$y = y_o + v_{yo}[(x - x_o)/v_{xo}] - \tfrac{1}{2}\, g\, [(x - x_o)/v_{xo}]^2$$

$$y = y_o + v_{yo}[(x - x_o)/v_{xo}] - \tfrac{1}{2}\, g\, [(x - x_o)/v_{xo}]^2$$

may not look parabolic!

In the spirit of the spherical cow, let's simplify this equation enough to appreciate its significance at a glance.

First, let's define

$$x_o = 0 \quad \& \quad y_o = 0$$

so that

$$y = v_{yo}[x/v_{xo}] - \tfrac{1}{2}\, g\, [x/v_{xo}]^2$$

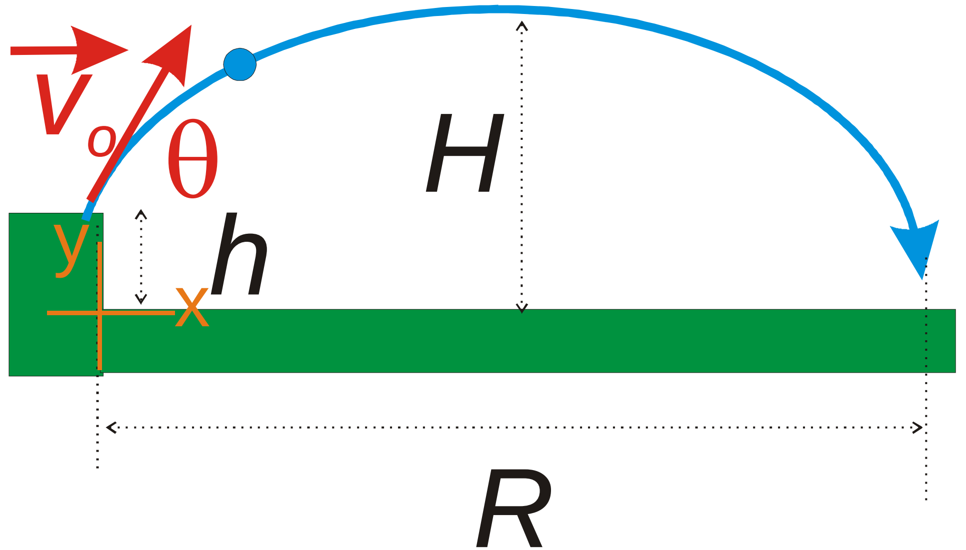
Next, we'll START (*set t = 0*) at the peak, so that $v_{yo} = 0$. Then

$$y = -\frac{g}{2v_{xo}^2}x^2$$

which is the equation for a parabola, opening downwards.

Projectile motion is parabolic motion.

The path for general projectile motion with v_o at some arbitrary angle is illustrated below.



h = difference in height for the starting point compared to the landing point $(h = y_o - y_f)$

H = maximum height reached during the motion ($v_y = 0$ there)

R = range/horizontal distance from the starting to the landing point

$\boldsymbol{v_o}$ = initial velocity **vector**

in terms of *components* $v_{xo} = |\boldsymbol{v_o}|\cos\theta$ & $v_{yo} = |\boldsymbol{v_o}|\sin\theta$

or *magnitude* $|\boldsymbol{v_o}| = v_o$ & *direction* θ (*most likely wrt horizontal*)

YOU ARE TYPICALLY GIVEN OR ASKED TO DETERMINE:

- $\boldsymbol{v_o}$ = initial velocity in terms of components v_x & v_y or magnitude and direction
- R = range or horizontal distance from the starting point to the landing point
- T = time the projectile is in the air
- $\boldsymbol{v_f}$ = final velocity when it hits the ground or passes some point in terms of components v_x & v_y or magnitude and direction
- H = maximum height reached during the motion
- T_H = time to reach the maximum height

PROJECTILE MOTION *cont'd*

What do you have to work with?

- *4 eqs. for constant acceleration for BOTH x and for y*

(8 equations total, only 4 for x are shown below)

$$v_x = v_{ox} + a_x t \qquad x = x_0 + v_{ox}t + ½\, a_x t^2$$

$$v_x^2 = v_{ox}^2 + 2a_x \Delta x \qquad x = x_o + ½(v_{ox} + v_x)t$$

- The acceleration in the horizontal (x) direction, $a_x = 0$, so

$$v_x = v_{xo} = |\mathbf{v_o}|\cos\theta \qquad x = x_o + v_{xo}t$$

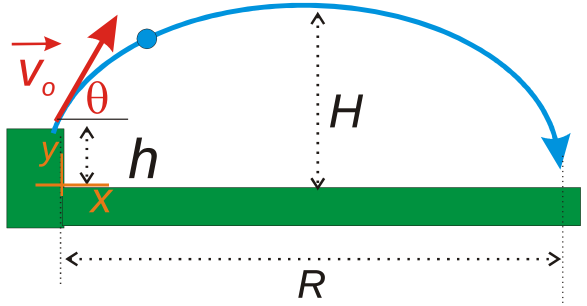
The projectile moves at constant speed in the x direction.

- The acceleration in the vertical direction is $g = 9.81$ m/s^2 downwards. Assuming $+y$ is defined as *up*:

$$v_{yo} = |\mathbf{v_o}|\sin\theta \qquad y = y_o + v_{yo}t - ½\, gt^2$$

- At the peak, $v_y = 0$

PROJECTILE MOTION EXAMPLE



A projectile is shot:

- from the edge of a cliff h = 140 m above the ground
- with an initial speed v_o = 60 m/s
- at an angle of 30° above horizontal.

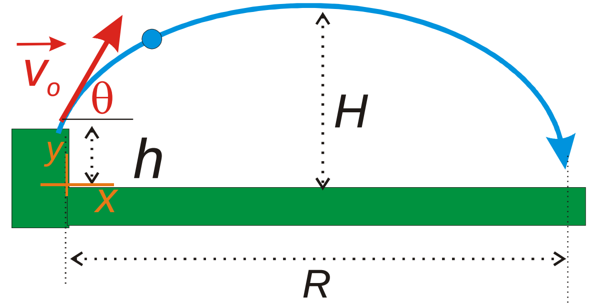
A. Determine the time taken by the projectile to hit the ground.

