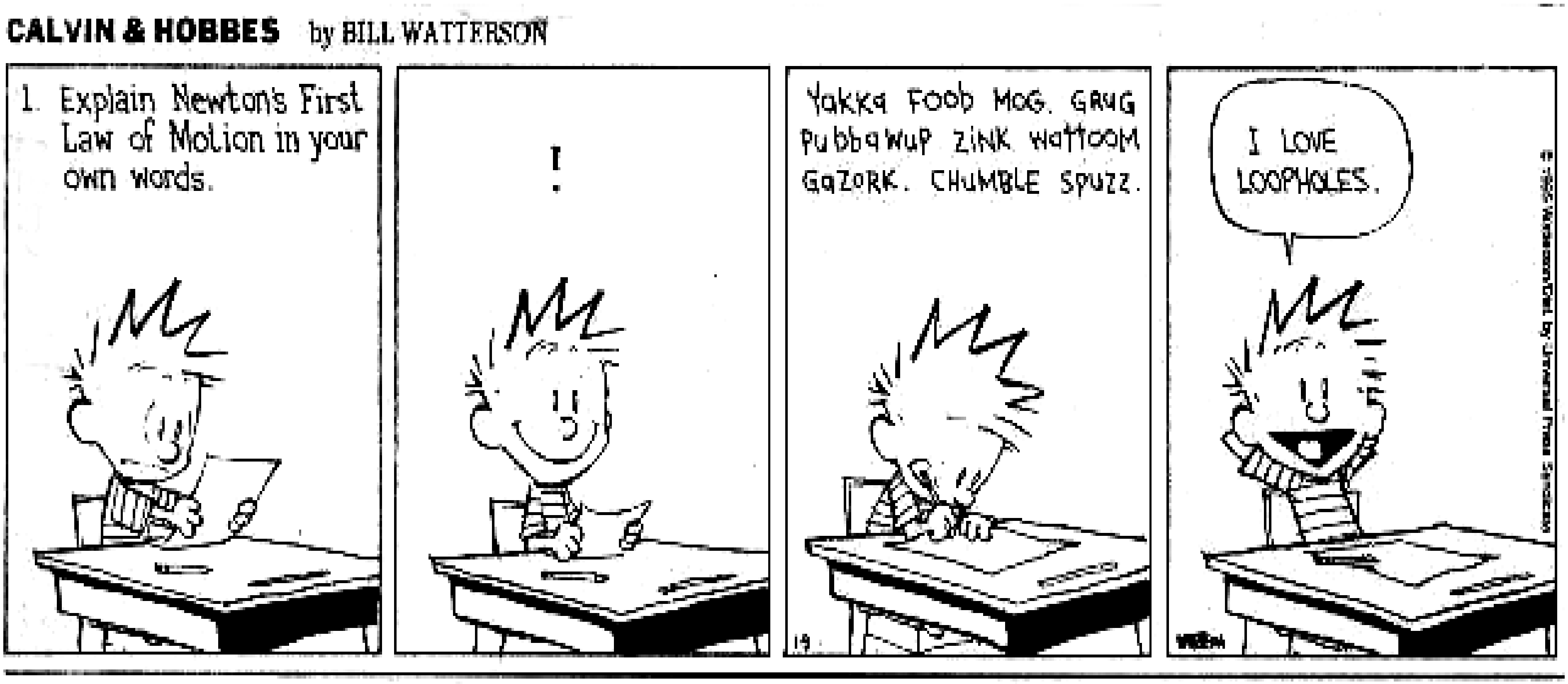
PHYS 121 – SPRING 2015



Chapter 5: Newton's Laws of Motion

version 02/06/2015, ~ 155 slides

We made it to slide #135 on Monday, February 9.

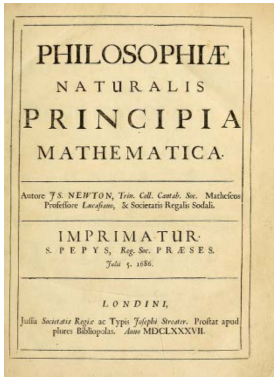
DYNAMICS!

DYNAMICS ≡ field of physics dealing with

forces that *cause* motion.

NEWTON'S (3) LAWS OF MOTION

- Newton's 1st Law (*inertia*)
- Newton's 2nd Law (*force = mass times acceleration*)
- Newton's 3rd Law (*action & reaction*)


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

http://en.wikipedia.org/wiki/Newton%27s_laws_of_motion#mediaviewer/File:Newtons_laws_in_latin.jpg


AXIOMATA
SIVE
LEGES MOTUS

Lex. I.

Lex. II.

Lex. III.

NEWTONS 1st LAW of MOTION

(*assuming constant mass & an inertial* reference frame*)

In the absence of a net external force, a body at rest remains at rest, and a body in motion continues to move at constant velocity.

$$\boldsymbol{F_{external} = 0 \quad \leftrightarrow \quad a = 0}$$

$$\text{since} \quad a = 0 \quad \leftrightarrow \quad v = \text{constant}$$

where the symbol ↔ means that the term on the left implies that the term on the right is true and *vice versa*

**inertial reference frames will be defined shortly*

NEWTONS 2nd LAW of MOTION

The acceleration of a body is proportional to the external *force* acting on it and inversely proportional to the *mass* of the body.

$$a = \frac{1}{m} F \qquad \text{or} \qquad F = ma$$

Once you have this second law, Newton's first law isn't very useful since it's just a special case of the second law with $F = 0$ and $a = 0$.

What are FORCE & MASS?

- We've defined acceleration independently, in terms of displacement, velocity & time.
- For practical purposes, in classical mechanics,

FORCE = cause of acceleration

MASS = resistance to acceleration (*inertia**)

**For our purposes, *inertia* is just a synonym for mass, and mass is the preferred term among physicists.

Where does *F* = *ma* come from?

a cultural aside

It's not just made up to explain experiment.

(*at least, not any more!*)

You'll see in a few weeks that F = ma can be derived from the principle of

CONSERVATION OF MOMENTUM

which itself comes from the

SYMMETRY OF SPACE

i.e. empty space looks the same everywhere.

More Culture

Where does 'mass' comes from?

- Mass comes from the interaction of certain particles with the "*Higg's Field* " that permeates the universe.

It's like moving through molasses.

- Some particles, like photons, don't have mass because they don't interact with the Higg's field.
- For more, see http://science.howstuffworks.com/higgs-boson.htm or http://en.wikipedia.org/wiki/Higgs_boson or http://www.nytimes.com/2012/07/05/science/cern-physicists-may-have-discovered-higgs-boson-particle.html?pagewanted=all

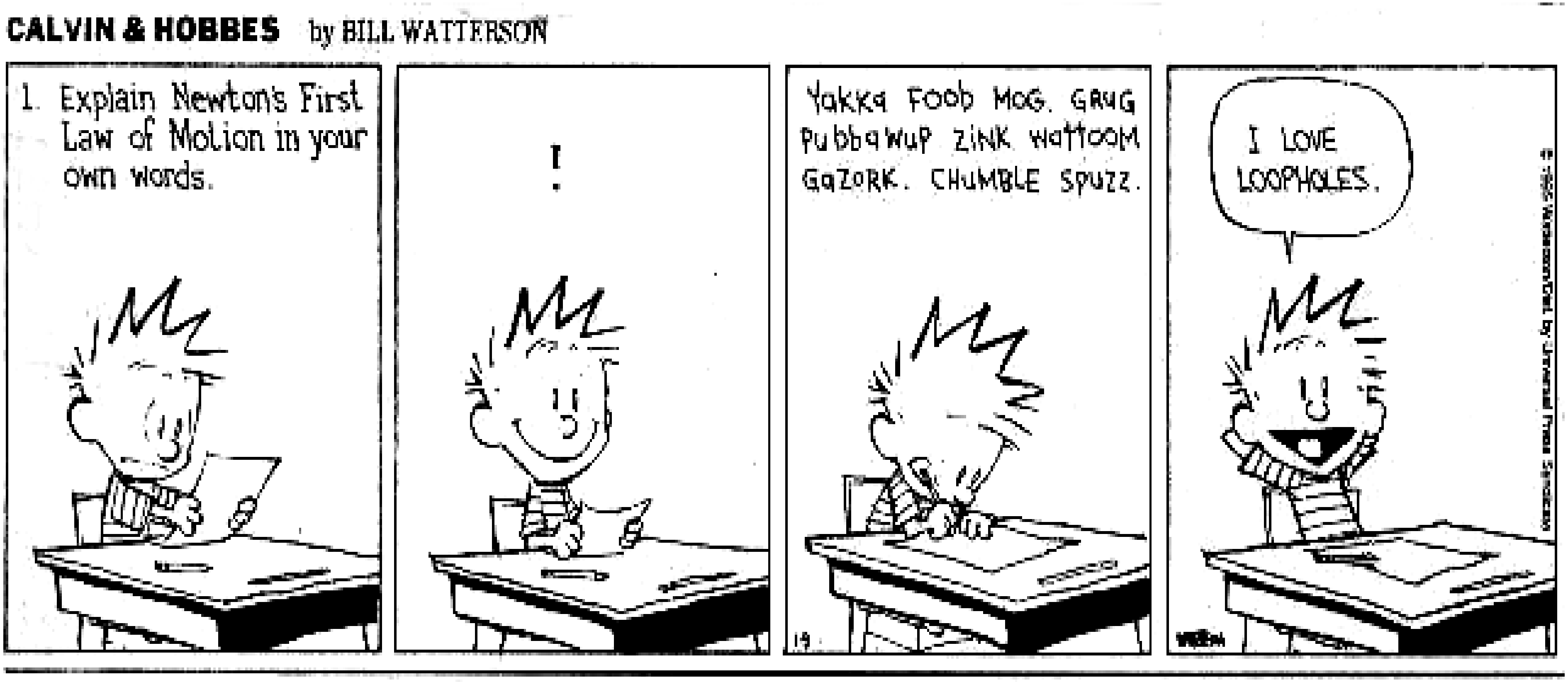
 or declare a major in physics.

- **GOOD NEWS: You don't have to understand where mass comes from for PHYS 121.**

Just think of mass as resistance to acceleration.

We made it to slide #7 on Wednesday, January 28, 2015.

PHYS 121 – SPRING 2015



Chapter 5: Newton's Laws of Motion

version 01/30/2015, ~ 155 slides

We made it to slide #7 on Wednesday, January 28, 2015.

FORCE

PHYS 121 DEFINITION: Given fundamental definitions of acceleration & mass, force is defined by Newton's 2^{nd} Law

$$\vec{F} = m\vec{a}$$

Note that this is THREE equations in 3D, for one each component.

- The SI units of force ($kg\text{-}m/s^2$) ≡ Newtons, abbreviated N.

  (*or pounds, lbs, in the 'British' system*)

- **F** is a vector because **a** is a vector. (*mass is a scalar*)
  - The direction of **F** is the same as the direction of **a**.
  - Like all vectors, forces add like displacements add.
- **SUPERPOSITION PRINCIPLE**: Multiple forces acting on a particle are equivalent to a single net force given by the vector sum of the individual forces.

  (*Stay tuned for examples of the superposition principle in action.*)

$$\sum \vec{F} \equiv \vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = m\vec{a}$$

INERTIAL REFERENCE FRAMES

$\mathbf{F} = m\mathbf{a}$ applies only in ***inertial reference frames***.

An ***inertial reference frame*** is one that is not accelerating (*significantly, for the problem at hand*).

INERTIAL REFERENCE FRAMES

You can see if a reference frame is *inertial* by checking for unexplained accelerations or inexplicable forces.

I. Do objects accelerate in some direction without any obvious explanation/force?
II. Does a plumb bob point at some angle other than straight down?
III. Is the surface of a fluid in a container NOT parallel to the surface of the earth?


http://www.plumb-bob.info/engl.html

INERTIAL REFERENCE FRAMES

EXAMPLE: If you are riding in a car ***at constant velocity***, a cup resting on the (*horizontal*) dashboard remains at rest – relative to the frame of reference of the car (*and you in it*).

- This is an **inertial** frame of reference.
- If you brake, accelerate or turn, the cup will accelerate forwards, backwards or *in the opposite direction* of the turn - although no force causes the acceleration of the cup.
- An accelerating car is a **noninertial** frame of reference.

An object at rest does not remain at rest!

INERTIAL FRAMES OF REFERENCE

$$\vec{a}_{AB} = \vec{a}_{AC} + \vec{a}_{CB}$$

There's an ice cube on the (*horizontal*) dashboard of your car and you are traveling with a constant velocity.

There's no way to transmit a horizontal force to frictionless ice.

The ice cube will remain motionless,

from YOUR point of view in the driver's seat,

until you accelerate, brake or go around a corner.

$$\vec{a}_{ice-car} = \vec{a}_{ice-ground} + \vec{a}_{ground-car}$$

In the horizontal plane:

$$\vec{a}_{ice-ground} = 0$$

$$\vec{a}_{ice-car} = +\vec{a}_{ground-car}$$

$$\vec{a}_{ice-car} = -\vec{a}_{car-ground}$$

<u>INERTIAL FRAMES OF REFERENCE</u>

$$\vec{a}_{ice-car} = -\vec{a}_{car-ground}$$

multiply both sides of the above equation by m_{ice}

$$\vec{F}_{ice-car} = -m_{ice}\vec{a}_{car-ground}$$

As seen from the reference frame of the car, there is a mysterious "***<u>apparent</u>***" force acting on the ice whenever the car accelerates.

BUT this force is an *<u>artifact</u>* of

using a *noninertial frame of reference,* the car!

In Uniform Circular Motion,

<u>centrifugal forces</u> (*outward*)

are an artifact of centripetal acceleration (*inward*).

INERTIAL FRAMES OF REFERENCE

We normally think a reference frame
attached to the surface of the earth is *inertial*
although we know that there is a centripetal acceleration
towards **the axis** of the earth
(*not the CENTER of the earth*)
but this acceleration/force is normally small
$\sim 0.022 \text{ m/s}^2$
compared to other accelerations/forces we are considering
like $g = 9.8 \text{ m/s}^2$
and we'll often ignore it.

Suppose you are lying in a sensory depravation chamber (*the chamber is dark and soundproof*) in an airplane.

Which of the following can you detect?

more than 1 answer might be correct

A. whether the plane is flying or is at rest on the runway
B. the speed of the plane
C. the plane climbing or descending at a constant rate
D. any change in the speed of the plane
E. whether the plane is turning

Suppose you are lying in a sensory depravation chamber (*the chamber is dark and soundproof*) in an airplane. Which of the following can you detect?

A. whether you are the plane is flying or is instead at rest on the runway
B. the speed of the plane
C. the plane climbing or descending at a constant rate
D. any change in the speed of the plane
E. whether the plane is turning

NEWTON'S 3rd LAW

When one object exerts a force on another object, the latter exerts an ***equal and opposite*** force *on the former*

≡ **ACTION & REACTION**.

$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$$

Note that the action & reaction forces act on different objects!

NEWTON'S 3rd LAW

ACTION & REACTION

$$\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$$

Internal vs. *External* forces

- *Internal forces:* interactions within a "*rigid body*" must cancel.
  - Otherwise, different pieces of the object will accelerate apart.
  - And your head might fly off your neck while you watch this lecture!
  - **GOOD NEWS! Since $\Sigma F_{internal} = 0$, we can ignore *internal* forces.**
- Only *external forces* cause macroscopic motion of an object.
- *Defining the system* you are considering includes categorizing forces as internal or external.
  - We'll provide LOTS of practice with this shortly.

ACTION/REACTION

If you push against a wall, the wall pushes back on you with an identical force (*except in the opposite direction*).

How can we prove this?

Why don't YOU accelerate backwards?

