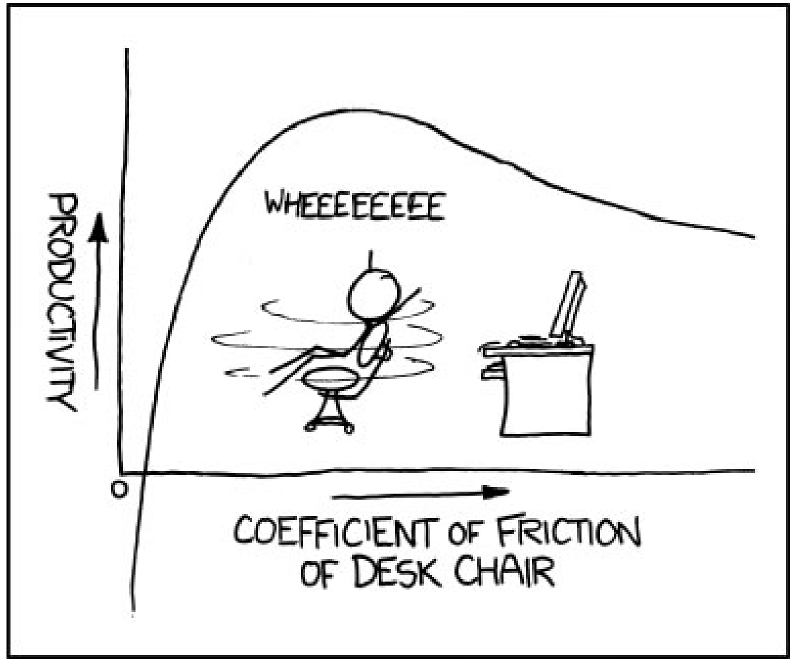
# PHYS 121 – SPRING 2015




# Chapter 6: Further Applications of Newton's Laws

***FRICTION, AIR RESISTANCE, SPRINGS & UNIFORM CIRCULAR MOTION*** *(~118 slides )*

*version 02/16/2015*

We made it to slide # 78 on Wednesday, February 18.

# FRICTION

- At a fundamental level
  - Friction is due to chemical bonds and/or micro/nanoscale bumps and depressions.
  - However, friction was handled 100’s of years ago (~*Leonardo da Vinci*) before people knew about atoms.
- That’s how we’ll handle friction this semester, just like they did in the 19th century.
  - That’s why they call this class **Classical Mechanics**!

# CULTURAL ASIDE

- The field of study of friction, and engineering solutions to problems caused by friction, is called *tribology*.
- NASA-Glenn on the west side of Cleveland has world renown expertise in tribology (*& employs many CWRU grads*)

  See http://www.grc.nasa.gov/WWW/StructuresMaterials/TribMech/index.html

  For more, see http://web.mae.ufl.edu/tribology/Links/tribologyLinks.HTML.
- One of the leading companies in tribology is located on the east side of Cleveland. http://www.lubrizol.com/
  - Lubrizol = Smith (*co-founders of Dow Chemical*) https://www.lubrizol.com/timeline/
  - Campus buildings with 'Smith' in their name were built with Lubrizol $$$.
- Friction involves atoms on one surface interacting with atoms on another surface, my research specialty.

# <u>PRACTICAL FRICTION</u>

We'll use a simple, <u>macroscopic</u> model of friction.

- The magnitude of the force of friction between two objects is proportional to the contact (*normal*) force between them.

$$F_{friction} = \mu N$$

- The proportionality constant ≡ *coefficient of friction* $\mu$ (*Greek letter mu*)

*Things are more complicated in reality, but this simple equation will get you through 90% of the situations you might encounter in your career – and 100% of the problems in PHYS 121.*

- $\mu$ is dimensionless.
- $F = \mu N$ is <u>NOT</u> a vector equation; $\boldsymbol{F}$ and $\boldsymbol{N}$ are <u>perpendicular</u> to each other.
- The force of friction is always <u>parallel to the direction of motion</u> of the object and oriented so as to <u>**oppose**</u> that motion.
- Friction is a contact force. If $N = 0$ between two objects, they aren't in contact and there is no friction between them!

*Some of the best bearings in turbine and other high speed applications rely on magnetic forces or air to separate moving surfaces.*

# PRACTICAL FRICTION

$$F_{friction} = \mu N$$

- $\mu$ ~ 0.04 for Teflon sliding on Teflon
- $\mu$ ~ 0.6 for steel sliding on steel
- $\mu$ ~ 1 for rubber on concrete

  *Bigger $\mu$ is better for tires if you want to stay on the road!*
- $\mu$ CAN be smaller than 0.04 or greater than 1

# PRACTICAL FRICTION

$$F_{friction} = \mu N$$

- There's usually more friction between objects at rest ***wrt* each other** (*static*) compared to objects in relative motion (*kinetic*).

  We account for this by defining separate coefficients

$$\mu_{static} = \mu_s \qquad \& \qquad \mu_{kinetic} = \mu_k$$

- Friction does *not* depend on velocity or contact area.
- Friction comes in *action-reaction* pairs acting in opposite directions on surfaces in contact.

We made it to slide #6 on Friday, February 13.

# PHYS 121 ANNOUNCEMENTS

**Pick up Homework #4 & Exam #1. No homework due today!**

**EXAM GRADES = 82.0 ± 14.8%** , 7 × 100% perfect grades

Homework #4 grades: 88.2 ± 12.7%.

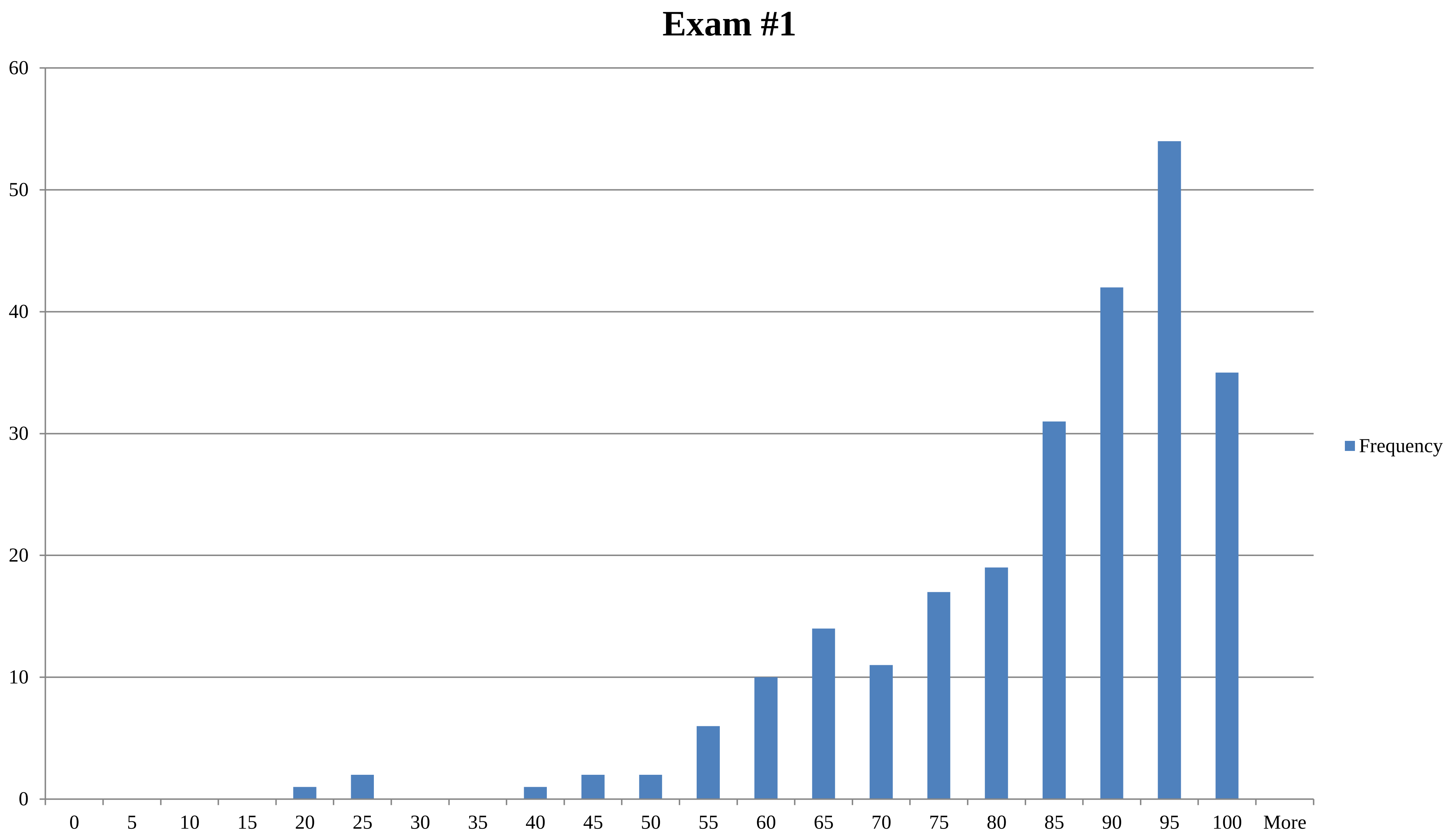
We made it to slide # 6 on Friday, February 13.

Get your clickers ready.

# Exam #1 Grade Distribution

*100 = 95 to 100, etc.*

**82.0±14.8%** , seven 100% grades



# PRACTICAL FRICTION $F_{friction} = \mu N$

- $\mu_s > \mu_k$ usually. (*They might be equal but* $\mu_s$ *is never* $< \mu_k$)
- The force of static friction is **not** in general **equal** to $\mu_s N$ but rather is **less than or equal** to $\mu_s N$.
  - $\mu_s N$ gives a **MAXIMUM** for the value of static friction,

$$F_{static\ friction} \leq \mu_s N$$

  - The actual value of $F_{static\ friction}$ will range from 0 up to $\mu_s N$, at which point static friction is overwhelmed and the objects in contact start to move with respect to each other.
  - Until then, $F_{static\ friction} = -\Sigma F_{trying\ to\ overcome\ static\ friction}$
  - But you often use $\mu_s N$ while solving a problem.

# BASIC FRICTION

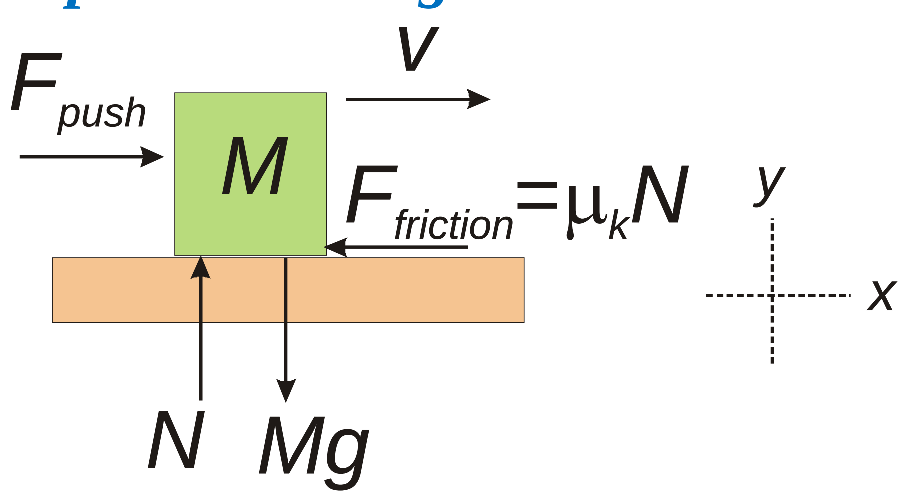
A block of mass *M* moves along a *rough* horizontal surface, pushed by a horizontal force $F_{PUSH}$ *in the direction of the block's motion*.

**What is the acceleration *a* of the block?**

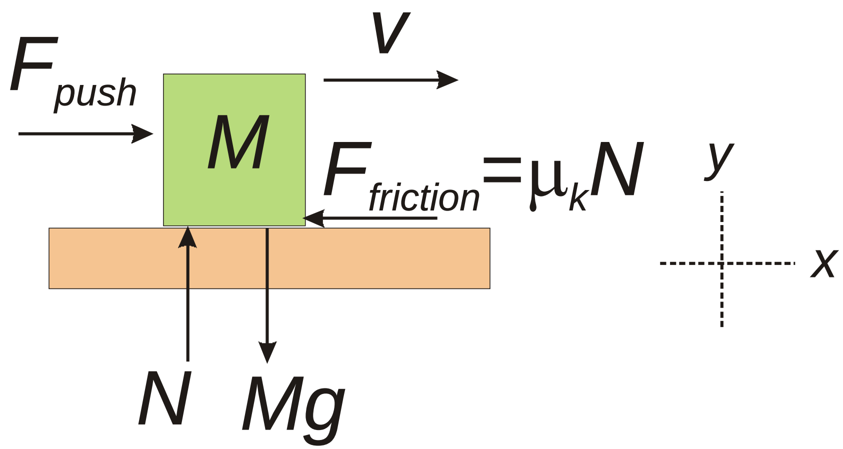
**v** in the figure below is useful but not part of the FBD.

- The **direction of motion** tells you the **direction of kinetic friction**.
- ***ROUGH*** is a code word for *include friction in your analysis*.
- ***SMOOTH*** implies that you should ignore friction but ideally this is spelled out explicitly, as in *smooth, frictionless surface*.

***Note that a can be positive or negative in this situation.***



# BASIC FRICTION




$$\sum F_y = N - Mg = 0 \quad \Rightarrow \quad N = Mg$$

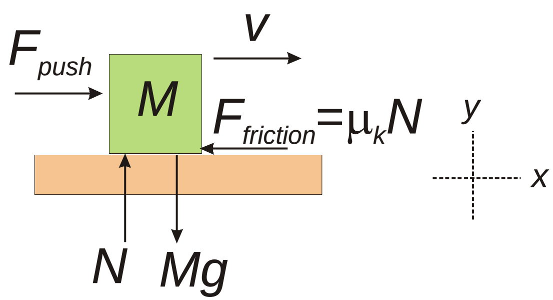
$$\sum F_x = F_{push} - F_{friction} = F_{push} - \mu_k N = F_{push} - \mu_k Mg = Ma_x$$

$$\Rightarrow \; a_x = \frac{F_{push} - \mu_k Mg}{M} = \frac{F_{push}}{M} - \mu_k g$$

Note that $F_{push}$ must be $> \mu_k Mg$ or $a \leq 0$.

**What does this mean?**

# BASIC FRICTION




$$a = \frac{F_{push}}{M} - \mu_k g$$

What does $a = 0$ or a negative value mean?

If $a = 0$, the block will continue to move at a constant speed,

$$F_{push} = F_{friction}$$

If $a$ is negative, $F_{friction} > F_{push}$ & the block slows.

**IF THE BLOCK STOPS,**

**YOU MUST SWITCH TO STATIC FRICTION.**

The same $F_{push}$ will not be able to get the block moving again.

$F_{push} < F_{static\ friction\ MAX}$ since $F_{static\ friction\ MAX} \geq F_{kinetic\ friction}$

$$F_{static\ friction\ ACTUAL} = F_{push} \Rightarrow a = 0 \Rightarrow v = 0$$

5 options

Steve is trying to push a box across a *rough* floor. (*'rough'* $\Rightarrow$ *friction matters*)

Which of the following statements describes the situation correctly?

A. The box moves forward because Steve pushes forward slightly harder on the box than the box pushes backward on Steve.

B. Because action always equals reaction, Steve cannot push the box; the box pushes back just as hard as Steve pushes forward & there is no motion.

C. Steve gets the box to move by giving it a sudden, sharp push so that the force on the box is momentarily greater than the force exerted by the box on Steve.

D. The force of the box on Steve equals Steve's force on the box, but the floor exerts a greater force on Steve than the box does.

E. Steve can push the box forward only if he weighs more than the box does.

5 options

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D. The force of the box on Steve equals Steve's force on the box, but the floor exerts a greater force on Steve than the box does.

E. Steve can push the box forward only if he weighs more than the box does

The force Steve exerts on the box is equal to that exerted by the box on the Steve; these are an action-reaction pair.

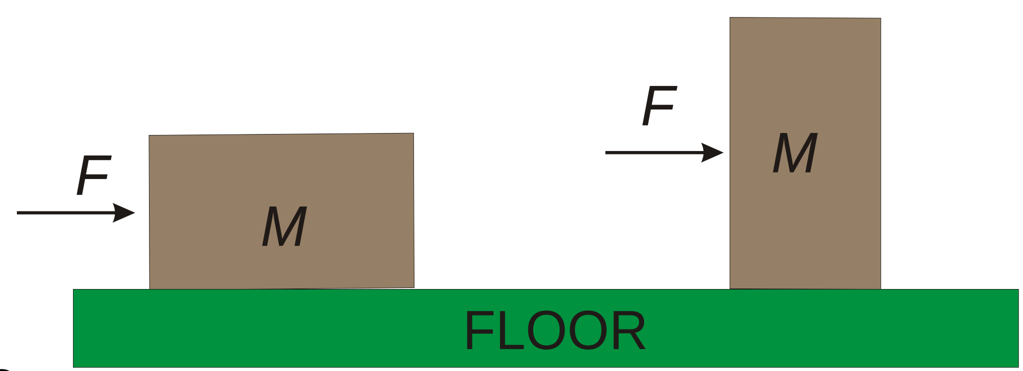
Steve moves forward because of a forward frictional force exerted by the floor as a reaction to Steve's pushing on the floor. He needs to push on the floor with a greater force than friction exerts on the box. (~cart & horse).

The box moves because the force on IT by Steve is greater than the force that friction from the floor exerts ON THE BOX.

5 options

You are pushing a wooden crate across the floor at constant speed. Dr. C. decides to turn the crate on its end, reducing the surface area in contact with the floor in half. How does the force you have to supply now compare to the original force, assuming you still push the crate at a constant speed?

