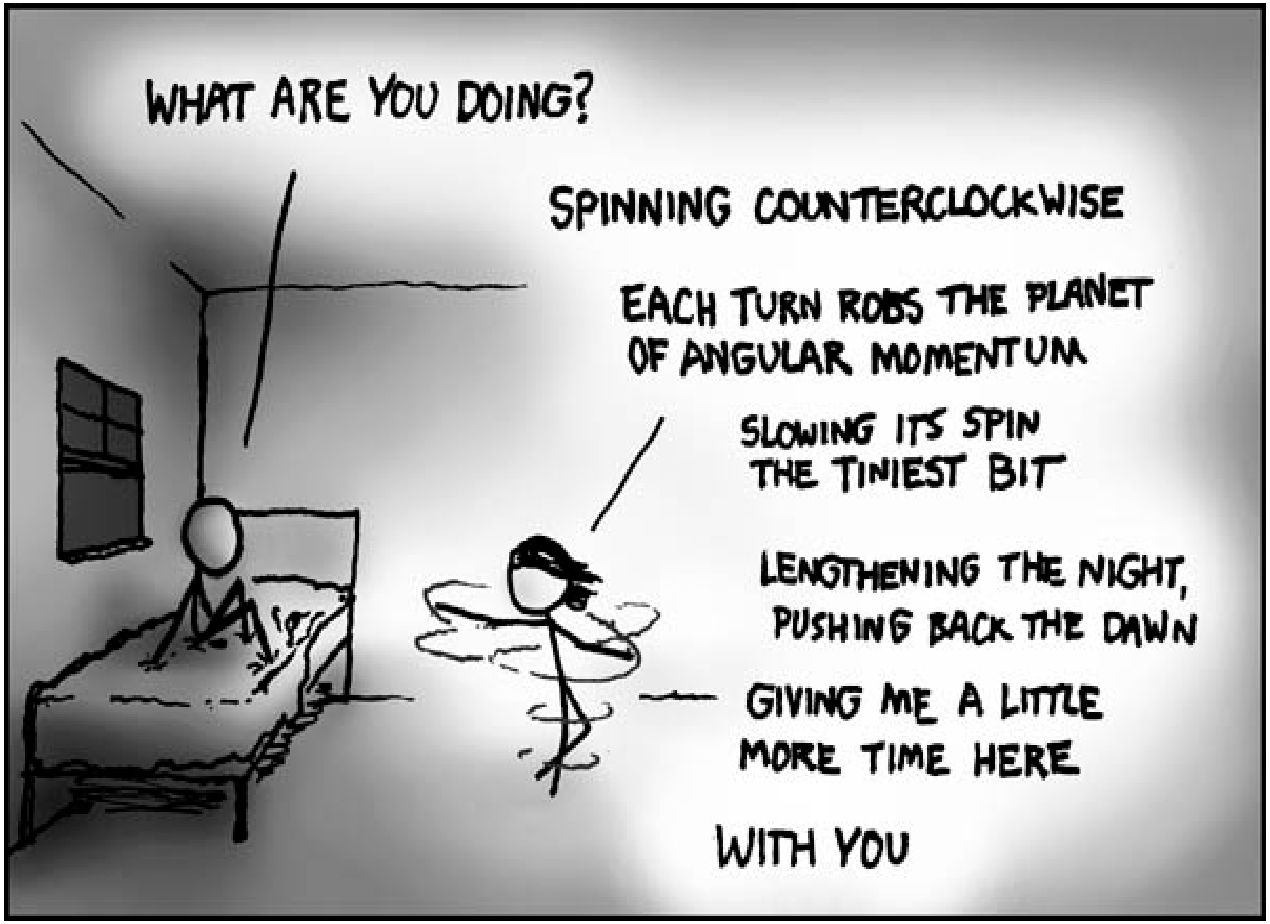
PHYS 121 – SPRING 2015



Chapter 13: Dynamics of a Rigid Body

version 04/10/2015 ~ 103 slides

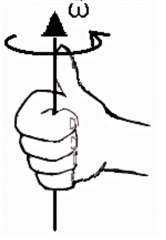
We completed this material on Friday, April 10.

PREVIOUSLY

in PHYS 121

We introduced circular kinematics with

$$\omega = \frac{d\theta}{dt}$$



$$\alpha = \frac{d\omega}{dt}$$

$$a_{radial} = \frac{v^2}{R} = \omega^2 R$$

$$a_{\tan} = \alpha R$$

v = ωR   **ω = 2π*f***   **T = 1/f = 2π/ω**

$$K = \tfrac{1}{2} I\omega^2 \qquad \mathrm{I} = \sum m_i r_i^2$$

ROTATIONAL DYNAMICS

ROTATIONAL DYNAMICS is based on the rotational equivalent of Newton's 2^{nd} Law for *translational* motion.

- *Translational acceleration* requires a force

$$\mathbf{F} = m\mathbf{a}$$

- Likewise, something must cause a *rotational* acceleration $\boldsymbol{\alpha}$.
- But that something isn't just a force, since
  - a force applied on the axle of a wheel won't make it spin at all
  - while the same force applied on the rim ***might*** make it spin
  - but only if that force has some component tangent to the rim
  - and not at all if that force points towards the axle.

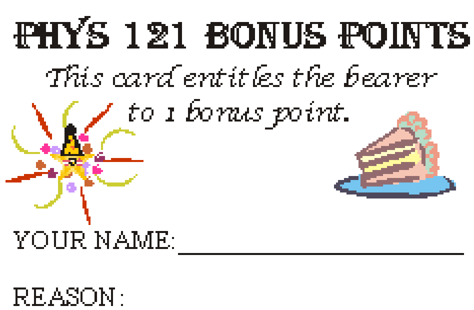
DEMO

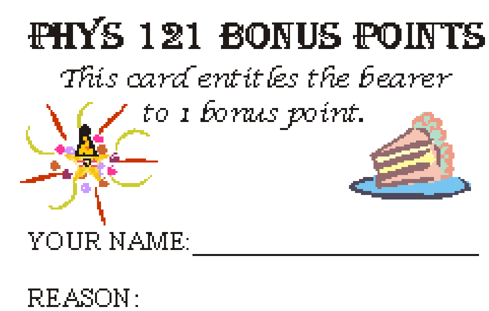
2 bonus points are available

This concept is easy to demonstrate if I can get two volunteers, one who is very strong and someone else with a delicate touch.

Your mission is to open the exit door at the front of the auditorium so that people can leave -

but with certain constraints on where & how you may push.





TORQUE

The physical quantity

(*equivalent to force, F, for translational motion*)

that causes rotational acceleration α is called

TORQUE

and is given the symbol

τ

(Greek letter *tau*)

TORQUE

Instead of $\mathbf{F} = m\mathbf{a}$ as for translational dynamics, rotational dynamics is described by

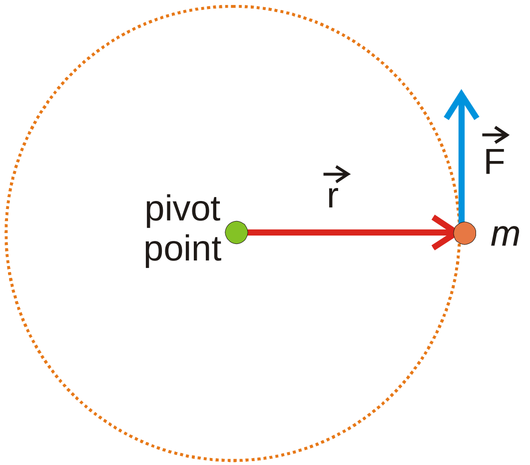
$$\boldsymbol{\tau} = \mathbf{I}\boldsymbol{\alpha}$$

where

$$\alpha = \frac{d\omega}{dt}$$

is the angular acceleration.

ROTATIONAL DYNAMICS for a POINT PARTICLE



Consider a point particle of mass *m*

constrained to move on a circular path of radius *r*.

The constraint might be a string or m might be a bead on a hoop or car on a track or part of a rigid body.

$\vec{r}$ is a vector that points from the center of the circle to the point at which the force is applied.

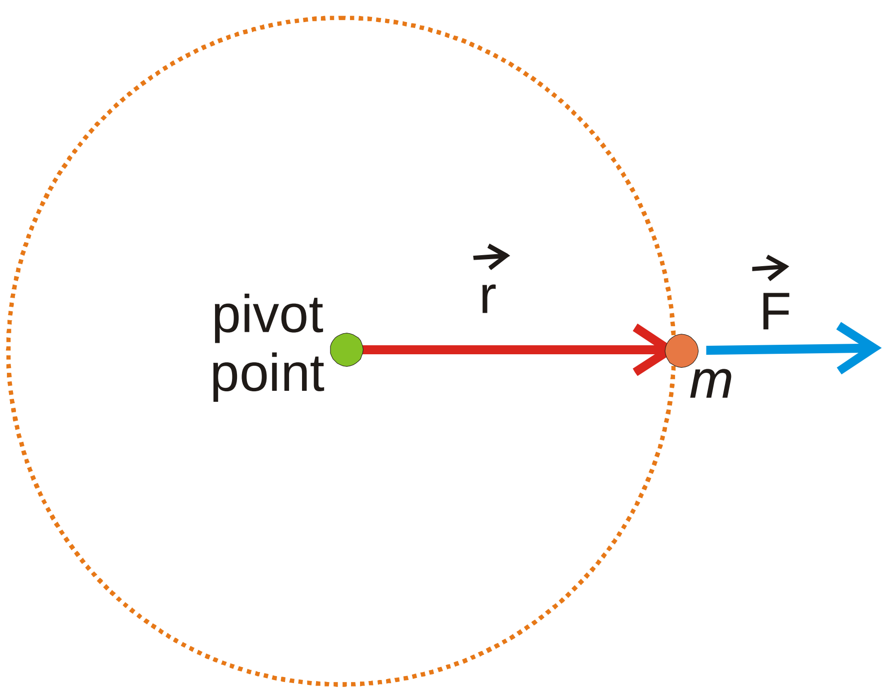
To change the tangential/angular velocity, you need to apply a force that is tangent to the path, perpendicular to $\vec{r}$

$$\vec{F} \perp \vec{r}$$

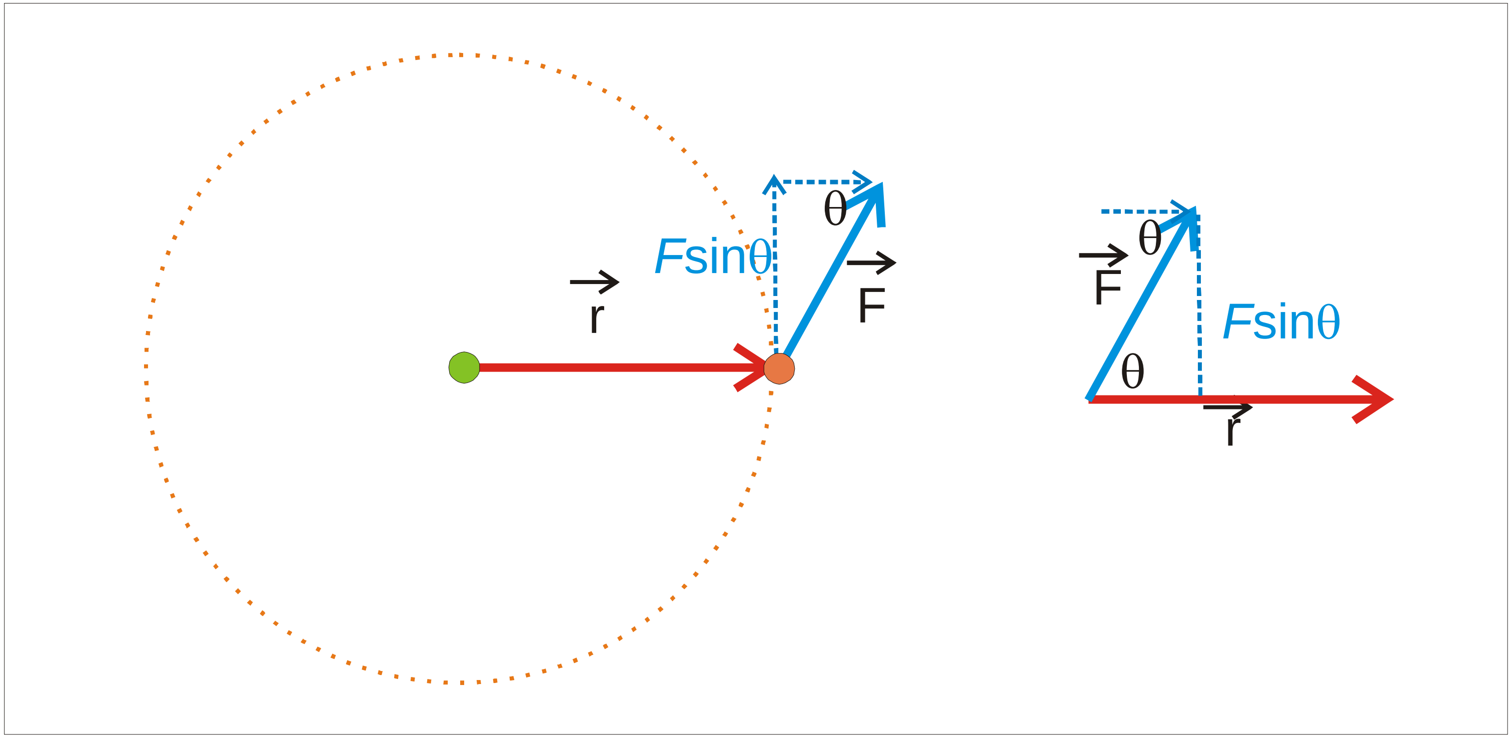
ROTATIONAL DYNAMICS for a POINT PARTICLE

A component of $\vec{F}$ parallel to $\vec{r}$ will have no effect on the rotational motion;

it will simply increase the force supplied by the constraint (*string*, *rod*, *track*, *etc*.).



If a force is applied at some arbitrary angle, only the component of $\vec{F}$ that is perpendicular to $\vec{r}$ will contribute to any angular acceleration.



θ is the angle between **F** and **r**, as shown.

The component of **F** perpendicular to **r** is *F*sinθ.

- The effect of a force in rotational dynamics also depends on how far it acts from the pivot point. Its effect increases linearly as r increases.
- These factors are combined in the definition of torque

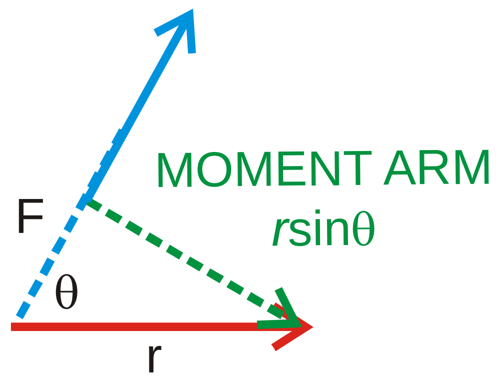
$$\tau = rF\sin\theta$$

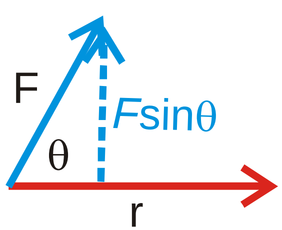
- The units for torque are Newton-meters, **N-m**. (NOT = JOULES!)
- We can think of this as either $r(F\sin\theta)$ or $F(r\sin\theta)$

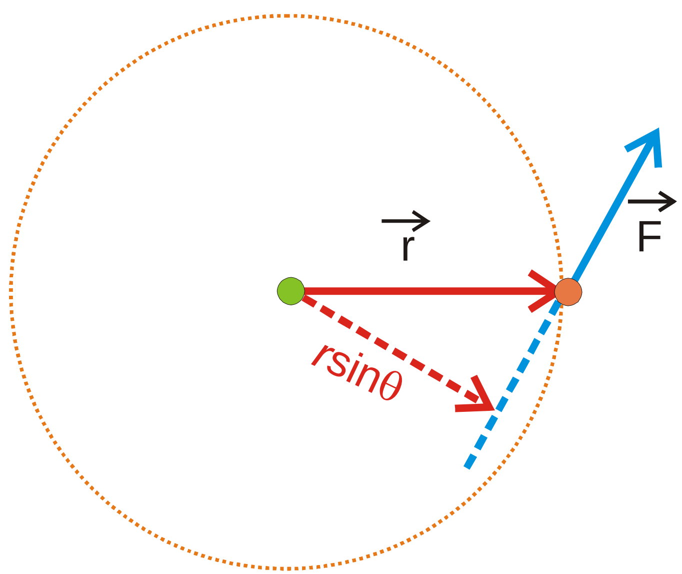
$(F\sin\theta)$ = component of **F** perpendicular to **r**

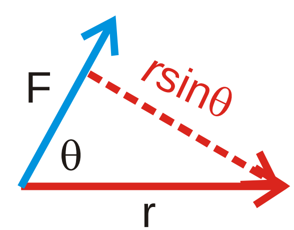
$(r\sin\theta)$ = component of **r** perpendicular to **F**

$r\sin\theta \equiv$ ***moment arm*** for the torque.





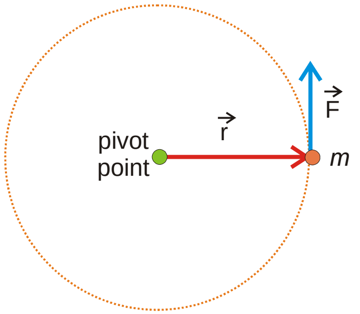




PROOF: $\tau = I\alpha$ *with* $\tau = rF\sin\theta$

For a simple point mass moving around a circular path while a force is applied <u>tangent</u> to the path.

F will cause a *tangential* acceleration $a_{tangential}$ which translates to an *angular* acceleration



$$\alpha = a_{tangential}/r$$

$$\mathbf{F} = m\mathbf{a}_{\mathbf{tangential}}$$

The torque associated with this force is

$$\tau = Fr \quad since\ \theta = 90°$$

Using $a_{tangential} = \alpha r$ & $I = mr^2$

$$\tau = Fr = (ma_{tangential})r = (m[\alpha r])r = (mr^2)\alpha = I\alpha$$

QED

τ as a VECTOR

A general formula for **τ** should take into account:

- τ is proportional to $|r|$
- τ is proportional to $|F|$
- τ depends on the angle between **r** & **F**
- τ is a vector that points in the direction of the angular acceleration α $\quad \vec{\tau} = I\vec{\alpha}$

The ***VECTOR*** or ***CROSS PRODUCT***

satisfies all of these requirements.

VECTOR DOT PRODUCT *review*

The vector **DOT** product

$$\vec{A}\bullet\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{AB} = A_x B_x + A_y B_y + A_z B_z$$

was useful in the context of WORK.

$$W = \vec{F}\bullet\vec{d}$$

Only the component of **F** <u>in the direction of displacement</u> does work

or, equivalently,

only the component of displacement <u>in the direction of **F**</u> does work.

Dot product $\rightarrow$ these two statements are equivalent.

The dot product yields a *scaler.*

VECTOR CROSS PRODUCT

There's another type of vector multiplication that plays a critical role in rotational dynamics.

This is called the ***CROSS PRODUCT***.

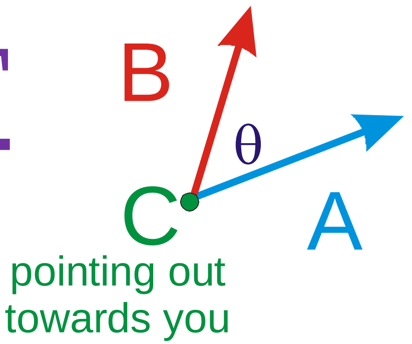
Instead of a dot, it uses a cross as in $\vec{A}\times\vec{B}$

The cross product can be defined in two forms, similar to

$\vec{A}\bullet\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\cos\theta_{AB}$ and $\vec{A}\bullet\vec{B}=A_xB_x+A_yB_y+A_zB_z$

However, the cross product of two vectors is a vector rather than a scalar.

