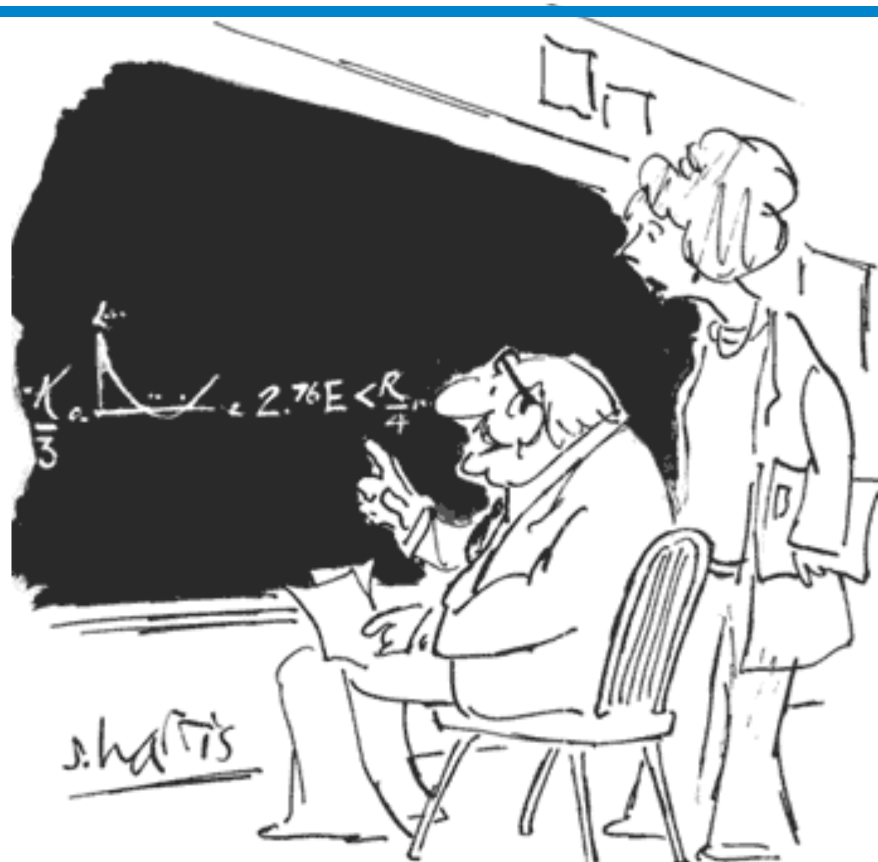
PHYS 121 – SPRING 2015

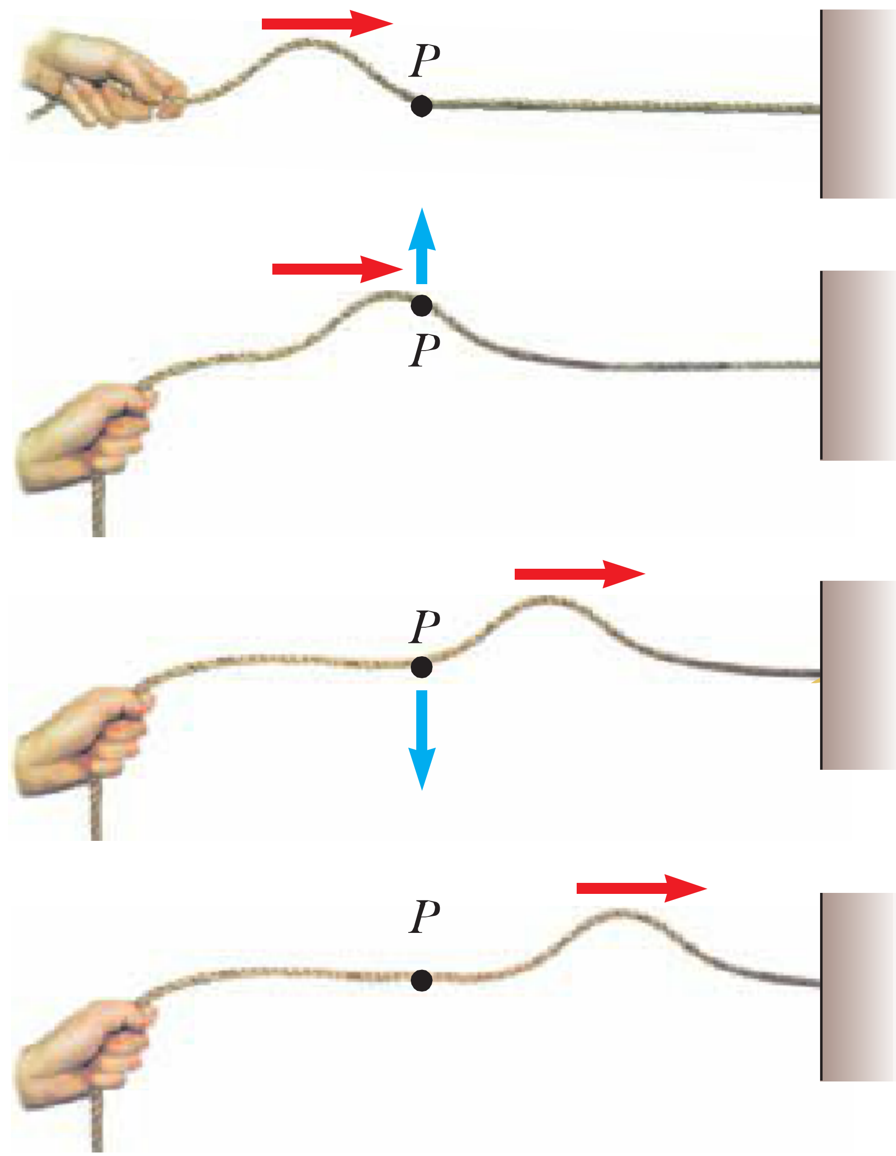


"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."

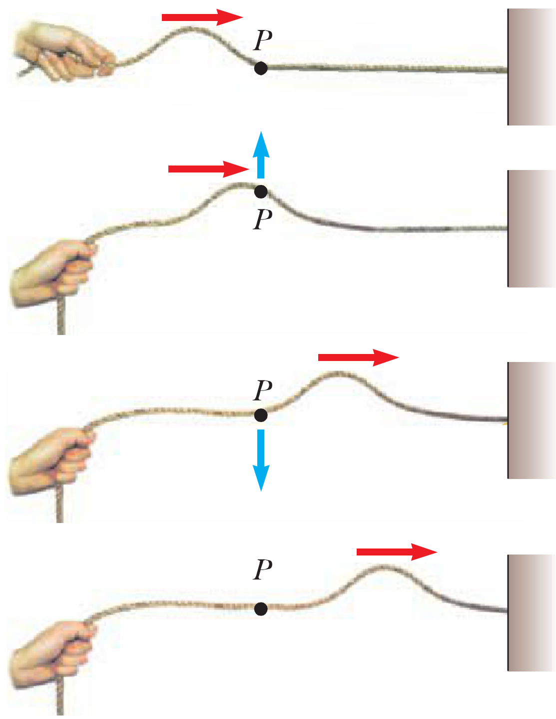
Chapter 16: Waves

Version 4/22/2015

Travelling Pulse on String



Travelling Pulse on String



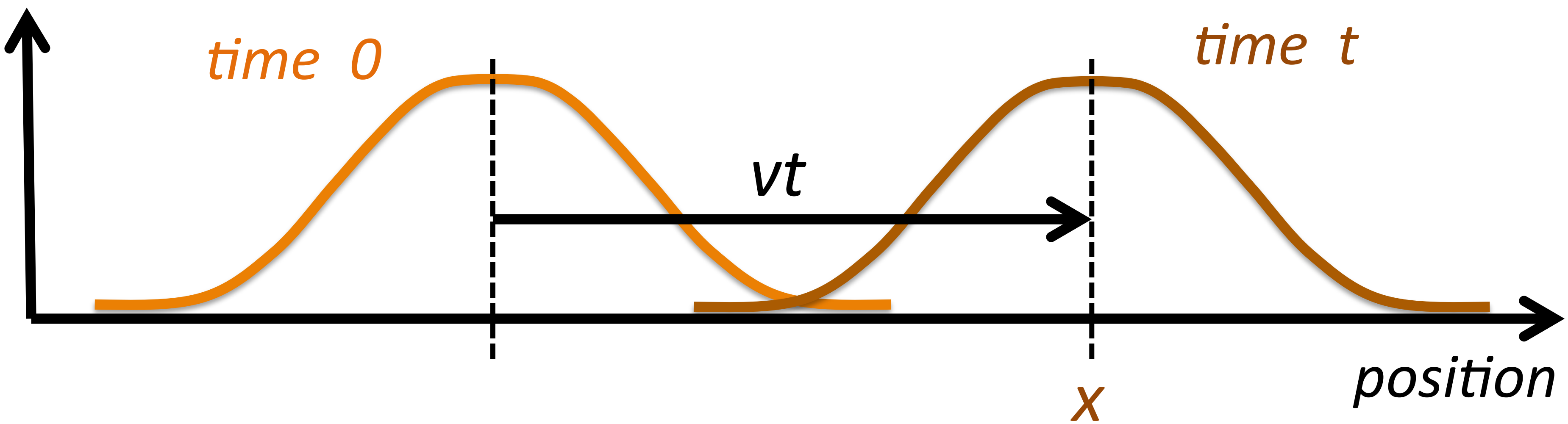
Height of the string is a function of position along the string (*x*), and time (*t*).

$$height = h(x,t)$$

A point on the string does not move horizontally, only up and down. Only the pulse "disturbance" moves horizontally.

Equation for Height

Assume the pulse travels with a velocity *v*.



In a time *t*, the peak will move a distance *vt*.

If the peak is at a position *x* at time *t*, then the peak *was* at a position *x-vt* at time 0.

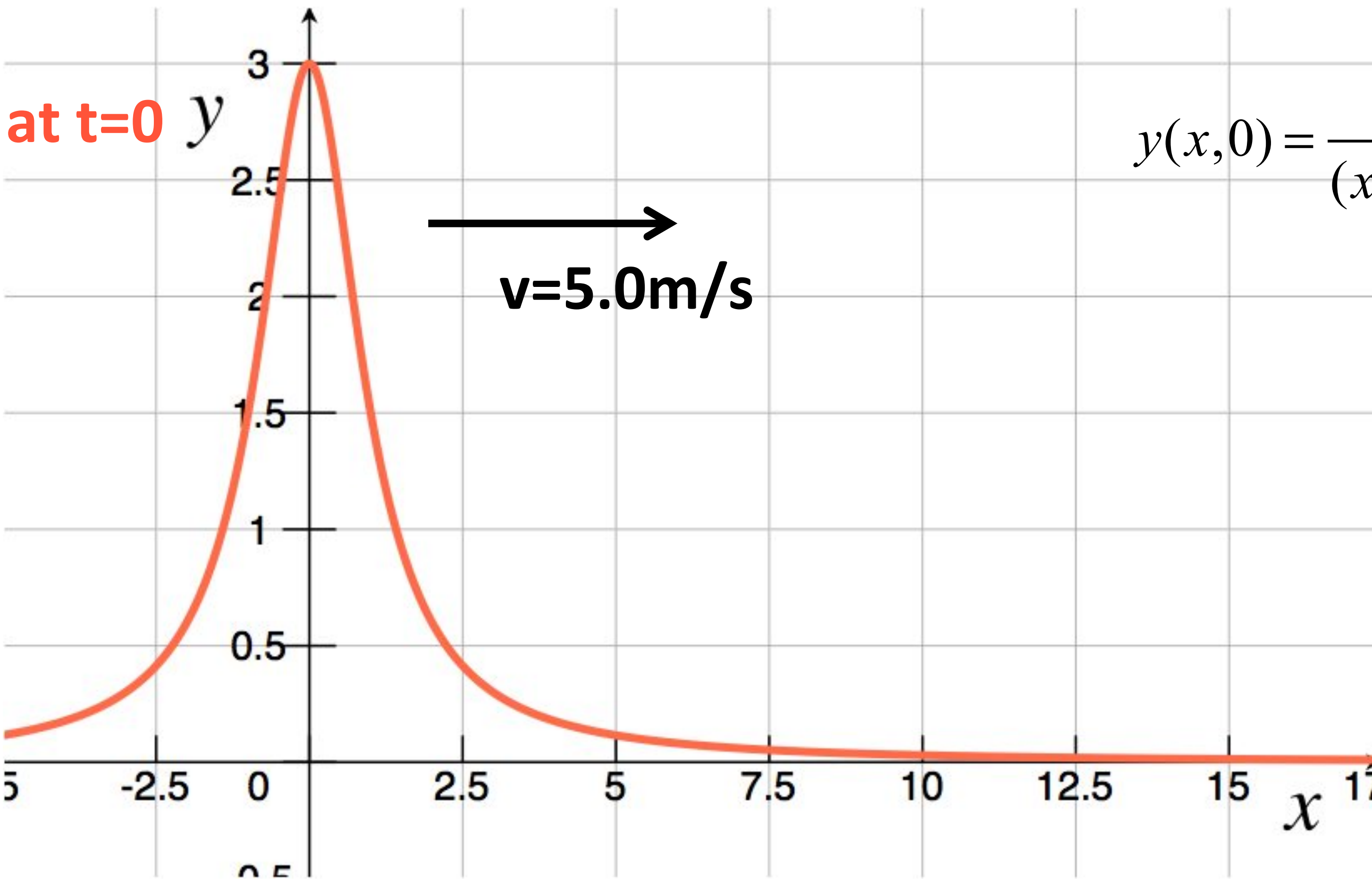
$$h(x,t) = h(x - vt, 0)$$

This means, in general, a travelling pulse will have an *x-vt* in the equation somewhere!

Travelling Pulse Example

Suppose a pulse moving to the right is represented by the function:

$$y(x,t) = \frac{3.0}{(x - 5.0t)^2 + 1}$$

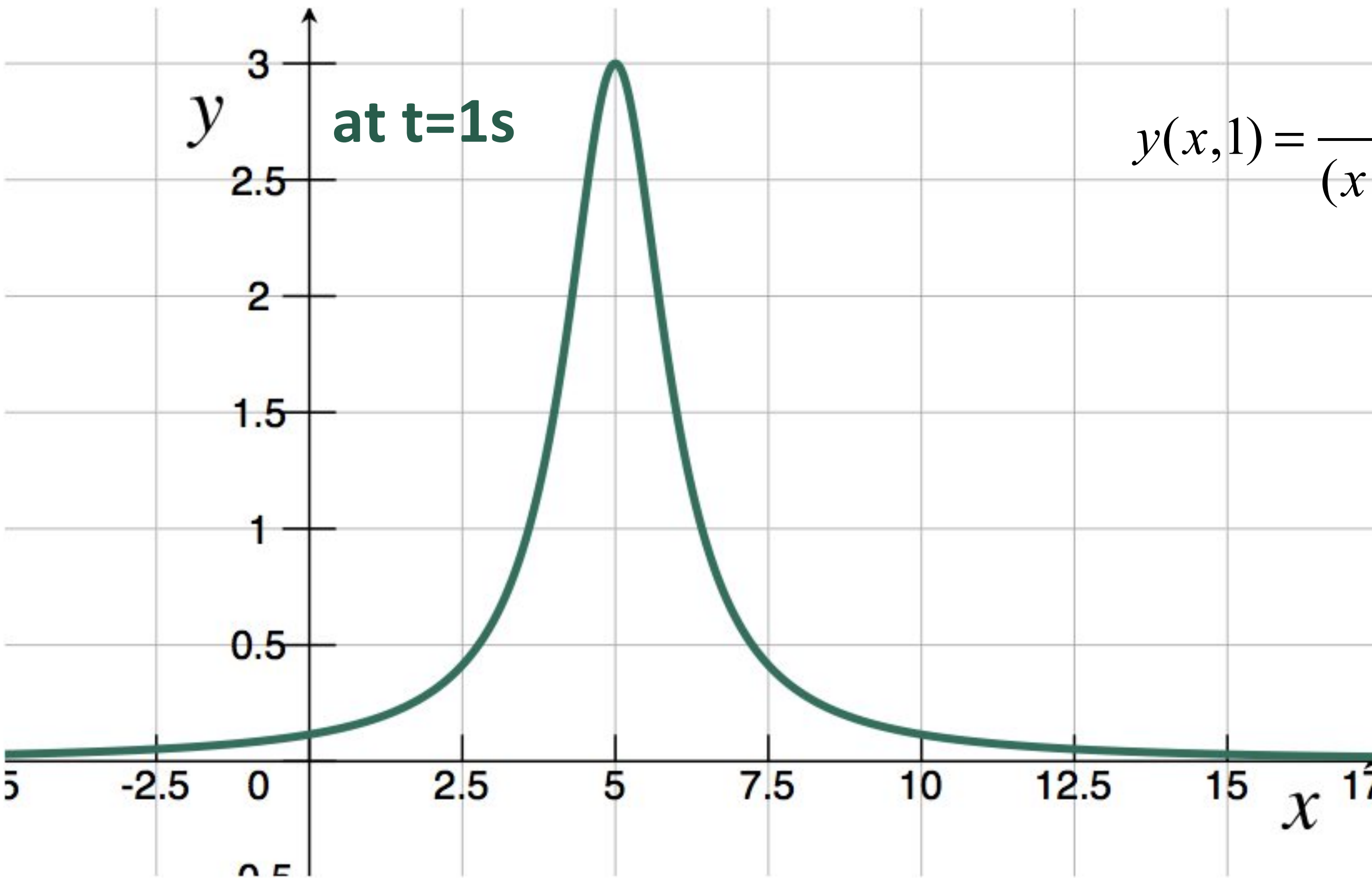


$$y(x,0) = \frac{3.0}{(x)^2 + 1}$$

Travelling Pulse Example

Suppose a pulse moving to the right is represented by the function:

