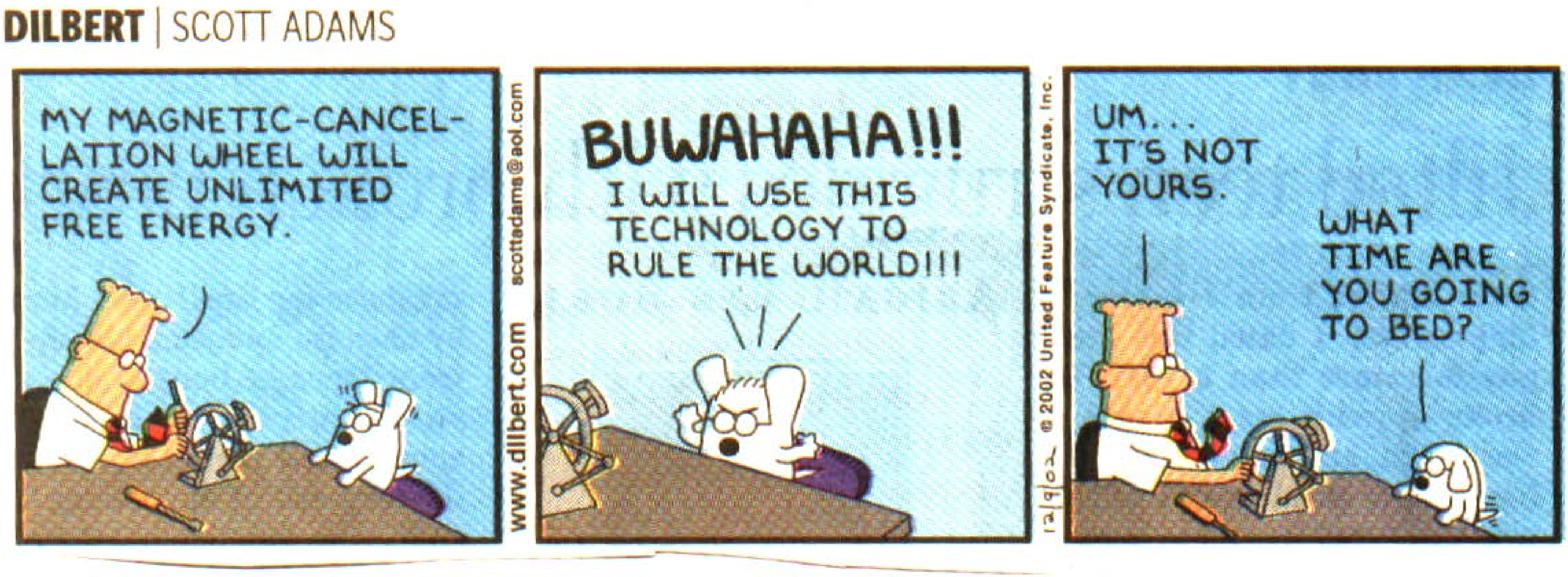
PHYS 121 – SPRING 2015



Chapter 7: Work & Energy

version 02/25/2015

~ 47 slides. We made it to slide #9 on Monday, February 23.

SERMON ON ENERGY

There are many ways to understand dynamics.

- We started with Newton’s Laws because:
  - historically, they came first
  - they are *relatively* easy for people to understand
- It’s possible, however, to use other concepts to explain the motion of objects.

SERMON ON ENERGY

WORK, ENERGY & MOMENTUM

are arguably better methods for understanding dynamics.

1. It's often simpler to use these concepts rather than $F = ma$.
2. It can be impractical or practically impossible to solve some problems with $F = ma$.
3. It's possible to derive energy & momentum methods directly from more fundamental concepts in physics (*symmetry*).

   and you can derive $F = ma$ from momentum.
4. Energy and momentum are useful in the realm of the small (*quantum mechanics*) and the fast (*special relativity*),

   where $F = ma$ doesn't work.
5. *$F = ma$ vs. work-energy-momentum* ~ algebra *vs.* calculus-based physics.

There's a learning curve but a BIG PAYOFF!

WORK

You can change the velocity of an object by letting a force act on it as it moves.

WORK, ***W***, is defined (*by physicists*), in a 1D system, as force, F, times displacement, $\Delta x = d$.

For a force of constant magnitude in a 1D universe:

$$W = F\Delta x = Fd$$

- Work can be positive (*F & d in the same direction*) or negative (*F & d in opposite directions*).
- Work has units of *force times distance* = N·m

  In the context of work, you should refer to this as **Joules, J**

$$1 \text{ N·m} = 1 \text{ kg-m}^2/\text{s}^2 \equiv \mathbf{1\ J}$$

It's easy to confuse Work with Weight ⇒ try to use Mg for weight.

N·m ≠ J will be used for torque.

WORK

$$W = F\Delta x$$

If a particle doesn't move while a force acts on it, $\Delta x = 0$

$\Rightarrow$ the force does no work on the object.

You do no work holding a heavy book at arm's length!

- This is counter-intuitive; it takes EFFORT to hold a heavy object,
- but you could be replaced by a table that, once in place, requires no *energy* to operate
- and you wouldn't say that the table is doing any work, would you,

  or that you have to supply energy so that the table will continue to hold the book off the floor?

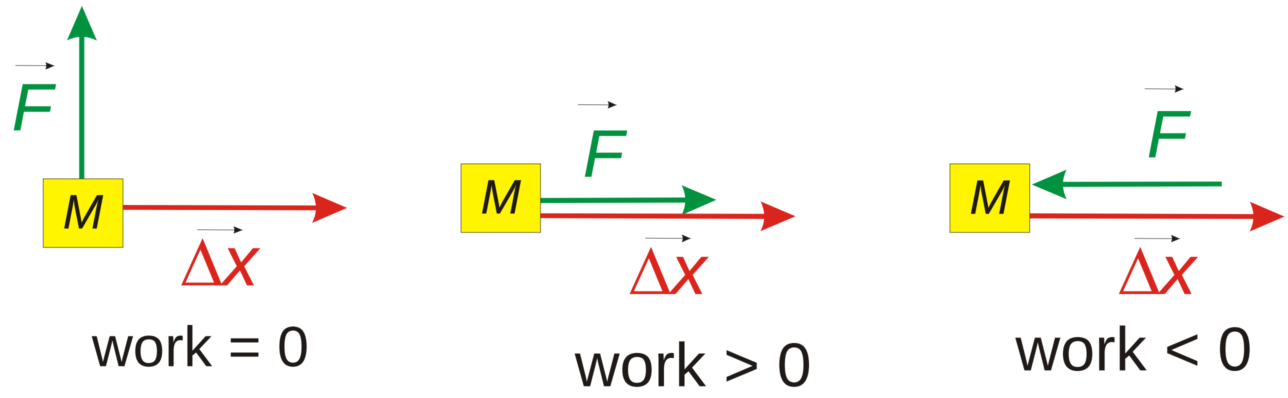
<u>WORK & DIRECTION</u>

$W = F\Delta x$

<u>DISPLACEMENT & FORCE ARE VECTORS.</u>

<u>WORK IS A SCALAR.</u>

- A force in some direction does <u>no</u> work on an object moving in a perpendicular direction.
- The <u>normal force</u> from a frictionless table does no work on a book as the book slides along the surface of the table.
- The only component of a force that has an effect on an object's motion is <u>the component of the force along the direction in which the object moves</u>.
- Work can have a positive or negative sign, if the direction of the force points in or opposite to the direction of motion.