VECTOR CROSS PRODUCT



$$\vec{C} = \vec{A} \times \vec{B} \equiv \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta_{AB}$$

pointing in a direction given by the Right Hand Rule

RHR = fingers first along **A**, then bend them along **B** $\rightarrow$ thumb gives **C**.

In the figure, **A** & **B** are both in the plane of this slide

$\Rightarrow$ their cross product **C** points out of the slide toward you.

If |**A**| = 4 m |**B**| = 3 N $\theta_{AB} = 30°$

then |**C**| = (4)(3)(sin30°) = 6 N-m

- Shift vectors translationally to line up tail to tail to determine θ.
- Apply the RHR to the SMALLER angle between the two vectors.
- The order of multiplication is IMPORTANT

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

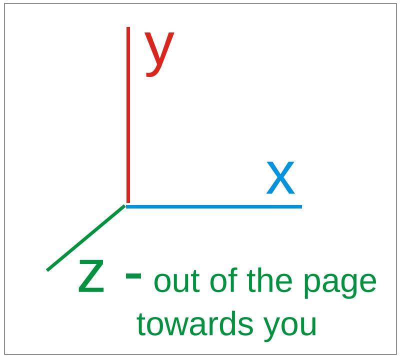
because the direction of **C** is reversed.

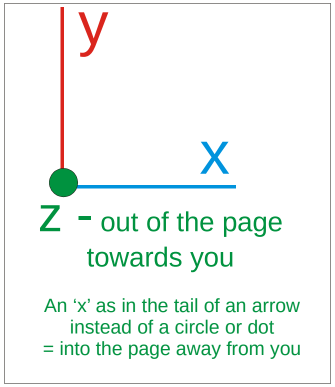
VECTOR CROSS PRODUCTS & UNIT VECTORS

$$\vec{A}\times\vec{B} \equiv \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta_{AB}$$

pointing in a direction given by

the Right Hand Rule





$\hat{x}\times\hat{x} = \left|\hat{x}\right|\left|\hat{x}\right|\sin\theta_{xx} = 0$ The same is true for $\hat{y}\times\hat{y}$ & $\hat{z}\times\hat{z}$

All other combinations of cross products of unit vectors have magnitude = 1

Only their directions change, as given by the Right Hand Rule.

$$\hat{x}\times\hat{y} = \hat{z} \qquad \hat{y}\times\hat{x} = -\hat{z}$$

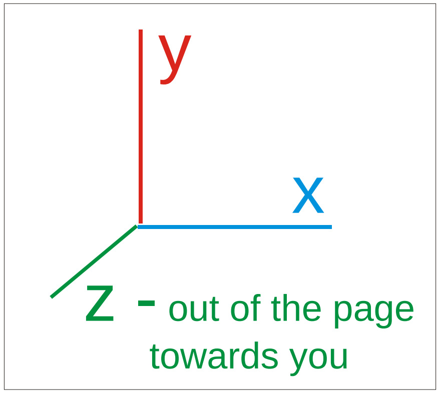
All (known) physicists in the universe

have agreed that all our Cartesian coordinate systems will always have $\hat{x}\times\hat{y} = \hat{z}$.

This turns out to be important in Electricity & Magnetism where you need a uniform convention in order for everyone to get the same results.

$\equiv$ Right Handed Coordinate Systems

VECTOR CROSS PRODUCTS IN COMPONENTS



$$\vec{A}\times\vec{B}\equiv\left|\vec{A}\right|\left|\vec{B}\right|\sin\theta_{AB}$$

can be written in terms of components

using the rules for unit vectors derived on the previous slide.

$$\vec{A}\times\vec{B}=\left(A_x\hat{x}+A_y\hat{y}+A_z\hat{z}\right)\times\left(B_x\hat{x}+B_y\hat{y}+B_z\hat{z}\right)$$

$$=\left(A_yB_z-A_zB_y\right)\hat{x}+\left(A_zB_x-A_xB_z\right)\hat{y}+\left(A_xB_y-A_yB_x\right)\hat{z}$$

If you know how to handle determinants, you can use the following aid for writing the cross product in terms of components

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$=\left(A_yB_z-A_zB_y\right)\hat{x}+\left(A_zB_x-A_xB_z\right)\hat{y}+\left(A_xB_y-A_yB_x\right)\hat{z}$$

DETERMINANT *review*

2×2 determinants are evaluated as shown below

$$\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \equiv A_y B_z - A_z B_y$$

3×3 determinants are broken down into 2×2 determinants as shown below, with each term excluding the two corresponding components below it.

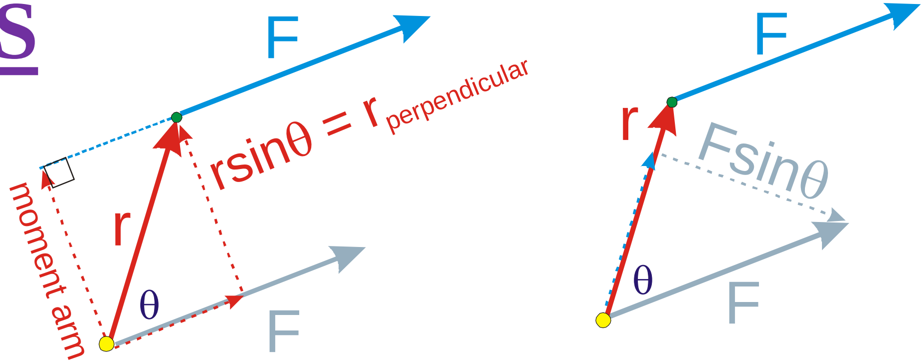
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{x}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{y}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{z}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \left(A_y B_z - A_z B_y\right)\hat{x} - \left(A_x B_z - A_z B_x\right)\hat{y} + \left(A_x B_y - A_y B_x\right)\hat{z}$$

WHY DO WE CARE ABOUT VECTOR CROSS PRODUCTS IN PHYS 121*?

$$\vec{\tau} = \vec{r} \times \vec{F}$$



- This is the standard definition of torque.
- It yields the magnitude of torque $\tau = rF\sin\theta_{RF}$
- It provides the correct direction of the torque *via* the RHR.
- $F\sin\theta_{RF}$ is the component of F that leads to angular acceleration.
- It incorporates *the moment arm*

  $r_{perpendicular} = r\sin\theta_{RF} =$ component of **r** perpendicular to the force.
- Both ways of thinking about torque are correct – and equivalent – and **you should be able to think in either way.**

$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta_{rF} = r_{\perp}F = rF_{\perp}$$

PS – We'll also use vector cross products for angular momentum.

-5 options

A 1.0 kg ball is tied to the end of a string and swung at a constant angular velocity $\omega = 2\pi$ rad/s in a horizontal circle of radius 1.0 m.

What is the torque on the stone due to the tension in the string?

A. 0 N·m

B. 2π N·m

C. $(2\pi)^{-1}$ N·m

D. 1.0 N·m

E. none of the above

-5 options

A 1.0 kg ball is tied to the end of a string and swung at a constant angular velocity $\omega = 2\pi$ rad/s in a horizontal circle of radius 1.0 m.

What is the torque on the stone due to the tension in the string?

A. 0 N·m since the tension T is parallel to the r vector. $\theta = 0° \Rightarrow \sin\theta = 0$

B. 2π N·m

C. $(2\pi)^{-1}$ N·m

D. 1.0 N·m

E. none of the above

4 options

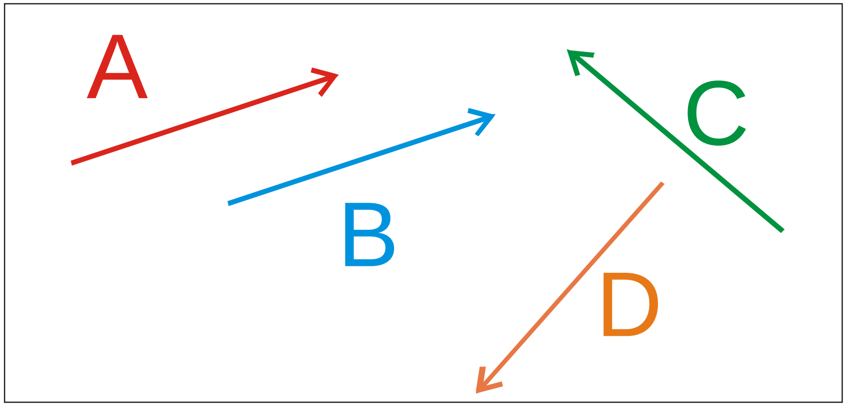
Suppose vectors **A** and **B** are parallel while vectors **C** and **D** are perpendicular. Which of the following statements are true?

A. $\vec{A}\bullet\vec{B}=0 \quad \& \quad \vec{C}\bullet\vec{D}=0$

B. $\vec{A}\bullet\vec{B}=0 \quad \& \quad \vec{C}\times\vec{D}=0$

C. $\vec{A}\times\vec{B}=0 \quad \& \quad \vec{C}\bullet\vec{D}=0$

D. $\vec{A}\times\vec{B}=0 \quad \& \quad \vec{C}\times\vec{D}=0$



4 options

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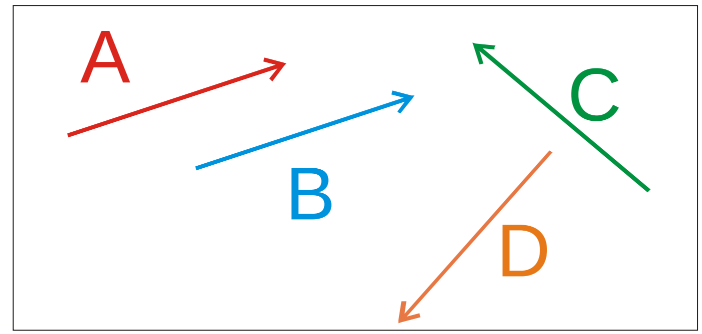
A. $\vec{A}\bullet\vec{B}=0 \quad \& \quad \vec{C}\bullet\vec{D}=0$

B. $\vec{A}\bullet\vec{B}=0 \quad \& \quad \vec{C}\times\vec{D}=0$

C. $\vec{A}\times\vec{B}=0 \quad \& \quad \vec{C}\bullet\vec{D}=0$

D. $\vec{A}\times\vec{B}=0 \quad \& \quad \vec{C}\times\vec{D}=0$



A is parallel to **B** so their dot product is NOT 0. This rules out (A) & (B).

(B) is also ruled out because the cross product of **C** and **D** is not 0.

C is perpendicular to **D** so their cross product is not 0. This rules out (D).

5 options

You are using a wrench, with an optional extension rod, to loosen a stuck bolt. Rank the arrangements drawn below in order of descending effectiveness (*rank them from biggest to smallest torque*).

<u>Biggest to Smallest</u>

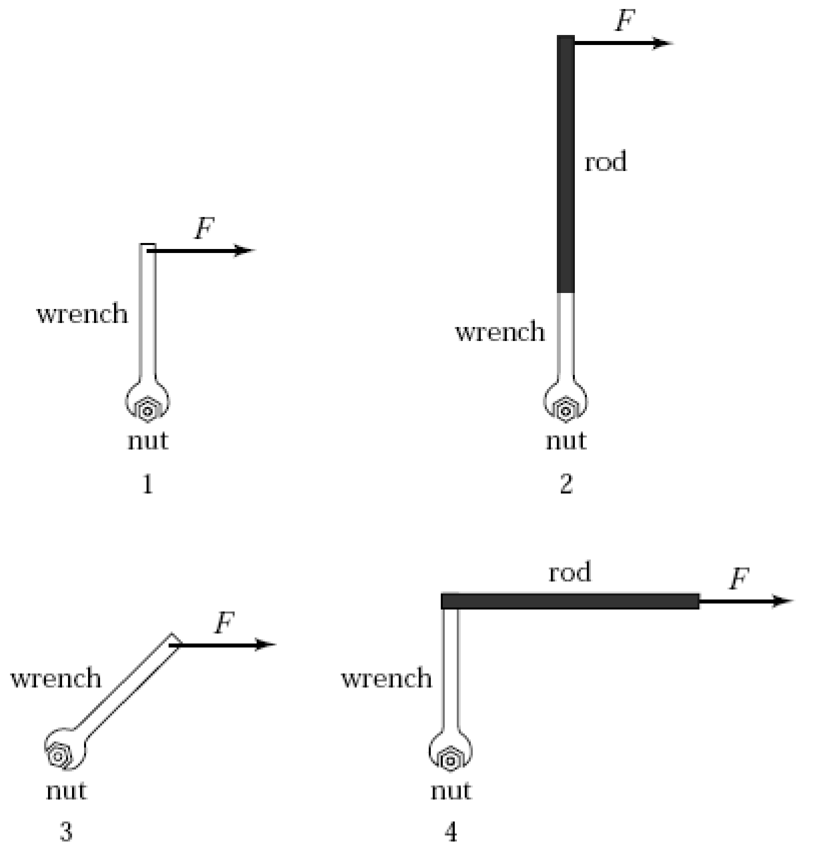
A. 2, 4, 1, 3

B. 3, 1, 4, 2

C. 2, 1 = 4, 3

D. 2, 1, 3, 4

E. The correct answer is not listed.



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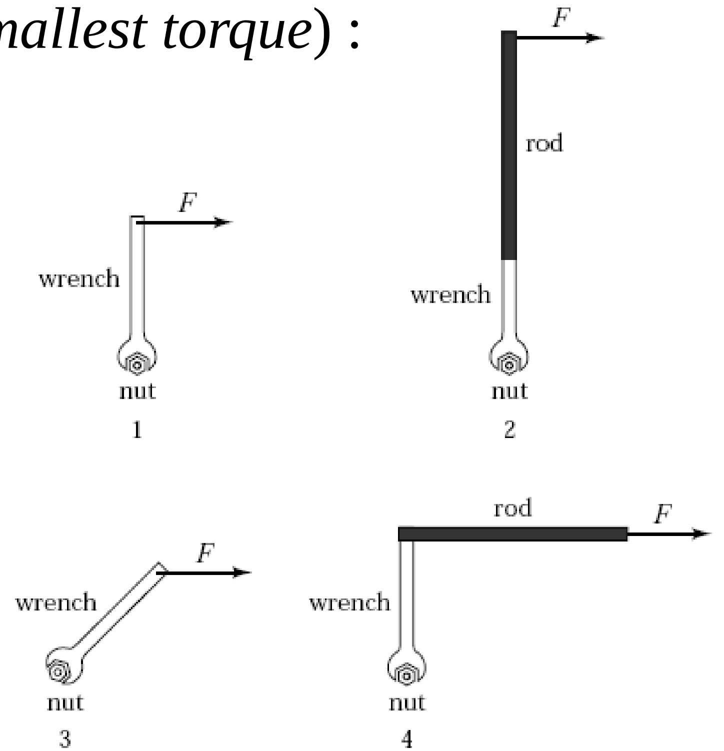


Biggest to Smallest

A. 2, 4, 1, 3

B. 3, 1, 4, 2

C. 2, 1 = 4, 3

D. 2, 1, 3, 4

E. The correct answer is not listed.

To increase a torque, you can increase either the applied force or the moment arm.

Here the force is the same in all four situations, and so this question boils down to comparing moment arms.

5 options

Two wheels, each of mass 1 kg and initially at rest, have forces applied as shown. F_1 = 1 N but F_2 is unknown. Assume that the hubs and spokes are massless so that the rotational inertia is $I = mR^2$.

How large is F_2 if both wheels have the same angular acceleration α?

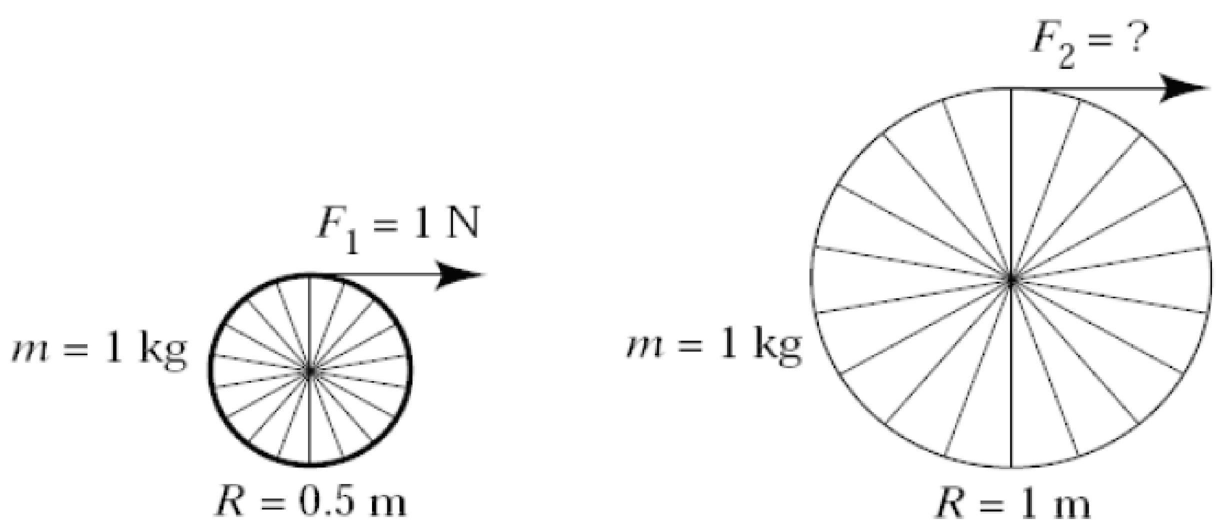
A. 0.25N

B. 0.5N

C. 1 N

D. 2 N

E. 4 N



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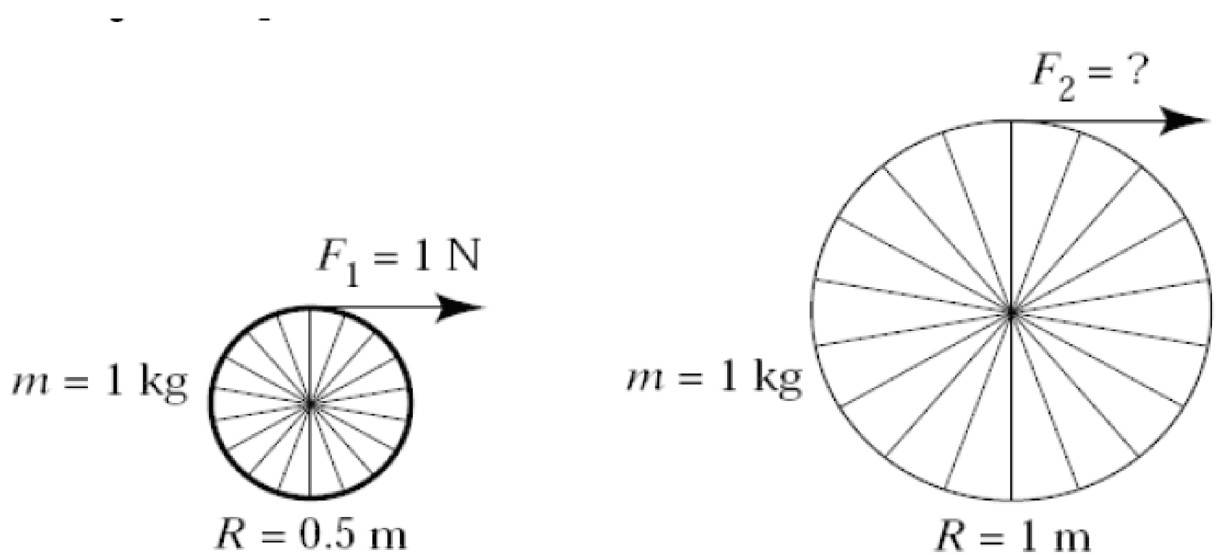


The torque is $\tau = RF = I\alpha = mR^2\alpha$

$\Rightarrow$ F scales as mR for any given α.