$$y(x,t) = \frac{3.0}{(x - 5.0t)^2 + 1}$$

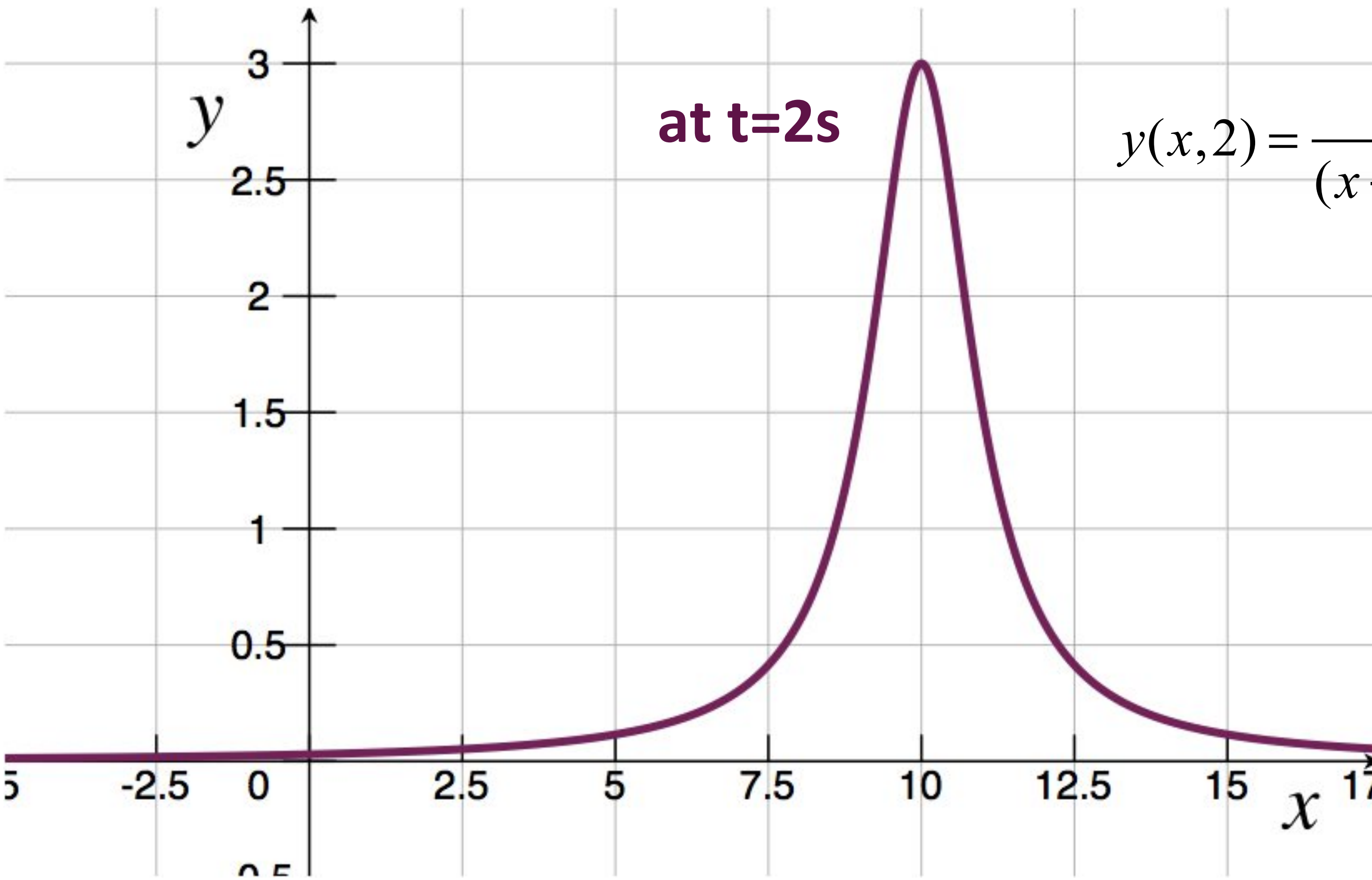


$$y(x,1) = \frac{3.0}{(x-5)^2 + 1}$$

Travelling Pulse Example

Suppose a pulse moving to the right is represented by the function:

$$y(x,t) = \frac{3.0}{(x - 5.0t)^2 + 1}$$

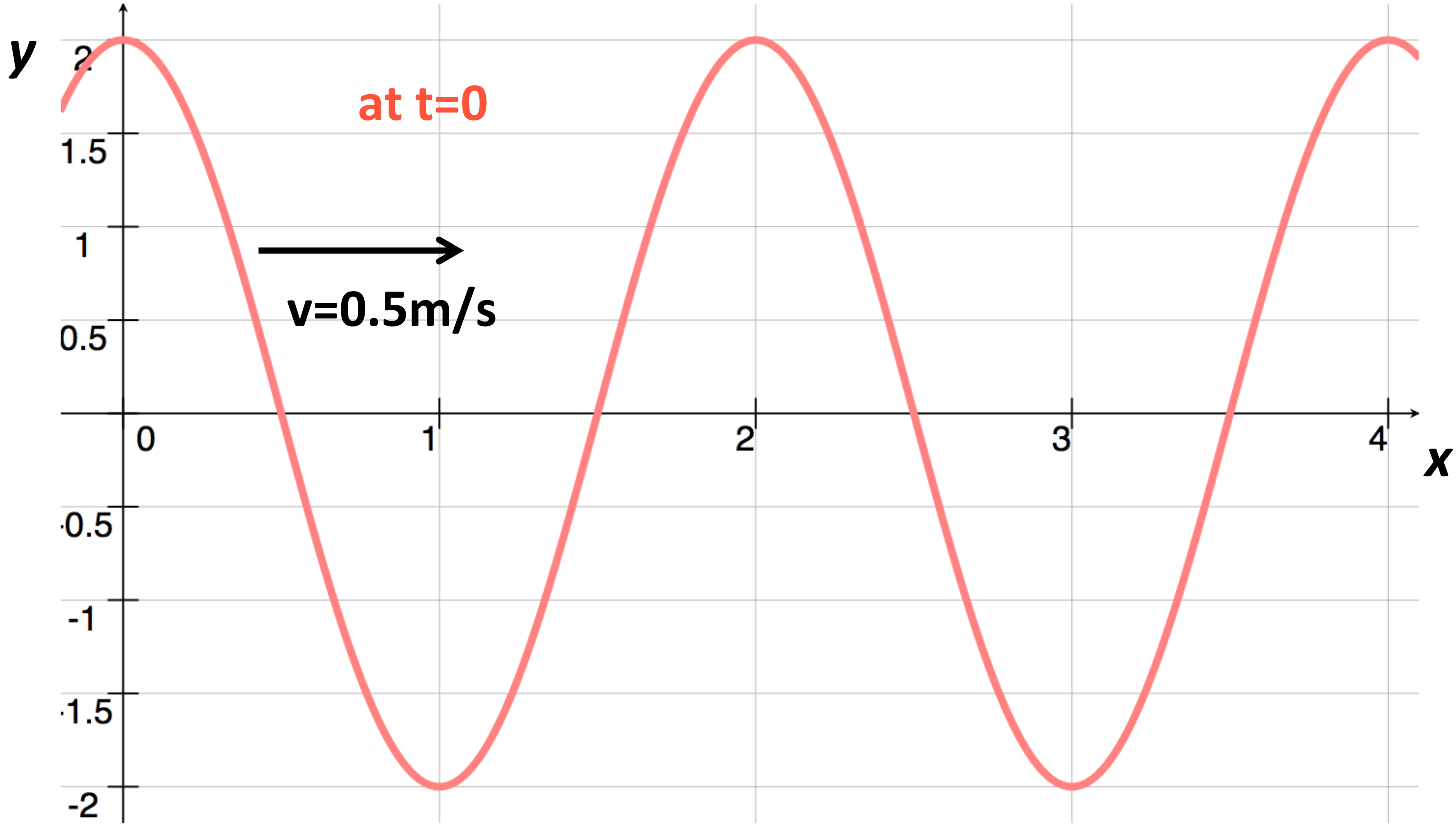


$$y(x,2) = \frac{3.0}{(x - 10)^2 + 1}$$

Travelling Sine Wave Example

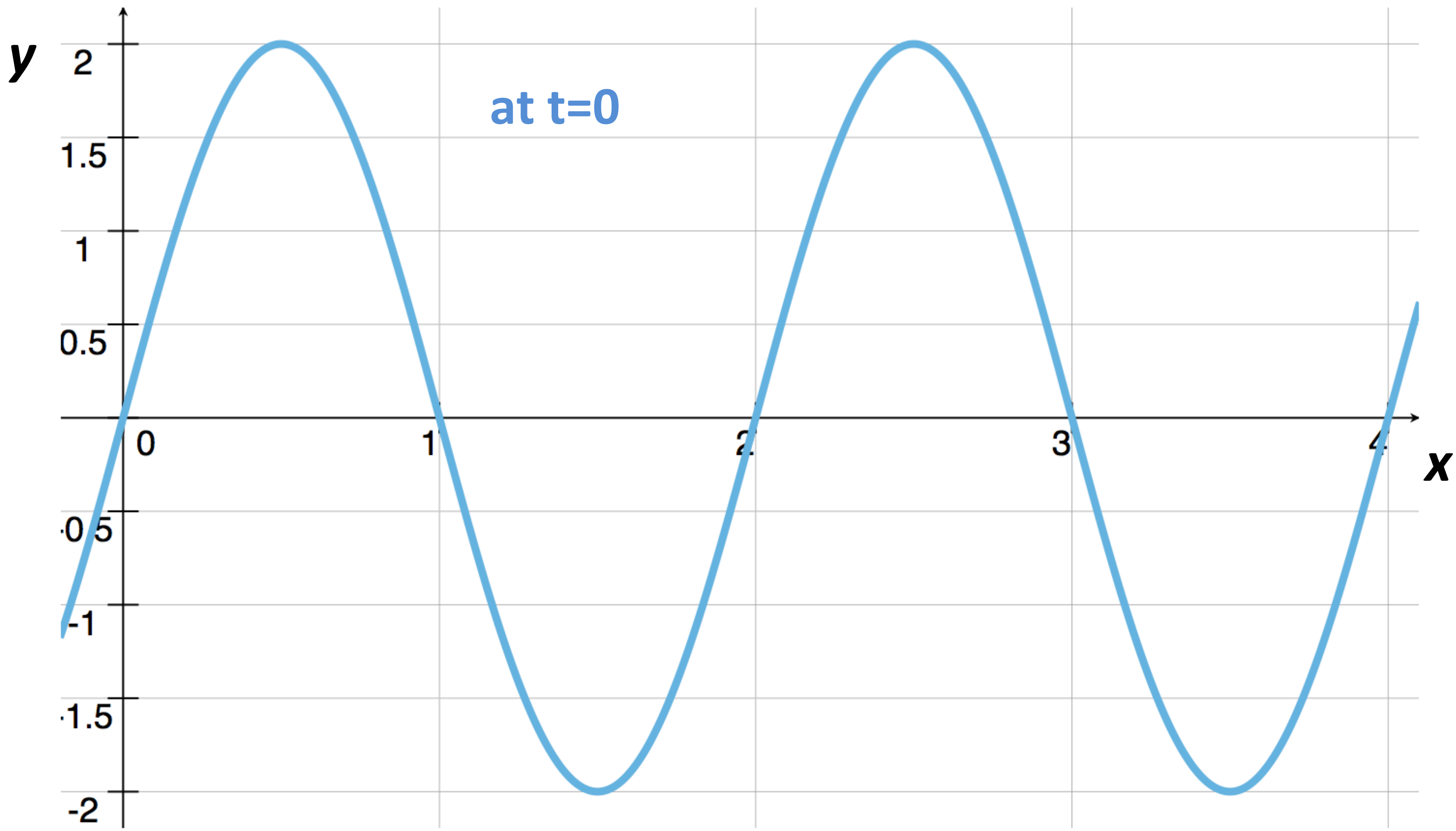
$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$

If we look at this function vs. position at specific points in time:



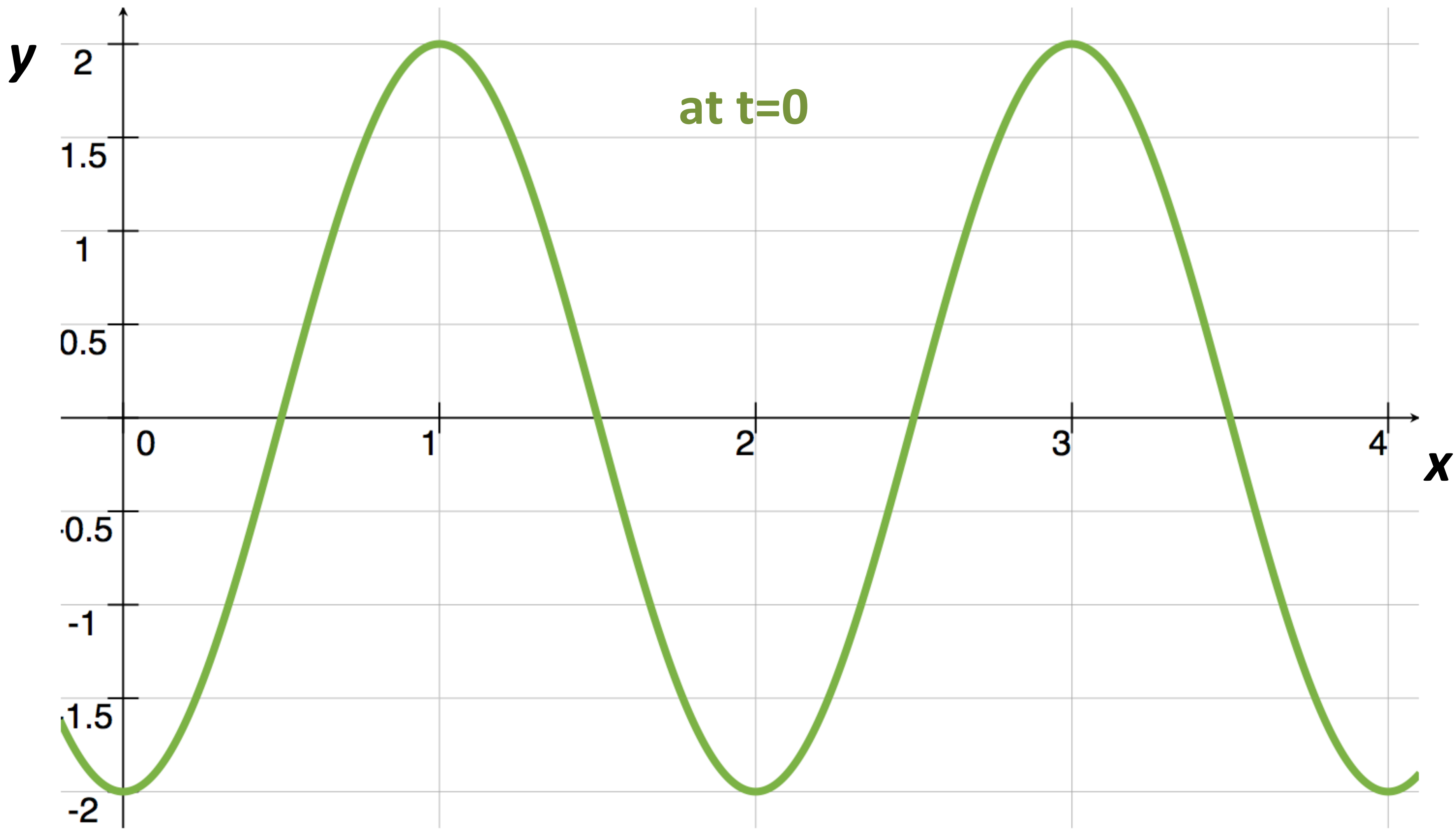
Travelling Sine Wave Example

$$y(x,t) = 2.0\cos\left(\pi(x-0.5t)\right)$$



Travelling Sine Wave Example

$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$



PHYS 121 – SPRING 2015

Chapter 16: Waves

Version 4/24/2015

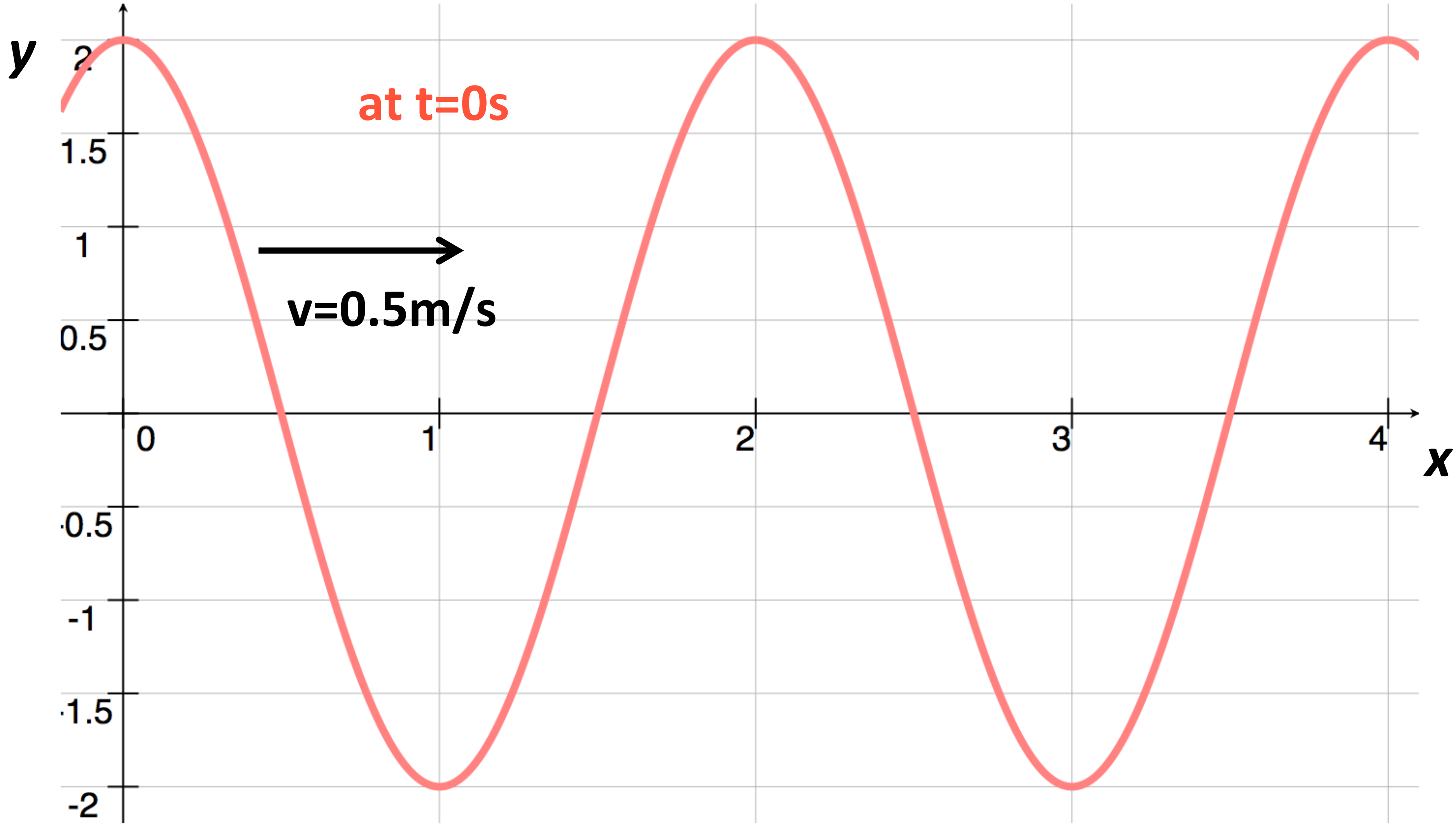
Waves Concepts

- A wave is a travelling disturbance (pulse on a string, water waves on a lake).
- The height of the string, or the height of the water, will be a function of x-vt