- Imagine that there's some very light object between your hand and the wall, say a piece of paper.
  - What would happen to the paper if you or the wall exerts a greater force on it?
  - *i.e.* what is the acceleration of an object, $a = F/M$, with $M \sim 0$ if you apply a net force to it?

Checking for *absurdities* is a common technique in physics.

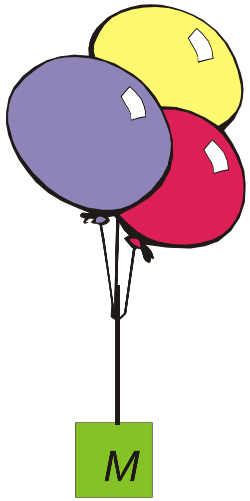
- If you push against a wall, you *would* move backwards if not for friction from the floor!

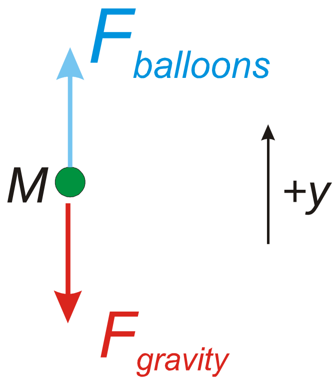
$$\mathbf{\Sigma F_{on\ you} = 0} \quad \textbf{because} \quad \mathbf{F}_{\text{floor-friction on YOU}} = \mathbf{-\,F}_{\text{wall on YOU}}$$

SIMPLE EXAMPLE of $F = ma$ in 1D

- A trio of balloons exerts a lift force $F_{balloons}$ raising mass M up.
- Gravity pulls M downward.

Describe the motion of *M*.





- I've chosen up as my positive **y** direction.
- The net force on M is $F_{balloons} - F_{gravity}$
- The acceleration of M is

$$a_y = (\Sigma F)/M = (F_{balloons} - F_{gravity})/M$$

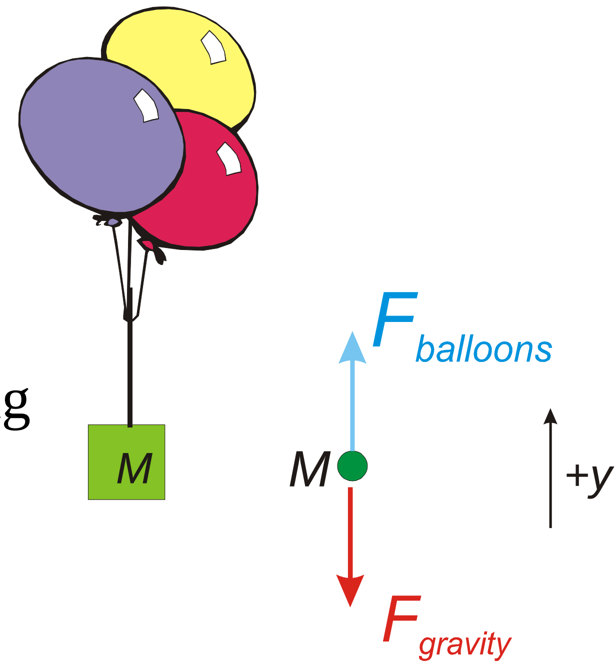
FREE BODY DIAGRAMS

The right portion of the figure is a

FREE BODY DIAGRAM *aka* FBD

sometimes called a **FORCE DIAGRAM**.

- Objects (*masses*) are represented ~ points.
- A coordinate system is indicated for each mass, including positive directions for each axis.
- Forces that act on objects are drawn as acting on those points.
- The forces are drawn as vectors, with lengths proportional to the magnitude of the force (*if known*) and pointing in the appropriate direction.
- Drawing a Free Body Diagram is generally the FIRST STEP in solving a dynamics problem.

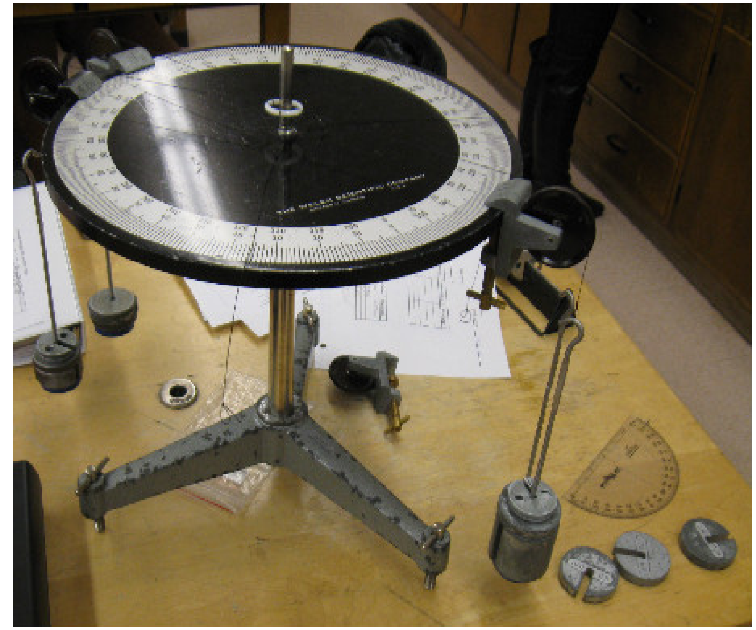


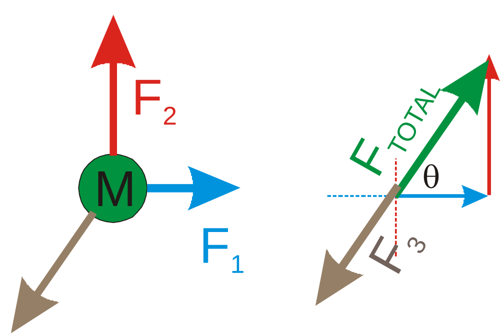
ΣF = *m*a in 2D: FORCE BALANCE

The **Force Balance** is a standard intro physics lab experiment.

Arrange 3 forces to keep a ring centered on a pin in the middle.

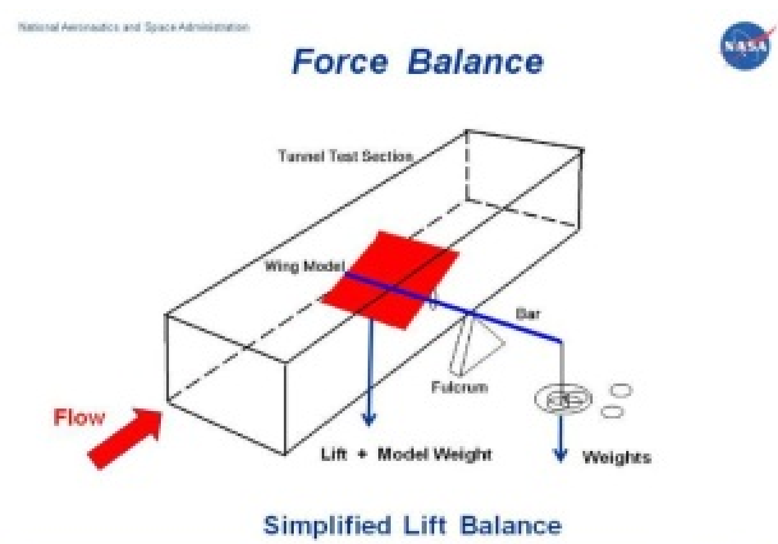
http://www.foothill.edu/~marasco/4alabs/lab2/forcetable.jpg

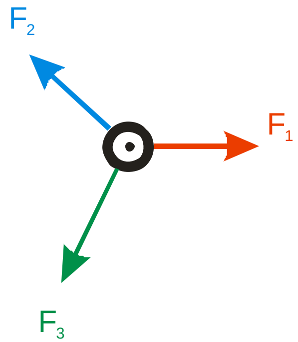




NASA uses a variation of the force balance for aerospace research.

http://www.grc.nasa.gov/WWW/k-12/airplane/tunbal.html





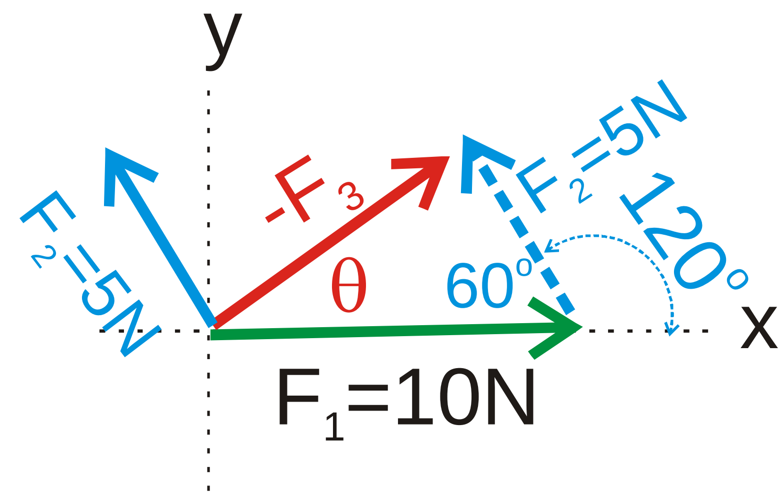
FORCE BALANCE EXAMPLE

Arrange three forces around a horizontal table so that a small ring stays centered on a pin at the center of the table.

(*Each force is supplied by a string tied via a pulley to a hanging mass*)

- Force $\mathbf{F_1}$ = 10 N points in the direction of 0°.
  - Align your coordinate system with one of the forces if you can!
  - Choose wisely!
- $\mathbf{F_2}$ = 5 N at 120°.
- What $\mathbf{F_3}$ will balance the ring in your force balance?
- This is the same as asking for the sum of $\mathbf{F_1} + \mathbf{F_2}$ since $\mathbf{F}_3$ will just be the negative of this sum.

FORCE BALANCE



“Graphical” solution

- Solve for $\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$
- Draw $(\mathbf{F_1} + \mathbf{F_2})$ using head to tail rule for vector addition.
- Use *Law of Cosines and the Law of Sines to find the length & direction θ of* $(\mathbf{F_1} + \mathbf{F_2})$
- $\mathbf{F_3}$ will be directly opposite $(\mathbf{F_1} + \mathbf{F_2})$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$F_3^2 = F_1^2 + F_2^2 - 2F_1F_2\cos 60^0$$

$$F_3 = \sqrt{10^2 + 5^2 - 2\cdot 10\cdot 5\cdot (0.5)} = 8.66N$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

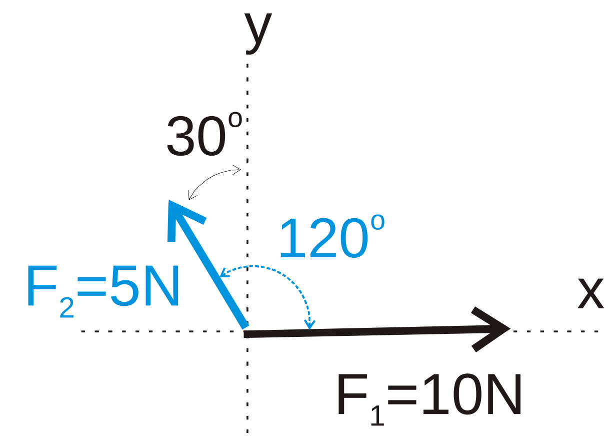
$$\frac{5}{\sin\theta} = \frac{8.66}{\sin 60^o}$$

$$\theta = \sin^{-1}\left(\frac{5}{8.66}\sin 60^o\right) = 30^o$$

FORCE BALANCE

Component Method

(recommended by Dr. C.)

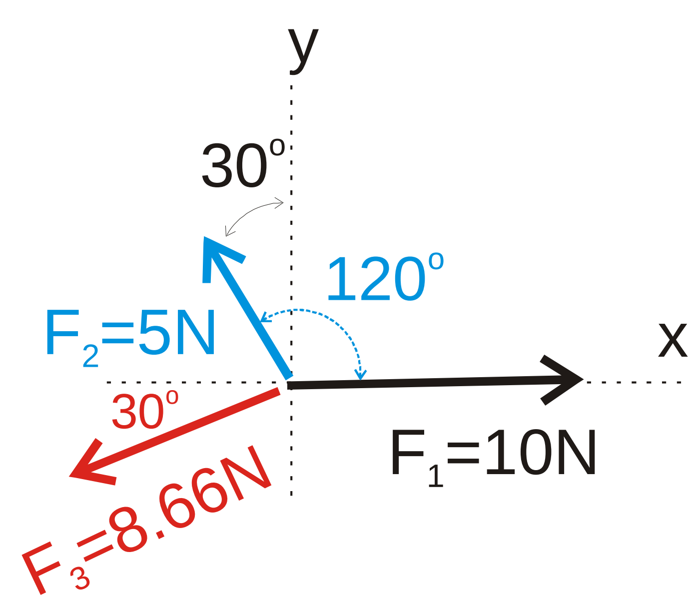


$$\sum F_x = F_1 - F_2 \sin 30^o = 10 - 5 \times 0.5 = 7.5 \text{ N}$$

$$\sum F_y = F_2 \cos 30^o = 5 \times 0.867 = 4.33 \text{ N}$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{7.5^2 + 4.33^2} = 8.66 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = 30^o$$



- **F_3** will be directly opposite this **F.**
- It will have the same magnitude

  but point at 30°+180° = 210° CCW *wrt* the positive x axis

  or 150° CW or -150° .

SOME COMMON FORCES

- Gravity, $\mathbf{F_G}$, near the surface of the earth.
- Contact force ≡ force between surfaces of two objects in contact with each other.
  - Normal force, **N**, ≡ force *perpendicular* to a surface that prevents things from sinking into that surface.
  - Friction $\mathbf{F_f}$ = force *parallel* to a surface that resists motion along that surface.
- Tension **T** ≡ pull from a string, rope or rod.

GRAVITY/WEIGHT

- Gravity *near the earth's surface* results in

$$F_G = mg \quad (down)$$

Later we'll investigate gravity over larger distance scales.

- $F_G \equiv$ **WEIGHT** $= W = m \times 9.81\ \text{m/s}^2$ *near the earth's surface*.
  - Weight on other celestial bodies uses a different *g*.
- Weight is a vector *force*, mass is neither a force nor a vector.
  - Mass is *inertia*, resistance to acceleration by a force (*including gravity*).
  - Weight is a *force* due to gravity in your neighborhood.
- You usually measure weight by balancing it against some other force, such as a spring (*which we'll examine later this semester*).

GRAVITY/WEIGHT

The US system of units is confusing.

- *Pounds* (*pounds-force*) is a weight *or* force, not a mass. 1 lb_f = 4.482 N
- *Pounds* (*pounds-mass*) is the mass of a pound-force.
- 1 lb_m = 4.482 kg / 9.81 m/s^2 = 0.453 kg
- Mass in the US system may also be given as *slugs*.

  1 slug = F/a = (1 lb_f) / (1 ft/s^2) = 32.2 lb_m.
- Physicists avoid using the US system!
- Engineers might have to use it!

