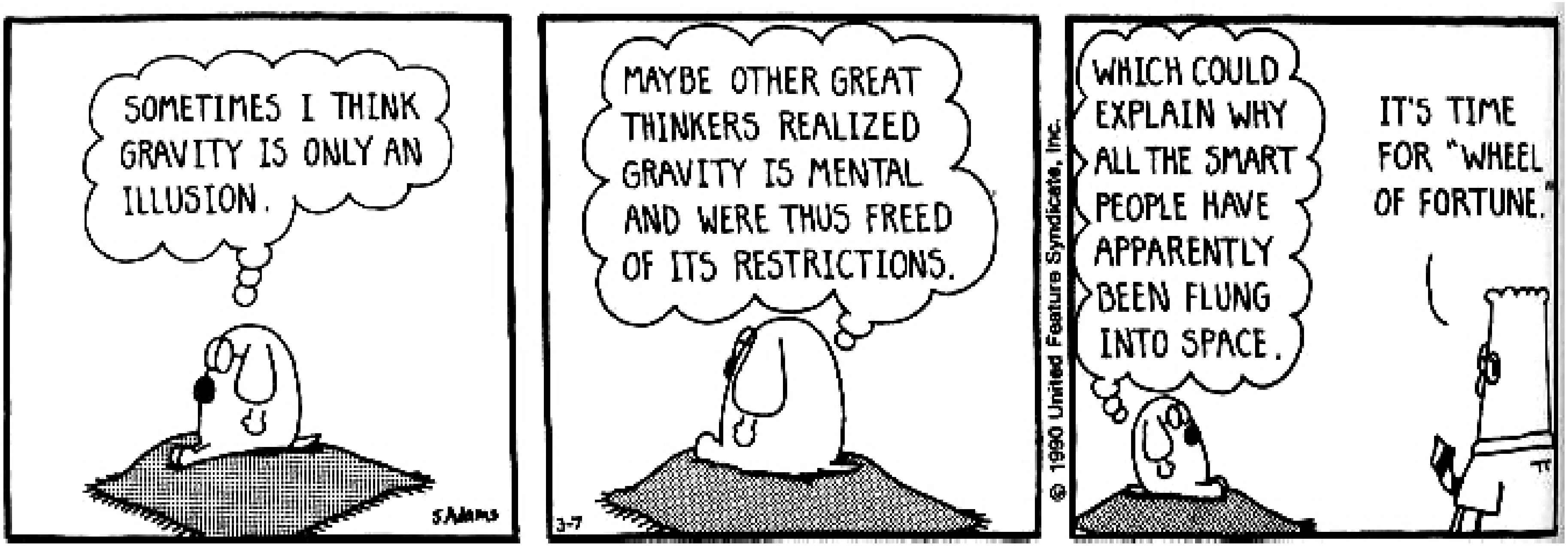
PHYS 121 – SPRING 2015



Chapter 9: Gravitation

version 04/13/2015

~ 66 slides total

no clickers today

ANNOUNCEMENTS

- Course Evaluations open on Wednesday & close 2 weeks later.
  - 1 Bonus Point for everyone if participation reaches 75%
  - 1 trivial problem on the final exam if participation reaches 85%
- The **PHYS 121 Grade Estimator** has been posted on Blackboard under General Information. You can use this file to estimate the scores you need on Exam #3 and the Final Exam to earn the grade you desire.
- Our final exam is scheduled for **Monday, May 4 from 4 – 7 PM**.

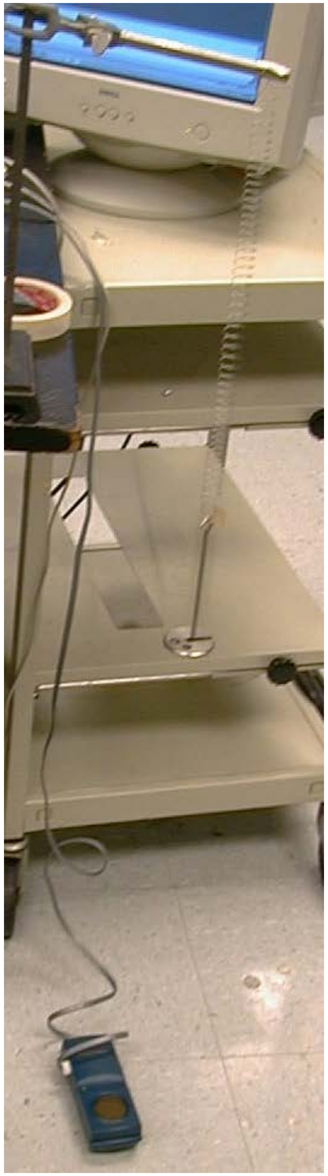
  The Registrar had the wrong date listed at the beginning of the semester but has since corrected it in the Searchable Schedule of Classes. The correct date is listed on Blackboard under General Information and in the syllabus posted there.

  Make sure you know the correct date and time or there will be dire consequences!
- Wednesday's class will be dedicated to review for Exam #3.

  You can see a sample formula sheet by looking at last year's Exam #3 on Blackboard, but note that exam covered slightly more material.

LAB

- April 1 – 9 Simple Harmonic Motion
- April 15 – 23 Waves
- April 20 – 24 **LECTURES** on these topics



NEWTON'S APPLE TREE

A descendant of Newton's famous apple tree is growing on the north side of the Rockefeller Building.

"The Case tree is a 7th generation descendant of the one legendarily used by Newton in stating his theory of gravity."

http://blog.case.edu/case-news/2006/04/26/look_out_below_case_western_reserve_university_to_dedicate_planting_of_sir_isaac_newton_apple_tree_descendant_on_campus

The tree had apples last fall!

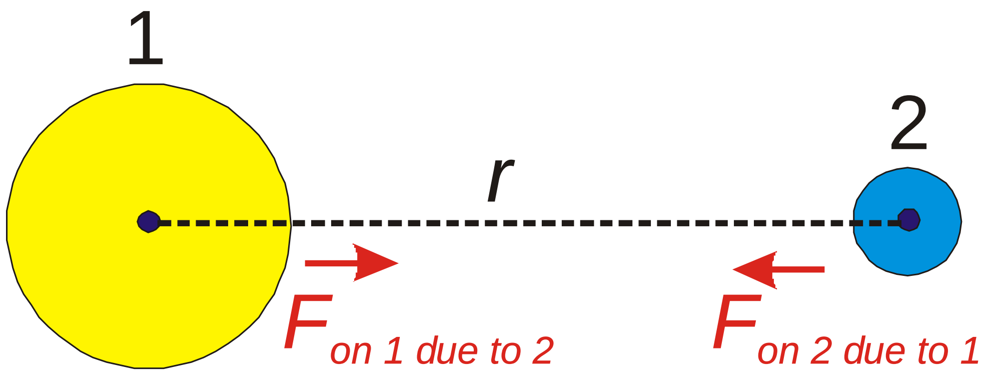
Most fell downwards,

towards the center of the earth.

