Rotational Kinetic Energy

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Abstract

In this lab, my group and I tested a theory of rotational kinetic energy. My group derived equations of conservation of energy in order to calculate conservation of energy derivation of momentum in a suitable way with measurable values. We then utilized this derivational process to find inertia in a spinning wheel with and without massed loads attached. The inertia of a spinning wheel with weights measured 0.065 Kgm^2 and the inertia of a spinning wheel without weights measured 0.034 Kgm^2. These two values aided in finding the inertia of energy to be 0.03 ± 0.02 Kgm^2. This value was then compared to the value of inertia for the massed loads. This value of inertia was 0.029 ± 0.001 Kgm^2. These values did not agree because their difference, 0.002 Kgm^2, was not within their uncertainties, so human error was at play.

Introduction and Theory

The theory supporting laws of inertia, specifically concerning inertia of a spinning wheel, is simply derived through equations of conservation of energy. In the context of this lab, conservation of energy is defined by the conversion of gravitational potential energy (U_w) to kinetic energy of a falling object (K_w), rotational kinetic energy of a wheel (K_r), and work done by friction. However, in regards to our procedure, all friction was countered and will held as negligible.

$$\Delta U_w + K_w + K_r = W_f$$

Substituting our known equations for these variables, our equation transforms into a measurable quantity. This transformed equation includes known values of the hanging mass (M), the mass of hanging paperclips (m), the radius of a wheel (r), a vertical displacement (y), the gravitational constant (g), the velocity in which the wheel rotates (v), and the inertia of the wheel (I).

$$W_f = -Mgy + mgy + (0.5)Mv^2 + (0.5)I\omega^2$$

Because the weight of our paper clips counteracts friction, the energy from the weight of paper clips equals the work done by friction. And these two values can be excluded from our equation.

$$W_f = mgy$$

$$\Delta U_w = mgy + Mgy$$

$$\Delta U_w + ½(M+m)v2 + ½I\omega 2 = 0$$

Mg is the Force of Gravity that accelerates the wheel.
Substituting $\omega = v/r$.

$$-Mgy + (0.5)Mv2 + (0.5)I\ (v/r)\ 2 = 0$$

Simplify for v^2.

$$v^2 = 2gy/(1+I/Mr^2)$$

Rearrange for Inertia.

$$I = Mr^2(2gy/v^2 - 1)$$

The slope of our experimental procedure, represented by the graph of "v^2 vs. y", is equal to y/v^2. We shall denote this slope as S.

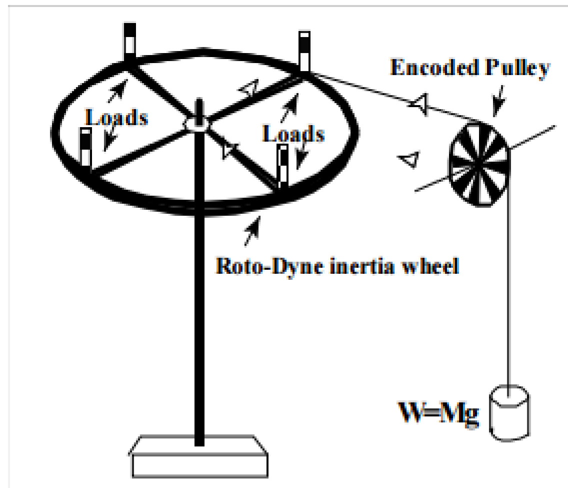
$$I = Mr^2(2g/S - 1)$$

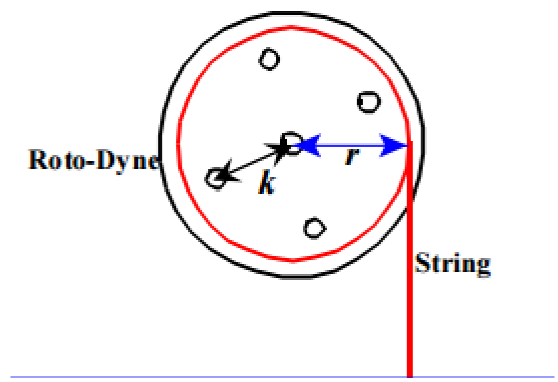
Now that we have derived Inertia of a spinning wheel, we can compute it using our known values. Moreover, this derivation can be confirmed through another calculation for Inertia.

$$I_p = \int r^2 \, dM$$

We can compare a value from the first equation for Inertia to the second equation which calculates Inertia by parts. These two equations are congruent and should produce a equivalent result.

Experimental Procedure





This section denotes the steps in which my group and I measured and recorded the data produced in this lab. The first step in producing our results involved the setup of the experiment. The conductor of the lab should collect all necessary items and construct the following: A frictionless Roto Dyne Inertial Wheel should be placed parallel to the floor, on its base, at some height above the floor. The pole the wheel lies on should also accommodate an attachment that can support an encoded pulley to take measurements. The wheel should have a mass attached to it by a string. This mass, of 60 g, should rest over the encoded pulley.

The first step of this lab pertained to taking measurements of masses and recording these values. Using an electronic balance, our group measured the mass of paperclips needed to counteract the frictional forces. We first hung the paperclips on the massless string and gave the wheel a slight push. When the paper clips moved at a constant speed, friction had been balanced. We proceeded to mass the amount of paper clips and record the value. Our setup needed 5 paperclips to balance the friction, approximately 0.002 ± 0.001 Kg. Next step included measurements with a ruler. The radius of the wheel measured 0.200 ± 0.005 m. The distance from the center of the wheel to the load holes measured 0.180 ± 0.005 m. Because these measurements proved difficult, I believe my judgment was accurate within this uncertainty.