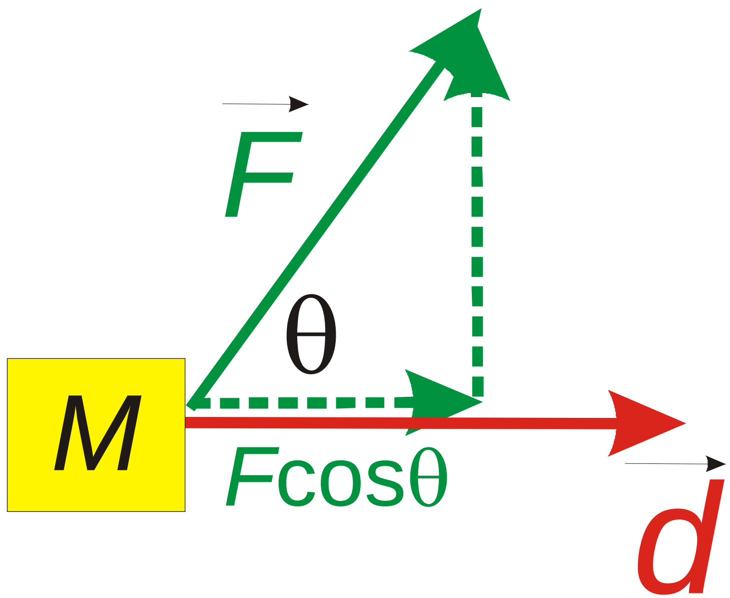
WORK & DIRECTION

The only component of a force that does work on an object is the component of the force that lies along the direction in which the object moves.

- This component is given by $F\cos\theta$

  where $\theta \equiv$ angle between $\boldsymbol{F}$ and $\boldsymbol{d}$, when you arrange these vectors tail-to-tail (*and take the smaller angle between them*)

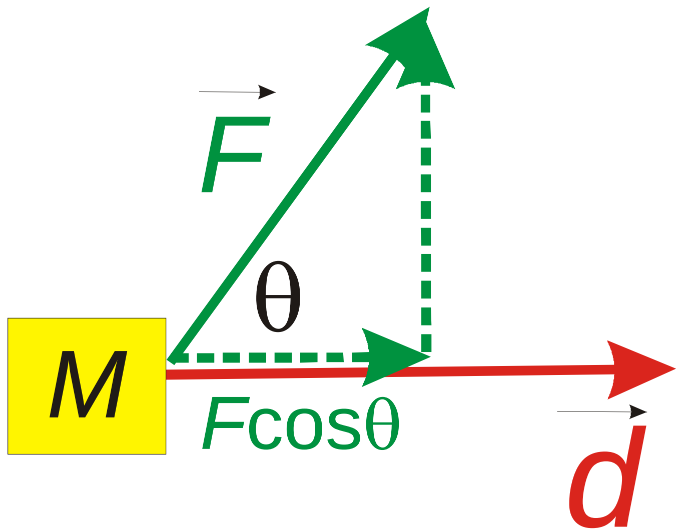
- $W = (F\cos\theta)d = Fd\cos\theta$

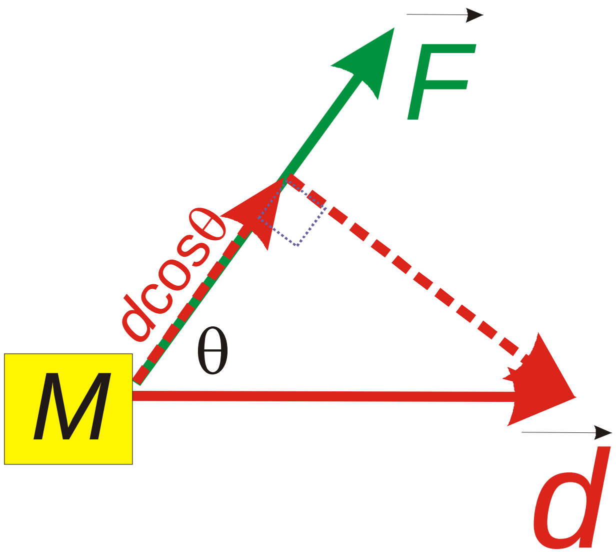


WORK & DIRECTION

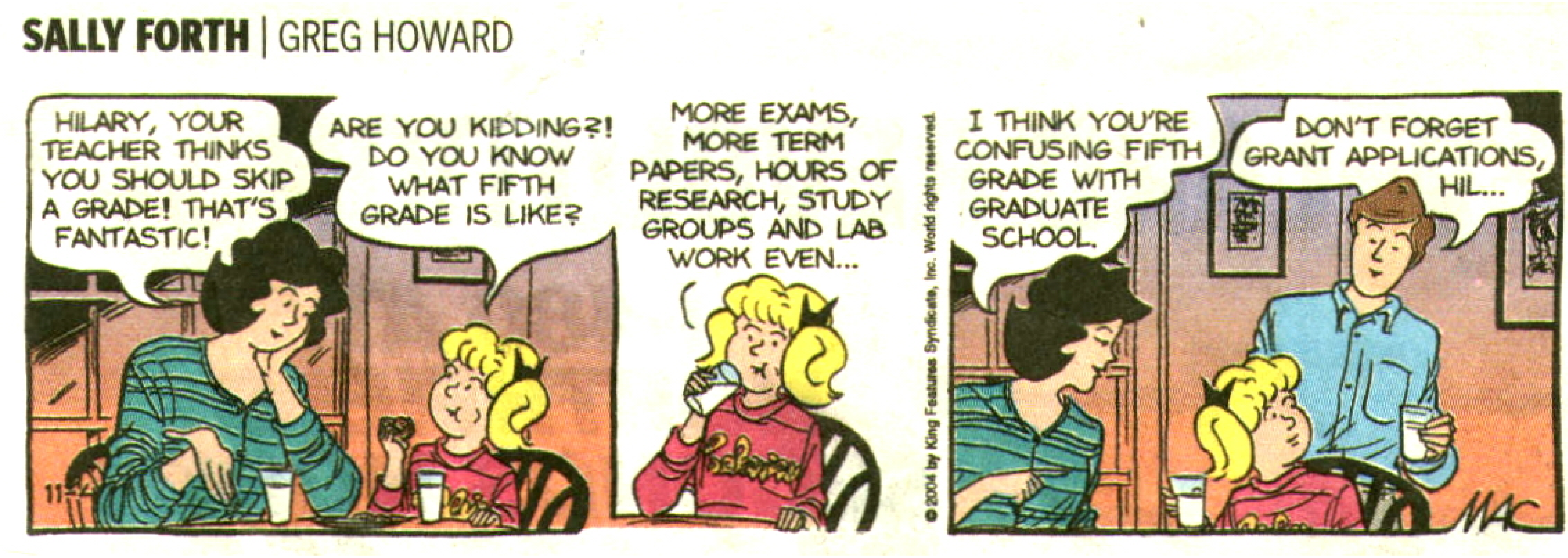
$Fd\cos\theta$ can also be interpreted as $F(d\cos\theta)$

- $d\cos\theta$ is the component of the displacement in the direction of the force.
- Both interpretations are equally valid.
- But sometimes one will be more convenient than the other.





PHYS 121 – SPRING 2015



Chapter 7: Work & Energy

version 02/25/2015

~ 47 slides. We made it to slide #9 on Monday, February 23.

Get your clickers ready.

ANNOUNCEMENTS

EvaluationKit course evaluation participation:

as of today, 11:00 AM =85/257 = 33%, far from 66%.