Travelling Cosine Wave Example

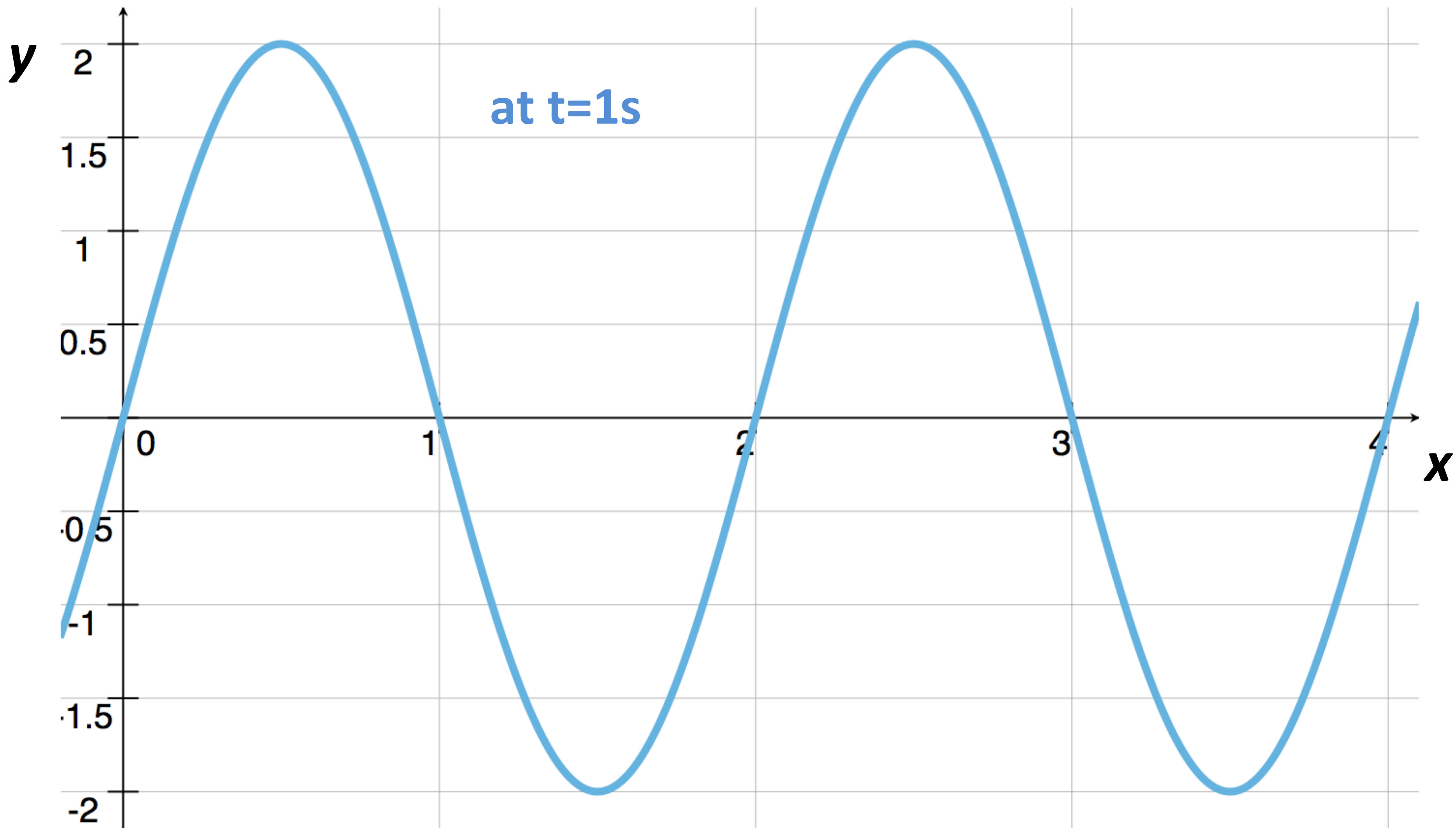
$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$

If we look at this function vs. position at specific points in time:



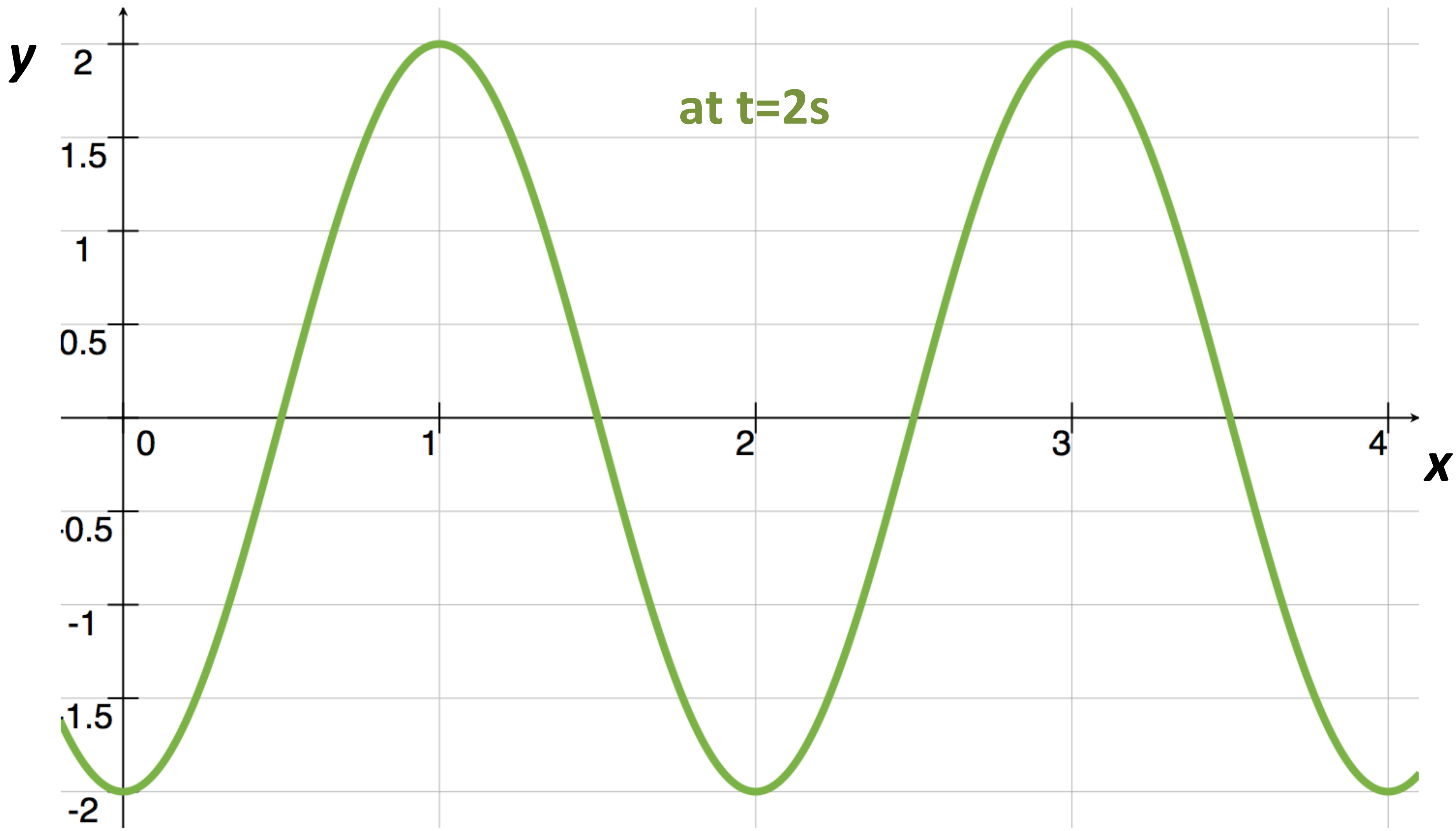
Travelling Cosine Wave Example

$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$



Travelling Cosine Wave Example

$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$



Waves Concepts

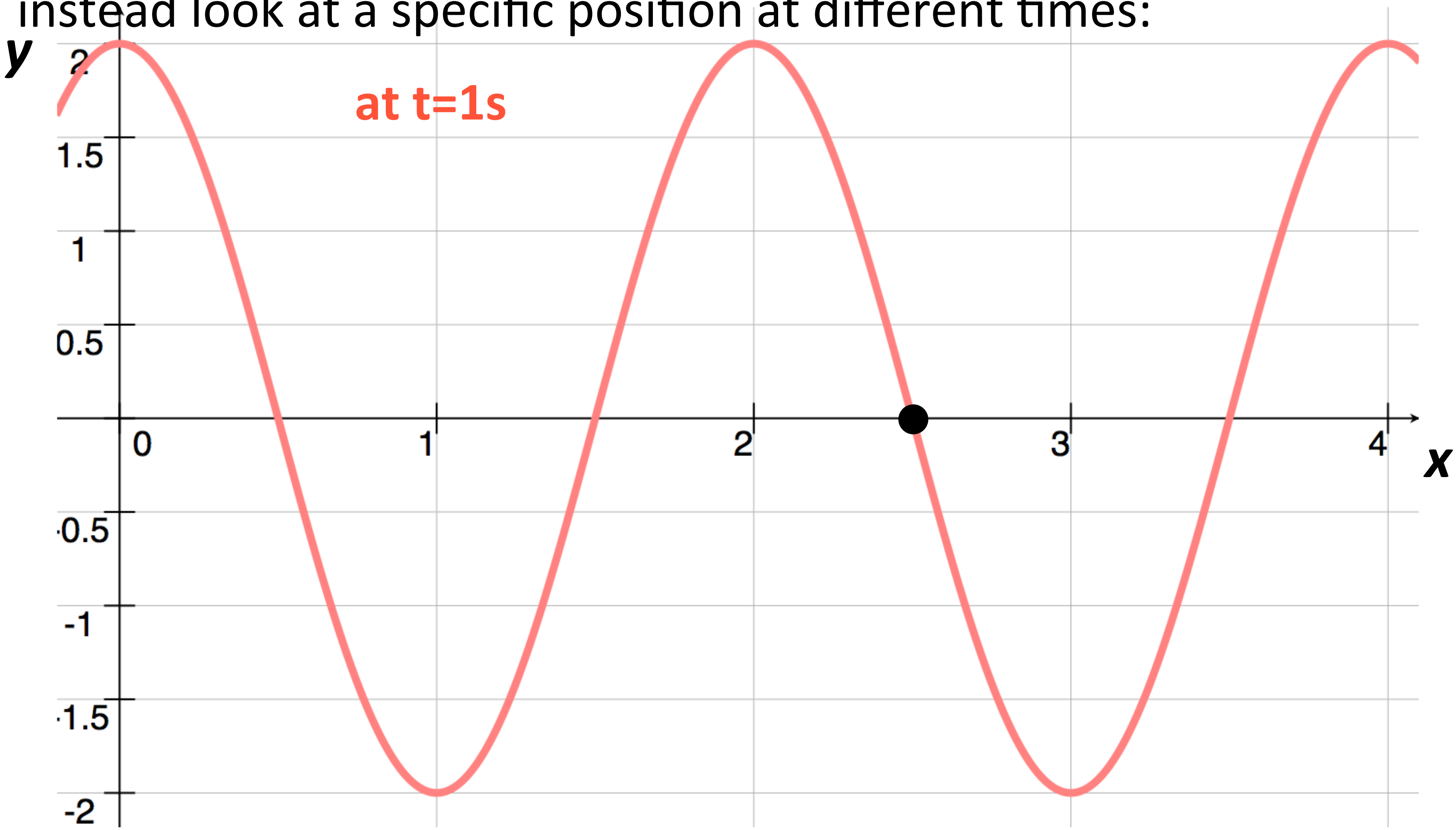
- A wave is not the same as the medium through which it moves. A wave is like an abstract movement of energy.

- A wave travels horizontally through a medium, leading to up-and-down motion of the medium.

Travelling Cosine Wave Example

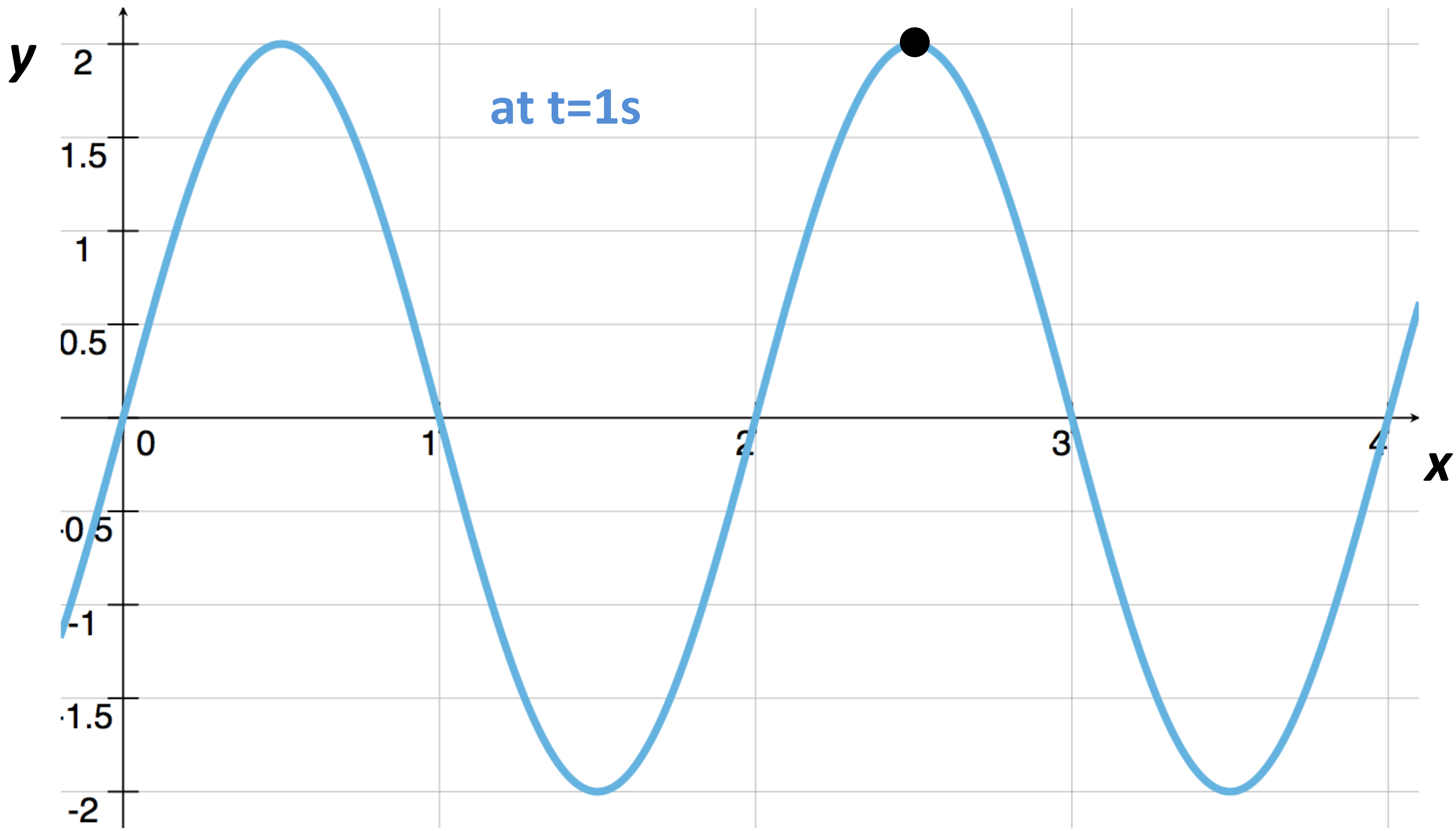
$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$

These plots are of *y vs. position* for specific points in time. If we instead look at a specific position at different times:



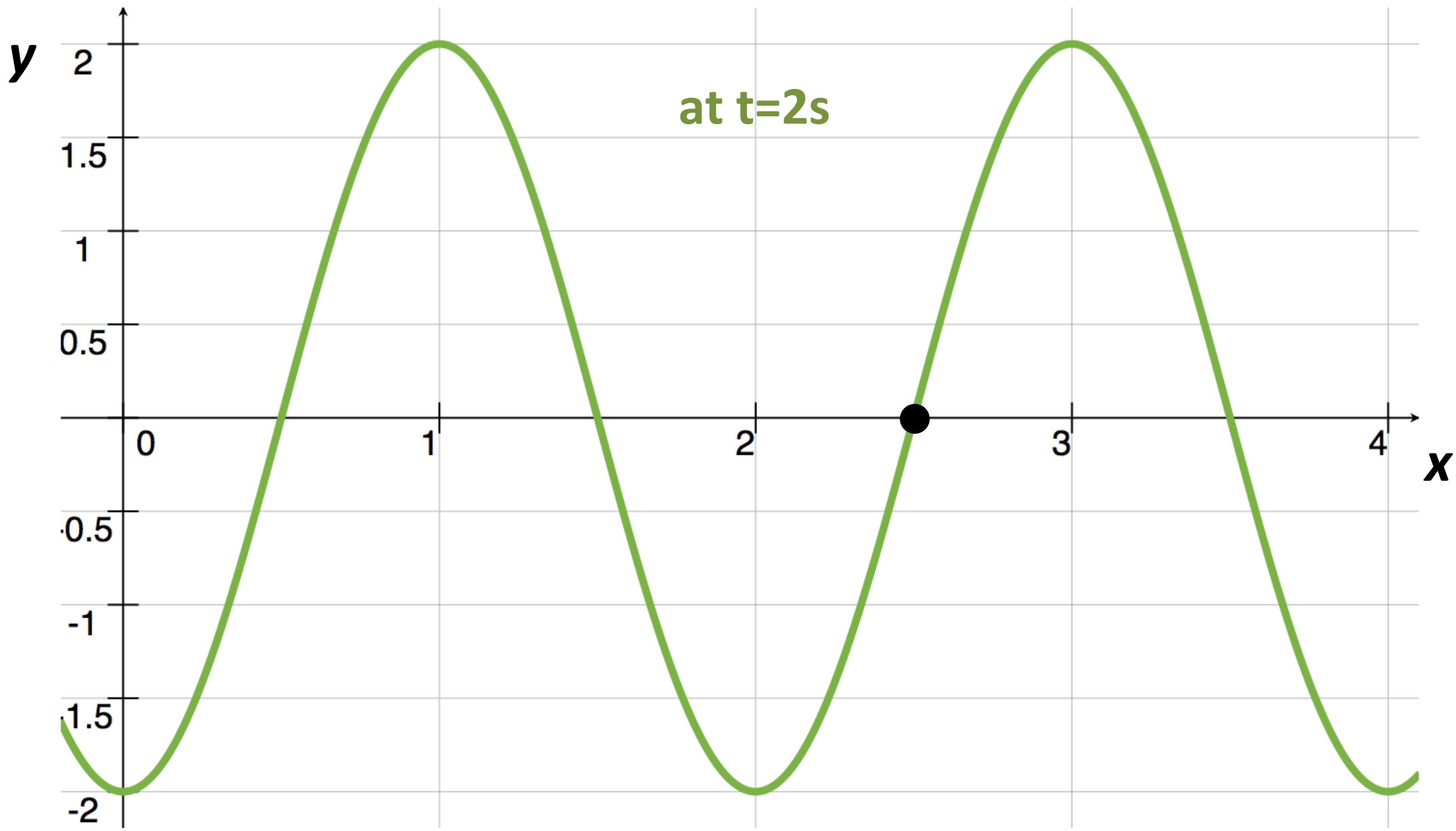
Travelling Cosine Wave Example

$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$



Travelling Cosine Wave Example

$$y(x,t) = 2.0\cos\left(\pi(x - 0.5t)\right)$$



Travelling Cosine Wave Equation

$$y(x,t) = A\cos\left(k(x - vt)\right)$$
$$= A\cos\left(kx - \omega t\right)$$

y is the height: this is a *transverse wave*.

A is the amplitude = maximum height of a point on the string.

k is the wave number = $2\pi/\lambda$

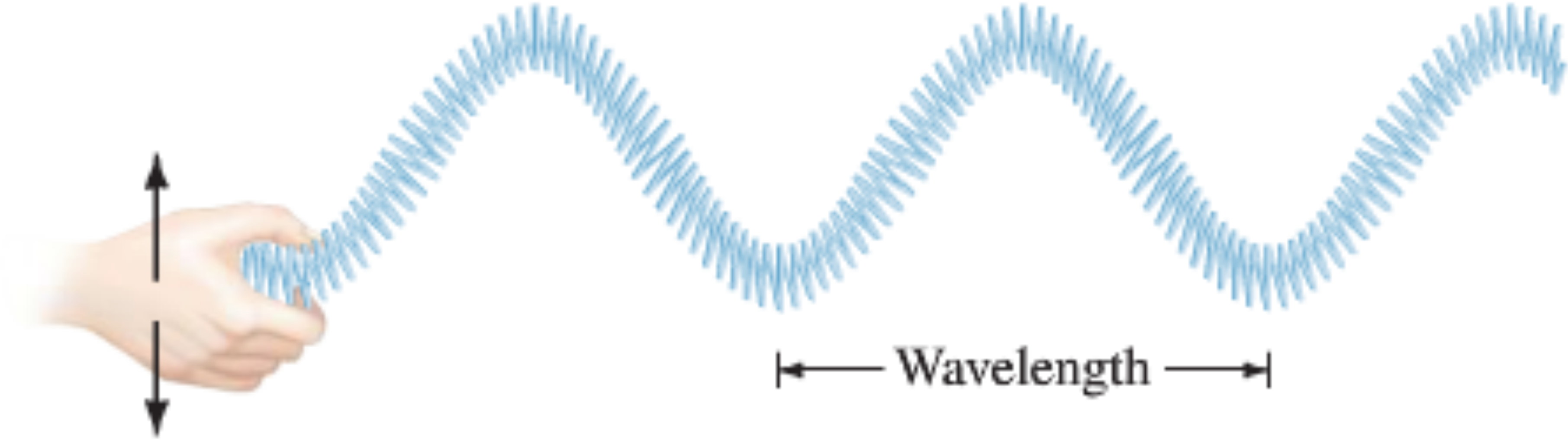
> ***λ*** is the distance (length) between two similar points on the wave (two peaks, two troughs...)

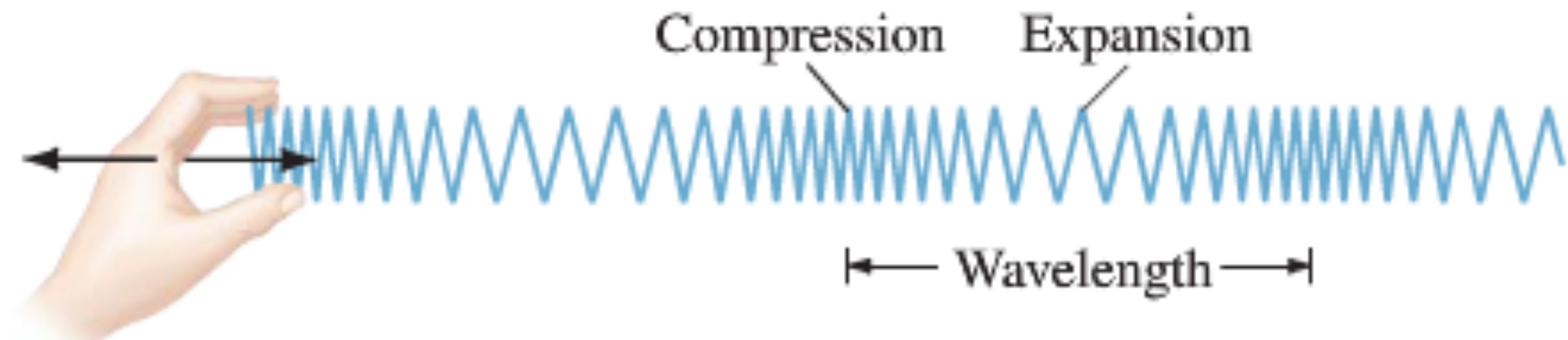
v is the wave speed (m/s); ***ω*** is the angular frequency

$$v = \frac{\omega}{k} \qquad v = 2\pi f \cdot \frac{\lambda}{2\pi} \Rightarrow \boxed{v = \lambda f} \qquad \omega = \frac{2\pi}{T}$$

Longitudinal vs. Transverse

Transverse (displacement perpendicular to velocity)





Longitudinal (displacement parallel to velocity)

Velocities of Cosine Wave

$$y(x,t) = A\cos\left(k(x - vt)\right)$$

Horizontal velocity of wave: $v_{wave} = v\ \hat{i}$ (- sign means to the right)

Vertical velocity of medium:
(transverse velocity)

$$v_y = \frac{dy}{dt} = Akv\sin\left(k(x - vt)\right)$$

$$v_{y,max} = Akv = A\omega$$

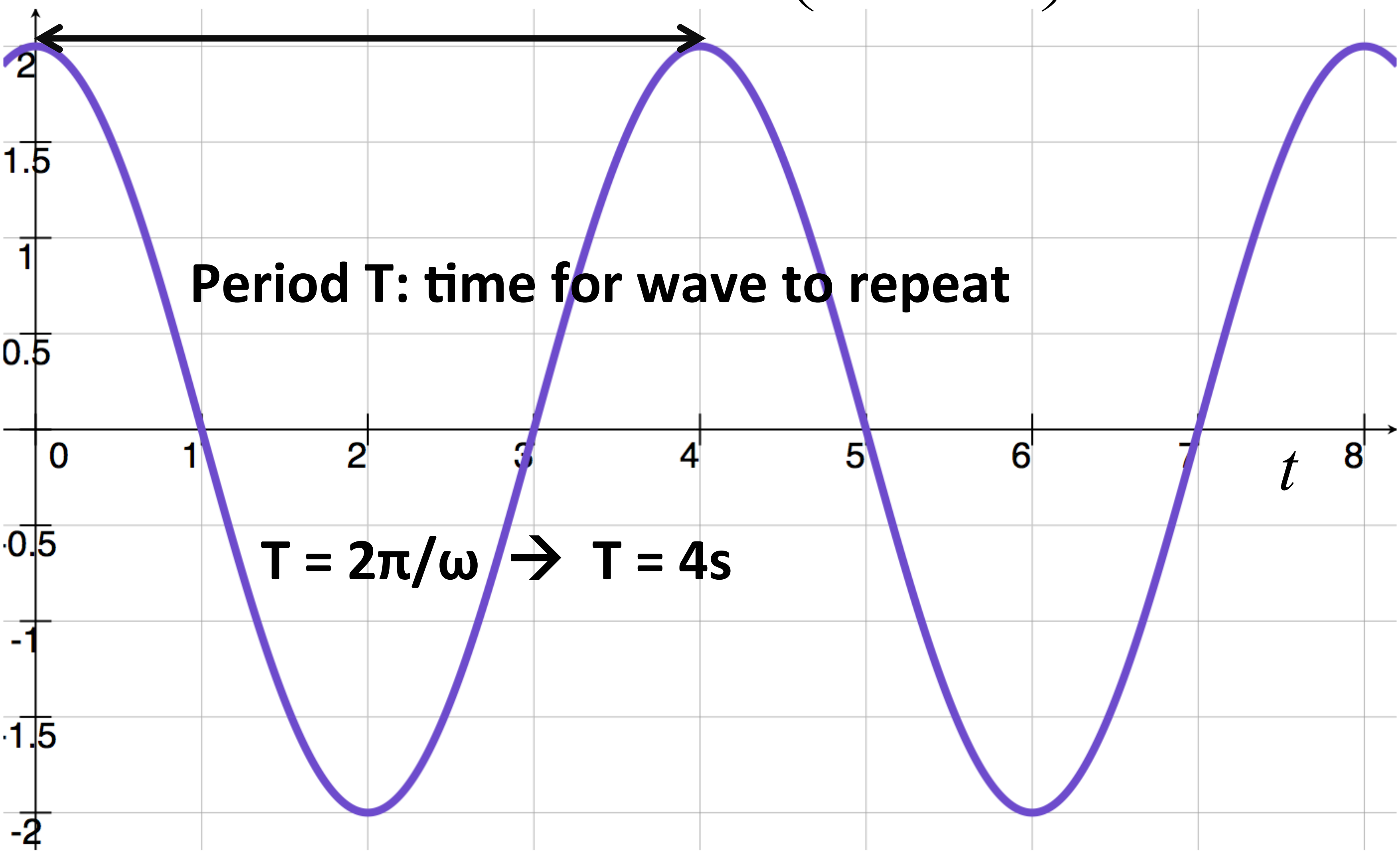
Vertical acceleration of medium:
(transverse acceleration)

$$a_y = \frac{dv_y}{dt} = -A(kv)^2\cos\left(k(x - vt)\right)$$

$$a_{y,max} = A(kv)^2 = A\omega^2$$

Wave Parameters: ω

$$y(x,t) = 2.0\cos\left(\pi x - \frac{\pi}{2}t\right)$$

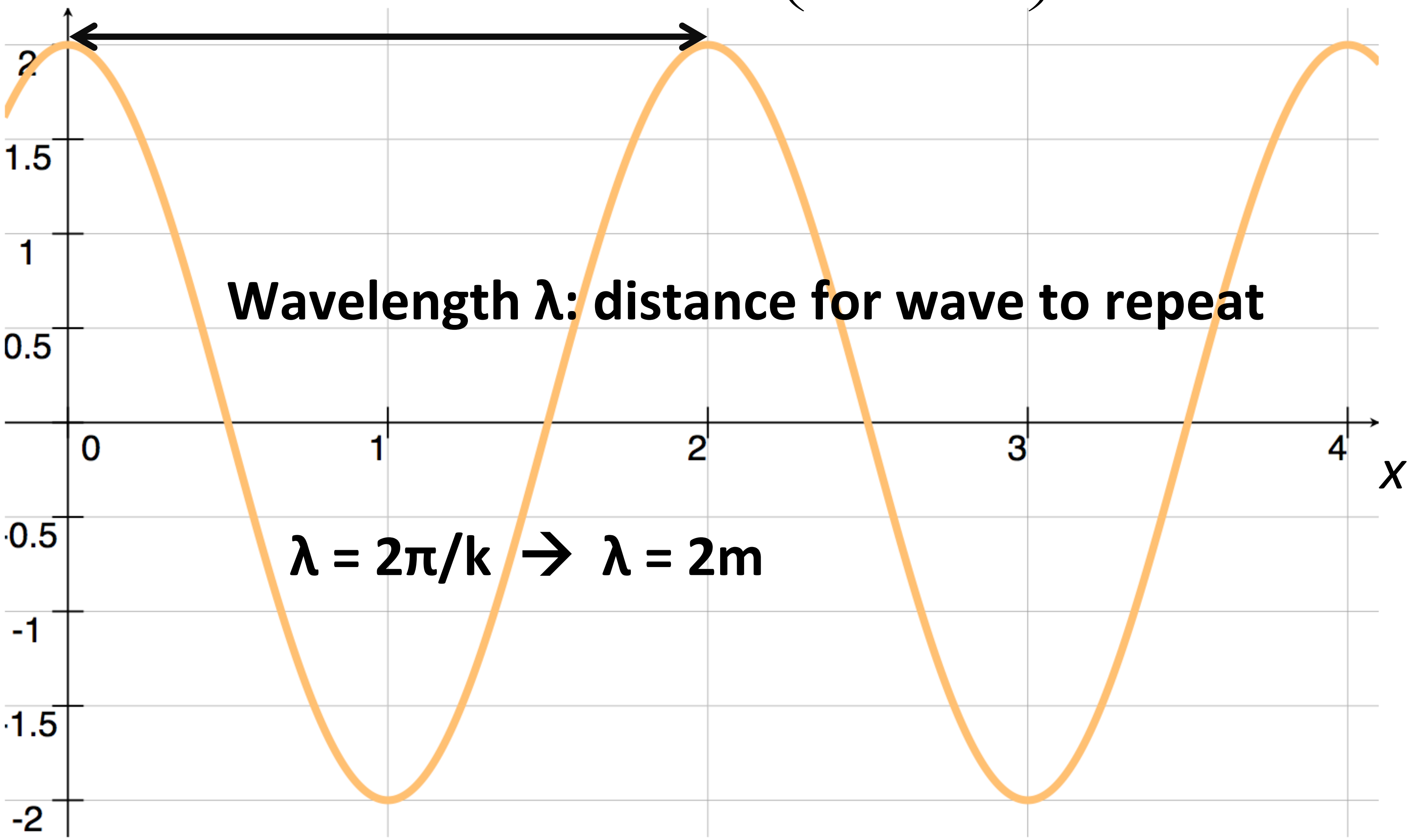


Period T: time for wave to repeat

T = 2π/ω → T = 4s

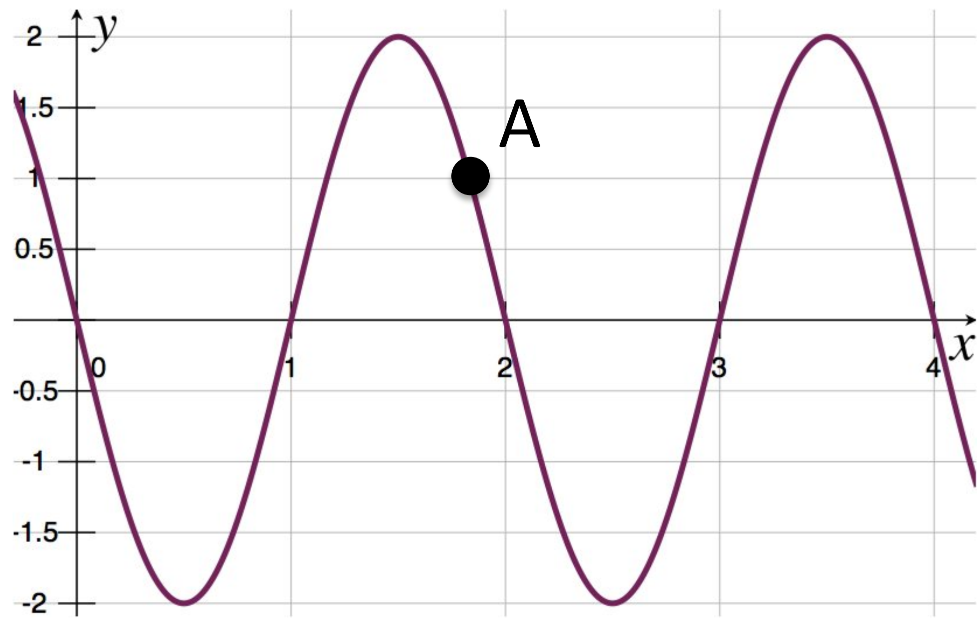
Wave Parameters: k

$$y(x,t) = 2.0\cos\left(\pi x - \frac{\pi}{2}t\right)$$



Clicker

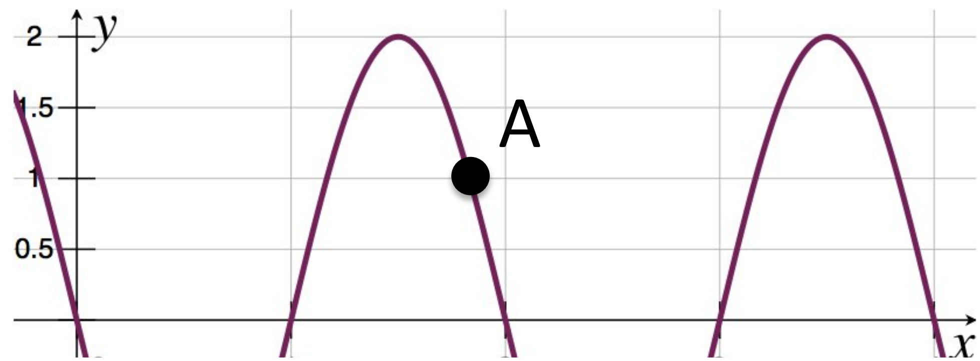
A transverse wave is traveling along a string to the right. The height of the string vs. position is shown in the diagram at one instant in time. Consider the point on the string labeled A. At this instant, in what direction is this part of the string moving?



a. Up
b. Down
c. Left
d. Right
e. None of the above, or need more info

Clicker

A transverse wave is traveling along a string to the right. The height of the string vs. position is shown in the diagram at one instant in time. Consider the point on the string labeled A. At this instant, in what direction is this part of the string moving?