B. Determine the range *R* of the projectile as measured from the base of the cliff.

C. Determine the maximum height *H* reached by the projectile.

D. Determine the projectile's velocity (*magnitude & direction*) when it hits the ground.

PROJECTILE MOTION EXAMPLE



$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

STRATEGY

Motion in the *x* & *y* directions are *quasi-independent*

BUT the projectile only moves WHILE it is in the air.

So let's solve for the time it's in the air first.

$$y = y_o + v_{yo}t + \tfrac{1}{2}at^2$$

$$(y_o - y) + v_{yo}t - \tfrac{1}{2}gt^2 = 0$$

$$h + (|\mathbf{v_o}|\sin\theta)t - 4.9t^2 = 0$$

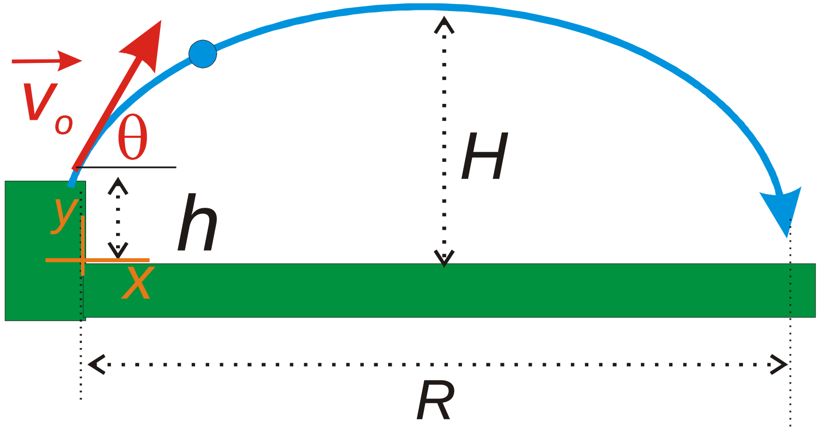
$$-(4.9\ m/s^2)t^2 + (60\ m/s)(\sin 30°)t + (140\ m) = 0$$

The solutions of this quadratic equation are

$t = -3.11\ s$ or $\underline{t = +9.22\ s}$

$t = -3.11\ s$ is not relevant; it's when $y = 0$ for negative x.

PROJECTILE MOTION EXAMPLE



Given the total time in the air, it's easy to calculate R.

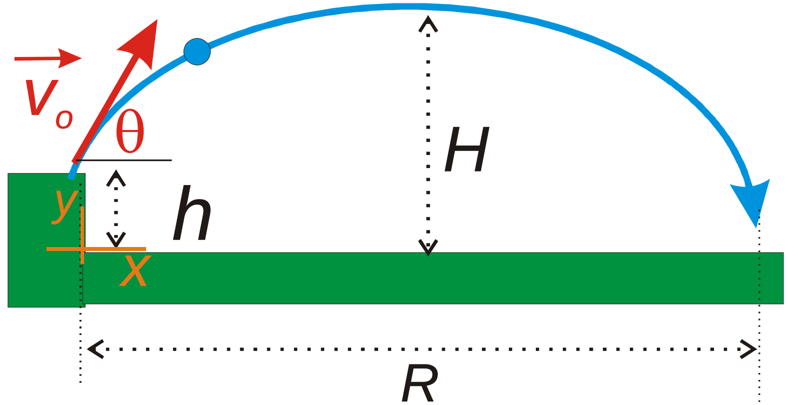
$$x = x_o + v_x t$$

$$v_x = v_{xo} = |\mathbf{v_o}|\cos\theta$$

$$R = x - x_o = v_{xo}t = (|\mathbf{v_o}|\cos\theta)t$$

$$= (60\ m/s)(\cos 30°)(9.22\ s) = \underline{479\ m}$$

PROJECTILE MOTION EXAMPLE



To solve for the maximum height, *H*,

use $v_y = 0$ at $\Delta y = H$.

$$v^2 = v_o^2 + 2a\Delta y$$

$$\Delta y = (v_y^2 - v_{oy}^2)/2(-g)$$

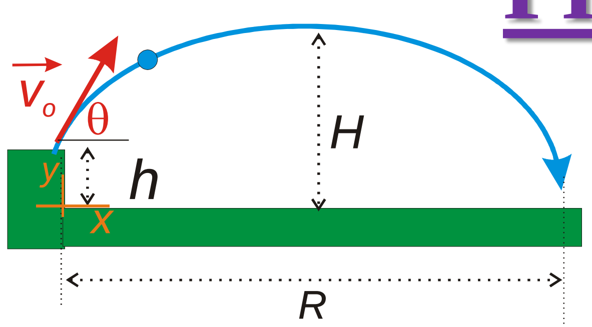
$$= (0 - v_o^2 \sin^2 30°)/2(-g) = 45.9\ m$$

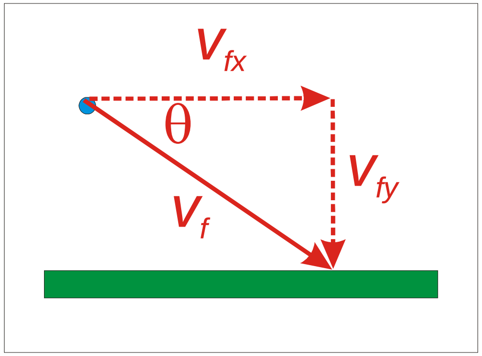
This is relative to the starting height $y_o = h$

$$H = h + \Delta y = 140\ m + 45.9\ m$$

= 185.9 *m*

PROJECTILE MOTION EXAMPLE





To solve for the velocity at which the projectile hits the ground:

Note that the velocity in the *x*-direction never changes

$$v_x = v_{xo} = |\mathbf{v_o}|\cos\theta = (60\ m/s)(\cos 30°) = \underline{\mathbf{52\ m/s}}$$

The velocity in the y-direction is given by

$v_y = v_{yo} - gt$ with $v_{yo} = |\mathbf{v_o}|\sin\theta$ and we found t already

$$v_y = (60\ m/s)\sin 30° - (9.8\ m/s^2)(9.22\ s) = \underline{\mathbf{-60.4\ m/s}}$$

This gives **v** in terms of its components. *How about magnitude & direction*?

$$|\mathbf{v}| = (v_x^2 + v_y^2)^{1/2} = (52^2 + 60.4^2)^{1/2} = \underline{\mathbf{79.7\ m/s}}$$

$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}(60.4/52) =$ **49 ° below horizontal** (*draw a figure*)

REALITY CHECK: ***The projectile was launched at 60 m/s so a 79.7 m/s landing makes sense, as does a steeper angle.***

We made it to slide #25 on Friday, January 23, 2015.