System closes Friday at 11:59 PM.

Blackboard Grade Columns:

H1 - H4 = raw scores, not %, for each assignment.

Htotal = total % homework score

Exam1 = Exam #1 %

BonusPts = raw #of bonus points

BP/Max = Bonus Points divided by largerst # earned by any student = 6.

This value will be added to your % grade at the end of the semester.

The maximum bonus is 1%.

Blackboard might display other, meaningless columns such as total course grades, although I've tried to turn those displays off.

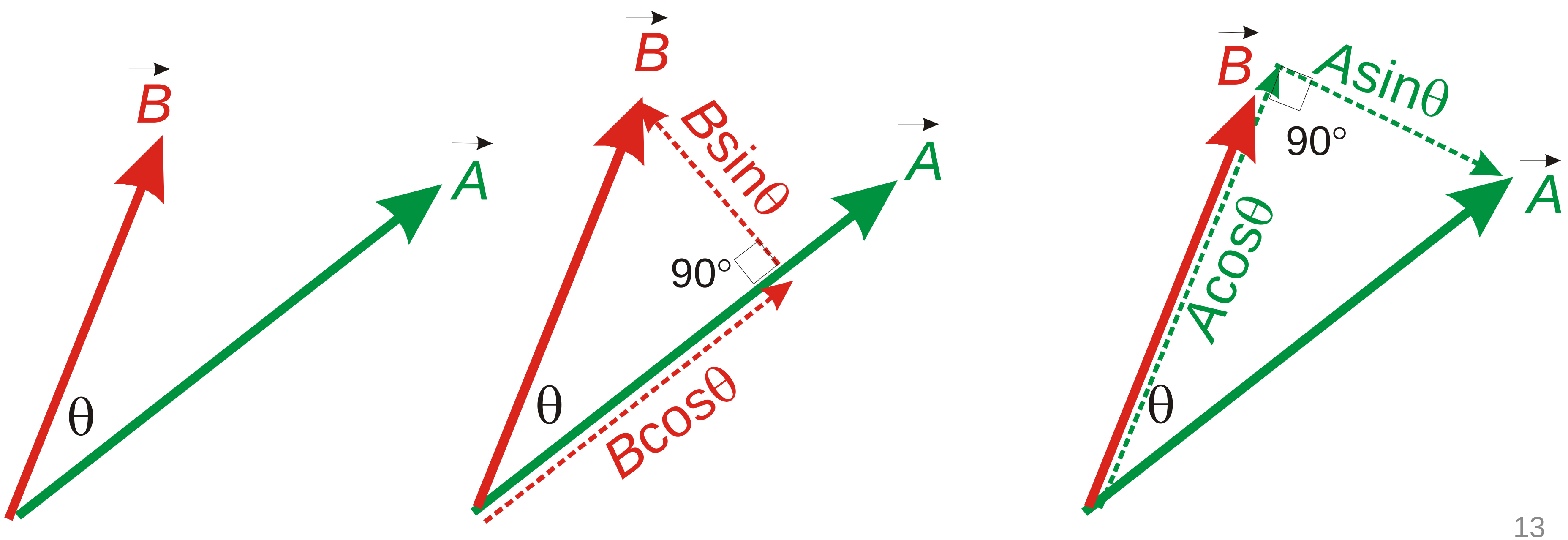
THE VECTOR DOT PRODUCT

Ohanian Chapter 3.4

This connection between the force & displacement vectors is described by a ***dot*** or ***scalar*** product.

The dot product between two vectors **A** and **B** ≡

$$\vec{A}\bullet\vec{B} \equiv AB\cos\theta_{AB} = A\left(B\cos\theta_{AB}\right) = B\left(A\cos\theta_{AB}\right)$$



DOT PRODUCT with COMPONENTS

instead of magnitude & direction

$$\vec{A}\bullet\vec{B}=|\mathrm{A}||\mathrm{B}|\cos\theta_{AB}$$

Applied to unit vectors $\Rightarrow \quad \hat{x}\bullet\hat{x}=1=\hat{y}\bullet\hat{y} \quad$ since $\cos 0^o = 1$

$$\hat{x}\bullet\hat{y}=0 \quad \text{since} \quad \cos 90^o = 0$$

In other words, x̂ has no component in the direction of ŷ

If $\quad \vec{A}=a_x\hat{x}+a_y\hat{y} \qquad \& \qquad \vec{B}=b_x\hat{x}+b_y\hat{y} \quad$ *then*

$$\vec{A}\bullet\vec{B}=\left(a_x\hat{x}+a_y\hat{y}\right)\bullet\left(b_x\hat{x}+b_y\hat{y}\right)$$

$$\vec{A}\bullet\vec{B}=a_x\hat{x}\bullet b_x\hat{x}+a_x\hat{x}\bullet b_y\hat{y}+a_y\hat{y}\bullet b_x\hat{x}+a_y\hat{y}\bullet b_y\hat{y}$$

$$\vec{A}\bullet\vec{B}=a_xb_x+a_yb_y$$

In 3D, $\quad \vec{A}\bullet\vec{B}=a_xb_x+a_yb_y+a_zb_z$

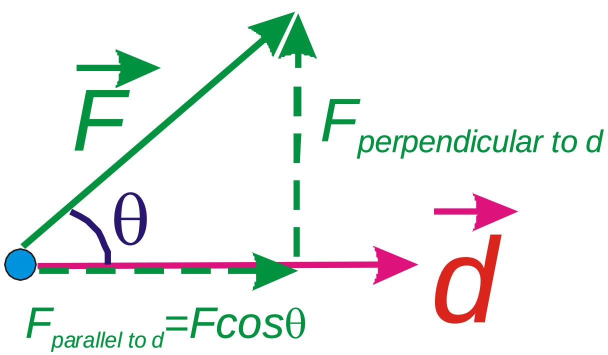
THE DOT PRODUCT & WORK

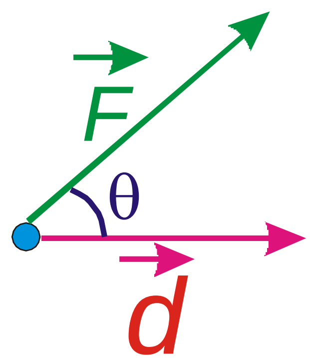
<u>Only the component of a force parallel to a path does</u> **<u>work.</u>**

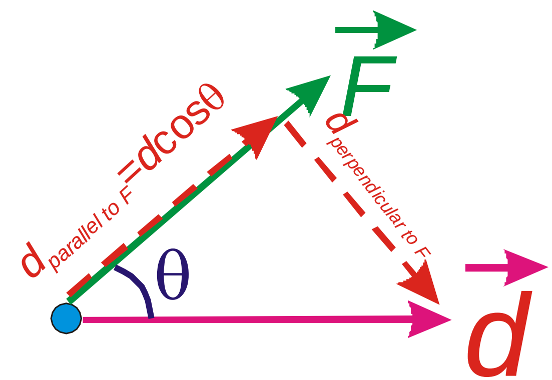
For a vector displacement ***d*** at some angle θ from a vector force ***F***

$$W = \vec{F} \bullet \vec{d} = Fd\cos\theta_{Fd}$$

$$= (F_{parallel})d \;\text{ or }\; F(d_{parallel})$$







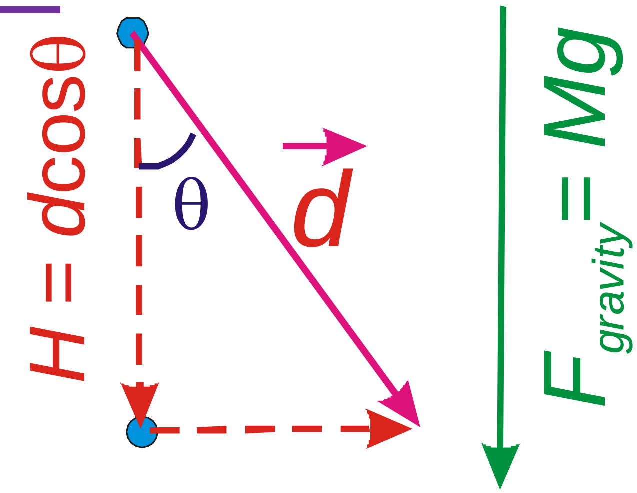
$$W \underset{\substack{\text{1-D}\\\text{constant F}}}{=} F\Delta x \underset{\substack{\text{higher D}\\\text{constant F}}}{\rightarrow} \vec{F} \bullet \vec{r} = Fr\cos\theta_{Fr}$$

