EECS 281, February 5, 2015

Example:

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

z \ xy	00	01	11	10
0		1		
1	1		1	1

X

Z

Y

$xy'z$

xyz

$$xy'z + xyz = xz\underbrace{(y'+y)}_{1} = xz$$

Minimizing Sum of Products

Each input combination with a "1" corresponds to a minterm in canonical sum.

Pairs of adjacent "1" cells in the map have minterms that differ in only one variable. ⇒ minterm pairs can be combined into a single product term.

Example:

$x'yz'$

$z \backslash xy$	00	01	11	10
0		1		
1	1		1	1

xz $\quad$ $y'z$

$$F = x'yz' + xz + y'z$$

In general, we can simplify by:

- first combine pairs of adjacent 1-cells (minterms)
- then select a set of product terms that cover all 1-cells.
- Sum them.

Cell combining can be extended to more than two 1-cells into a single product term. Number of cells combined should always be a <u>power of 2.</u>

In general 2^i 1-cells may be combined to form a product term with $n-i$ literals.

Rule: Set of 2^i cells may be combined if there are i variables of the logic function that take on all 2^i combinations within that set, while $n-i$ variables have same value throughout. Corresponding product term has $n-i$ literals.

For each variable that are not changing:

- if a variable is 0 in the area ⇒ use complement
- if " " " 1 " " " ⇒ use variable.
- if " " " 0 or 1 " " " ⇒ it does not appear in the product term.

Example:

z \ xy	00	01	11	10
0	1		1	1
1	1			1

$\to xz'$

$\to y'$

$$F = y' + xz'$$

Definitions:

Minimal sum: sum-of-products expression for F such that no sum-of-products expression for F has fewer product terms, and any sum-of-products expression with the same number of product terms has at least as many literals.

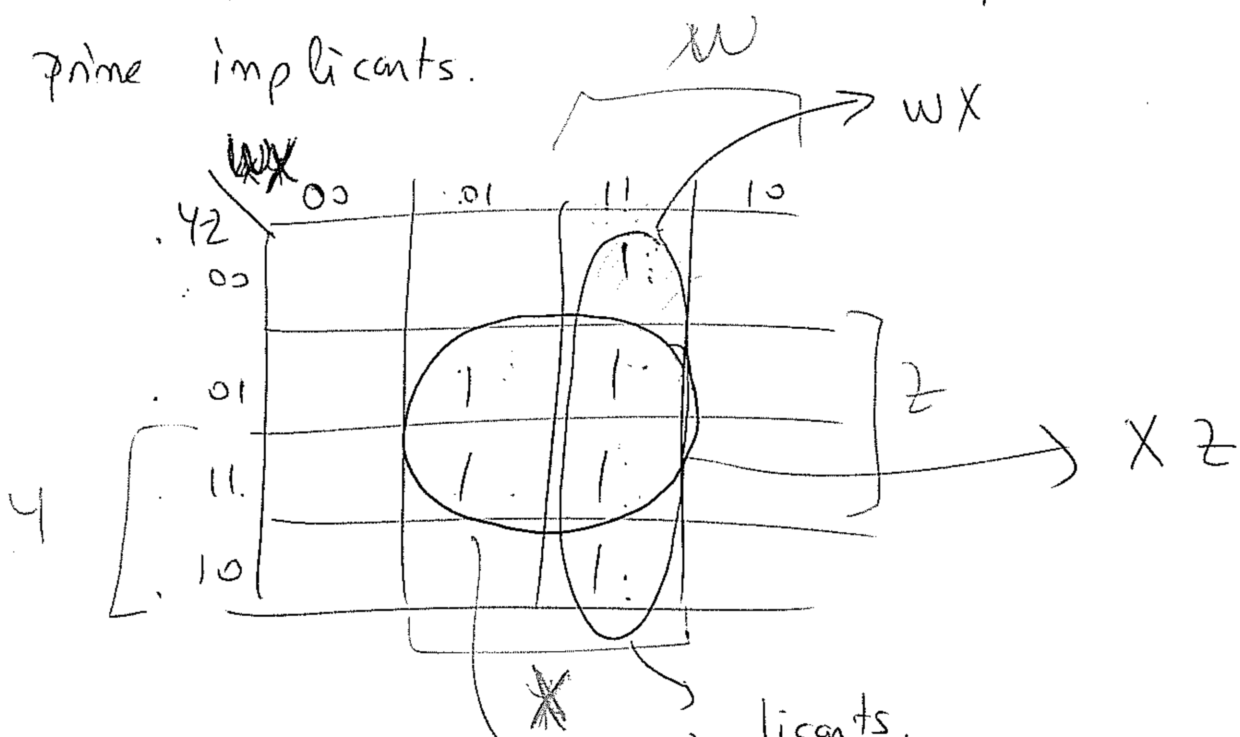
Imply: A logic function $P(X_1, \dots, X_n)$ implies a logic function $F(X_1, \dots, X_n)$ if for every input combination such that $P=1$, then $F=1$ also.

$$P \Rightarrow F.$$

Prime implicant of $F(x_1, \ldots, x_n)$ is a normal product term $P(x_1, \ldots, x_n)$ that implies F.