PHYS 121 – SPRING 2015

Chapter 4: Motion in 2 & 3 Dimensions

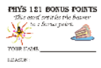
version 01/23/2015 , ~ 80 slides

We made it to slide #25 on Friday, January 23, 2015.

Get your clickers ready!

ANNOUNCEMENTS

- Pick up your graded homework's from the 3 bins before or after class or from the Rockefeller 104 hallway.
- There's a special red folder for work with no name $\Rightarrow$ we need your name!
- The syllabus describes how complaints about grading are handled.
- Submit today's assignment in one of the 6 color-coded folders on stage.
- ~ 24 students did not '*activate*' a clicker or app by last Friday & have forfeited a bonus point.
- I will start allocating bonus points for using a clicker in lecture on random class days.
- The average grade on Homework #1 was 50.2 ± 7.1 out of 55 points or 91.3±12.9%.
- Get your clicker/app ready.



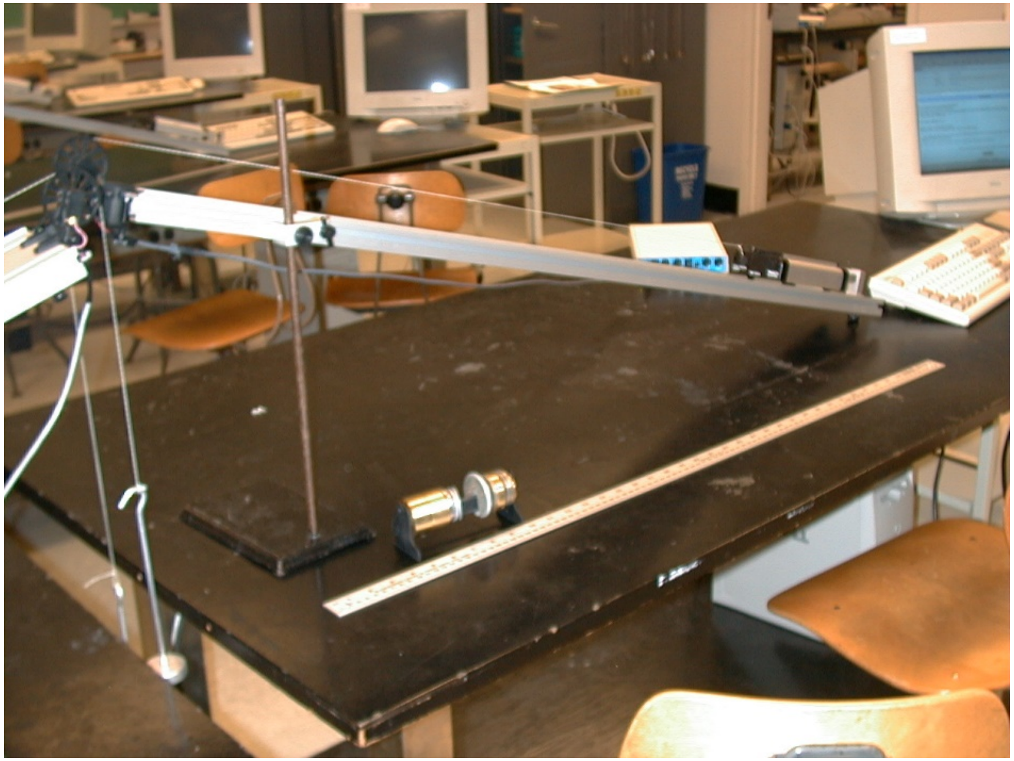
Lab #2: Inclined Plane

January 28 – February 5

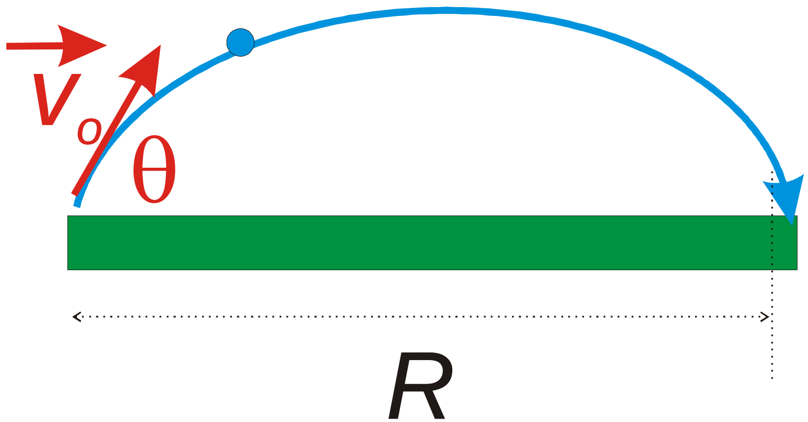
You will check the validity of Newton's Second Law by measuring the motion of a cart as it accelerates up an inclined plane.

We'll get to Newton's 2^{nd} Law in lecture early this week.

Inclined planes will be introduced later this week.



PROJECTILE RANGE



Assuming $\Delta y = 0$, at what angle should you launch a projectile for maximum range;

i.e. Find the MAXIMUM RANGE given some initial speed.

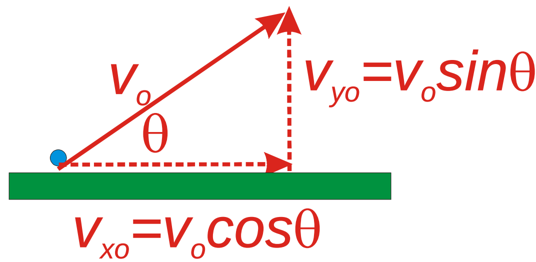
How do you identify a maximum for some quantity $y(x)$?

Calculate dy/dx and set the result = 0.