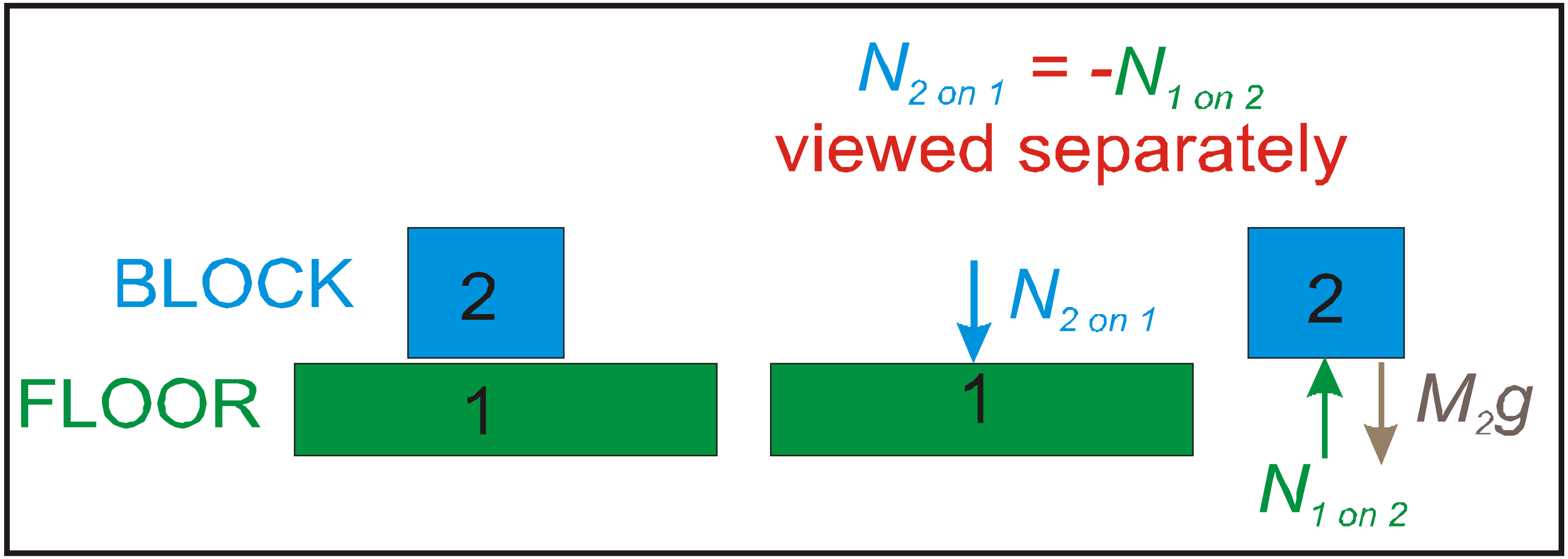
NORMAL FORCES

Normal force N ≡ force *perpendicular* to a surface that keeps things from sinking into that surface.

- **N** is always perpendicular to a surface or interface, whether that surface is horizontal, vertical or anywhere in between.
- **N** is due to repulsive electric forces between atoms/molecules.
- There will be an action-reaction PAIR of normal forces acting separately on each object in contact.

$$N_{2\ on\ 1} = -\ N_{1\ on\ 2}$$

NORMAL FORCES



- Gravity does **NOT** exert a force on the floor due to block #2.

If $a_y = 0$, $\Sigma F_{y\ on\ 2} = N_{1\ on\ 2} - M_2 g = 0 \Rightarrow N_{1\ on\ 2} = M_2 g$

ACTION-REACTION $\Rightarrow N_{2\ on\ 1} = -N_{1\ on\ 2}$

$$\Rightarrow N_{2\ on\ 1} = -M_2 g$$

- Gravity only **INDIRECTLY** exerts the force M_2g on the floor.
- If $a_y \neq 0$, perhaps it's an elevator floor, $N_{1\ on\ 2} \neq M_2 g$

NORMAL FORCES

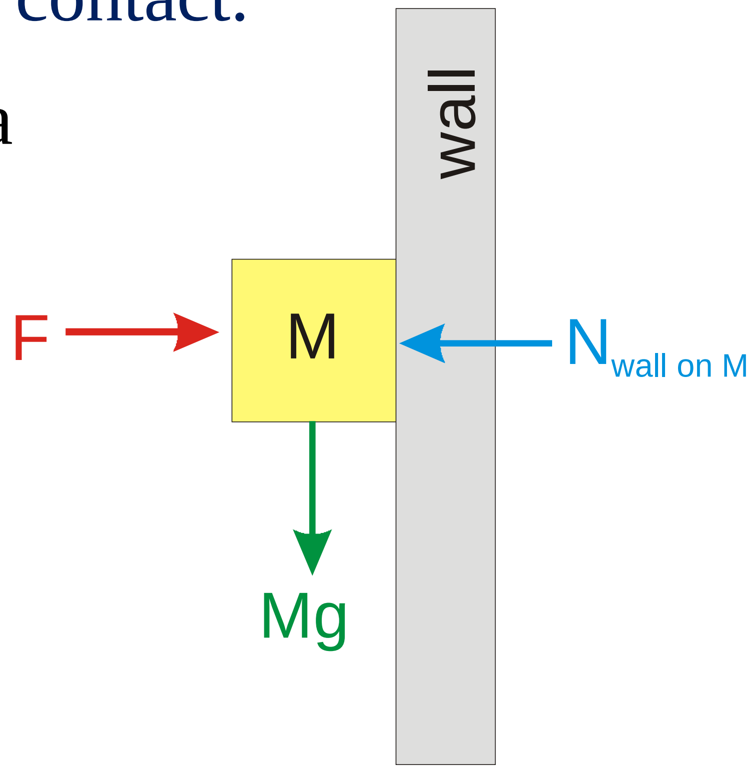
- Normal forces can point in any direction

  but must be perpendicular to surfaces in contact.
- In the figure, a block is pressed against a <u>frictionless</u> wall by a force F.
- *Since $a_x = 0$*

$$N_{\text{wall on M}} = -F_{\text{pushing M against the wall}}$$

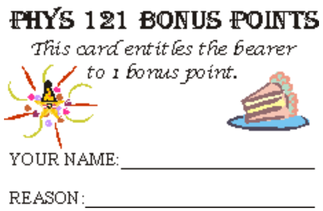
- Nothing counteracts $F_{\text{gravity}} = Mg$

  $\Rightarrow M$ accelerates downwards at $|a| = g$.
- $\Sigma\mathbf{F} = m\mathbf{a}$ is a *vector* equation & is *independently* true for each vector component.



NORMAL FORCES DEMO

a bonus point is available



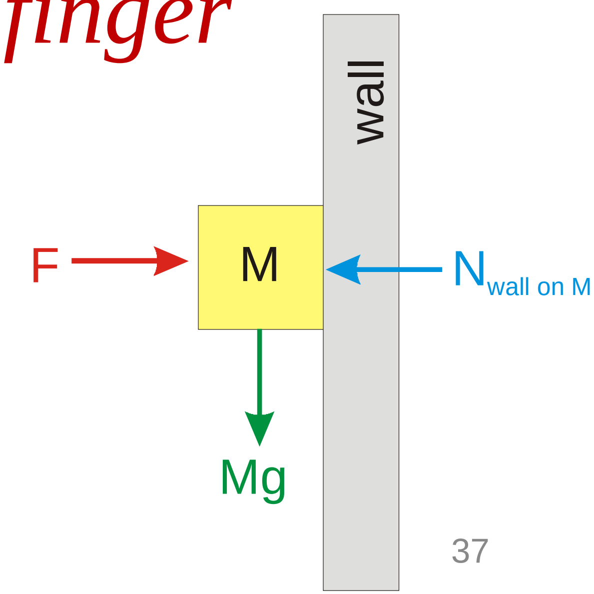
We can demonstrate this effect with the assistance of a **strong** member of the class

who must hold an Ohanian text against the smooth wooden wall of Strosacker

pushing ONLY horizontally with one finger

& ***keep the text from falling!***

(2 bonus points if you can overcome the normal force and push the book INTO the wall!)



FRICTION

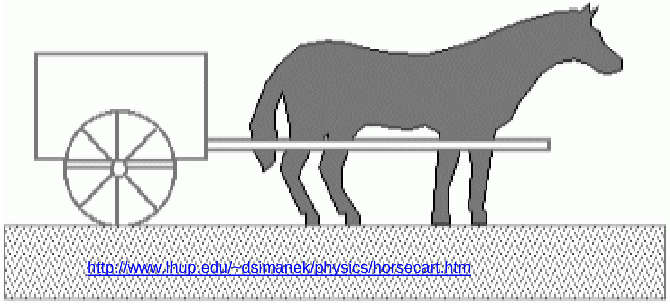
Friction F_f = force *parallel* to a surface that *resists* motion along that surface

- Friction always opposes the motion of one object with respect to another object in contact with it.
- Friction is related to (*proportional to*) the normal force between objects.
- Friction acts between a *pair* of surfaces, pointing in opposite directions for each; an action-reaction pair of forces.

Friction is covered in detail in Chapter 6 of Ohanian.

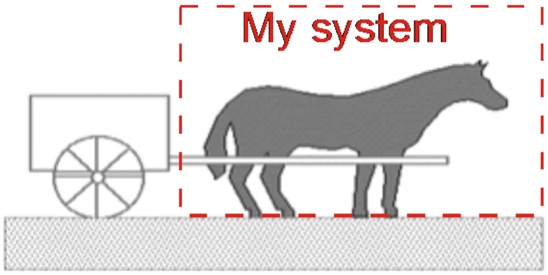
CONTACT FORCES as ACTION-REACTION PAIRS

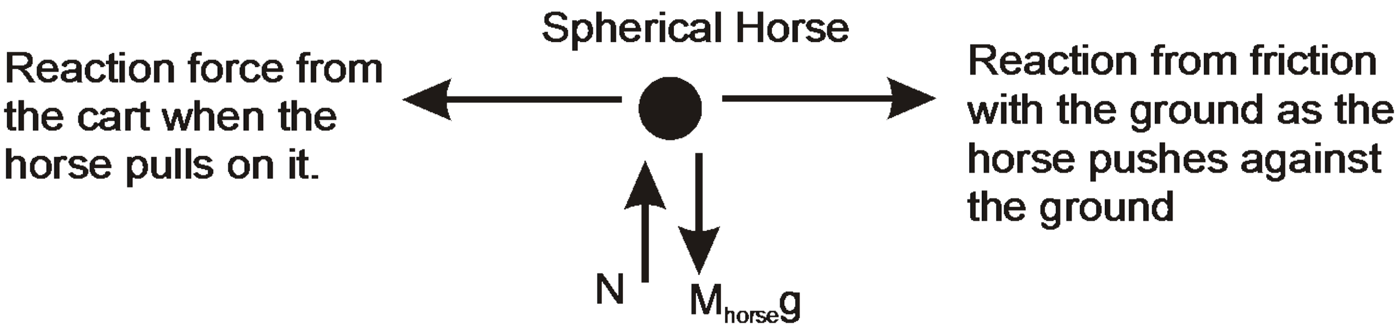
- The two forces in an action-reaction pair
  - have the same magnitude but opposite directions
  - **act on different objects**
  - may have different effects on those objects.
- If you push against a wall,
  - the wall (*likely*) won't move
  - but you might fall backwards if you push too hard.
- Classic "*paradox*": How can a horse pull a cart if the cart pulls back on the horse with the same (*reaction*) force which the horse exerts on the cart?

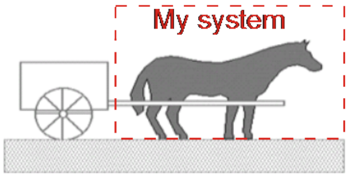


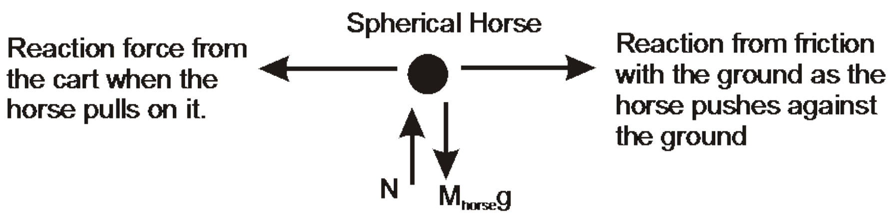
CART BEFORE THE HORSE?

- It's critical to consider **ALL** the forces acting on **each** object and to **define your 'system' carefully**.
- The **force of friction** is an action-reaction force that results from the horse pushing its hooves against the ground.
  - A horse standing on ideal ice can't move the cart!
- Define the horse as your system; draw its Free Body Diagram.







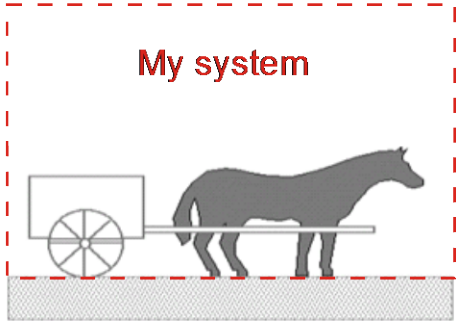


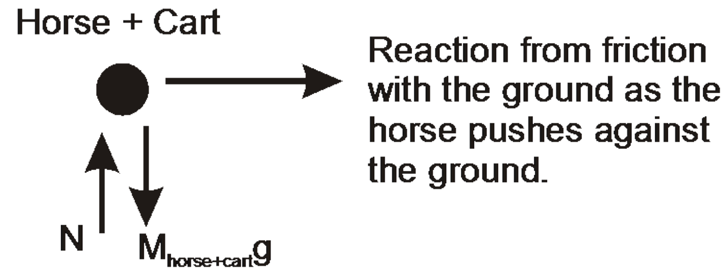
- If the two horizontal forces are equal, the cart and horse won't accelerate – although they could move at constant velocity
- To get things going, the force from the earth against the horse's hooves must be > the force from the cart.

⇒The horse has to supply a greater force <u>against the ground</u> (*leading to a reaction force on the horse*) than it feels from the cart resisting its motion.

P.S. The reaction force from the cart exists because

a force <u>on</u> the cart is needed to make it accelerate.





- If you *instead* define your system as the horse + cart, the action-reaction forces between the horse & cart are irrelevant.
- The acceleration of the horse and cart are determined by

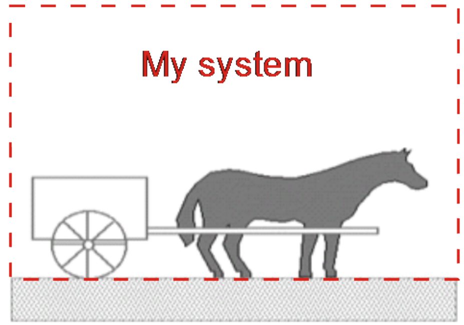
$$\Sigma F_x = F_{\text{friction from the ground acting on the horse}} = m_{cart+horse}\, a_{cart+horse}$$

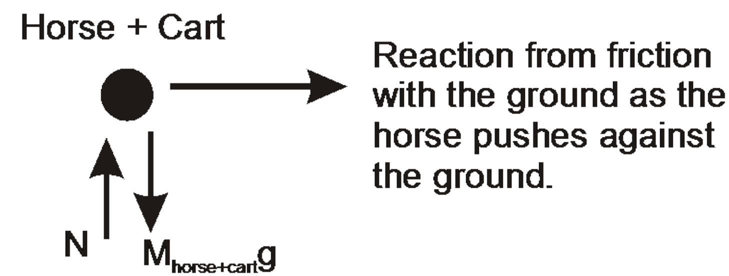
or

$$a_{cart+horse} = -(F_{\text{horse pushing against the ground}})/m_{cart+horse}$$

where the minus sign yields a positive acceleration because the horse pushes in my negative x direction.

Which explanation/system is easier?





Don’t get confused by the fact that the force the horse exerts on the ground is equal but opposite to the force the ground exerts on the horse!

These forces don’t cancel; they act on different objects!

Only the second force acts on the horse,

causing it (*and the cart*) to accelerate.

What does the force from the horse on the ground do?

It changes the rotation of the earth – VERY LITTLE!

- 4 options

Shaquille O’Neal (*7’1” tall and 325 pounds*) is trying to push you out of his way so he can get to the front of the line at McDonalds. You almost certainly have less mass than Shaq does. You find yourself moving at a constant velocity, against your will, in the direction that Shaq is pushing you.

Which of the following is correct?