NEWTON'S LAW OF GRAVITATION

Newton's Law of Universal Gravitation describes the gravitational attraction between a pair of objects of mass m_1 and m_2 as

$$\left|F_{gravity}\right| = \frac{Gm_1m_2}{r^2}$$

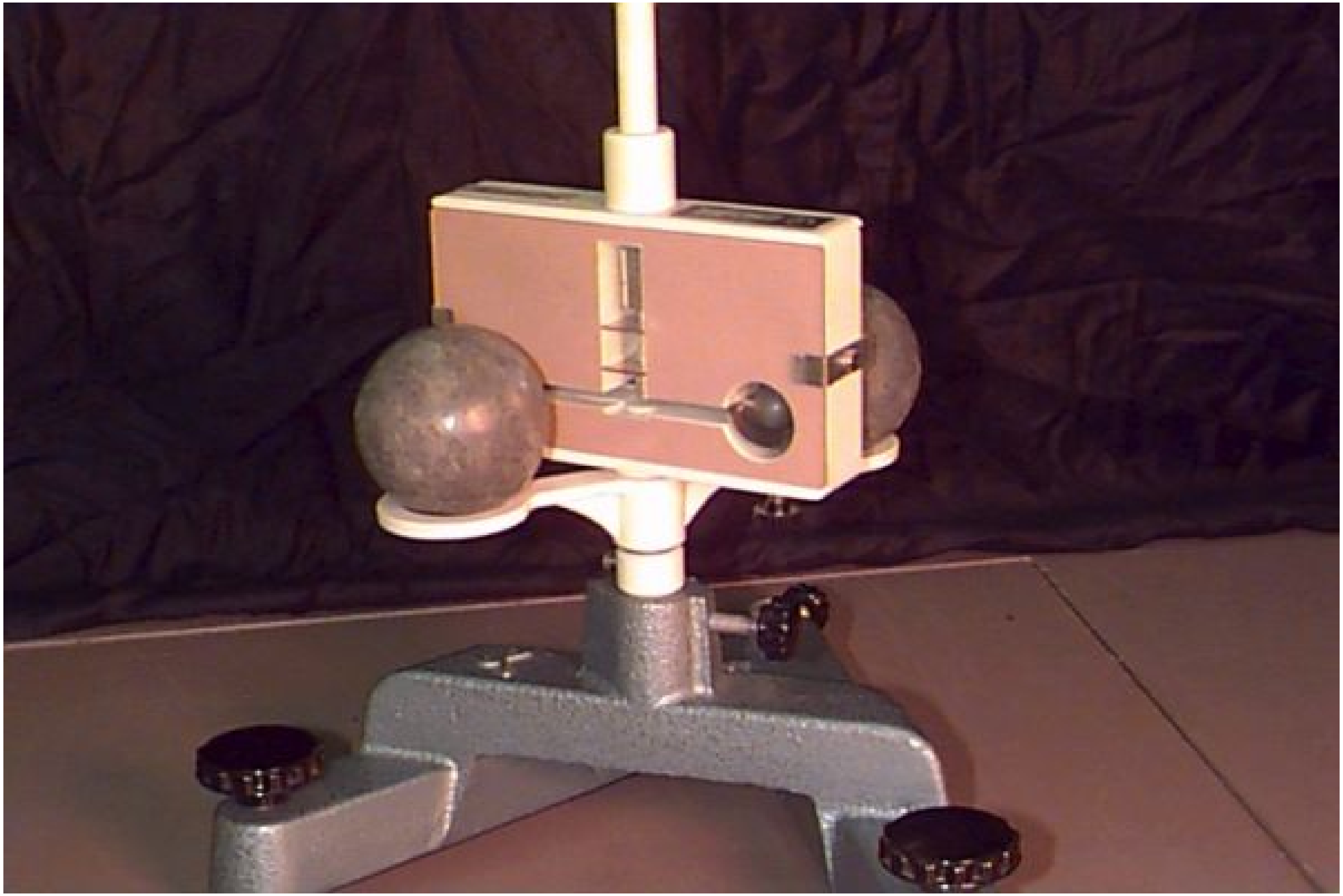


$$\vec{F}_{\text{on 2 due to1}} = -\frac{Gm_1m_2}{r^2}\hat{r}_{\text{from 1 to 2}}$$

- $G \equiv$ Universal Gravitational Constant = $6.67\text{x}10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
- r is the distance between the (*centers of the*) two objects.
- The direction of the force is attractive along a line joining them.
- Ohanian uses $m_1 \rightarrow m$ and $m_2 \rightarrow M$ ~ small & large masses.

Cavendish ‘Torsion Balance” like one in Rockefeller Building’s “Junior Lab”.

Students measure G with it!



‘LARGE-SCALE’ GRAVITY

Away from the earth’s surface, the acceleration due to gravity is no longer approximately constant.

$\Rightarrow$ **DON’T USE *g* = 9.81 m/s²** unless you know all the action is taking place near the earth’s surface!

g is found from Newton’s Law of Gravitation as

$$F_{gravity} = \frac{GmM}{r^2} \xrightarrow[M=M_{earth}]{} a_m = g_{surface\ of\ earth} = \frac{F_{gravity}}{m} = \frac{GM_{earth}}{R_{earth}^2} = 9.81\ \mathrm{m/s^2}$$

g varies ~ 0.7% due to:

- altitude above sea level
- effects of earth’s non-spherical shape
- non-uniform density
- how the rotation of the earth modifies apparent weight.

These effects are easily measurable but we won’t worry about them because we’re concerned about much larger effects (*distances*) now.

GRAVITY for SPHERES

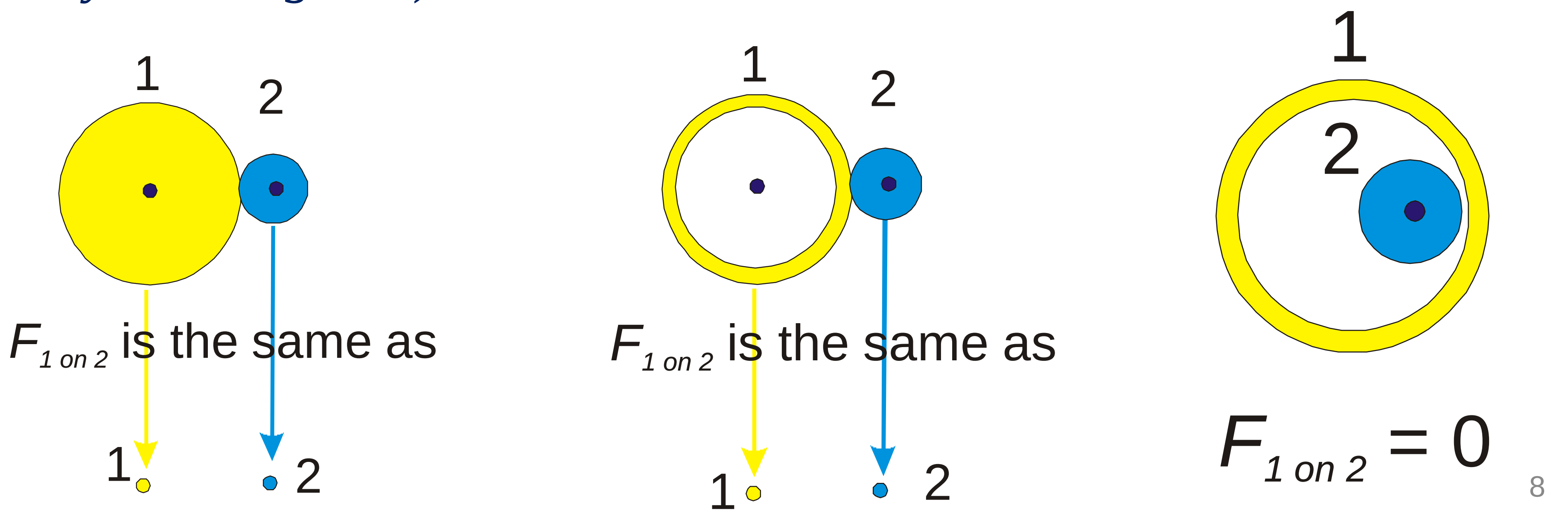
The effect of a spherical mass on an object ***outside*** that sphere is equivalent to a point source of the same mass at the center of the sphere.

The same principle applies to a spherical shell.

But the force of gravity = 0 ***INSIDE*** a spherical shell.