If R doubles while the mass remains constant, $F_2 = 2N$

WORK & TORQUE

Translational motion: $\boldsymbol{W = F\Delta d = \Delta K}$

The kinetic energy of a rotating rigid body is:

$$\boldsymbol{K = \tfrac{1}{2} I\omega^2}$$

The work $W = \Delta K$

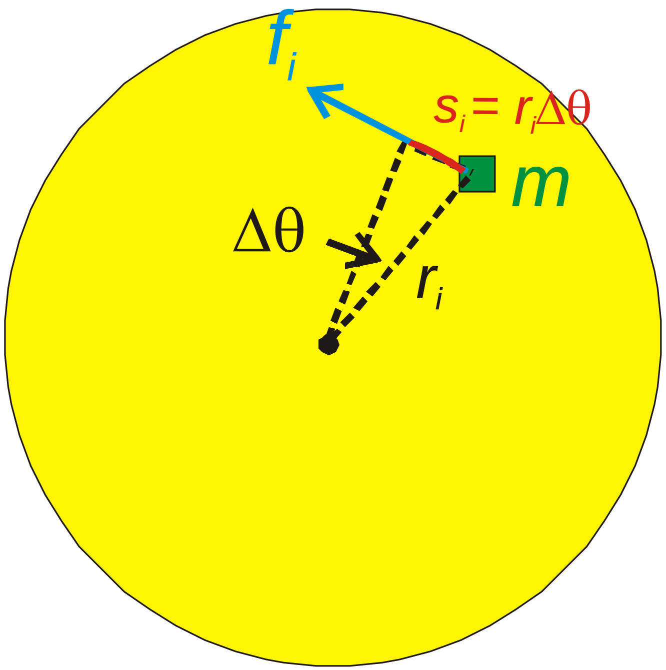
done by torque

$$\boldsymbol{W = \tau\Delta\theta}$$

PROOF: $W = \tau\Delta\theta$

- Add up the work done on each m_i as an object rotates by an angle $\Delta\theta$ due to a torque τ.
- τ = sum of the individual elements of torque τ_i acting on each m_i
- Each τ_i is a force f_i acting on m_i through moment arm r_i
- Using arc length $s_i = \Delta\theta r_i$ & $\tau = \Sigma\tau_i = \Sigma(f_i\,r_i)$

$$\text{Work} = Fd = \Sigma f_i s_i = \Sigma f_i\,(\Delta\theta r_i) = (\Sigma f_i\,r_i)\Delta\theta = \tau\Delta\theta$$



WORK & TORQUE

If the torque τ is constant:

$$W = \tau\Delta\theta$$

If τ varies as a function of θ, this relation applies only to very small $\Delta\theta$:

$$dW = \tau\, d\theta$$

$$W = \int_{\theta_i}^{\theta_f} \tau(\theta)\, d\theta$$

TORQUE & POWER

Power ≡ Work per Time

and $dW = \tau \, d\theta$

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(\tau d\theta\right) \underset{\text{if } \tau \text{ is constant}}{=} \tau \frac{d\theta}{dt} = \tau\omega$$

This should remind you of the equivalent formula for translational motion

$$P = \vec{F} \bullet \vec{v}$$

EXAMPLES

You apply a torque $\tau = 5$ N-m to a hoop of mass $M = 7$ kg and radius $R = 3$ m for 1 complete revolution of the wheel.

- How much work do you do?
- How much kinetic energy does the hoop gain?
- What is the angular velocity of the hoop when you are done, assuming the hoop started from rest?

You apply a torque $\tau = 5$ N-m to a hoop of mass $M = 7$ kg and radius $R = 3$ m for 1 complete revolution of the wheel.

- **How much work do you do?**

$$W = \tau\Delta\theta = (5 \text{ N-m})(2\pi \text{ radians}) = 10\pi \text{ Joules}$$

Note that I switched the units to Joules, a unit of work & energy instead of N-m, which suggests torque.

- **How much kinetic energy does the hoop gain?**

$$W = \Delta K = 10\pi \text{ Joules}$$

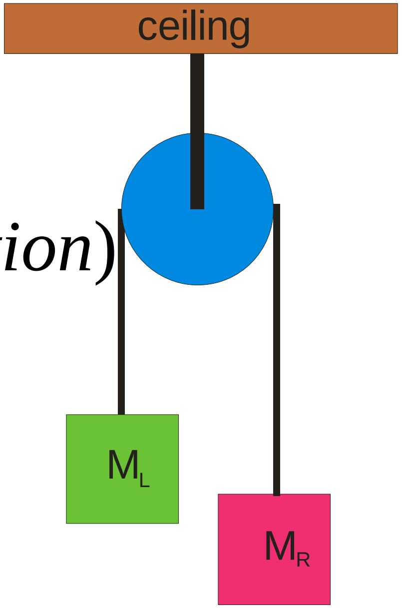
- **What is the angular velocity of the hoop when you are done?**

$$K = \tfrac{1}{2} I\omega^2 \quad \& \quad W = \Delta K$$

$$\text{so } W = \tfrac{1}{2} I\omega^2 \quad \text{with } I = MR^2$$

$$\omega = (2W/I)^{1/2} = \{(2)(10\pi \text{ Joules})/[(7 \text{ kg})(3 \text{ m})^2]\}^{1/2} = 1.23 \text{ sec}^{-1}$$

<u>“REAL” (= *massive*) PULLEYS</u>



‘Ideal” pulleys have no mass.

Real pulleys do have mass

But pulleys normally only rotate (*no translational motion*)

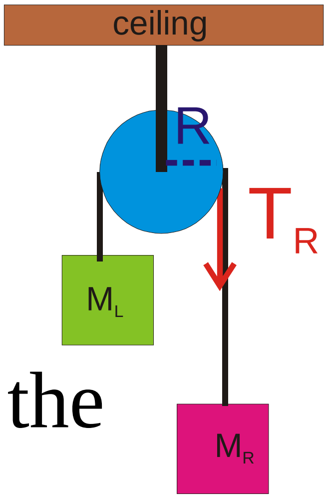
so <u>what matters is their moment of inertia I</u>.

Problems with ‘real’ pulleys require:

- One additional free body diagram
  - *one for each mass + one more for the pulley*
  - *The FBD for a pulley shows the forces AND their moment arms.*
- One more equation of the form $\Sigma F = Ma$

  except that it’s $\Sigma\tau = I\alpha$ for a rotating object like a pulley.

MASSIVE PULLEYS



Keep in mind:

- τ for the pulley is given by $\tau = \Sigma(R_i T_i)$ where T_i is the tension in the rope(s) running over the pulley.
- τ and α have directions.
- If the pulley is accelerating *(angular acceleration)*,

  $\Rightarrow$ There must be a net torque acting on it.

  $\Rightarrow$ The tension in the rope on either side of the pulley must be different! $T_L \neq T_R$ *(2 unknowns rather than 1)*
- α for the pulley is related to a for the rope and masses attached to the rope by $a = \alpha R$.

A wise person chooses $+\alpha$ corresponding to $+a$.

PULLEY EXAMPLE *Ohanian 13.96*

A block hangs from a pulley via an ideal rope that is wrapped around the pulley several times and then fastened to the pulley (*like a bucket for a well*).

Given:

R = radius of the *disc-like* pulley

M_P = mass of the pulley

m = mass of the hanging block

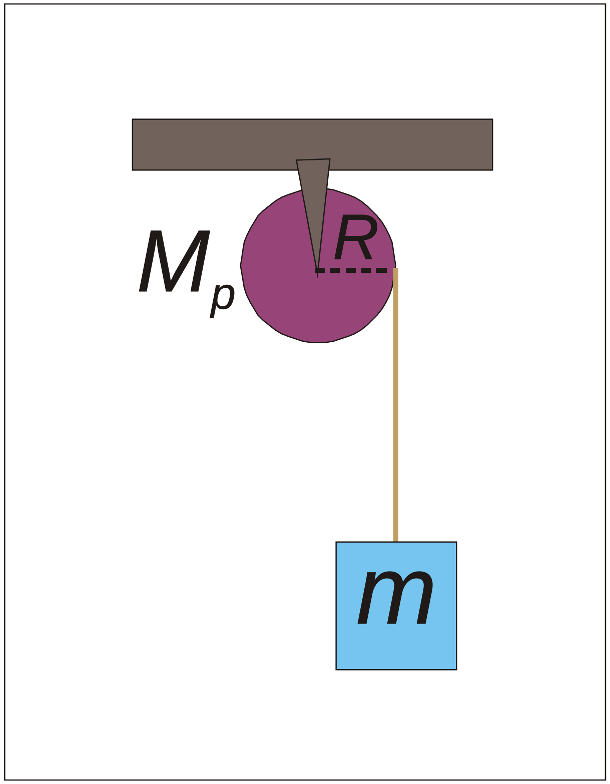
g = acceleration of gravity

Calculate:

a = acceleration of m

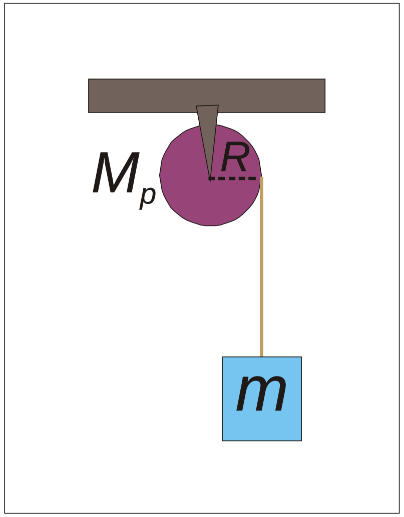
α = angular acceleration of the pulley

T = tension in the rope.

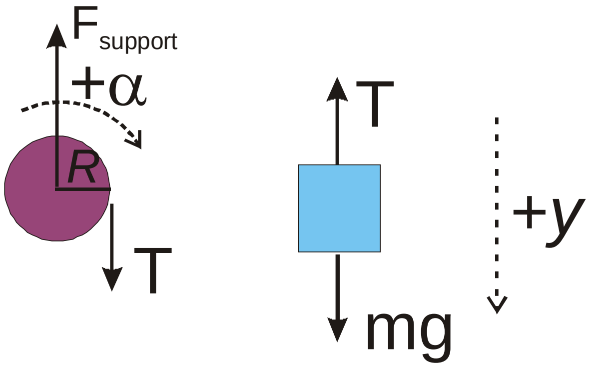


INVENTORY of PARAMETERS

7 parameters with 3 unknown → need 3 equations/physical relationships



PULLEY EXAMPLE

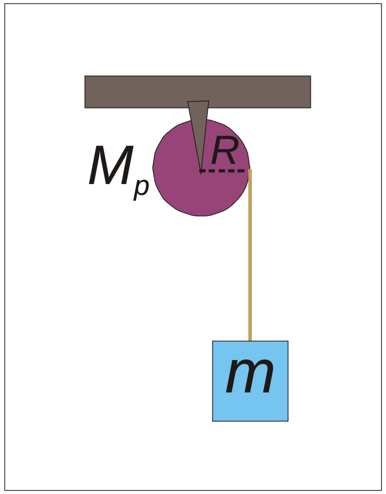


$$\sum F = ma \text{ for the BLOCK: } mg - T = ma \qquad \text{(Eq. \#1)}$$

$$\sum \tau = I\alpha \text{ for the PULLEY: } TR = \left(\frac{1}{2} M_p R^2\right)\frac{a}{R}$$

$$T = \frac{M_p}{2} a \qquad \text{(Eq. \#2)}$$

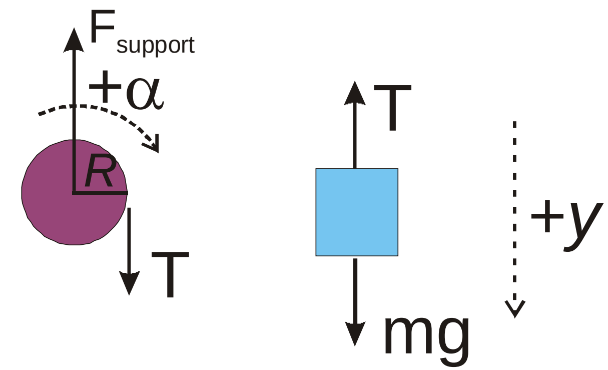
$$\alpha = \frac{a}{R} \text{ was Eq. \#3}$$



PULLEY EXAMPLE

$$mg - T = ma \qquad \text{(Eq. \#1)}$$

$$T = \frac{M_p}{2} a \qquad \text{(Eq. \#2)}$$

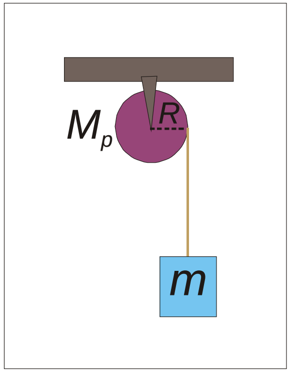


You can solve for *T* and *a* in a variety of ways.

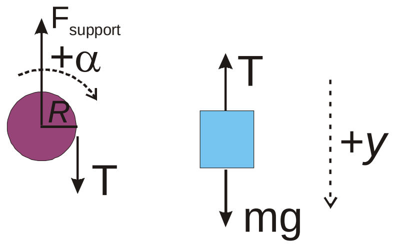
Add Eq #1 + Eq #2 to eliminate **T** or simply plug **T** from Eq #2 into Eq. #1.

$$mg = ma + \frac{M_p}{2} a$$

$$a = \left(\frac{m}{m + \frac{M_p}{2}} \right) g \qquad \text{so} \qquad T = \frac{M_p}{2} a = \frac{M_p}{2} \left(\frac{m}{m + \frac{M_p}{2}} \right) g = \left(\frac{M_p m}{2m + M_p} \right) g$$



PULLEY EXAMPLE



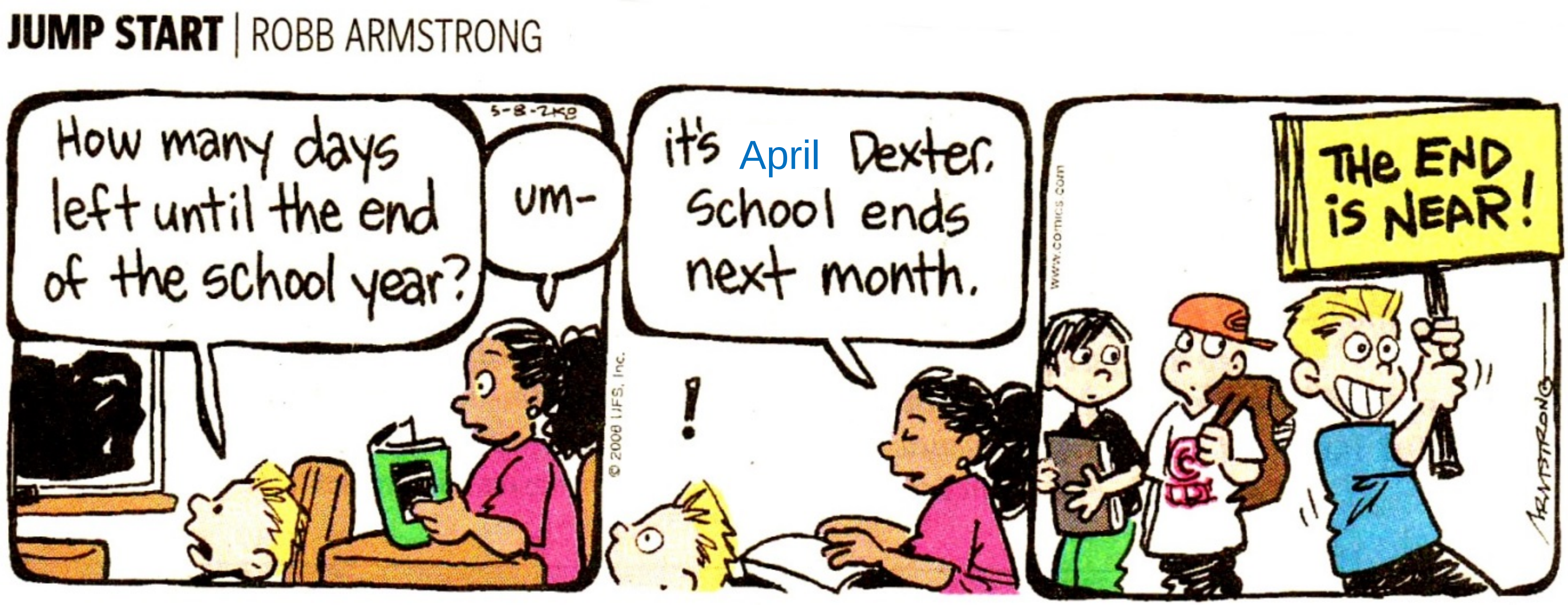
$$a = \left(\frac{m}{m + \frac{M_p}{2}} \right) g \qquad T = \frac{M_p}{2} a = \frac{M_p}{2} \left(\frac{m}{m + \frac{M_p}{2}} \right) g = \left(\frac{M_p m}{2m + M_p} \right) g$$

REALITY CHECK: *You should understand each of the following points.*

- A more massive uniform pulley slows the acceleration – but *usually* by less than if M_P was hanging on the other side of a massless pulley.
  - *If the pulley was a hoop, it would act like a mass of M_P instead of $M_P/2$*
- For a *massive* M_p, $a \to 0$ & $T \to mg \Rightarrow$ *the pulley ~ fixed point.*
- For a *light* pulley, $a \to g$ & $T \to 0 \Rightarrow$ *m is in free fall.*
- ‘*light*’ or ‘*massive*’ is relative; M_p compared to m.

We made it to slide #44 on Monday, April 6.

PHYS 121 – SPRING 2015



Chapter 13: Dynamics of a Rigid Body

version 04/08/2015 ~ 103 slides

We made it to slide #44 on Monday, April 6.

Get your clickers ready.

ANNOUNCEMENTS

- Clicker bonus points have been awarded for last Wednesday & last Friday.

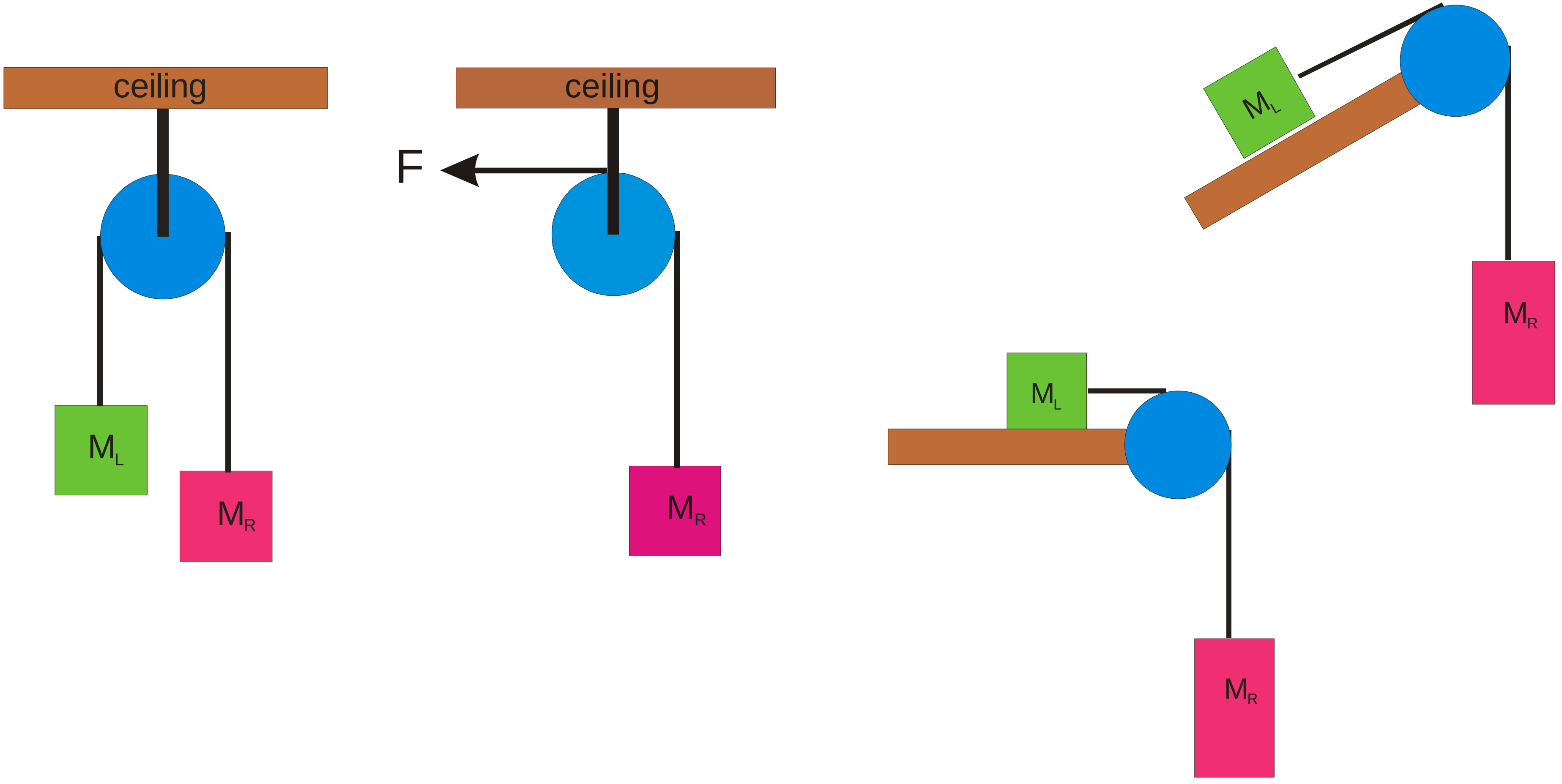
  The maximum # of bonus points in the class = 12.

