PHYS 121 – SPRING 2015

Chapter 14: Statics & Elasticity

version 04/10/2015

~ 36 slides

ANNOUNCEMENTS

- **GOOD NEWS:** The homework assignment due Monday includes no homework from Chapter 14.
- **BAD NEWS:** There's a homework assignment due a week from Monday, shortly after Exam #3, that will cover Chapter 14 + Chapter 9 on Gravitation, which we will start & finish on Monday.
- **GOOD NEWS:** There's only one more PHYS 121 assignment after that!
- **BAD NEWS:** That last assignment is due on the last day of classes.

ENGR 200

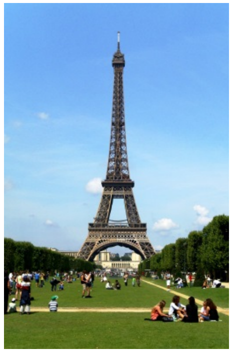
STATICS AND STRENGTH OF MATERIALS

Many of you will be taking ENGR 200 next year

Prereq: PHYS 121.

An introduction to the analysis, behavior and design of mechanical/structural systems. Course topics include:

- *concepts of equilibrium;*
- *geometric properties*
- *distributed forces;*
- *stress,*
- *strain*
- *mechanical properties of materials;*
- *linear elastic behavior of elements.*



STATICS & EQUILIBRIUM

Statics = study of rigid objects with translational & rotational acceleration = 0.

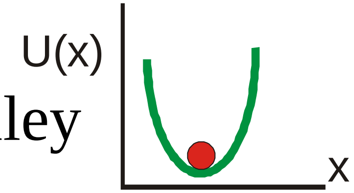
- Examples: buildings & bridges that ideally won't twist or fall.
- Constant translational or angular velocity is allowed ~ our earth.

Static equilibrium ⇒ 'naturally' stable, without continuous adjustment.

→ corresponds to a potential energy curve opening upwards.

→ returns to equilibrium if disturbed slightly, like a ball in a valley

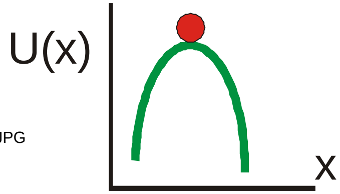
~ mass on a spring



Unstable equilibrium ⇒ "tenuous" equilibrium

→ corresponds to a potential energy curve opening downwards.

→ moves away from equilibrium if disturbed, like a ball on a mountain top or a house of cards



http://upload.wikimedia.org/wikipedia/commons/3/3b/Eiffel_Tower_Paris_01.JPG

http://upload.wikimedia.org/wikipedia/commons/c/cd/Brooklyn_Bridge_20080501.JPG

STATIC EQUILIBRIUM

TWO KEY EQUATIONS

$$\vec{a} = 0 \quad \Leftrightarrow \quad \sum F = 0$$

$$\vec{\alpha} = 0 \quad \Leftrightarrow \quad \sum \tau = 0$$

STATIC EQUILIBRIUM

$$\vec{a} = 0 \iff \sum F_{external} = 0$$

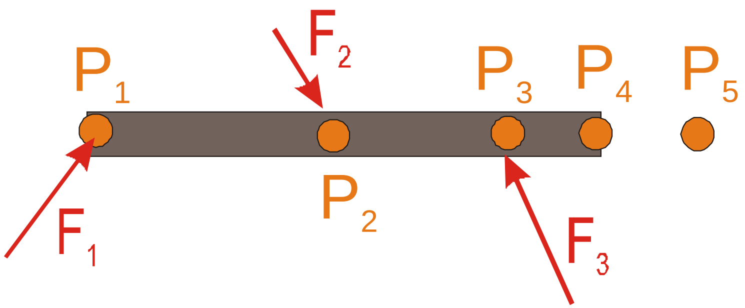
$$\vec{\alpha} = 0 \iff \sum \tau_{external} = 0$$

Torque can be calculated about any axis.

– **But it's CRITICAL that you choose wisely!**

– **You can eliminate a torque term by using a reference point where that force is applied.**

– **P_1, P_2, & P_3 are wise choices. P_4 & P_5 are poor choices.**



STATIC EQUILIBRIUM EXAMPLES

- SEE-SAW
- BOOMS: hanging objects from beams & ropes
- LADDERS
- AVOIDING FALLS or “FACE-PLANTS”

https://americaexplained.files.wordpress.com/2011/01/face-plant.jpg

https://www.youtube.com/watch?v=J6i9Un7bDyQ

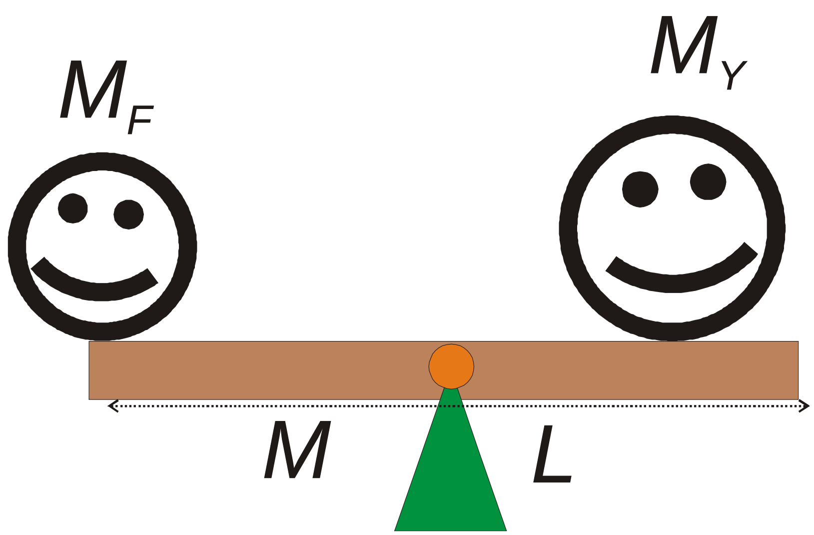
SEE-SAW – *a simple example*

A seesaw made of a uniform board of mass M and length L is supported by a frictionless bearing (*or fulcrum*) at its center.

Your friend, whose mass M_F < your mass M_Y sits at the far end.

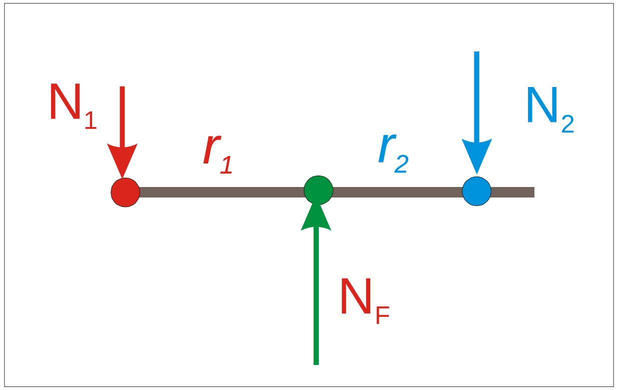
#1: Where should you sit in order to balance the see-saw?