This is a general property of $1/r^2$ forces & applies to electrical forces too.

The proof is slightly advanced for PHYS 121 but a "*hand-waving*" justification is possible. You'll see a real proof (*with surface integrals*, *flux and maybe divergence*) in PHYS 122!

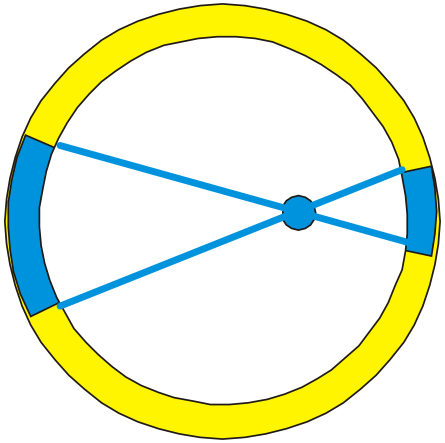


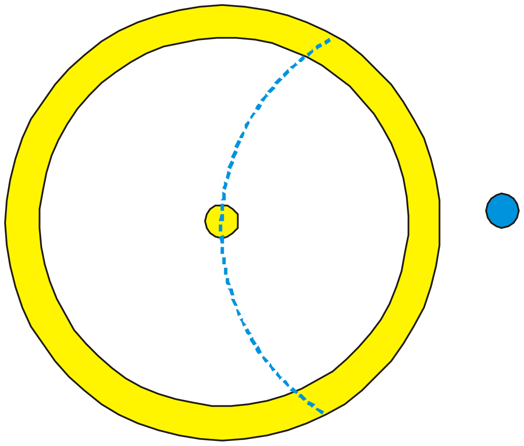
‘LARGE-SCALE’ GRAVITY

“*HAND-WAVING*” JUSTIFICATION

A point mass inside a spherical shell feels a gravitational force of attraction from sectors of the sphere on opposite sides.

- These forces cancel because the element on the far side is weaker by $1/r^2$ but its mass is bigger by a compensating r^2 factor.
- A similar argument explains why $F_{gravity}$ outside a sphere equals the force from a point with the same mass.

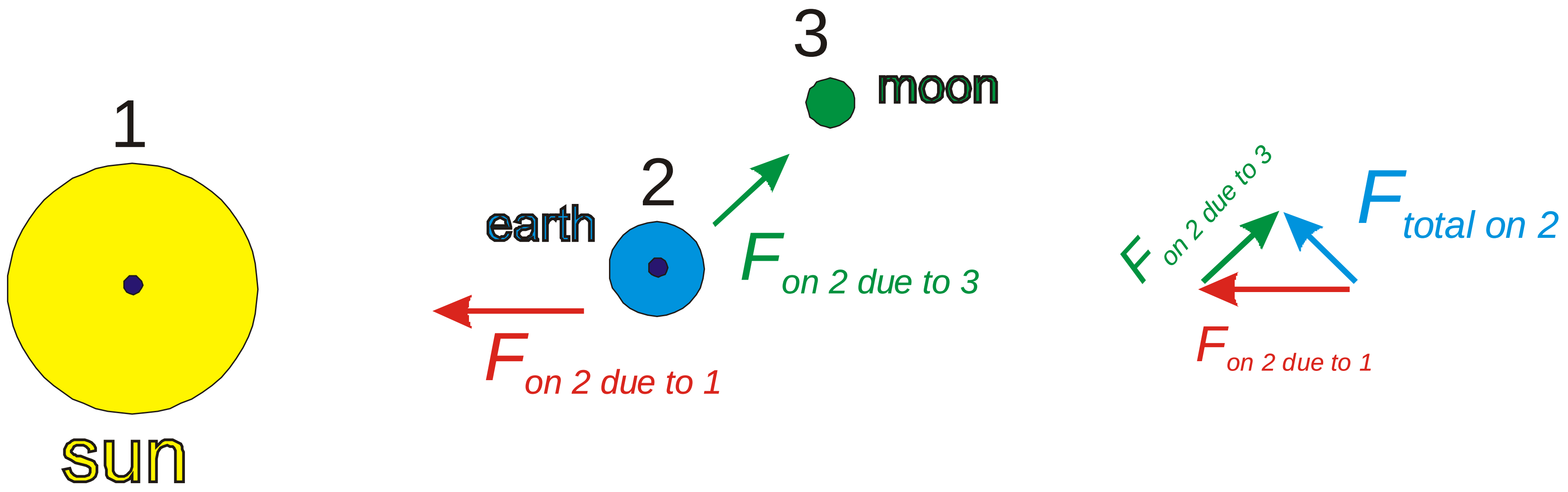




GRAVITY AS A VECTOR

Gravity obeys the **Principle of Superposition**

Gravitational force vectors add.



$$\vec{F}_{\text{total on 2}} = \vec{F}_{\text{1 on 2}} + \vec{F}_{\text{3 on 2}}$$

CULTURAL INTERLUDE I

$$\vec{F}_2 = -\frac{Gm_1m_2}{r^2}\hat{r}_{\text{from 1 to 2}}$$

IMPORTANT CHARACTERISTICS

- $F \propto 1/r^2$

  If it was $1/r^{2.0000000000000000000000000001}$ our universe would not exist!

  $F \bullet 4\pi r^2$: integral of F over a sphere centered on M is independent of r.

- F_{gravity} is weaker than the other 'fundamental' forces (*electromagnetism & nuclear*) by ~ 30 orders of magnitude.
- A toddler exerts a force larger than the gravitational pull of the entire earth when she or he stands up.
- Gravity seems strong in part because all mass is 'positive', unlike charges in Coulomb's Law which appear in both positive and negative forms which cancel each other's effects.
- The effects of gravity are not infinitely fast and require a field-carrying particle, the graviton. It's similar to the photon for electromagnetic fields (*light*).

CULTURAL INTERLUDE II

$$\vec{F}_2 = -\frac{Gm_1m_2}{r^2}\hat{r}_{\text{from 1 to 2}}$$

Coulomb's Law for electrostatics is

$$\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$$

where k is a constant and q_i is an electric charge.

It's important to understand Newton's Law of Gravitation even if you plan to stay on the surface of the earth because it helps you understand Coulomb's Law, which is critical to understanding matter, chemistry & biology.

CULTURAL INTERLUDE III

$$\vec{F}_2 = -\frac{Gm_1m_2}{r^2}\hat{r}_{\text{from 1 to 2}}$$

- Where does this equation comes from?

  Newton dreamt it up and proved it explained planetary orbits.

- Today, we could argue that orbits are explained by Einstein's Theory of General Relativity which says that mass 'warps' space and the effect of this warping is identical to an acceleration due to gravity. To learn more, take PHYS 365 your senior year.
- A fundamental understanding of gravity has eluded physicists to this day and is one of the holy grails of the profession.

Gravity's properties might be related to alternate universes; gravity leaking off of our *membrane*.

CULTURAL INTERLUDE IV

- There are major efforts (*2.5 mile long apparatus*) to detect gravitational waves and gravitons - http://en.wikipedia.org/wiki/LIGO .
- Prof. Ruhl of our Department of Physics is investigating indirect evidence of gravity waves in the *Cosmological Microwave Background Radiation* from the *Big Bang* that permeates our universe. He uses a telescope located at the South Pole. See http://cmb.phys.cwru.edu/ruhl_lab/

KEPLER'S LAW'S – *a history lesson*

- **FIRST LAW:** All planets move in elliptical orbits, with the sun at one focus.
- **SECOND LAW:** The line connecting the planet to the sun sweeps out equal areas in equal times.
- **THIRD LAW:** The square of the period, T of any planet (its "year!") is proportional to the cube of the semi-major axis, a, of the elliptical orbit: $T^2 \propto a^3$.