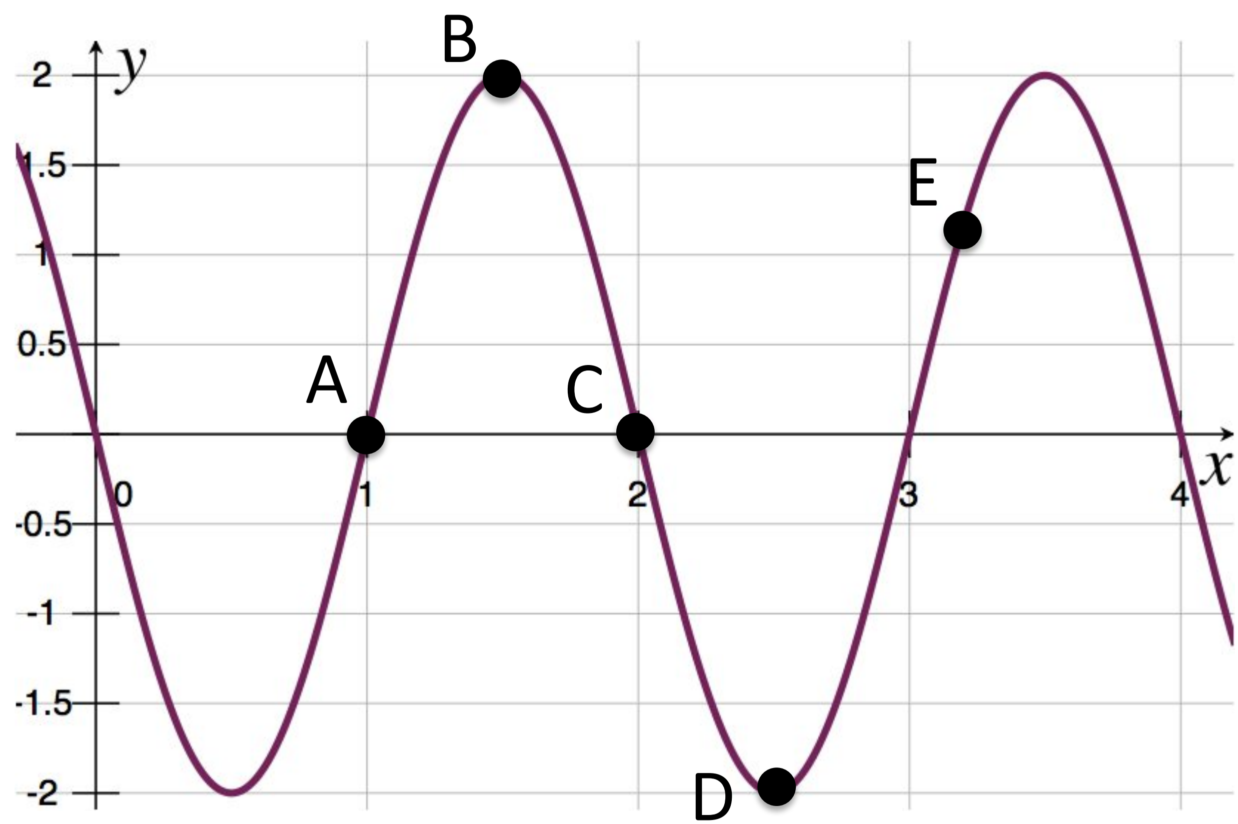


a. Up
b. Down
c. Left
d. Right
e. None of the above, or need more info

The graphs shows the height of the string vs. position, not time. We are asking some time later, in what direction as A moved. Some time later, the wave has moved a bit to the right, bringing the peak of the wave closer to A. Thus, A will move up the peak as the wave moves right.

Clicker

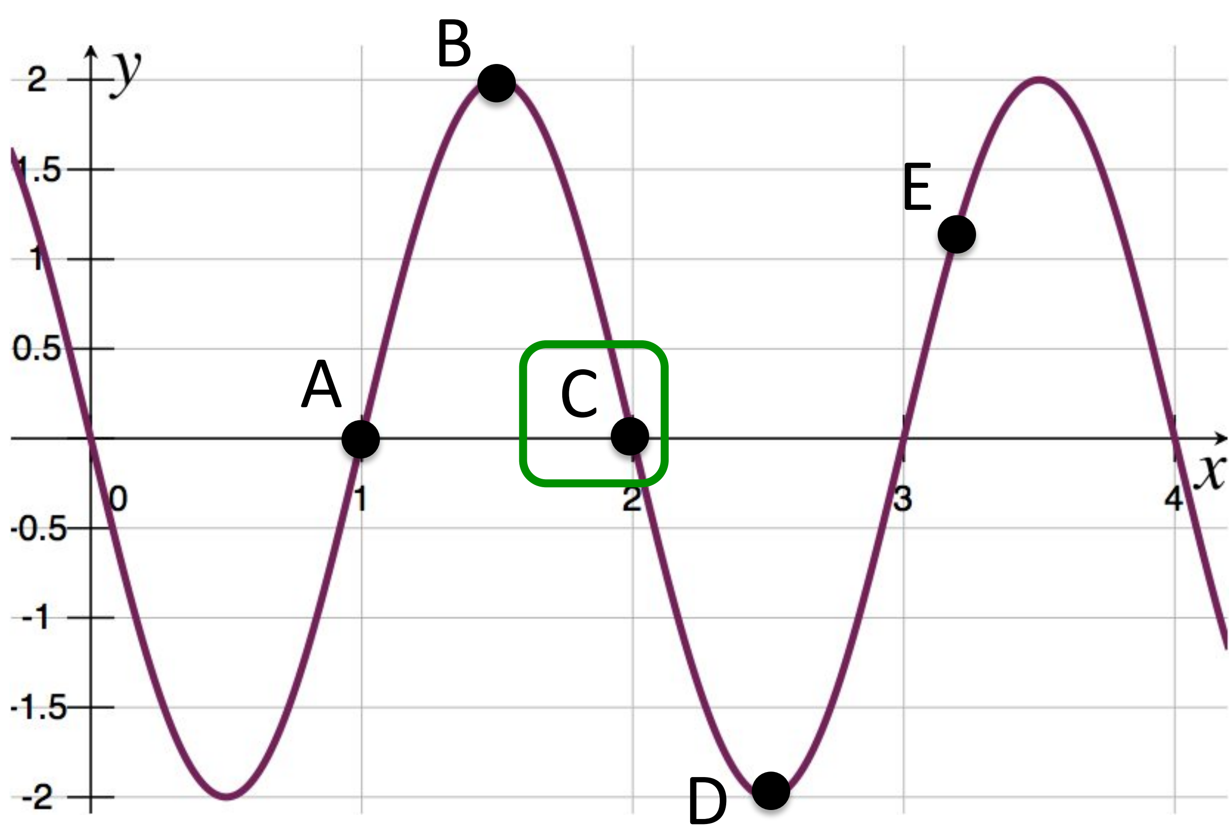
A transverse wave is traveling along a string to the right. The height of the string is shown in the diagram at one instant in time. Different points on the string are labeled. At this instant, which part of the string has the greatest upward speed?



Clicker

As the wave moves to the right, points A, E, and B will move downward, so it is not them. Points C and D will move up as the wave moves right. Since D is near the min of the wave, which is mostly horizontal, it will not move very far. C is at a transition point of the wave which is changing greatly. So a short time later, it will move the largest distance upward.

speed?



Speed of Wave on String

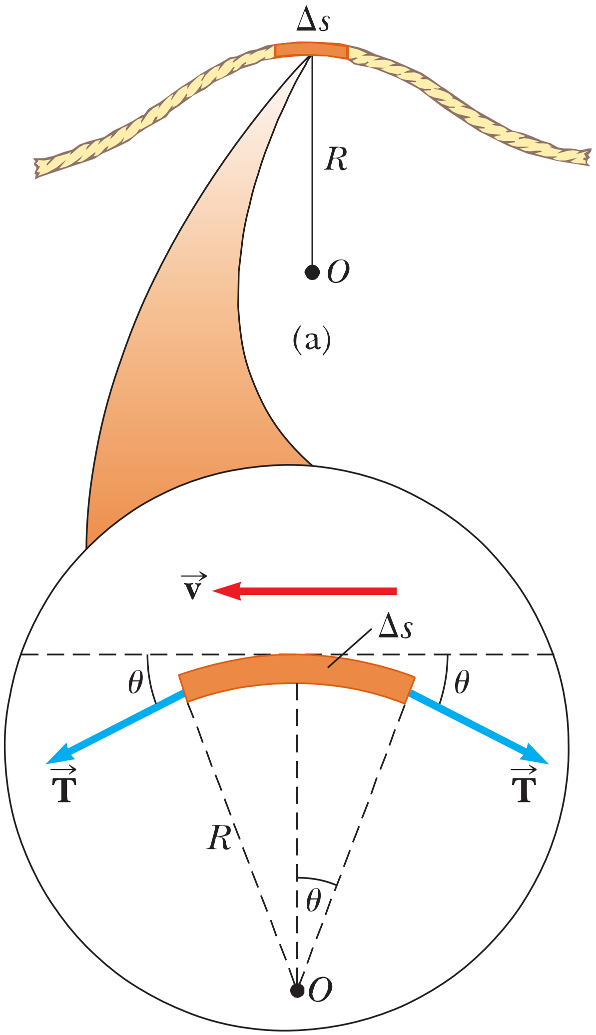
The speed of a wave in a medium is usually determined by the properties of the medium.

For waves on a string:

$$v = \sqrt{\frac{T}{M/L}}$$

T is the tension in the string and *M/L* is the mass per length.

Get a fast wave if tension is large and mass is small. Large T and small M leads to large accelerations, meaning quick up and down motion.



Speed of Wave on String

$$\sum \vec{F} = m\vec{a}$$

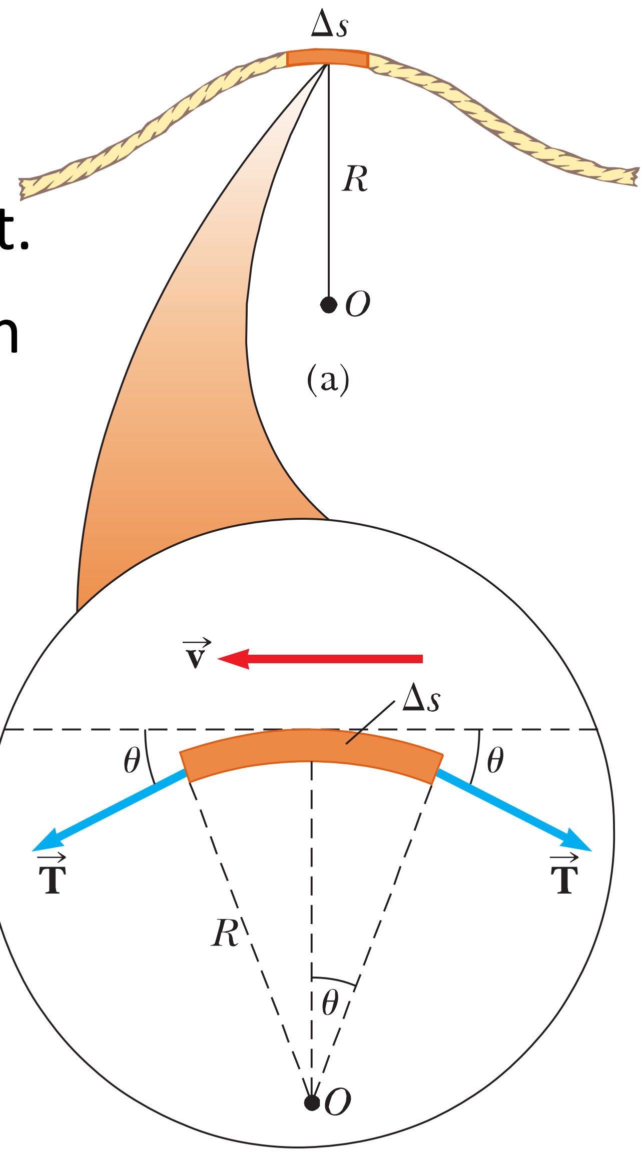
- Tension T on both sides of our segment.
- Segment subtends an angle 2θ. Tension makes an angle θ with the horizontal.

$$\sum F_x = T\cos(\theta) - T\cos(\theta) = 0$$

$$\sum F_y = 2T\sin(\theta)$$

As we move past this segment, in the region of the peak, it appears to be moving in circle.

$$a_y = \frac{mv^2}{R}$$



Speed of Wave on String

$$2T\sin(\theta) = \frac{mv^2}{R}$$

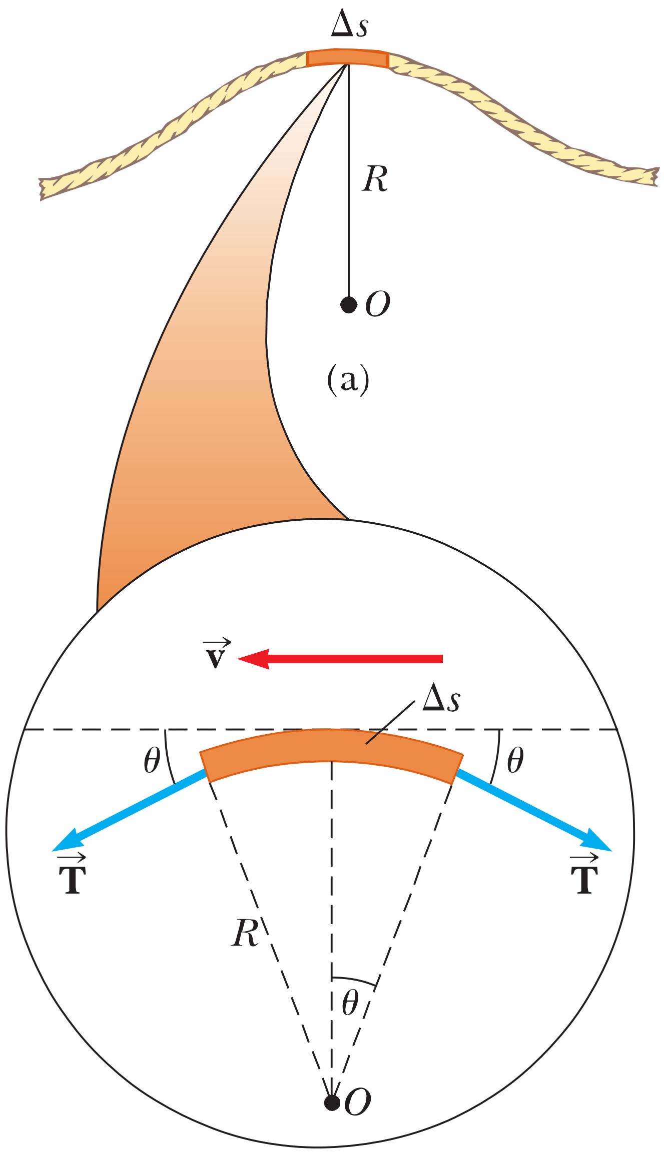
If the length Δs is very small, then

$$\sin(\theta) \approx \theta \quad \rightarrow \quad 2T\theta = \frac{mv^2}{R}$$

Say that the string has a mass density of μ (mass per length), then $m = \mu\Delta s$

$$\cancel{2}T\cancel{\theta} = \frac{\mu\Delta s \cdot v^2}{R} = \frac{\mu(\cancel{R}\cdot\cancel{2}\cancel{\theta})\cdot v^2}{\cancel{R}}$$

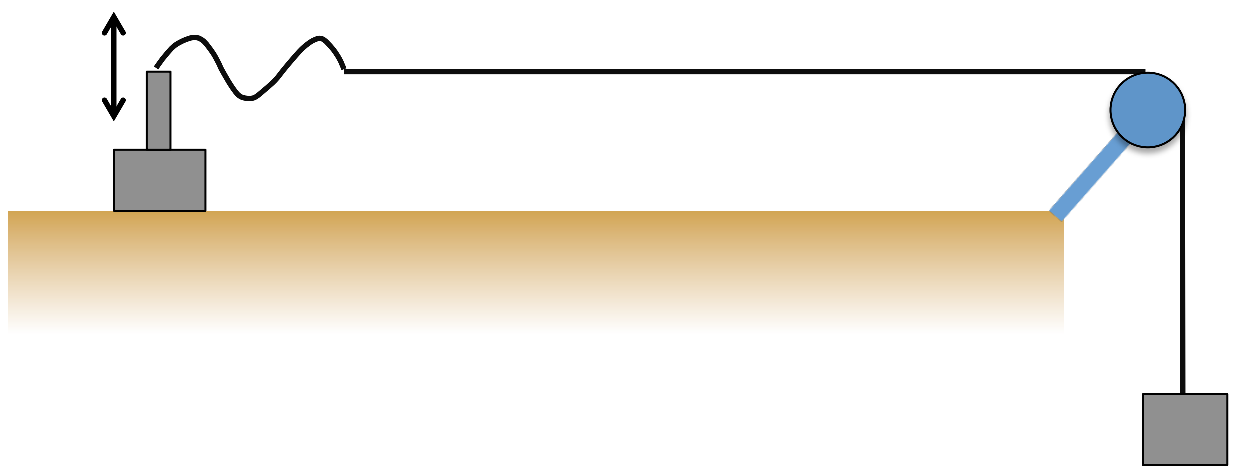
$$T = \mu v^2 \qquad \boxed{v = \sqrt{\frac{T}{\mu}}}$$



Clicker

A string is attached on one end to a mechanical arm that oscillates up and down. At the distant other end, the string passes over a pulley and a mass *M* is hung from it. Assume the string has a linear mass density of μ. The oscillator is originally moving up and down at a frequency *f*.