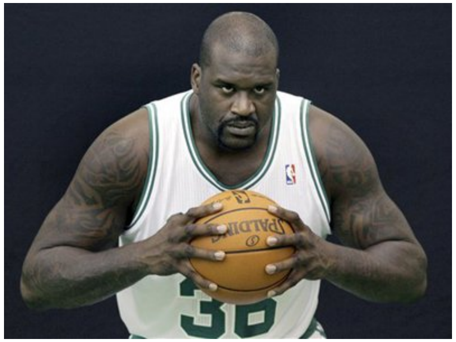
A. The force from Shaq pushing on you is greater than the force you are supplying in resistance.

B. The force you are supplying is greater, because you are working harder to resist Shaq while he barely notices you.

C. These forces are equal in magnitude.

D. May the force be with you, because you’ll need it if you’re between Shaq and his lunch!

- 4 options



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C. These forces are equal in magnitude.

D. May the force be with you, because you'll need it if you're between Shaq and his dinner!

Trust in Newton's 3rd Law

& learn to identify action-reaction force pairs.

Would the answer to the above question change if you found yourself *accelerating* (*flying across the room*) in response to Shaq's push?

No, you should still trust Newton's 3rd Law.

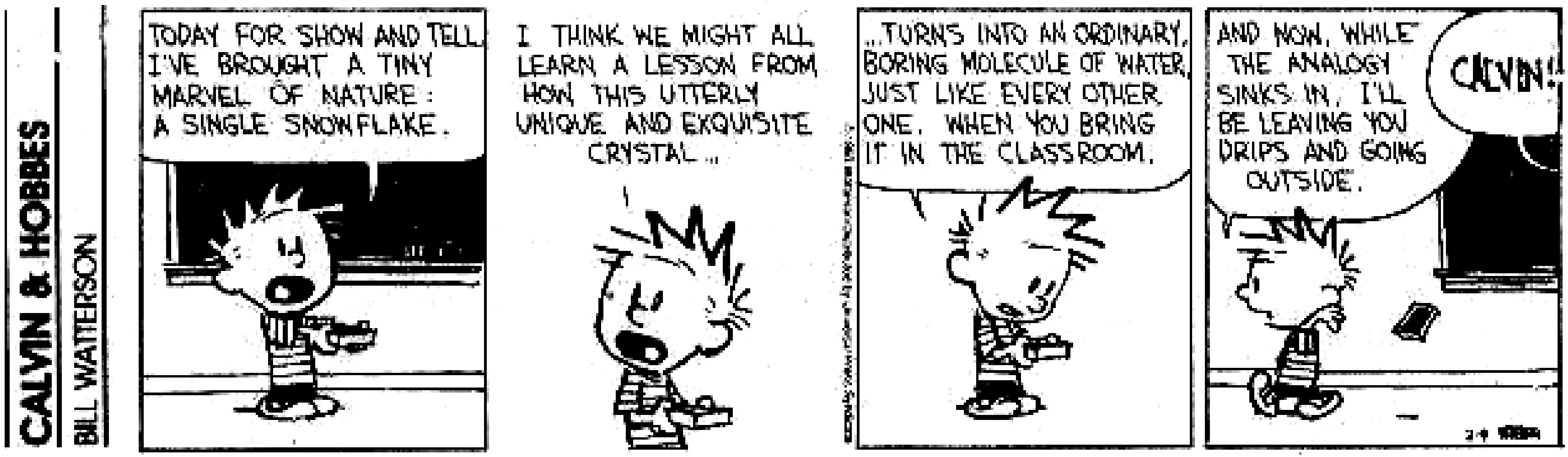
It's just that the ***effect*** of the force ***on you*** is greater than its effect *on Shaq* – plus you have to take into account the effect that friction with the ground has on each of you.

What would happen if Shaq is standing on ice while you are standing on gravel?

In this case, Shaq is the one who would go flying.

We made it to slide #48 on Friday, January 30, 2015.

PHYS 121 – SPRING 2015



Chapter 5: Newton’s Laws of Motion

version 02/04/2015, ~ 155 slides

We made it to slide #48 on Friday, January 30, 2015.

Get your clickers ready!

ANNOUNCEMENTS

HOMEWORK #2 Class average = $(44.5 \pm 7.5)/50 = (89.1 \pm 15.0)\%$

HOMEWORK #4 is posted on Blackboard, includes some challenging problems & is due two days before Exam #1, so get an early start and attend the SI sessions for help!

EXAM #1

Tentative formula sheet is posted on Blackboard. *You may request additions!*

Informal review in class Monday, February 9.

On Wednesday, sit in every *second* seat, starting with *aisle* seats.

CLICKER BONUS POINTS

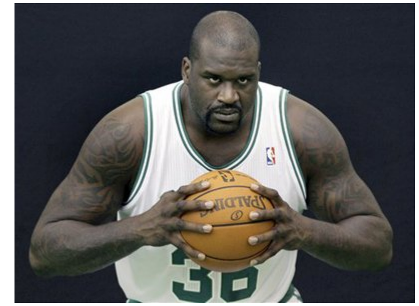
Collected Monday, January 26.

Should I send emails to students alerting them when they earn or forfeit clicker bonus points or rely on occasional Blackboard postings?

A. Send email.

B. Leave me alone!

- 4 options



You're pushing against a wall with a force F = 10 N. The wall doesn't move, and you believe in Newton's 3rd Law, so you conclude that the wall must be pushing back on you with the same 10 N force but in the opposite direction.

We magically replace the wall with Shaq. Instead of pushing on the wall, you are pushing on Shaq – and he doesn't move either. The force you are supplying in this case is:

A. = 10 N

B. > 10 N, since Shaq is pushing back.

C. < 10 N

D. ***VERY DANGEROUS***, because Shaq doesn't like people trying to push him around.

- 4 options

You're pushing against a wall with a force F = 10 N. The wall doesn't move, and you believe in Newton's 3rd Law, so you conclude that the wall must be pushing back on you with the same 10 N force but in the opposite direction.

We magically replace the wall with Shaq. Instead of pushing on the wall, you are pushing on Shaq – and he doesn't move either. The force you are supplying in this case is:

A. = 10 N

B. > 10 N, since Shaq is pushing back

C. < 10 N

D. Very dangerous, because Shaq doesn't like people trying to push him around.

Do you believe Newton's 3rd Law yet?

Would it matter if the wall or Shaq DID move when you pushed?

NO!

The *reaction* force would be still be equal and opposite to the *action* force.

4 options

Consider a car at rest. We can conclude that the downward gravitational pull of Earth on the car and the upward contact force of Earth on the car are equal and opposite because:

a. the two forces form an action-reaction pair.

b. the net force on the car is zero.

c. both (a) & (b).

d. neither (a) nor (b).

4 options

Consider a car at rest. We can conclude that the downward gravitational pull of Earth on the car and the upward contact force of Earth on the car are equal and opposite because:

a. the two forces form an action-reaction pair.

b. the net force on the car is zero.

c. both (a) & (b).

d. neither (a) nor (b).

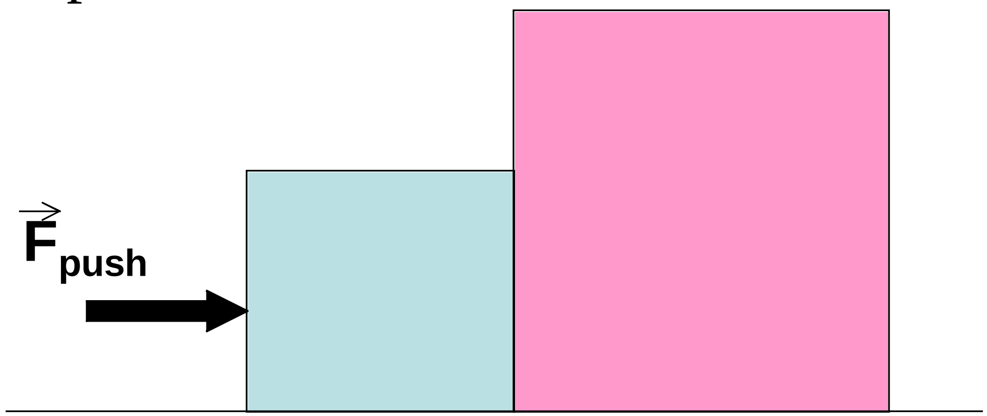
They can't be an action-reaction pair since they act on the same object. The reaction to gravity on the car is gravity on the earth!

The car is at rest, its acceleration in the vertical direction is constant (*and zero*), so the net force on the car must be zero.

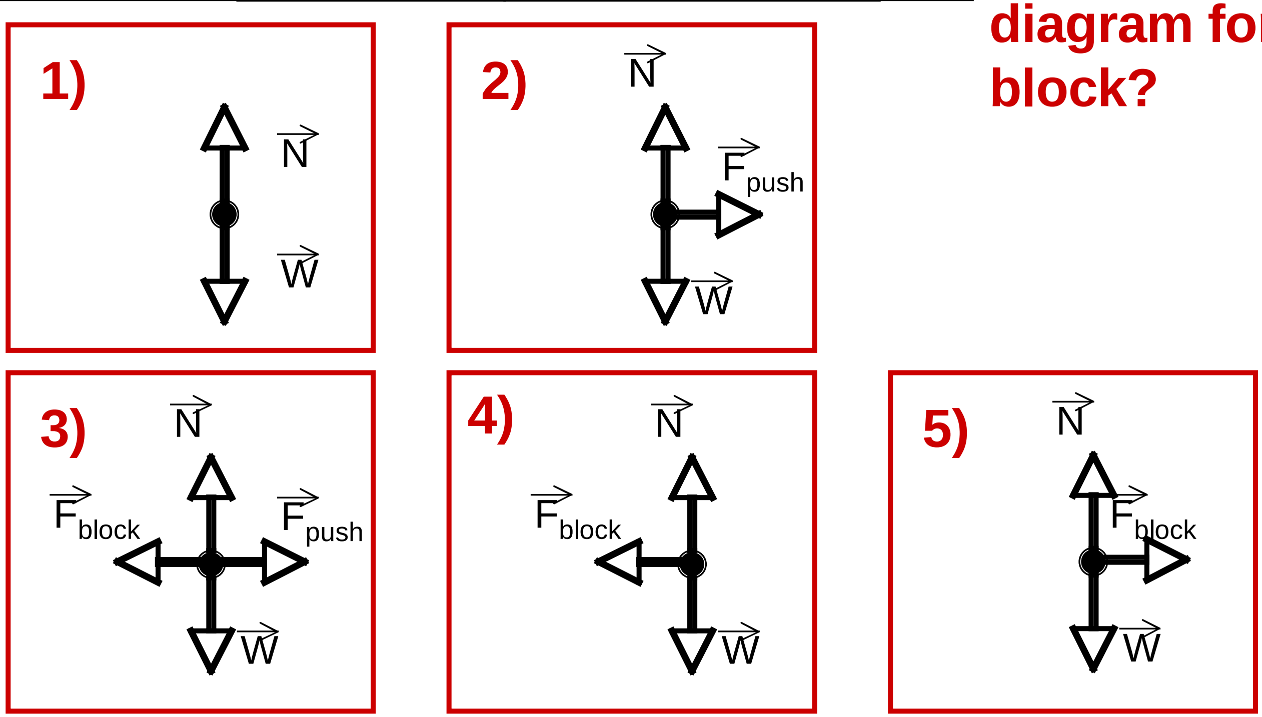
This means that the two forces must be equal and opposite.

PRACTICE IN DRAWING A FREE BODY DIAGRAM

- 5 options

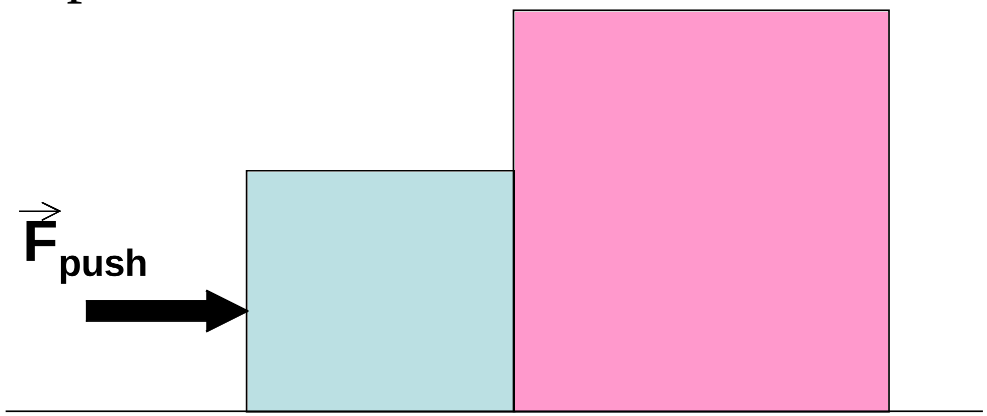


Two blocks sit on a frictionless surface. A pushing force F_{push} is applied to the left block (blue). What is the correct force diagram for the pink block?



PRACTICE IN DRAWING A FREE BODY DIAGRAM

- 5 options



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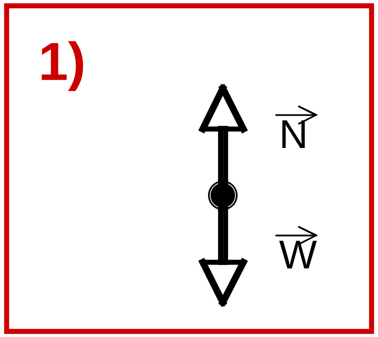
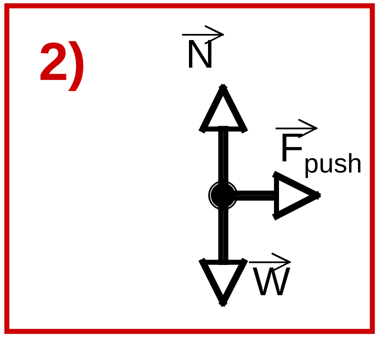
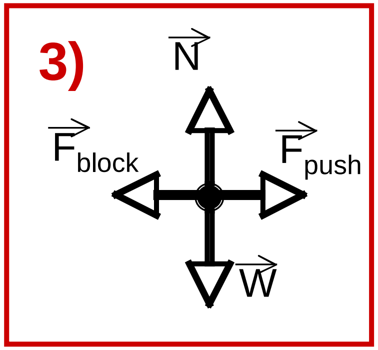
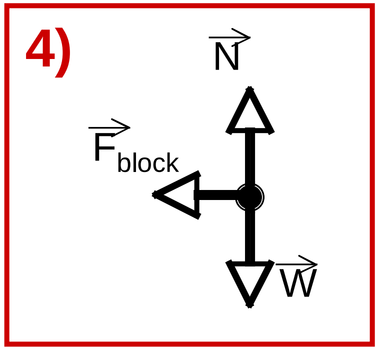
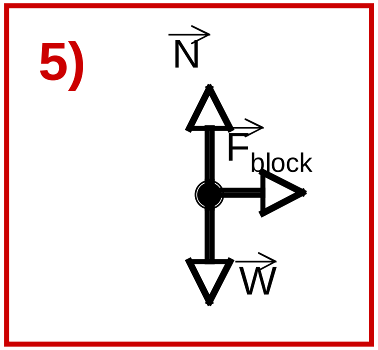










The answer is

5

PRACTICE WITH CONTACT FORCES

~ Ohanian 5.47

4 options

Boxes #1 and #2 in the figure have masses $M_1 > M_2$.

The boxes are being pushed across a frictionless horizontal surface by a force F. In case A, the force is applied to the larger box while in case B, the same (*magnitude*) force is applied to the smaller block.

In which case is the contact force between the blocks larger?