A. four times as much force
B. twice as much force
C. the same force
D. half as much force
E. one-fourth as much force



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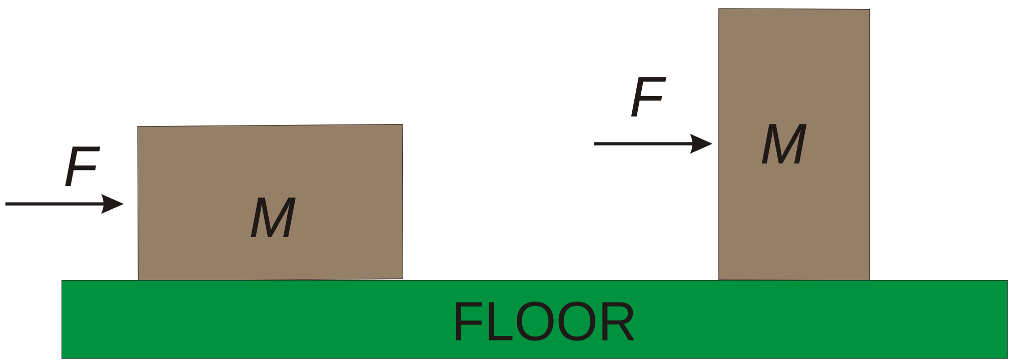
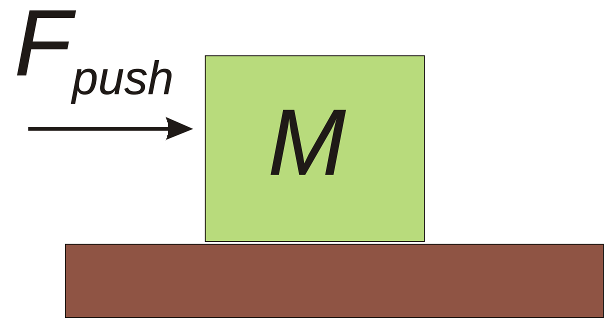



**Friction = $\mu_k N = \mu_k Mg$**

**The orientation and shape of *M* doesn't matter.**

$N = Mg$

- 6 options




A block of mass $M$ is at rest on a surface with a coefficient of *static* friction $\mu_s$. A force $F$ pushes against the block but the block does not move.

Which of the following expressions is always correct for the magnitude of the force of static friction in this situation?

A. $|F_{static\ friction}| = |Mg|$

B. $|F_{static\ friction}| = |\mu_s Mg|$

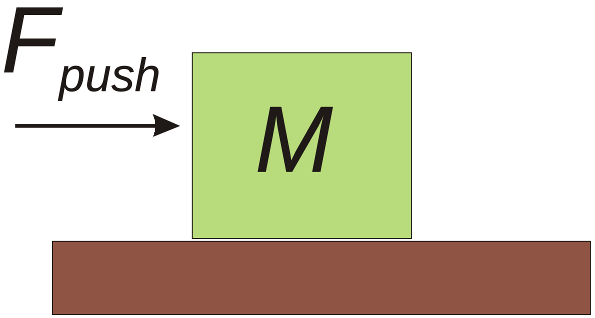
C. $|F_{static\ friction}| = |\mu_k Mg|$

D. $|F_{static\ friction}| = |\mu_s Mg - Mg|$

E. $|F_{static\ friction}| = |F_{push}|$

F. $|F_{static\ friction}| = |F_{push} - \mu_s Mg|$

- 6 options

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C. $|F_{static\ friction}| = |\mu_k Mg|$

D. $|F_{static\ friction}| = |\mu_s Mg - Mg|$

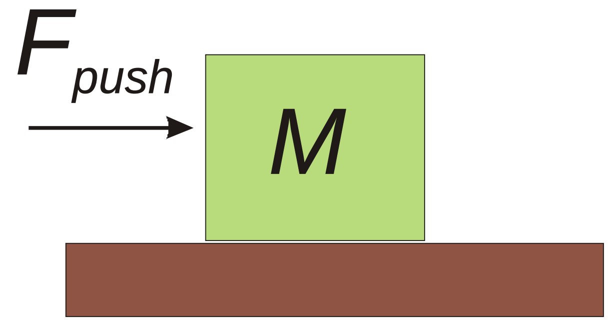
**E. $|F_{static\ friction}| = |F_{push}|$**

F. $|F_{static\ friction}| = |F_{push} - \mu_s Mg|$

**Since the object is not moving $\Rightarrow$ $a = 0$, the magnitude of the force of static friction must equal the pushing force.**

**$|F_{static\ friction}|$ can be as large as $|\mu_s Mg|$ but can also be as small as 0.**

- 5 options




You are pushing a box across a rough floor at <u>constant velocity</u> $v_o$.

If you <u>double</u> the force you are applying, the box will move:

A. At double the speed $v_o$

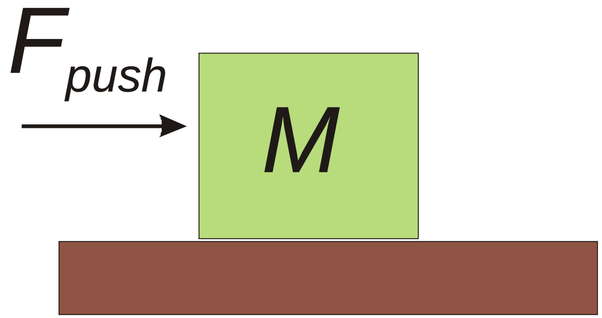
B. At a constant speed that is greater than $v_o$ but not twice as great

C. At a constant speed for awhile, then with increasing speed

D. With increasing speed for awhile, then with a new constant speed

E. With a continuously increasing speed indefinitely

- 5 options




You are pushing a box across a rough floor at a constant velocity $v_o$.

If you double the force you are applying, the box will move:

A. with a constant speed that is double the speed $v_o$

B. with a constant speed that is greater than $v_o$ but not twice as great

C. for a while with a speed that is constant and greater than $v_o$, then with an increasing speed

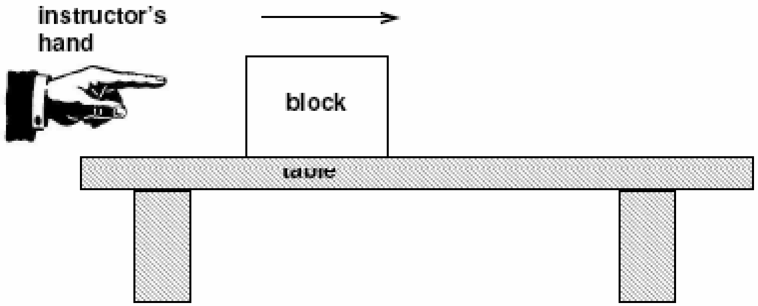
D. for a while with an increasing speed, then with a constant speed

E. with a continuously increasing speed indefinitely

The box was originally moving with constant velocity, which means that the force you applied was equal to the force of kinetic friction opposing the motion.

Once you increase the force you apply there is a net force on the box. The frictional force isn't increasing, so the box accelerates and its speed increases indefinitely.

- 2 options each, 8 questions;
TRUE = 1, FALSE = 2




To demonstrate sliding friction, an instructor takes a rectangular block of wood and gives it a quick tap (*not a continuous push*) so that the block slides across a table, eventually coming to rest because of friction. The following statements <u>may</u> describe the motion of the block after it has been released and before it comes to rest.

<u>Is each of the following 1 = TRUE or 2 = FALSE?</u>

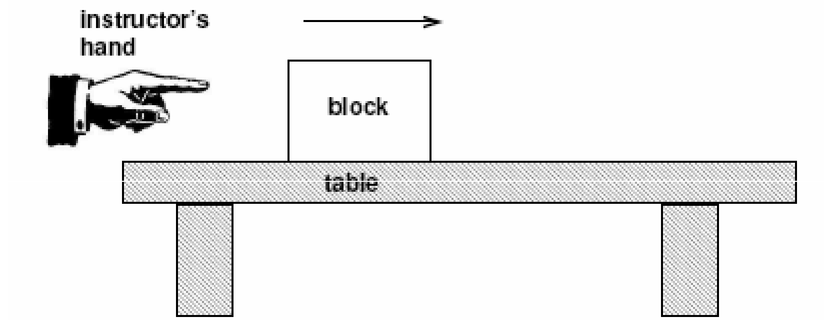
Immediately after the block leaves the instructor's hand, the net force on the block is zero.

**FALSE, there's a force of friction.**

At the instant the block leaves the instructor's hand, the velocity of the block remains constant for awhile and then gradually decreases.

**FALSE, the velocity starts decreasing immediately in response to the force of friction.**

- 2 options each;
TRUE = 1, FALSE = 2




The acceleration (*deceleration*) of the block depends on its mass.

**FALSE, $M$ divides out of $F = Ma$: $-\mu N = -\mu Mg = Ma \Rightarrow a = -\mu g$**

The acceleration of the sliding block depends on what material it's made of and its surface treatments (*sanding*, *polishing*, *etc*.).

**TRUE, since these factors help determine $\mu_k$.**

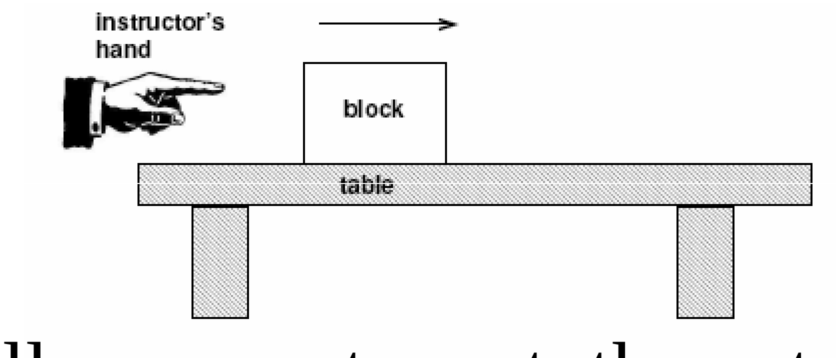
The normal force and the weight are equal in magnitude.

**TRUE, the sum of the forces in the *y*-direction equals 0.**

There is a constant force of friction <u>on the table</u> in the direction of the motion of the block.

**TRUE, Newton's 3$^{rd}$ Law on action-reaction applies to friction too. If the block feels a frictional force in the –*x* direction, the table feels a reaction force in the +*x* direction.**

 - 2 options each;
TRUE = 1, FALSE = 2




At the moment the block finally comes to rest, the net horizontal force on the block is $F_{friction} = \mu_s N$.

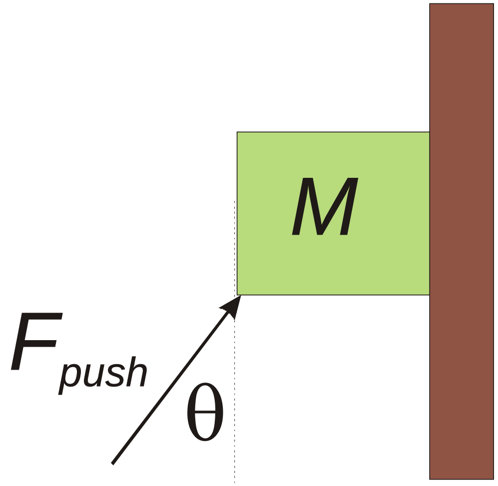
**FALSE. There is no force of static friction if there is no force trying to make the block move.**

At the moment the block finally comes to rest, the net horizontal force on the block is zero.

**TRUE. If v = 0 and *a* = 0 you'd better have $F_{net} = 0$**

# VERTICAL FRICTION

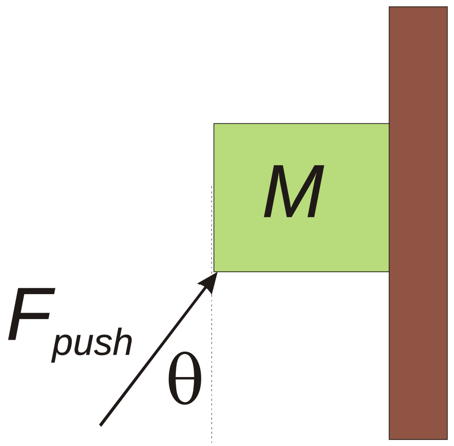
**~ Ohanian 6.29** but with kinetic friction**

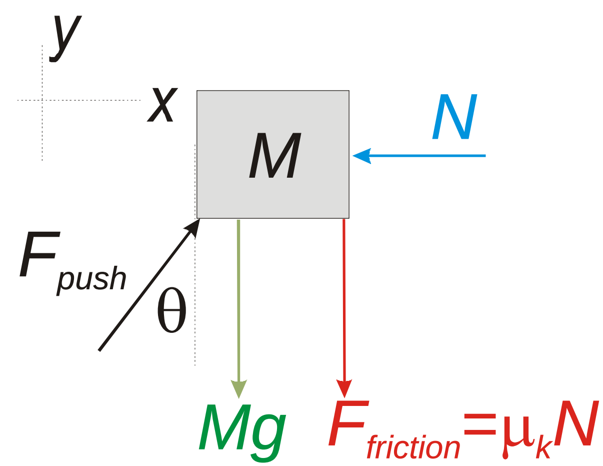



You are pushing a block of mass $M$ up a vertical surface with a force $F$ that's applied at an angle $\theta$ *wrt* vertical.

The coefficients of friction between the block and the wall are $\mu_k$ and $\mu_s$.

If the block is moving up at constant velocity, what is the force $F$ that you must be supplying, in terms of the variables listed above and $g$?




- Draw a free body diagram and define your coordinate system.
- Note that the block is moving, so it's $\mu_k$ that we want.
- Apply $\Sigma \boldsymbol{F} = m\boldsymbol{a}$ in the $x$ and $y$ directions with $a = 0$ in this case.

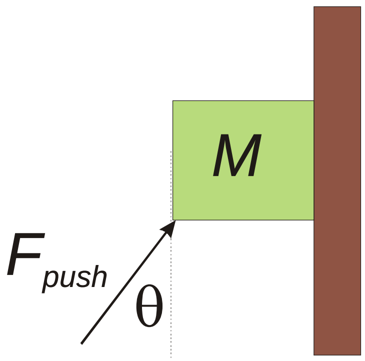
$$\Sigma F_x = F_{push}\sin\theta - N = 0$$

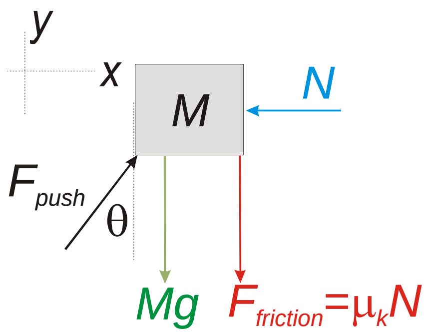
$$\Sigma F_y = F_{push}\cos\theta - Mg - \mu_k N = 0$$

→ 2 equations, $\theta$, $M$, & $g$ are presumably known but $F$ & $N$ are not.

You can't solve for $F$ without solving for $N$ and *vice versa*.

You can solve two equations for two unknowns in a variety of ways.




$$\Sigma F_x = F_{push}\sin\theta - N = 0$$

$$\Sigma F_y = F_{push}\cos\theta - Mg - \mu_k N = 0$$

- Let's start by eliminating $N$.
  - From the $\Sigma F_x$ equation we have $N = F_{push}\sin\theta$
  - Plug this into the $\Sigma F_y$ equation to get

$$F_{push}\cos\theta - Mg - \mu_k (F_{push}\sin\theta) = 0$$

  - Rearrange to get the $F_{push}$ term on one side of the equation

$$F_{push}(\cos\theta - \mu_k \sin\theta) = Mg$$

- And we have our answer $F_{push} = \dfrac{Mg}{\cos\theta - \mu_k \sin\theta}$

# THINK!

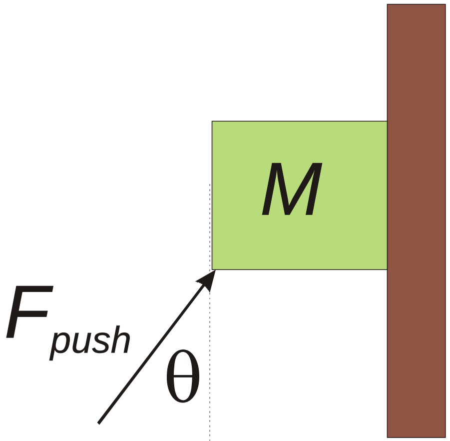
$$F_{push} = \frac{Mg}{\cos\theta - \mu_k \sin\theta}$$

What happens as $\theta \to 0°$?

$F_{push} = Mg/\cos\theta = Mg$

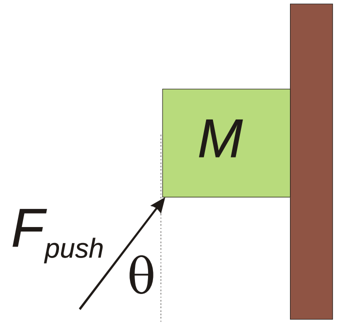
$F_{push}$ is just balancing the weight of the book

Friction is irrelevant because $N \to 0$. $N = \mathrm{F}_{\mathrm{push}} \sin 0°$



# THINK!

$$F_{push} = \frac{Mg}{\cos\theta - \mu_k \sin\theta}$$




What happens as $\theta \rightarrow 90°$, a horizontal pushing force?

$F_{push} = -Mg/\mu_k$ is not possible since it reverses $F$!

**⇒ There's no way to push the book up with a horizontal force!**

As you increase $\theta$ from 0° towards 90°,

*the cosine term gets smaller as the sine term increases &*

at some point $(\cos\theta - \mu_k \sin\theta) = 0$ & $F_{push} = Mg/0$

There is a '*critical angle*' $\boldsymbol{\theta = \tan^{-1}(1/\mu_k)}$

→ You need an **infinite** force at this θ to keep the block moving!

**It is hard to supply an infinite force! Try it.**

Note that you could measure $\mu_k$ this way.