Next, using the *Logger Pro* program, we recorded data from the encoded pulley. By dropping the mass with paperclips, the program recorded displacement, velocity, and acceleration of the system. These results were then transferred over to another program titled *Origin*. This new program required equations for velocity squared and the change in velocity squared. *Origin* graphed these results as "v^2 vs. y," or velocity squared versus vertical displacement. This graph can be found at the end of this report. The slope of this graph 1.297 ± 0.001 m/s^2. This slope results in an inertia of 0.034 Kgm^2 using our derived equation for inertia.

At this point, we again had to set up the lab. We massed four loads we attached to the top of the wheel. Each measured 0.225 ± 0.001 Kg. These loads were attached to the wheel. And again, *Logger Pro* and *Origin*, were used to take measurements for this setup. This graph of velocity squared versus displacement resulted in 0.702 ± 0.001 m/s^2. This slope results in an Inertia of 0.065 Kgm^2.

After obtaining these values, our group calculated the experimental value of inertia represented by $I_E = I_1 - I_2$, which results in 0.031Kgm^2. This value is compared to the value of inertia for the individual body of mass, which was 0.029 Kgm^2. These values are very close, so there difference can be attributed to errors. All of these values should be recorded in a journal of some form.

Results and Analysis

By graphing our results, we came upon the slopes for unweighted rotation and weighted rotation; these values are 1.297 ± 0.001 m/s^2 and 0.702 ± 0.001 m/s^2 respectively. The graphs are attached at the end of this report. By calculating the slope of these graphs, we were able to find Inertia for both cases. These values were 0.034 Kgm^2 for unweighted rotation (I_2) and 0.065 Kgm^2 for weighted rotation (I_1).

$$I_1 = Mr^2(2g/S_1 - 1)$$
$$I_1 = (.6)(.2^2)[(2*9.81)/.702 - 1]$$
$$I_1 = 0.034\ Kgm^2$$
$$I_2 = Mr^2(2g/S_2 - 1)$$
$$I_2 = (.6)(.2^2)[(2*9.81)/1.297 - 1]$$
$$I_2 = 0.065\ Kgm^2$$

These calculations led to the conclusion of I_E as 0.031 ± 0.015 Kgm^2, or more appropriately, 0.03 ± 0.02 Kgm^2. This value is determined by the following derivations.

$$I_E = I_1 - I_2 \quad \text{or} \quad I_E = 2Mgr^2[B_1^{-1} - B_2^{-1}]$$
$$I_E = .065 - .034 = .031\ Kgm^2$$
$$\delta I_E = \text{sqrt}[(\delta I_{E,r})^2 + (\delta I_{E,B1})^2 + (\delta I_{E,B2})^2]$$
$$\delta I_{E,r} = 4Mgr[B_1^{-1} - B_2^{-1}]*\ \delta r$$
$$\delta I_{E,r} = 4(.6)(9.81)(.2)[(0.702)^{-1} - (1.297)^{-1}] * 0.005 = 0.015$$
$$\delta I_{E,B1} = 2Mgr^2(-1/B_1^2) *\delta B_1$$
$$\delta I_{E,B1} = 2(.6)(9.81)(.2^2)(-0.702^{-1}) * 0.001 = -0.0007$$
$$\delta I_{E,B2} = 2Mgr^2(1/B_2^2) *\delta B_2$$
$$\delta I_{E,B2} = 2(.6)(9.81)(.2^2)(1.297^{-1}) * 0.001 = 0.0004$$
$$\delta I_E = \text{sqrt}((.015)^2 + (-.0007)^2 + (.004)^2) = 0.015\ Kgm^2$$

We compared these values to the value of the four point body masses calculated below. These equations include intertia (I), total mass of the loaded weights (m), the radius of the wheel (r), and distance to the load holes (k). The final result was 0.029 ± 0.001 Kgm^2.

$$I = mr^2$$
$$I = \int R^2 dm$$
$$I_p = mk^2$$
$$I_p = 4(.225)(0.18^2) = 0.029\ Kgm^2$$
$$\delta I_p = sqrt((\delta I_{p,m})^2 + (\delta I_{p,k})^2)$$
$$\delta I_{p,m} = k^2 * \delta m$$
$$\delta I_{p,m} = (0.18^2)*.001 = 0.00003$$
$$\delta I_{p,k} = 2mk * \delta k$$
$$\delta I_{p,k} = 2(.6)(.18)*.005 = 0.001$$
$$\delta I_p = sqrt((0.00003^2) + (0.001^2)) = .001\ Kgm^2$$

These calculations provided a comparison of these two values for inertia. These two values are very close and most likely marginally different in consequence of environmental and human error. The difference in the first inertia and the second inertia is approximately .002 Kgm^2, a value that could be from a combination of air resistance, hidden friction, and use of a non-ideal string.

Conclusion

The two values that were produced can be used in an analysis of conservation of momentum in our lab's system, dealing with rotational kinetic energy. Our produced inertia from the experiment was 0.03 Kgm^2 with an uncertainty of 0.02 Kgm^2. The inertia of the four loads resulted in an inertia of 0.029 Kgm^2 with an uncertainty of 0.001 Kgm^2. These values do not crossover from there uncertainties and therefore there is error in the experiment. They do not completely agree, but these values are extremely close. Based off theory, conservation of rotational kinetic energy should be conserved. From this, we also know conservation of momentum allowed us to derive equations of Inertia, whose value we calculated. Yet, because the two values did not agree within their uncertainties, it can be said there was sources of errors. These sources of errors may have been attributed to air resistance of additional unaccounted for friction which provide an opposing force and reduction in kinetic energy. Also, an uncertainty of 5 mm for measurements with the ruler may have been two small. The lab setup was at a strange height and proved difficult to place the ruler on.

Acknowledgements

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References

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