θ_{Fr} is the angle between $\vec{F}$ & $\vec{r}$ when they are placed tail to tail.

$$\vec{F} \bullet \vec{r} = F_x r_x + F_y r_y$$

WORK DONE BY GRAVITY

$$W = \vec{F} \bullet \vec{d}$$



Gravity points straight down.

ONLY the component of displacement up or down leads to work.

$$H = d\cos\theta \Rightarrow \quad W_{gravity} = MgH$$

where H is the distance an object falls.

Forces obey the **PRINCIPLE OF SUPERPOSITION**

$\Rightarrow$ so does **work**.

$$W_{total} = \Sigma W_{each\ segment\ of\ a\ path} \text{ and/or } \Sigma W_{each\ force\ acting\ on\ an\ object}$$

EXAMPLES

You lift your textbook from the floor

and set it on a table of height *H*.

How much work do you do? (*assume up is positive*)

How much work does gravity do?

How much total work is done on the book?

The force you need to supply is infinitesimally more than the force of gravity pulling the book down, *mg*.

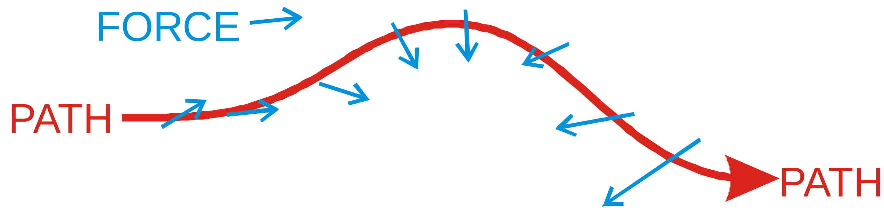
YOU: $W_{you} = F\Delta y = (+mg)\Delta y = mgH$

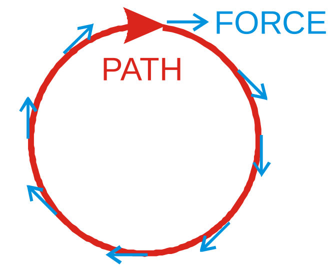
GRAVITY: $W_{gravity} = F\Delta y = (-mg)\Delta y = -\,mgH$

The total work done on the book is 0.

Notice that the velocity is 0 at the beginning & end.

VARIABLE FORCES





If a force is **NOT** constant in magnitude & direction

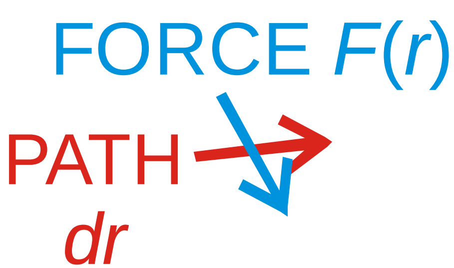
or the displacement is **NOT** a straight line, then we can't use

$$W \underset{\text{in 1D}}{=} F\Delta x \underset{\text{in 3D}}{=} \vec{F}\bullet\Delta\vec{r}$$

But we CAN say that this is true for an ***infinitesimal*** element of displacement dx or $\boldsymbol{dr}$.

The force might vary as a function of position, so it should be written as $F(x)$ or $\boldsymbol{F(r)}$.

The infinitesimal amount of work dW done over this infinitesimal displacement is given by



$$dW \underset{\text{in 1D}}{=} F_x\left(x\right)dx \underset{\text{in 3D}}{=} \vec{F}\left(\vec{r}\right)\bullet d\vec{r}$$

VARIABLE FORCES

$$dW \underset{\text{in 1D}}{=} F_x(x)\,dx \underset{\text{in 3D}}{=} \vec{F}(\vec{r})\bullet d\vec{r}$$

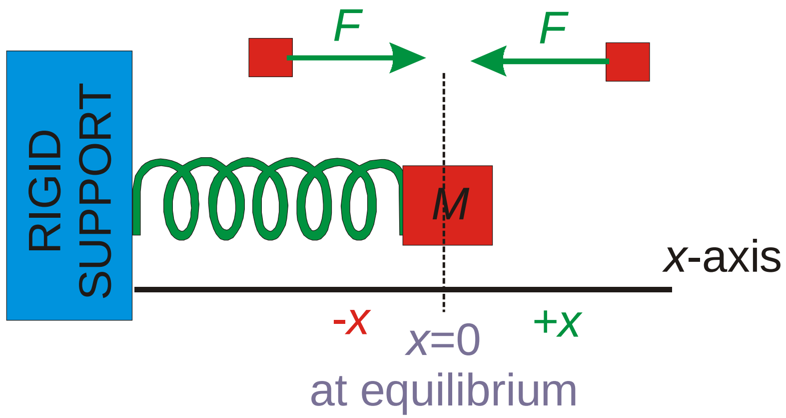
The TOTAL work done from the beginning to the end of a finite displacement is found by summing (*integrating*) the small infinitesimal elements *dW*. (*Thanks to the Principle of Superposition.*)

$$W = \sum_{\substack{original\\position}}^{\substack{final\\position}} dW \underset{1D}{=} \int_{x_o}^{x_f} F_x(x)\,dx \underset{3D}{=} \int_{\vec{r}_o}^{\vec{r}_f} \vec{F}(\vec{r})\bullet d\vec{r}$$

The 3D form might look intimidating but don't worry since you'll only encounter simple examples in PHYS 121.

SPRING FORCE

review



The canonical example of a 1D variable force is the spring.

An **ideal spring** pushes or pulls towards its equilibrium position with a (*non-constant*) force proportional to the displacement from equilibrium.

This force is described by **Hooke's Law**

$$F = -kx$$

- k is the spring constant, with units of force/distance or N/m

  $k \sim$ *strength* of the spring

- x is the displacement from equilibrium, $x_{equilibrium} \equiv 0$
- The negative sign $\Rightarrow$ the force pulls to the left if the spring is stretched to the right & *vice versa*.

SPRING FORCE WORK

$$W \underset{\text{in 1D}}{=} \int_{x_o}^{x_f} F(x)\,dx$$

The work done **by** an ideal spring, $F = -kx$, is

$$W = \int_{x_o}^{x_f} F(x)\,dx = \int_{x_o}^{x_f} (-kx)\,dx = -\frac{1}{2}kx^2\Big|_{x_o}^{x_f} = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_o^2$$

If we define $x_o = 0$ and let $x_f \equiv x$

then the work done BY THE SPRING in moving to a position x from $x_o = 0$ is

$$W = -\frac{1}{2}kx^2$$

The work done by a force which moves the spring is $+\frac{1}{2}kx^2$.