A. Case A

B. Case B

C. The contact force is the same in both cases.

D. There's not enough information to tell.

PRACTICE WITH CONTACT FORCES

4 options

Boxes #1 and #2 in the figure have masses $M_1 > M_2$.

The boxes are being pushed across a frictionless horizontal surface by a force F. In case A, the force is applied to the larger box while in case B, the same (*magnitude*) force is applied to the smaller block.

In which case is the contact force between the blocks larger?

A. Case A

B. Case B

C. The contact force is the same in both cases.

D. There's not enough information to tell.

- The acceleration is the same in both cases and for both masses but, for the block not subject directly to F, that acceleration must be supplied by the contact force between the blocks.
- A given acceleration requires a larger contact force for the larger mass.

DEMO

bonus points are available

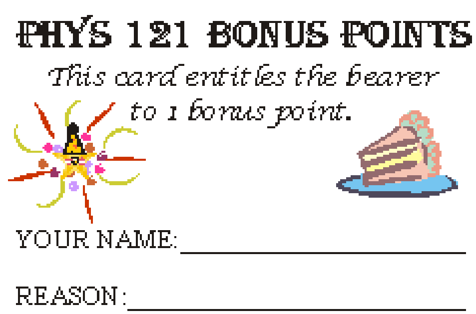
Two pushy students are needed,

preferably one who is relatively massive and another who is not!

Their charge is to try to push each other around.

Blocks and Bathroom Scales

https://www.youtube.com/watch?v=ckFm3s3DI3Q&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr&index=3



APPLYING NEWTON'S LAWS

- ropes & pulleys
- masses on inclined planes
- blocks pushing blocks
- ropes WITH mass
- bootstrapping (*pulling yourself up with a pulley*)
- mechanical advantage

ROPES & TENSION

- **TENSION**, ***T***, on or in an object ≡ force trying to pull each element/piece of that object **apart**.
- Tension in PHYS 121 is usually supplied by a rope or string, *not by exams.*
- The tension is the same everywhere within an ideal rope.
- This same tension appears at both ends of an ideal rope, pulling objects attached to the ends of the rope **inwards**.

  as a reaction to the force applied to the rope
- The opposite of *tension* is *compression*.

  An object in *compression* pushes outwards at its ends.

  An ideal rope can't push on an object, only pull.
- Tension is due to molecular/atomic bonds inside the rope resisting attempts to stretch those bonds.

IDEAL ROPES

- have no mass of their own
- are held perfectly straight by tension
- cannot stretch $\Rightarrow$ same acceleration at both ends, if those ends are *free*
- can exert a force only parallel to their length

<u>IDEAL ROPES</u>

Why does an ideal, <u>massless</u> rope have the same tension on both ends and, internally, everywhere along its length?

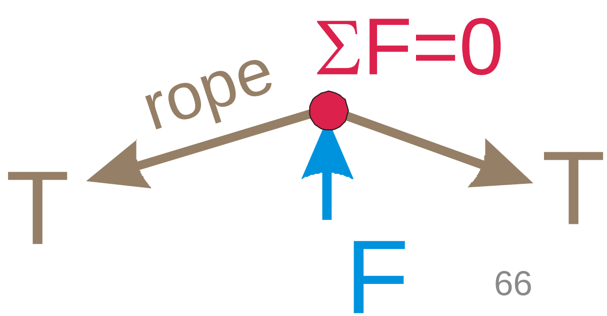
- The key is the word *massless.*

  You need to get good at spotting key phrases like this.

- If the net force on any massless element is not zero, the acceleration of that part of the rope = ∞!

The magnitude of the tension is constant if you push somewhere along a rope

- but that constant changes compared to the original **T**,
- the direction of the tension changes &
- $\Sigma \mathbf{F} = 0$ where you push or $\mathbf{a} \rightarrow \infty$



<u>IDEAL PULLEYS</u>

<u>Pulleys are used to change the direction of tension in a rope.</u>

Ideal pulleys have

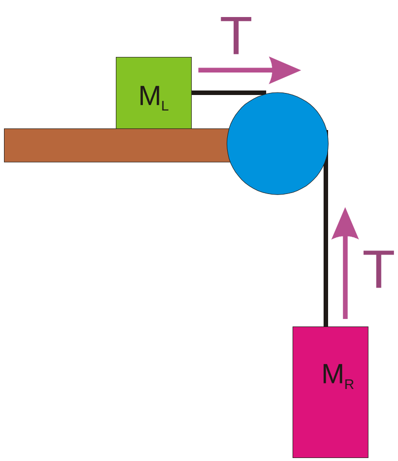
- <u>no mass</u> of their own
- <u>no friction in their bearings</u>

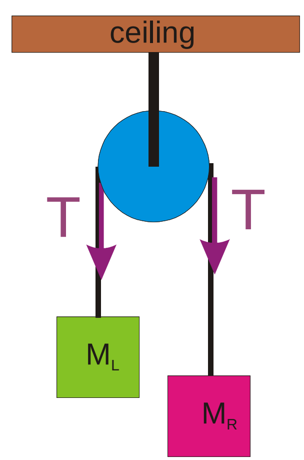
Tension in an **<u>ideal</u>** rope is the same on both sides of an **<u>ideal</u>** pulley.

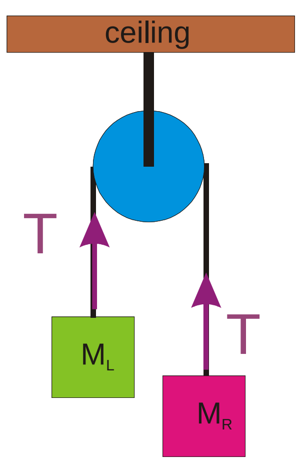
Otherwise the massless pulley would have ∞ rotational acceleration.

Keep an eye out for the term "ideal" for ropes & pulleys.

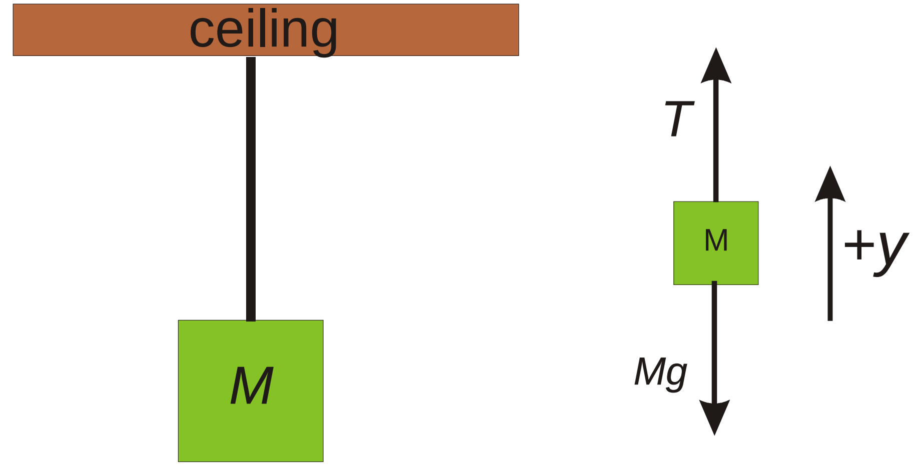
We'll consider "realistic" ropes and pulleys soon.







ROPES



Consider a mass *M* hanging from a rope and either *at rest* or *moving with a constant velocity* in the vertical direction ($a_y = 0$).

$$\Sigma F_y = Ma_y = 0$$

$$T - Mg = Ma_y = 0$$

$$\Rightarrow \quad T = Mg$$

HANGING BY A THREAD (or rope)

A mass M is hanging from the ceiling of an elevator by a rope.

What is T in the rope if the elevator accelerates up at $g/2$?

$$\Sigma F_{ext} = Ma \;\rightarrow\; T - Mg = M(g/2) \qquad \text{so} \qquad T = 3/2\, Mg$$

What is T in the rope if the elevator accelerates down at $g/2$?

$$T - Mg = -M(g/2) \qquad \text{so} \qquad T = 1/2\, Mg$$

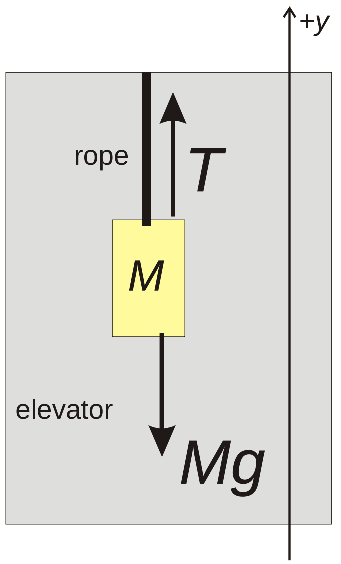
What is T in the rope as the elevator accelerates down at $g/2$ if I switch my y-axis so that down is positive?

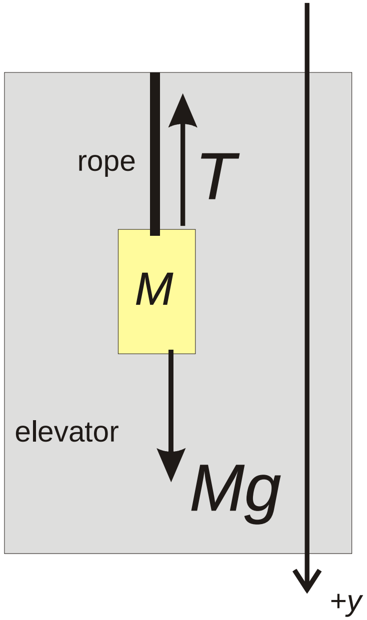
$$-T + Mg = Mg/2 \qquad \text{so} \qquad T = 1/2\, Mg$$

Physically observable quantities can't change just because you choose a different coordinate system!

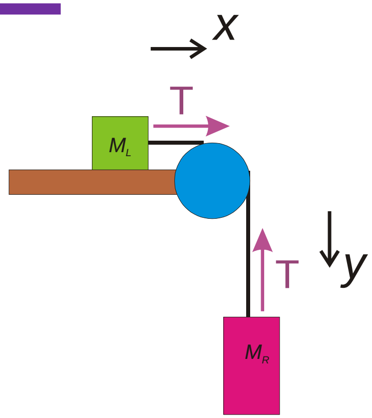
What is T if the elevator moves up at a high, but constant speed?

$$a = 0 \;\Rightarrow\; T = Mg$$





APPLYING NEWTON'S LAWS



1. Make a sketch representing the problem.
2. Draw a free body diagram for each object (mass).
3. Choose your *system*
   - This will distinguish internal from external forces.
   - You might change the system for different parts of a calculation.
4. Choose the coordinate system for each object
   - You can use different coordinate systems for different objects in a system, as long as you 'connect' the forces and motion correctly.
5. Write $\Sigma \mathbf{F} = m\mathbf{a}$ for each relevant component of each object.
   - You need one independent equation for each unknown quantity.

     (*If there are 2 unknown quantities in the system, perhaps one mass and one acceleration, you will need 2 independent equations.*)

APPLYING NEWTON'S LAWS

6. Solve the resulting set of equations.
   - Identify known and unknown quantities + what you want to know.
   - Take your shoes off & do the math.

7. **CHECK YOUR ANSWER!**
   - The dimensions/units must be appropriate.
     - Don't submit an answer like $a = 5\ m/s$.
   - The magnitudes must be *reasonable*.
     - Don't submit an answer like $a = 3 \times 10^{57}\ m/s^2$.
   - The results must be *self-consistent*.
   - Don't ignore an answer that you should know is wrong.
     - Even if you can't find the error, point out that you realize the concern.

APPLYING NEWTON'S LAWS

8. THINK ABOUT YOUR ANSWER

– What can you learn about the behavior of the system?

– How do different terms (M, g, v_o θ, *etc*.) show up?

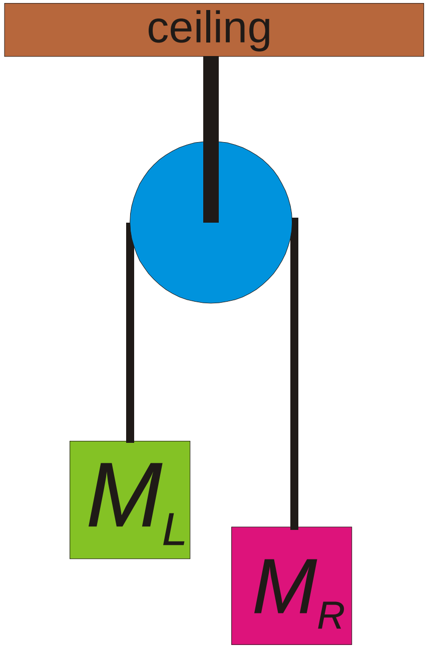
– Consider *limiting cases* as important quantities grow

very large or very small.

What is the PHYSICS of the system

(*in contrast to the math*)?

NEWTON'S LAWS EXAMPLE

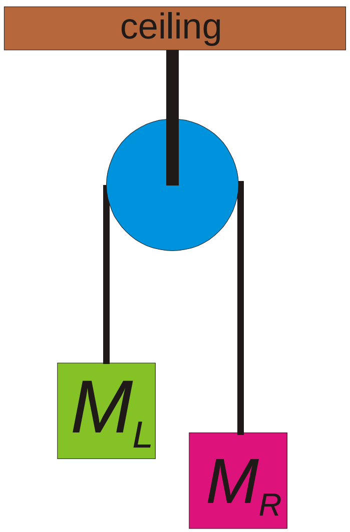


An **Atwood's machine** is two masses connected by an ideal rope hanging on either side of an ideal pulley.

There are 4 quantities of interest.

- M_L = mass on left side of pulley
- M_R = mass on right side of pulley
- T = tension in the rope (*same on both sides of the pulley*)
- a = acceleration of the masses (*same magnitude for both*)

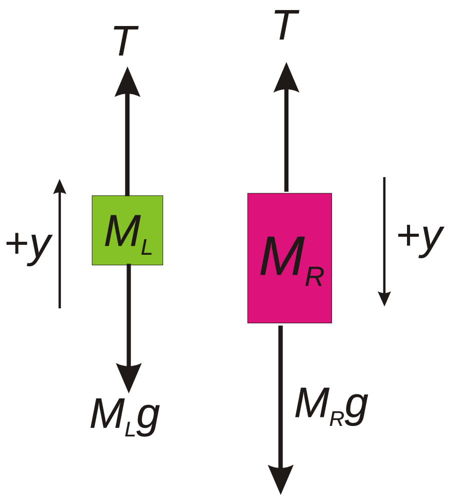
ATWOOD’S MACHINE



- ✓ Make a sketch representing the problem.
- ✓ Draw a free body diagram for each object (*mass*).
- ✓ Choose your *system* (2 *separate masses*)
- ✓ Choose the coordinate system for each object.

I’ve picked the positive y-direction up for M_L and down for M_R so that I can use the same a for both masses. $a_L = a_R = a$

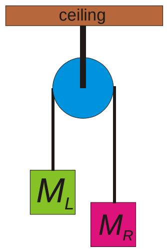
- ✓ Write out $\mathbf{F} = m\mathbf{a}$ for each relevant component of each object.



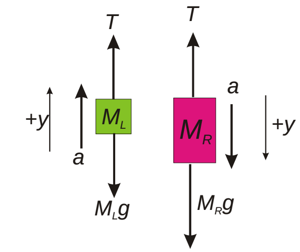
Newton’s 2nd law for each mass:

$T - M_Lg = M_La$ for the system consisting of M_L

$-T + M_Rg = M_Ra$ for the system consisting of M_R



ATWOOD'S MACHINE



✓ Solve the resulting set of equations.

- Identify known and unknown quantities + what you want to know.
- You need one independent equation for each unknown quantity. We have

$$T - M_L g = M_L a \qquad\qquad -T + M_R g = M_R a$$

- ➢ There are potentially 4 unknowns in these 2 equations; M_L, M_R, T & a.
- ➢ We have two independent equations connecting these quantities.