(*This gives maxima and minima but it's usually easy to distinguish between them.*)

We need an equation for $R(\theta)$ so we can use $dR/d\theta = 0$

PROJECTILE RANGE



$$y = y_o + v_{yo}t + \tfrac{1}{2}a_y t^2$$

$$0 = (y_o - y) + (v_o\sin\theta)t - \tfrac{1}{2}gt^2$$

If $h = \Delta y = 0$

$$t = 2(v_o\sin\theta)/g$$

$$x = x_o + v_{xo}t + \tfrac{1}{2}\cancel{a_x}t^2$$

$R = \Delta x = x - x_o = v_{xo}t = (v_o\cos\theta)t$ gives us

$$R = \frac{2v_o^2\sin\theta\cos\theta}{g}$$

$$\frac{dR}{d\theta} = \left(\frac{2v_o^2}{g}\right)\left(\cos^2\theta - \sin^2\theta\right) = 0$$

This derivative is 0 if θ = 45°

(*and this clearly provides a* ***maximum***, *not a minimum*)

The range R for $\theta = 45°$ (*since* $\sin 45° = \cos 45° = 2^{-1/2}$) is v_o^2/g

4 options

If a projectile is launched towards the right from the edge of a cliff at an angle θ above horizontal, using a coordinate system where positive *x* is to the right and positive *y* is upwards, then we describe the initial horizontal and vertical components of the velocity as

$$\mathbf{(v_{xo}\ ,\ v_{yo}) =\ v_o\cos\theta\ ,\ v_o\sin\theta}$$

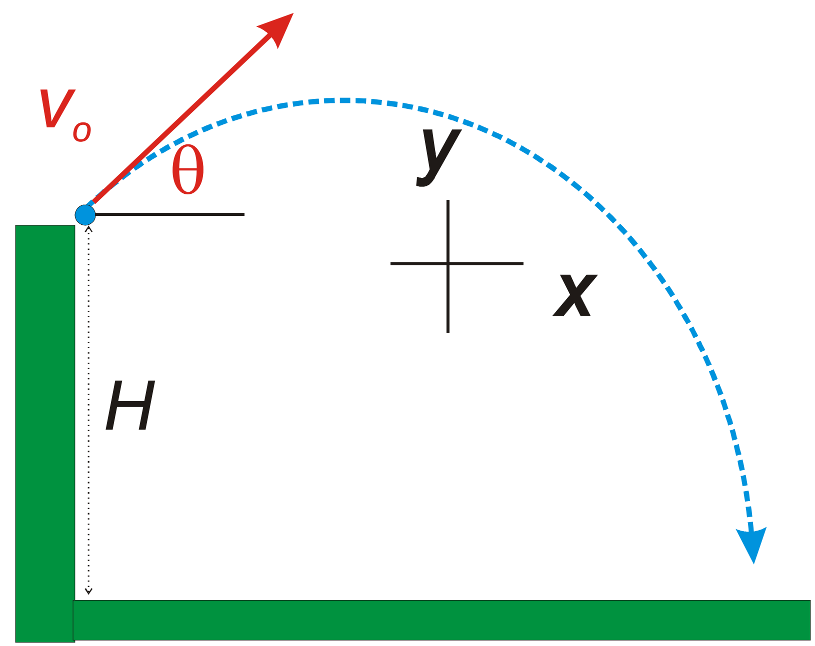
If we instead launch the projectile downwards ***below*** horizontal at an angle θ, the initial velocity should be described as

A. $v_o\cos\theta\ ,\ v_o\sin\theta$

B. $-v_o\cos\theta\ ,\ v_o\sin\theta$

C. $v_o\cos\theta\ ,\ -v_o\sin\theta$

D. $-v_o\cos\theta\ ,\ -v_o\sin\theta$



4 options

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$$(v_{xo}, v_{yo}) = v_o\cos\theta, v_o\sin\theta$$

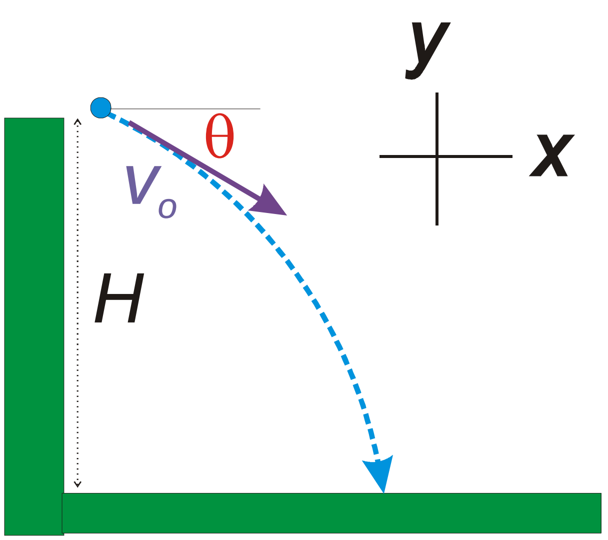
If we instead launch the projectile downwards *below* horizontal at an angle θ, the initial velocity should be described as

A. $v_o\cos\theta$, $v_o\sin\theta$

B. $-v_o\cos\theta$, $v_o\sin\theta$

C. $v_o\cos\theta$, $-v_o\sin\theta$

D. $-v_o\cos\theta$, $-v_o\sin\theta$



- 5 options

Two ballplayers, Timmy and Tammy, are standing next to each other. They each throw a ball to a teammate, either Sue or Sam.

Sue is 10 *m* away and the ball she catches rises to a maximum height of 20 *m*.

Sam is 20 *m* away and the ball thrown to him only reaches a peak of 10 *m*.
Which fielder catches a ball first?

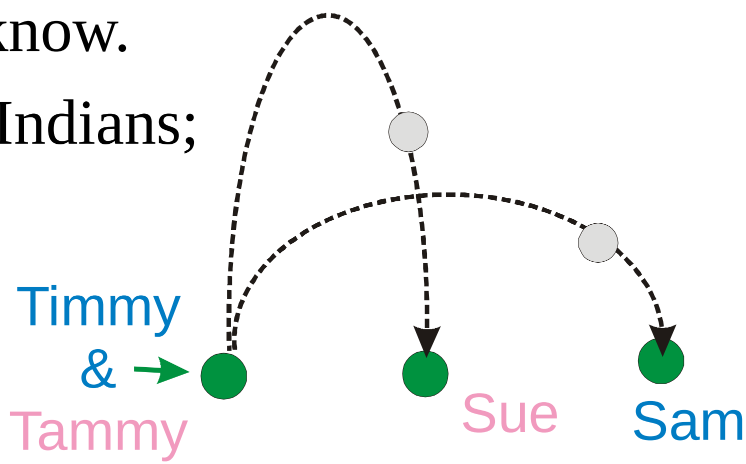
A. Sue catches a ball first.