#2: What is the net force on the bearing/fulcrum?



http://lifeisbutadrink.com/wp-content/uploads/2012/06/seesaw.jpg

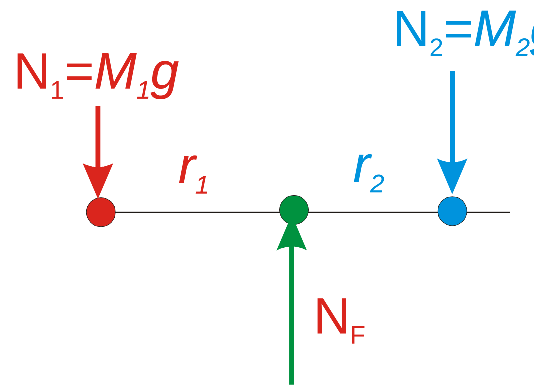
SEE-SAW



- Pick your system, to what will you apply $\Sigma F = 0$ & $\Sigma \tau = 0$.
  - My system is the seesaw, not the base or people.
- Draw the free body force/torque diagram.
- Choose (*wisely*) your reference point for torque calculations.

  (*NOTE: a frictionless bearing or hinge cannot transmit a torque*)
- There are 3 forces on the seesaw.
  - By shifting our focus/system temporarily to each person in turn, and noting that $\Sigma F = 0$, we can say that $N_1 = M_1 g$ and $N_2 = M_2 g$.
  - The force that the fulcrum applies to the seesaw is labeled as N_F.

SEE-SAW



- N_F is unknown

$\Rightarrow$ Apply $\Sigma\tau = 0$ about the fulcrum, with positive $\equiv$ CCW.

$$\Sigma\tau = M_1 g r_1 - M_2 g r_2 = 0$$

so

$$r_1/r_2 = M_2/M_1$$

SIT HERE: $r_2 = (M_1/M_2)r_1 = (M_1/M_2)(L/2)$

Solve for $N_F \rightarrow$ use $\Sigma F = 0$.

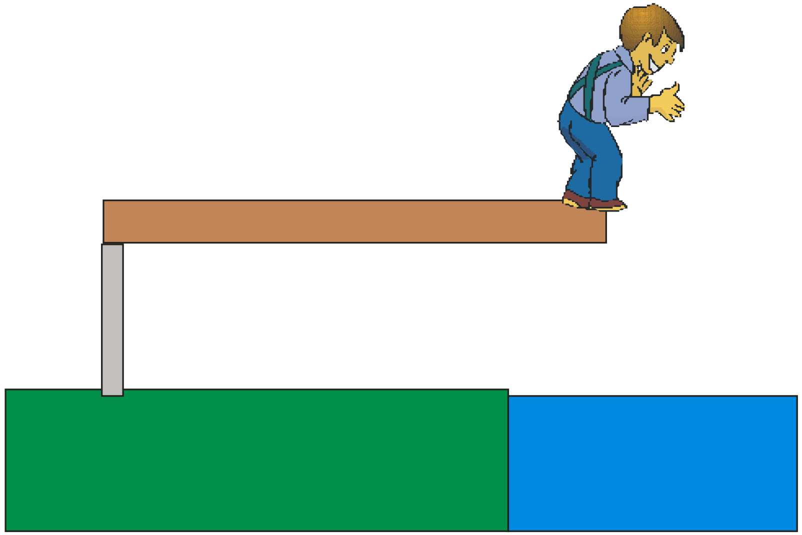
$$N_F - M_1 g - M_2 g = 0$$

so the normal force of the fulcrum acting on the seesaw is

$$N_F = (M_1 + M_2)g$$

- You likely could have guessed the answer to this problem but the method used above will work for problems whose answers you cannot guess.

POOR ENGINEERING

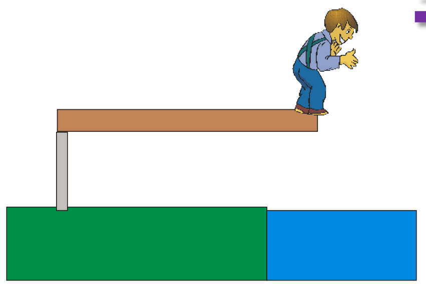


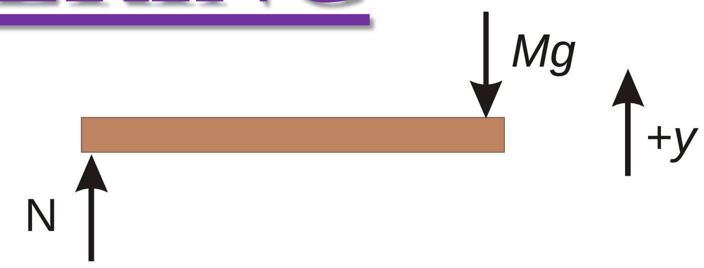
A BAD engineer (*with a degree from some generic online university*) designs the diving board shown above,

with an elegant but very thin vertical support.

Explain why this is a engineering disaster waiting to happen.

BAD ENGINEERING





Consider the free body diagram for the diving board.

Ignore the weight of the diving board; this won't change our result.

The normal force *N* (*on the board due to the support*) must point up in order for $\Sigma F_y = 0$.

But if N points up, there's no way for $\Sigma\tau = 0$; both forces tend to spin the board CW.

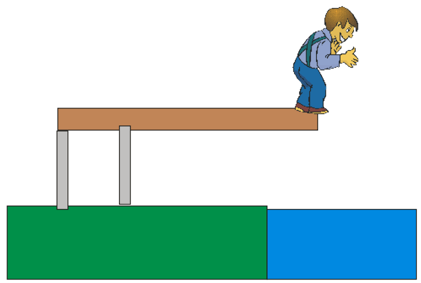
If you choose your reference point at the support, the torque due to *Mg* will force the board to break loose!

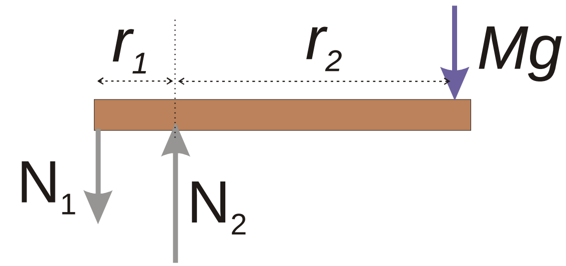
N can't point up and down at the same time!

$\Rightarrow$ This is not a stable static situation;

the diving board <u>will</u> break free from the support.

CWRU-TRAINED ENGINEER





You need another element in order to support the diving board. The design from a student who earned her degree from CWRU, after passing PHYS 121 and ENGR 200, is shown above.

Using the point of contact of N_2 as our reference for torque

$$\Sigma\tau = 0 \rightarrow N_1 r_1 = (Mg)r_2$$

$$N_1 = Mg(r_2/r_1)$$

$$\Sigma F = 0 \rightarrow \quad N_2 = N_1 + Mg = Mg[(r_1+r_2)/r_1]$$

$\Rightarrow$ Larger r_1 makes the normal forces smaller.

P.S. - the two separate supports could be a single wide support.

BOOM!

A boom is attached to a wall with a frictionless pivot.

Its other end is supported by a rope.