- More opportunities to collect bonus points are coming.
  - more demos
  - more clicker questions
  - course evaluations
  - course methods survey

- Would you like me to post an Excel "GRADE ESTIMATOR" on Blackboard?

PULLEY PROBLEMS

The pulley systems shown below are all "*fair game*".



PULLEY EXAMPLE – ATWOOD's MACHINE

Ohanian Example #5

EIGHT PARAMETERS:

M_L, M_R, M_P, R, g, T_L, T_R a, α

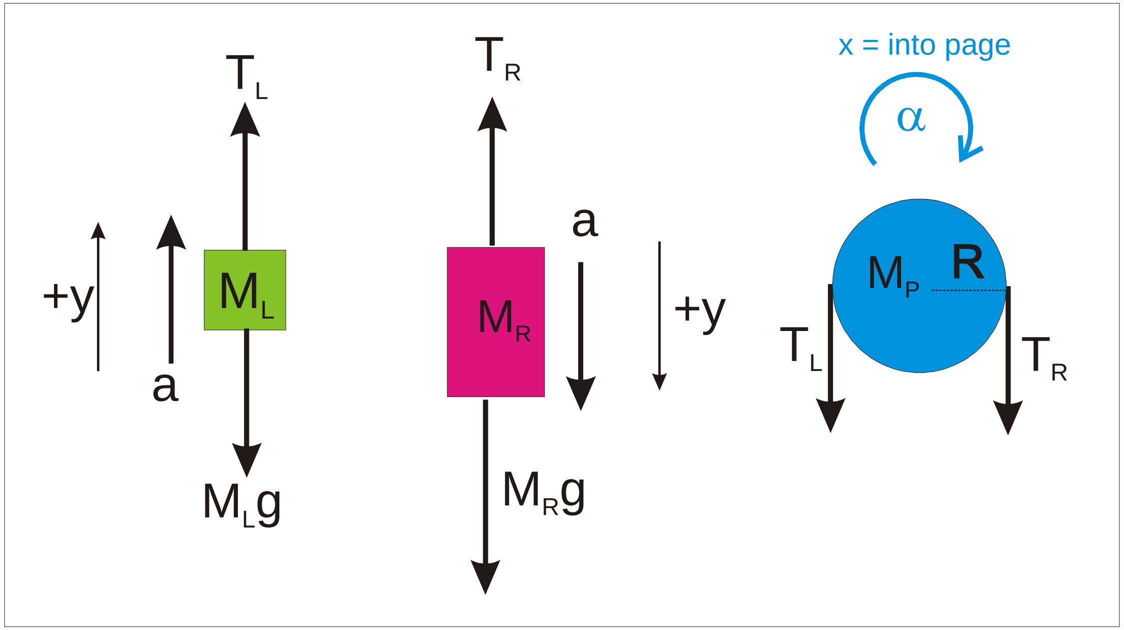
FOUR EQUATIONS

M_L: $T_L - M_L g = M_L a$

M_R: $-T_R + M_R g = M_R a$

M_P: $T_R R - T_L R = I\alpha$

$\alpha = a/R$

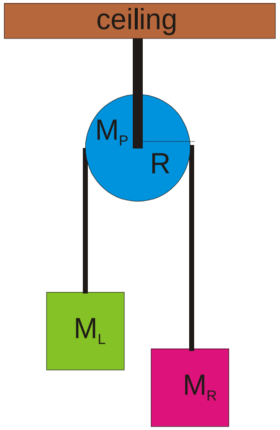


Given all but 4 of the parameters in these equations, you can solve these 4 equations for those 4 unknowns.

Commonly you're given the 3 masses + *R* and asked to solve for the 2 tensions and 2 accelerations (*a* & α).

But you might be given *a* and asked to solve for *g*.

The Atwood's machine is ideally suited to measure g since you can slow the motion considerably.



4 options

Two blocks are connected to each other by an ideal rope over a pulley that has mass *M*. The incline is frictionless.

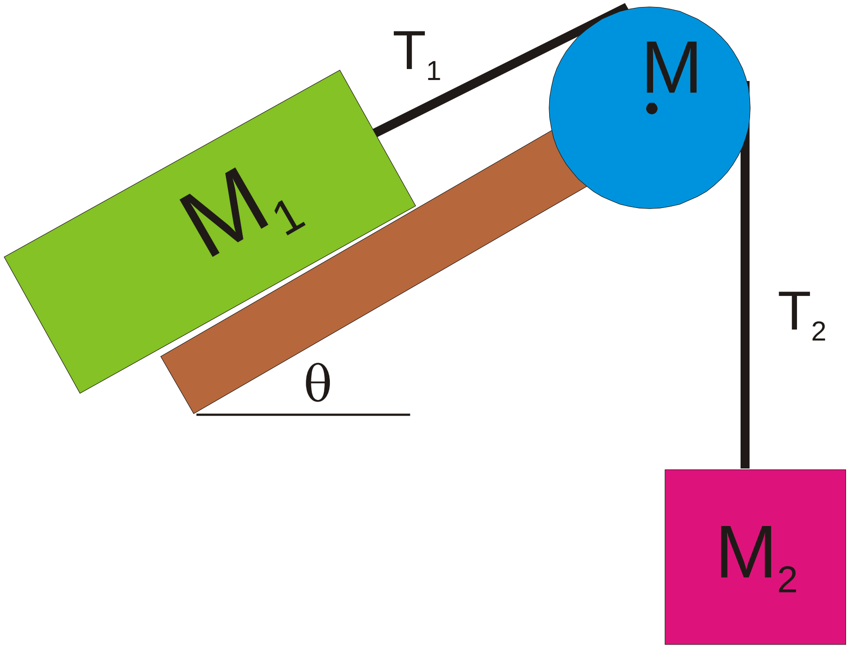
The relationship between T_1 and T_2 is described best by which of the following statements:

A. It depends on which is larger, $m_1 g\sin\theta$ or $m_2 g$.

B. $T_1 = T_2$

C. $T_1 > T_2$

D. $T_1 < T_2$



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Two blocks are connected to each other by an ideal rope over a pulley that has mass *M*. The incline is frictionless. The relationship between T_1 and T_2 is described best by which of the following statements:

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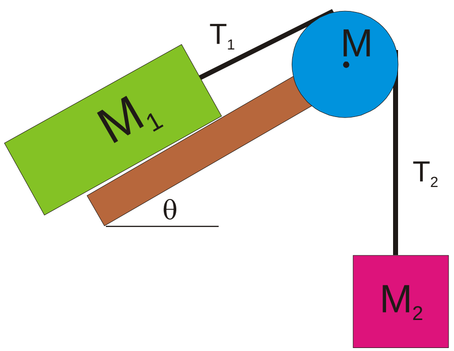


A complete analysis is provided on the next slide BUT

all you need to know to answer this question is which way the pulley is turning since this tells you the net torque and therefore the relation of T_1 to T_2.

The pulley is turning CW $\leftrightarrow$ $T_2 > T_1$.

The pulley is turning CCW $\leftrightarrow$ $T_1 > T_2$.

The pulley is not turning $\leftrightarrow$ $T_1 = T_2$.

You can tell which way the pulley is turning if you know the relationship between $m_1g\sin\theta$ and m_2g.

COMPLETE SOLUTION

$$m_2 g - T_2 = m_2 a \quad \text{[Sum of Forces on m}_2\text{ in } y\text{-direction]}$$

$$-m_1 g \sin\theta + T_1 = m_1 a \quad \text{[Sum of Forces on m}_1\text{ along incline]}$$

$$T_2 R - T_1 R = I\alpha = I\frac{a}{R} \underset{\text{divide by R}}{\longrightarrow} T_2 - T_1 = \frac{I}{R^2}a \quad \text{[Sum of Torques on pulley]}$$

or summing the three eqs.

$$m_2 g - m_1 g \sin\theta = (m_1 + m_2 + \frac{I}{R^2})a$$

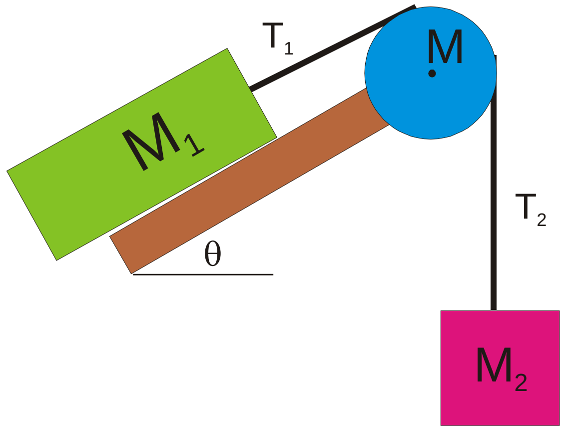
You can use this to write a and then solve for the T's

It's not necesary to do this to see that, for

$$m_2 > m_1 \sin\theta \;\Rightarrow\; a > 0 \;\Rightarrow\; T_2 > T_1$$

$$m_2 < m_1 \sin\theta \;\Rightarrow\; a < 0 \;\Rightarrow\; T_2 < T_1$$

$$m_2 = m_1 \sin\theta \;\Rightarrow\; a = 0 \;\Rightarrow\; T_2 = T_1$$



4 options

A force F is applied to a dumbbell for a time interval Δt. F is applied either to the center of the dumbbell as in (*a*) below or to one of the masses as in (*b*).

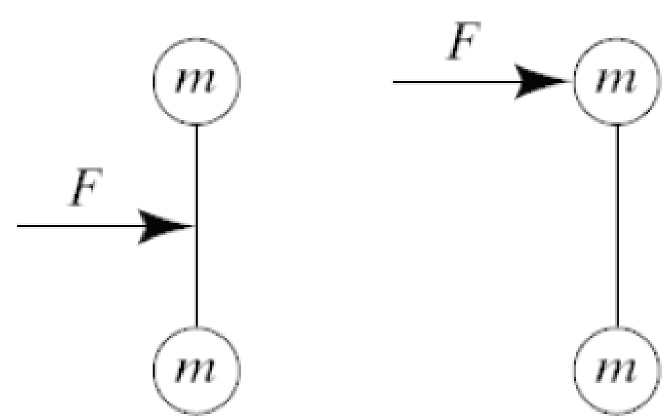
Which of the following is true for the resulting velocity or speed of the CM of the dumbbell?

A. They'll have the same CM velocity (*speed and direction*).

B. (*a*) will lead to the larger CM speed.

C. (*b*) will lead to the larger CM speed.

D. The CM speeds will be the same but the CM velocities (*directions*) will be different.



(*a*) (*b*)

4 options

A force F is applied to a dumbbell for a time interval Δt. F is applied either to the center of the dumbbell as in (*a*) below or to one of the masses as in (*b*). Which of the following is true for the resulting velocity or speed of the CM of the dumbbell?

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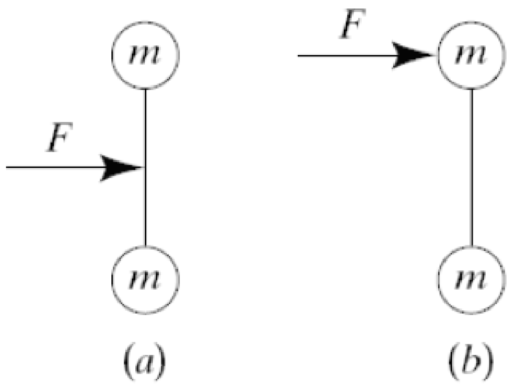


F is an external force so the acceleration of the CM of the dumbbell system is $a = F/m$ & the velocity of the CM is $v_{CM} = a\Delta t$ no matter where F is applied.

It's important that the force is applied for the same <u>time</u>, not the same <u>distance</u>.

F does lead to a torque in (*b*) and therefore a rotation but that's an issue for a different clicker question.

4 options

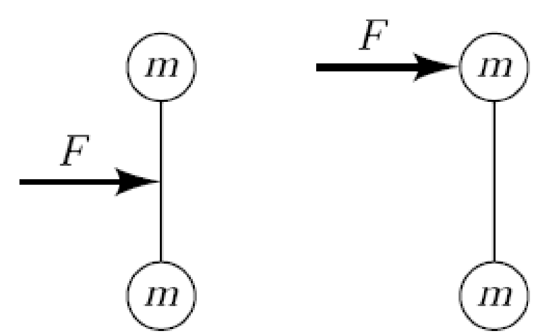
A force F is applied to a dumbbell for a time interval Δt, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater kinetic energy?

A. (a)

B. (b)

C. no difference

D. the answer depends on the rotational inertia of the dumbbell.



4 options

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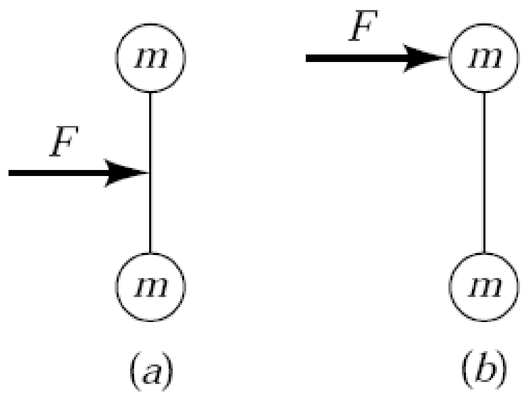


A. (a)

B. (*b*)

C. no difference

D. The answer depends on the rotational inertia of the dumbbell.

We've just shown that the velocities of the CM will be the same for both cases $\Rightarrow$ the kinetic energy due to translational motion is the same.

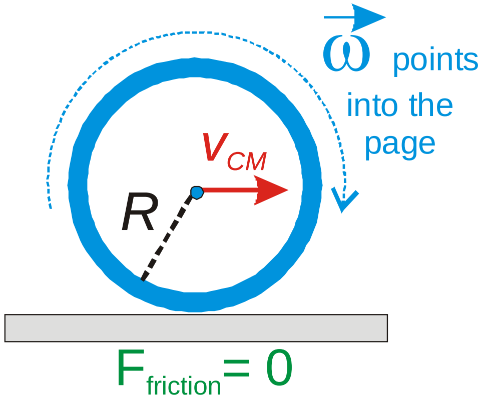
HOWEVER, **F** leads to a torque in (b) and therefore a rotation $\Rightarrow$ there is also kinetic energy of rotation in this case, hence more overall kinetic energy.

Note that the WORK done by F depends on the DISTANCE over which it acts (*both translationally and rotationally*) and this will equal the overall change in kinetic energy.

F works over a larger distance as the dumbbell rotates. (*This is actually due to the* τ *&* $\Delta\theta$ *that are added.*)

ROLLING WHEELS

What are the forces & torques?



- If a wheel rolls **at constant speed** on a horizontal surface:
- $\Sigma F_{horizontal} = 0$ since $a = 0$ & $\Sigma \tau_{CM} = 0$ since $\alpha = 0$

  $\Rightarrow$ There is no friction from the ground!
- Remember that $F_{static\text{-}friction} \leq \mu_s N \Rightarrow F_{static\text{-}friction} = 0$ is fine

  ~ book at rest on a table.
- $F_{static\text{-}friction}$ only comes into play when you accelerate a wheel.
- $F_{kinetic\text{-}friction}$ only comes into play if the wheel slips/slides without rolling or without as much rolling as needed to match v_{cm}.
- We will not cover wheels that both slip and roll in PHYS 121
  - The transition between slipping and rolling is an interesting topic

    - for a more advanced course or for a Dr.C. homework problem!

ROLLING WHEELS *that* ACCELERATE!

Consider two ways to make a wheel accelerate.

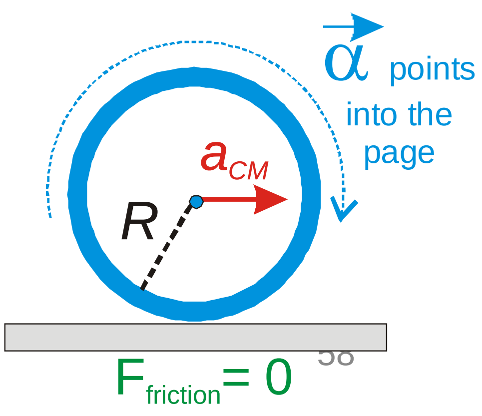
- Apply a <u>horizontal force</u> to the CM/axle

  ~ rear wheels on a front wheel drive car.

- Apply a <u>torque</u> about the axle

  ~ rear wheels on a rear wheel drive car.

Engines & drive axles supply a TORQUE to make wheels rotate (*faster*).

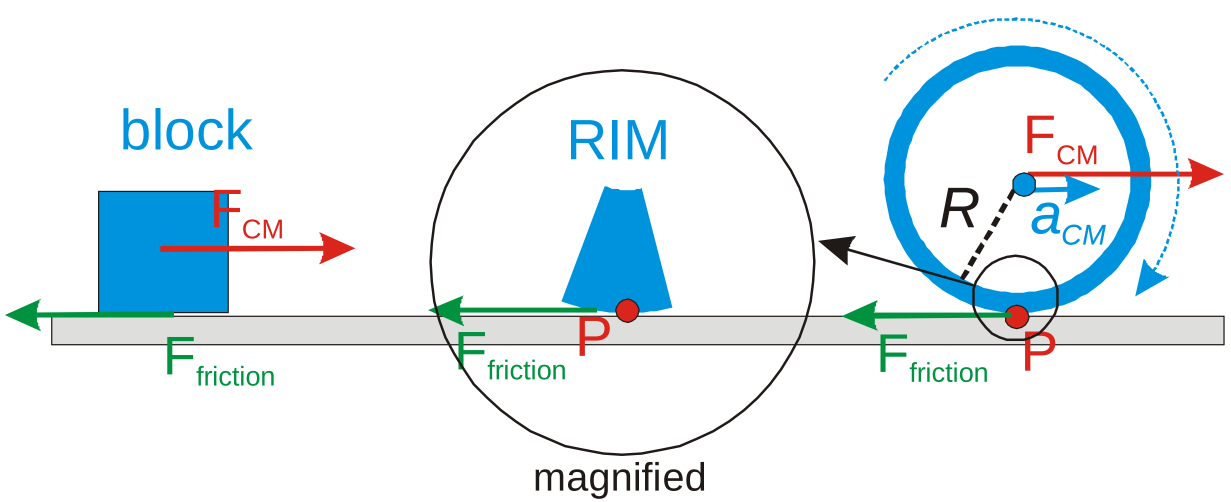


ACCELERATING WHEELS *with* FORCE

- A horizontal force is applied to the right on an axle/CM.

A block is shown for comparison in the figure below.

- If the wheel doesn't start rotating (*faster*) point P will slide to the right.
- $F_{static\text{-}friction}$, pointing to the left, opposes acceleration of the piece of rim in contact with the road, forcing the wheel to "rotate" about point P.
- To calculate a_{CM} you need $\Sigma F_{horizontal} = F_{CM} - F_{static\text{-}friction}$.
- But we don't know the magnitude of $F_{static\text{-}friction}$.
- We can work around this using $\Sigma\tau_P$ since $\tau_{friction} = 0$ about point P.