For example: $\mu_k = 1 \leftrightarrow \theta = 45°$ *Smaller* $\mu_k \Rightarrow$ *larger* $\theta$

# HOLDING THE BOOK

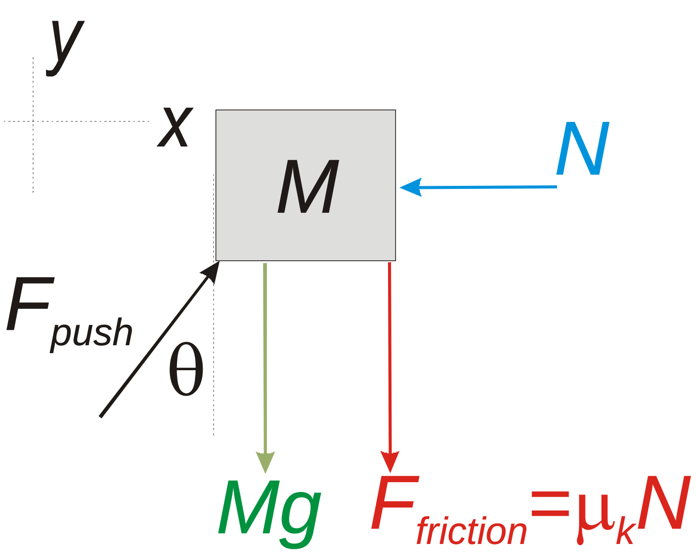
What changes if you are simply trying to keep the book from sliding **down** the wall?

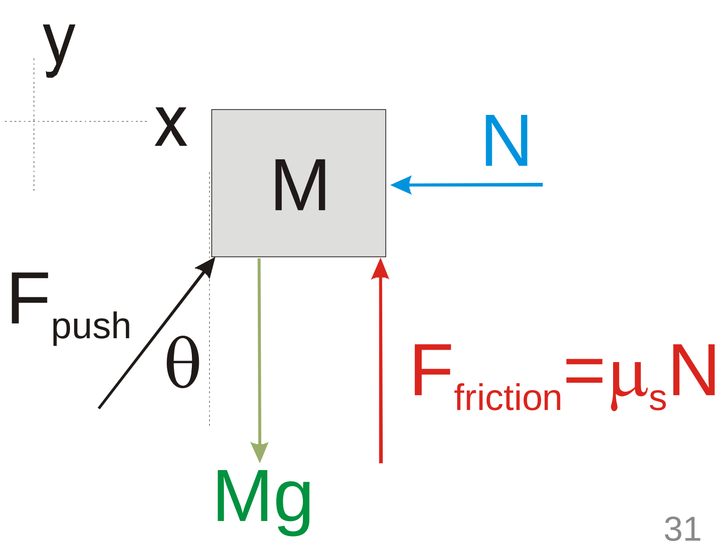
You're dealing with **static** rather than **kinetic** friction.

The direction of the frictional force is up instead of down *assuming you don't push too hard upwards*.

Push slightly too hard and $F_{static\text{-}friction}$ will change direction!

($too\ hard = F_{push}\cos\theta > Mg$)

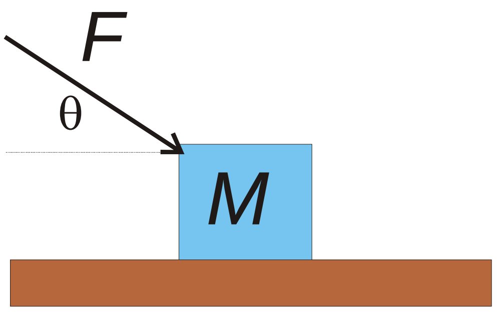


# TRANSITION BETWEEN STATIC & KINETIC FRICTION

Consider the system shown below. You are given

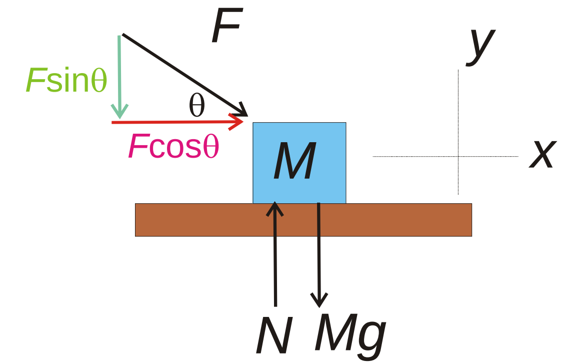
- $F$ in terms of magnitude & direction = 3 N at 30°
- $M$ = 5 kg
- $g$ = 9.8 m/s$^2$
- $\mu_s$ = 0.8 & $\mu_k$ = 0.6




but you are NOT told whether you should use kinetic or static friction.

**HOW DO YOU PROCEED?**

# FRICTION DECISION TREE




#1: Is $M$ moving initially (*in the direction of F* )? YES or NO/?

**YES**

#1Y: Use <u>kinetic</u> friction to calculate the acceleration.

$$Ma_x = \Sigma F_x = F\cos\theta - \mu_k N = F\cos\theta - \mu_k(Mg + F\sin\theta)$$

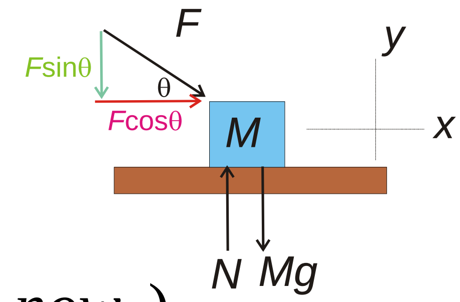
**DONE** – unless $M$ is decelerating → it will stop moving at some point.

**NO/?** (*Note that this is <u>static</u> friction now.* )

#1N: Is $F\cos\theta > \mu_s(Mg + F\sin\theta)$ **YES** or NO

CONTINUED ON NEXT SLIDE

# FRICTION DECISION TREE




**NO/?** (from previous slide) (*Note that this is* ***static*** *friction now.* )

#1N: Is $F\cos\theta > \mu_s(Mg + F\sin\theta)$ YES or NO

**YES**

#2Y: $F$ will overcome static friction & $M$ will start accelerating.

Use the equation in #1Y with $\mu_k$ to calculate the acceleration.

**NO**

#2N: $M$ will remain at rest with $F\cos\theta = F_{static\ friction}$

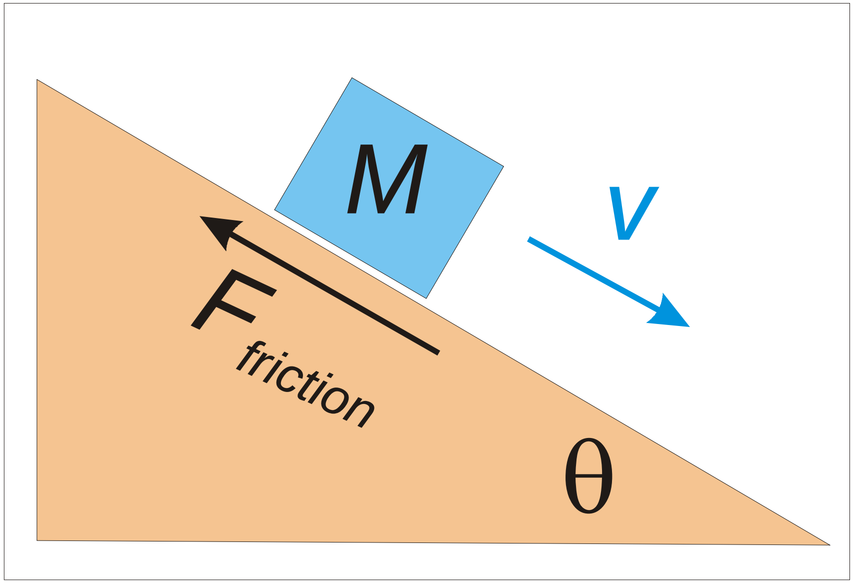
$F_{static\ friction}$ will be equal to or less than its maximum value $\mu_s(Mg + F\sin\theta)$

**DONE**

*This becomes more complicated if the block is initially moving in a direction opposite to F, but the basic analysis process is the same & M is guaranteed to stop for at least a moment sometime since both F & $F_{friction}$ oppose the motion.*

# <u>INCLINES with FRICTION</u>

Consider a block on a ‘rough’ inclined plane




**What is the acceleration of this block?**

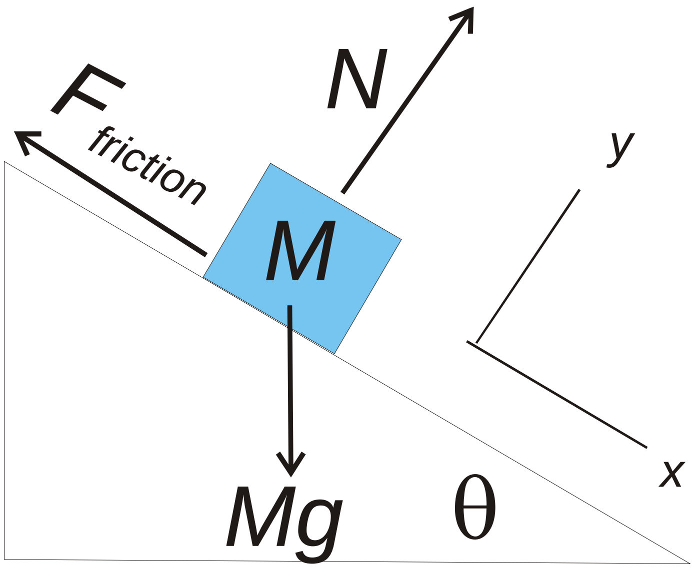
*assuming it is sliding <u>down</u> the inclined plane*

*which lets me draw the direction of $F_{friction}$ as shown.*

# DEFINE YOUR SYSTEM & COORDINATE SYSTEM

# &

# DRAW THE FREE BODY DIAGRAM



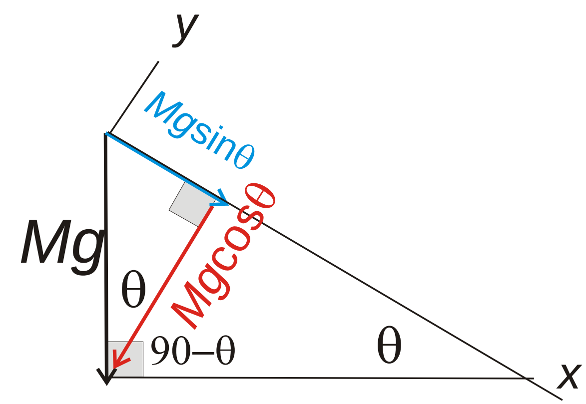

There are 3 forces acting on the block: $Mg$, $N$ & $F_{friction}$

$N$, $F_{friction}$ & $a$ are all easier with an angled coordinate system.

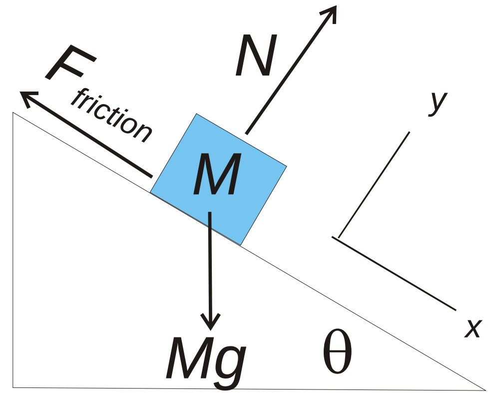
*Only Mg is vertical → the tilted coordinate system is the best.*

Choosing +***x*** pointing down rather than up the plane is arbitrary.

*The components of Mg in the tilted xy coordinate system are shown to the right.*




Apply $\Sigma \boldsymbol{F} = M\boldsymbol{a}$ for each vector component of $\boldsymbol{F}$




$\Sigma F_y = N - Mg\cos\theta = Ma_y = 0$

This gives us the normal force $N$

**Eq. #1:** $N = Mg\cos\theta$

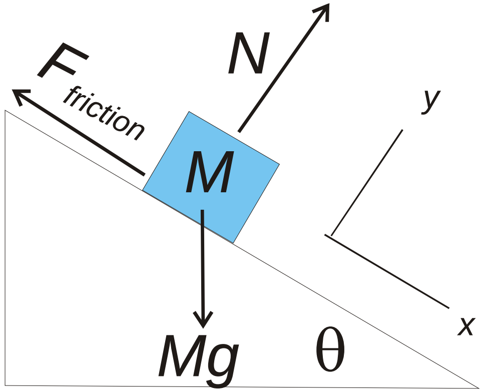
$\Sigma F_x = Mg\sin\theta - F_{friction} = Ma_x$

Note that $F_{friction} = \mu_k N = \mu_k Mg\cos\theta$ (*after plugging in N from Eq. #1*)

**Eq. #2:** $Mg\sin\theta - \mu_k Mg\cos\theta = Ma_x$

ANSWER: $\boldsymbol{a_x = (\sin\theta - \mu_k\cos\theta)g}$

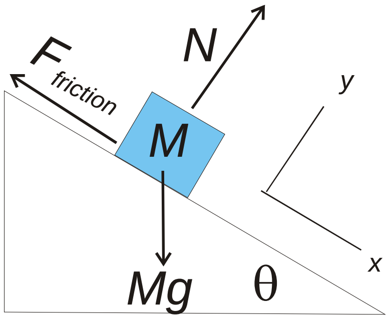
ANSWER: $a_x = (\sin\theta - \mu_k \cos\theta) g$




**COMMON SENSE (*reality*) CHECKS:**

- *The dimensions/units are correct; the answer is an acceleration.*
- *The answer behaves properly as $\mu_k \rightarrow 0$; just like a frictionless inclined plane.*
- *The answer behaves properly as $\theta \rightarrow 90°$; $a \rightarrow g$*
- *It appears that we can trust this answer.*

**EXCEPT**



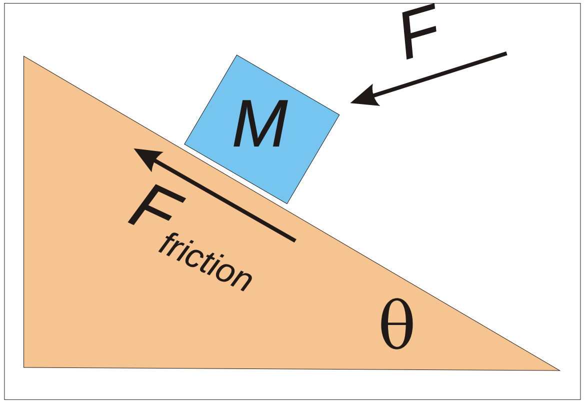

ANSWER: $\boldsymbol{a_x = (\sin\theta - \mu_k \cos\theta) g}$

**Except what's the significance of $(\sin\theta - \mu_k\cos\theta) = 0$ *or* <0 for small $\theta$?**

- $a < 0$ suggests the block is accelerating in a direction UP the incline.

  This is OK, it means the block is slowing as it moves down the incline.
- a = 0 is okay; the block can move at constant velocity down the incline.
- But if $v = 0$, we need to redo the problem with $\mu_s$ instead of $\mu_k$.
- If we had started with a negative $v$, pointing up the incline, we'd have to redo our analysis with the direction of $F_{friction}$ reversed.

# VARIATIONS ON A THEME

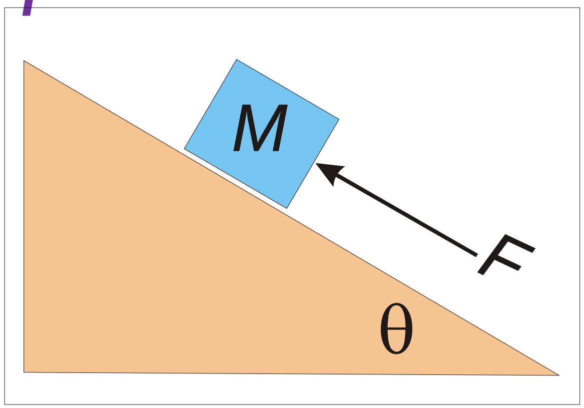


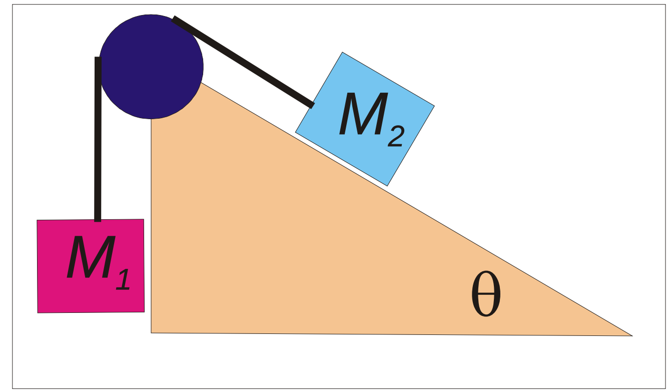

**You should be able to handle the following questions using the concepts just presented.**

- If the block is initially at rest, for what maximum $\theta$ will it remain at rest?
- What $\theta$ would result in the block sliding down the plane at *constant* speed?
- What will change if you apply additional forces in arbitrary directions?

# VARIATIONS ON A THEME

## *friction on an incline ~ Ohanian 6.33***




The two problems shown above are similar except that the explicit force pushing in the left figure is replaced by a hanging block in the right figure.