L = length of the boom

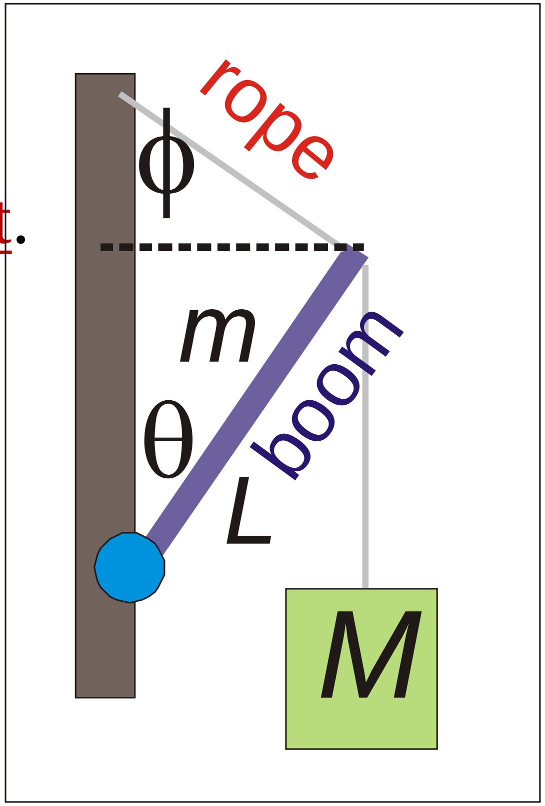
θ = angle between the boom & wall

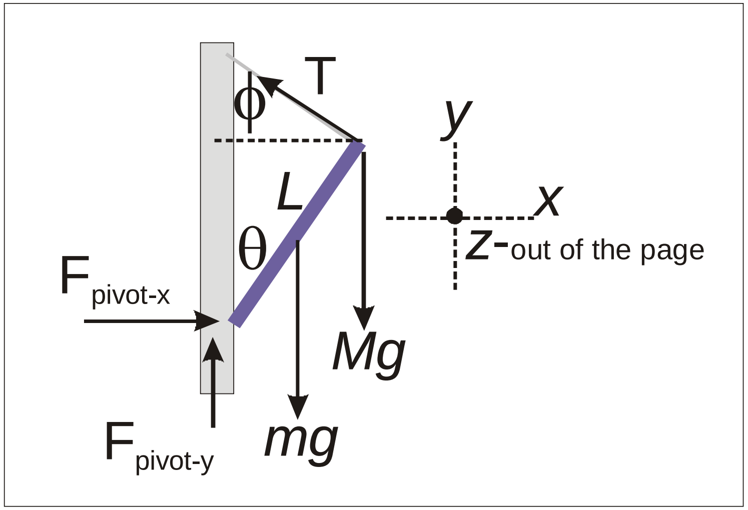
ϕ = angle between the rope & wall

m = mass of the boom

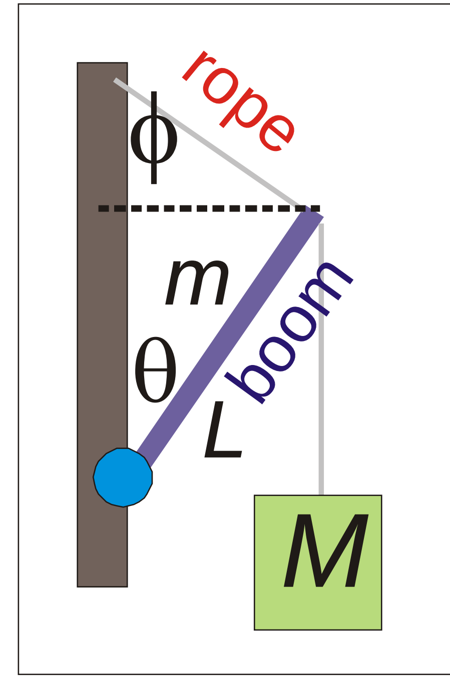
M = mass of the object supported by the boom.

What is the tension *T* in the rope supporting the boom?





BOOM!



The four forces acting on the boom are shown above.

mg, Mg, T & F_{pivot} (*two components*)

(*Mg* = *tension in the rope attached to M*)

You can calculate the torque about any point you wish.

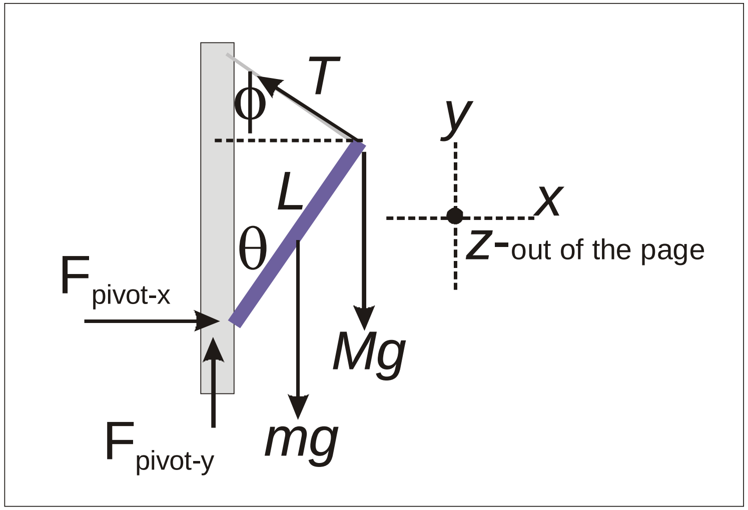
Choose wisely!

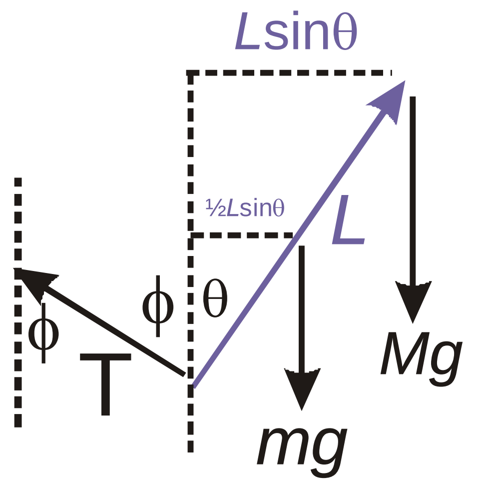
The pivot is a wise choice; it eliminates F_{pivot}

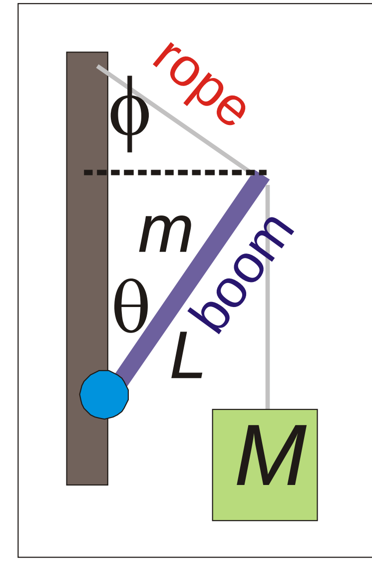
although you might be asked for the force exerted by the pivot – but it's easy to get this from $\Sigma F = 0$ *on the pivot once you know T.*

BOOM!