  $\Rightarrow$ you need to be given two of the unknowns to solve for anything.
- ➢ Conventionally, students are given the masses and asked to solve for the acceleration and the tension in the string – and that's what we'll do now.

  But you could be given any two of the parameters & be asked to solve for the other two.

ATWOOD'S MACHINE MATH

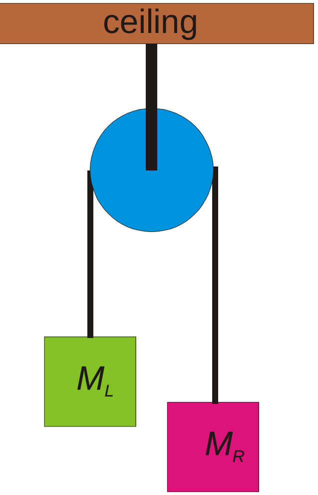
$T - M_L g = M_L a$ $\qquad$ $-T + M_R g = M_r a$

One method to solve this pair of equations is to add them, so that *T* goes away, and then solve what remains for *a*.

$$-M_L g + M_R g = M_L a + M_R a$$
$$g\left(M_R - M_L\right) = a\left(M_R + M_L\right)a$$
$$a = \left(\frac{M_R - M_L}{M_R + M_L}\right)g$$

Plug *a* into either of the original equations and solve for *T*.

$$T = M_L\left(a + g\right) = M_L\left(\left(\frac{M_R - M_L}{M_R + M_L}\right)g + g\right)$$
$$= M_L g\left(\left(\frac{M_R - M_L}{M_R + M_L}\right) + \left(\frac{M_R + M_L}{M_R + M_L}\right)\right)$$
$$= \left(\frac{2M_R M_L}{M_R + M_L}\right)g$$



ATWOOD'S MACHINE

So we have the answers for a and for T.

$$a = \left(\frac{M_R - M_L}{M_R + M_L} \right) g \qquad T = \left(\frac{2M_R M_L}{M_R + M_L} \right) g$$

BUT YOU ARE NOT DONE!

Check your answers & consider limiting cases.

- Do they have the right dimensions?

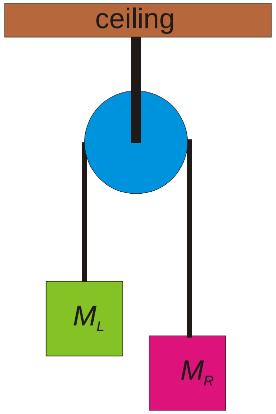
  ✓a is an acceleration and T is a force.

- Does the system behave as expected if $M_L = M_R$?

  ✓$a \rightarrow 0$ and $T \rightarrow Mg$, just as you would expect (*I hope*) if the masses were just hanging there.

Does anyone think T should equal 2Mg because you have M hanging from each side?

If so, apply Newton's 2nd law to one mass at a time.



ATWOOD'S MACHINE

So we have the answers for *a* and for *T*.

$$a = \left(\frac{M_R - M_L}{M_R + M_L} \right) g \qquad T = \left(\frac{2M_R M_L}{M_R + M_L} \right) g$$

- Does the system behave properly if $M_R >> M_L$?

  ✓ $a \rightarrow g$ and $T \rightarrow 2M_L g$

  The acceleration makes sense.

  T is the answer we found earlier when you use a rope to pull M_L up with an acceleration of *g*.

- Does the system behave properly if $M_L >> M_R$?

  ✓ $a \rightarrow -g$ and $T \rightarrow 2M_R g$

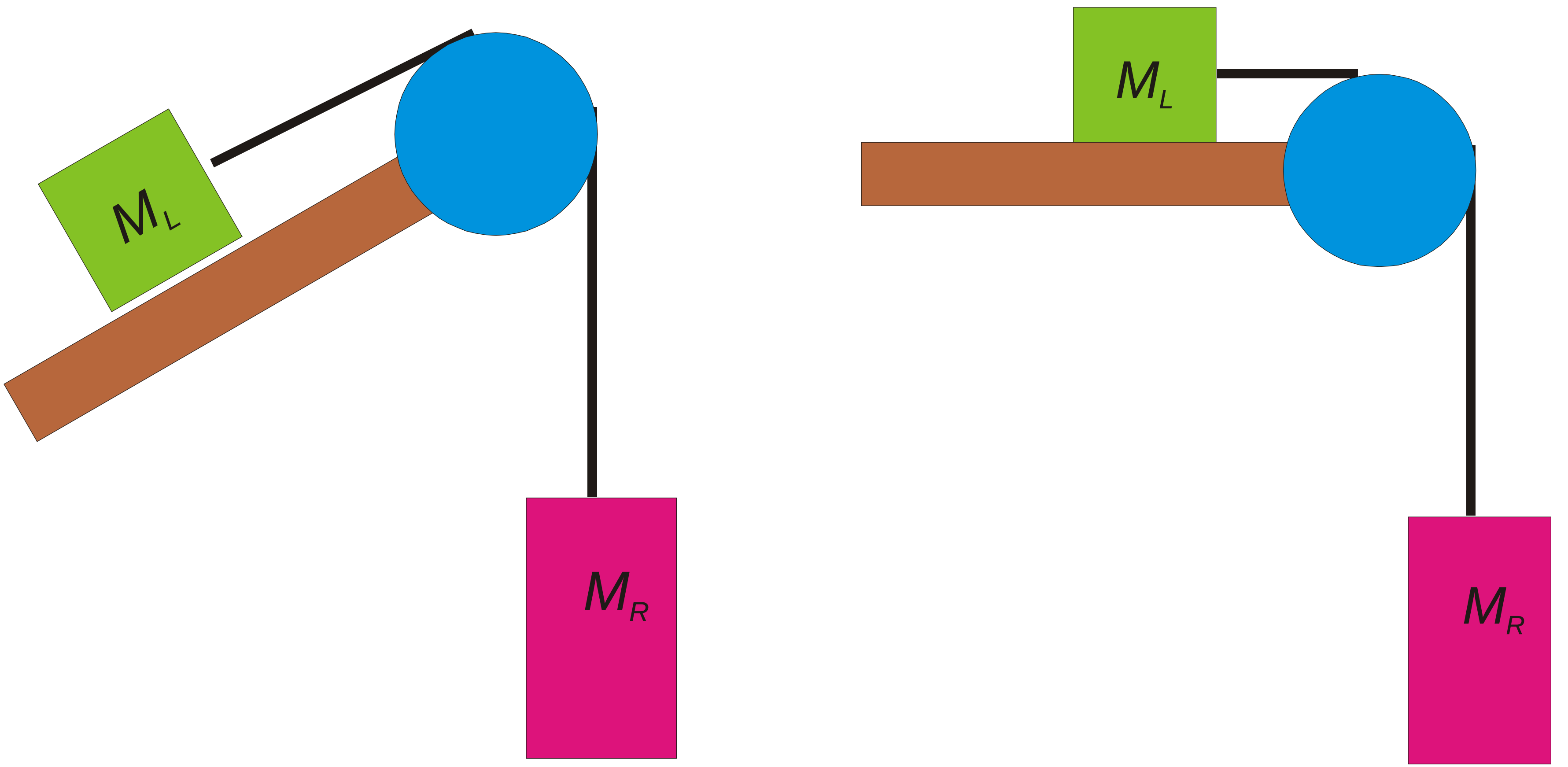
NUMBERS *vs.* *SYMBOLS*

Note the advantages of working with symbols rather than plugging in numbers near the beginning of a problem.

- You can't understand the general behavior of this system if all you have are a pair of numbers for acceleration and tension.
- It's easy to plug different numbers into a formula to see, for example, what would happen with a different mass – or a different planet and *g*.
- It's easier to identify and fix mistakes since you don't have to redo all your numerical calculations.

$$a = \left(\frac{M_R - M_L}{M_R + M_L} \right) g \qquad T = \left(\frac{2M_R M_L}{M_R + M_L} \right) g$$

Once you understand Atwood's machine, analyzing variations like the following should be straight-forward.

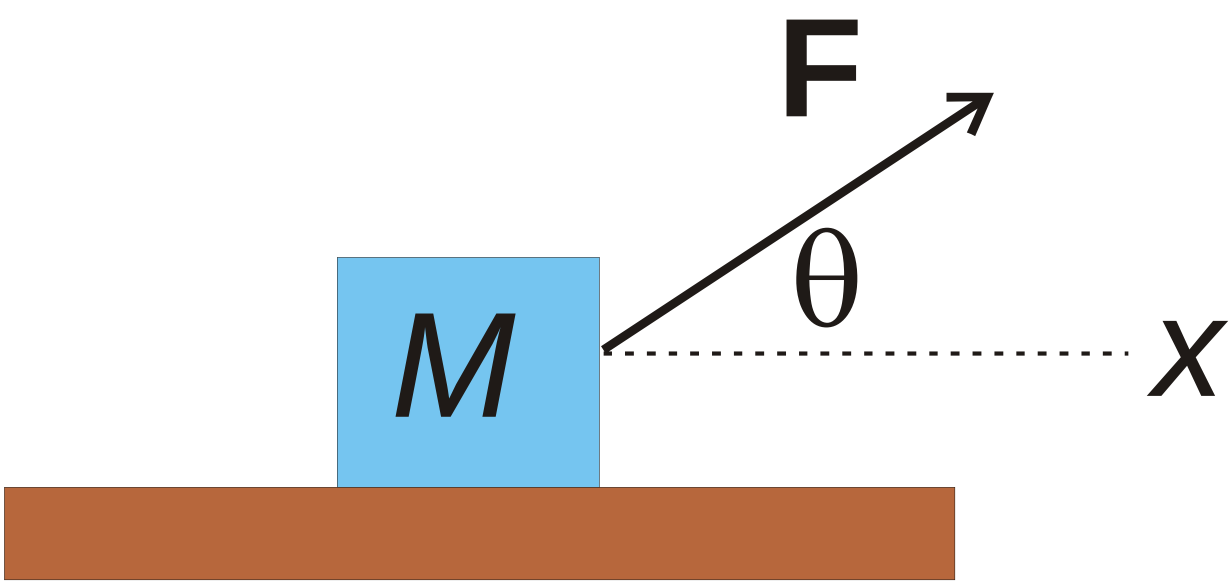


FORCES at ANGLES

Consider a mass that is pulled along a table by a rope at an angle θ above horizontal.

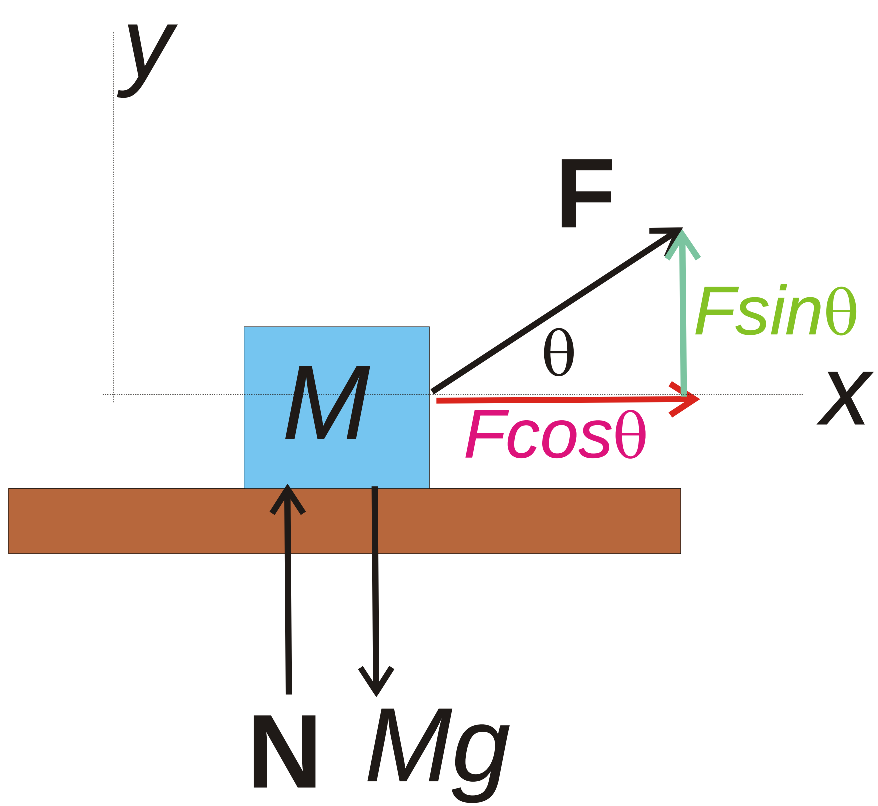
What is its acceleration?

What is the normal force between the mass and table?

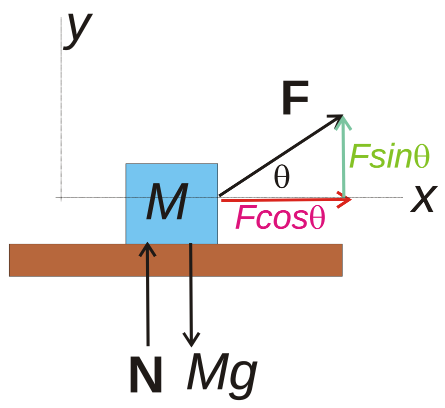


FORCES at ANGLES

- Draw the free body diagram.
- Resolve **F** into its vector components.



FORCES at ANGLES



$$\vec{F} = M\vec{a}$$ is TWO equations in 2D

$$\sum F_x = F\cos\theta = Ma_x \Rightarrow a_x = \frac{F}{M}\cos\theta$$

$$\sum F_y = F\sin\theta + N - Mg = Ma_y = M \times 0 = 0$$

$$\Rightarrow N = Mg - F\sin\theta$$

DOES THIS MAKE SENSE?

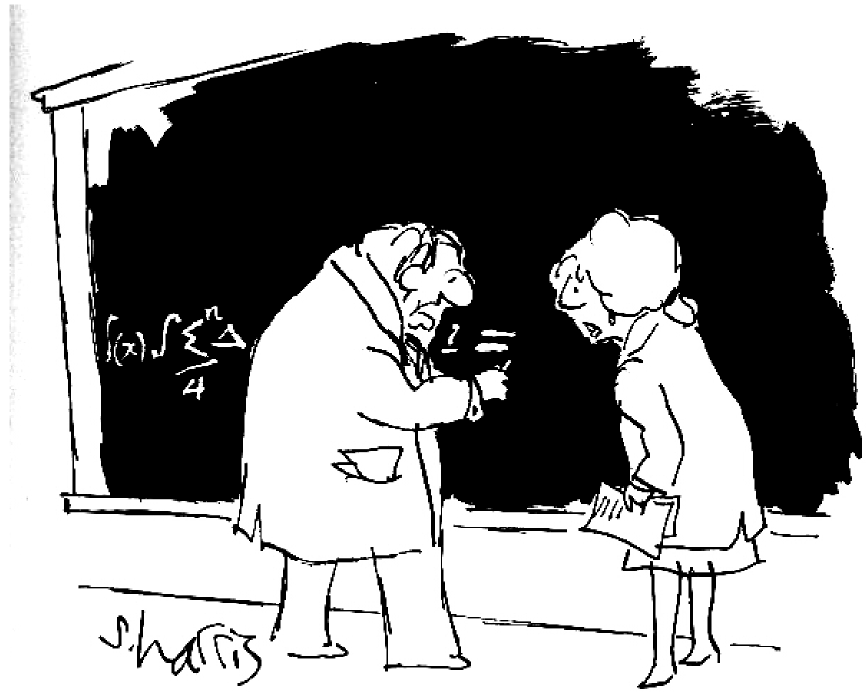
What happens as θ goes to 0° or 90°?