KINETIC ENERGY

KINETIC ENERGY (*K*) ≡ "*energy*" associated with the motion of an object.

$$K \equiv \tfrac{1}{2} m v^2$$

v *is a vector but* $\mathbf{v}^2$ *is a scalar*

$$\vec{v}^{\,2} = \vec{v} \bullet \vec{v} = vv\cos 0^o = |v|^2 = v^2$$

K is a scalar.

Only an object's speed matters, not its direction.

WORK & KINETIC ENERGY

Doing net (*non-zero*) **work** on a mass

$\Rightarrow$ a net force is acting on the mass

$\Rightarrow$ the mass is accelerating (*accelerations can have negative signs*)

$\Rightarrow$ the velocity of the mass is changing

$\Rightarrow$ the kinetic energy of the mass is changing

In our discussion of constant acceleration, we showed that

$$v^2 - v_o^2 = 2a\left(x - x_o\right) \quad \rightarrow \quad \Delta v^2 = 2a\Delta x$$

So, in 1D with a constant force $\Rightarrow$ constant acceleration:

$$W \equiv F\Delta x = [ma]\Delta x = m[a\Delta x] = m[\tfrac{1}{2}\Delta v^2] = \Delta\left(\tfrac{1}{2}mv^2\right)$$

Justifying our definition: **KINETIC ENERGY** $\equiv \tfrac{1}{2}mv^2$

Work done on a mass changes its kinetic energy.

Ohanian demonstrates that this is true even if *a* isn't constant.

The

WORK-ENERGY THEOREM

is one of the most important principles in classical mechanics.

$$W = \Delta K = K_{final} - K_{initial}$$

Dr. C. prefers to think of this as

$$K_{final} = K_{initial} + W$$

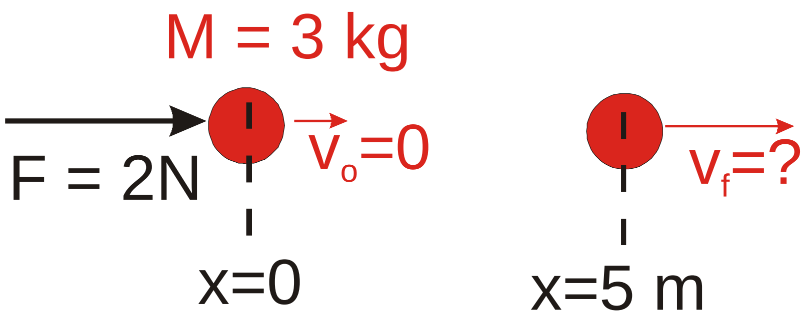
- Work done by an external force on a mass changes the kinetic energy of that mass.

  The external force could be gravity, friction, tension, *etc*.

- Internal forces (*holding an object together*) don't contribute.

  Internal forces must sum to zero, $\Sigma F_{internal} = 0 \Rightarrow \Sigma W_{internal} = 0$.

EXAMPLE



A force F = 2 N pushes a mass M = 3 kg towards the right, starting from rest.

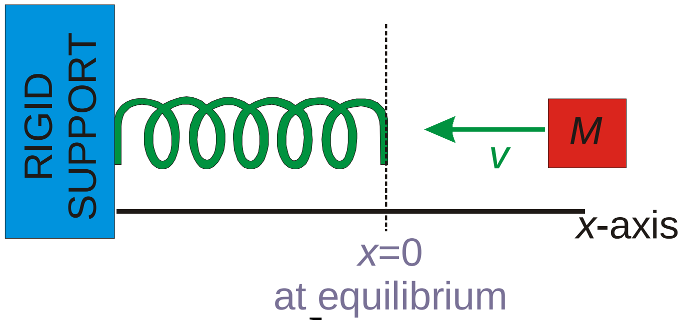
How fast is the mass moving after it travels a distance x = 5 m?

$$K_{final} = K_{initial} + W$$

$$\tfrac{1}{2}mv^2 = 0 + Fx$$

$$v = (2Fx/m)^{1/2} = [(2 \bullet 2\mathrm{N} \bullet 5\mathrm{m})/3\mathrm{kg})]^{1/2} = 2.58\ \mathrm{m/s}$$

SPRING EXAMPLE #1



A mass m is moving with a velocity v in the negative x-direction when it comes into contact with an ideal spring relaxed at its equilibrium position.

How far, L, will the spring compress before the mass is brought momentarily to rest?

The WORK-ENERGY THEOREM tells us that

$$K_{final} = K_{initial} + W_{spring}$$

$K_{final} = 0$ since the mass is brought to rest

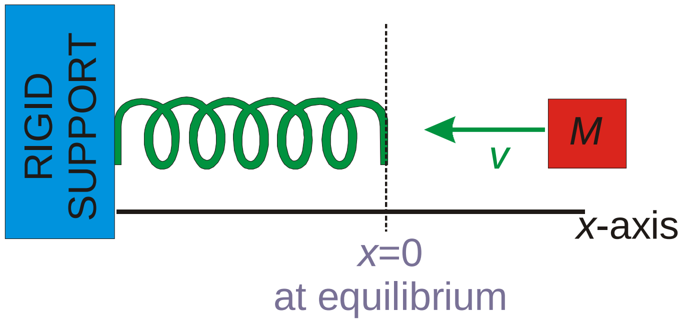
$$K_{initial} = ½mv^2$$

$$W_{spring} = -\ ½\ kL^2$$

negative because F_{spring} points to the right but displacement points to the left

$$0 = ½mv^2 - ½\ kL^2$$

SPRING EXAMPLE #1



$$0 = \tfrac{1}{2}mv^2 - \tfrac{1}{2}kL^2$$

$$L = \left(\sqrt{\frac{m}{k}}\right)v$$

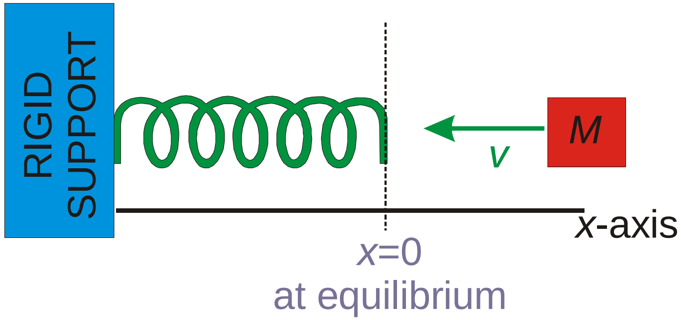
Note that equations relating v^2 and x^2 lose track of the signs of both.

You need to put these in explicitly if you want to say $x_{final} = -L$.

P.S. The units work out as shown below.

$$L = \left(\sqrt{\frac{m}{k}}\right)v \rightarrow \left(\sqrt{\frac{m}{F/x}}\right)v \rightarrow \left(\sqrt{\frac{m}{\left(\frac{m \bullet x}{t^2}\right)/x}}\right)\left(\frac{x}{t}\right) \rightarrow x$$

SPRING EXAMPLE #1



Would this problem have been easier or harder with $F = ma$?

You CANNOT use the equations

for <u>constant acceleration</u> with a spring!