$$\vec{\tau} = \vec{r} \times \vec{F} \underset{magnitude}{=} r_{\perp} F$$





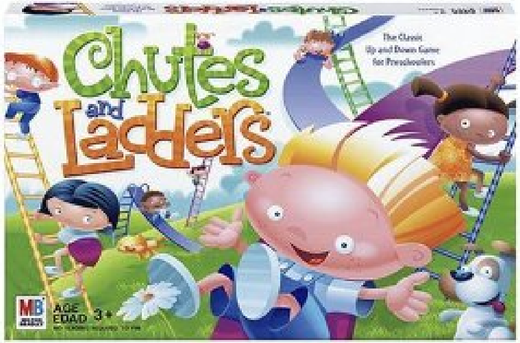


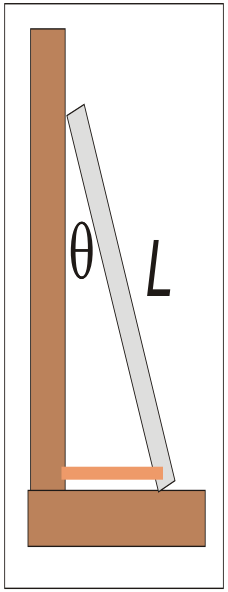
$$\sum \vec{\tau} = (mg)\left(\frac{L}{2}\right)(\sin\theta)(-\hat{z}) + (Mg)(L)(\sin\theta)(-\hat{z}) + (T)(L)\sin(\theta+\phi)(+\hat{z}) = 0$$

- The sinθ terms in the first two torque terms are easily explained using the concept of moment arms = $r_{\text{perpendicular.}}$
- For the third term, shift *T* so that it's tail to tail with ***L*** & then find the angle between them.
- Given *m*, *M*, *L*, *g*, θ & ϕ, this equation can be solved for *T*.

http://ecx.images-amazon.com/images/I/51XTMJMB4CL._SX300_.jpg

LADDERS

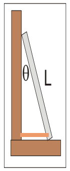


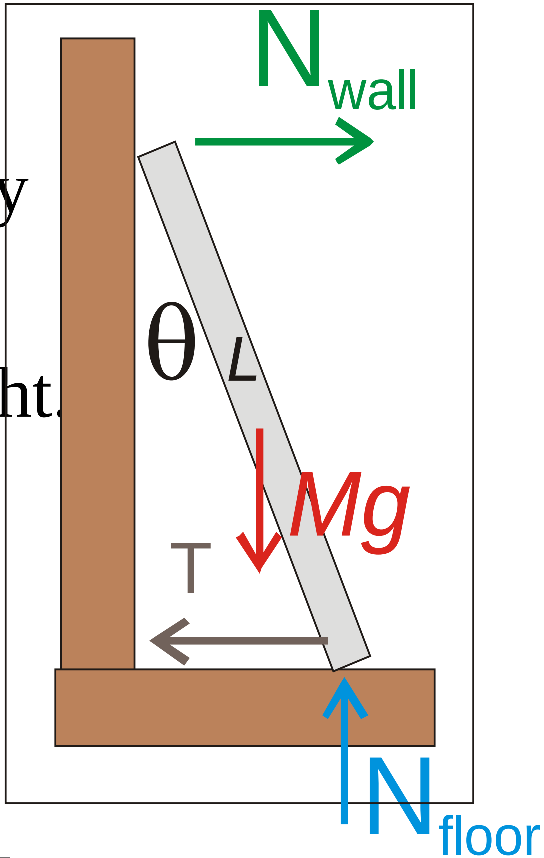


A ladder is leaning against a wall.

- The wall and/or floor might or might not be frictionless.
- There might or might not be a restraint (~ *rope*) keeping the bottom of the ladder from moving.
- A person might be climbing the ladder and you could be asked to calculate how far up the ladder the person gets before the ladder slips.

We'll start with the easiest case – no person, no friction and a rope holding the bottom of the ladder in place.

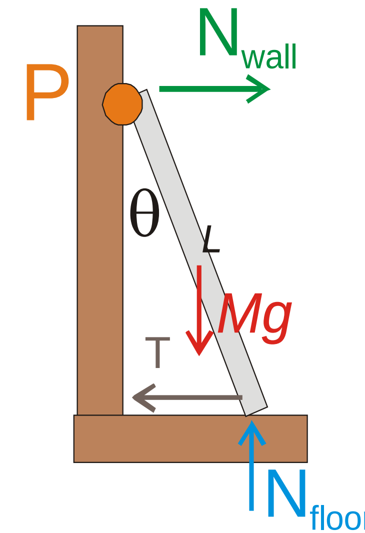
LADDERS 



Given a ladder of length L and mass M supported by a frictionless wall & floor at an angle θ, calculate the tension in the rope at its base that keeps it upright.

- The system is the ladder.
- There are 4 forces acting on the ladder
  - The contact force of the wall at the top, N_{wall}.
  - The contact force of the floor at the bottom, N_{floor}.
  - The force of gravity acting at the middle of the ladder, Mg.
  - The tension in the rope acting at the bottom, T.
- What point should you use for calculating $\Sigma\tau$?
  - We want T so don't pick the point where the rope is attached to the ladder since this eliminates T from our torque equation.
  - Choose wisely = the point where the ladder touches the wall so that we don't have to calculate that force at all.

LADDERS



τ about the point P where the ladder touches the wall, with positive *into the page* = clockwise, is:

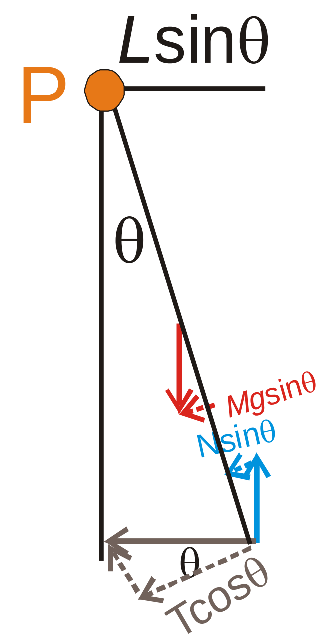
- $\tau_P = Mg(L/2)\sin\theta - N_{floor}L\sin\theta + TL\cos\theta = 0$

Divide out L and plug in $N_{floor} = Mg$ (*from* $\Sigma F_y = 0$)

- $(Mg/2)\sin\theta - Mg\sin\theta + T\cos\theta = 0$

Solve for T

- $T = [Mg\sin\theta - (Mg/2)\sin\theta] / \cos\theta = \underline{\boldsymbol{(Mg/2)\tan\theta}}$



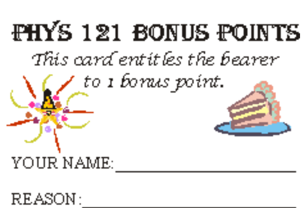
REALITY CHECK

✓ The dimensions/units are right.

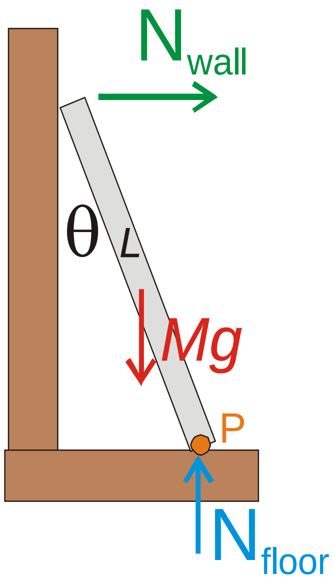
✓ As $\theta \to 0°$ $T \to 0$ vertical is easy

✓ As $\theta \to 90°$ $T \to \infty$ horizontal is impossible

Anyone in the class who is infinitely strong can earn a bonus point.