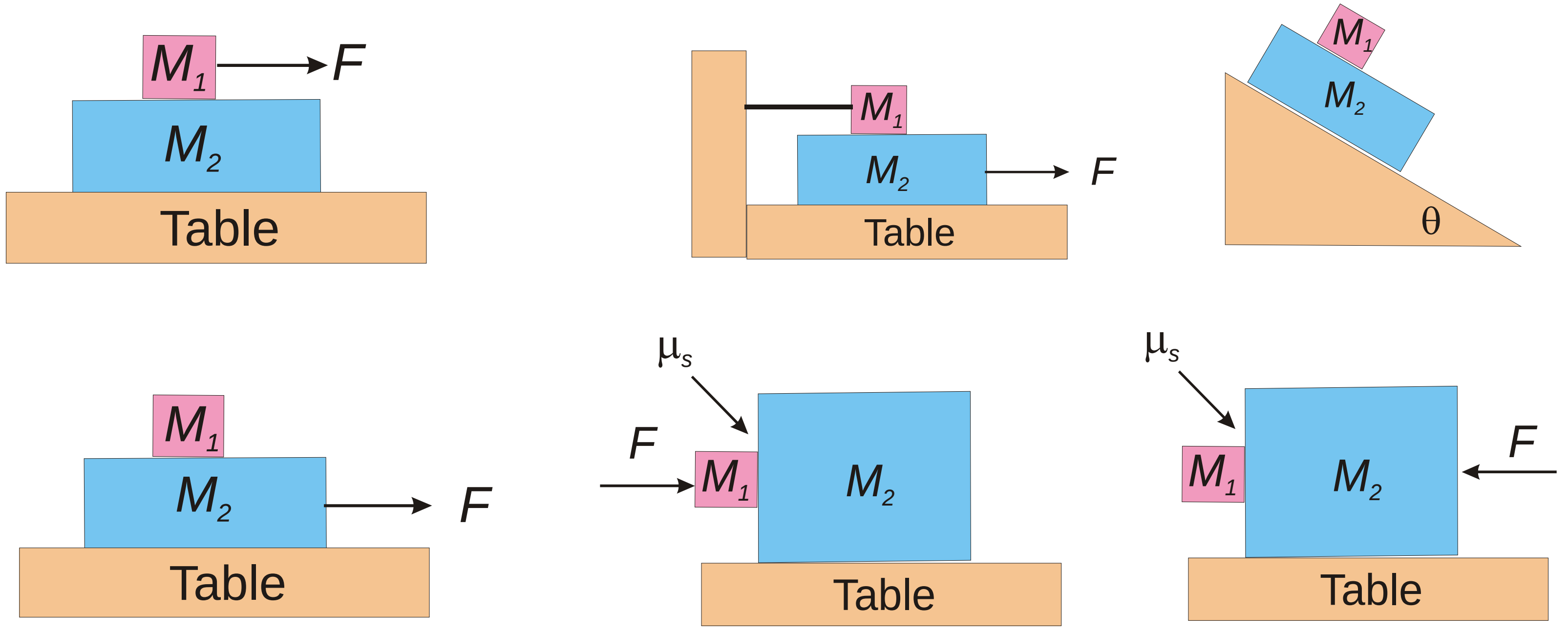
- **The tension in the string is NOT $T = M_1 g$.**

  This tension is modified by the acceleration of the blocks.

- You have to solve a pair of coupled $\Sigma F = Ma$ equations, one for each mass.
- But this is still VERY similar to other problems you've seen.

# BLOCKS + BLOCKS

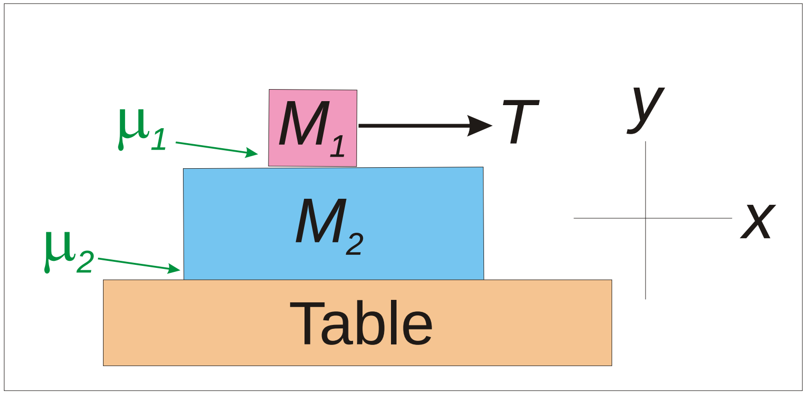
The drawings below show variations of block on block problems. Friction (*kinetic and/or static*) might ≠ 0 any pair of surfaces.




We'll examine two of these systems in lecture.

**YOU** *should be able to analyze the others*

*for homework and/or exams*.



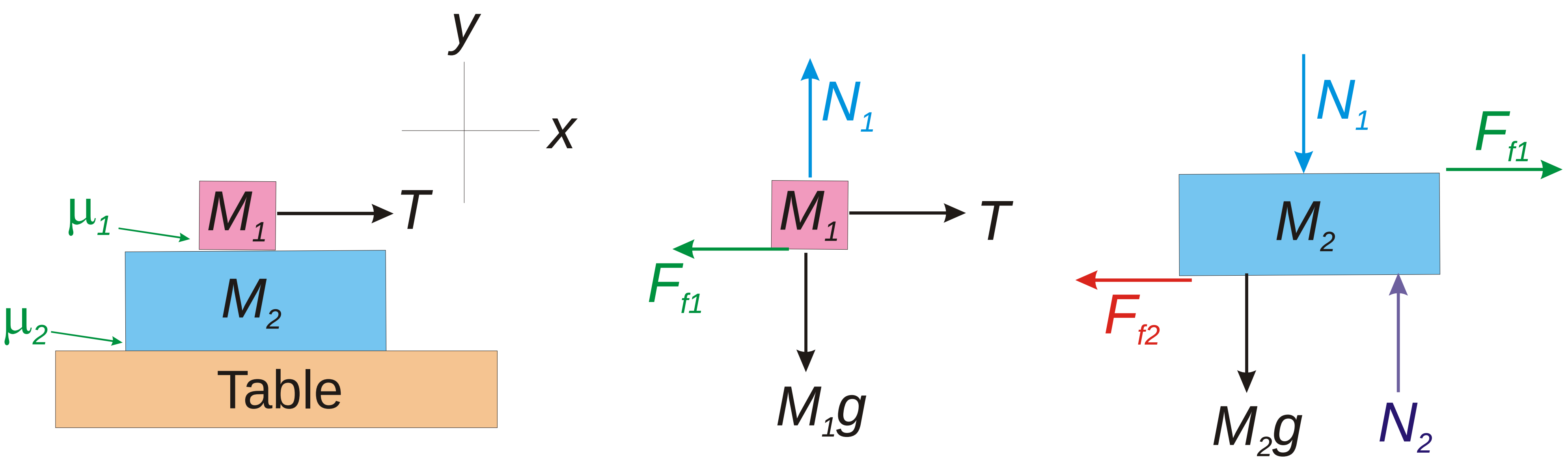

Ohanian 6.32**: A force $T$ pulls $M_1$ to the right.

$T$ and the coefficients of static friction are set so that both blocks <u>move</u> *wrt* to each other & to the table.

*(This problem is EASIER if static friction pins one or both blocks in place.)*

Given $M_1$, $M_2$, $T$, $\mu_1$ and $\mu_2$,

what is the acceleration of each mass?

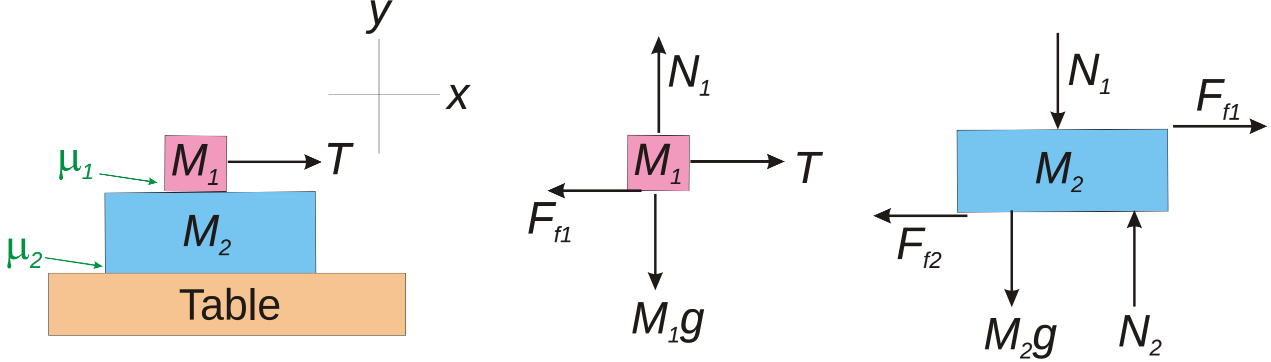



**Draw the free body diagrams & choose your coordinate system.**

*The figure above shows the forces & my choice of coordinate system plus labels for the various quantities.*

Note that the force of friction between the two blocks acts on both blocks but in opposite directions.

This is an action-reaction pair.




Apply $\Sigma \boldsymbol{F} = M\boldsymbol{a}$ for each vector component of $\boldsymbol{F}$ for each mass.

$M_1$: $\Sigma F_y = N_1 - M_1 g = 0$

so $N_1 = M_1 g$ & $F_{f1} = \mu_1 N_1 = \mu_1 M_1 g$

$M_2$: $\Sigma F_y = N_2 - N_1 - M_1 g = 0$

so $N_2 = N_1 + M_2 g = (M_1 + M_2)g$ & $F_{f2} = \mu_2 N_2 = \mu_2 (M_1 + M_2)g$

$M_1$: $\Sigma F_x = T - F_{f1} = T - \mu_1 M_1 g = M_1 a_1$

$a_1 = (T/M_1) - \mu_1 g$ **We have the acceleration for $M_1$**

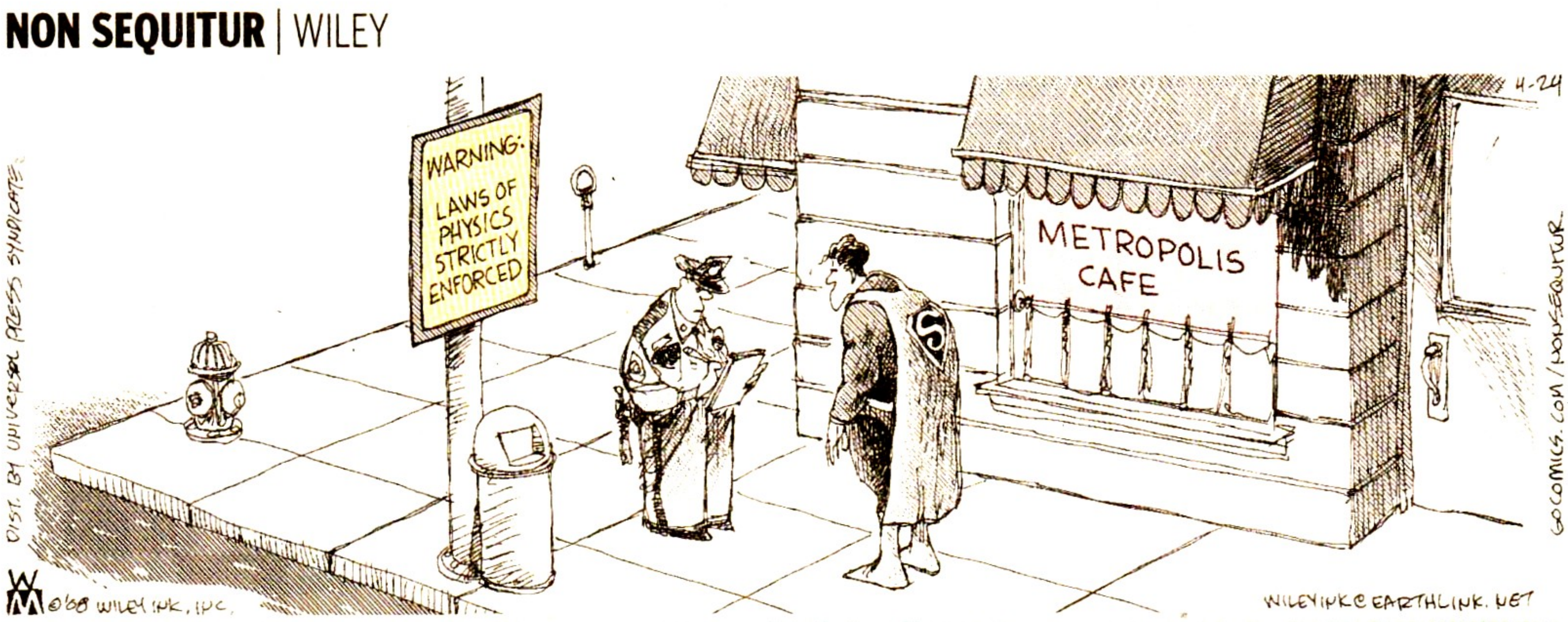
$M_2$: $\Sigma F_x = F_{f1} - F_{f2} = \mu_1 M_1 g - \mu_2 (M_1 + M_2)g = M_2 a_2$

$a_2 = \mu_1 g(M_1/M_2) - \mu_2 g(M_1 + M_2)/M_2$ **We have the acceleration for $M_2$**

**DONE!** *(except for thinking, which I'll leave to you!)*

We made it to slide #47 on Monday, February 16.

# PHYS 121 – SPRING 2015




**Chapter 6: Further Applications of Newton's Laws**

***FRICTION, AIR RESISTANCE, SPRINGS & UNIFORM CIRCULAR MOTION***

*(~118 slides )*

*version 02/18/2015*

We made it to slide # 47 on Monday, February 16.

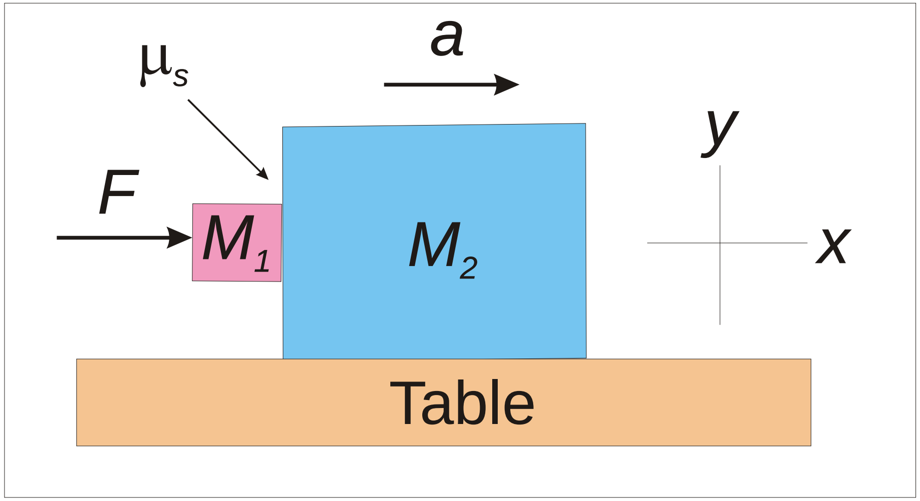
*Get your clickers ready.*

# ANNOUNCEMENTS

- The average for EXAM #1 has increased to 82.6 ± 14.5 %.

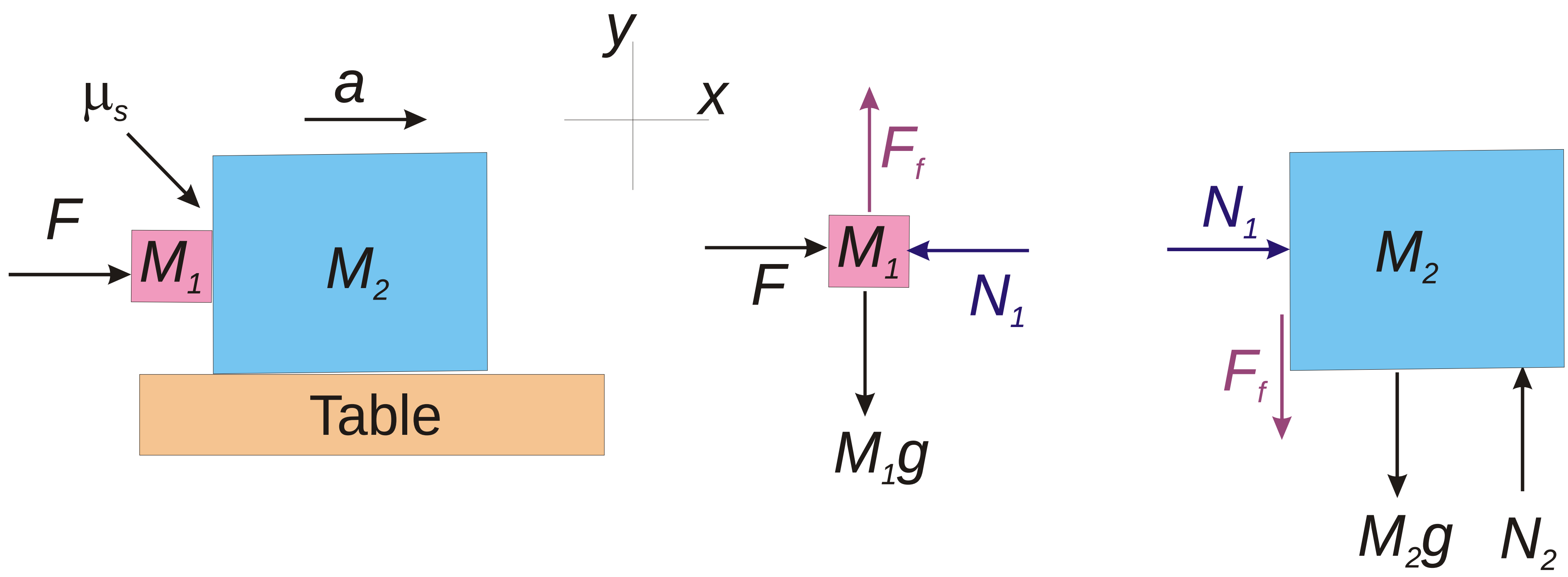
  There are now NINE perfect scores on the exam!
- Grades for homework, Exam #1 and bonus points will be posted on Blackboard by next week.
- Evaluation Kit midterm trial next week
  - Test of new course evaluation system in four ‘special’ courses.
  - 1 Bonus Point for everyone in PHYS 121 if our participation > 66%!

# FRICTION *vs.* GRAVITY




- Block $M_2$ is on a frictionless table.
- Block $M_1$ is pressed against block $M_2$ by a horizontal force $F$.
- The coefficient of static friction between the blocks is $\mu_s$.
- The blocks accelerate together to the right.