KEPLER'S LAWS – TRANSLATED

briefly – explanations are expanded later

- *FIRST LAW: All planets move in elliptical orbits with the sun at one focus.*

  Solve for the equation of motion $\boldsymbol{r}(t)$ given $\vec{F} = m\vec{a} = m\dfrac{d^2\vec{r}}{dt^2} = -\dfrac{GMm}{r^2}\hat{r}$

- *SECOND LAW: The line connecting the planet to the Sun sweeps out equal areas in equal times.*

  Angular momentum is conserved.

- *THIRD LAW: The square of the period, T of any planet (its "year") is proportional to the cube of the semi-major axis, a, of the elliptical orbit: $T^2 \propto a^3$.*

  $F = ma$ works for the orbits found above, just solve for the period T.

CIRCULAR ORBITS – *a special (easy) case*

$$\vec{a} = \frac{\vec{F}}{m} = -\frac{GM}{r^2}\hat{r}$$

If r is a constant, then;

- r = radius of the orbit $\equiv R$
- The *magnitude* of the acceleration a is a constant = GM/R^2.
- The *direction* of the acceleration always points towards the center of the circle.

This is a prescription for Uniform Circular Motion.

Given $|a| = GM/R^2$ we can solve for ω, T, v_{tan}, *etc.*

CIRCULAR ORBITS

$$\vec{a} = \frac{\vec{F}}{m} = \frac{GM}{r^2}\hat{r}$$

Given $|a| = GM/R^2$ we can solve for v_{tan} & T.

$$\vec{a} = \frac{GM}{r^2}\hat{r} \quad \Rightarrow \quad \frac{v^2}{r} = \frac{GM}{r^2} \quad \Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow \text{ period } T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$$

Justifying Kepler's 3rd Law: $T^2 \propto a^3$

CENTRAL FORCES

Newton's Law of Universal Gravitation $\vec{F}_2 = -\frac{Gm_1m_2}{r^2}\hat{r}_{\text{from 1 to 2}}$

is an example of a ***central*** force.

A central force points towards one particular point in space.

The force between two charges q_1 and q_2 is also a central force.

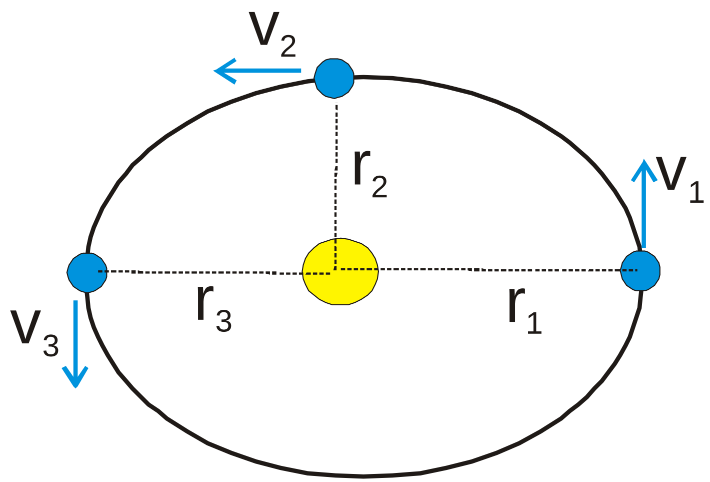
Coulomb's Law, which you will encounter in PHYS 122, says

$$\vec{F} = -\frac{kq_1q_2}{r^2}\hat{r}$$

CONSERVATION OF ANGULAR MOMENTUM IN ORBIT

What is the angular momentum (*with respect to the sun*) of the blue planet of mass m when it is in the given positions in its (*elliptical*) orbit?

$$\vec{L} = \vec{r} \times \vec{p}$$



$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

$L = mv_1 r_1 = mv_{2\perp} r_2 = mv_3 r_3$ = CONSTANT

Gravity is a *central force*, always pointing opposite to ***r***,

so it can't supply a torque and ***L* IS CONSERVED**.

Planets speed up as they approach the sun.

You can use this to prove Kepler's 2nd Law: *The line connecting the planet to the Sun sweeps out equal areas in equal times.*

CENTRAL FORCES *continued*

The orbits due to a central force like

$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

must be two dimensional, confined to a plane.

Why?

2 straight lines define a plane.

For an orbiting object, it's initial velocity defines one line and the **r** vector in the force law defines another line, Δv.

Also, ***L*** is conserved; its vector direction defines a plane.

ORBITAL EQUATIONS OF MOTION

Calculating $\vec{r}(t)$ from $\dfrac{d^2\vec{r}}{dt^2} = -\dfrac{GM}{r^2}\hat{r}$ is POSSIBLE if you know the *initial conditions*, **r**(0) & **v**(0)

BUT

A. easier if you understand polar/spherical coordinates & differential eqs.

B. mathematically more advanced than expected for students in PHYS 121.

C. not necessary to understand important aspects of the solutions.

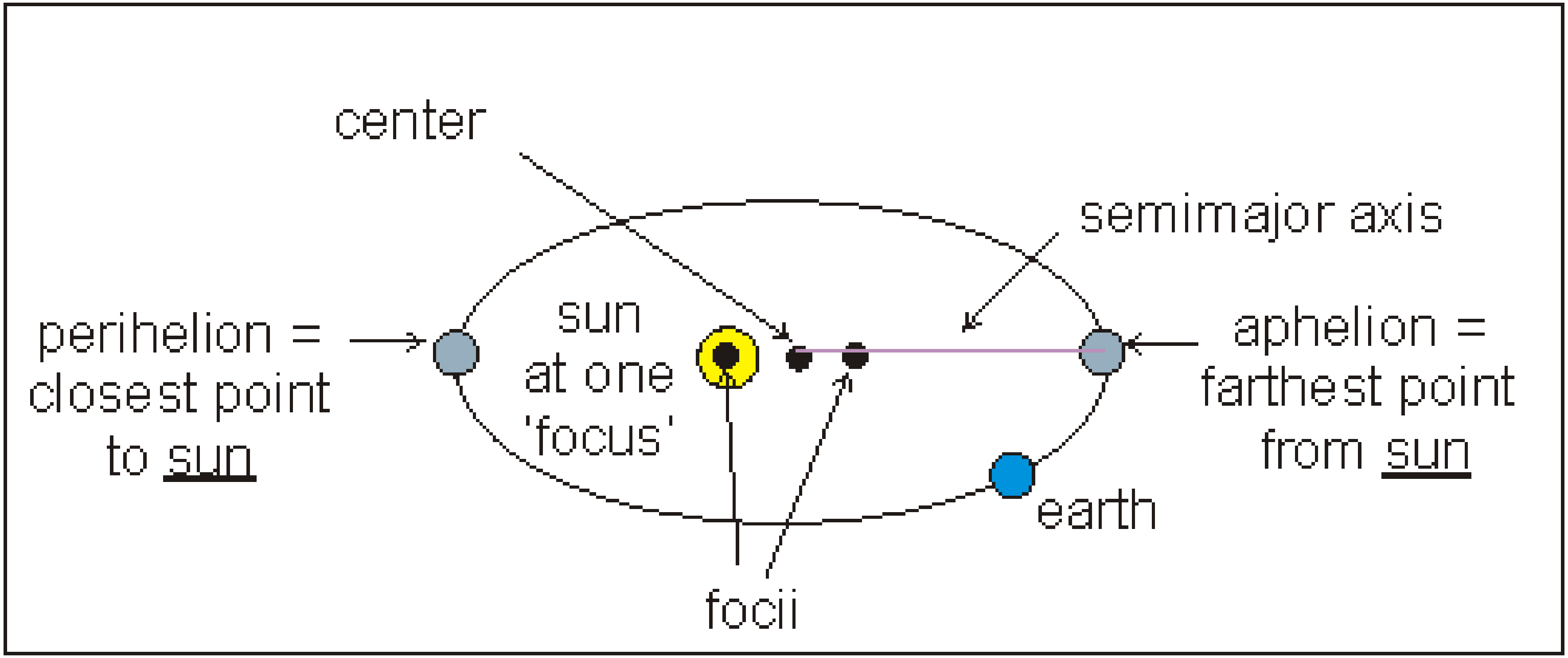
ORBITAL EQUATIONS OF MOTION

There are three classes of solutions (*conic sections*), depending on the *initial conditions or total energy*;