LADDERS



What changes if you remove the rope?

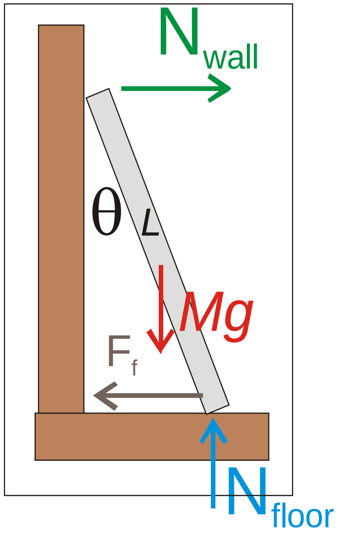
The ladder will slip & fall! Why?

- Consider the torque about the contact point with the floor.
- $Mg(L/2)\sin\theta - N_{wall}L\cos\theta = 0$
- $N_{wall} = (Mg/2)\tan\theta$
- But $\Sigma F_x \neq 0$ since N_{wall} is the only force in the x-direction

  $\Rightarrow$ the ladder accelerates to the right & falls.
- **You'd better check that there's friction with the floor before you climb a ladder!**

LADDERS

What changes if you remove the rope
but add friction with the floor?



- Note that $F_{friction} \leq \mu_s N_{floor}$ & $N_{floor} = Mg$
- Otherwise this is similar to the problem with a rope holding the base in place.

Calculating τ about the point where the ladder touches the wall, with + into page

$$\tau = Mg(L/2)\sin\theta - N_{floor}L\sin\theta + F_{friction}L\cos\theta = 0$$

$$\cancel{Mg}(\cancel{L}/2)\sin\theta - (\cancel{Mg})\cancel{L}\sin\theta + \mu_s\cancel{Mg}\cancel{L}\cos\theta = 0$$

$$-½\sin\theta + \mu_s\cos\theta = 0$$

Stability requires $\mu_s = ½\tan\theta$

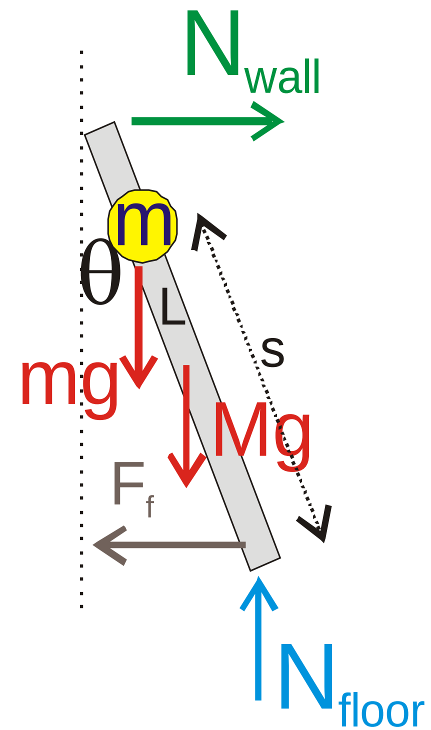
As $\tan\theta \rightarrow$ larger, θ increases & the ladder falls if $\tan\theta > 2\mu_s$

A large value for μ_s is 1, for which $\theta = \tan^{-1}(2) = 63°$

PS – many sources choose θ at the base of the ladder and get $\mu_s = ½\cot\theta$ or $1/(2\mu_s) = \tan\theta$

LADDERS

What changes if you add a mass *m* somewhere long the ladder a distance *s* from the bottom?
(*as in a person climbing the ladder*)



$$F_{friction} = \mu_s N_{floor}$$

$$\Sigma F_y = N_{floor} - (M + m)g = 0$$

$$\Sigma F_x = N_{wall} - F_{friction} = N_{wall} - \mu_s(M + m)g = 0$$

$$\Sigma\tau_{\mathrm{wall}} = Mg(L/2)\sin\theta - N_{floor}L\sin\theta + mg(L\text{-}s)\sin\theta + \mu_s N_{floor}L\cos\theta = 0$$

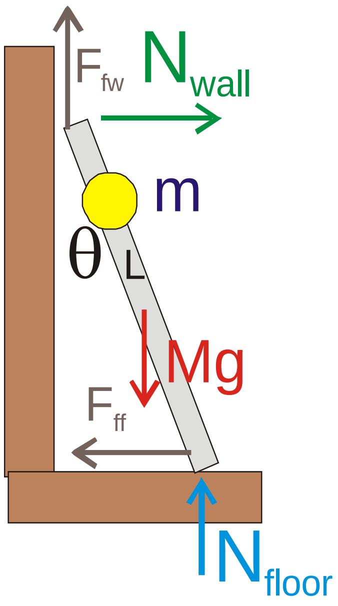
$$\Sigma\tau_{\mathrm{floor}} = -Mg(L/2)\cos\theta + N_{wall}\mathrm{L}\cos\theta - mg(L\text{-}s)\cos\theta = 0$$

6 parameters: M, m, L, θ, μ_s, s with 5 equations connecting them

→ you should be able to solve for any 1 parameter given the other 5.

Given all the parameters (*except s*), how high up can you climb before the ladder slips? A question for YOU to answer!

LADDERS



What changes if you include friction with the wall?

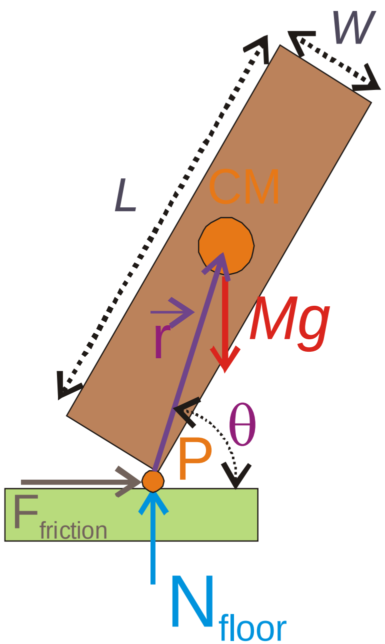
A lot of PHYS 121 students drop the course!

Actually, you <u>should</u> be able to handle this question for homework, **but not on an exam!**

$\Sigma F = 0$ & $\Sigma\tau = 0$ will still provide the equations you need but the algebra will get more annoying.

$F_{friction\text{-}wall}$ depends on N_{wall} but $N_{wall} = -F_{friction\text{-}floor}$

FALLING DOWN



What conditions determine whether a tilting object will fall?