In Karnaugh maps: circled set of 1-cells satisfying combining rule.

A minimal sum is a sum of prime implicants.



Complete sum: sum of all prime implicants.

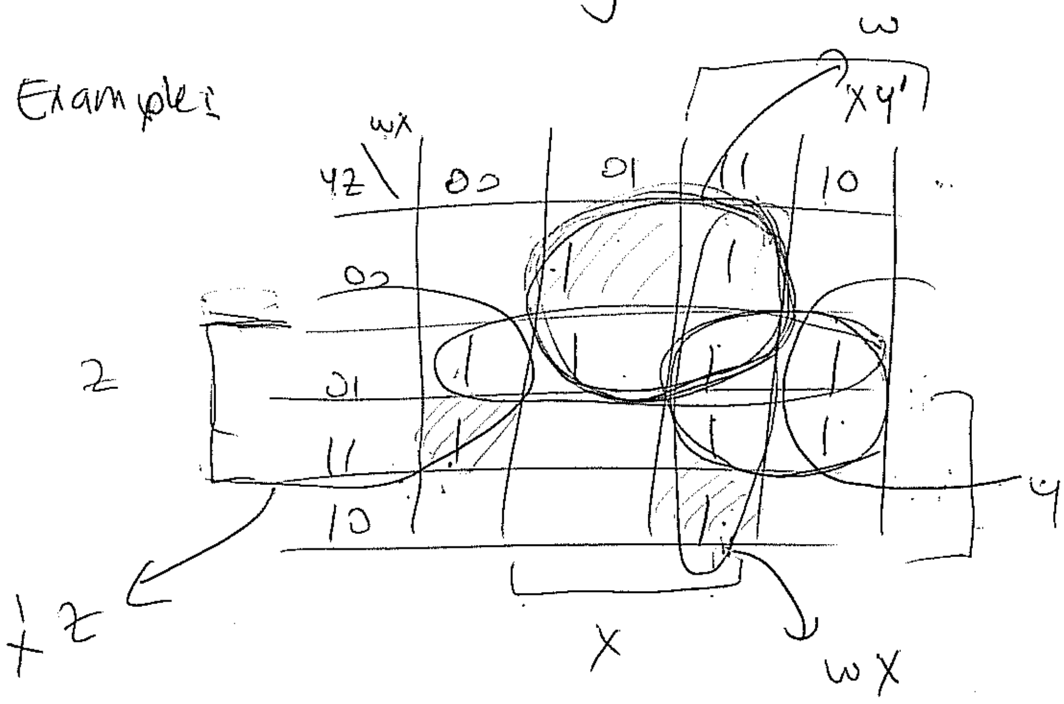
Distinguished 1-cell: input combination that is covered by only one prime implicant.

Essential prime implicant: if it covers one or more distinguished 1-cells.

To select prime implicants to include in the sum:

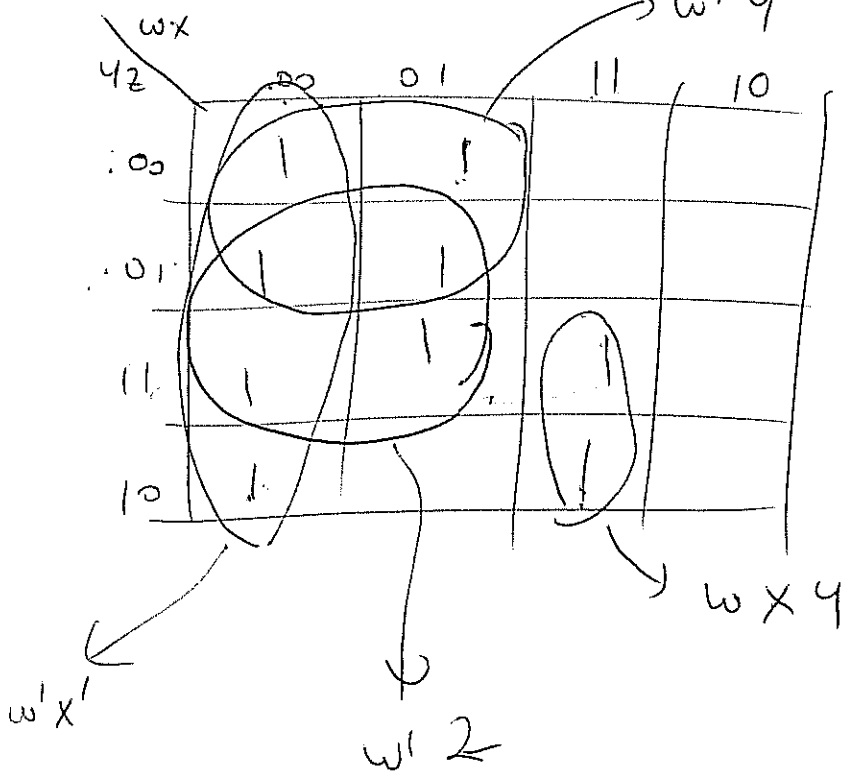
- identify distinguished 1-cells and corresponding prime implicant.
- include essential prime implicants in the minimal sum.
- Then determine how to cover 1-cells not covered by essential prime implicants