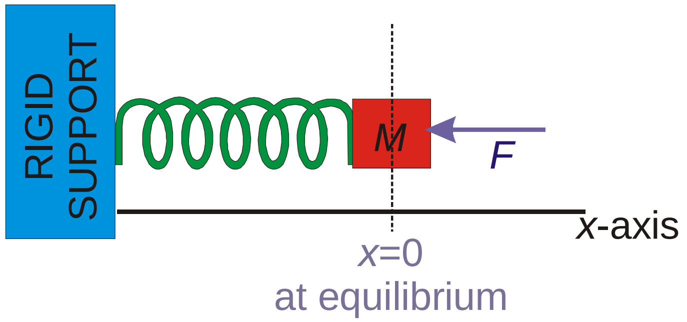
We can solve it with work-energy simply by writing

$$K_{final} = K_{initial} + W$$

$$0 = \tfrac{1}{2}mv^2 - \tfrac{1}{2}kL^2$$

$$L = \left(\sqrt{\frac{m}{k}}\right)v$$

SPRING EXAMPLE #2

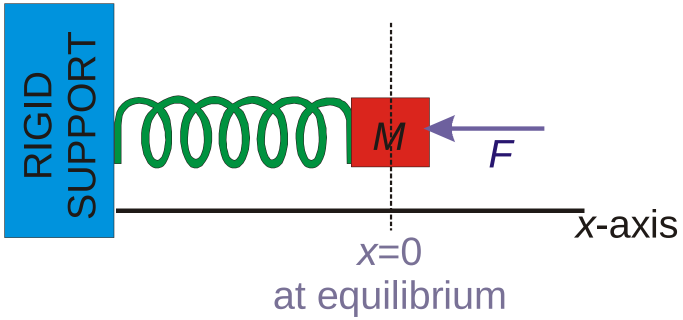


A mass *M* is at rest against an ideal spring at its equilibrium position when a constant external force *F* begins compressing the mass against the spring.

#1: How far, *L*, will the spring compress before the mass comes to rest?

#2: If the external force is suddenly removed, how fast will the mass be moving when the spring returns to its equilibrium position?

SPRING EXAMPLE #2



A mass M is at rest against an ideal spring at its equilibrium position when a constant external force F begins compressing the mass against the spring.

How far, L, will the spring compress before the mass comes to rest?

The WORK-ENERGY THEOREM tells us that

$$K_{final} = K_{initial} + W_{external\ forces}$$

$K_{initial} = K_{final} = 0$ since M starts & ends at rest.

$$W_{spring} = -\tfrac{1}{2} kL^2 \quad \text{while} \quad W_F = FL$$

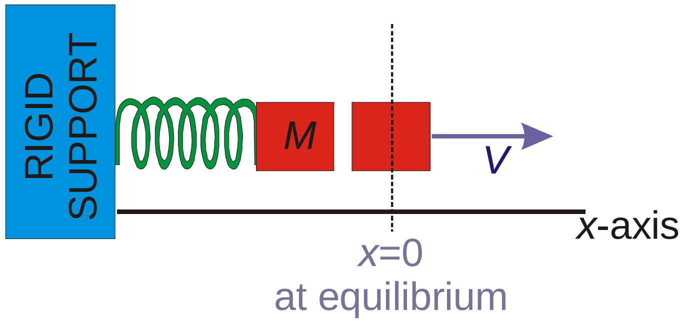
$$0 = 0 + (-\tfrac{1}{2} kL^2 + FL)$$

$$\tfrac{1}{2} kL = F \quad \rightarrow \quad \underline{\mathbf{L = 2F/k}}$$

Note that setting $F_{spring} = -F$ so $\Sigma F = 0$ gives $-kL = F$ or $\underline{L = F/k}$. Why?

M is moving when $\Sigma F = 0$. $F > F_{spring}$ until then $\Rightarrow$ K increases.

SPRING EXAMPLE #2



If the external force is suddenly removed, how fast will the mass be traveling when the spring returns to its equilibrium position?

The WORK-ENERGY THEOREM tells us that

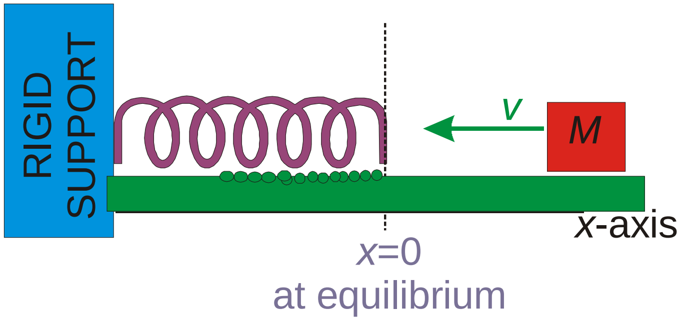
$$K_{final} = K_{initial} + W_{spring}$$

$$K_{initial} = 0 \qquad K_{final} = \tfrac{1}{2}Mv^2 \qquad W_{spring} = +\tfrac{1}{2}kL^2$$

$$\tfrac{1}{2}Mv^2 = 0 + \tfrac{1}{2}kL^2$$

$$v = \left(\sqrt{\frac{k}{M}}\right)L$$

SPRING EXAMPLE #3



A mass M moving at velocity v in the negative direction encounters a spring at its equilibrium position. The surface to the right of the eq. position is frictionless but the surface to the left has a coefficient of kinetic friction μ_k.
Where will the block come momentarily to rest?

The WORK-ENERGY THEOREM tells us that

$$K_{final} = K_{initial} + W_{spring} + W_{friction}$$

$$K_{initial} = \tfrac{1}{2}Mv^2 \qquad K_{final} = 0$$

$$W_{spring} = -\tfrac{1}{2} kL^2 \qquad W_{friction} = -F_{friction}L = -\mu_k NL = -\mu_k MgL$$

$$0 = \tfrac{1}{2}Mv^2 - \tfrac{1}{2} kL^2 - \mu_k MgL$$

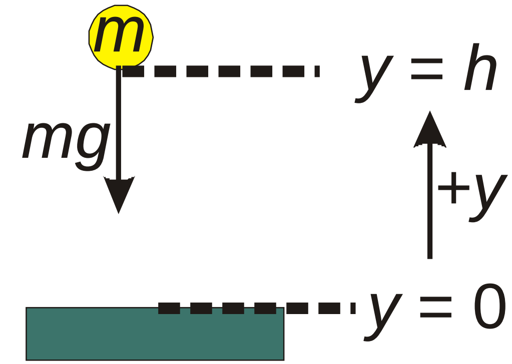
$$kL^2 + 2\mu_k MgL - Mv^2 = 0$$

$$L = \frac{-2\mu_k Mg \pm \sqrt{(2\mu_k Mg)^2 + 4kMv^2}}{2k}$$

The + sign gives the correct answer, a positive result for L which we assumed for $W_{friction} = -F_{friction}L$.

WORK DONE BY GRAVITY

again, but now with kinetic energy



Near the earth's surface, $F_{grav} = mg$ downward.

- If an object falls a distance h, the work done *by* gravity is

$$W_{gravity} = \vec{F} \bullet \vec{d} = F_{gravity} \Delta y = (-mg)(-h) = mgh$$

- Gravity always points down ⇒ only changes in **height** matter.
- Components of displacement parallel to the earth are irrelevant.
- Gravity & displacement both point in the same direction in this case, so their dot product is positive.
- The work-energy theorem tells us $K_{final} = K_{initial} + W_{gravity}$

$\frac{1}{2}mv^2 = mgh$ (*if it starts with* $v_o = 0$)

- Any object which falls a height h from rest ends up with
- Mass is irrelevant (*ignoring air resistance*)!

$$v = \sqrt{2gh}$$

$$v = \sqrt{2gh}$$

- You can get this result from one of our four equations for kinematics with constant acceleration,

$$v^2 - v_o^2 = 2a\left(x - x_o\right)$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

- but the work-energy theorem is easier to use

(eventually, if not yet for you)

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh}$$

GRAVITY & POTENTIAL ENERGY

Ohanian introduces *potential energy* for gravity in Chapter 7.4

BUT

Potential Energy & Conservation of Energy

are covered more carefully in Chapter 8

&

we'll leave these concepts for later.