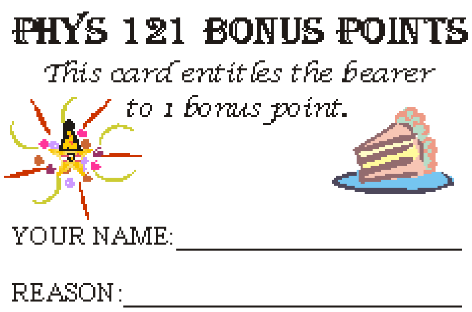
- Choose point P for the torque calculations.
  $\Rightarrow$ the torques from the normal force & friction with the floor = 0
  - Positive τ points into the page (CW) and makes the object fall.
  - Negative torque restores it to a stable, standing position
- $\tau_{gravity}$:

$$\tau_{gravity} = rF\sin\theta_{rF} = \pm\left(\sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{W}{2}\right)^2}\right)(Mg)\sin\theta$$

- The ± is needed because the CM, relative to a vertical line above the point P of contact with the floor, can be:
  (+) CM to the right of P $\Rightarrow$ it falls
  (-) CM to the left of P $\Rightarrow$ it doesn't fall

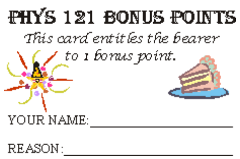
DEMOS

- Sliding ladder demo http://www.youtube.com/watch?v=6Y2GVnbwZSA&feature=em-share_video_user
- Walk a mass up a ladder

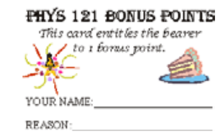


TOUCHING YOUR TOES

1 bonus point available if you can touch your toes.

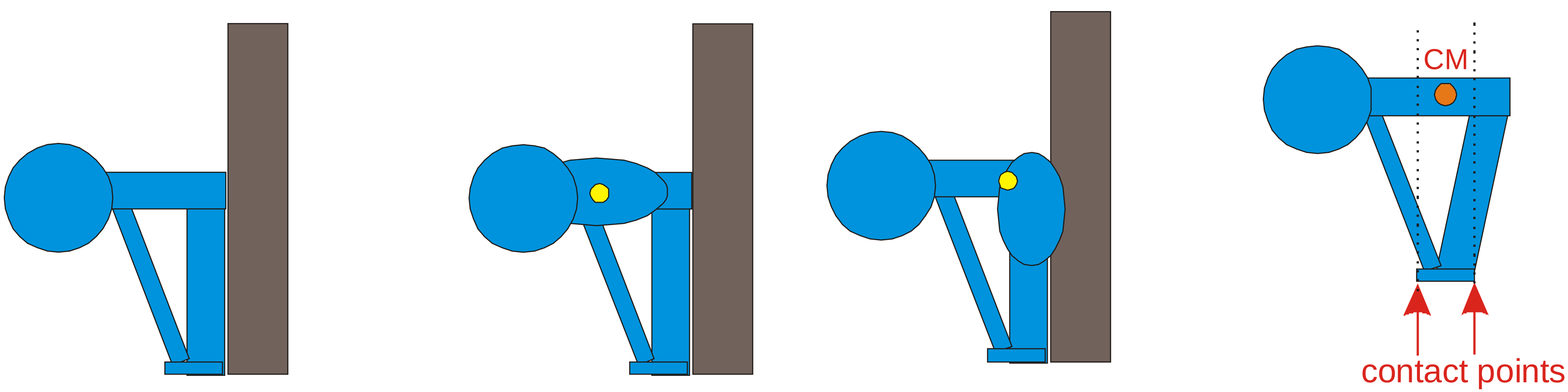


I need 2 volunteers, 1 male + 1 female



who can touch their toes without bending their knees or falling over while standing with the back of their legs against a wall.

- Where is your CM?
- What torque is needed and what is the pivot point?



4 options

A gymnast walks on a beam supported by two columns A & B as shown. As she moves past column B towards the right end of the beam she reaches a point where the beam is just on the edge of tipping.

Which of the following is **NOT** true at that critical tipping point?

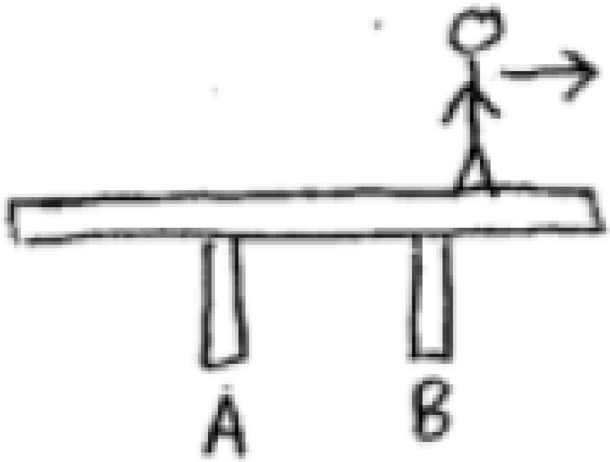
A. The contact force between the beam & column B vanishes.

B. The contact force between the beam & column A vanishes.

C. The torque due to the beam's weight cancels the torque due to the woman's weight around any point.

D. The CM of the system made up of the beam and the woman is located over column B.

http://www.thetopamerican.com/wp-content/uploads/2013/08/Top-American-Sports.jpg



4 options

A gymnast walks on a beam supported by two columns A & B as shown. As she moves past column B towards the right end of the beam she reaches a point where the beam is just on the edge of tipping.

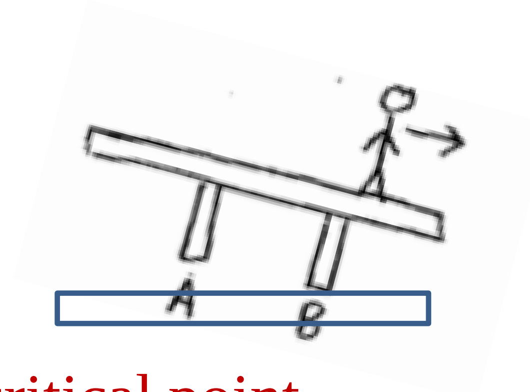
Which of the following is **NOT** true at that critical tipping point?

A. The contact force between the beam & column B vanishes.

B. The contact force between the beam & column A vanishes.

C. The torque due to the beam's weight cancels the torque due to the woman's weight around any point.

D. The CM of the system made up of the beam and the woman is located over column B.



The other answers are true when the gymnast is at the critical point.

The contact force from column B must equal the weight of the woman and beam or the beam won't tip, it will collapse straight down!

4 options

A thin uniform beam of mass *m* and length *L* is supported in a horizontal position by a pivot A at its left end and a cord with tension T on the right, positioned at an angle of 30 degrees from vertical. *The positive direction of torque is defined as out of the page towards you.*