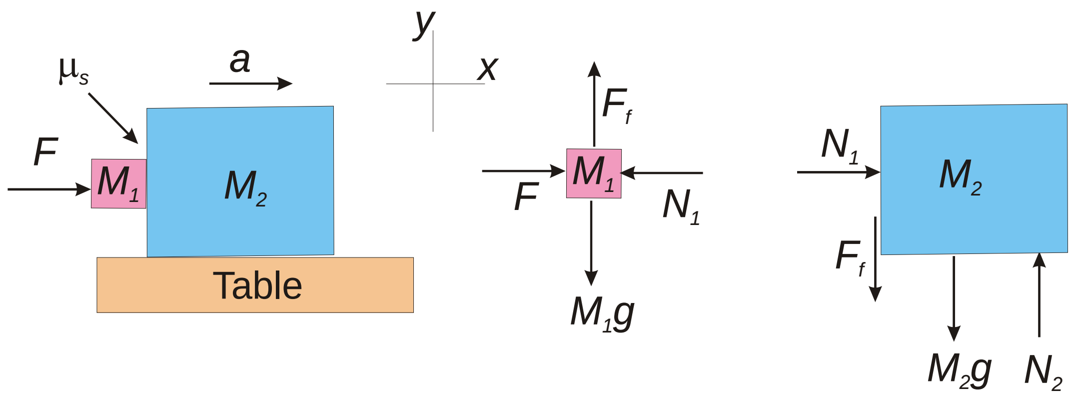
**What minimum $F$ will keep $M_1$ from falling?**




**Draw the free body diagram and choose your coordinate system.**

*The figure above shows the forces & my choice of coordinate system and labels for all the quantities.*

Note that the normal force between the blocks is an action-reaction pair.




Consider the combined $M_1 + M_2$ system:

F is the only *external* force in the *x*-direction,

so $a = F/(M_1 + M_2)$

Next focus on $M_1$:

$\Sigma F_y = F_f - M_1 g = 0$ (*assuming $M_1$ doesn't slip* )

but max $F_f = \mu_s N_1 \Rightarrow \mu_s N_1 = M_1 g$ & $N_1 = M_1 g/\mu_s$

$\Sigma F_x = M_1 a \rightarrow F - N_1 = F - M_1 g/\mu_s = M_1[F/(M_1 + M_2)]$

*Solve for F.*

$$F - \frac{M_1 g}{\mu_s} = F\frac{M_1}{M_1 + M_2}$$

group all terms with F on the left

$$F\left(1 - \frac{M_1}{M_1 + M_2}\right) = \frac{M_1 g}{\mu_s}$$

$$F\frac{M_2}{M_1 + M_2} = \frac{M_1 g}{\mu_s}$$

$$F = \left(\frac{M_1 M_2}{M_1 + M_2}\right)\left(\frac{g}{\mu_s}\right)$$

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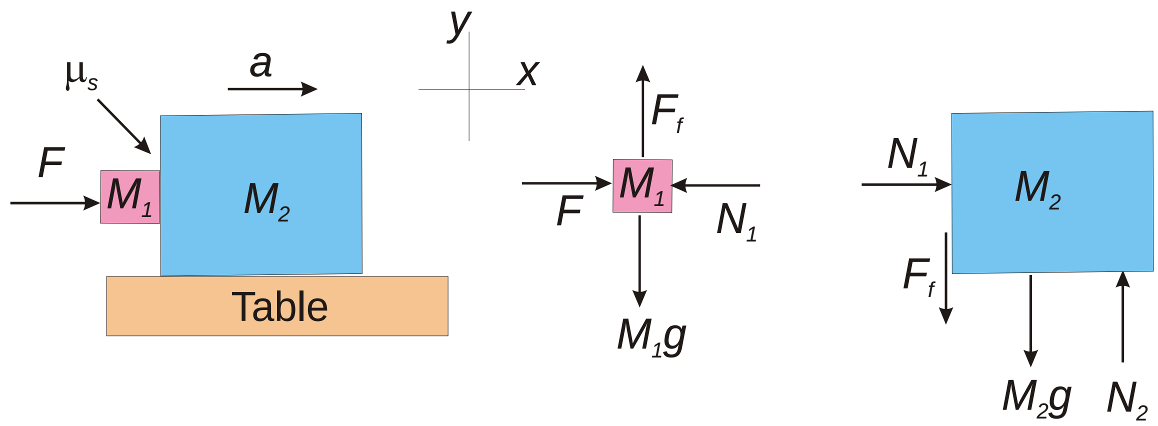



$$F = \left(\frac{M_1 M_2}{M_1 + M_2}\right)\left(\frac{g}{\mu_s}\right)$$

- $F$ is the minimum force for $M_1$ to remain at its original height.

  *corresponding to the* MAXIMUM $F_{static\ friction} = \mu_s N$

- Note that larger $\mu_s$ corresponds to a smaller $F$.
- Note that we didn't need $\Sigma \boldsymbol{F}_{M2}$ to solve this problem!

  but we would have needed it if there was friction between $M_2$ and the table (*& note that* $N_2 \neq M_2 g$)

  What changes if $F$ pushes on $M_2$ ~ upcoming homework?

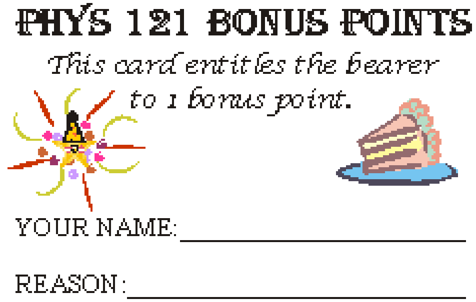
# DEMO

*A bonus point is available to a speedy PHYS 121 student who can demonstrate the effect we've just discussed.*

Can YOU run fast enough to defeat gravity with speed!
Actually with *acceleration* rather than *speed* ?
Are you clever enough to supply a constant acceleration without running?

**Running with erasers**

https://www.youtube.com/watch?v=86Bx668o-kI&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr&index=52



4 options

An object is held in place by friction on an inclined surface. The angle of inclination is increased until the object starts moving. Assume $\mu_s > \mu_k$ .

**If the surface is kept at this steeper angle, the object**

A. slows down.

B. moves at uniform speed down the incline.

C. speeds up.

D. none of the above

4 options

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**If the surface is kept at this steeper angle, the object**

A. slows down.

B. moves at uniform speed down the incline.

C. speeds up.

D. none of the above

Once you overcome static friction there is a net force on the object.

A force leads to an **acceleration** so the **velocity** increases.

4 options

There is friction between the two blocks shown below, but not between the table and the lower block.

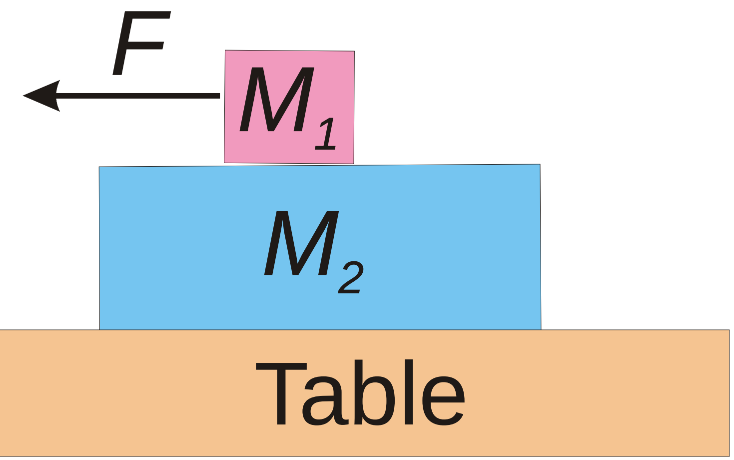
A horizontal force F is applied on the top block. **What are the directions of the friction forces on $M_1$ & $M_2$ , respectively?**

a) left and left (*on* $M_1$ & $M_2$)

b) left and right

c) right and right

d) right and left



4 options

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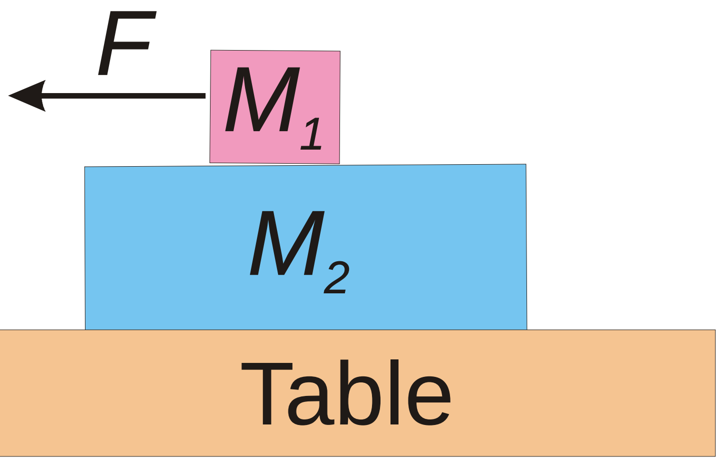
b) left and right

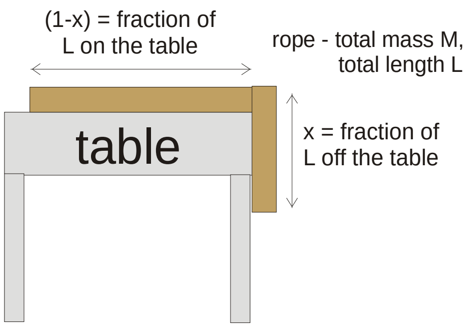
c) right and right

**d) right and left**




**These are an action-reaction pair and the force on $M_2$ is clearly to the left, dragging it along to the left.**

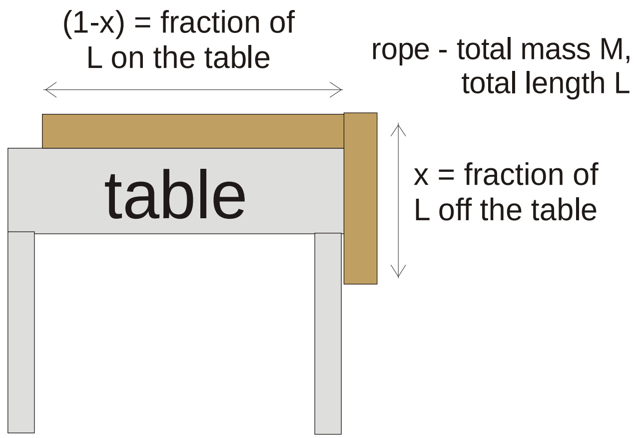
# Rope Trick




One end of a rope of length $L$ and mass $M$ is lying stretched out on a table. The coefficients of static and kinetic friction between the rope and the table are $\mu_s$ and $\mu_k$ respectively.

- What fraction, $x$, of the rope could hang over the edge of the table without pulling the rest of the rope over the edge?
- Once the rope starts sliding, what will its acceleration be as a function of $x$, the fraction of the length that is already off the table?

What fraction, *x*, of the rope could hang over the edge of the table without pulling the rest of the rope over the edge?




$$F_{friction} = F_{gravity}$$

$$\mu_s N = xMg$$

$$\mu_s (1 - x) Mg = xMg$$

$$\mu_s = x(1 + \mu_s)$$

$$x = \frac{\mu_s}{1 + \mu_s}$$

**THINK!**

*x* varies from 0 if friction is weak to 1 if friction is very strong.

But $\mu_s$ is rarely much bigger than 1, so *x* is probably less than ½.

Once the rope starts sliding,
what will its acceleration be as a function of *x*,
the fraction that is already off the table?

$$Ma = \sum F = F_{gravity} - F_{friction}$$
$$= xMg - \mu_k (1 - x) Mg$$

so

$$a = xg - \mu_k (1 - x) g$$
$$a = \left[ x - \mu_k (1 - x) \right] g$$

As $x \to 1$, $a \to g$ as we expect.

Bigger $\mu_k$ leads to slower acceleration.

# FRICTION in a FLUID:

**FLUID** = gas or liquid, but most likely air in PHYS 121.

**DRAG** ~ friction from moving through a *viscous* fluid (*like* $H_2O$ *or air*).

**TERMINAL VELOCITY** ≡ maximum speed at which an object can move through a fluid, given some fixed propulsive force (*like gravity*).

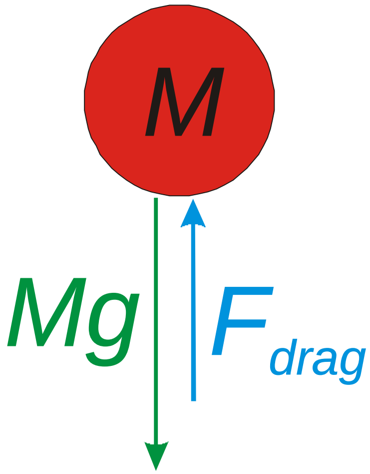
# TERMINAL VELOCITY

*for falling objects* (Ohanian 2.6)

- Terminal velocity is achieved when the force of gravity is balanced by the force of air resistance (*drag*).

$$\Sigma F = 0 \;\leftrightarrow\; a = 0 \Rightarrow v = \text{constant} \equiv v_{terminal}$$

$$|F_{drag}| = |F_{gravity}|$$




- $F_{gravity} = Mg$ is <u>independent of velocity</u>

*but*

- $F_{drag}$ <u>increases with velocity</u>.

# (*Low Speed*) AIR RESISTANCE

At low speeds , like that of dust floating in air, air resistance is linearly proportional to velocity

$$F_{drag} = - bv$$

where *b* is a constant that takes into account the density of air and the shape of the falling object.

Newton's 2nd Law $Ma = \Sigma F$ yields

$$Ma_y = Mg - bv_y \qquad \text{or} \qquad M\frac{d^2y}{dt^2} = Mg - b\frac{dy}{dt}$$

This can be solved analytically for $y(t)$ & $v(t)$, Ohanian 6.26**.

It is easy to solve for $v_{terminal}$ since it occurs when $a_y = 0$

$$v = \frac{Mg}{b}\left(1 - e^{-\frac{b}{M}t}\right) \qquad v_{y-\text{terminal}} = \frac{Mg}{b}$$

# “NORMAL” AIR RESISTANCE

For higher speeds and larger objects, like baseballs or people jumping from airplanes, *drag* is better described as

$$F_{drag} = ½\, C_d \rho_{air} A v^2$$ Ohanian Eq. 6.10

with

- $\rho_{air}$ = density of air (ρ = Greek letter *rho*)
- $A$ = cross-sectional area of the projectile ($\perp \boldsymbol{v}$)
- $v$ = velocity ($F_{drag} \sim v^2 \Rightarrow v$ *is destined to dominate*)
- $C_d$ = coefficient of drag, the projectile’s shape

($C_d$: *cube = 1.05, sphere = 0.47, streamlined = 0.04*)

http://en.wikipedia.org/wiki/Automobile_drag_coefficient

Honda Odyssey = 0.39, Porsche = 0.29, Prius = 0.23

but AREA counts as much as $C_d$!

# TERMINAL VELOCITY

$$F_{gravity} = F_{drag}$$

$$Mg \;= ½\, A\rho_A C_d v^2_{\,terminal}$$

$$v_{terminal} = (2Mg/A\rho_A C_d)^{1/2}$$

Plugging in $g = 9.8$ m/s$^2$ & $\rho_{Air} = 1.3$ kg/m$^3$

$$v_{terminal} = 3.88(M/AC_d)^{1/2}$$

where $v$ will be in m/s if $M$ is in kg & $A$*rea* is in m$^2$.

Air resistance limits the top speed of cars, bicycles, *etc*. horizontally too.

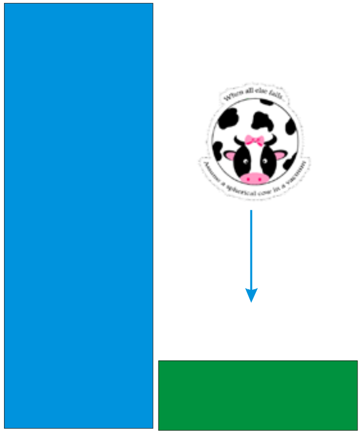
In fact, it's generally THE most important limiting factor for the top speed for a car or cyclist, which is why aerodynamics is so important.

http://ranuvp.files.wordpress.com/2013/02/drafting.jpg

Cycling records: http://en.wikipedia.org/wiki/Cycling_records

133.78 km/h (83.13 mph) on an open flat road

268 km/h (167 mph) behind a vehicle to "break" the wind.



# TERMINAL VELOCITY

$$v_{terminal} = 3.88(M/AC_d)^{1/2}$$

Imagine your *(former)* best friend is a spherical cow of radius 25 cm and mass 60 kg.

If you throw him/her/it off a tall building, what is $v_{terminal}$?

Plug in $C_d$ for a sphere = 0.47, $A = \pi r^2 = 0.196\ m^2$

$v_{terminal}$ = 99 m/s → (2.24 mph = 1 m/s) → 222 mph

**OUCH!**

# TERMINAL VELOCITY

$$v_{terminal} = 3.88(M/AC_d)^{1/2}$$

- How will this change with mass or shape?

A heavier object DOES '*fall faster*', proportional to $M^{1/2}$, after awhile.

'*fall faster*' $\Rightarrow v_{terminal}$ is larger, but $a_o = -g$ & $a_f = 0$.

- For objects of similar density (*people* ~ *water*)

$v_{terminal}$ scales as $r^{1/2}$

since $M$ scales as volume $r^3$ while $A$ scales as area $r^2$

→ a raindrop falls slower than you do, by $(1\text{ mm}/10\text{ cm})^{1/2} = 1/10$

(*fortunately or every rainstorm would be hazourdous*)

- Spreading your body can increase $C_d \sim 1$ & $A \sim (2\text{ m})(0.5\text{ m}) = 1\text{ m}^2$

$v_{terminal} = 30$ m/s → 67 mph **OW!**

- Adding a parachute with $r = 5$ m, $v_{terminal} = 3.4$ m/s → 7.6 mph

**LAND WITH YOUR KNEES BENT!**

# TERMINAL VELOCITY

Can you calculate $y(t)$ or how FAR an object falls from rest before reaching terminal velocity or how LONG it takes to get there?

Not unless you are handy with MatLab or a similar program.

