EECS 281, January 15, 2015

$$\overset{3\ 2\ 1\ 0}{1\ 0\ 1\ 1}_2 = 1\times 8 + 0\times 4 + 1\times 2 + 1\times 1 = 11_{10}$$

Octal numbers: Radix : 8.

Digits: 0 - 7

Hexadecimal numbers: Radix : 16

$2^3 = 8$

0: 000_2

1: 001

2: 010

3: 011

4: 100

5: 101

6: 110

7: 111

Hexadecimal numbers: 0 - 9, A - F

A = 10, F = 15

Binary	Decimal	Octal	3-Bit String	Hexadecimal	4-Bit String
0	0	0	000	0	0000
1	1	1	001	1	0001
10	2	2	010	2	0010
11	3	3	011	3	0011
100	4	4	100	4	0100
101	5	5	101	5	0101
110	6	6	110	6	0110
111	7	7	111	7	0111
1000	8	10	—	8	1000
1001	9	11	—	9	1001
1010	10	12	—	A	1010
1011	11	13	—	B	1011
1100	12	14	—	C	1100
1101	13	15	—	D	1101
1110	14	16	—	E	1110
1111	15	17	—	F	1111

Table 2-1
Binary, decimal, octal, and hexadecimal numbers.

$100\,11\,00\,110_2 = ?_8$

4 3 1 6

100

8

$000\,1\,1\,0\,1\,1\,0\,1\,1\,0\,1\,0\,1\,001_2 = ?_{16}$

1 D B A 9

1101₂

13₁₀

?

16

1011

1010

$76_8 = ?_2$

↓

$00\,111\,110_2$

↓

$3E_{16}$

$010\,10\,11\,00\,10\,11\,00_2 = ?_8$

$2\,5\,4\,5\,4_8$

$BEAD_{16} = 1011\ 1110\ 1010\ 1101_2$

$B_{16} = 11 \quad 1011$

Byte: 8-bit.

4 bit hexadecimal digit is called a nibble.

0x : denotes a hexadecimal number.

Value of a number:

$$D = \sum_{i=-n}^{p-1} d_i \cdot r^i$$

Example: $1CE8_{16} = ?_{10}$

$$\begin{array}{cccc} 16^3 & 16^2 & 16^1 & 16^0 \\ 1 & C & E & 8 \\ & \{ & \{ & \\ & (12) & (14) & \end{array} =$$

$$1 \times 16^3 + 12 \times 16^2 + 14 \times 16 + 8 \times 1 = 7400_{10}$$

$$D = \left(\cdots \left(\left(d_{p-1} \times r + d_{p-2} \right) \times r + d_{p-3} \right) r + \cdots + d_1 \right) r + d_0$$

$$\begin{array}{cccc} 16^3 & 16^2 & 16^1 & 16^0 \\ F & 1 & A & C \\ \{ & & \{ & \{ 16 \\ 15 & & 10 & 12 \end{array} = \left(\left(\left(15 \times 16 \right) + 1 \right) \times 16 + 10 \right) \times 16 + 12$$

To convert decimal to radix r:

$ar + d_0$

$$D = \underbrace{\left(\cdots \left(d_{p-1} \times r + d_{p-2} \right) r + \cdots \right) r + d_1}_{} \Big) r + d_0$$

Divide by r:

remainder: d_0

quotient $q = \left(\cdots \left(d_{p-1} \times r + d_{p-2} \right) r + \cdots \right) r + d_1$

$\vdots$

successive divisions by r.

$179_{10} = ?_2$

LSB

$179 \div 2 = 89$ remainder 1

$89 \div 2 = 44$ remainder 1

$44 \div 2 = 22$ remainder 0

$22 \div 2 = 11$ remainder 0

$11 \div 2 = 5$ remainder 1

$5 \div 2 = 2$ " 1

$2 \div 2 = 1$ " 0

$1 \div 2 = 0$ " 1

MSB

$10\ 11\ 00\ 11_2$

$467_{10} = ?_8$ $\quad 723_8$

$467 \div 8 = 58$ remainder 3 (LSDigit)

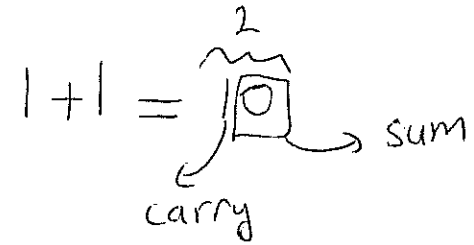
$58 \div 8 = 7$ " 2

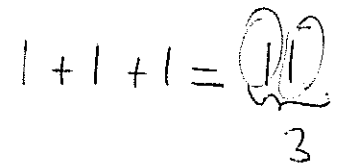
$7 \div 8 = 0$ " 7 (MSDigit)