B. Sam catches a ball first.

C. Both catch a ball at the same time.

D. There's not enough information to know.

E. Sam & Sue play for the Cleveland Indians; neither can catch a ball.



All ballplayers are shown in the spherical cow approximation.

- 5 options

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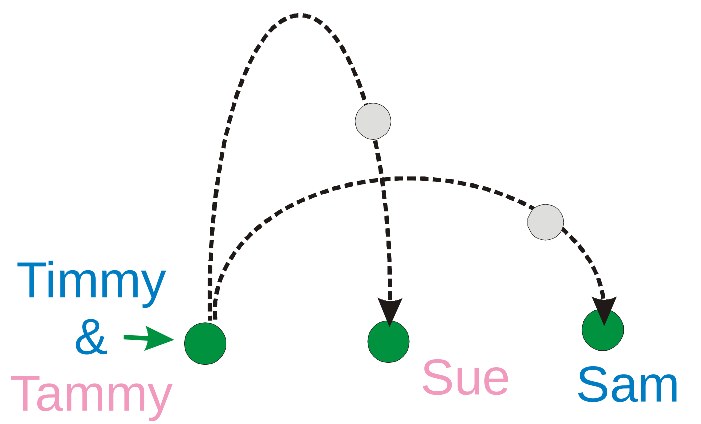


All ballplayers are shown in the spherical cow approximation.

The ONLY thing that matters for the time of flight is the vertical component of motion.

A higher peak equals a longer time in the air, no matter what is happening with the horizontal component of the velocity.

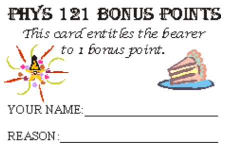
DEMO – “*SEEING IS BELIEVING*”

3 volunteers are needed,

preferably from the CWRU baseball and/or softball teams.

You can earn a bonus point if you don’t drop the ball

or “throw it into the stands”.



Let’s test whether the concepts described in the previous clicker question actually work in the real world.

- Two students standing at opposite sides of the stage will play catch.
- Meanwhile, another student will throw a ball straight up and catch it.

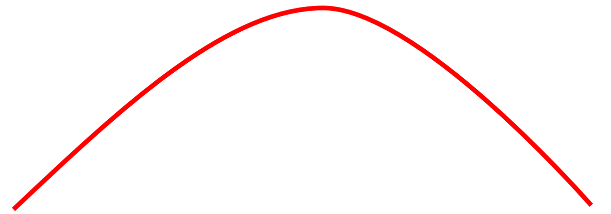
Who catches a ball first if:

1. If the lone student throws her ball higher?
2. If the lone student throws her ball lower?
3. If the lone student matches the height of the pair playing catch?

Playing Catch With a Baseball

https://www.youtube.com/watch?v=28do9IC-gI0&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr

3 options



Shooting a projectile at 45° above horizontal will give it the longest possible range for any given initial speed.

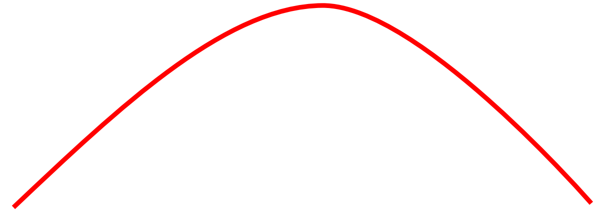
Given some initial speed, the time the projectile spends in the air is (*pick one of the following*):

A. longer than for any other initial angle, $\theta \neq 45°$.

B. shorter than for any other initial angle, $\theta \neq 45°$.

C. longer than some, shorter than other initial angles.

3 options



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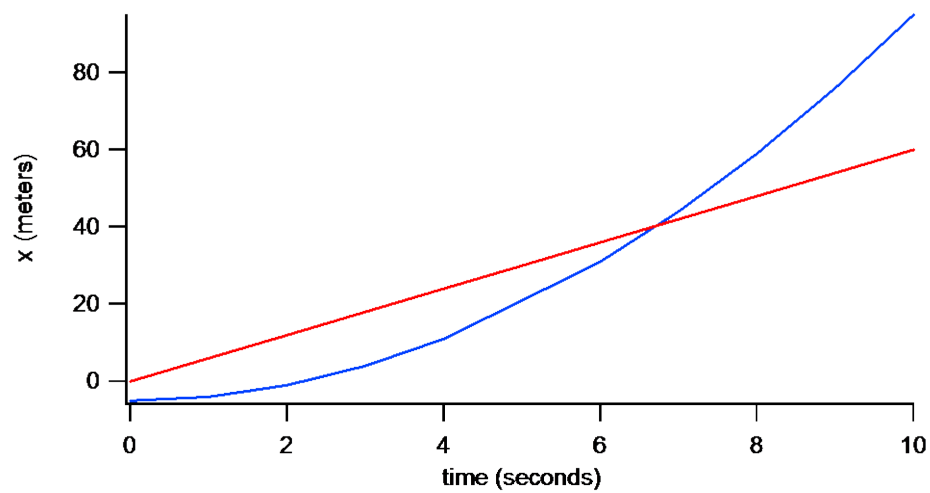
The higher the top of the projectile's trajectory, the longer it spends in the air.

The time in the air is given by $t = 2v_o\sin\theta/g$ which has a minimum at $\theta = 0°$ and a maximum at $\theta = 90°$.

INTERSECTIONS

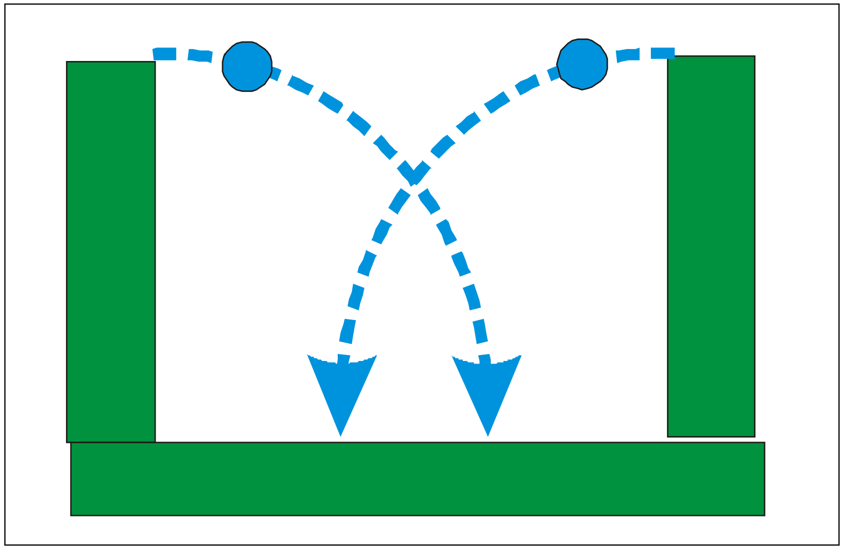
in ballistic motion

The Chapter 2 lecture notes described intersecting paths for objects moving in 1D with different initial positions, velocities and acceleration, as shown below.