The equation of motion is ( *with positive y down* )

$$Ma = F_{gravity} - F_{drag} = Mg - \tfrac{1}{2} A\rho_A C_d v^2$$

or

$$M\frac{d^2y}{dt^2} = Mg - \frac{1}{2}A\rho_A C_d\left(\frac{dy}{dt}\right)^2$$

and this doesn't have a simple analytic solution for $y(t)$;

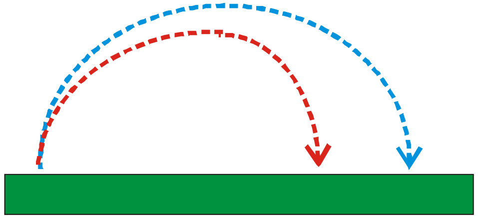
numerical analysis is necessary.

# PROJECTILE RANGE

## What about air resistance for projectiles?

Including $F_{drag} = ½\ \rho A C_d v^2$ opposite to a projectile's instantaneous velocity everywhere along its path:

1. Changes the shape of the path from a parabola to a path where the fall is sharper than the rise.
2. Shortens the maximum height and the range.
3. Lowers the angle of fire for maximum range from $\theta = 45°$ to ~ 43° for common parameters, ignoring effects like the spin of a ball, wind, etc.



3 options

A ball is thrown vertically upwards, quite far, and caught at the same height where it was released. Air resistance is NOT negligible.

Compared to the time it takes the ball to reach its maximum height on the way up, the time it takes to fall back down is:

A. longer

B. the same

C. shorter

3 options

A ball is thrown vertically upwards, quite far, and caught at the same height where it was released. Air resistance is NOT negligible.

Compared to the time it takes the ball to reach its maximum height on the way up, the time it takes to fall back down is:

A. **longer**

B. the same

C. shorter

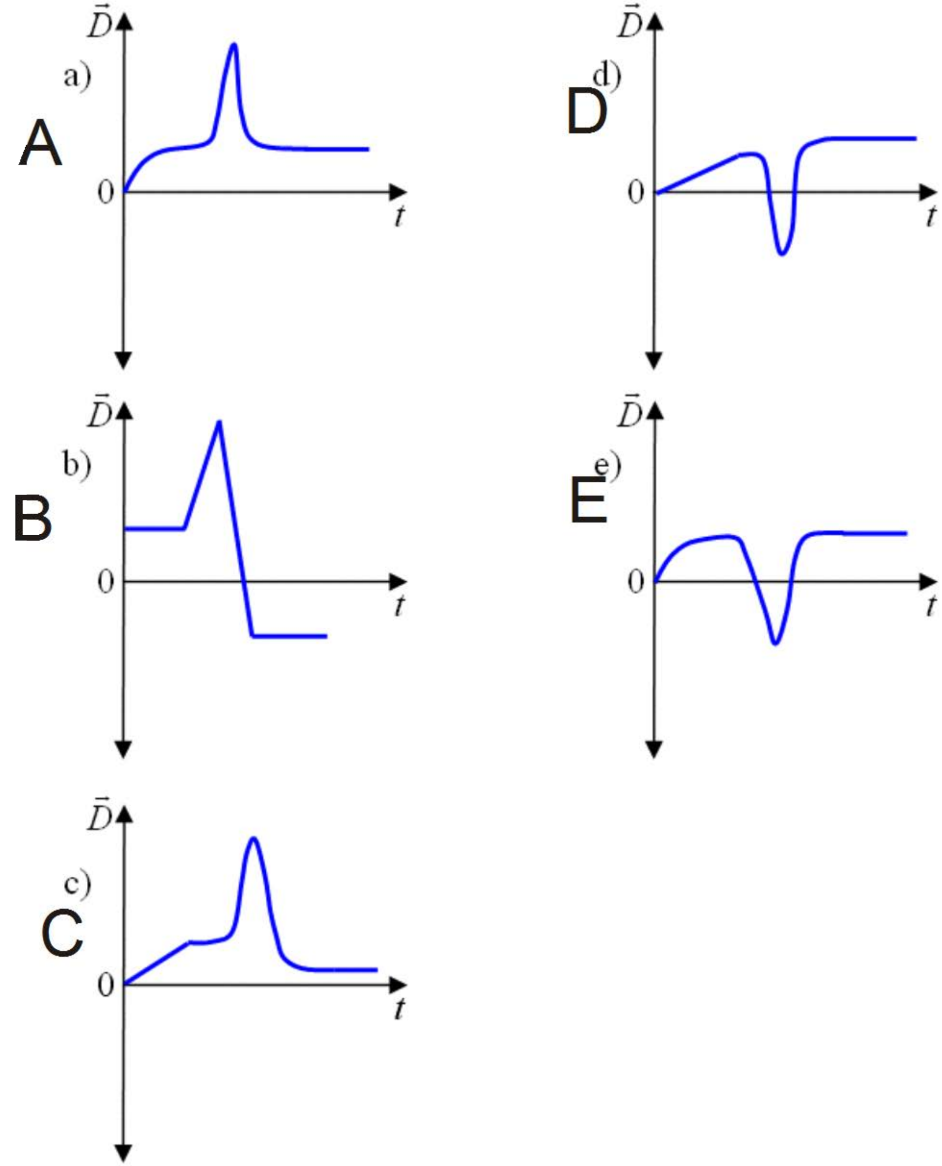
Drag acts to slow the ball & always points opposite to $v$.
At any given height on the path, the ball is moving slower on way down that it was on the way up.
Slower coming down = longer coming down.

5 options

A sky diver jumps from an airplane and falls for several seconds before she reaches terminal velocity.

She then opens her parachute, reaches a new terminal velocity, and continues her descent to the ground.

Which one of the graphs of the <u>drag force</u> *vs*. time best represents this situation?



5 options

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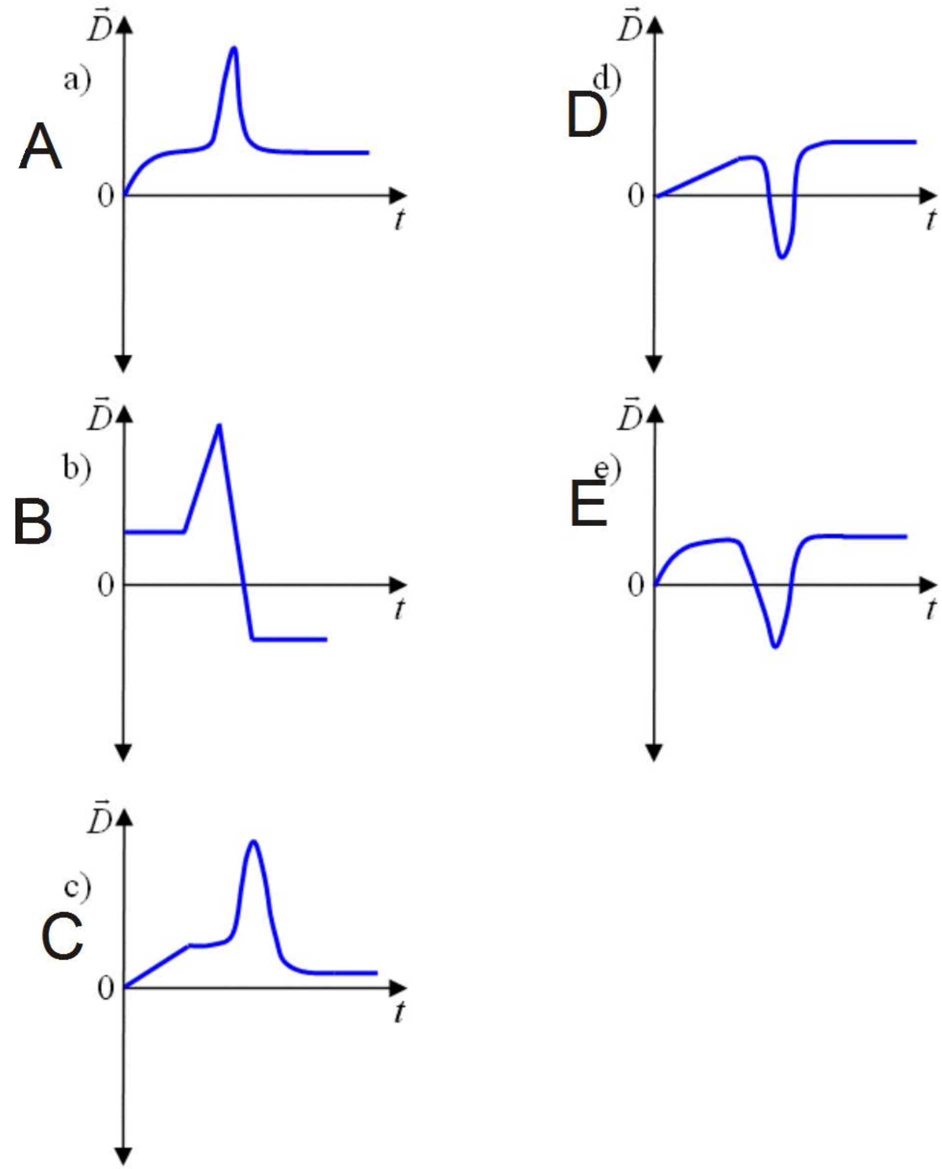



The answer is A.

The drag force = 0 at $t$ = 0.

The drag force can't be negative.

Both times she reaches terminal velocity, the drag force must = $mg$.

C is tempting but it's the terminal VELOCITY that's smaller with the parachute, not $F_{DRAG} = -mg$. Acceleration can = 0 with a range of $v_{terminal}$.

# 5 options

A feather and a truck are dropped from a height of 100 meters. The truck reaches the ground much faster than the feather.

Which of the following statements concerning this situation is true?

A. The truck has a larger terminal velocity than the feather because it experiences less air resistance than the feather.

B. The truck encounters a smaller force of air resistance than the feather and thus accelerates downwards faster.

C. Both experience the same amount of air drag, but the truck feels a greater force of gravity.

D. The feather experiences a larger air drag force than the truck and has a smaller terminal velocity.

E. None of the above statements are true.

5 options $F_{drag} = \frac{1}{2} A\rho_{air}C_d v^2$ $v_{terminal} = 3.88(M/AC_d)^{1/2}$

A feather and a truck are dropped from a height of 100 meters. The truck reaches the ground much faster than the feather.

Which of the following statements concerning this situation is true?

A. The truck has a larger terminal velocity than the feather because it experiences less air resistance than the feather. The truck experiences MORE air resistance!

B. The truck encounters a smaller force of air resistance than the feather and thus accelerates downwards faster. The truck feels a LARGER drag force!

C. Both experience the same amount of air drag, but the truck feels a greater force of gravity. They do NOT experience the same drag force.

D. The feather experiences a larger air drag force than the truck and has a smaller terminal velocity. The feather feels a SMALLER drag force at any given velocity.

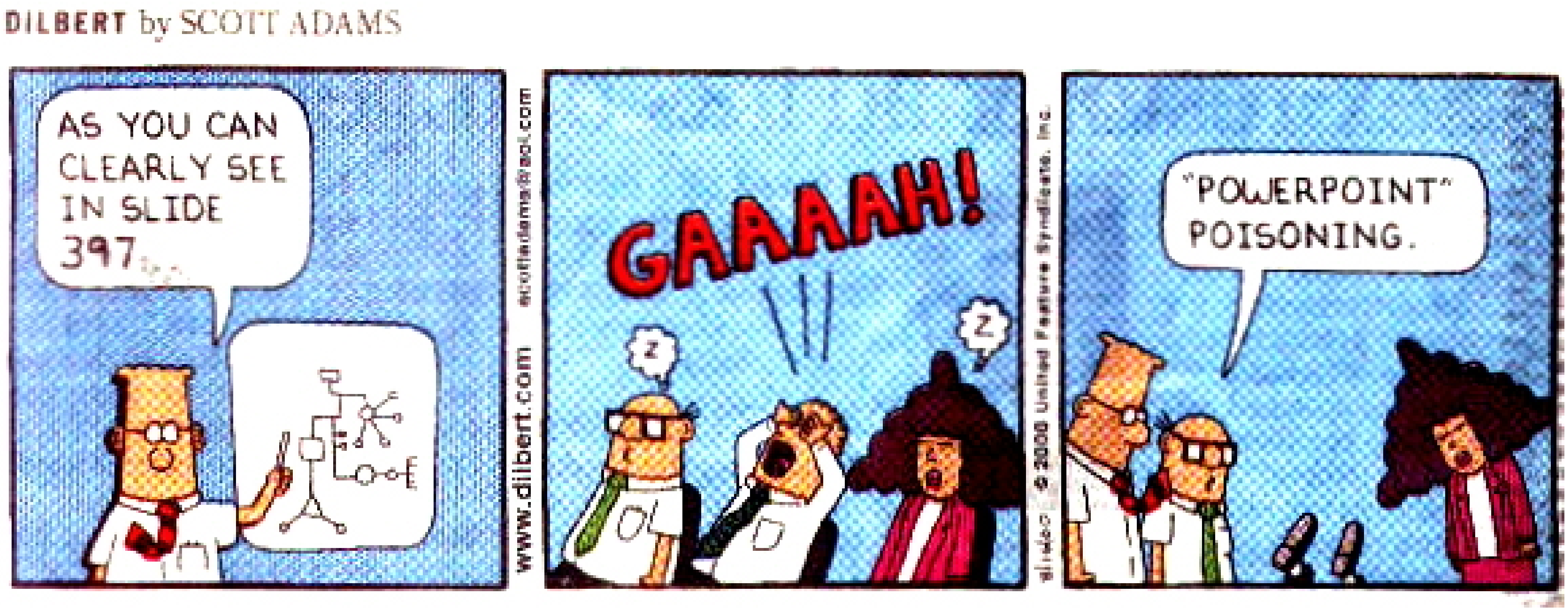
**E. None of the above statements are true.**

**The feather has a smaller terminal velocity because its ratio of *M/A* is smaller.**

**This can be traced back to a RATIO of *Mg* to $\frac{1}{2}A\rho_{air}C_d$**

We made it to slide #78 in lecture on Wednesday, February 18.

# PHYS 121 – SPRING 2015




**Chapter 6: Further Applications of Newton's Laws**

***FRICTION, AIR RESISTANCE, SPRINGS & UNIFORM CIRCULAR MOTION***

*(~118 slides )*

*version 02/23/2015*

We made it to slide # 78 on Wednesday, February 18.

*Get your clickers ready.*

# ANNOUNCEMENTS

- **Class is NOT canceled today due to winter weather!**
- "EvaluationKIT, a new course evaluation system being assessed mid-semester, will open for student responses on 2/23/2015. The evaluation period will close at 11:59 p.m. 2/27/2015."

  There's 1 Bonus Point for everyone if participation for PHYS 121 > 66%!

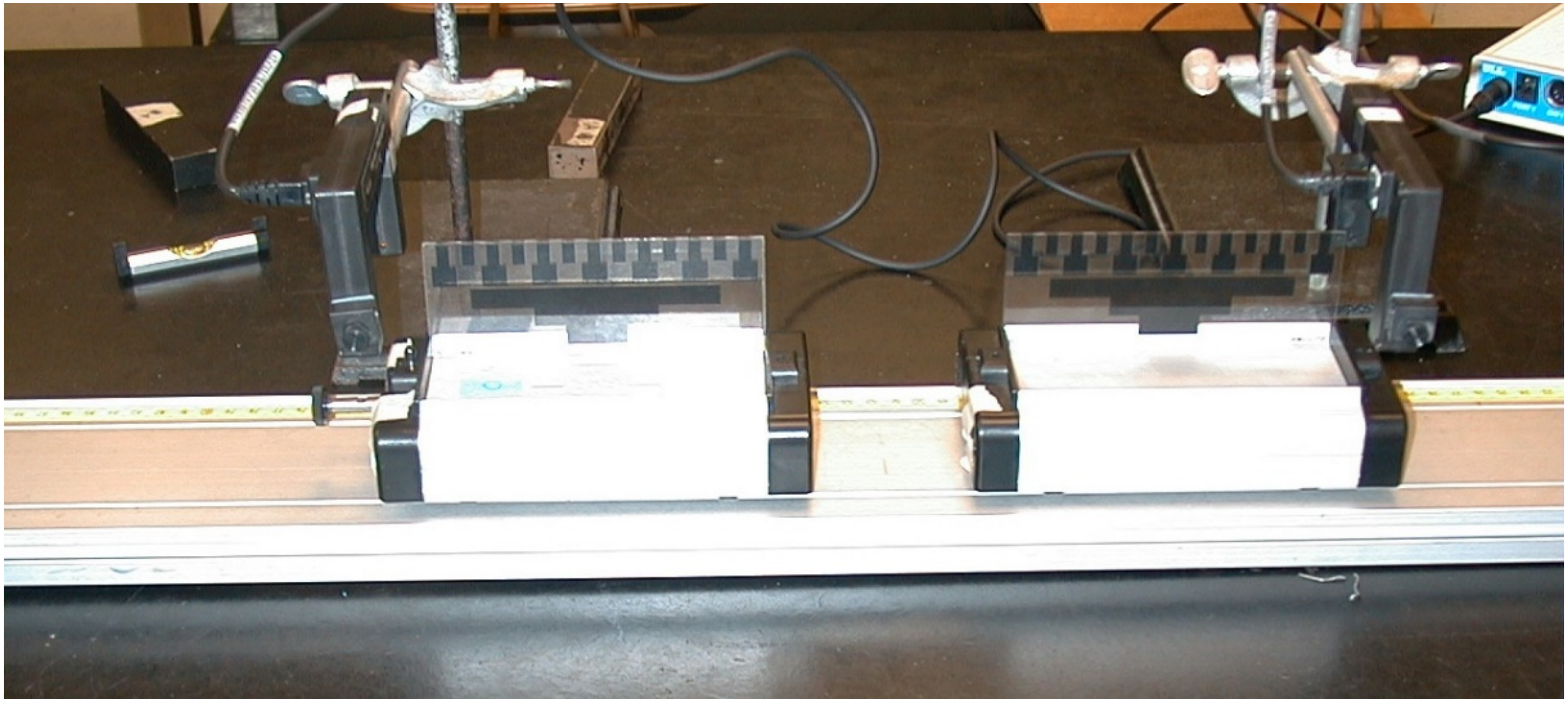
  **As of 7:45 AM, we're at 18/257 = 7%.** *We need 170 responses, total.*
- Grades will be posted on Blackboard today. This includes Exam #1, Homework & cumulative Bonus Points.