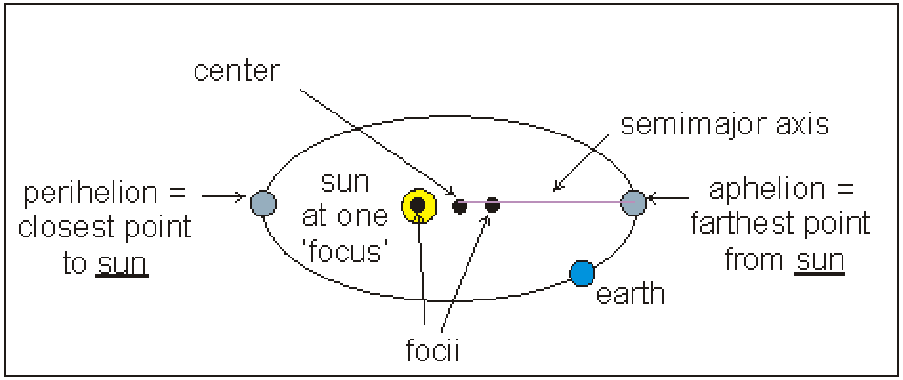
A. Elliptical orbits (*Kepler's First Law*)
B. Circular orbits (*a special case of elliptical orbits*)
C. Parabolic/Hyperbolic 'orbits'

'orbit' → *the object only passes by once & then leaves the system forever*

ELLIPTICAL ORBITS



ELLIPTICAL ORBITS



The *eccentricity of an orbit is defined as* $e = (r_{ap} - r_{peri})/(r_{ap} + r_{peri})$

Earth: e = 0.0167 to 0.058 → very nearly circular, within 2 parts in 100.

The elliptical nature is VERY exaggerated in the figure above.

The ellipse does NOT explain seasons; they are due to the tilt of earth's axis.

Mercury's orbit is the most 'elliptical' of the planets, with e = 0.2056

GRAVITATIONAL POTENTIAL ENERGY

$$dW = \vec{F}_{gravity} \bullet d\vec{r} = -\left(\frac{Gm_1m_2}{r^2}\right)dr$$

Gravity does negative work if r increases.

$$W = -\int_{\substack{initial\\position}}^{\substack{final\\position}} \frac{Gm_1m_2}{r^2}\,dr$$

Gravity is a conservative force;

the work done by gravity does NOT depend on the path,

only on the change in $|r|$,

so we can define a potential energy for gravity on large distance scales as well as near the surface of the earth, where $\Delta U = mgh$

$$\Delta U_{gravity} = -\,W_{gravity}$$

GRAVITATIONAL POTENTIAL ENERGY

$$\Delta U_{gravity}(r) = -W_{gravity} = \int_{r_o}^{r_f} \frac{GMm}{r^2} dr$$

$$= GMm \int_{r_o}^{r_f} \left(\frac{1}{r^2} \right) dr = \text{-}GMm \frac{1}{r} \bigg|_{r_o}^{r_f} = -GMm \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

Only CHANGES in potential energy are determined by work.

If we define $U(r) \equiv 0$ at $r_{\mathrm{i}} = \infty$, then

$$U(r) = -\frac{GMm}{r}$$

TOTAL ENERGY

There are two contributions to the total energy

$$E = U + K$$

Let $m_1 = M$ be a large mass like the earth.

Let $m_2 = m$ represent a satellite moving in the earth's vicinity.

For the satellite:

$$U_{gravity} = -GMm/r \qquad K = \tfrac{1}{2} mv^2$$

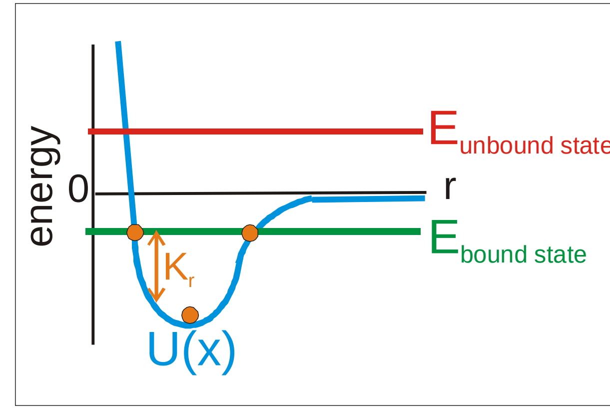
For a circular orbit, $F_{gravity} = F_{centripetal}$

$$GMm/r^2 = m(v^2/r) \;\rightarrow\; mv^2 = GMm/r$$

$$\Rightarrow K = \tfrac{1}{2} mv^2 = GMm/2r \quad (\textit{larger } r = \textit{slower!})$$

$$E = U + K = -GMm/r + GMm/2r = -GMm/2r$$

TOTAL ENERGY



$$E = U + K = -GMm/2r$$

An object in a circular orbit has < 0 energy

where 0 J is the energy it would have if it was at rest infinitely far away

where we defined $U(r) = 0$.

This does make sense.

Satellites (*and planets*) are TRAPPED in their orbits

and can't escape to infinity

unless you provide some extra energy.

ESCAPE VELOCITY

Escaping the gravitational pull of a body like the sun or the earth requires a total energy $E = K + U = 0$

assuming the idea is to escape to infinity

or a close facsimile thereof.

- Starting from finite r with a gravitational $U(r) = -GMm/r$

  where r might correspond to the surface of the planet

  or the radius of an orbit

- The kinetic energy needed to make $E = 0$ is

  $K = \frac{1}{2}mv^2 = E - U = 0 - (-GMm/r) = GMm/r$

- You need a velocity to escape of $v_{escape} = (2GM/r)^{1/2}$

ESCAPE VELOCITY

$$v_{escape} = (2GM/r)^{½}$$

- The escape velocity from the surface of the earth is
  $[(2)(6.67\text{x}10^{-11})(5.98\text{x}10^{24})/6.37\text{x}10^{6})]^{½} = 11.2\text{x}10^{3}$ m/s ~ 25,000 mph.
- The escape velocity from the surface of the moon is
  $[(2)(6.67\text{x}10^{-11})(7.35\text{x}10^{22})/1.74\text{x}10^{6})]^{½} = 2.38\text{x}10^{3}$ m/s ~ 5,310 mph.
- The escape velocity from the surface of Mars is
  $[(2)(6.67\text{x}10^{-11})(6.42\text{x}10^{23})/3.39\text{x}10^{6})]^{½} = 5.03\text{x}10^{3}$ m/s ~ 11,200 mph.
- The escape velocity from sun, starting at the radius of the earth's orbit is
  $[(2)(6.67\text{x}10^{-11})(1.99\text{x}10^{30})/1.5\text{x}10^{11})]^{½} = 42.1\text{x}10^{3}$ m/s ~ 91,400 mph.

HOLE-Y EARTH

$$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$$

An elegant way to get from Cleveland to (*somewhat west of*) Australia would be to bore a hole through the center of the earth and jump in. See http://en.wikipedia.org/wiki/Gravity_train .