Now consider intersections of two objects which experience the *same* acceleration, $a = -g$

Two projectiles are launched towards each other from the same height with horizontal initial velocities in opposite directions.



They will collide halfway between their initial starting points, in terms of the horizontal component.

(*assuming they reach the halfway point before hitting the ground*)

You can prove this using only symmetry.

- 3 options

A projectile is thrown horizontally at another projectile which is dropped from rest at the same instant.

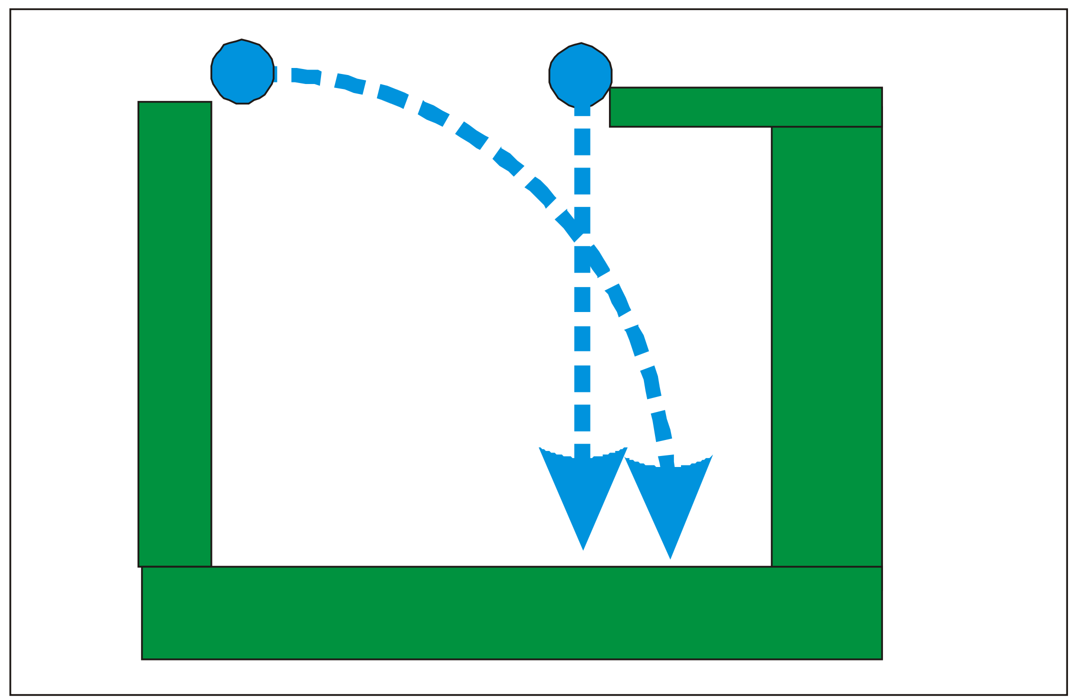
Note that the left projectile has a lot more distance to cover.

Will they collide if they don't first hit the ground?

A. Yes

B. No

C. Maybe



- 3 options

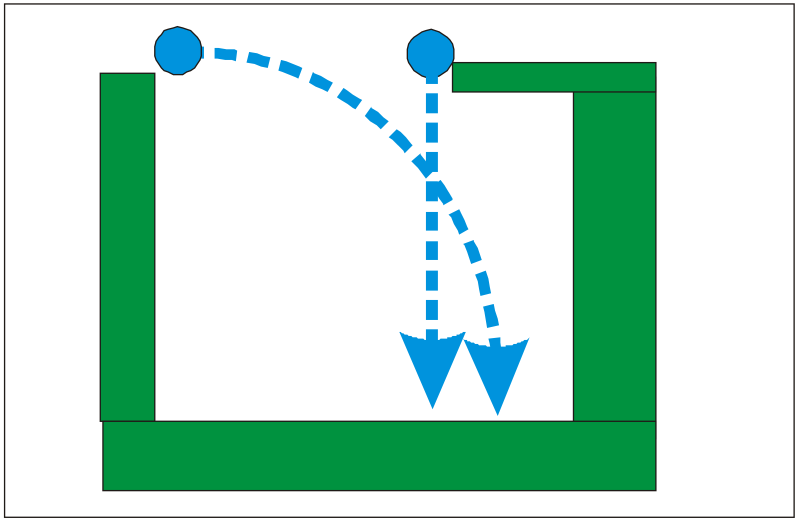
A projectile is thrown horizontally at another projectile which is dropped from rest at the same instant Note that the left ball in the figure has a lot more distance to cover.

Will they collide if they don't first hit the ground?

A. Yes

B. No

C. Maybe



- They have exactly the same equations of motion in the y-direction,

$$y = y_o + v_{yo}t - ½ gt^2$$

- and their positions in the x-direction will overlap at some time.
- Conversely if you release them at different times, they CANNOT intersect since they are always at different *y*'s at any given time.

- 5 options

A tranquilizer dart gun is aimed at a monkey hanging from a tree branch. The instant the dart is fired, the monkey lets go and falls to the ground.

Which of the following is correct?