# Lab #4: Collisions - Conservation of Momentum

*February 25 – March 5*

"You will consider collisions between 2 carts on a low-friction track."

Collisions won't be covered in lecture until Chapter 11 ~ March 23.

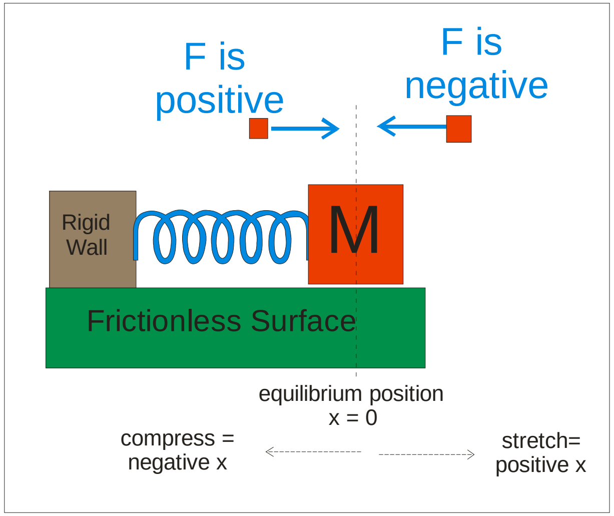


# SPRING FORCE: HOOKE'S LAW

*a brief interlude*

Ideal springs, or *elastic* materials, obey Hooke's Law: $F_{spring} = -kx$

An ideal spring tries to:




- *restore* the position of its free end
- to an *equilibrium* (*relaxed*) position
- with a force that is *directly proportional*
- to the *displacement from equilibrium*,
- *i.e.* ***F is linearly proportional to x.***

The proportionality constant ≡ spring constant *k*.

The direction of the force is opposite to the direction of the displacement.

We'll examine spring systems in detail in Chapter 15 of Ohanian but introduce Hooke's Law now so we can use it sooner.

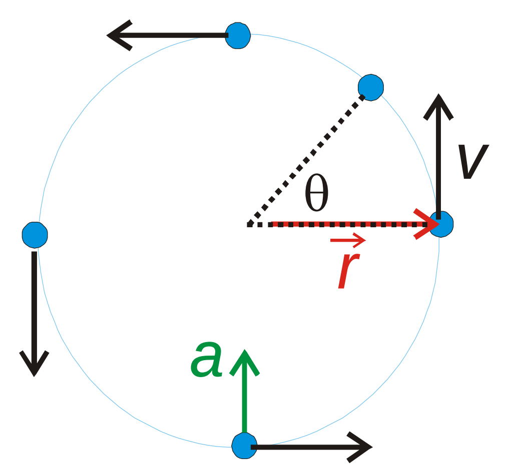
# UNIFORM CIRCULAR MOTION *REVIEW*

## UNIFORM CIRCULAR MOTION, UCM,

$\equiv$ an object moves around a circle of fixed radius at constant *speed*

## CENTRIPETAL ACCELERATION

$$\vec{a}_{centripetal} = -\left(\frac{v^2}{r}\right)\hat{r} \underset{v=\omega r}{=} -\left(\omega^2 r\right)\hat{r}$$




$-\hat{r}$ tells us the direction is radially inward.

$\hat{r}$ is a unit vector pointing outwards from the center of the circle.

# CENTRIPETAL FORCE

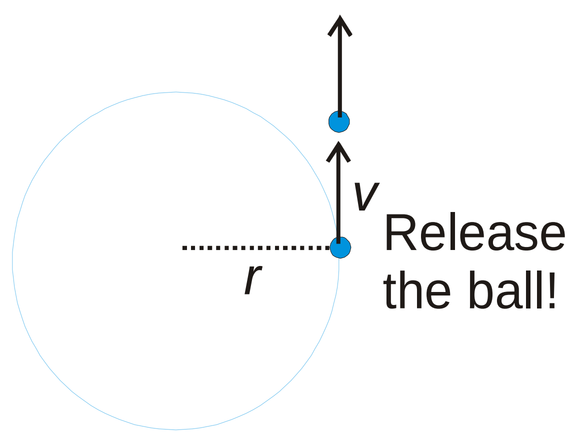
- Newton's 2nd Law: centripetal acceleration $\Rightarrow$ ***centripetal force***.
- A centripetal force = some regular, identifiable force such as gravity, the tension in a string, the normal force due to a track or the friction from tires.

**It is not some new, mysterious, invisible force!**

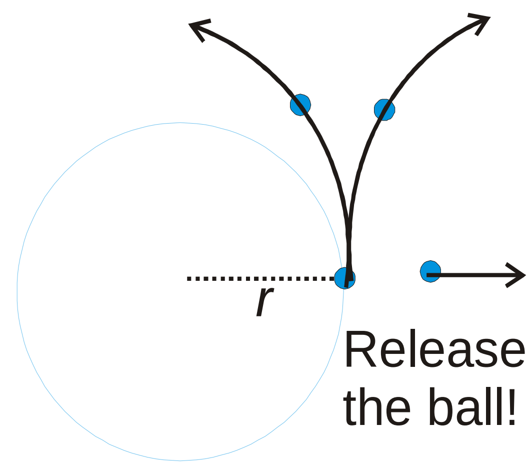
- If the centripetal force is removed, the object will move in a straight line at speed $v_{tan} = 2\pi R/T$, tangent to the path of the circle at the point of release.

**It will NOT move on a circular arc!** ***(viewed from an inertial frame of reference)***

http://www.admavericks.com/wp-content/uploads/full_davidgoliath.jpeg

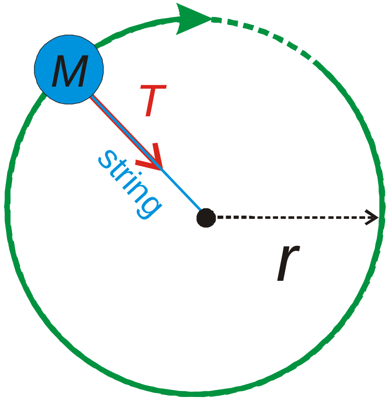



YES!




NO!

# CENTRIPETAL FORCE = TENSION




A mass $m$ in deep space (*ignore gravity*) spins at the end of a string of length $r$ at a frequency $f$ (*revolutions/second*) = ω/(2π) (*angular frequency in radians/sec*) .

What is the tension in the string?

Note that the tension in the string <u>is</u> the centripetal force for this motion.

We know: $\sum \vec{F} = m\vec{a}$

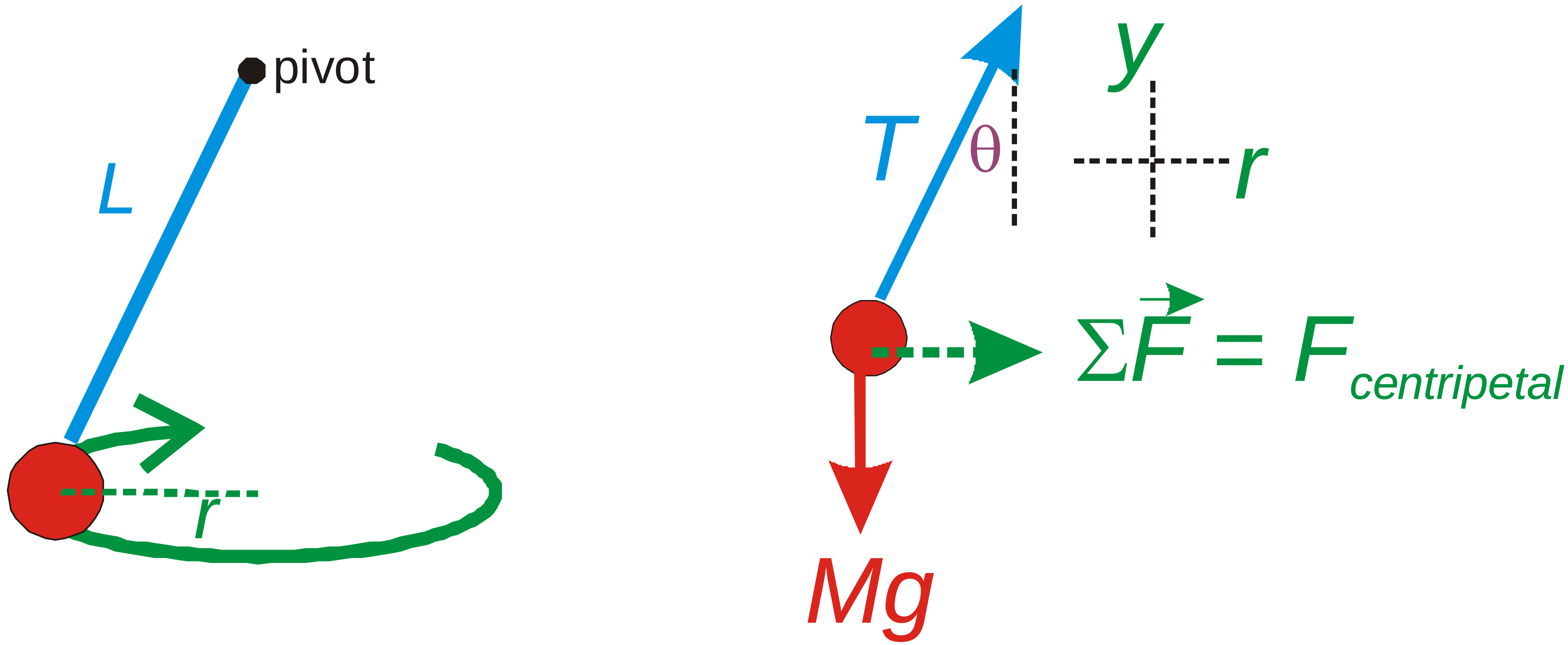
$$\sum \vec{F} \underset{UCM}{=} \vec{F}_{centripetal} \underset{\text{in magnitude}}{=} \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T_{period}} = 2\pi r f = \omega r$$

Therefore:

$$T_{tension} = m\frac{v^2}{r} = m\frac{(2\pi r f)^2}{r} = m(2\pi f)^2 r = m\omega^2 r$$

# What is the net force acting on the mass in a *conical pendulum*? (~Ohanian 6.65)



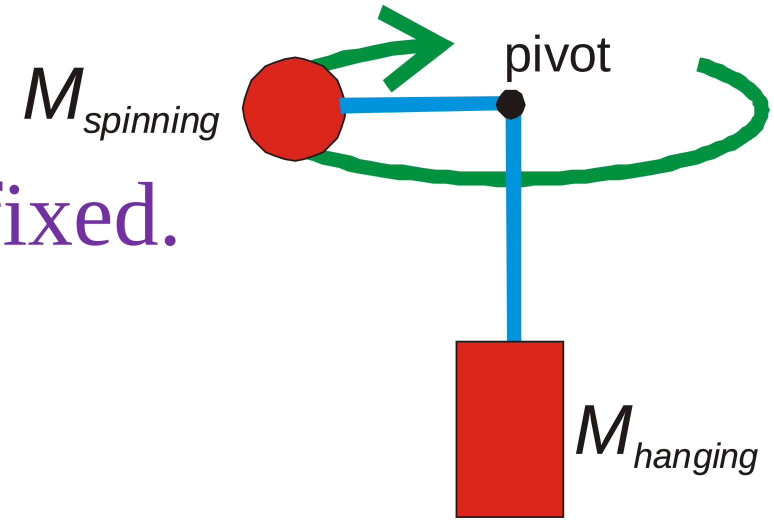

$$\sum F_r = F_{centripetal} = T\sin\theta = \frac{mv^2}{r} = m\left(\frac{2\pi r}{T_{period}}\right)^2 \frac{1}{r} = m\left(2\pi f\right)^2 r = m\omega^2 r$$

$$\Sigma F_y = T\cos\theta - mg = 0$$

# DEMOS (~Ohanian 6.88)

What is the tension in the string for the system shown below; two masses connected by an ideal string bent 90° over a frictionless pivot?

$$T_{tension} = F_{centripetal} = M_{hanging}g.$$




If $M_{\text{hanging}}$ is **fixed**, $F_{centripetal}$ is fixed.

smaller $r \Leftrightarrow$ larger $f$

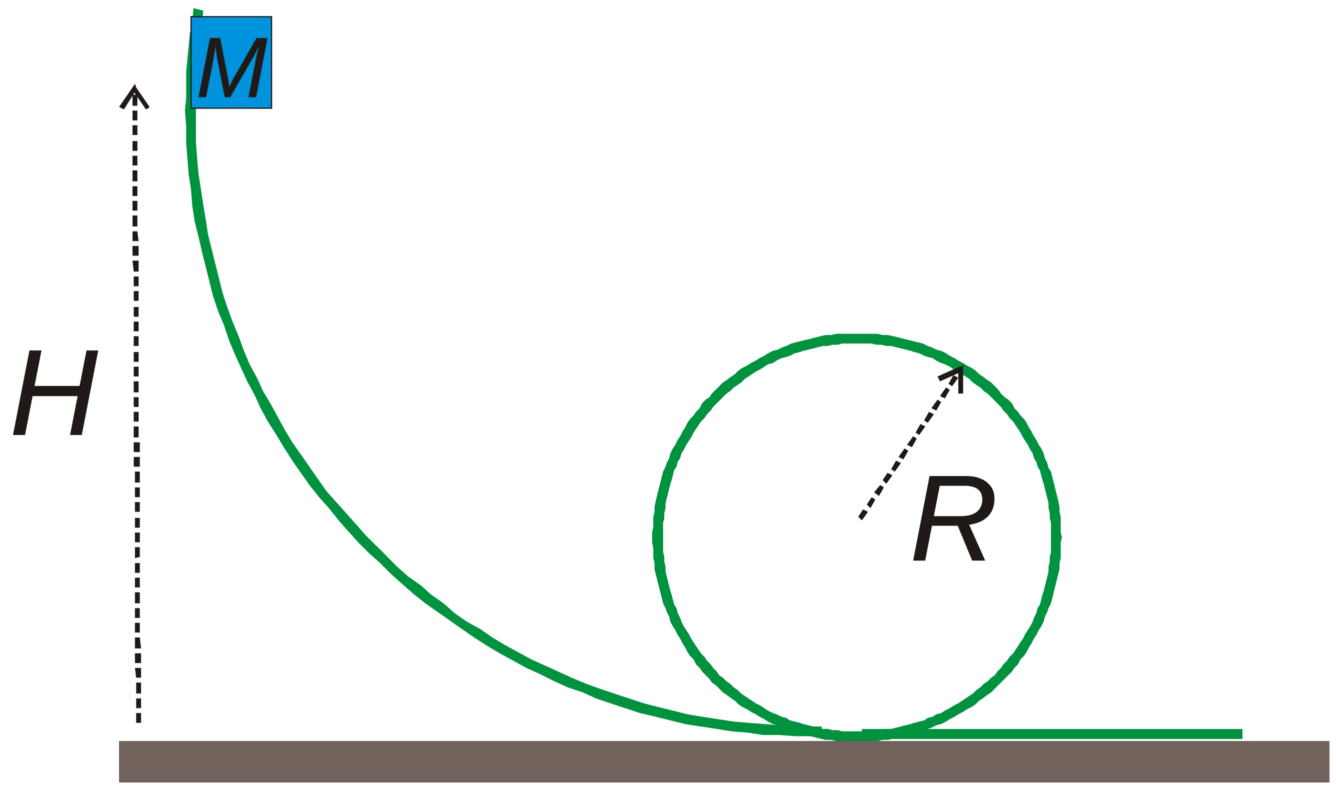
larger $r \Leftrightarrow$ smaller $f$

$$T_{tension} = mg = F_{centripetal} = \frac{mv^2}{r} = m\left(\frac{2\pi r}{T_{period}}\right)^2 \frac{1}{r} = m\left(2\pi f\right)^2 r = m\omega^2 r$$

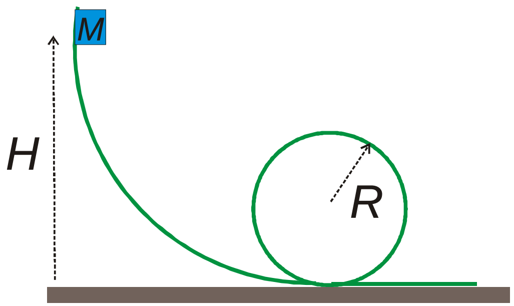
# CENTRIPETAL FORCE = NORMAL FORCE

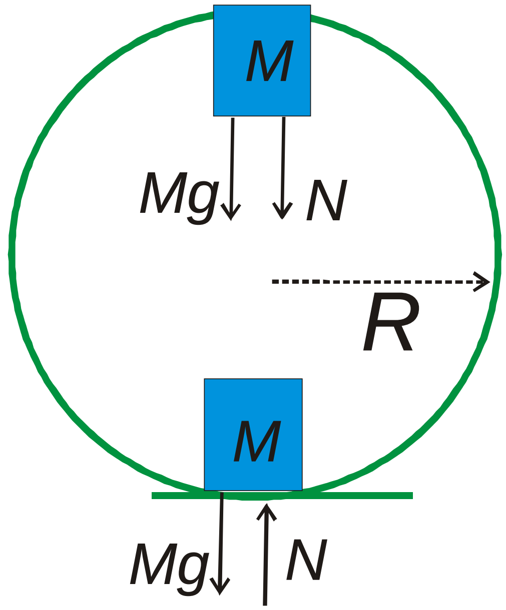
A mass *M* is traveling around a frictionless vertical circular track of radius *R*.

What velocity does it need to have at the top of the circle if it's not going to *fall off* the track?



# LOOPS




The only *real* forces acting on the block are:

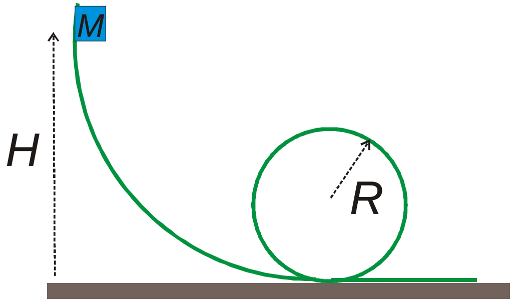
gravity, which always points down

normal force, which always points perpendicular to the track.