If the frequency changes from *f* to *2f*. How does the speed *v* of the wave on the string change?



a. *v* goes to *2v*

b. *v* goes to *v/2*

c. *v* is unchanged

d. None of the above, or need more info

Clicker

A string is attached on one end to a mechanical arm that oscillates up and down. At the distant other end, the string passes over a pulley and a mass *M* is hung from it. Assume the string has a linear mass density of μ. The oscillator is originally moving up and down at a frequency *f*.

The speed is only a property of the material, it depends only on the tension and mass density of the string. Changing the frequency does not affect the speed.

a. v goes to $2v$

b. v goes to $v/2$

c. $\boxed{v \text{ is unchanged}}$

d. None of the above, or need more info

Clicker

A string is attached on one end to a mechanical arm that oscillates up and down. At the distant other end, the string passes over a pulley and a mass M is hung from it. Assume the string has a linear mass density of μ. The oscillator is originally moving up and down at a frequency f.

If the frequency changes from f to $2f$. How does the wavelength λ of the wave on the string change?

a. λ goes to 2λ

b. λ goes to $\lambda/2$

c. λ is unchanged

d. None of the above, or need more info

Clicker

A string is attached on one end to a mechanical arm that oscillates up and down. At the distant other end, the string passes over a pulley and a mass M is hung from it. Assume the string has a linear mass density of μ. The oscillator is originally moving up and down at a frequency f.

The speed is unchanged since it only depends on the material. The speed v = fλ. So if v is unchanged, but f increases by 2, then the wavelength must decrease by 2.