- This was the basis of a recent sci-fi movie, the 2012 version of Total Recall. http://en.wikipedia.org/wiki/Total_Recall_(2012_film) /
- It's also a common problem on physics proficiency exams.
- If you ignore air resistance and jump into the hole (*and survive – which is unlikely*) conservation of energy tells us that you should 'fall' to the opposite side of the earth before coming (*momentarily*) to a stop.
- Question #1: Find the equation of motion.
- Question #2: Calculate the transit time.

TUNNEL THROUGH THE EARTH

Find the equation of motion

If you are a distance r from the center of the earth, only the part of the earth < r is exerts a gravitational pull on you.

A spherical shell exerts no net force on an object inside the shell for $1/r^2$ forces.

Assuming (*incorrectly*) the density of the earth is constant $\rho = \dfrac{M_E}{\frac{4}{3}\pi R_E^3}$

Y = you, E = earth, r = distance from center of earth

Sign of the force on you = - sign of $\boldsymbol{r}$ from the center of the earth.

$$F_{gravity\ on\ you} = -\frac{Gm_Y M_{E<r}}{r^2} = -\frac{Gm_Y}{r^2}\rho_E\left(\frac{4}{3}\pi r^3\right) = -\frac{Gm_Y}{r^2}\left(\frac{M_E}{\frac{4}{3}\pi R_E^3}\right)\left(\frac{4}{3}\pi r^3\right)$$

$$= -\frac{Gm_Y M_E}{R_E^3} r \qquad \Rightarrow \qquad a = \frac{d^2 r}{dt^2} = \frac{F_{gravity}}{m_Y} = -\frac{GM_E}{R_E^3} r$$

TUNNEL THROUGH THE EARTH

$$F_{gravity} = -\frac{Gm_Y M_E}{R_E^3} r \quad \Rightarrow \quad F_{gravity} = -kr \quad \text{with} \quad k = \frac{Gm_Y M_E}{R_E^3}$$

Calculate the transit time.

- This equation is just like Hooke's Law for a spring, $F = -kx$, except that $x \rightarrow r$ & $k \rightarrow (Gm_Y M_E/R^3_E)$
- Your motion will be just like a mass on a spring (*Ohanian chapter 15*) $r = R_E\cos(\omega t)$ with $\omega = (k/m_Y)^{1/2} = (GM_E/R^3_E)^{1/2}$
- The time to get to the other side of the earth is half the period $= T/2 = 1/(2f) = \pi/\omega = 42$ minutes.
- *This result applies to any tunnel from one point to another, Cleveland to Detroit or Cleveland to Peking or Strosacker to your dorm.*

There's no need to tunnel to the center of the earth but you do have to remove air and other sources of drag/friction from your tunnel.

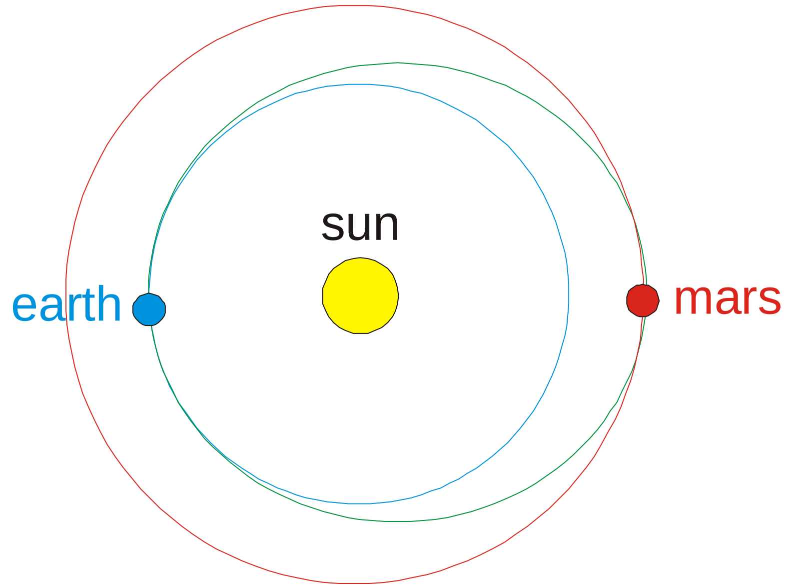
We made it to here in lecture on Monday, April 13.

The following slides are not ‘required’ for homework or exam purposes.

INTERPLANETARY TRAVEL

cultural interlude

- $E = U + K$ will tell us how much energy is needed to travel between orbits/planets.
- $L = mvr$ will constrain how we apply that energy most efficiently.

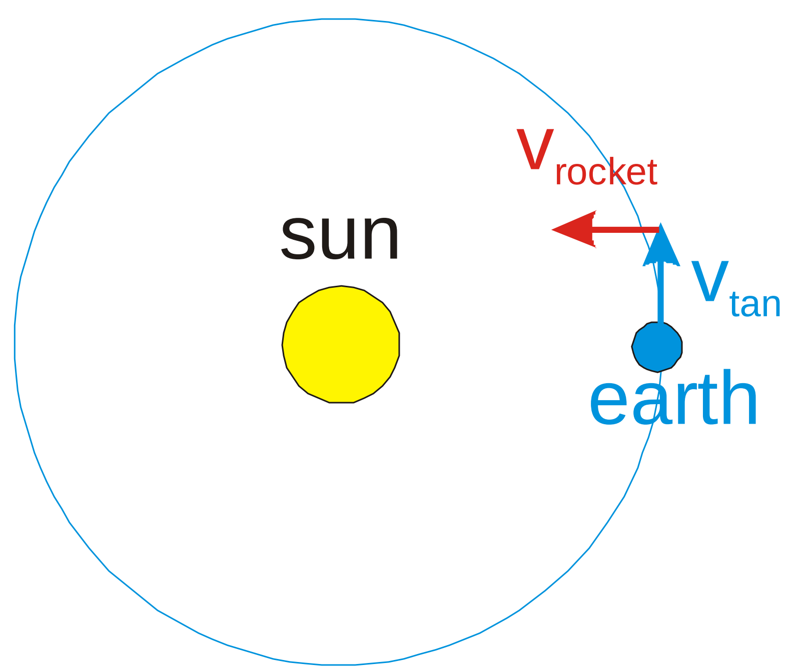


TRAVEL TO THE SUN

What would it take to 'fall' into the sun from the earth's orbit?

- Googling "*nuclear waste sun disposal*" yields 988,000 hits.

This is a VERY poor choice!



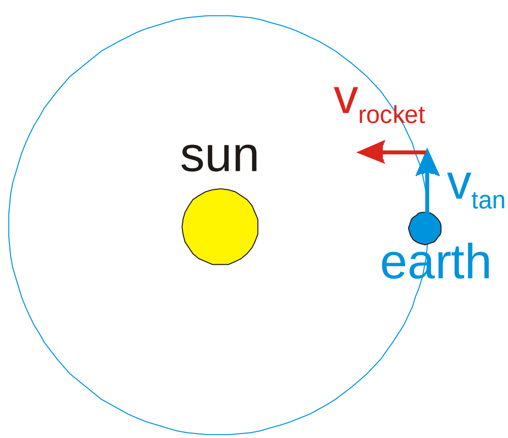
TRAVEL TO THE SUN

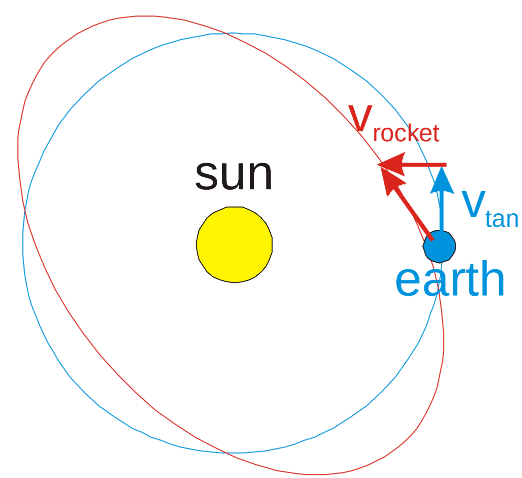
- Aiming a rocket towards the sun doesn't work.