EXAMPLES

A block of snow m = 1 kg falls a distance y = 15 m from the top of the Rockefeller Building before hitting an unsuspecting student (*you?*) on the head.

How fast is the snow traveling when it hits you?

Use: $K_{final} = K_{initial} + W$

with $y_o = 0$ & $v_o = 0$

$$\tfrac{1}{2}mv^2 = 0 + mgy$$

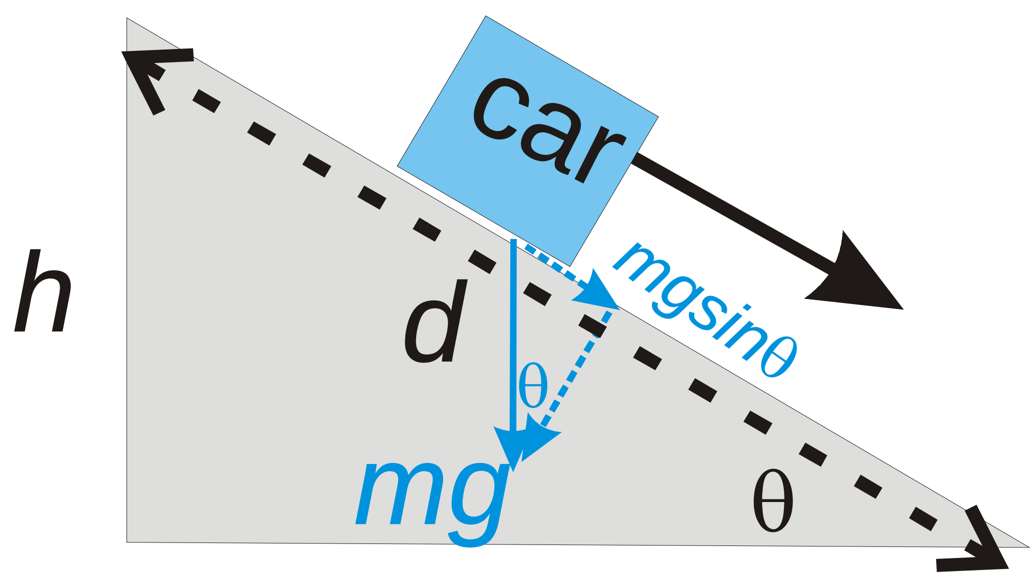
$$v_{final} = (2gy)^{1/2} = (2 \bullet 9.8 \bullet 15)^{1/2} = 17.1 \text{ m/s} = 38 \text{ mph}$$

4 options

A car travels <u>down</u> a hill a distance d measured along the hill. The incline of the road is θ above horizontal. The component of gravity parallel to the road is $mg\sin\theta$ while the vertical component of the car's displacement has a magnitude $|h| = d\sin\theta$.

Which of the following is <u>NOT</u> a correct expression for the work done <u>ON</u> the car <u>BY</u> gravity during the descent?

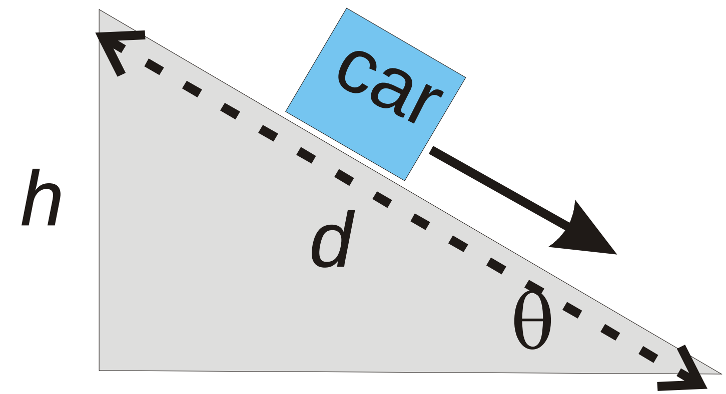
1) $+mgd$
2) $+mg(d\sin\theta)$
3) $+mgh$
4) $+(mg\sin\theta)d$



4 options

A car travels <u>down</u> a hill a distance d measured along the hill. The incline of the road is θ above horizontal. The component of gravity parallel to the road is $mg\sin\theta$ while the vertical component of the car's displacement has a magnitude $|h| = d\sin\theta$.

Which of the following is <u>NOT</u> a correct expression for the work done <u>ON</u> the car <u>BY</u> gravity during the descent?



1) $+mgd$

2) $+mg(d\sin\theta)$

3) $+mgh$

4) $+(mg\sin\theta)d$

Expressions 2 – 4 all say the same thing since $h = d\sin\theta$.

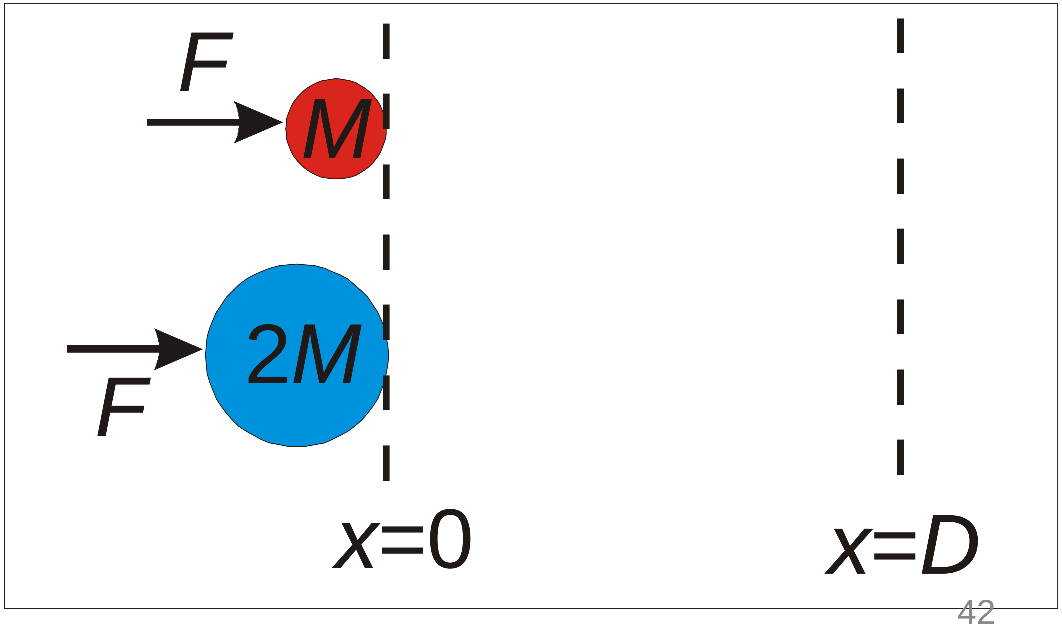
#1 is wrong since it's the displacement PARALLEL to gravity that is associated with work done by gravity. A component of displacement perpendicular to gravity is irrelevant.

- 4 options

Two discs sit at rest on a frictionless horizontal surface. One disc has mass M while the other disc has mass $2M$. A constant force F is applied to each disc until they cross the finish line a distance D from where they started.