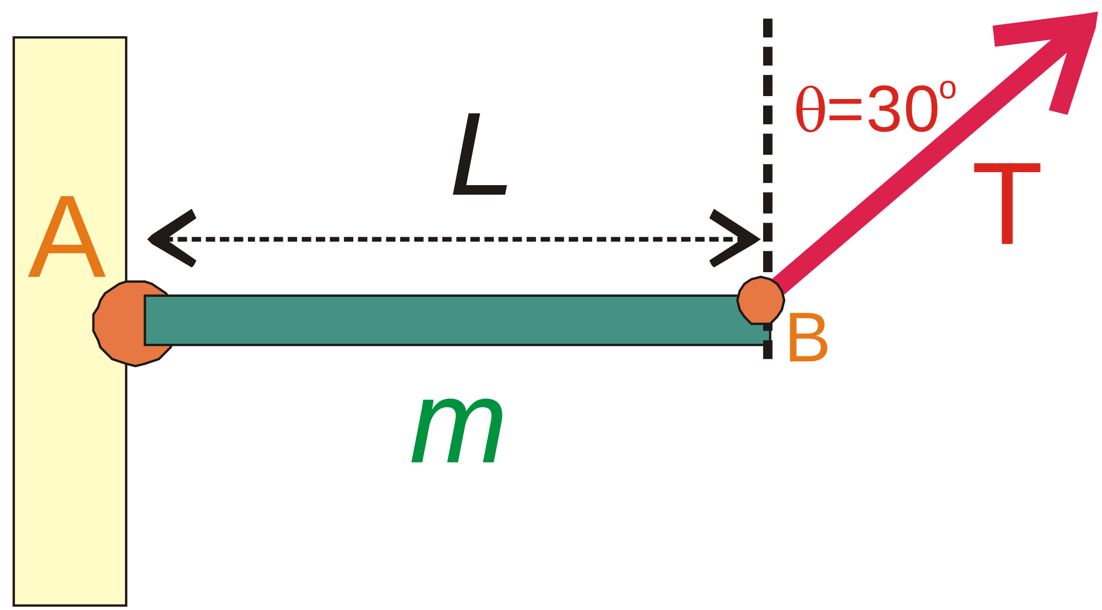
Which of the following is a correct expression for the torque due either to T or to gravity about point A?

A. $+mgL$

B. $+T\cos30°$

C. $-mgL/2$

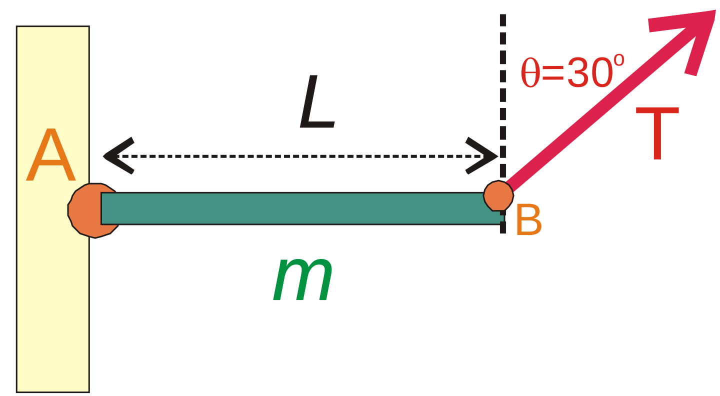
D. $-TL\cos30°$



4 options

A thin uniform beam of mass m and length *L* is supported in a horizontal position by a pivot A at its left end and a cord with tension T on the right, positioned at an angle of 30 degrees from vertical. *The positive direction of torque is defined as out of the page towards you.*

Which of the following is a correct expression for the torque due either to T or to gravity about point A?



A. $+mgL$

B. $+T\cos 30°$

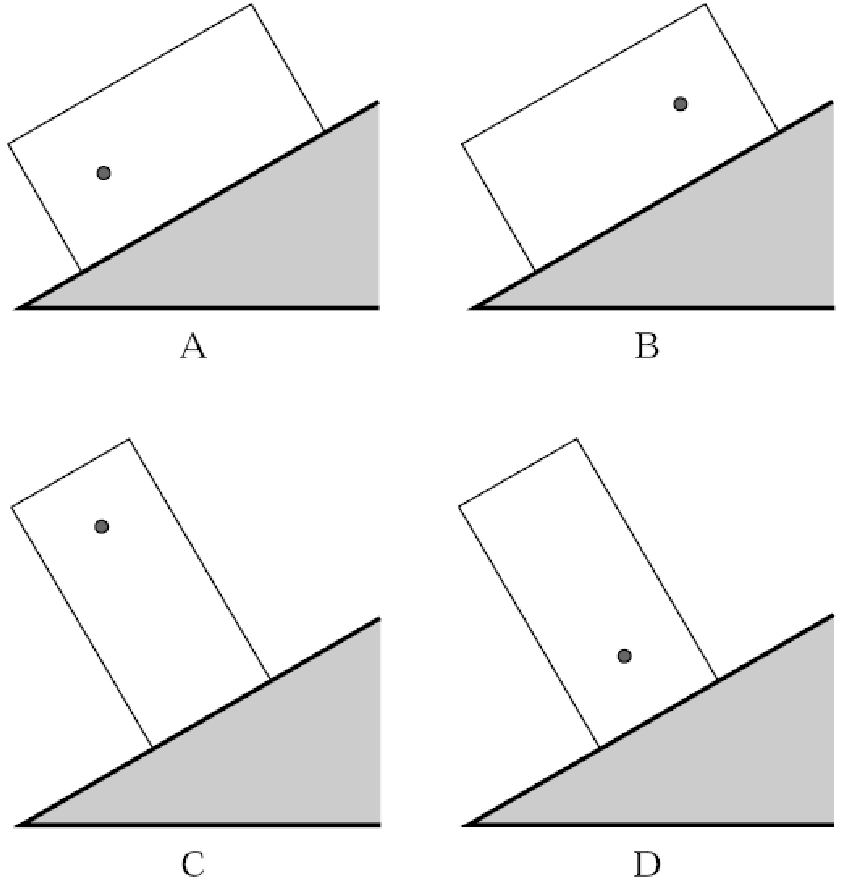
C. $-mgL/2$

D. $-TL\cos 30°$

- (A) is wrong since since the moment arm of gravity is $L/2$, not L.
- (B) is wrong; it doesn't even have the correct dimensions for a torque.
- (D) is wrong only because the sign of the torque is wrong. **L**x**T** points out of the page towards you and is a positive torque instead of the negative torque in this option.

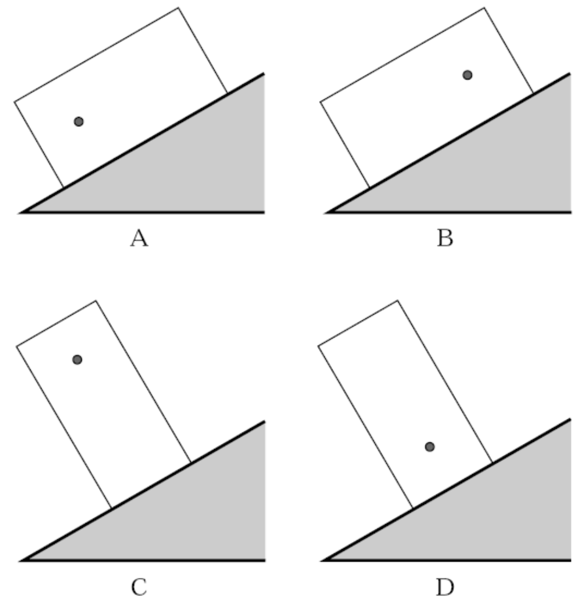
 - 4 options

A box, with an off-center center-of-mass, indicated by a dot in the figure below, is placed on an inclined plane.
In which of the four orientations shown is the box unstable so that it tips over about its leading edge?



- 4 options

A box, with an off-center center-of-mass, indicated by a dot in the figure below, is placed on an inclined plane.
In which of the four orientations shown is the box unstable so that it tips over about its leading edge?