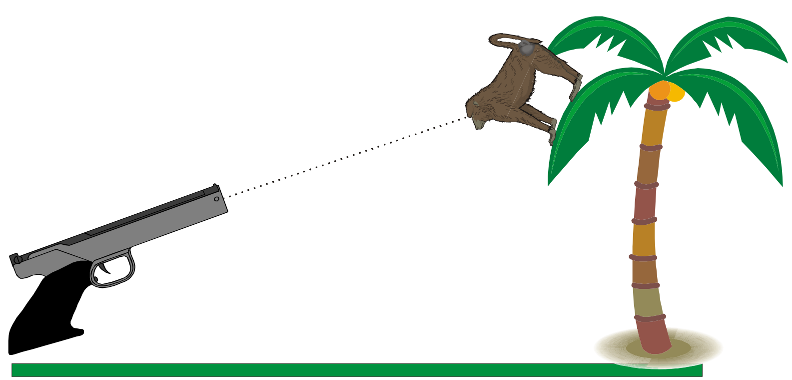
A. The dart passes above the monkey without hitting it.

B. The dart passes below the monkey without hitting it.

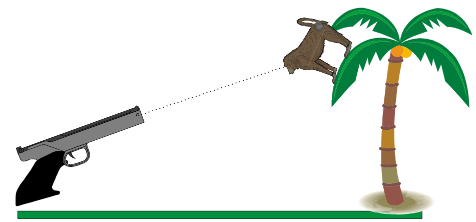
C. The dart hits the monkey if the dart can make it as far as the tree before hitting the ground.

D. The dart hits the monkey only if the dart's speed v_o is high enough so that gravity is negligible.

E. The dart misses; the monkey grabs the dart gun and shoots you!



- 5 options



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- Think about the terms in $y = y_o + v_{yo}t - \frac{1}{2}gt^2$ for the dart.
- If there was no gravity, $a_y = 0$, you would just have $y = y_o + v_{yo}t$, a straight line from the firing point to the original position of the monkey, and the dart will go where you aimed it.
- The <u>actual</u> path of the dart <u>DIFFERS</u> from a straight line, *i.e.* will fall below that line, by $\Delta y = -\frac{1}{2}gt^2$
- but this is exactly where the monkey will be at any given time.

$y_D = \cancel{y_{oD}} + v_{yoD}t - \frac{1}{2}gt^2$ $y_M = y_{oM} + \cancel{v_{yoM}}t - \frac{1}{2}gt^2$ with $v_{yoD} = y_{oM}/t$

How do you solve a general problem with two projectiles #1 & #2 launched at each other with

- different initial velocities
- from different heights
- at different angles
- & different launch times, $t_2 = t_1 \pm \Delta t$
- to keep it simple, they both move in the same plane.

Under what conditions will these projectiles collide?

Can you design a missile defense system?

There is no guarantee that these paths WILL intersect!

The only way to know for certain is to solve the problem

$y_1(t) = y_2(t)$ & $x_1(t) = x_2(t)$

for some value of t, using

$$x_1(t) = x_{1o} + v_{x1}t \qquad x_2(t) = x_{2o} + v_{x2}(t \pm \Delta t)$$

$$y_1(t) = y_{1o} + v_{y1o}t - \frac{1}{2}gt^2 \qquad y_2(t) = y_{2o} + v_{y2o}(t \pm \Delta t) - \frac{1}{2}g(t \pm \Delta t)^2$$

You may be given all the parameters except v_{x2} , v_{y2o}, t & Δt.

Solve for these 4 parameters given 4 equations above.

If the answers for t are negative and/or imaginary numbers

$\Rightarrow$ the projectiles will not collide.

GOOD NEWS: You won't be expected to solve a general problem like this in PHYS 121!

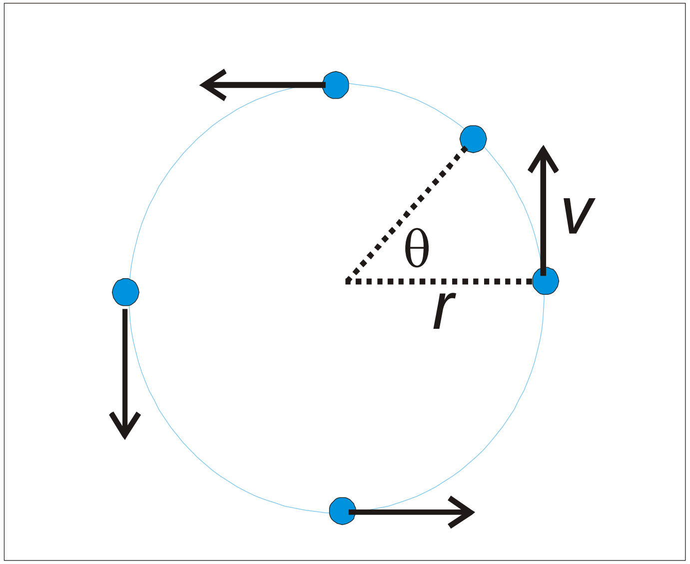
But you **SHOULD** be able to, after writing out the equations given above; it's just math – or MatLab!

Actually I was thinking of giving you a homework problem like this, pitting residents of one dormitory/fraternity/sorority versus another. Do you want it?

Ohanian DOES include some homework problems like this.

UNIFORM CIRCULAR MOTION

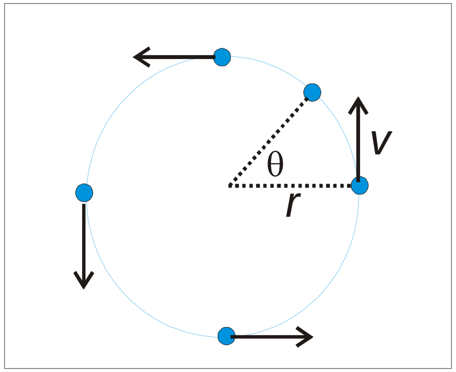
- **UNIFORM CIRCULAR MOTION, UCM** ≡ an object moves around a circle of fixed radius at constant *speed*.
  - Constant *speed* is NOT the same thing as constant *velocity*.
  - *Velocity* is a vector and has a direction.
  - The direction of the velocity vector in UCM rotates 360° as the object moves around a circle.
- **UNIFORM CIRCULAR MOTION** has *interdependent* *x* and *y* components.



JARGON: PERIOD & FREQUENCY

- **PERIOD** $\equiv$ ***T*** is the time required to complete one transit of the circular path.
- The *velocity* in UCM is constant in *magnitude* (*speed*) but is constantly changing direction.