Example:



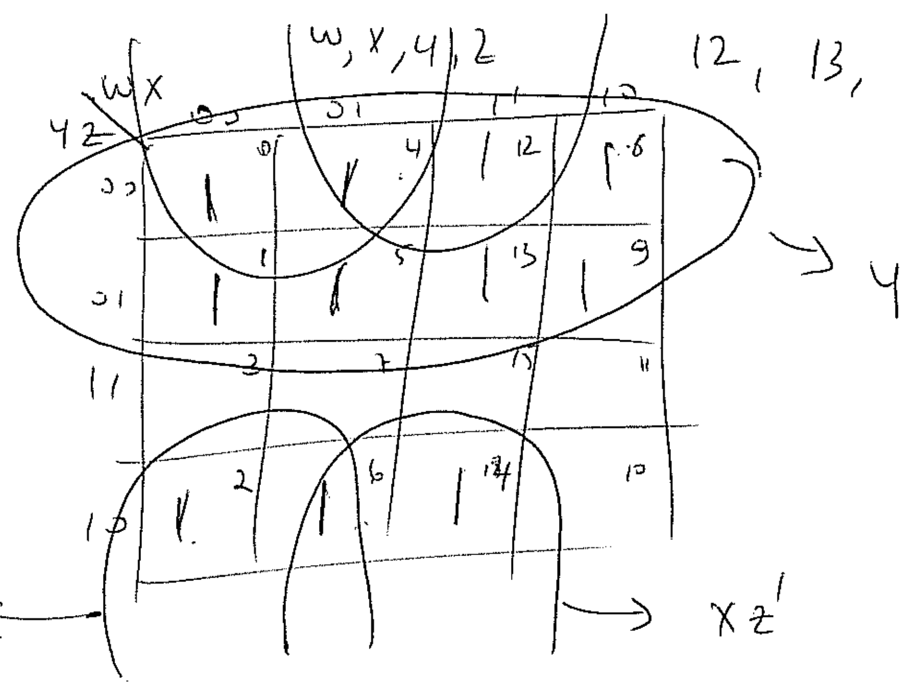
$$F = x'z + wx + xy'$$

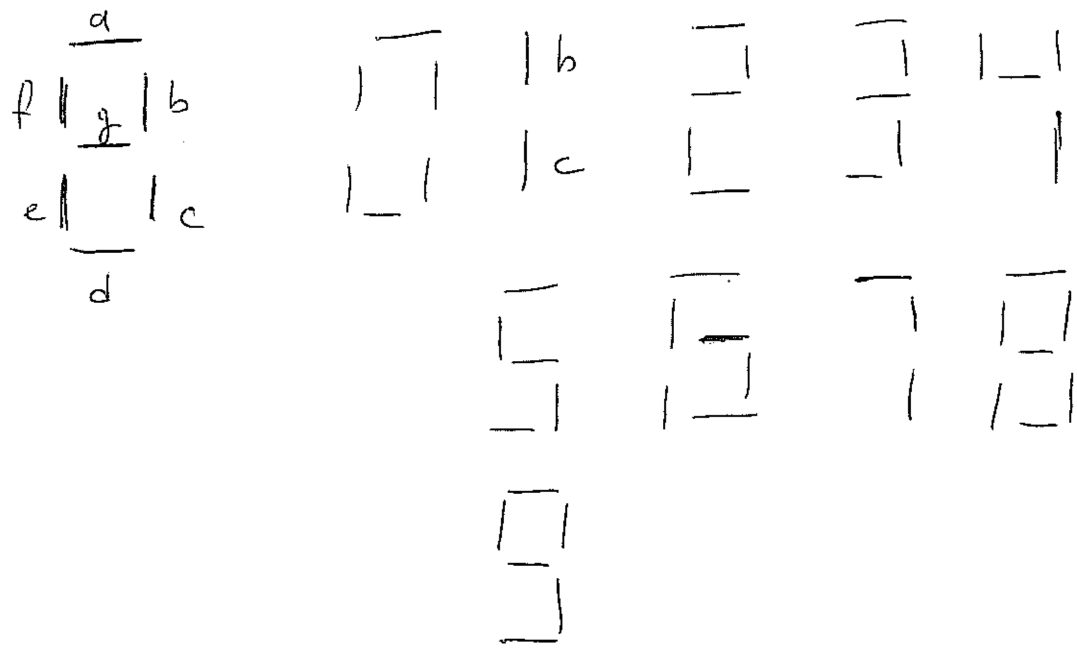
Example:



$$F = w'x' + w'z + wxy + w'y'$$

Example: $F = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$





input: D3-D0 , S_a .

D3 D2 D1 D0

D3-D0	S_a	S_b	S_c	...	S_g .
0000	1	1	1		0
0001	0				
0010	1				
0011	1				
0100	0				
0101	1				
0110	1				
0111	1				
1000	1				
1001	1				
all other	0	0	0	...	0