  It just puts the rocket into an elliptical orbit that will return to a point in the earth's orbit periodically.

  You can see this with vector addition of the velocities of the earth & rocket.

  You probably won't like it when your nuclear waste returns.
- To reach the sun, the orbit has to be so elliptical that it brushes the surface of the sun. This takes $v_{rocket} \gg v_{tan}$





TRAVEL TO THE SUN

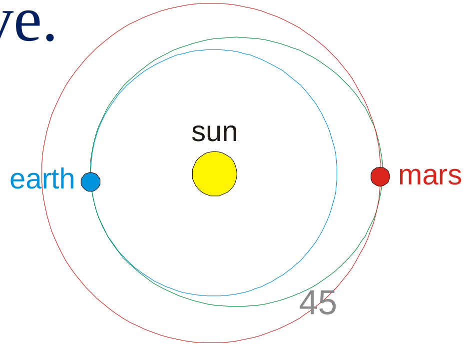
- The most efficient way to send a rocket into the sun is to give it $-v_{tangential\text{-}earth\text{-}orbit}$ so it falls straight in – there's no need to supply any extra velocity or energy.
- This requires $E = ½mv^2$, not counting the escape velocity from the earth. $v_{tangential\text{-}earth\text{-}orbit} \sim 30$ km/sec.
- It takes less energy to send the waste in the other direction; the escape velocity to ∞ from earth's orbit is 42.1×10^3 m/s but you have a 30 km/sec head start!

ELLIPTICAL *vs.* CIRCULAR ORBITS & ANGULAR MOMENTUM

- The difference between circular orbits compared to elliptical orbits that are tangent to the circular orbit at r is the velocity $v_{tangent}$.

  This is the same as saying they have different angular momentum at r.
  - *Note that $U(r)$ is the same at any given r.*
  - *Note that $v_{perpendicular} = 0$ at max/min; the velocity is all $v_{tangent}$.*
  - *$L = mvr_{tangent}$ at these points.*
- The most (*energy*) efficient way to travel between planets is to convert from a circular to an elliptical orbit and back again – since you don't waste any energy on unnecessary $v_{perpendicular}$.
- You still need an overall energy $E(r_2) – E(r_1) = (GMm/2)(1/r_1 – 1/r_2)$.
- You need to time things so that Mars is there when you arrive.

  This is a major reason that *launch windows* are important.



ORBITS & THE CM

- Objects don’t orbit other objects, they orbit the CM of their system.
- For a binary star system with two stars of equal mass, the stars rotate about their midpoint.
- We don’t orbit the sun; the earth & sun orbit their joint CM (*actually the CM of the entire solar system*).

ORBITS & THE CM

Counting just the earth & sun (*ignore the other planets*) and measuring from the center of the sun, the earth-sun CM is a distance

$r = r_{EARTHORBIT} M_E/(M_S + M_E) =$

$(1.5\text{x}10^8 \text{ km})(5.97\text{x}10^{24}\text{kg})/[1.99\text{x}10^{30} \text{ kg} + 5.97\text{x}10^{24}\text{kg}]$

$=$ 450 km from the center of the sun, whose radius ~ $7\text{x}10^5$ km

For earth/moon system, the CM is a distance

$r = r_{MOONORBIT} M_M/(M_M + M_E) =$

$(3.84\text{x}10^5 \text{ km})(7.35\text{x}10^{22}\text{kg})/[7.35\text{x}10^{22}\text{kg} + 5.97\text{x}10^{24}\text{kg}]$

$=$ 4670 km from the center of the earth, whose radius ~ 6371 km

6 options

The Moon does not accelerate towards the Earth because

A. It is in Earth’s gravitational field.

B. The net force on it is zero.

C. It is beyond the main pull of Earth’s gravity.

D. It is being pulled by the Sun and planets as well as by Earth.

E. all of the above

F. It does accelerate towards the earth. L**OOK OUT, the moon’s falling!**

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D. It is being pulled by the Sun and planets as well as by Earth.

E. all of the above

F. It does accelerate towards the earth (LOOK OUT, the moon is falling!)

It is accelerating towards Earth because of the gravitational attraction between the two.

This attraction supplies the centripetal force necessary to keep the Moon in orbit. Otherwise the moon would take off on a straight line somewhere and we’d sorely miss it.

- 5 options

$$\left|F_{gravity}\right| = \frac{Gm_1m_2}{r^2}$$

How far would you have to travel above the surface of the earth before your acceleration from gravity fell to half its normal value?

r_{earth} = *radius of the earth in the options below*

A. ½ r_{earth} above r_{earth}

B. $2r_{earth}$ above earth's surface

C. $2^{1/2}r_{earth}$ above earth's surface

D. None of the above is the right answer

E. Halfway to the orbiting space station, where astronauts are weightless.

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r appears in Newton's Law of Gravity as $1/r^2$ and the force will decrease by half if the r increases by $2^{1/2}$.

- 5 options $F_{gravity} = \frac{GMm}{r^2}$ density $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$

If we could increase the Earth's radius *R* while retaining its average mass density, the gravitational acceleration *g* at the surface of this larger earth would

A. remain constant.

B. increase in proportion to *R*.

C. increase in proportion to R^2

D. decrease as $1/R$

E. decrease as $1/R^2$

- 5 options

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$$g = \frac{GM}{R^2} = \frac{G\frac{4\pi}{3}\rho R^3}{R^2} \propto R$$

This is why you can jump much higher on the moon than you can on earth; the earth has a larger radius & a higher mass but the radius dominates.

4 options

$$F=-\frac{dU(r)}{dr}$$

A 3-D potential energy grows linearly with r as $U(r) = +Cr$.

What is the magnitude and direction of the force this produces?

A. C and outward radial

B. $\frac{1}{2}Cr^2$ and outward radial

C. C and inward radial

D. $\frac{1}{2}Cr^2$ and outward radial

4 options

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$$F = -\frac{dU(r)}{dr} = -\frac{d(Cr)}{dr} = -C$$

4 options

$$F = -\frac{dU(r)}{dr}$$

A 3-D potential energy grows linearly with r as $U(r) = +Cr$.

For circular orbits produced by this potential, what must the square of the velocity, v^2, be in terms of C and r?

A. $v^2 = 2Cr/m$

B. $v^2 = ½Cr^2/m$

C. $v^2 = Cr^2/m$

D. $v^2 = Cr/m$

Do you need a hint?