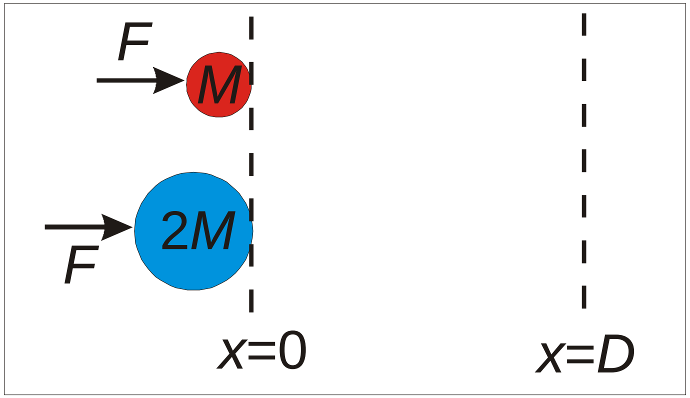
Which puck has the greater kinetic energy when it crosses the finish line (*which need not happen at the same time for both pucks*)?

1) M
2) $2M$
3) K is the same for both.
4) It's impossible to tell.



- 4 options

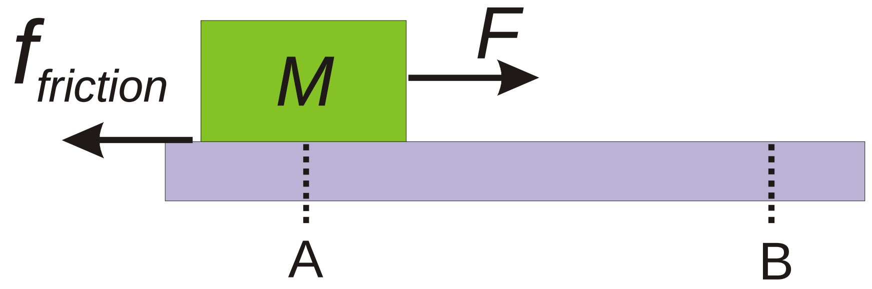
Two discs sit at rest on a frictionless horizontal surface. One disc has mass M while the other disc has mass $2M$. A constant force F is applied to each disc until they cross the finish line a distance D from where they started. Which puck has the greater <u>kinetic energy</u> when it crosses the finish line (*which need not happen at the same time for both pucks*)?



1) M
2) $2M$
3) ***K* is the same for both.**
4) It’s impossible to tell.

- Since the same force is applied to both over the same distance, the same work is done to both and the Work-Energy Theorem tells us that both will gain the same amount of kinetic energy.
- They won’t cross the finish line at the same time however; the heavier mass will have a smaller velocity in response to the force. Its kinetic energy is the same because its larger mass compensates for its smaller velocity. Note that velocity enters as a square in $K = ½\, mv^2$.

- 4 options

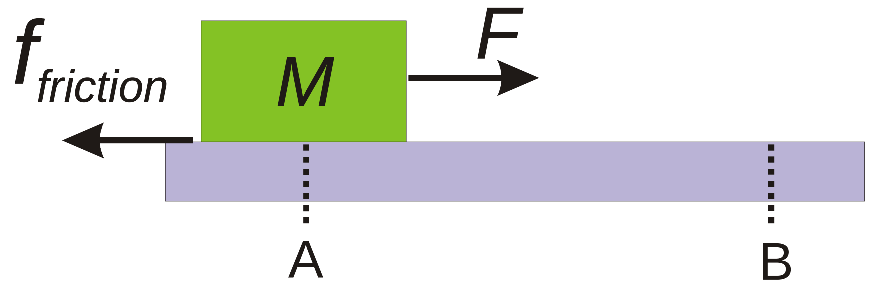


You use a rope to pull a box along a horizontal surface with a constant force *F*. Because of the force of friction, the box moves at constant velocity from position A to position B.

Which one of the following statements concerning the motion of the box from A to B is true?

1) The work done on the box by $f_{friction}$ is positive.
2) The total work done on the box by F and $f_{friction}$ is positive.
3) The magnitude of the work done on the box by F is equal to the magnitude of the work done by $f_{friction}$.
4) The magnitude of F is greater than the magnitude of $f_{friction}$.

- 4 options



You use a rope to pull a box along a horizontal surface with a constant force *F*. Because of the force of friction, the box moves at constant velocity from position A to position B.

Which one of the following statements concerning the motion of the box from A to B is true?

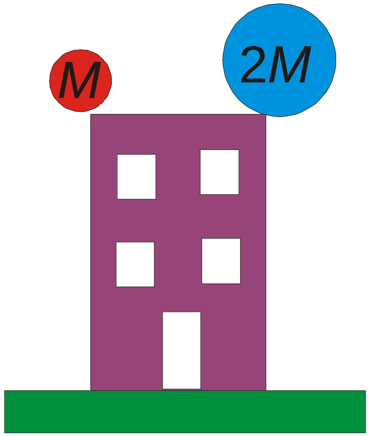
1) The work done on the box by $f_{friction}$ is positive.
2) The total work done on the box by *F* and $f_{friction}$ is positive.
3) **The magnitude of the work done on the box by *F* is equal to the magnitude of the work done by $f_{friction}$.**
4) The magnitude of *F* is greater than the magnitude of $f_{friction}$.

The box moves at constant velocity which means that there's no change in kinetic energy and therefore no net work. The two forces must therefore do equal but opposite amounts of work.

What would change if the box sped up or slowed down from A to B?

If it speeds up, there's a gain of kinetic energy and a net positive work, so the work done by *F* must be greater than the work done by *f*. The reverse is true if the box slows down.

- 4 options



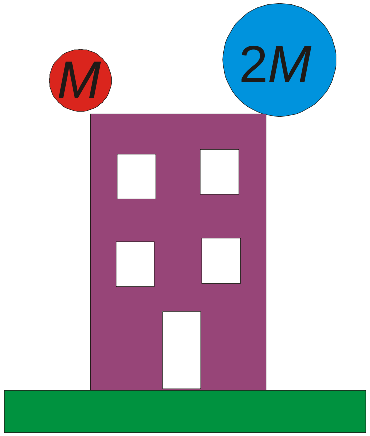
Two balls are dropped from the roof of a building.

One ball has twice the mass of the other.

Ignoring air resistance, so the balls hit the ground at the same time with the same speed, how do the kinetic energies of the two balls compare when the balls reach the ground?

1) The lighter one has one fourth as much kinetic energy.
2) The lighter one has one half as much kinetic energy.
3) The lighter one has the same kinetic energy.
4) The lighter one has twice as much kinetic energy.
5) The lighter one has four times as much kinetic energy.

- 4 options



Two balls are dropped from the roof of a building.

One ball has twice the mass of the other.

Ignoring air resistance, so the balls hit the ground at the same time with the same speed, how do the kinetic energies of the two balls compare when the balls reach the ground?

1) The lighter one has one fourth as much kinetic energy.

2) The lighter one has one half as much kinetic energy.

3) The lighter one has the same kinetic energy.

4) The lighter one has twice as much kinetic energy.

5) The lighter one has four times as much kinetic energy.

$$K_{final} = 0 + W_{gravity} = (\underline{mg})h$$

The balls will have the same VELOCITIES when they hit the ground, since the m's on both sides of $mgh = \frac{1}{2}mv^2$ cancel,

but the more massive ball has twice the kinetic energy.

THE END!

http://us.123rf.com/400wm/400/400/feverpitched/feverpitched0803/feverpitched080300154/2685279-the-end-road-sign-with-dramatic-blue-sky-and-clouds.jpg