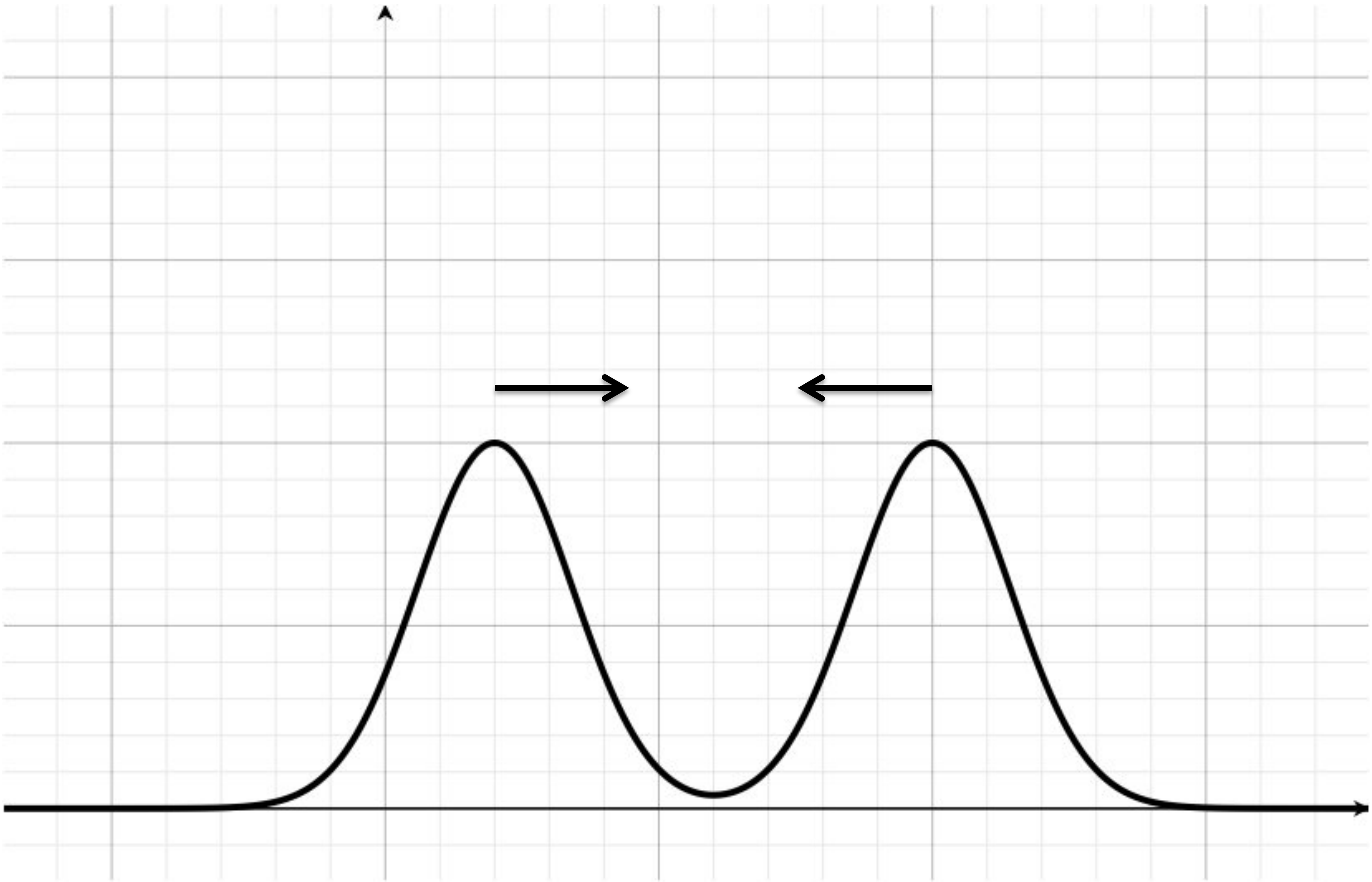
a. λ goes to 2λ

b. λ goes to $\lambda/2$

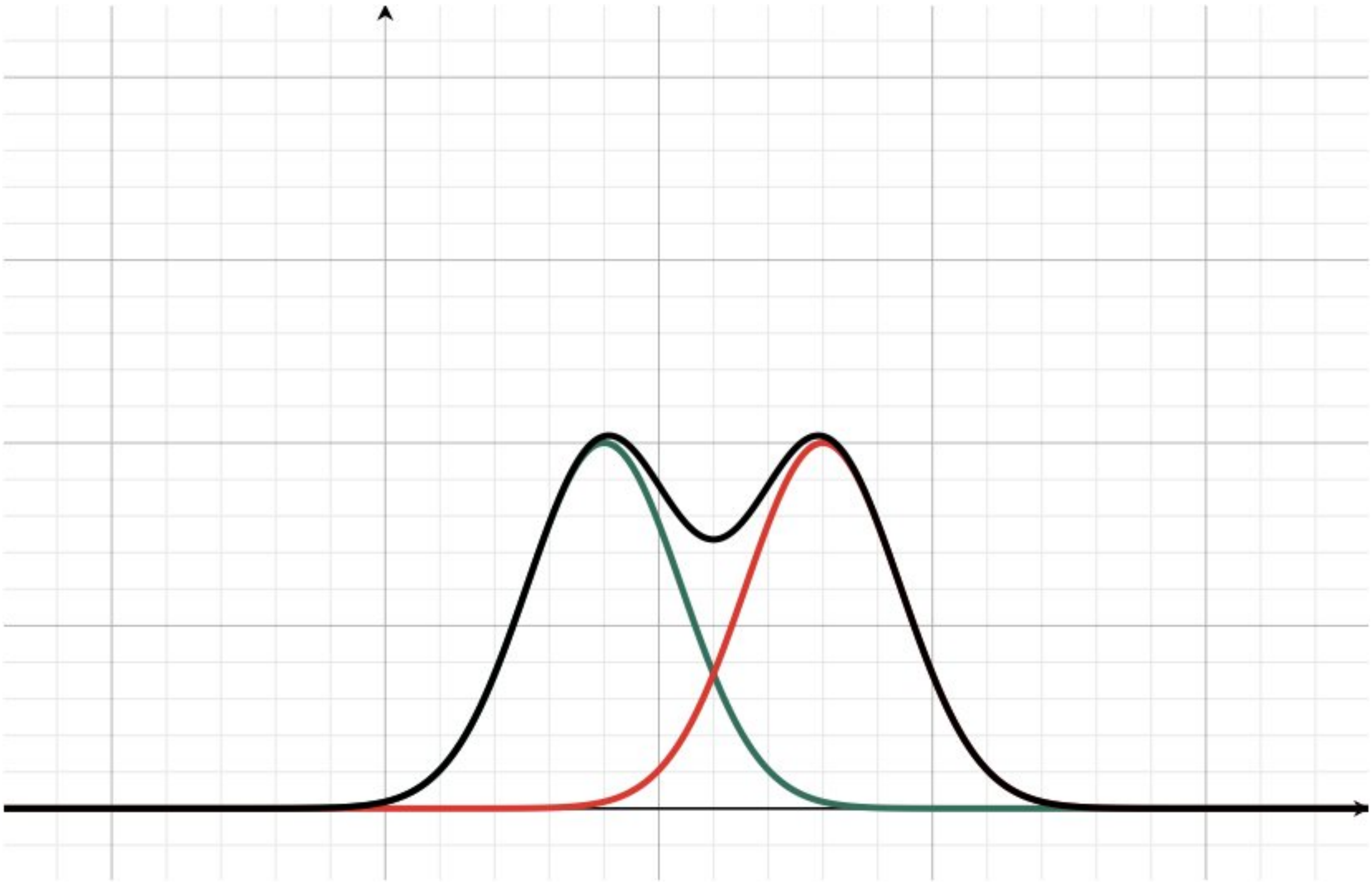
c. λ is unchanged

d. None of the above, or need more info

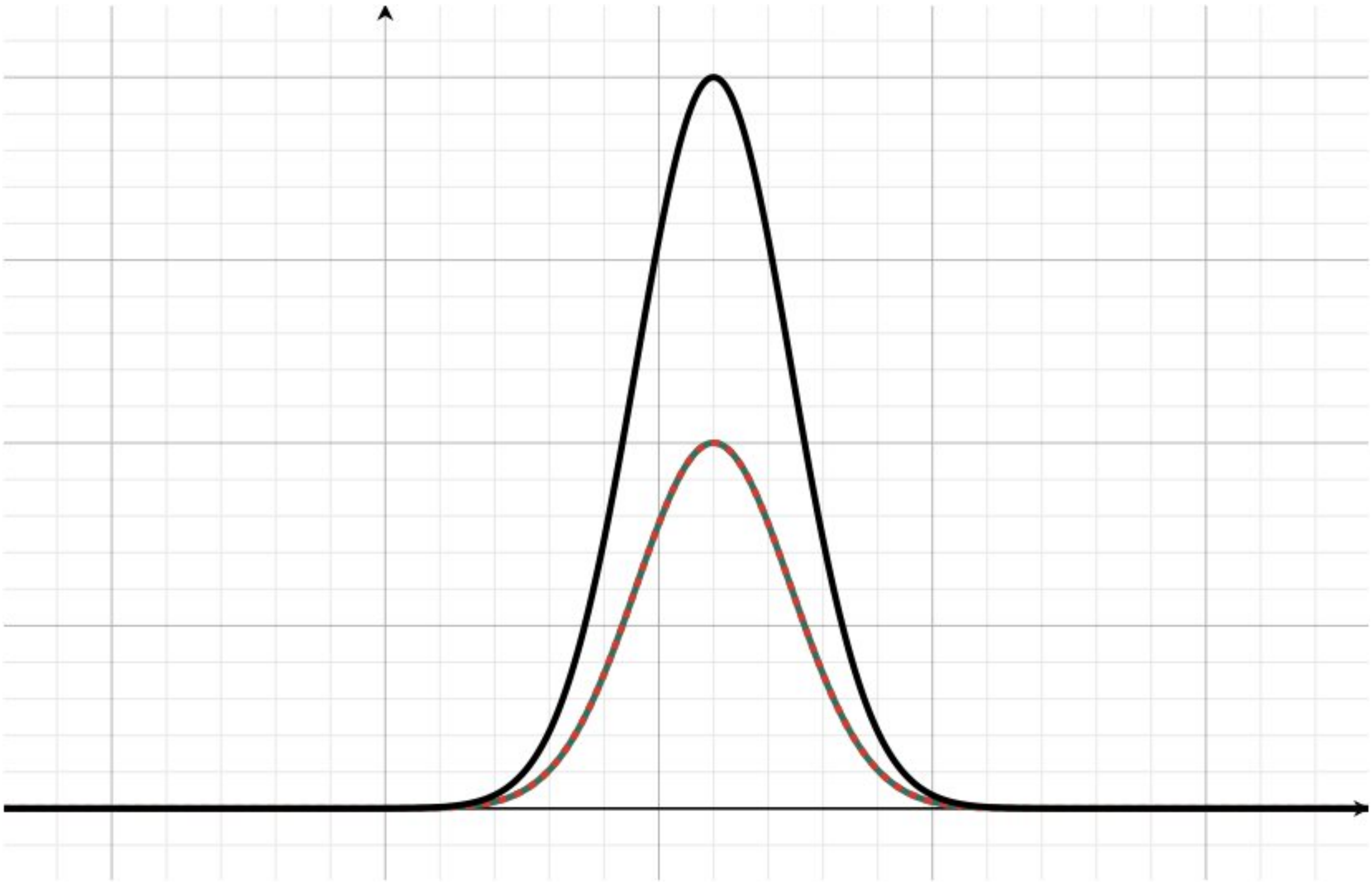
Superposition of 2 pulses



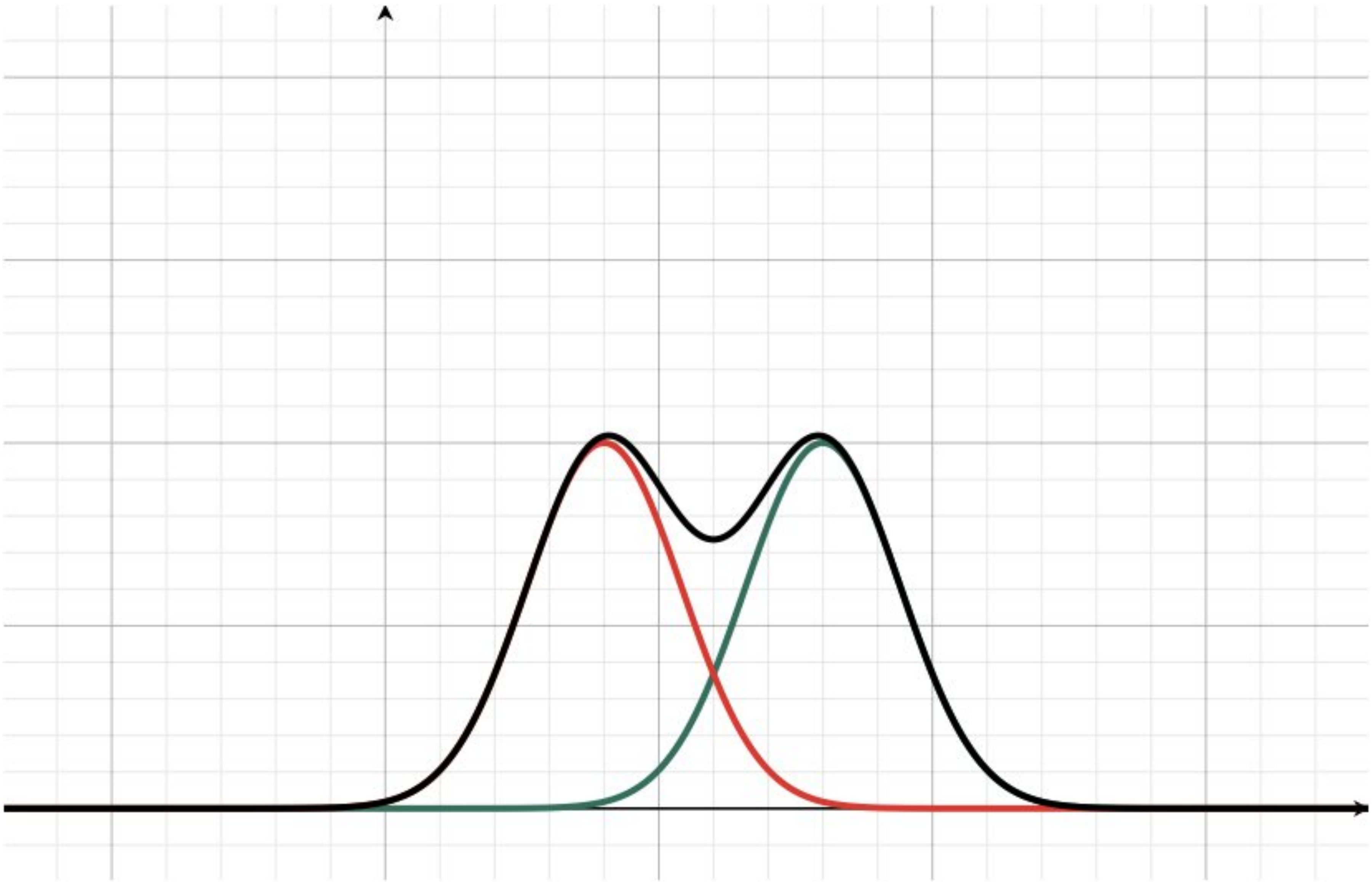
Superposition of 2 pulses



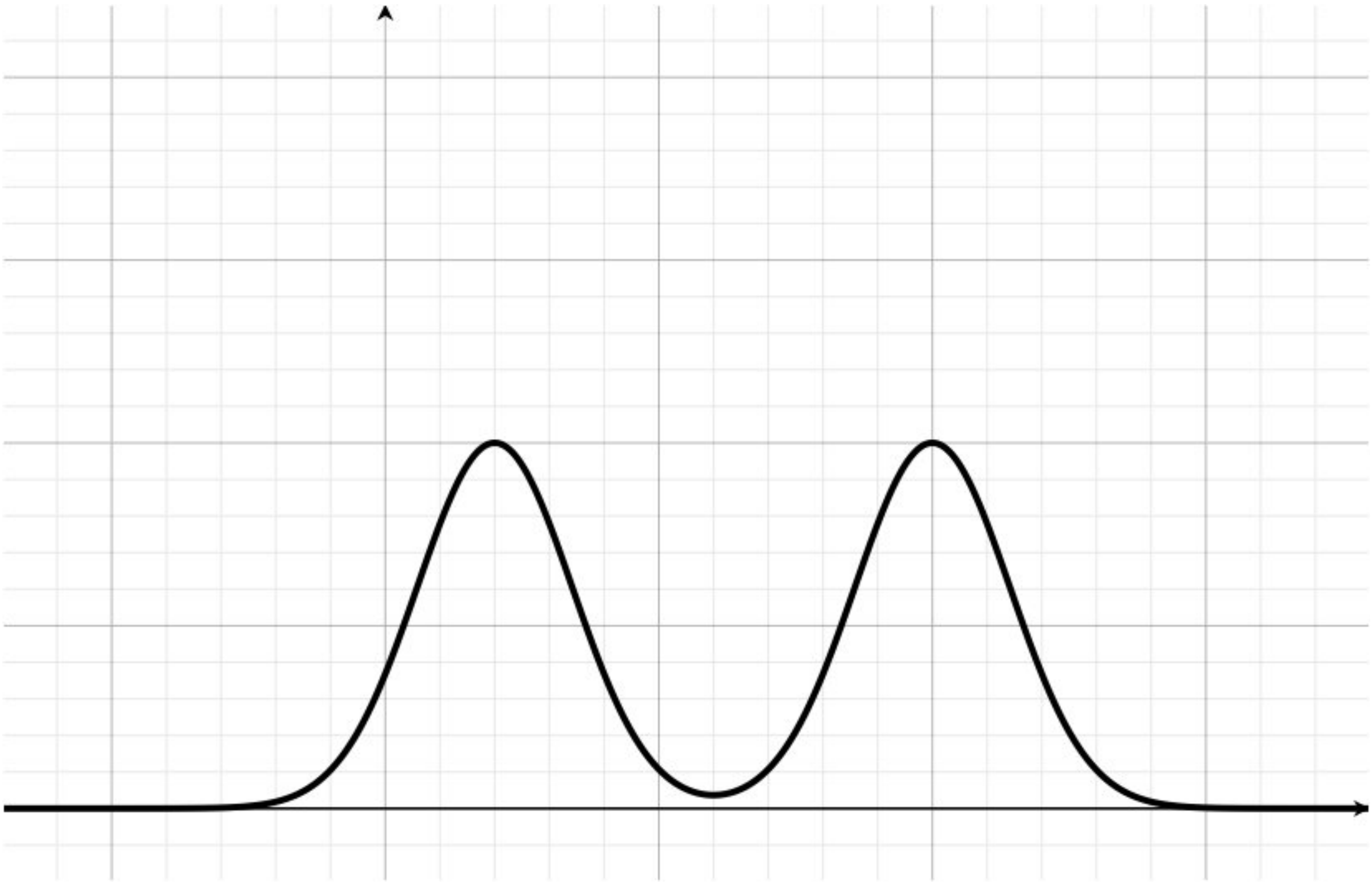
Superposition of 2 pulses



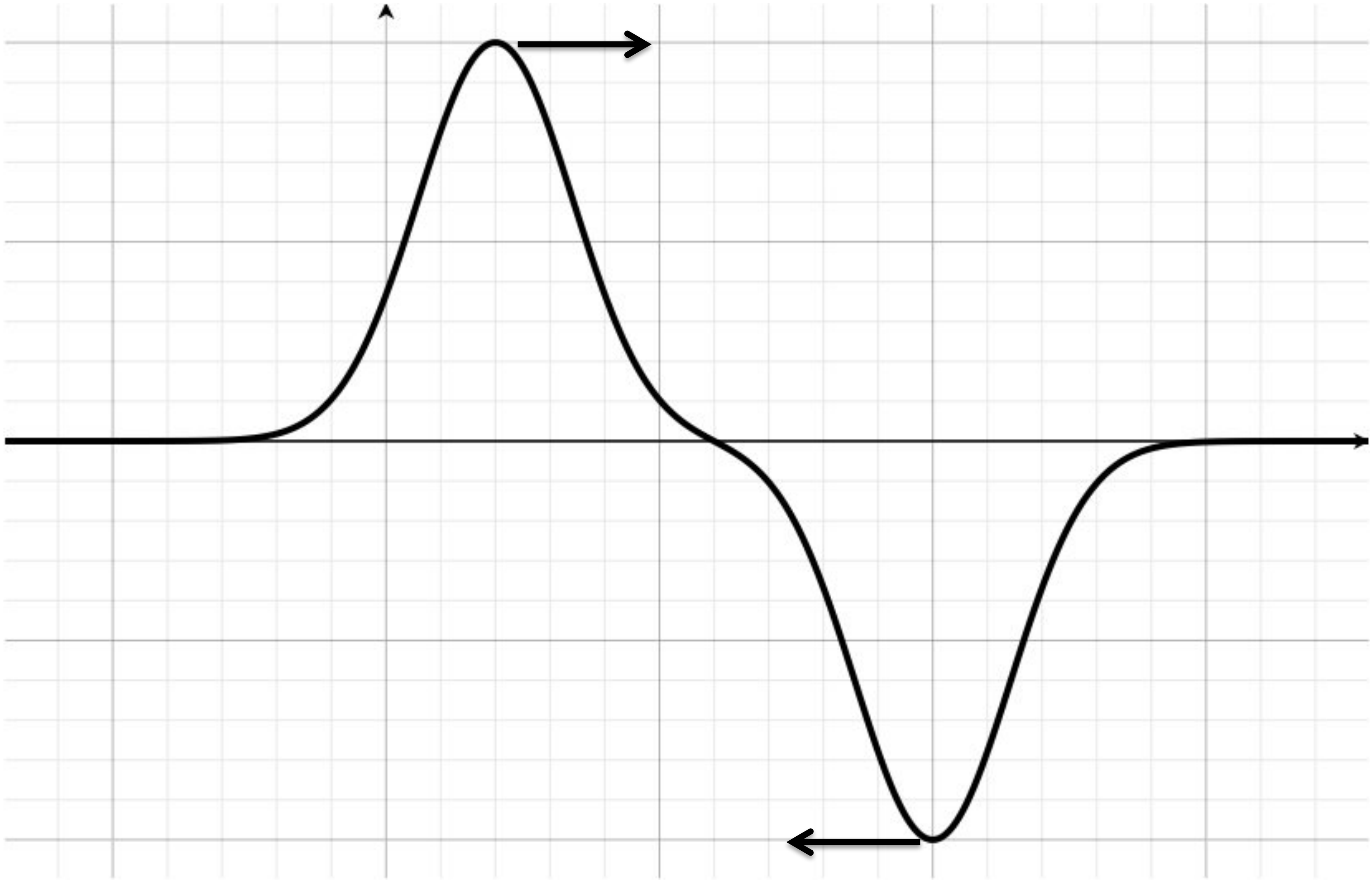
Superposition of 2 pulses



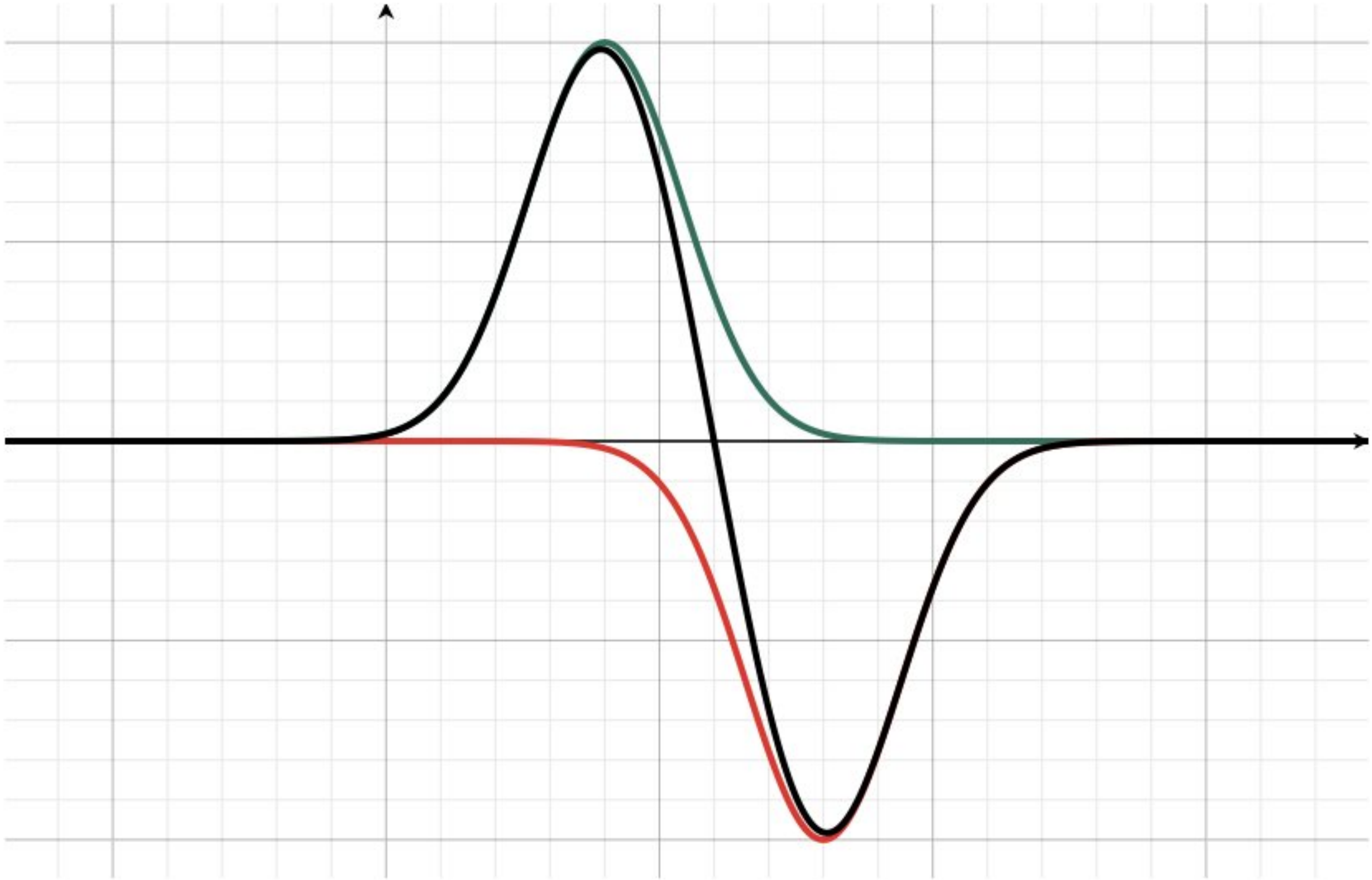
Superposition of 2 pulses



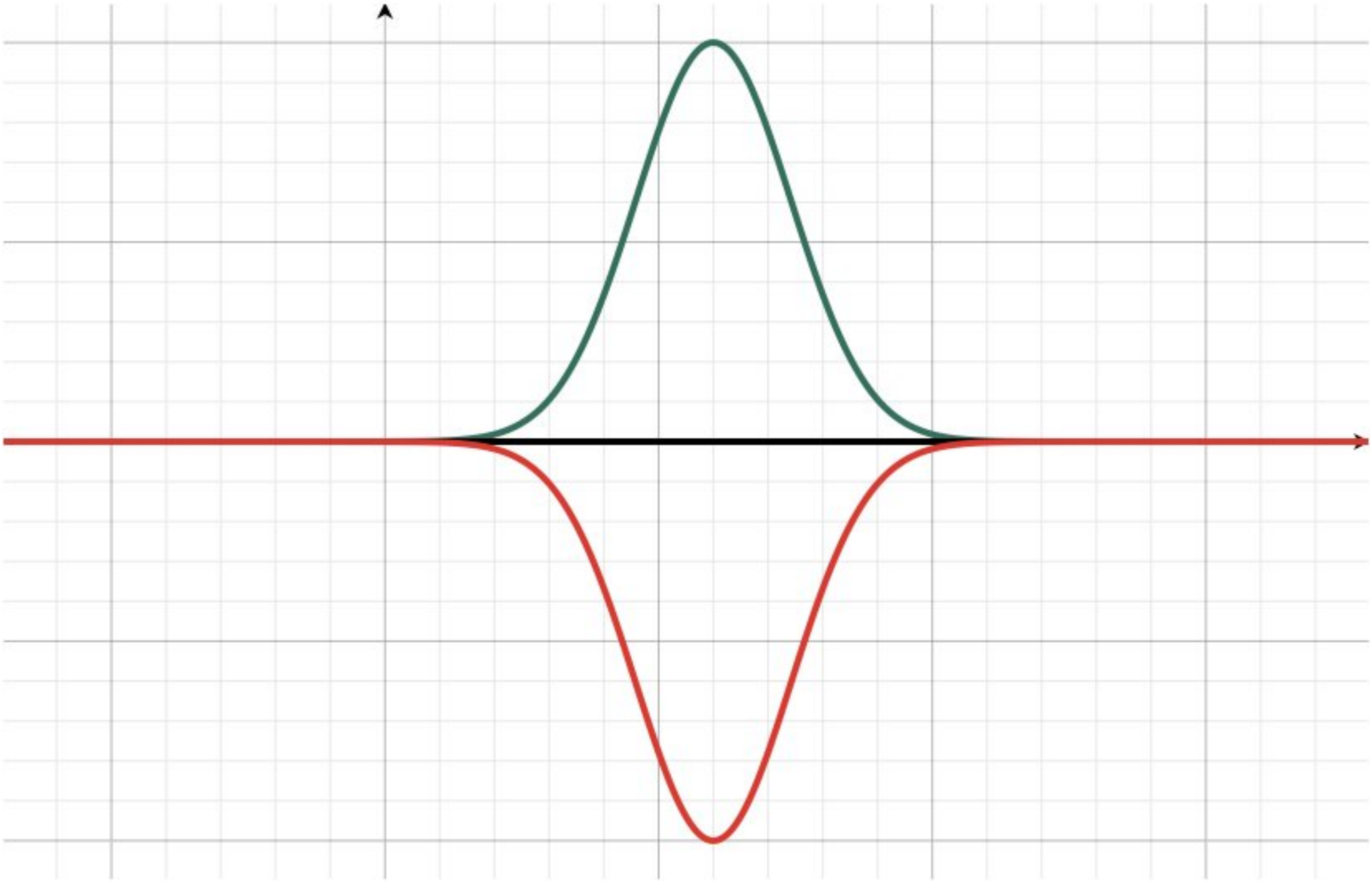
Superposition of 2 Pulses



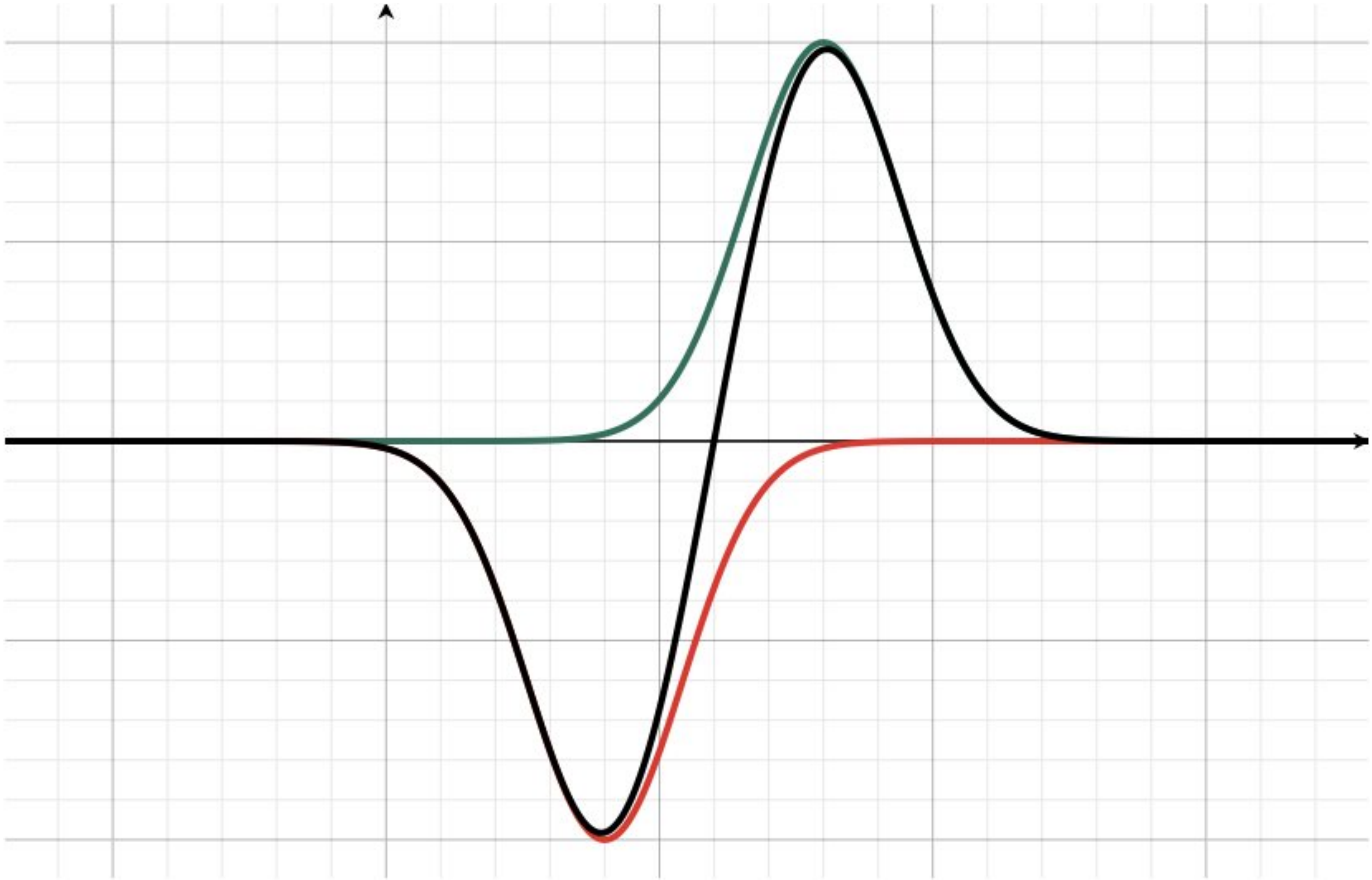
Superposition of 2 Pulses



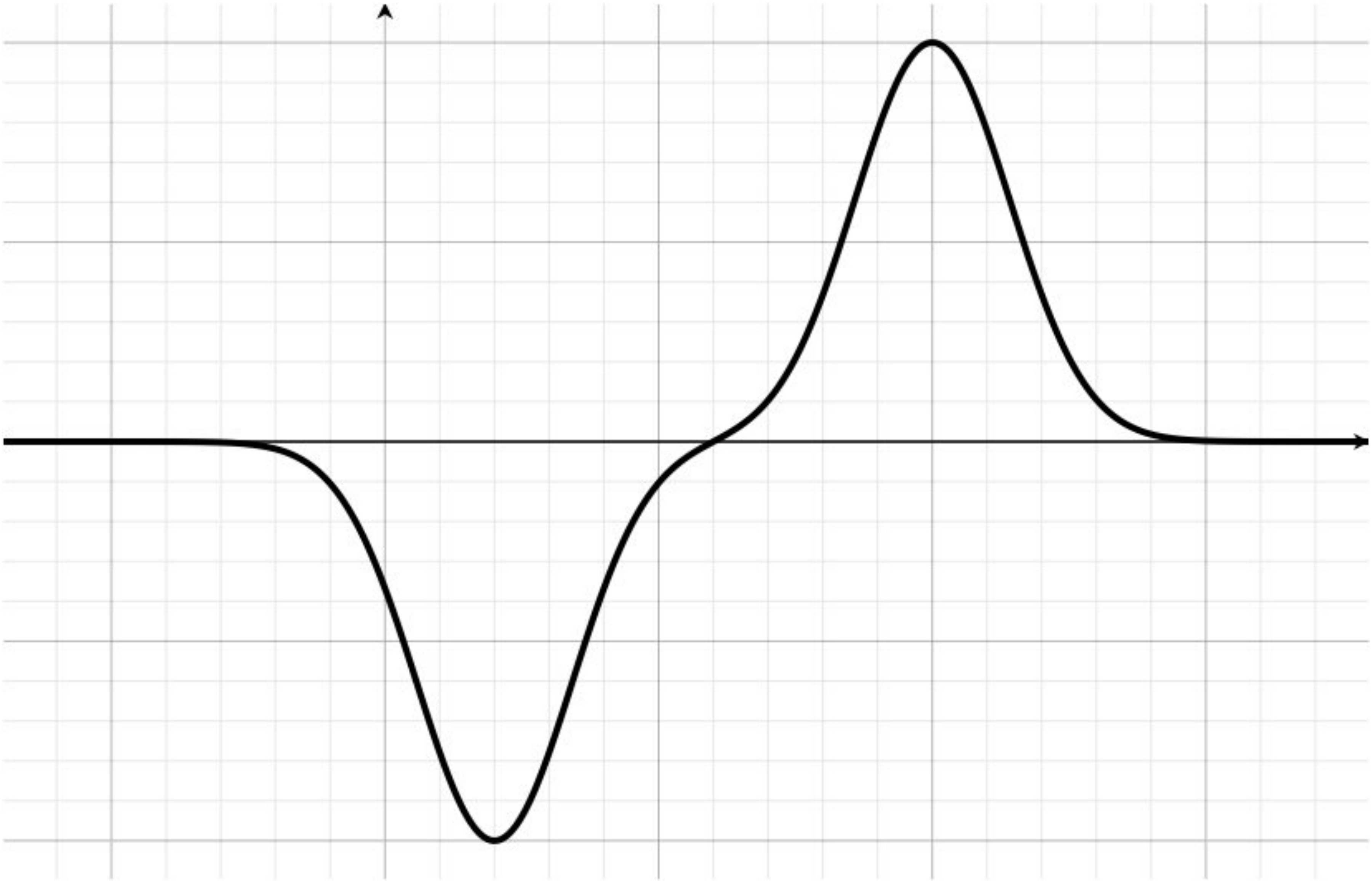
Superposition of 2 Pulses



Superposition of 2 Pulses



Superposition of 2 Pulses



Principle of Superposition

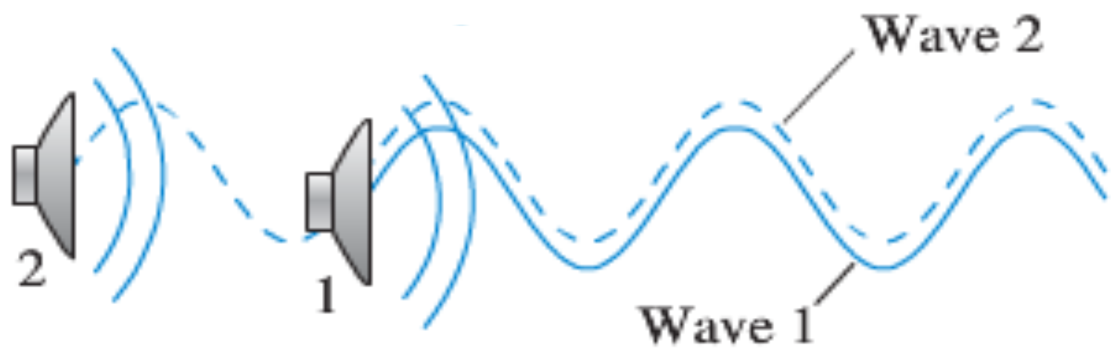
When two or more waves move in the same medium, the net displacement of the medium is the algebraic sum of all the individual waves.

$$y_{tot}(x,t) = y_1(x,t) + y_2(x,t) + \ldots$$

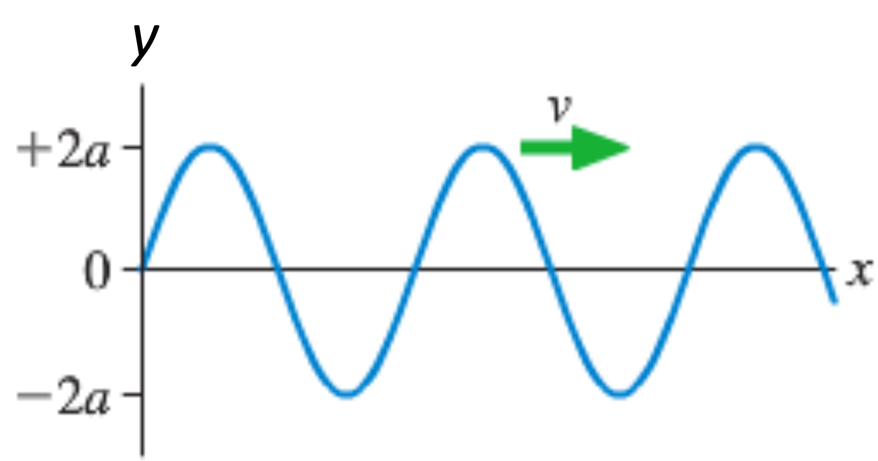
One wave does not affect in the any way the propagation of any other wave.

Superposition of Sine Waves

Take 2 speakers, which each produce a sound at the same frequency:



The wave from each speaker has an amplitude *a*. What is the resulting wave?

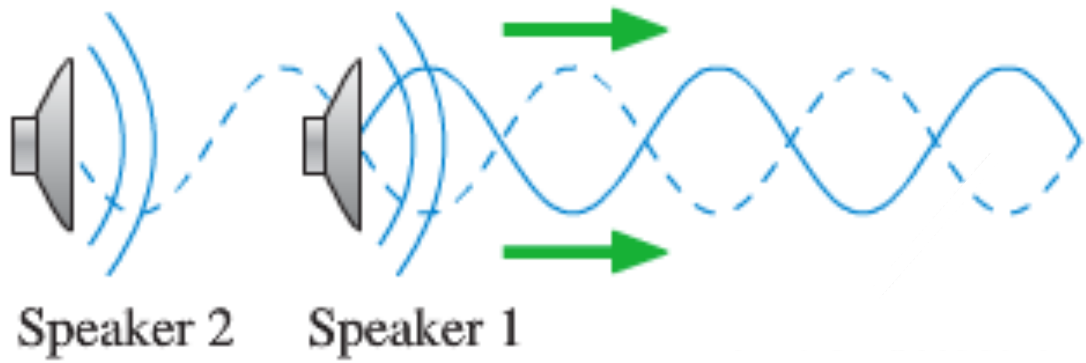


Wave with 2x the amplitude

"in-phase"

Superposition of Sine Waves

Take 2 speakers, which each produce a sound at the same frequency:



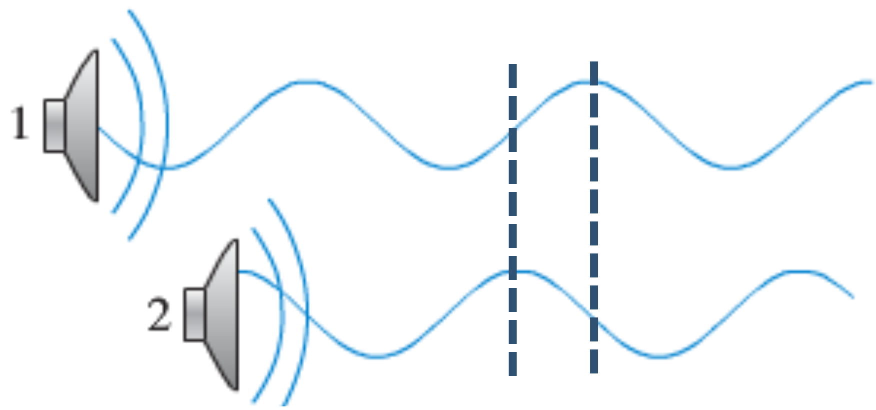
The wave from each speaker has an amplitude *a*. What is the resulting wave?

No wave!
Would hear no sound.

“out-of-phase”

Interference with Same Frequency

What if the two waves do not line up peak-to-peak, or trough-to-trough? What is the resulting wave?



$$y_1(x,t) = A\cos(kx - \omega t)$$

$$y_2(x,t) = A\cos(kx - \omega t + \delta)$$

$$y_{tot}(x,t) = y_1(x,t) + y_2(x,t)$$

$$= A\cos(kx - \omega t) + A\cos(kx - \omega t + \delta)$$

Sum of cosines with different arguments. Use the trig identity:

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha - \beta}{2}\right) \cdot \cos\left(\frac{\alpha + \beta}{2}\right)$$

Interference with Same Frequency

$$y_{tot}(x,t) = A\cos(kx - \omega t) + A\cos(kx - \omega t + \delta)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha - \beta}{2}\right) \cdot \cos\left(\frac{\alpha + \beta}{2}\right)$$

Using the trig identity, we get:

$$y_{tot}(x,t) = \underbrace{2A\cos\left(\frac{\delta}{2}\right)}_{\text{Amplitude}} \cdot \underbrace{\cos\left(kx - \omega t + \frac{\delta}{2}\right)}_{\text{Travelling wave}}$$

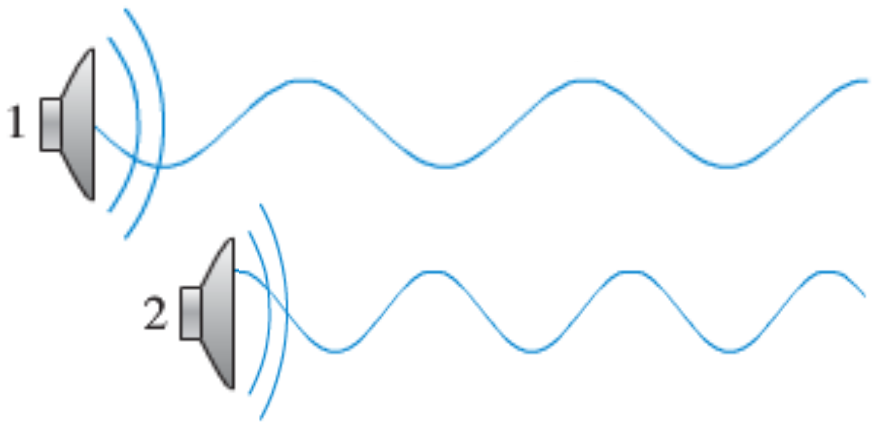
- There is still a travelling wave with the same frequency as the original from the speakers.

In-phase: $\delta = 0 \quad \Rightarrow 2A\cos(0) \quad = 2A$

Out-of-phase: $\delta = \pi \quad \Rightarrow 2A\cos(\pi / 2) \quad = 0$

Interference with Different Freq.

What if the two waves do not have the same frequency?



$$y_1(x,t) = A\cos(k_1 x - \omega_1 t)$$

$$y_2(x,t) = A\cos(k_2 x - \omega_2 t + \delta)$$

We still use the principle of superposition. To simplify the math, we are going to set *x=0*, and ignore the spatial variation. We will also say *δ=0* (at time=0 they both start at their peaks).

$$y_{tot}(x,t) = y_1(x,t) + y_2(x,t)$$

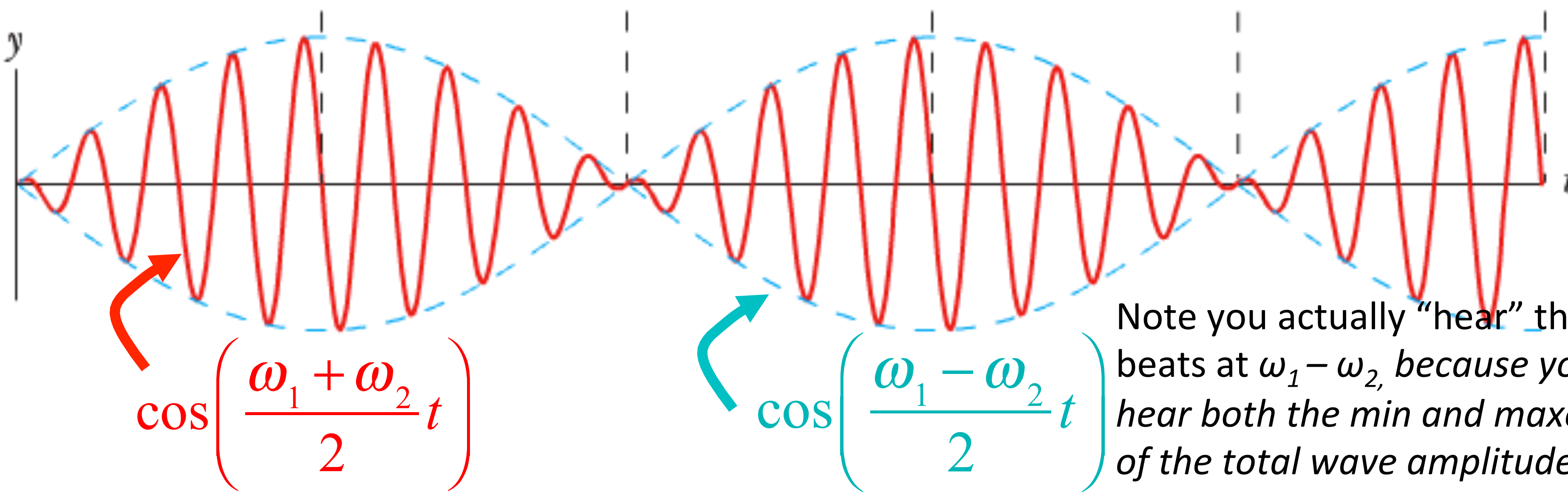
$$= A\cos(\omega_1 t) + A\cos(\omega_2 t)$$

$$= 2A\cos\left(\frac{(\omega_1 - \omega_2)}{2}t\right)\cdot\cos\left(\frac{(\omega_1 + \omega_2)}{2}t\right)$$

Interference with Different Freq.

$$\boxed{y_{tot}(0,t) = 2A\cos\left(\underbrace{\frac{(\omega_1-\omega_2)}{2}}_{\text{Small Freq.}}t\right)\cdot\cos\left(\underbrace{\frac{(\omega_1+\omega_2)}{2}}_{\text{Large Freq.}}t\right)}$$

You hear a sound at the $(\omega_1 + \omega_2)/2$ frequency, but the amplitude changes at rate given by $(\omega_1 - \omega_2)/2$. The changes in intensity you hear are called *Beats*.



Note you actually "hear" the beats at $\omega_1 - \omega_2$, *because you hear both the min and maxes of the total wave amplitude.*

Interference with Different Freq.

- Maximum intensity occurs when the two waves are temporarily in phase.
- Minimum intensity occurs when the two waves are temporarily out of phase.