- As θ → 0°, $a_x = \dfrac{F\cos\theta}{M} \rightarrow \dfrac{F}{M}$ & $N = Mg - F\sin\theta \rightarrow Mg$
- As θ → 90°, $a_x = \dfrac{F\cos\theta}{M} \rightarrow 0$ & $N = Mg - F\sin\theta \rightarrow Mg - F$

We made it to slide #88 on

Wednesday, February 4.

PHYS 121 – SPRING 2015



"This is the part I always hate."

Chapter 5: Newton's Laws of Motion

version 02/06/2015, ~ 155 slides

We made it to slide #88 on Wednesday, February 4, 2015.

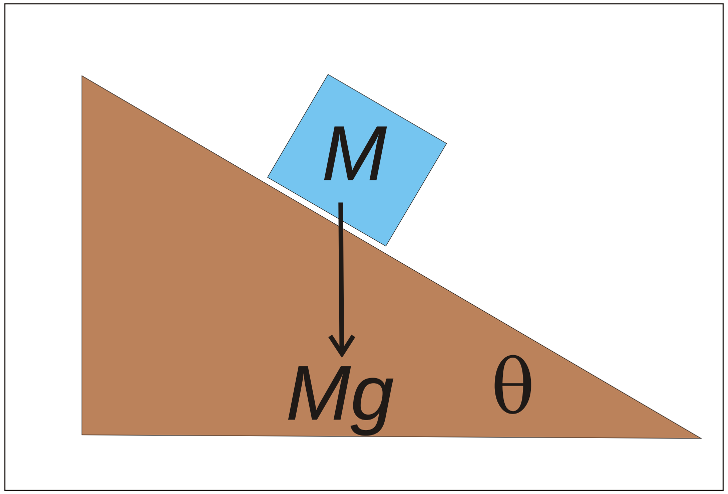
Get your clickers ready!

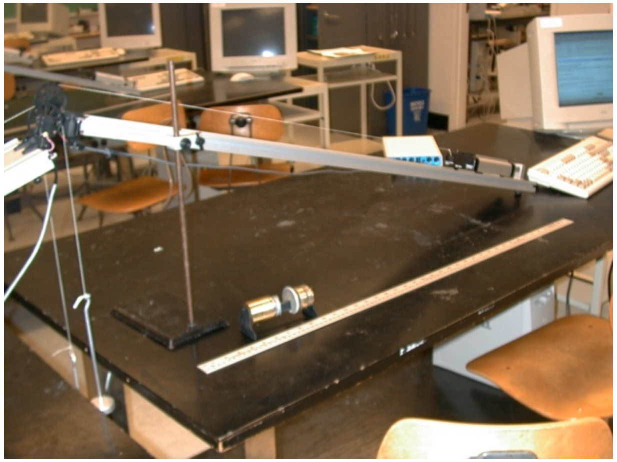
ANNOUNCEMENTS

EXAM #1

Tentative formula sheet is posted on Blackboard. *You may request additions!*

Review in class Monday, February 9, based on questions from the class.





Consider a block that is free to move under the influence of gravity along a frictionless inclined plane.

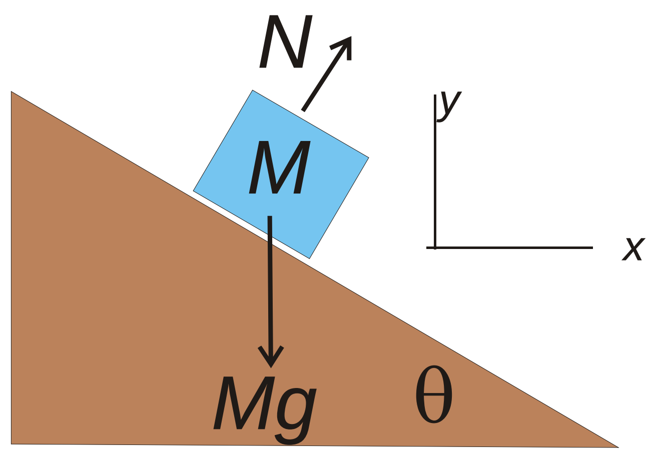
What is the acceleration of the block?

What is the normal force?

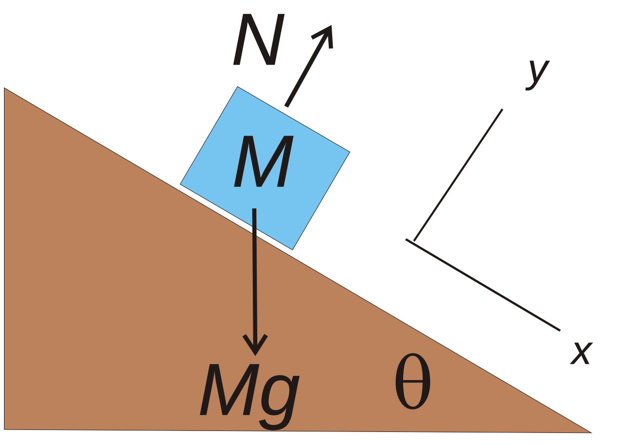
1. Decide on your system & coordinate system.

The system is the block *M* (*the inclined plane is excluded*).

Two options for the coordinate system are shown below.



A



B

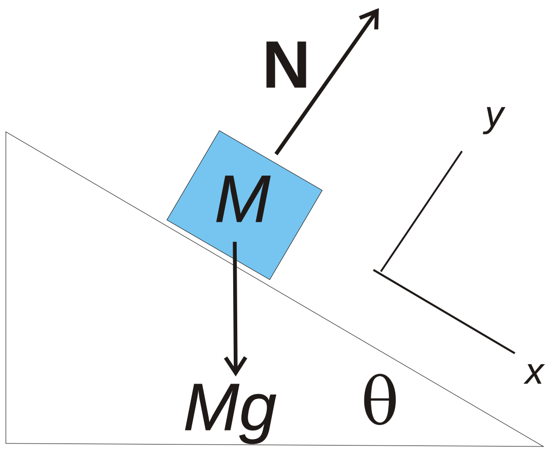
Which one would you choose: A, B, or C = *neither of the above?*

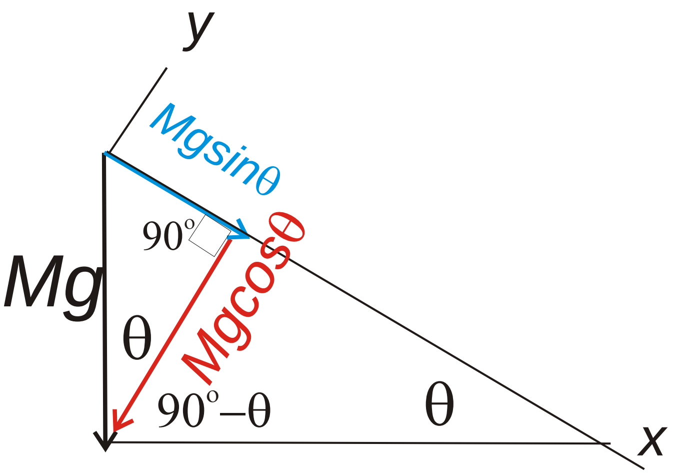
All three answers are "right", but B is the best IMHO

since *both* **N** & **a** are easier to describe in B.

If you picked C, your grade in the course will likely be no higher than a C.

2. Draw the Free Body (*force*) Diagram





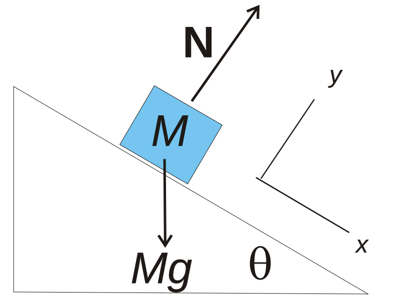
There are two forces acting on the block.

- $\mathbf{F}_{\mathbf{gravity}} = Mg$ down
- **N** = normal force from the inclined plane pointing in my +***y*** direction.

Resolve Mg into its components in my x and y directions.

- $F_{gx} = Mg\sin\theta$
- $F_{gy} = -Mg\cos\theta$

[HINT: *A trick to keep sine's and cosine's straight is to imagine which one works for $\theta = 0$, when the correct choice is usually obvious.*]

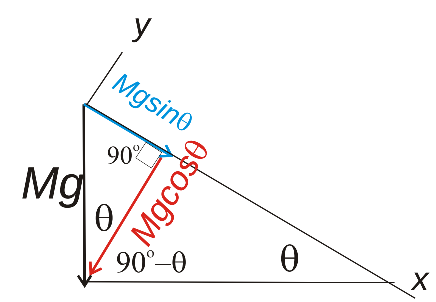


3. Apply ΣF = Ma for each component.

$$\Sigma F_y = N - Mg\cos\theta = Ma_y = 0$$

This tells us that the normal force **N** is **N = Mgcosθ**

- Note that **N** = *Mg* for a horizontal surface with θ = 0° and that **N** decreases as the surface is tilted.
- We won't normally need **N** unless we include friction.

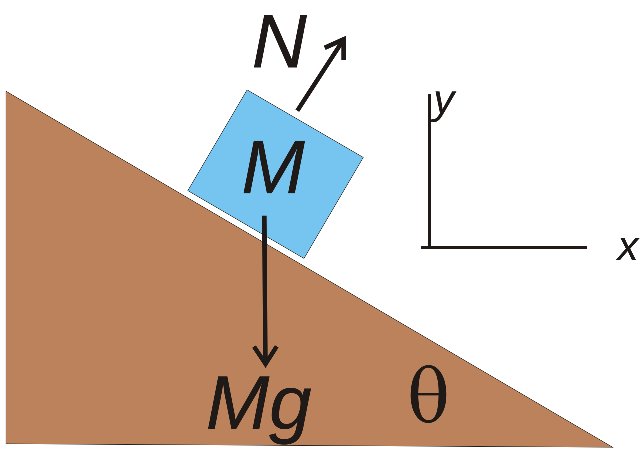


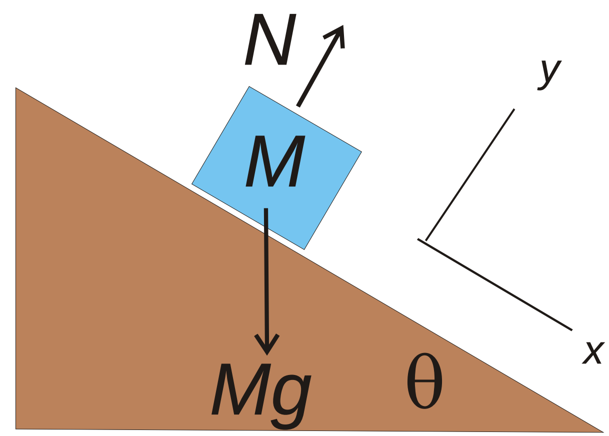
$$\Sigma F_x = Mg\sin\theta = Ma_x$$

This tells us that the acceleration in the *x*-direction, along the inclined plane, is $a_x = g\sin\theta$

- Note that $a \to 0$ for a horizontal plane with $\theta \to 0°$
- $a \to g$ for a vertical plane with $\theta \to 90°$

What would have changed if you had chosen the coordinate system on the left?





- *Mg* is purely in the –*y* direction. <u>GOOD; THAT MAKES IT SIMPLER</u>!
- The normal force **N** has components in both directions. <u>BAD</u> – *but even*!
- The acceleration *a* would also have components in both directions, a_x & a_y.

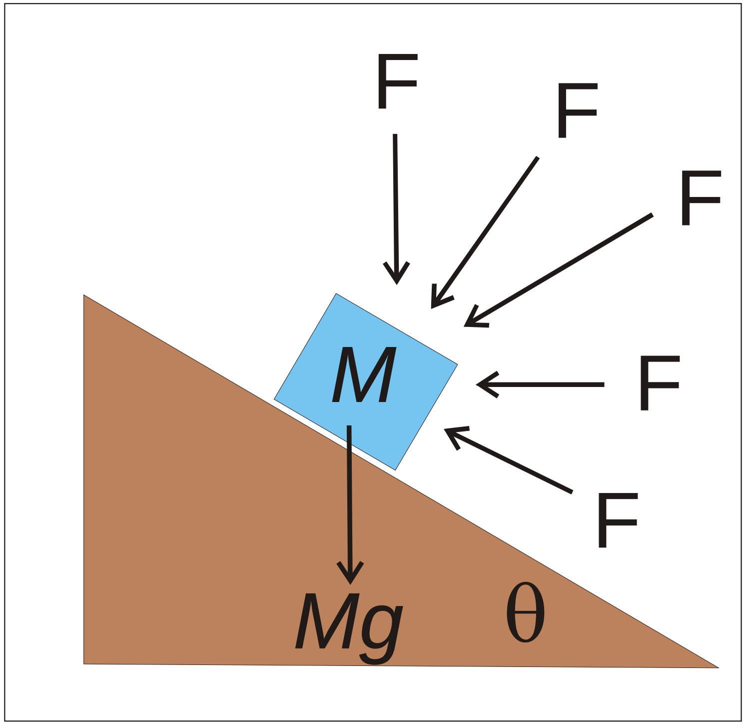
<u>!! VERY BAD !!</u>

Choose WISELY!

We can make this problem more interesting by adding another force like one of the options shown in the figure.

What would you do differently in each case?

$\Sigma F_y = N - Mg\cos\theta = Ma_y = 0$



$\Sigma F_x = Mg\sin\theta = Ma_x$

INCLUDE MORE COMPONENTS

in $\Sigma F_x = ma_x$ *&* $\Sigma F_y = ma_y$

Is this problem <u>conceptually</u> harder with another force?

<u>NOT IF YOU APPROACH IT SYSTEMATICALLY</u>

although the math is more annoying.

- 4 options

The positive horizontal axis is defined as pointing to the right in the system shown below. θ is defined *wrt* vertical and *F* points down and to the left.

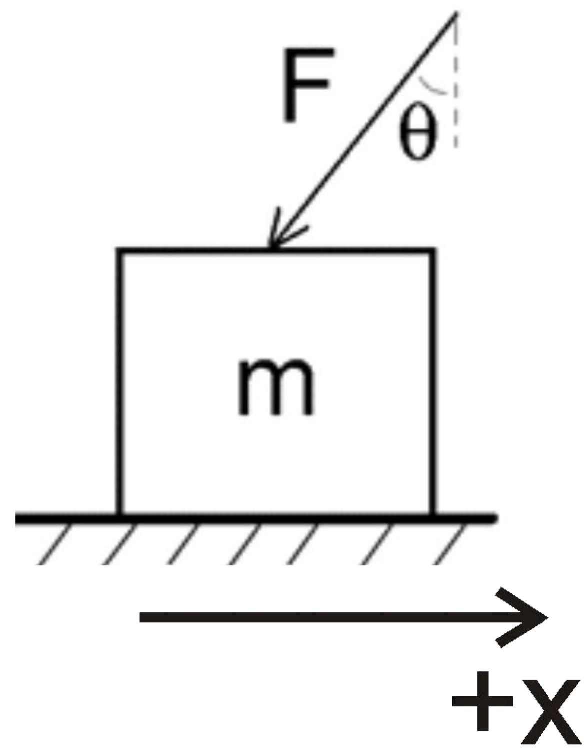
What is the *horizontal* component of the force *F*?