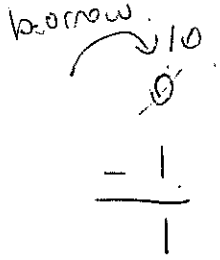
Conversion	Method	Example
Binary to		
Octal	Substitution	$1011101100l_2 = 10\ 111\ 011\ 001_2 = 2731_8$
Hexadecimal	Substitution	$1011101100l_2 = 101\ 1101\ 1001_2 = 5D9_{16}$
Decimal	Summation	$1011101100l_2 = 1\cdot 1024 + 0\cdot 512 + 1\cdot 256 + 1\cdot 128 + 1\cdot 64 + 0\cdot 32 + 1\cdot 16 + 1\cdot 8 + 0\cdot 4 + 0\cdot 2 + 1\cdot 1 = 1497_{10}$
Octal to		
Binary	Substitution	$1234_8 = 001\ 010\ 011\ 100_2$
Hexadecimal	Substitution	$1234_8 = 001\ 010\ 011\ 100_2 = 0010\ 1001\ 1100_2 = 29C_{16}$
Decimal	Summation	$1234_8 = 1\cdot 512 + 2\cdot 64 + 3\cdot 8 + 4\cdot 1 = 668_{10}$
Hexadecimal to		
Binary	Substitution	$CODE_{16} = 1100\ 0000\ 1101\ 1110_2$
Octal	Substitution	$CODE_{16} = 1100\ 0000\ 1101\ 1110_2 = 1\ 100\ 000\ 011\ 011\ 110_2 = 140336_8$
Decimal	Summation	$CODE_{16} = 12\cdot 4096 + 0\cdot 256 + 13\cdot 16 + 14\cdot 1 = 49374_{10}$
Decimal to		
Binary	Division	$108_{10} \div 2 = 54$ remainder 0 (LSB) $\div 2 = 27$ remainder 0 $\div 2 = 13$ remainder 1 $\div 2 = 6$ remainder 1 $\div 2 = 3$ remainder 0 $\div 2 = 1$ remainder 1 $\div 2 = 0$ remainder 1 (MSB) $108_{10} = 1101100_2$
Octal	Division	$108_{10} \div 8 = 13$ remainder 4 (least significant digit) $\div 8 = 1$ remainder 5 $\div 8 = 0$ remainder 1 (most significant digit) $108_{10} = 154_8$
Hexadecimal	Division	$108_{10} \div 16 = 6$ remainder 12 (least significant digit) $\div 16 = 0$ remainder 6 (most significant digit) $108_{10} = 6C_{16}$

Table 2-2

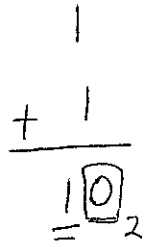
Conversion methods for common radices.







c_{in} **or** b_{in}	x	y	c_{out}	s	b_{out}	d
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	1	0	1
0	1	1	1	0	0	0
1	0	0	0	1	1	1
1	0	1	1	0	1	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1



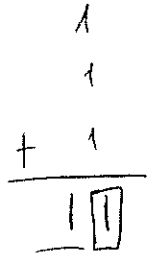


Table 2-3

Binary addition and subtraction table.

(1)

$$\begin{array}{r} 12 \\ +\ 9 \\ \hline 21 \end{array} \qquad \begin{array}{r} {}^{1}\cancel{2}^{11}1 \\ -\ 9 \\ \hline 12 \end{array}$$

Binary addition:

$0 + 0 = 0$

$0 + 1 = 1 + 0 = 1$

$1 + 1 = 10$ (carry 1, sum 0)

$0 + 1 + 1 = 10$ (c, s)

$1 + 1 + 1 = 11$ (c, s)

$a_3\ a_2\ a_1\ a_0$

$b_3\ b_2\ b_1\ b_0$

3 one bit:

$2^3 = 8$

c_{in}	x	y	c_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

c_{in},
x,
$+ \; y$,
0

0
1
+ 1
10

1
1
+ 1
11

```
  190        1 1 1 1
 +141      1 0 1 1 1 1 1 0
 ----    + 1 0 0 0 1 1 0 1
  331    ---------------
         1 0 1 0 0 0 1 1
```

```
  229        10 1 1 10
 - 46      0 X X 0 X 0 1
 ----    - 0 0 1 0 1 1 0
  183    ---------------
         1 0 1 1 0 1 1 1
```

19
+ 2
1

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 1\ 9\ B\ 9_{16} \\ +\ C\ 7\ E\ 6_{16} \\ \hline E\ 1\ 9\ F_{16} \end{array}$$

B 11
E 14
25 = 9+16

25
1 9

Example: $10110_2 = ?_{10}$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$1 \times 16 + 1 \times 4 + 2 \times 1 = 22_{10}$

Example: $EA_{16} = ?_{10}$

16^1

$14 \times 16 + 10 = 234$

$2ED_{16} = ?_8$

$2 = 0010$
$E = 1110$
$D = 1101$

$0010\ 1110\ 1101_2 = 1355_8$

Example:

$$\begin{array}{r} 1\ 7\ 7\ 6_{16} \\ +\ 1\ 4\ 3\ 2_{16} \\ \hline 2\ B\ A\ 8_{16} \end{array}$$