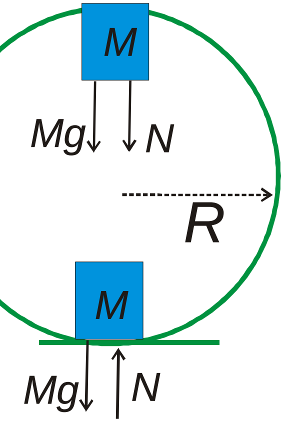
The *radial components* of these two forces must sum to provide whatever *centripetal force* is needed.

The phrase *not fall off the track* refers to the normal force ***N***.

$N$ must be $\geq 0$ or else the block is not in contact with the track.




# LOOPS




With $+\boldsymbol{r}$ defined as radially out,

$\Sigma F_{radial}$ at the top =

$$-Mg - N = -Mv^2_{top}/R$$

$N = 0$ at the top gives

$$-Mg = -Mv^2_{top}/R$$

$$v_{top} = \sqrt{gR}$$

At the bottom, the block will have more speed

& $Mg$ points in the opposite direction

$$-N + Mg = -Mv^2_{bottom}/R \rightarrow N = Mg + Mv^2_{bottom}/R$$

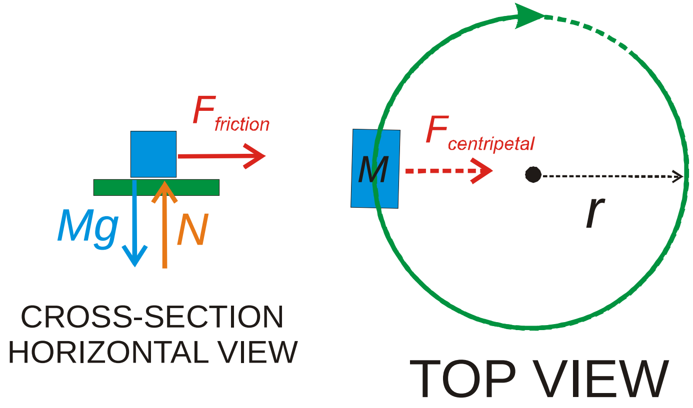
# CENTRIPETAL FORCE = FRICTION

A car of mass *m* is traveling at speed *v* around a (*flat*) circle of radius *r*.

What coefficient of friction is needed for the car to stay on the road?

$$\sum \vec{F} = m\vec{a}$$

$$\left|F_{centripetal}\right| = \frac{mv^2}{r}$$




There is a force of *static* friction acting *on the car* as a reaction to the car's outward force on the road surface.

It's static friction as the tires are not sliding along the road, as in a skid.

$$F_{static\ friction\text{-}max} = \mu_s N = \mu_s Mg$$

$$\mu_s Mg = \frac{Mv^2}{r} \Rightarrow \mu_s = \frac{v^2}{rg}$$

It's easier to go around a circle if the radius is larger and/or the velocity is smaller – and *v* is more important than *r*.

# **'AMUSING' FRICTION PROBLEMS** (~Ohanian 6.54)

An amusement park ride sometimes called the Rotor, Dropout or Graviton uses friction to keep the riders from falling to the bottom of a rotating cylinder.

What minimum coefficient of static friction is needed?

*http://www.cincinnativiews.net/images-4/Rotor%20Ride.jpg*
*http://en.wikipedia.org/wiki/Rotor_(ride)*
http://www.ride-extravaganza.com/intermediate/gravitron/
http://kylegrantham.files.wordpress.com/2010/07/krg_fairrides_0273.jpg

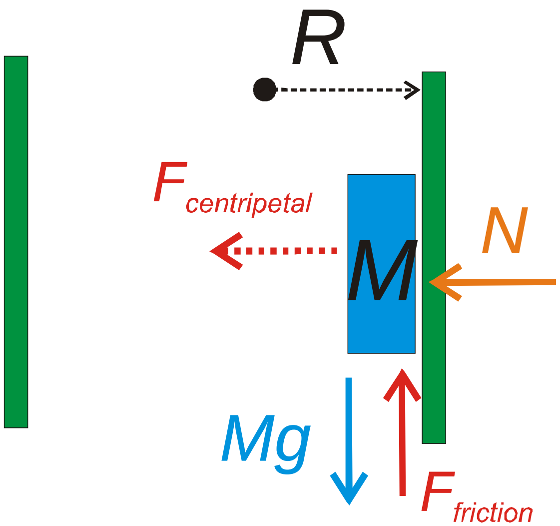
# 'AMUSING' FRICTION PROBLEMS

$f \equiv$ frequency of rotation, cycles/second

$R \equiv$ radius of the ride

$M \equiv$ mass of a rider

$\mu_s \equiv$ coefficient of static friction




$$\sum \vec{F} = m\vec{a}$$

$$\left|F_{centripetal}\right| = \frac{mv^2}{r}$$

The *normal force* supplies the centripetal force.

$F_{centripetal}$ in the FBD is **NOT** an ADDITIONAL FORCE

Static friction keeps the rider from falling, $|F_f| = Mg$.

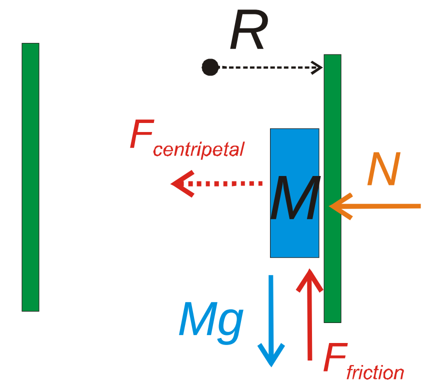
# ‘AMUSING’ FRICTION PROBLEMS

$f \equiv$ frequency of rotation, cycles/second

$R \equiv$ radius of the ride

$M \equiv$ mass of a rider

$\mu_s \equiv$ coefficient of static friction




$$\sum F_y = 0 \;\rightarrow\; F_{friction} \le \mu_s N = Mg \qquad \sum F_r = Ma_r \;\rightarrow\; N = \frac{Mv^2}{r}$$

$$\underset{\text{plug in N}}{\Rightarrow} \quad \mu_s \left( \frac{Mv^2}{r} \right) = Mg \;\Rightarrow\; \mu_s = g\frac{r}{v^2}$$

To replace $v$ with frequency, use $v = \dfrac{2\pi r}{T} = 2\pi r f$

$$\mu_s \ge g\frac{r}{(2\pi r f)^2} = \frac{g}{4\pi^2 f^2 r}$$

No mass!

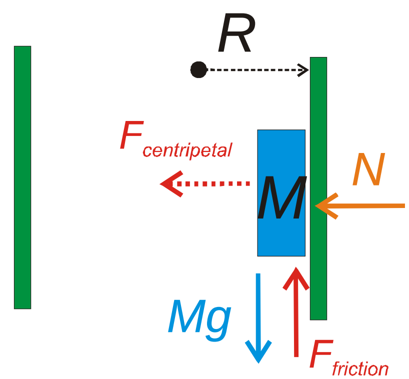
# ‘AMUSING’ FRICTION PROBLEMS- with numbers

$f \equiv 0.5$ cycles/second *or* $T = 2$ sec

$R \equiv 5$ m

$M \equiv$ doesn’t matter

$\mu_s \equiv$ coefficient of static friction = ?




$$\mu_s \geq \frac{g}{4\pi^2 f^2 r} = \frac{9.8 \text{ m/s}^2}{4\pi^2 \left(0.5 \text{ s}^{-1}\right)^2 (5 \text{ m})} = 0.20$$

Would you go on this ride?

Wearing Teflon-coated clothes?

($\mu_{Teflon} \sim 0.04$)

# The nonexistent CENTRIFUGAL FORCE

- Measurements made in non-inertial frames of reference can appear to show "*fictitious forces*".
- These forces look very real to observers in those reference frames.
- Recall from our discussion of relative motion

$$\boldsymbol{a}_{\textit{mass-observer}} = \boldsymbol{a}_{\textit{mass-proper}} + \boldsymbol{a}_{\textit{proper-observer}}$$

*Observer = someone in an accelerating (rotating) frame*

*Proper = someone in an inertial (lab) frame*

# The nonexistent CENTRIFUGAL FORCE

**Imagine standing on a spinning table while holding a mass on a string.**

$$\boldsymbol{a}_{mass\text{-}observer} = \boldsymbol{a}_{mass\text{-}proper} + \boldsymbol{a}_{proper\text{-}observer}$$

$$\Sigma \boldsymbol{F}_{mass\text{-}observer} = \Sigma \boldsymbol{F}_{mass\text{-}proper} + \Sigma \boldsymbol{F}_{proper\text{-}observer}$$

**Both observers agree that there is tension *T* in the string**

*$T$ makes an angle $\theta$ with vertical.*

# The nonexistent CENTRIFUGAL FORCE

You see the mass as stationary $\Rightarrow \Sigma F_{\textit{mass-observer}} = 0$

if you don't realize you are spinning & do an **incorrect analysis**!

**INCORRECT ANALYSIS:** $\Sigma F_{\textit{mass-observer}} = 0 = -T\sin\theta + ?$

***with*** $? = +mv^2/r$

*A mysterious force* $+ mv^2/r$ *is trying to push the mass away from you.*

This force seems very real to people in a non-inertial frame.

But it's not real & is not even a force as such, but rather $ma_{\textit{proper-observor}}$

**PROPER ANALYSIS:** In the radial direction:

$$\Sigma F_{\textit{mass-proper}} = -T\sin\theta = ma = -mv^2/r$$

# EARTH as a ROTATING FRAME

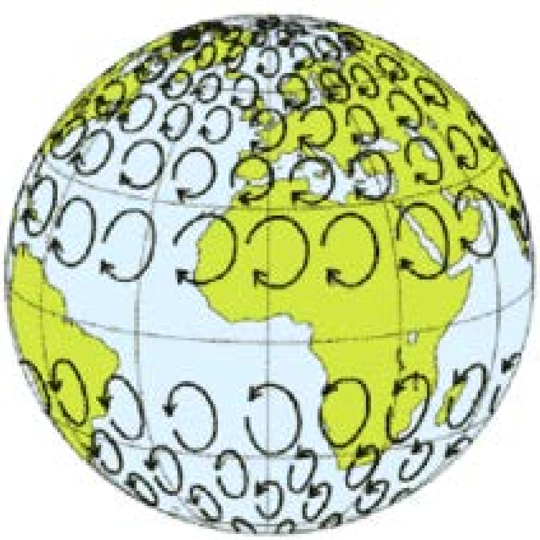
## *more culture*

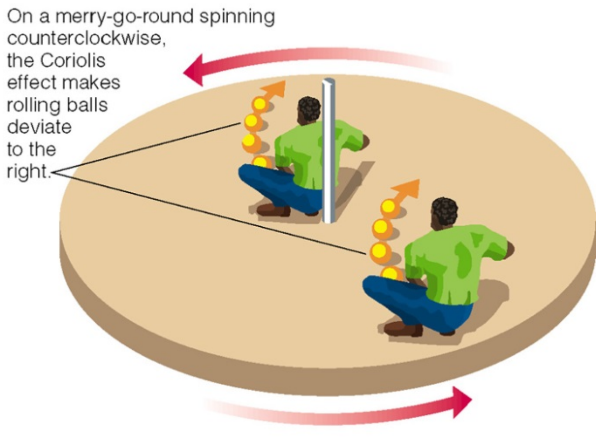
The "**coriolis effect**" is another fictitious, but useful force associated with the earth's rotation.

See http://en.wikipedia.org/wiki/Coriolis_effect.

The coriolis effect, tides *et al* are topics for sophomore physics majors.

We'll treat the earth as an inertial frame of reference in PHYS 121.





5 options

A car rounds a curve while maintaining a constant speed.

Is there a net force on the car as it rounds the curve?

A. No, since its speed is constant.
B. No, the vertical forces cancel and there are no horizontal forces *acting on the car*.
C. Yes, because of "*some principle of physics*".
D. It depends on the sharpness of the curve and/or the speed of the car.

5 options

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Is there a net force on the car as it rounds the curve?

A. No, since its speed is constant.

B. No, the vertical forces cancel and there are no horizontal forces *acting on the car*.

**C. Yes, because of some principle of physics**

D. It depends on the sharpness of the curve and/or the speed of the car.

**The principle of physics is $F = ma = Mv^2/r$.**

**Friction from the road acting on the tires supplies this force.**

5 options

You are a passenger in the front seat of a car, leaning gently against the door (*on the right*, *since this is the US*) when the car makes a sharp left turn (*without changing speed*). You notice a significant increase in pressure from the door.

Which is the correct analysis of the situation?

A. The door is pushing harder on you because of the turn.
B. You are pushing harder on the door due to the centrifugal force pushing you outward and what you feel is the reaction force.
C. Both of the above
D. Neither of the above
E. You're just imagining an increase in pressure; it's an artifact of your frame of reference.

5 options

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C. Both of the above

D. Neither of the above

E. You’re just imagining an increase in pressure; it’s an artifact of your frame of reference.

The door (*plus friction from your seat*) has to supply the centripetal force needed for you to get around the curve along with the car.

Remove the door and you might fly out of the car! But that’s only because you’d continue on a straight line path as the car rounds the curve.

B) isn’t completely wrong, except you aren’t ‘forced’ outward as seen from an inertial frame of reference.

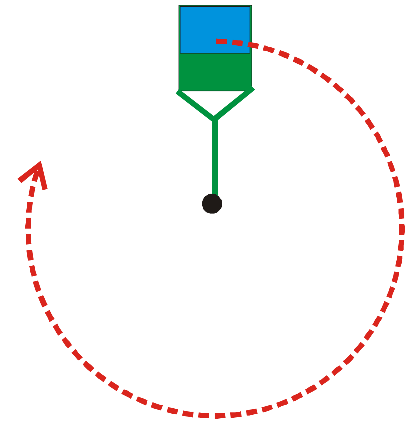
# DEMO (~Ohanian 6.62)

## SWINGING A BUCKET OF WATER

From the water's point of view, there IS a centrifugal force pushing it outwards.

That force better be > *mg* or the water will fall out of the bucket.

From the lab point of view, the water does accelerate down, but so does the bucket!



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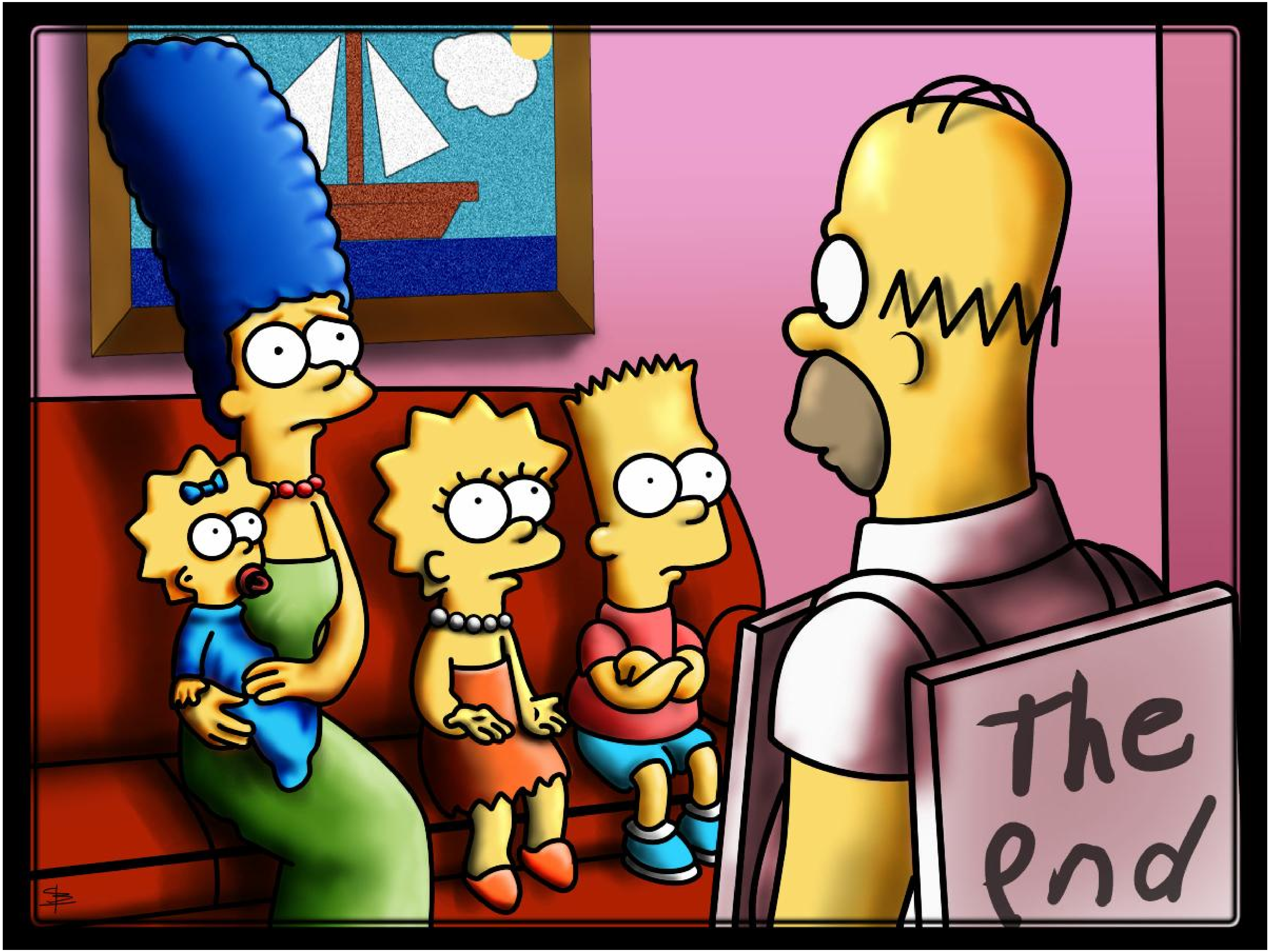


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