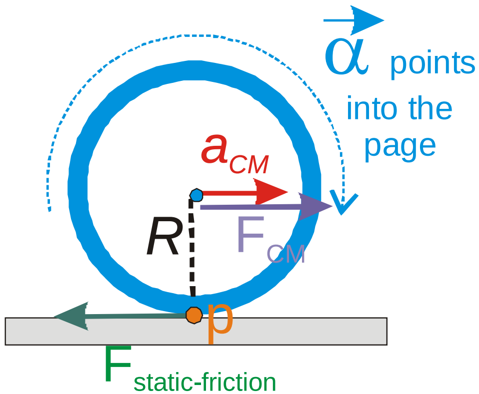
ACCELERATING WHEELS *with* FORCE

At any given instant (noting that $a_{CM} = \alpha_{CM} R$)

& using the parallel axis theorem, $I_P = I_{CM} + MR^2$

$$\tau_P = RF_{CM} = I_P \alpha = \left(I_{CM} + MR^2\right)\left(\frac{a_{CM}}{R}\right)$$

$$a_{CM} = \frac{R^2 F_{CM}}{\left(I_{CM} + MR^2\right)}$$



- ✓ If $I_{CM} = 0 \Rightarrow a_{CM} = F_{CM}/M$ as expected
- ✓ Larger $I_{CM} \Rightarrow$ smaller acceleration (*hoop is slower than a disc*)
- ✓ $F_{static\text{-}friction}$ reduces acceleration although μ_S doesn't appear in a_{CM}.
  - It does this by forcing the wheel to rotate, introducing the I_{CM} term.
- ✓ The wheel would accelerate faster without friction, without rolling at all.
  - But it would not accelerate at all if you apply a torque to the axle rather than a horizontal force to the CM; it would just spin in place.

ACCELERATING WHEELS *with* FORCE

How large is $F_{static\text{-}friction}$?

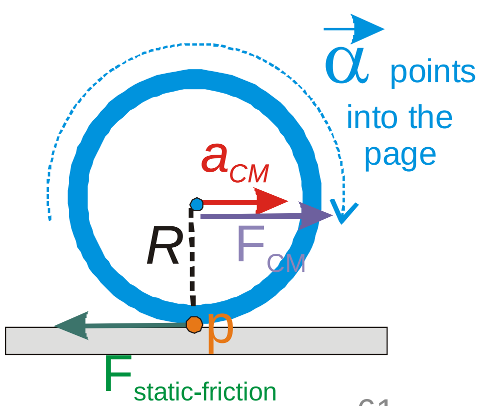
The net force acting on the wheel must satisfy $\Sigma F_x = Ma_x$

$$\sum F_x = F_{CM} - F_{static-friction} \qquad \& \qquad \sum F_x = Ma_{CM} = \frac{MR^2 F_{CM}}{\left(I_{CM} + MR^2\right)}$$

$$\Rightarrow F_{static-friction} = F_{CM}\left(1 - \frac{MR^2}{\left(I_{CM} + MR^2\right)}\right) = F_{CM}\left(\frac{\left(I_{CM} + MR^2\right)}{\left(I_{CM} + MR^2\right)} - \frac{MR^2}{\left(I_{CM} + MR^2\right)}\right)$$

$$F_{static-friction} = F_{CM}\left(\frac{I_{CM}}{I_{CM} + MR^2}\right)$$

This applies UP TO the maximum value $|F_{static\text{-}friction}| = \mu_s N$, after which the wheel starts slipping because you are trying to accelerate too quickly with too large a F_{CM}.



ACCELERATING WHEELS *with* TORQUE

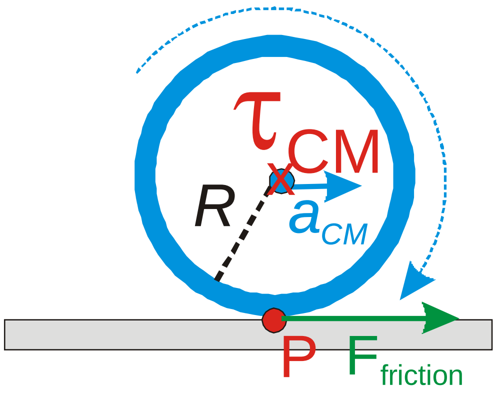
If you apply a torque τ_{CM} , pointing into the page, about the axle rather than a horizontal force at the axle:

- The only horizontal force making the wheel accelerate is $F_{static\text{-}friction}$.
- Without friction, the wheel would spin in place.
- The section of rim in contact with the ground would tend to move to the left.

$\Rightarrow$ $F_{static\text{-}friction}$ points to the right. (*NOT to the left as previously.*)

$$\tau_{CM} = I_{CM}\alpha = I_{CM}\left(\frac{a_{CM}}{R}\right)$$

$$\Rightarrow a_{CM} = \frac{\tau_{CM} R}{I_{CM}}$$



But there is a maximum possible a_{CM} given by the maximum possible

$$F_{static\text{-}friction} \le \mu_s N = \mu_s Mg$$

$$a_{max} = F_{static\text{-}friction\text{-}max}/M = \mu_s g$$

ROLLING DOWN A RAMP

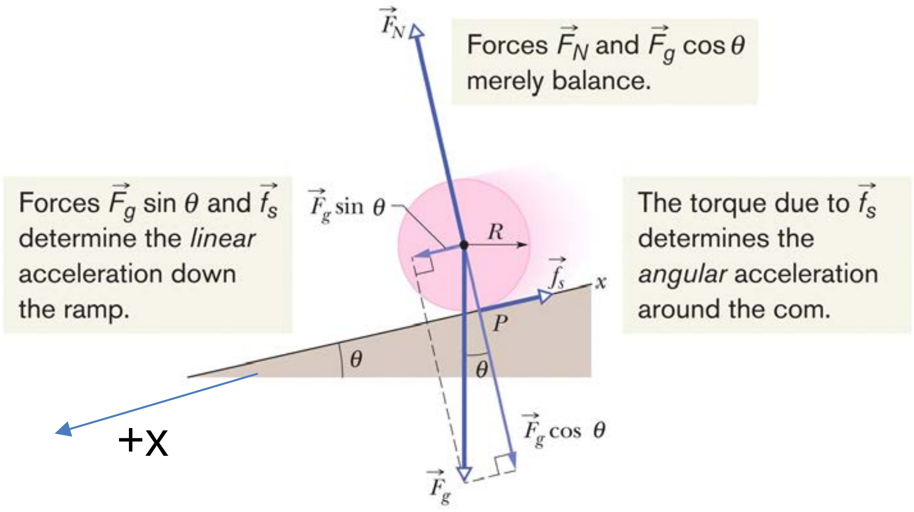
(~ Ohanian Example #7 to review on your own)

$$\sum F_x = Mg\sin\theta - F_{static-friction} = Ma_{CM}$$

$$\sum \tau_{CM} = RF_{static-friction} = I_{CM}\alpha_{CM}$$

$$a_{CM} = \alpha_{CM} R$$

$$F_{static-friction} = \frac{I_{CM}\alpha_{CM}}{R} = \frac{I_{CM}a_{CM}}{R^2}$$



Plug this into the first equation above.

$$Mg\sin\theta - \frac{I_{CM}a_{CM}}{R^2} = Ma_{CM}$$

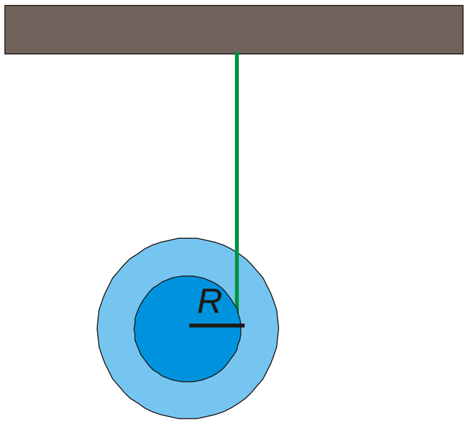
$$a_{CM} = \frac{g\sin\theta}{1+\frac{I_{CM}}{MR^2}}$$

This is the same answer we would find with our general solution

$$a_{CM} = \frac{R^2F_{CM}}{\left(I_{CM}+MR^2\right)}$$

$$\rightarrow a_{CM} = \frac{R^2Mg\sin\theta}{\left(I_{CM}+MR^2\right)} = \frac{g\sin\theta}{\left(1+\frac{I_{CM}}{MR^2}\right)}$$

YO-YO's



How fast is a yo-yo falling

***in terms of its velocity at height h*?**

Use CONSERVATION of ENERGY

After falling a height h from rest, the yo-yo's velocity is given by

$$E_o = E_f \rightarrow U_{gravity} = K_{translation} + K_{rotation}$$

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \quad \rightarrow \quad v = (2gh - I\omega^2/M)^{1/2}$$

If it simply fell, $v = (2gh)^{1/2}$ so $a < g$

This reasoning applies to any object that rotates as it travels with

$$v_{tan} = \omega R.$$

In terms of forces, the tension T in the string supplies an upward force that is absent if the yo-yo simply falls.

YO-YO's

How fast does a yo-yo accelerate?

We can use the general purpose formula derived earlier, with $F_{CM} = F_{gravity} = Mg$.

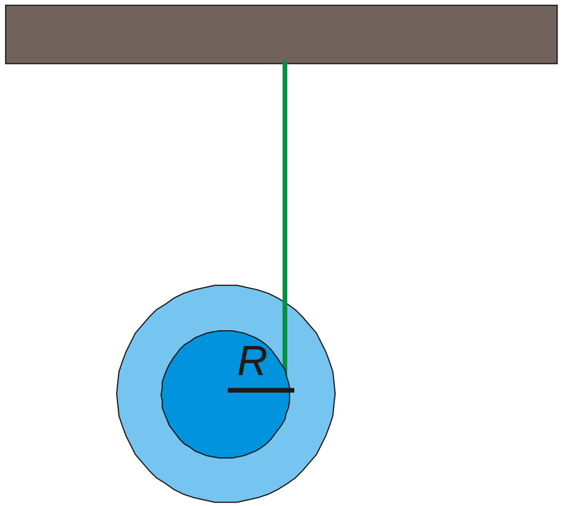
The yo-yo is essentially 'rolling' along its support string.

Note that the radius of the axle is used for $v_{CM} = \omega r$.

$$a_{CM} = \frac{R^2 F_{CM}}{\left(I_{CM} + MR^2\right)}$$

$$\rightarrow a_{CM} = \frac{-R^2 Mg}{\left(I_{CM} + MR^2\right)}$$

$$a_{CM} = -\frac{g}{\left(1 + \frac{I_{CM}}{MR^2}\right)}$$



YO-YO

How fast does a yo-yo accelerate?

Starting from 'scratch', you can analyze the motion as follows:

$$\sum F_y = T - Mg = Ma_{CM}$$

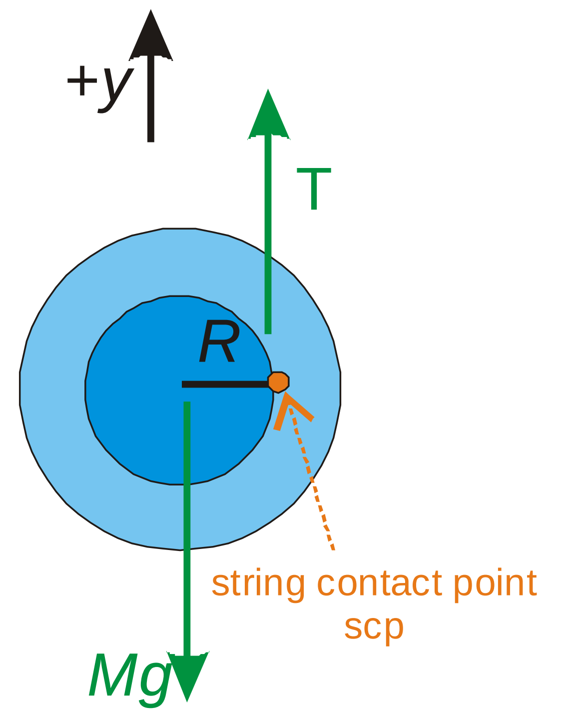
$$\sum \tau_{\text{string-contact-point}} = -MgR = I_{scp}\alpha = \left(I_{CM} + MR^2\right)\left(\frac{a_{CM}}{R}\right)$$

You could calculate τ about the CM if you prefer.

Note that we only need the second equation to find a_{CM}

$$\rightarrow a_{CM} = \frac{-MgR^2}{\left(I_{CM} + MR^2\right)}$$

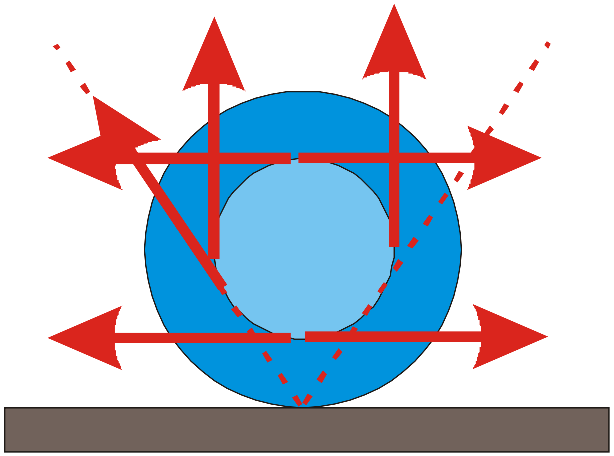
$$a_{CM} = -\frac{g}{\left(1 + \dfrac{I_{CM}}{MR^2}\right)}$$



Yo-Yo's / Bobbins / Spools

http://www.merrimacspool.com/images/Heavy%20Duty%20Spool.jpg

- A spool is at rest on a flat surface.
- Friction on the surface makes the spool roll without slipping.
- A rope wound around and attached to the spool can be pulled in various directions as shown in the figure.
- **In what direction will the spool roll? What does τ_{string} do?**
- **Will the rope wind or unwind as the spool rolls? What is ω?**

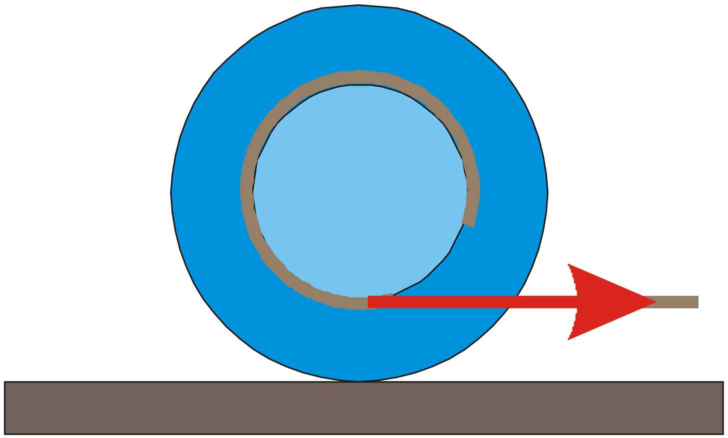


5 options

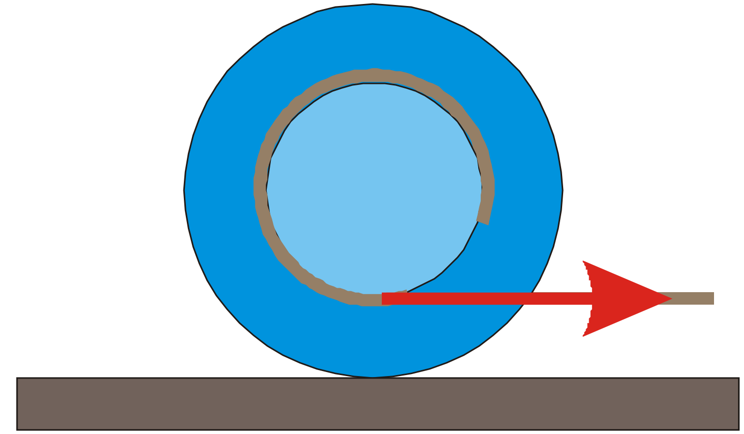
The force is applied from the bottom of the spool.

- In what direction will the spool roll?
- Will the rope wind or unwind as the spool rolls?

A. Rolls left & winds
B. Rolls left & unwinds
C. Rolls right & winds
D. Rolls right & unwinds
E. None of the above



5 options



The force is applied from the bottom of the spool.

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- Will the rope wind or unwind as the spool rolls?

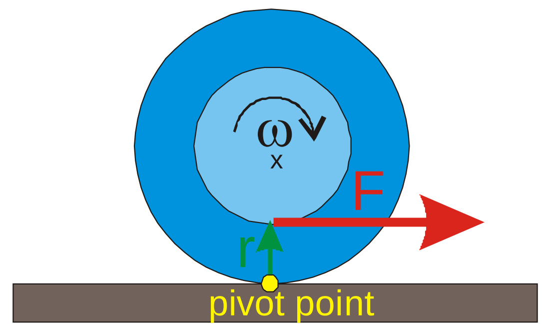
A. Rolls left & winds

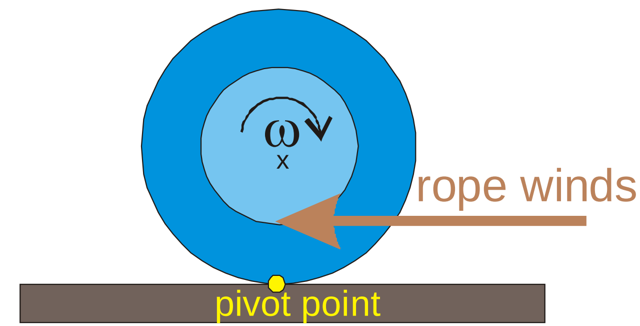
B. Rolls left & unwinds

C. Rolls right & winds

D. Rolls right & unwinds

E. None of the above





You can get the direction it rolls from the direction of the torque about the point of contact with the floor.

The torque makes it rotate about this point since the spool does not slip.

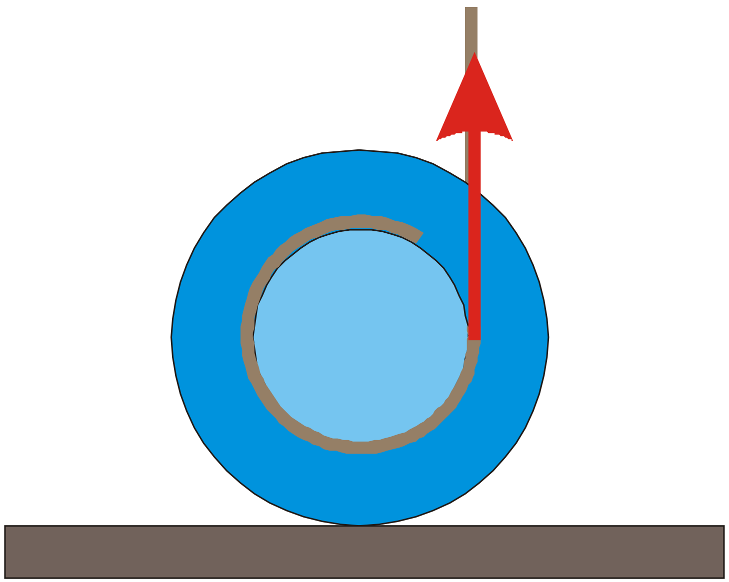
You can see the rope winds from the direction of ω_{CM} .

5 options

The force is applied vertically from the side of the spool.

- In what direction will the spool roll?
- Will the rope wind or unwind as the spool rolls?

A. Rolls left & winds
B. Rolls left & unwinds
C. Rolls right & winds
D. Rolls right & unwinds
E. None of the above



5 options

The force is applied vertically from the right side of the spool.

- In what direction will the spool roll?
- Will the rope wind or unwind as the spool rolls?