HINT: $a_{centripetal} = v^2/r = F/m$

4 options

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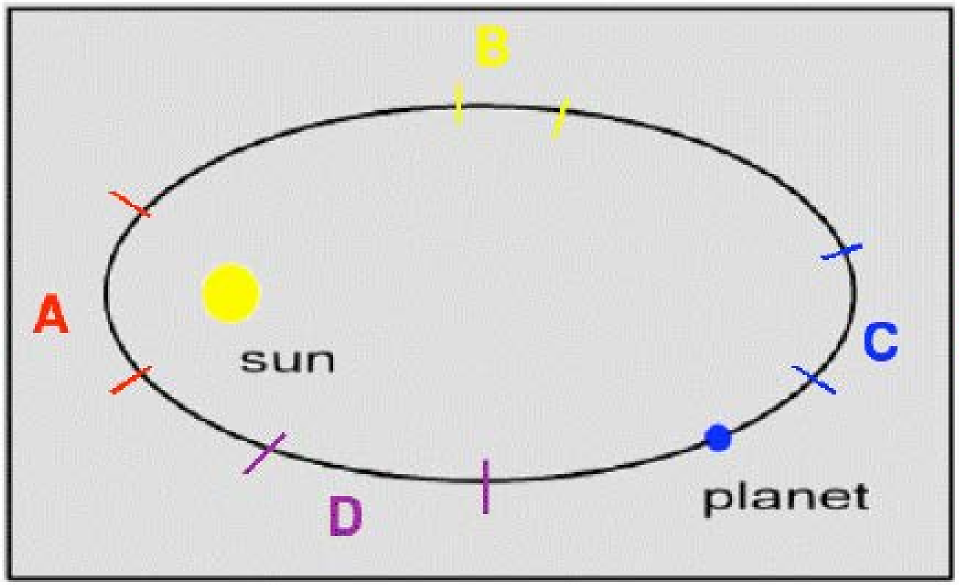
D. $\boldsymbol{v^2 = Cr/m}$

You can find the answer from the formula for centripetal acceleration

$$a_{centripetal} = v^2/r = F/m = C/m$$

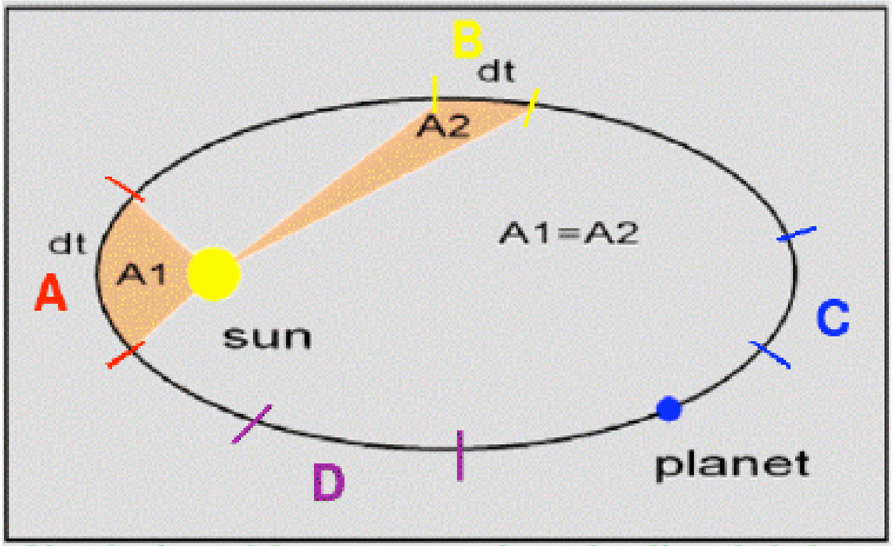
so $v^2 = Cr/m$

4 options



This figure shows a planet and its elliptical orbit. During which interval (A, B, C, or D) does the planet have the greatest velocity?

1) interval A
2) interval B
3) interval C
4) interval D



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Kepler's 2nd law states that the line joining the planet to the sun sweeps out equal areas in equal times as the planet travels around the ellipse. That means the planet moves faster when it is near the sun than away from the sun – its speed is not constant, it changes with its distance from the sun

You can also think in terms of:

- Angular Momentum: to remain constant, v and ω must increase as r decreases
- Energy: smaller r corresponds to more negative potential energy and that requires greater kinetic energy if the overall energy is to remain constant.

5 options

Two satellites *A* and *B* of the same mass are going around Earth in concentric orbits.

The distance of satellite *B* from Earth's center is twice that of satellite *A*.

What is the ratio of the centripetal force acting on *B* to that acting on *A*?

A. 1/8

B. 1/4

C. 1/2

D. $\sqrt{1/2}$

E. 1

5 options

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$$\vec{F} = -\frac{GMm}{r^2}\hat{r}$$

The centripetal force on each satellite is provided by the gravitational force between the satellite and Earth.

5 options

Two satellites *A* and *B* of the same mass are going around Earth in concentric orbits. The distance of satellite *B* from Earth's center is twice that of satellite *A*.

What is the ratio of the tangential speed of *B* to that of *A*?

A. $1/2$

B. $\sqrt{1/2}$

C. 1

D. $\sqrt{2}$

E. 2

5 options

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What is the ratio of the tangential speed of *B* to that of *A*?

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B. $\sqrt{1/2}$

C. 1

D. $\sqrt{2}$

E. 2

For each satellite, the centripetal force is equal to the gravitational force between the satellite and Earth, and is proportional to the square of the tangential velocity and the inverse of the distance.

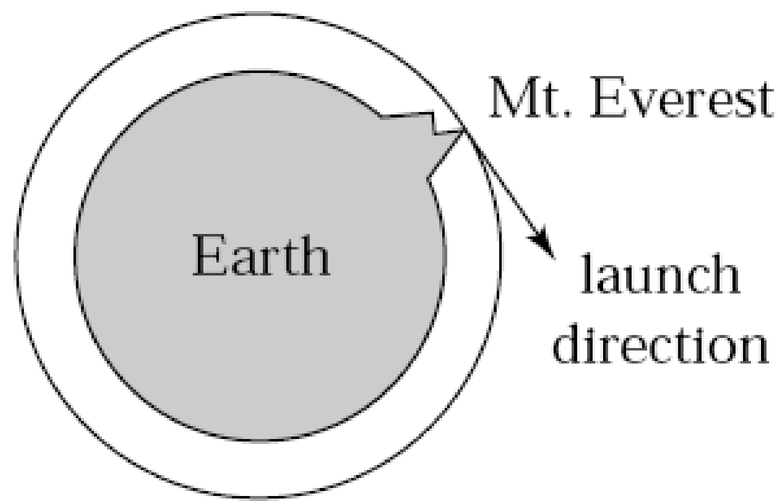
3 options

Suppose Earth had no atmosphere and a ball were fired from the top of Mt. Everest in a direction tangent to the ground. If the initial speed were high enough to cause the ball to travel in a circular trajectory around Earth, the ball's acceleration would

A. be much less than g (because the ball doesn't fall to the ground).

B. be approximately g.

C. depend on the ball's speed.



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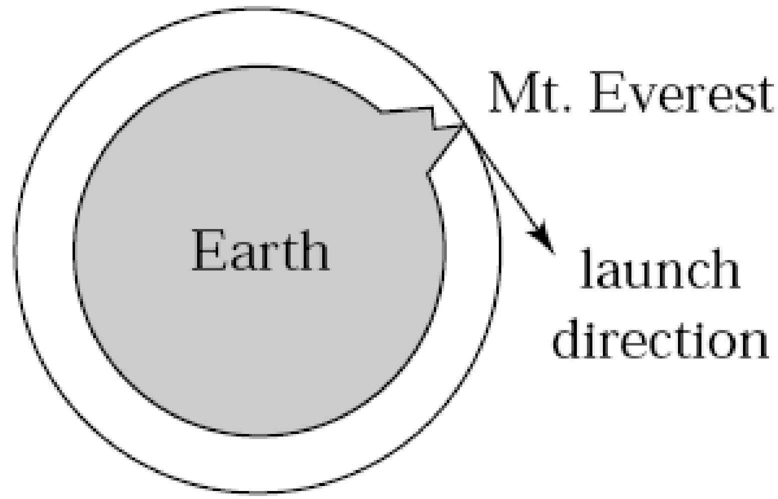

The ball's acceleration is caused by the only force exerted on the ball: gravitation.

Near the surface of Earth, the value of this acceleration is *g*

(*perhaps a little less because of the altitude of Mt. Everest*).

http://www.scifibloggers.com/wp-content/uploads/end-of-the-world.jpg