  This velocity is called a **TANGENTIAL VELOCITY** $\boldsymbol{v_{tan}}$ or just $\boldsymbol{v}$.
  - The ***magnitude*** of $\boldsymbol{v_{tan}}$ = ***speed*** of the object.
  - The ***direction*** of $\boldsymbol{v_{tan}}$ is ***tangent to the circle*** at every point of the path.
- The distance traveled in one period is the circumference of the circle = $2\pi r$.
- The relation between the magnitude of the tangential velocity, v, period T and circumference is v = distance/time = $2\pi r/T$
- **FREQUENCY,** $\boldsymbol{f}$ = *cycles* or *revolutions* per *time*
  - $f = 1/T$
- **ANGULAR FREQUENCY,** $\boldsymbol{\omega}$ = *radians* per *time*
  - 1 cycle = 2π radians $\Rightarrow \omega = 2\pi f$ $\quad \left(2\pi \frac{radians}{cycle}\right)\left(f \frac{cycles}{time}\right)$
  - $v = (2\pi r)/T = (2\pi r)f = (2\pi f)\, r = \omega r$



CENTRIPETAL ACCELERATION

$\boldsymbol{v} = \boldsymbol{v_{tan}}$, while constant in magnitude,
is constantly changing direction.

- Any change in velocity, including a change in direction, means an acceleration is involved.
- The *average* acceleration over a complete cycle is of little interest; the net change in velocity is 0 so

$$a_{avg} = 0$$

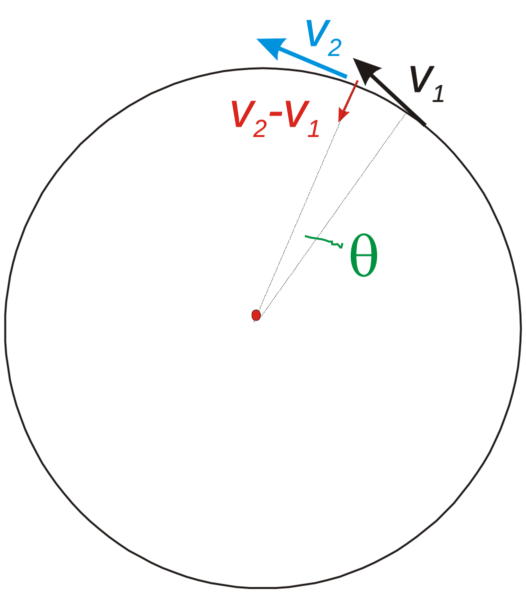
- **The *instantaneous* acceleration is not zero!**

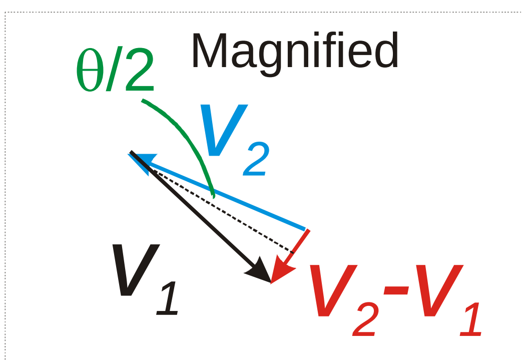
CENTRIPETAL ACCELERATION

Calculating $\boldsymbol{a}_{instantaneous}$ requires looking at $\boldsymbol{v}_{tan}$ at two closely spaced times, t_1 and t_2, a *moment* apart.

As you choose $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ ever closer together to calculate $\vec{a} = \frac{d\vec{v}}{dt}$

$\Delta\vec{v}$ or $d\vec{v}$ grows closer to being perpendicular to the circle.



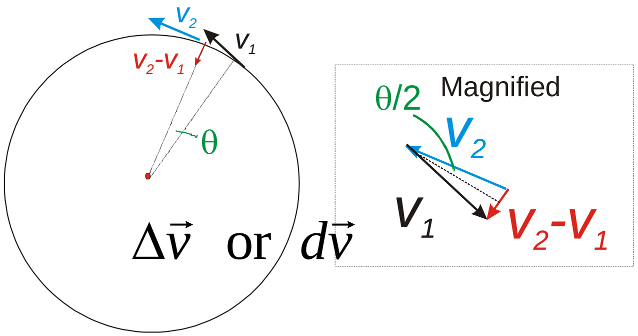


$$\Delta v = 2v\sin\left(\frac{\theta}{2}\right) \xrightarrow[\text{small } \theta]{} v\theta$$

$$\Delta t = \frac{\text{arc length}}{v} = \frac{\theta r}{v}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{v\theta}{\theta r / v} = \frac{v^2}{r}$$

CENTRIPETAL ACCELERATION



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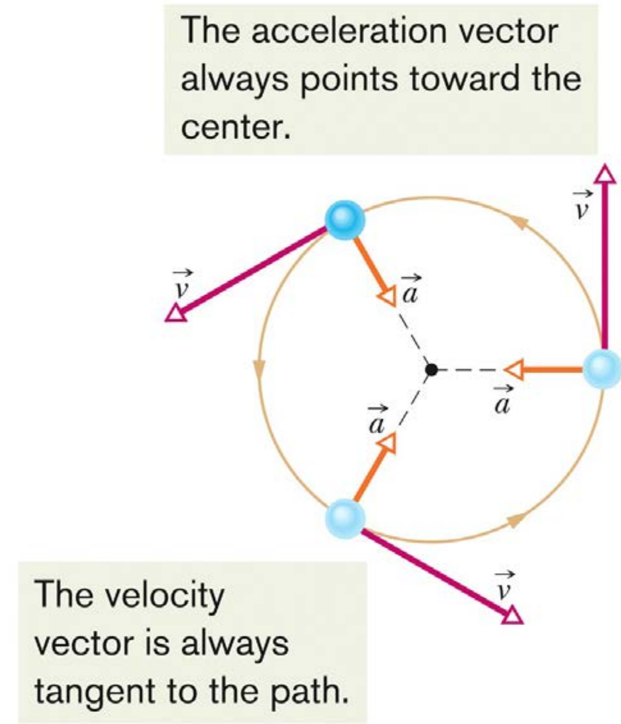

- The magnitude of this acceleration is $a_{radial} = \dfrac{v^2}{r} = \dfrac{(\omega r)^2}{r} = \omega^2 r$
- It is called the ***centripetal acceleration***.

  Centripetal means *pointing towards the center.*
- The direction of $\boldsymbol{a}_{UCM}$ is always towards the center, rotating 360° per revolution.

We made it to slide #52 on Monday, January 26, 2015.