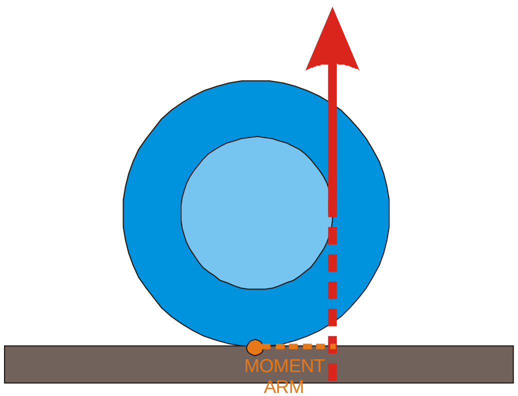
A. Rolls left & winds

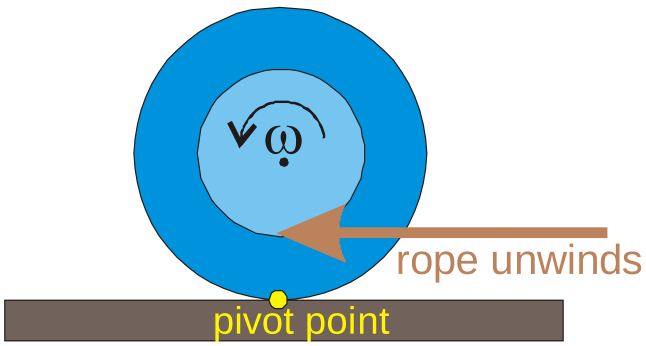
B. Rolls left & unwinds

C. Rolls right & winds

D. Rolls right & unwinds

E. None of the above





You can get the direction it rolls from the direction of the torque about the point of contact with the floor.

5 options

The force is applied vertically at an angle (*exactly*) as shown.

- In what direction will the spool roll?
- Will the rope wind or unwind as the spool rolls?

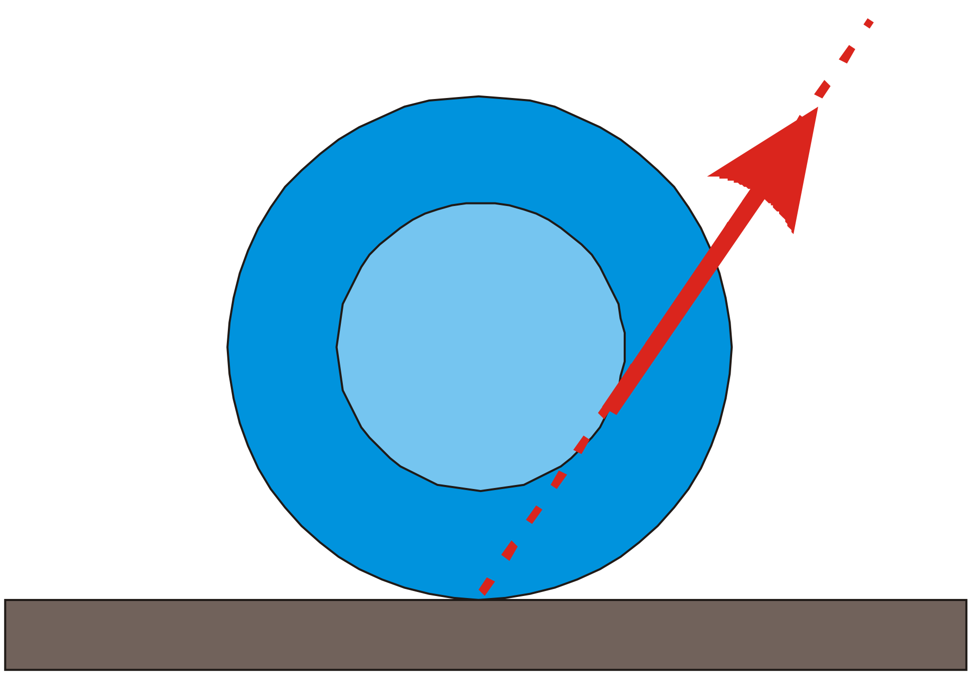
A. Rolls left & winds

B. Rolls left & unwinds

C. Rolls right & winds

D. Rolls right & unwinds

E. None of the above



5 options

The force is applied vertically from an angle (*exactly*) as shown.

- In what direction will the spool roll?
- Will the rope wind or unwind as the spool rolls?

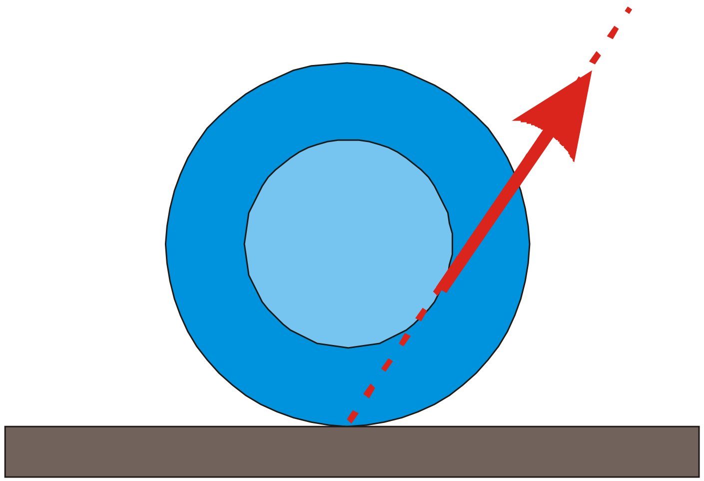
A. Rolls left & winds

B. Rolls left & unwinds

C. Rolls right & winds

D. Rolls right & unwinds

E. None of the above



There is no torque about the contact point with the floor.

If you pull with enough force to overcome static friction, the spool slides along the floor without rolling.

DEMO

SPOOL & ROPE

https://www.youtube.com/watch?v=S3D4YyBMwk4&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr&index=16

ANGULAR MOMENTUM

Translational momentum **P**

tells you that an object moving with some translational velocity **v** will continue moving at that velocity unless some ***force*** acts on it.

Angular momentum describes the fact that,

if a body is rotating at some angular velocity **ω**,

it will continue rotating at that angular velocity unless some ***torque*** acts on it.

$$\Delta P = \Delta\left(mv\right) = F\,\Delta t$$

ANGULAR MOMENTUM

OF A PARTICLE, SYSTEM OF PARTICLES & A RIGID BODY

The translational momentum of an object is

$$\vec{P} = M\vec{v} \quad \text{with} \quad \vec{F} = \frac{d\vec{P}}{dt}$$

In rotational dynamics

$$M \rightarrow \mathrm{I} \qquad v \rightarrow \omega \qquad \mathrm{F} \rightarrow \tau$$

$\Rightarrow$ **ANGULAR MOMENTUM** *might* be

$$\vec{L} = I\vec{\omega} \quad \& \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

Let's see if this holds true.

ANGULAR MOMENTUM OF A PARTICLE

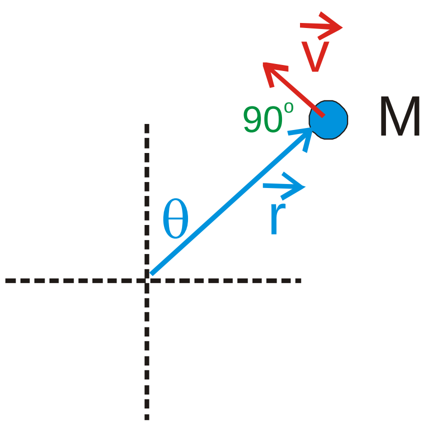
$$\vec{L} = I\vec{\omega} \quad \rightarrow \quad \ell = \left(mr^2\right)\left(\frac{v_\perp}{r}\right) = mrv_\perp$$

where $v_\perp = v_{\tan}$ is the component of $\vec{v}$ perpendicular to $\vec{r}$

$$\left|\vec{\ell}\right| = mrv_\perp = \left|m\left(\vec{r}\times\vec{v}\right)\right| = \left|\vec{r}\times\vec{p}\right|$$

$\vec{\ell} = \vec{r}\times\vec{p} = m\left(\vec{r}\times\vec{v}\right)$ where ℓ is a script, lower-case L

$\vec{\ell} \rightarrow$ single particle $\qquad \vec{L} \rightarrow$ more complex object



$$\left|\vec{\ell}\right| = rp\sin\theta_{rp} = mrv\sin\theta_{rp}$$

$$\left|\vec{\ell}\right| = mr\left(v\sin\theta_{rp}\right) = mrv_\perp$$

$$\left|\vec{\ell}\right| = m\left(r\sin\theta_{rp}\right)v = mr_\perp v$$

The direction of $\vec{\ell}$ comes from the RHR for cross-products

(*to review on your own*)

PROOF of $\vec{\tau} = \dfrac{d\vec{\ell}}{dt}$

$$\frac{d\vec{\ell}}{dt} = \frac{d}{dt}\left(\vec{r} \times \vec{p}\right) = \frac{d}{dt}\left(\vec{r} \times m\vec{v}\right) = m\frac{d}{dt}\left(\vec{r} \times \vec{v}\right)$$

$$\frac{d\vec{\ell}}{dt} = m\left(\frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}\right)$$

$$\frac{d\vec{\ell}}{dt} = m\left(\vec{v} \times \vec{v} + \vec{r} \times \vec{a}\right) = m\left(\vec{r} \times \vec{a}\right)$$

$$\frac{d\vec{\ell}}{dt} = \vec{r} \times \left(m\vec{a}\right) = \vec{r} \times \vec{F} = \vec{\tau}$$

$$\frac{d\vec{\ell}}{dt} = \vec{\tau}$$

QED

(*to review on your own*)

ANGULAR MOMENTUM OF A RIGID BODY

A rigid body is constructed of many particles moving with the same ω. Each particle has angular momentum

$$\vec{\ell}_i = m_i\left(\vec{r}_i \times \vec{v}_i\right) = \vec{r}_i \times \vec{p}_i$$

Since $\vec{v}_i$ is perpendicular to $\vec{r}_i$ for rigid body rotation

$$\ell_i = m_i r_i v_i$$

$$\text{but } \omega_i = \omega = \frac{v_i}{r_i} \quad \Rightarrow \quad v_i = \omega r_i$$

$$\ell_i = m_i r_i^2 \omega$$

$$\vec{L} = \sum_i m_i r_i^2 \omega = I\omega$$

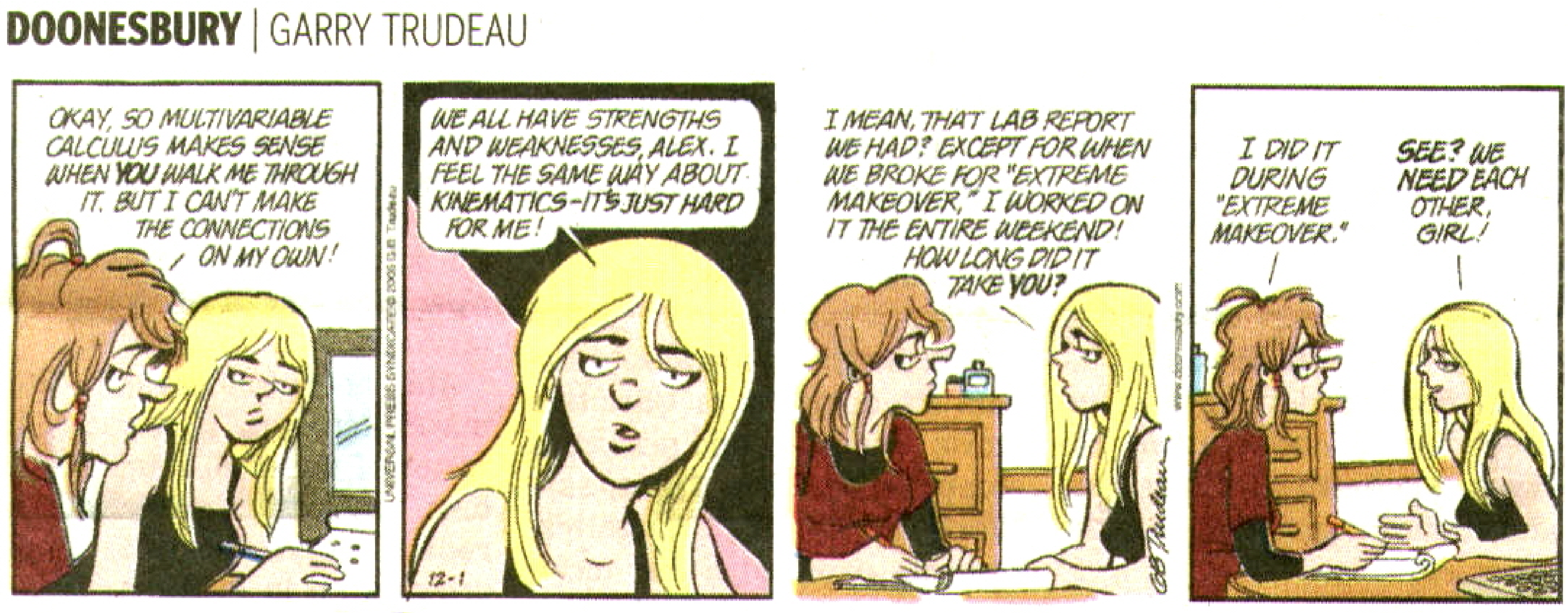
$$\vec{\tau}_i = \frac{d\vec{\ell}}{dt} \text{ for a single particle}$$

Summing over all the particles in a rigid body

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

We made it to slide #79 on Wednesday, April 8.

PHYS 121 – SPRING 2015



Chapter 13: Dynamics of a Rigid Body

version 04/10/2015 ~ 103 slides

We made it to slide #79 on Wednesday, April 8.

No clickers today.

ANNOUNCEMENTS

- Welcome high school visitors!
- Exam # 3 from spring 2014 is available on Blackboard, under COURSE DOCUMENTS<EXAMS<PRACTICE EXAMS.
- If you might miss Exam #3 next Friday for varsity sports or some other legitimate reason, let me know ASAP!

PREVIOUSLY in PHYS 121

ANGULAR MOMENTUM

- RIGID BODY: $\vec{L} = I\vec{\omega} \quad \sim \quad \vec{P} = m\vec{v}$

- PARTICLE: $\vec{\ell} = \vec{r} \times \vec{p} = m\left(\vec{r} \times \vec{v}\right) \quad : \quad \left|\vec{\ell}\right| = mrv_{\perp} = mr_{\perp}v$

- TORQUE: $\vec{\tau} = \dfrac{d\vec{L}}{dt} \quad \sim \quad \vec{F} = m\vec{a} = \dfrac{d\vec{p}}{dt}$

CONSERVATION OF *ANGULAR* MOMENTUM

$$\tau = \frac{dL}{dt}$$

If the net external torque acting on a system is zero,

that system's

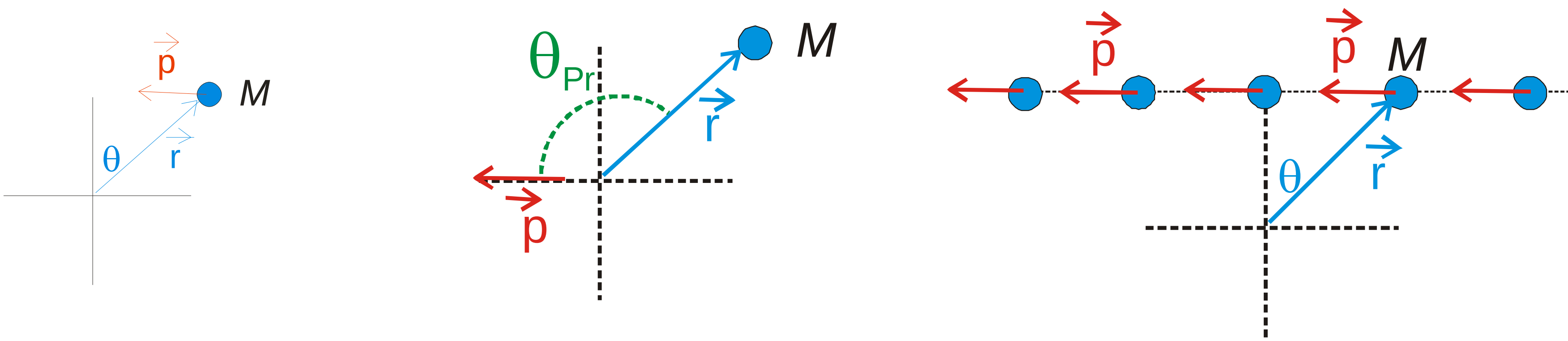
ANGULAR MOMENTUM IS CONSERVED.

[*SHORT CULTURAL ASIDE; thinking like a physicist, if there's no special angle in space, then angular momentum is conserved.*]

$$\tau = \frac{dL}{dt} = 0 \quad \Rightarrow \quad L \text{ is constant}$$

$$\vec{\ell} = \vec{r} \times \vec{p} \qquad \vec{\ell} = \vec{r} \times \vec{p} = m\left(\vec{r} \times \vec{v}\right) \;:\; \left|\vec{\ell}\right| = mrv_{\perp} = mr_{\perp}v$$

What is the angular momentum of M with respect to the origin as the particle moves along the linear path shown below with **constant p** = M**v?**



$$\ell = Mvr\sin\theta_{pr}$$

$$\ell = Mvr_{\text{perpendicular}} = Mvr\cos\theta$$

r & **θ** change but ($r\cos\theta$) is constant.

$\Rightarrow$ **ANGULAR MOMENTUM IS CONSERVED**

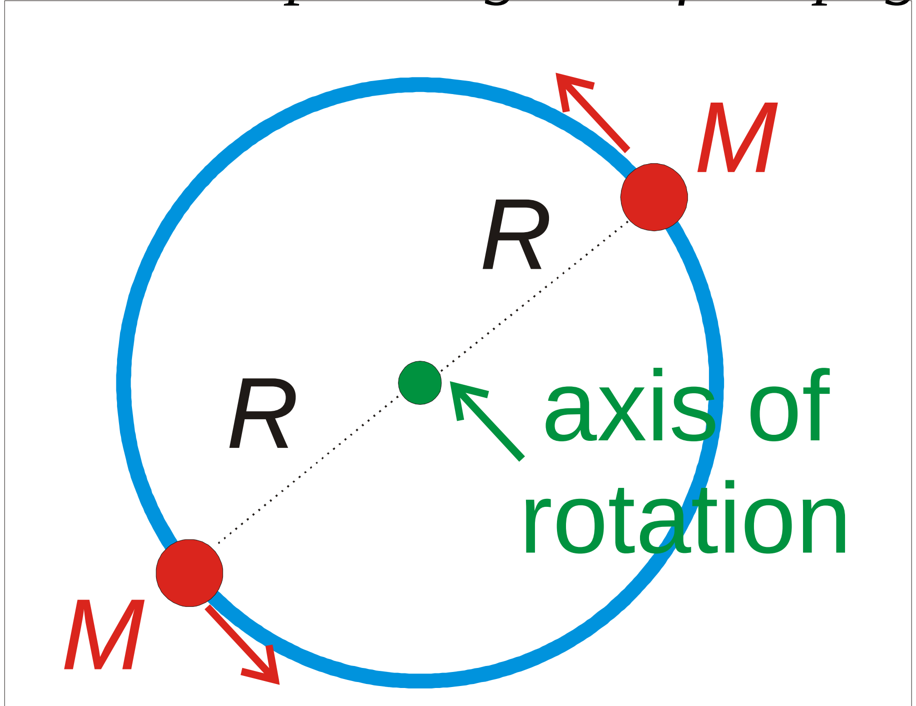
which we also know from $\tau = 0$.

ANGULAR MOMENTUM EXAMPLES

Two point masses *M* are circling an axis halfway between them, traveling counterclockwise in a circle of radius *R* at an angular frequency ω.

If those masses are pulled closer together in a radial direction, shrinking the circle to *R*/2, how does ω change?

(*Note that ω is a vector pointing out of the page at you.*)



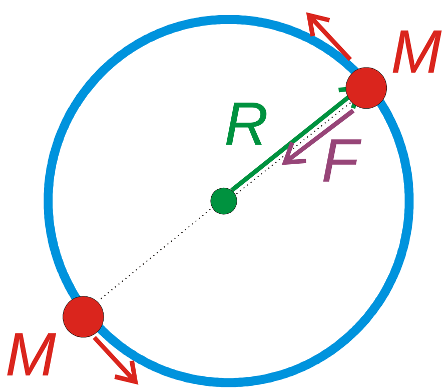
ANGULAR MOMENTUM EXAMPLES

First you need to say the *magic words* that justify the use of conservation of angular momentum.