The answer is C.

In order to tip over, the box must pivot about its bottom left corner.

Only in C does the force of gravity give a “CCW” *torque* about this pivot, tipping the box over rather than holding it onto the plane.

NOTE: Would you prefer to be driving a vehicle that resembles C up a hill?

- 5 options

The figure below shows 4 buckets of water “resting’ on a table (*without legs*) which the buckets support *via* a system of ropes and pulleys. Ignore the slight angle of the ropes in this setup, assume the ropes are vertical and massless and the pulleys are frictionless.

The condition for this system to remain at rest is best described as:

A. The mass of the water + buckets = mass of the table.

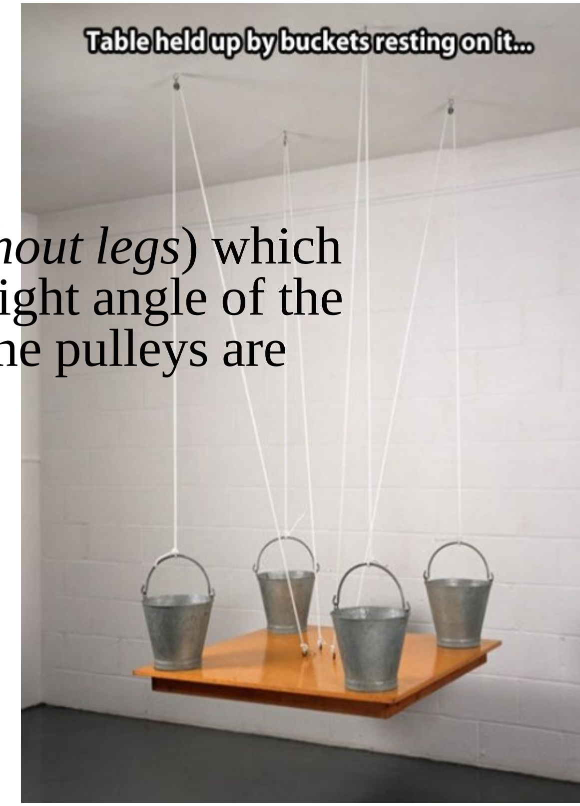
B. The mass of the water + buckets ≥ mass of the table.

C. The mass of the water + buckets ≤ mass of the table.

D. The weight of the water + buckets doesn’t’ matter.

E. This is a trick; this system can’t remain in equilibrium. Dr. C. probably glued the buckets to the table.

- 5 options



The figure below shows 4 buckets of water "resting' on a table (*without legs*) which the buckets support *via* a system of ropes and pulleys. Ignore the slight angle of the ropes in this setup, assume the ropes are vertical and massless and the pulleys are frictionless.

The condition for this system to remain at rest is best described as:

A. The mass of the water + buckets = mass of the table.

B. The mass of the water + buckets ≥ mass of the table.

C. The mass of the water + buckets ≤ mass of the table.

D. The weight of the water + buckets doesn't' matter.

E. This is a trick; this system can't remain in equilibrium.

The tension T in the ropes must be the same, so each rope supports 1/8 the combined mass of the table and buckets of water.

The condition that the buckets be in contact with the table requires this contact force be greater than 0. The buckets will be just barely in contact if this force is infinitesimally greater than 0.

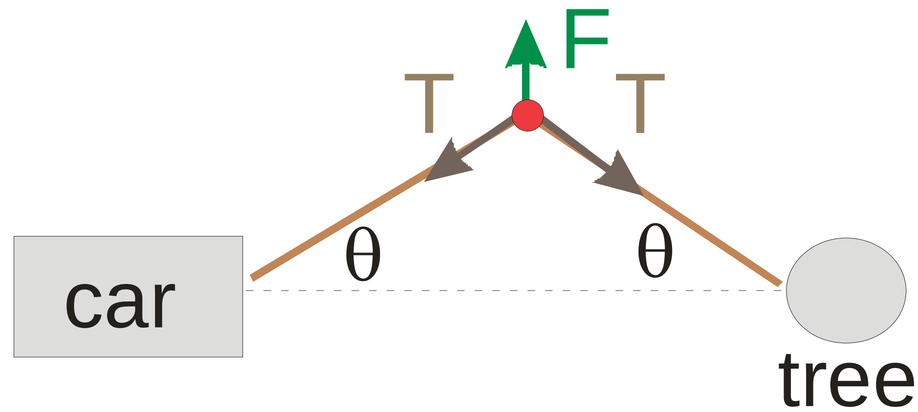
In this case, the total force on the table itself is $4T - W_{table} = 0$ while the total force on the 4 buckets is $4T – W_{buckets} = 0$.

So $W_{buckets} = W_{table}$.

This system will remain in equilibrium if more water is added to the buckets. The only change is that the normal force between the buckets and table will increase.

ROPE TRICK

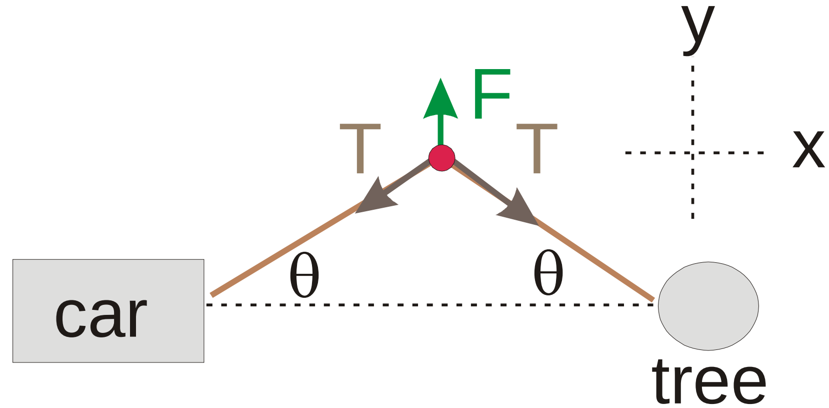
Ohanian 14.5



An old trick for moving heavy or 'stuck' objects is to tie one end of a rope to that object, tie the other end to an immovable object ~ a tree and push the middle of the rope, perpendicular to its length.

How much force can you generate *on* the stuck object if *you* push with a force F?

ROPE TRICK



Focus on the point where you are pushing, marked with a red dot above.

This point is static, so $\Sigma F = 0$ there.

$\Sigma\tau = 0$ too but all three forces exert 0 torque at that point, so all you would find is 0 = 0!

$\Sigma F_x = 0$ won't tell us anything useful, just T_x is the same on both sides.

$\Sigma F_y = 0$ tells us that $F - 2T\sin\theta = 0$

So $$T = \frac{F}{2\sin\theta}$$

For small θ, sinθ is also small, possibly approaching 0, and the tension can grow very big. $\theta = 5^\circ \rightarrow \sin\theta = 0.09 \sim 6$ times force multiplier

So you can use this effect to amplify your 'strength' by a large factor and perhaps pull a car out of the mud - or break ropes.

14.4 ELASTICITY

- We'll skip this material.
- It requires more time & attention than we have available.
- You'll see a LOT of it if you take ENGR 200.
- If you'll be taking ENGR 200, read this section in Ohanian!

http://img.photobucket.com/albums/v202/trblmn/the-end1.jpg