The force pulling the masses towards the center does not result in a torque,

since that force is parallel to ***r***.

Zero torque ⇒ angular momentum is conserved.



ANGULAR MOMENTUM EXAMPLES

The initial moment of inertia is $I_o = 2MR^2$

Call the initial angular velocity ω_o

$\rightarrow$ the initial angular momentum is

$$L_o = I_o\omega_o = 2MR^2\omega_o$$

The final angular momentum is

$$L_f = I_f\omega_f = 2M(R/2)^2\omega_f = \tfrac{1}{2}MR^2\omega_f$$

but conservation of angular momentum says

$$L_o = L_f$$

$$2MR^2\omega_o = \tfrac{1}{2}MR^2\omega_f$$

$$\omega_f = 4\omega_o$$

which is quite a difference!

It takes WORK = $\mathbf{F}\cdot\mathbf{d}$ to pull the masses in $\Rightarrow$ kinetic energy increases.

$K_o = \tfrac{1}{2}I_o\omega_o^2$ $\quad$ $K_f = \tfrac{1}{2}I_f\omega_f^2 = \tfrac{1}{2}[(1/4)MR^2][4\omega_o]^2 = 4K_o$

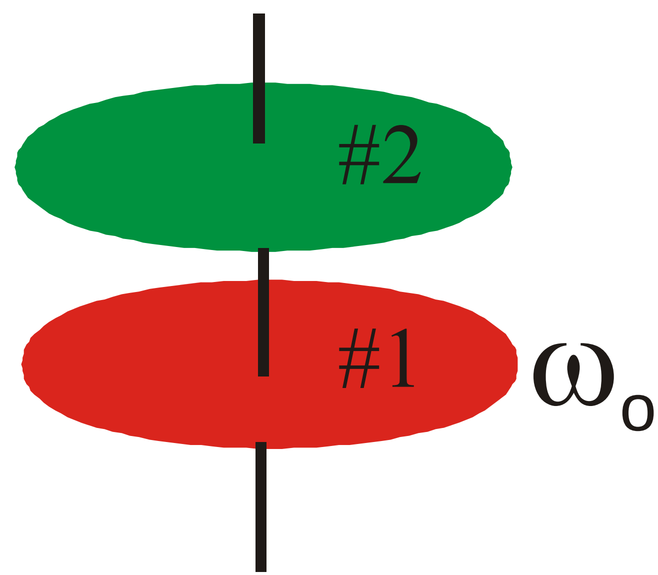
CONSERVATION OF ANGULAR MOMENTUM IN THE CLUTCH

Two discs, each of mass M & radius R are supported by a common vertical frictionless axle. (*similar to an old record changer*)

- Initially, disc #1 is spinning at angular velocity ω_o while disc #2 is at rest slightly above disc #1.
- Then disc #2 is released & drops onto disc #1.
- After a short time, friction brings both discs to the same (*angular*) speed. *The motor is off!*

What is that (*angular*) speed?

http://static.pascal-gienger.de/pgienger/w250.jpg



CONSERVATION OF ANGULAR MOMENTUM IN THE CLUTCH

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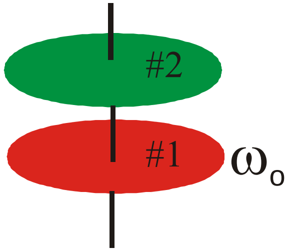


magic words: Friction is an <u>internal</u> force; there are no external torques

$\Rightarrow$ angular momentum is conserved.

$$L_{final} = L_{initial}$$

with $L = I\omega$ & $I = \frac{1}{2}MR^2$

$$(\tfrac{1}{2}MR^2 + \tfrac{1}{2}MR^2)\omega_f = (\tfrac{1}{2}MR^2)\omega_o$$

$$\omega_f = \tfrac{1}{2}\omega_o$$

Kinetic energy $K = \frac{1}{2}I\omega^2$ is NOT conserved

$K_f = \frac{1}{2}(2I)(\omega/2)^2 = \frac{1}{2}K_o \Rightarrow \frac{1}{2}K_o$ is lost to heat.

How hard would this problem have been with $\tau = I\alpha$, worrying about friction forces?

http://static.pascal-gienger.de/pgienger/w250.jpg

CONSERVATION OF ANGULAR MOMENTUM IN SPACE

The orientation of satellites is generally controlled by *reaction wheels* rather than rockets.

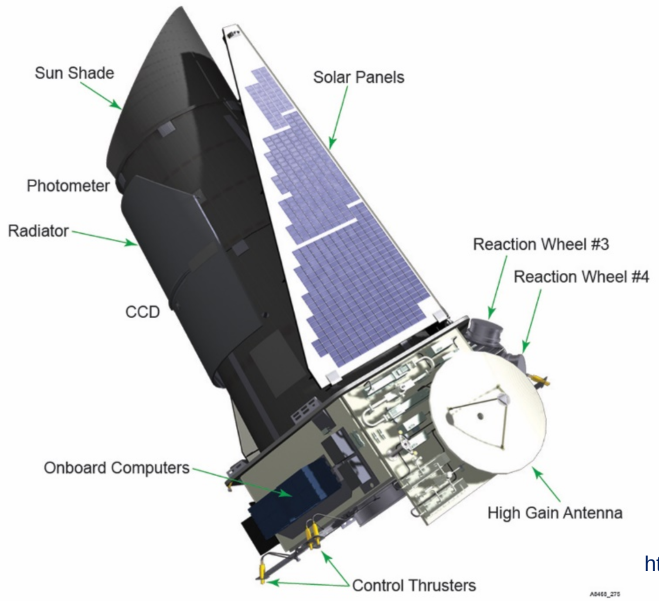
There are no *external* torques acting on a satellite.

$\Rightarrow$ its angular momentum is conserved

$\Rightarrow$ if a motor spins an internal flywheel CW, the rest of the satellite must spin CCW.

$\Rightarrow$ when the reaction wheel stops spinning, the satellite stops spinning too, with a new orientation.

$\Rightarrow$ Dr. C. homework problem on exchanging a hula hoop in space.



http://www.nasa.gov/mission_pages/kepler/news/keplerm-20130521.html

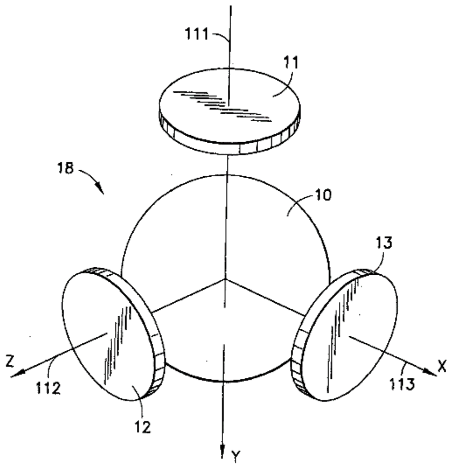
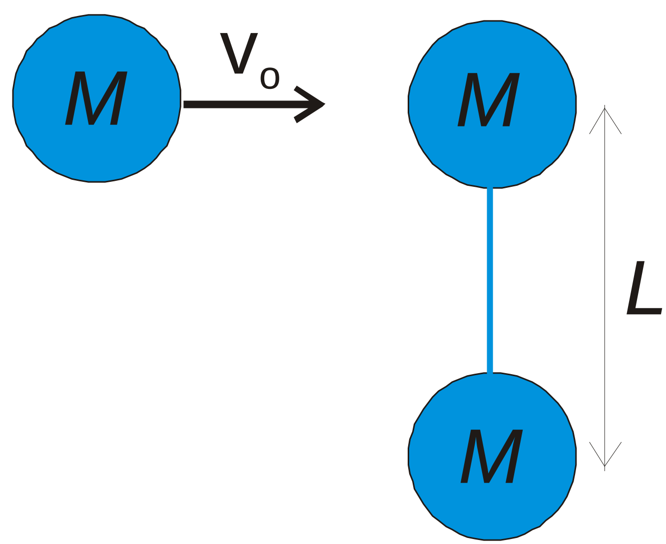


FIG.3

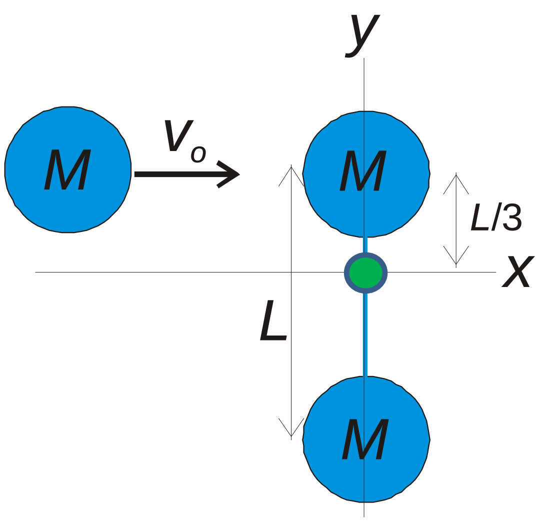
COLLISIONS WITH DUMBBELLS

A dumbbell constructed of two masses M separated by a light rod of length L is at rest on a sheet of ice when it is struck at one end by a hockey puck of mass M moving at velocity v_o which sticks to one of the masses.

Describe the subsequent motion of this system.



COLLISIONS WITH DUMBBELLS



Choose your coordinate system wisely.

Use the Center of Mass!

Place the x-axis, $y = 0$, at the height of the CM a distance $L/3$ from the top M.

magic words → Conservation of Linear Momentum

$Mv_o = 3Mv_{f\text{-}CM}$ → $v_{f\text{-}CM} = (1/3)v_o$

magic words → Conservation of Angular Momentum

$L_o = Mv_o(L/3)$

$L_f = I\omega_{CM} = [(2M)(L/3)^2 + M(2L/3)^2]\omega_{CM} = (2/3)ML^2\omega_{CM}$

$L_o = L_f$ → $\omega_{CM} = ½v_o/L$

Could you solve a similar problem

with a uniform rod instead of a dumbbell?

This has been given as an exam problem in the past!

4 options

$$\vec{L} = \vec{r} \times \vec{p} \qquad \left|\vec{L}\right| = rp\sin\theta_{rp}$$

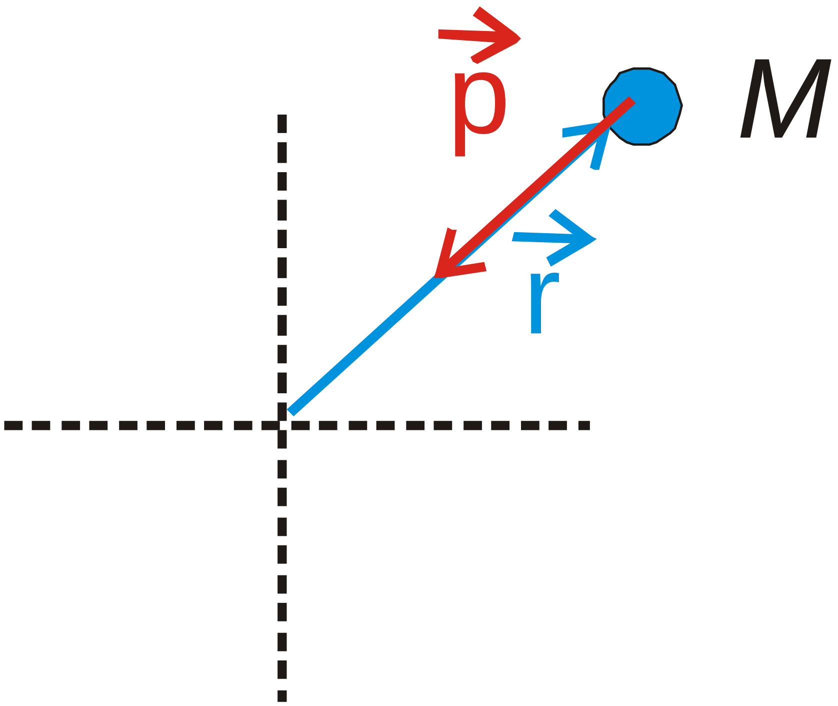
What is the angular momentum of *M* with respect to the origin in the situation shown below, assuming **p** = *M***v**?

A. L = 0

B. L = Mvr

C. L = $Mvr\cos\theta$

D. L = $Mvr\sin(\theta + \phi)$



4 options

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B. L = Mvr

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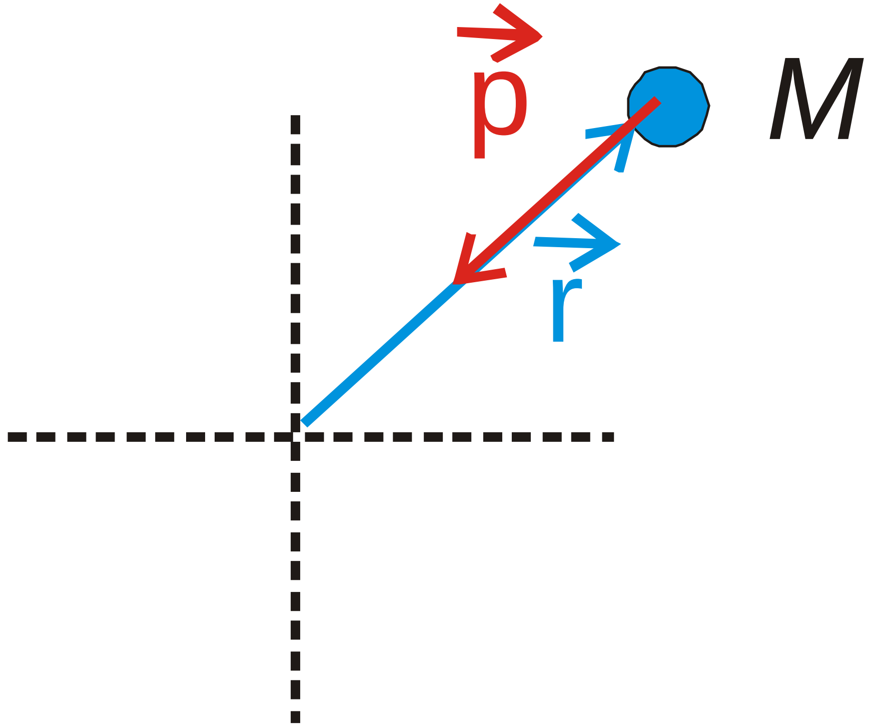


4 options

$$\vec{L} = \vec{r} \times \vec{p} \qquad \left|\vec{L}\right| = rp\sin\theta_{rp}$$

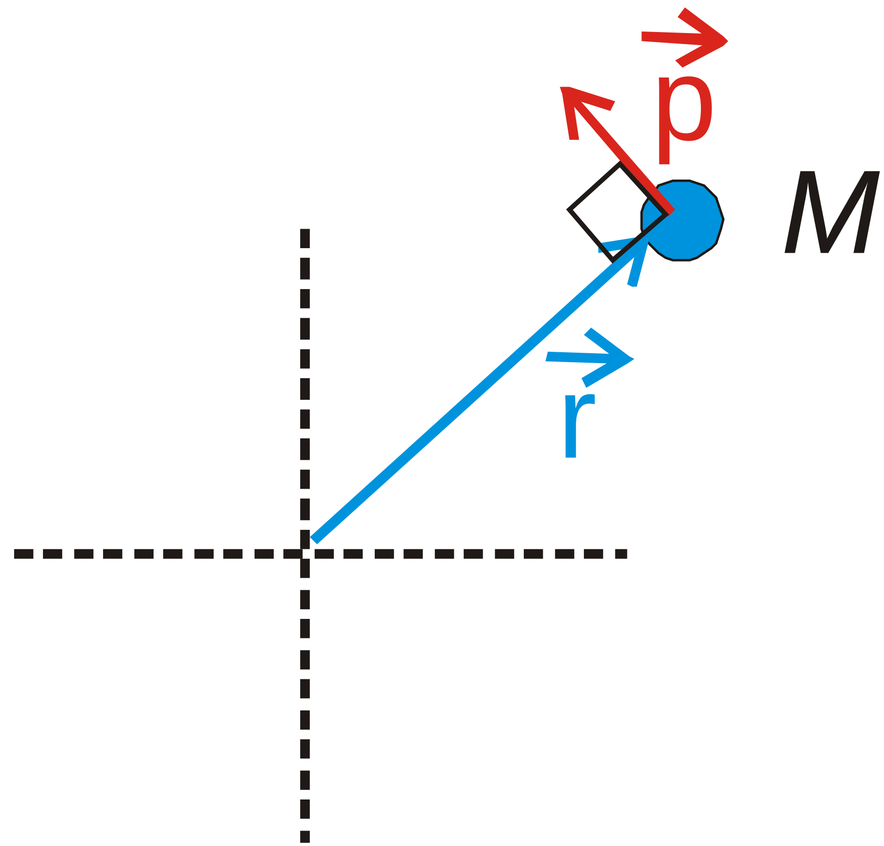
What is the angular momentum of *M* with respect to the origin in the situation shown below, assuming p = *Mv*?

A. L = 0

B. L = *Mvr*

C. L = *Mvr*cosθ

D. L = *Mvr*sin(θ + ϕ)



4 options

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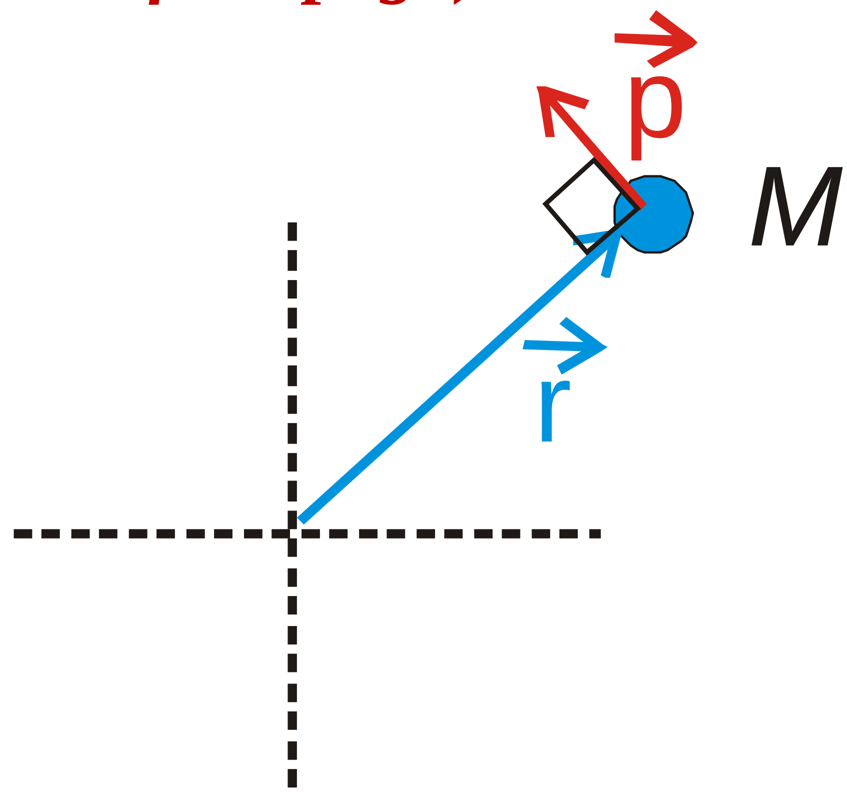
What is the angular momentum of *M* with respect to the origin in the situation shown below, assuming $p = Mv$?

A. L = 0

B. L = *Mvr* *(direction is out of the page)*

C. L = $Mvr\cos\theta$

D. L = $Mvr\sin(\theta + \phi)$



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B. $L = Mvr$

C. $L = Mvr\cos\theta$

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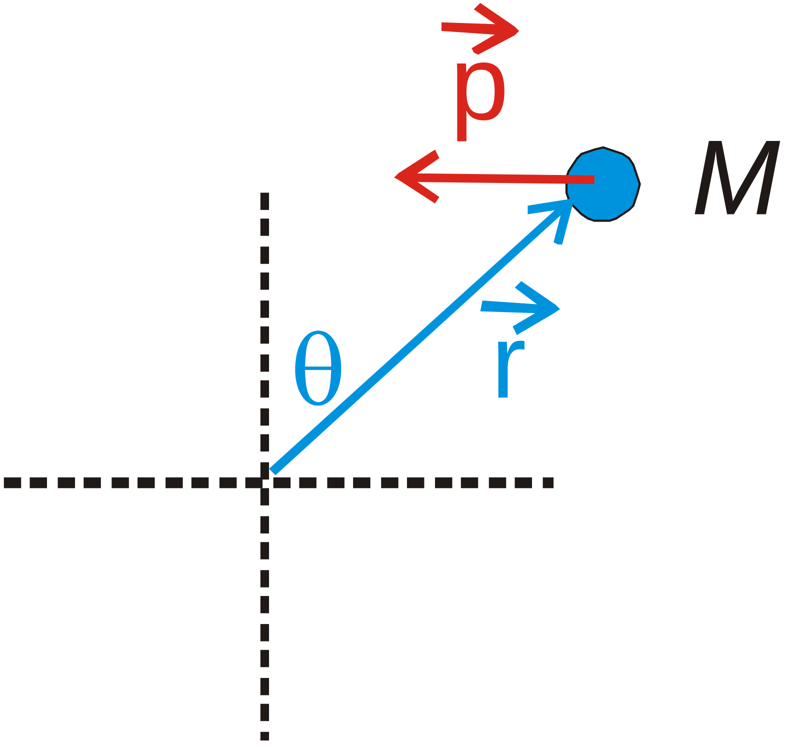

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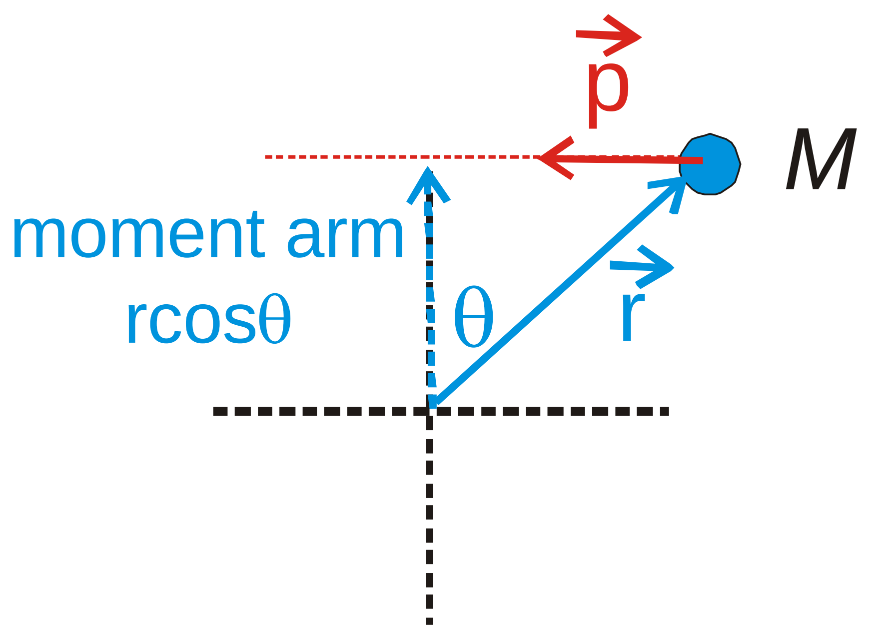
What is the angular momentum of *M* with respect to the origin in the situation shown below, assuming $p = Mv$?

A. L = 0

B. L = Mvr

C. L = $Mvr\cos\theta$ ***(thinking in terms of a moment arm, dir. out of page)***

D. L = $Mvr\sin(\theta + \phi)$



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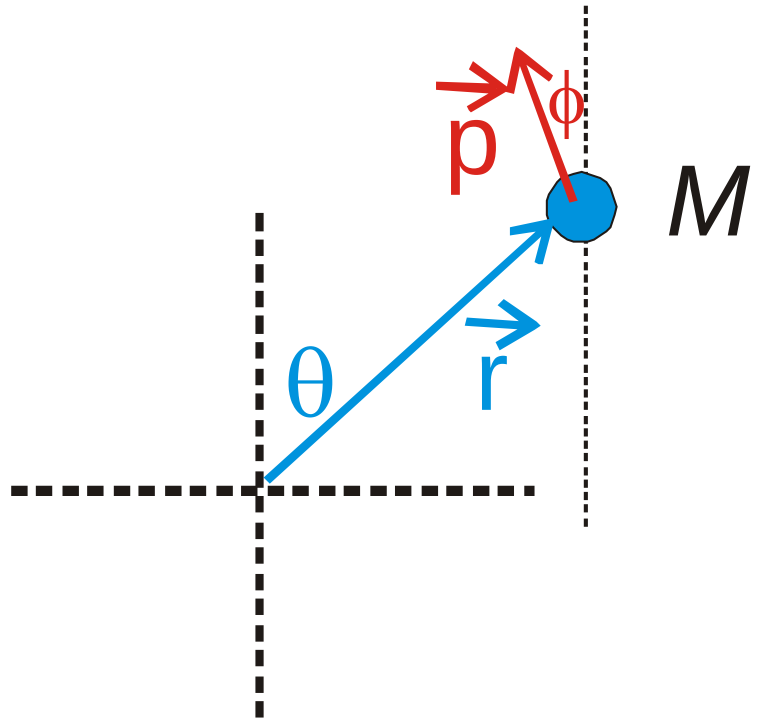
C. L = $Mvr\cos\theta$

D. L = $Mvr\sin(\theta + \phi)$



4 options

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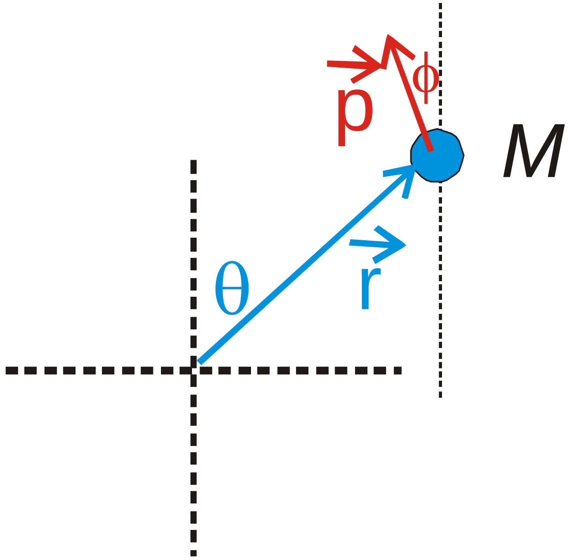
What is the angular momentum of M with respect to the origin in the situation shown below, assuming $p = Mv$?

A. L = 0

B. L = Mvr

C. L = $Mvr\cos\theta$

D. L = $Mvr\sin(\theta + \phi)$ *(shift p to the origin, dir. out of page)*



- 5 options

A freshman PHYS 121 student standing on the edge of a merry-go-round (*which is spinning with its motor disengaged*) walks towards the center.

Which one of the following statements is correct and why?

A. The angular speed of the system decreases because the moment of inertia of the system has increased.

B. The angular speed of the system increases because the moment of inertia of the system has increased.

C. The angular speed of the system decreases because the moment of inertia of the system has decreased.

D. The angular speed of the system increases because the moment of inertia of the system has decreased.

E. The angular speed of the system remains the same because the net torque on the merry-go-round is zero.

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E. The angular speed of the system remains the same because the net torque on the merry-go-round is zero.

There are no outside torques, so angular momentum Iω is conserved.

When the student moves towards the center, more mass is concentrated at the center, so the moment of inertia $I = MR^2$ decreases.

This means that angular speed must increase to preserve the same total angular momentum.

CONSERVATION OF ANGULAR MOMENTUM DEMO

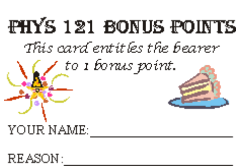
8 VOLUNTEERS NEEDED, 2 AT A TIME

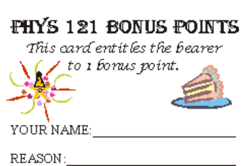
BONUS POINTS ARE AVAILABLE

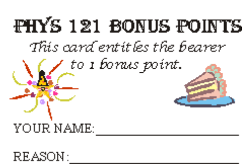
1. Alter your moment of inertia and ω by moving your arms (*holding weights*).
2. Alter your angular momentum by flipping a spinning wheel.
3. Orient your body to face the class by spinning a wheel. (*winner takes all*)
4. Rotate 360° by shifting arms & weights. (*winner takes all*)
5. Jump backwards from the balcony of Strosacker & land (*safely*) on your feet on the main floor, like a cat. **Don't do that!**

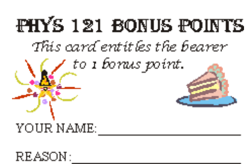
FROM: http://en.wikipedia.org/wiki/Cat_righting_reflex :

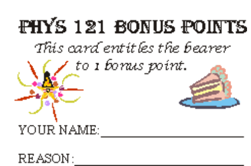
- "*a cat … relies on conservation of angular momentum to set up for landing*"
- "*injuries per cat increased depending on the height fallen up to seven stories but decreased above seven stories*"
- Don't try this at home! http://www.telegraph.co.uk/news/science/10865832/How-cats-always-fall-on-their-feet.html https://www.youtube.com/watch?v=aAZYrJiDEPc&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr

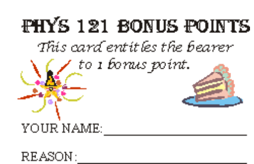












PRECESSION OF A GYROSCOPE

(*to review on your own*)

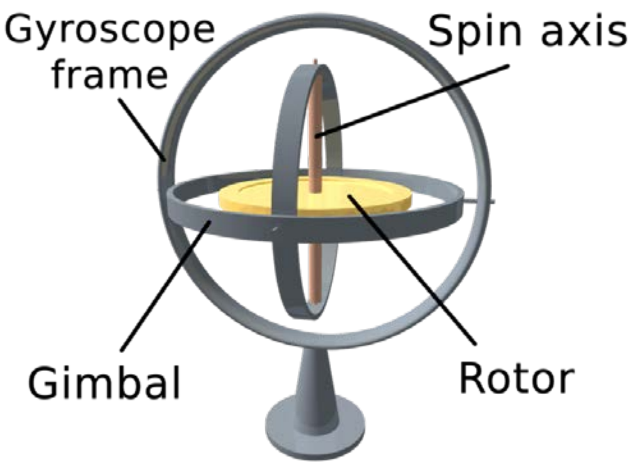
Classic gyroscopes are fast-spinning wheels with a large angular momentum.

Gyroscopes are very important for measuring changes in orientation, as in changing a display from landscape to portrait orientation.

See http://www.ifixit.com/Teardown/iPhone+4+Gyroscope+Teardown/3156

Gyroscopes illustrate odd aspects of rotational dynamics.

A bicycle wheel can serve as a gyroscope.



(*to review on your own*)

PRECESSION OF A GYROSCOPE

The key concepts are based on

$$\vec{L} = I\vec{\omega} \quad \& \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

Assume the axis is oriented horizontally, supported on one end as in the figure

The torque about the support due to gravity points horizontally, to the left in the figure.

$\Rightarrow$ **L** moves towards the left

$\Rightarrow$ The wheels rotates to the left rather than falling down!

This time of motion ≡ **precession**.

Spinning tops precess until they slow enough to topple over.

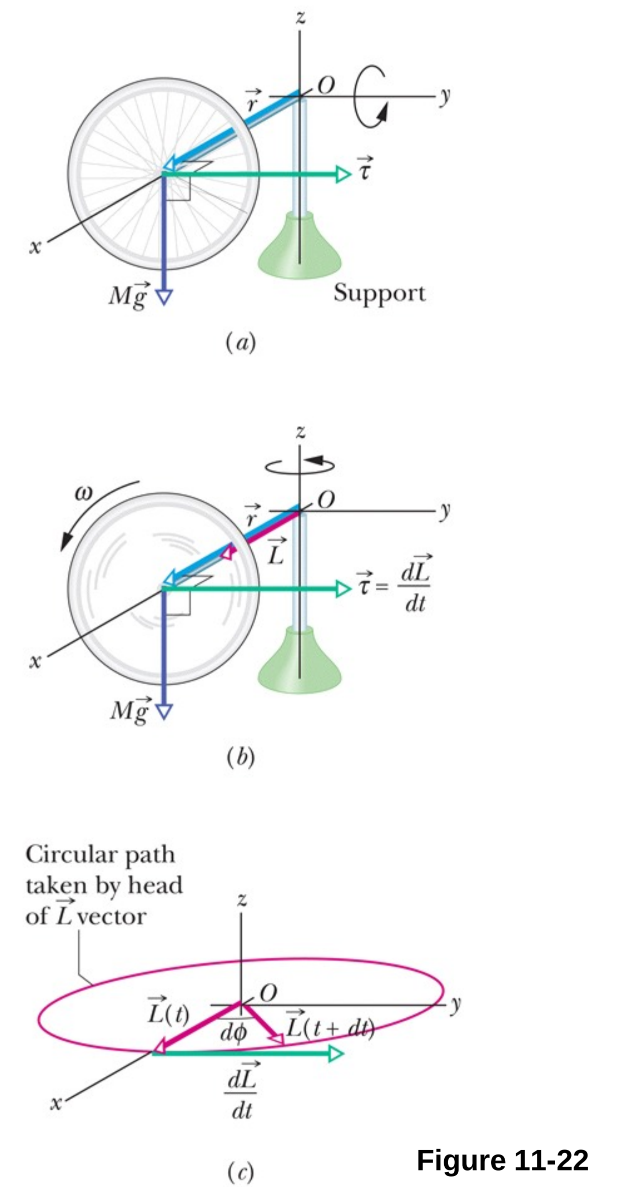
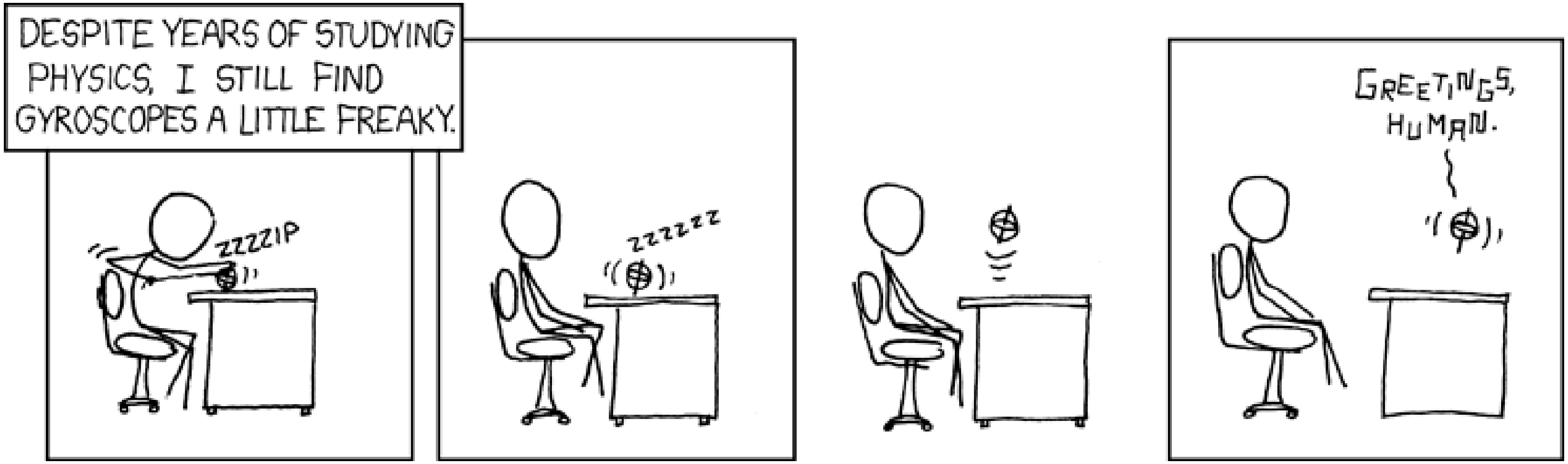


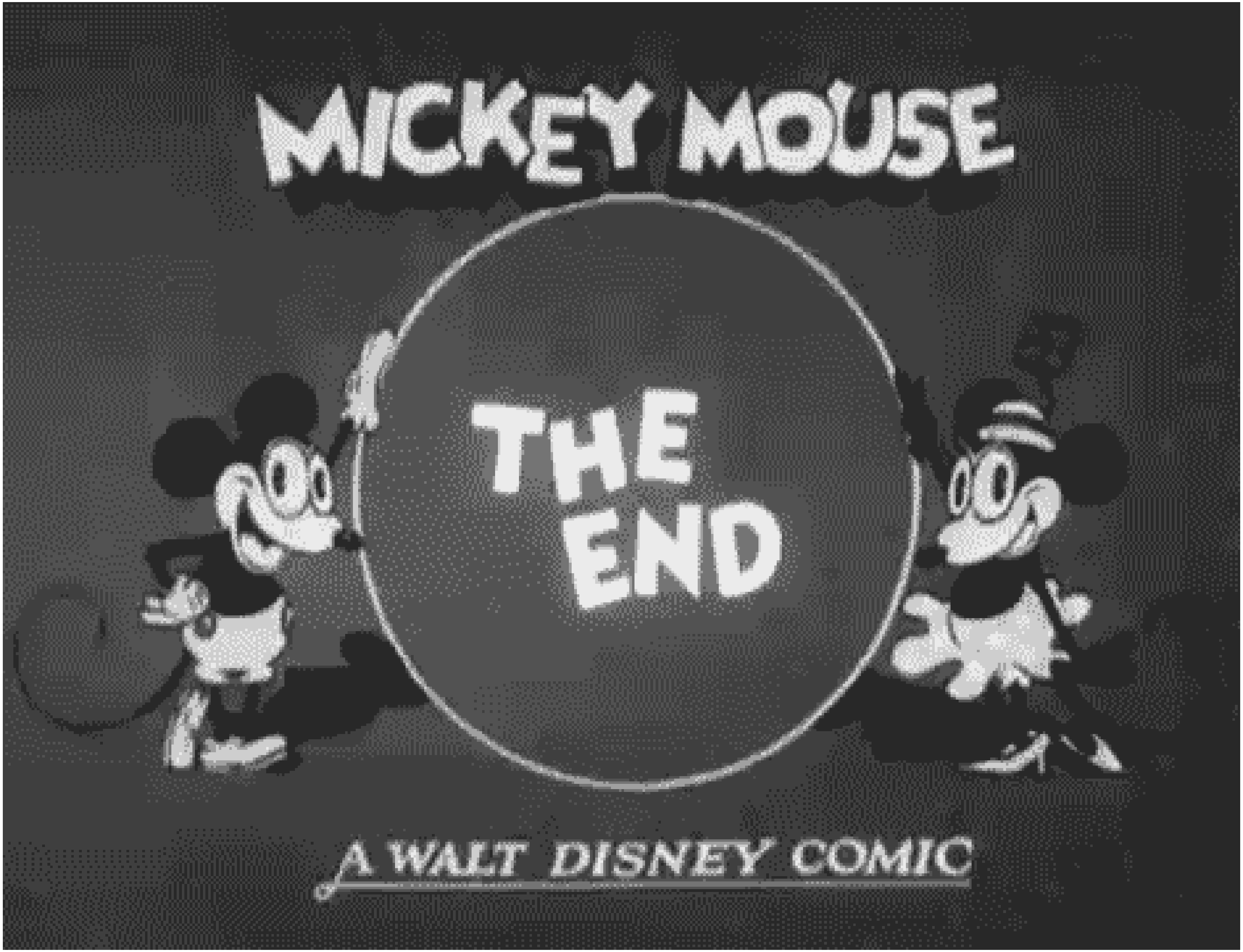
Figure 11-22

DEMO

Precession of a wheel

https://www.youtube.com/watch?v=IdE_yc_GXv4&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr





http://data3.whicdn.com/images/29984186/original.gif