A. $F\sin\theta$

B. $F\cos\theta$

C. $-F\sin\theta$

D. $-F\cos\theta$



- 4 options

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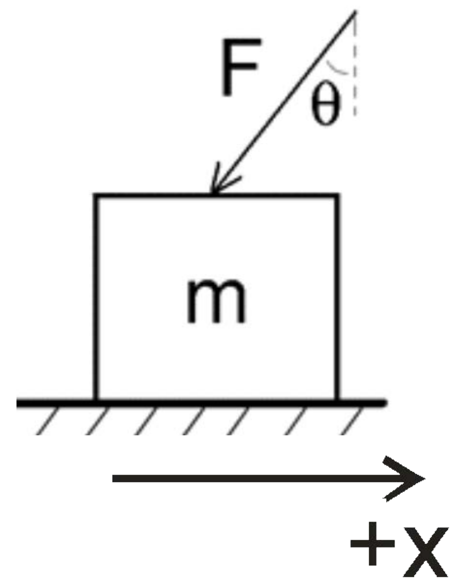


$\sin\theta$ = opposite/hypotenuse

- 4 options

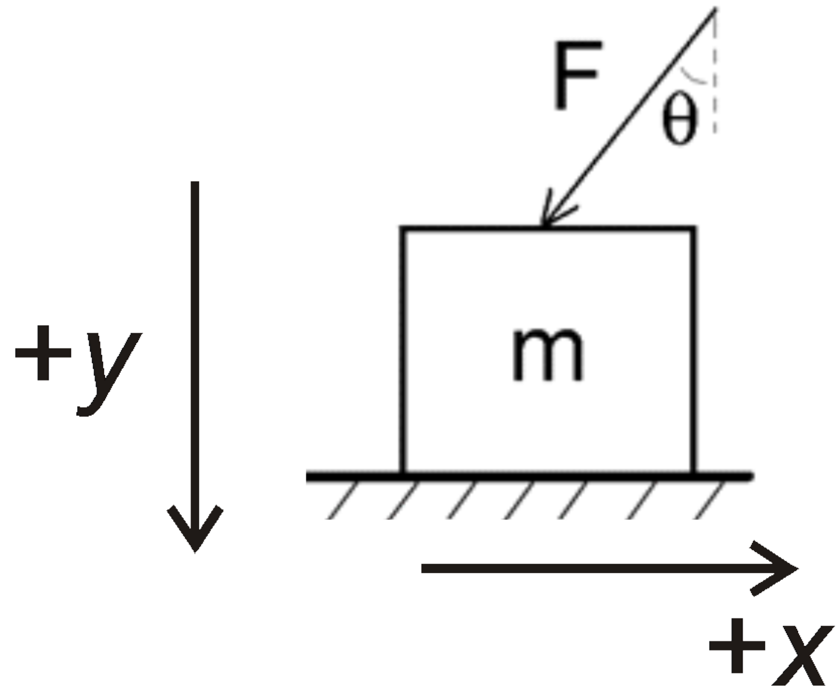
For the same figure, which of the following is true for the normal force N acting on the block due to the table, if down is called positive?

A. $N = mg + F\sin\theta$

B. $N = mg - F\sin\theta$

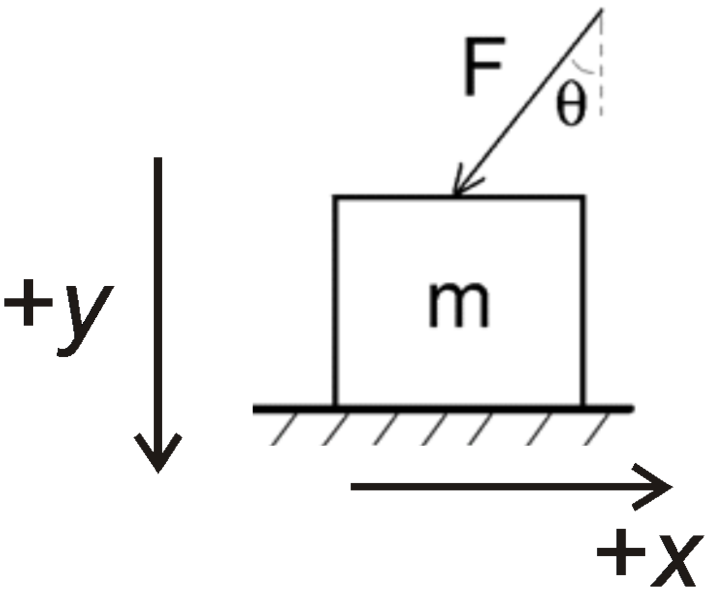
C. $N = mg - F\cos\theta$

D. $N = mg + F\cos\theta$



- 4 options

For the same figure, which of the following is true for the normal force N acting on the block due to the table, if down is called positive?



A. $N = mg + F\sin\theta$

B. $N = mg - F\sin\theta$

C. $N = mg - F\cos\theta$

D. $N = mg + F\cos\theta$

Adding up the forces in the +y direction

$$-\mathbf{N} + mg + F\cos\theta = 0$$

$$\mathbf{N} = mg + F\cos\theta$$

4 options

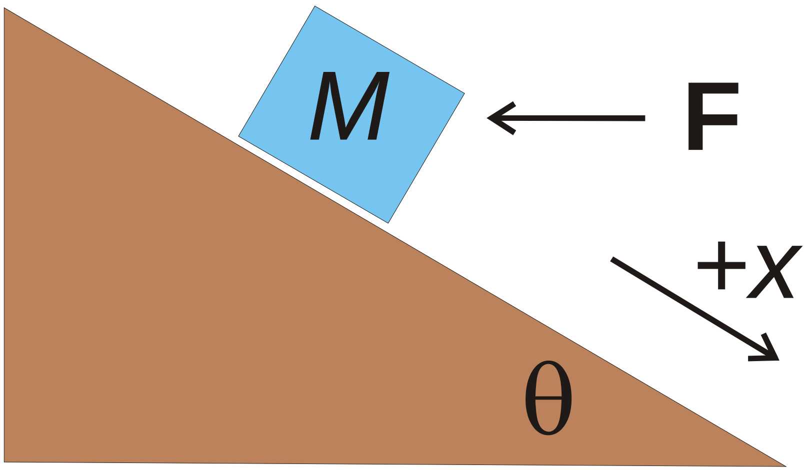
A horizontal force pushes a block up an incline that makes an angle θ from horizontal. If we align our x-axis along the incline and define positive as down the incline, what is the component of **F** in this direction, F_x?

A. $F\sin\theta$

B. $F\cos\theta$

C. $-F\sin\theta$

D. $-F\cos\theta$



4 options

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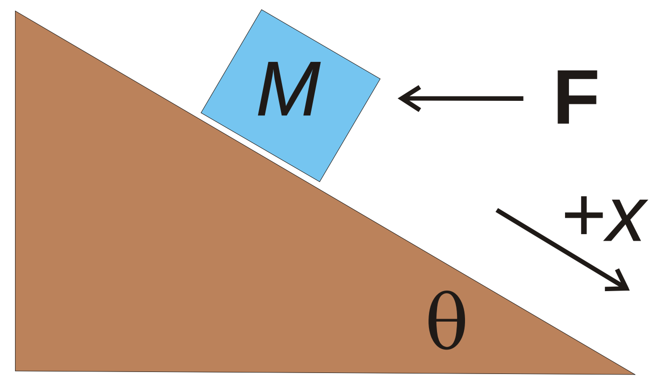
A. $F\sin\theta$

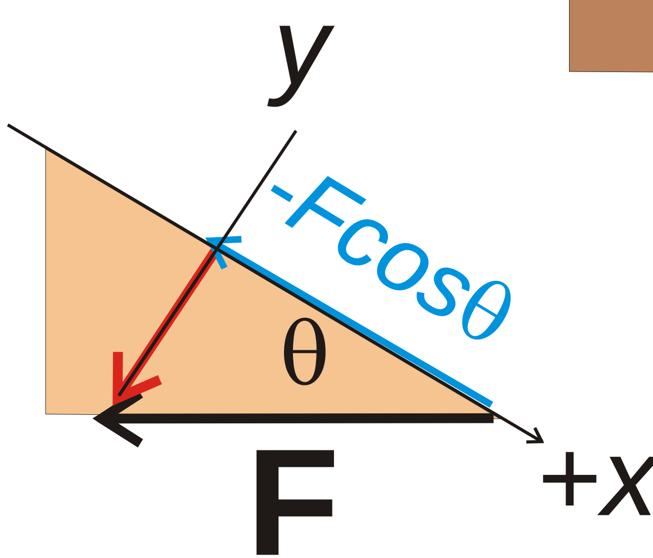
B. $F\cos\theta$

C. $-F\sin\theta$

D. $-F\cos\theta$

As $\theta \rightarrow 0$, $F_x \rightarrow -F$



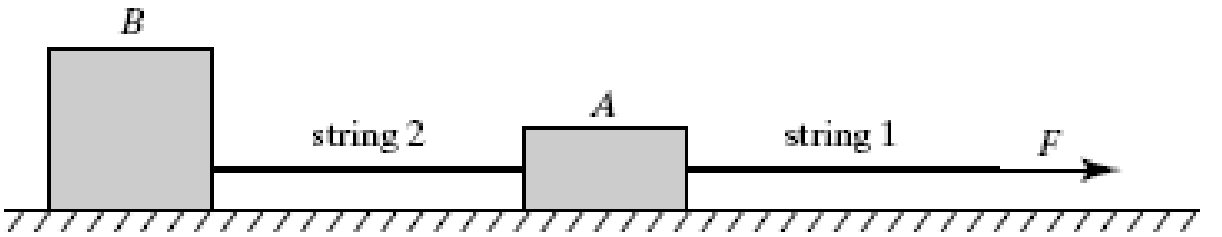


BEYOND THE SPHERICAL COW!

Systems more complex than 1 or 2 point particles.

- Multiple ropes between multiple masses
- Blocks pushing on blocks
- Ropes *with* mass

- 4 options



The mass of block B on the left is much larger than the mass of block A.

When both blocks are accelerated to the right by a pair of ideal massless strings as shown above, how does the tension of string 1 on the right compare with the tension of string 2 on the left?

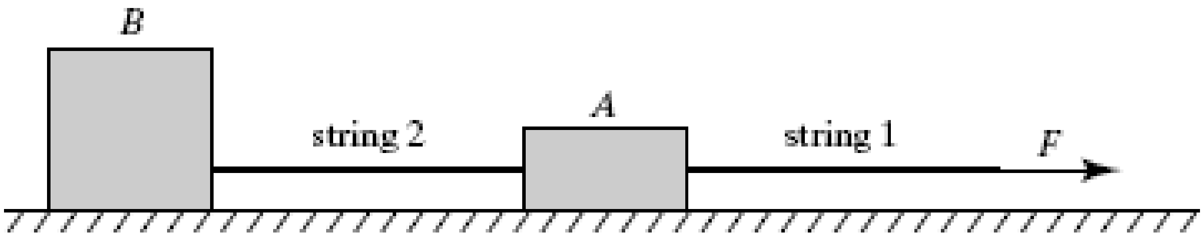
A. The tension of string 1 on the right is larger.

B. The tension of string 2 on the left is larger *since it's pulling the larger mass*.

C. The tensions are the same *since they form an action-reaction pair acting on block A.*

D. It's not possible to tell without more information.

- 4 options



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B. The tension of string 2 on the left is larger (*since it's pulling the larger mass*).

C. The tensions are the same, since they form an action-reaction pair acting on block A.

D. It's not possible to tell without more information.

- String 1 has to accelerate both masses while string 2 only accelerates B. To prove this, define your system first as BOTH blocks and then as just block B.
- Alternatively, focus on block A, draw a FBD for it, and show that T_1 must be greater than T_2 if block A is to accelerate to the right.

- 6 options

Three blocks, each of mass *M*, are hung from three ideal ropes as shown. The tensions in the three ropes are T_1, T_2, & T_3.

How do these three tensions compare?

A. $T_1 > T_2 > T_3$

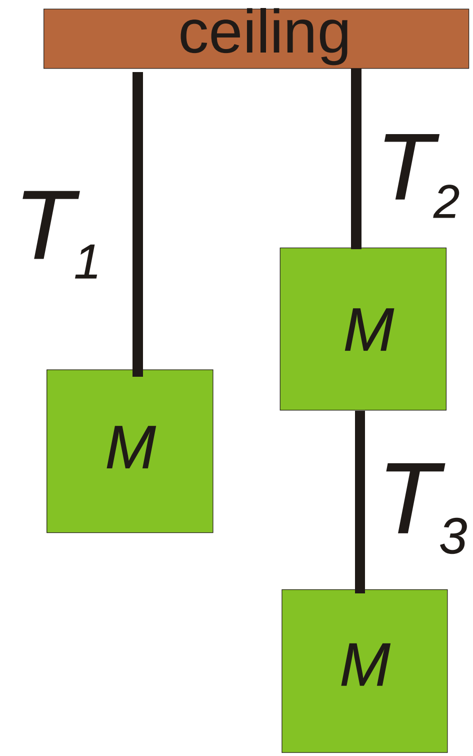
B. $T_2 > T_3 > T_1$

C. $T_2 = T_3 < T_1$

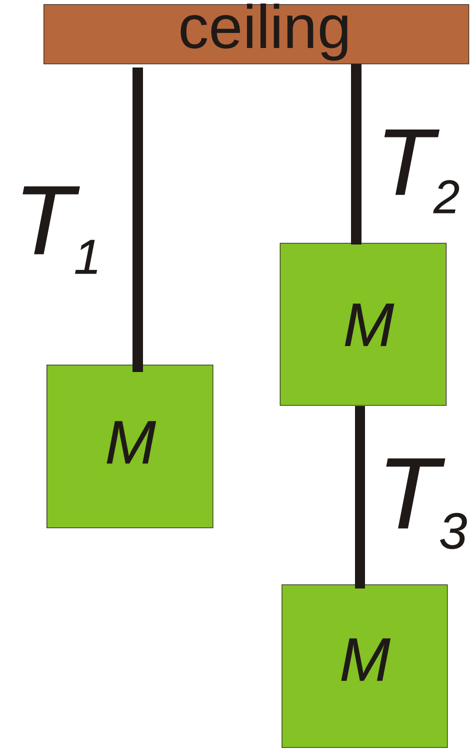
D. $T_2 = T_3 > T_1$

E. $T_2 > T_3 = T_1$

F. $T_2 = T_1 > T_3$



- 6 options



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D. $T_2 = T_3 > T_1$

E. $T_2 > T_3 = T_1$

F. $T_2 = T_1 > T_3$

- **$T_1 = T_3$ since both ropes are supporting Mg at rest.**
- **T_2 is larger than T_3 since it's supporting 2Mg.**

- 5 options

A 5-lb (*2.27 kg*) fish is hung from a pair of light spring scales, each of negligible weight, as shown in the figure.

What will the scales read?

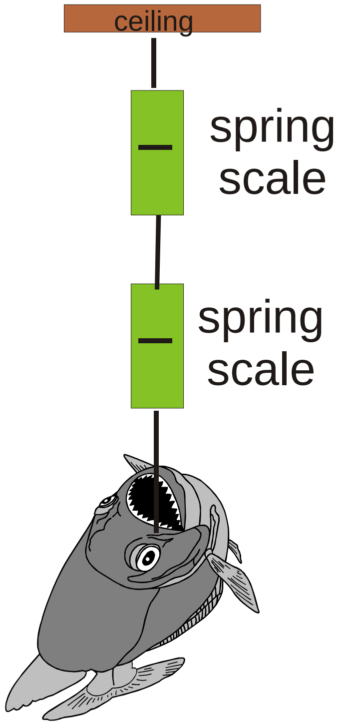
[NOTE: *The reading on any light spring scale is just the tension in the wire below it. Also, since the scale is light, this tension is the same as that in the wire supporting the scale.*]

A. The bottom scale will read 5 lb.; the top scale will read 0.

B. The bottom scale will read 0; the top scale will read 5 lb.

C. The sum of the two readings will be 5 lb. but the individual readings are uncertain.

D. Both scales will read 5 lb; their sum is 10 lb.



- 5 options

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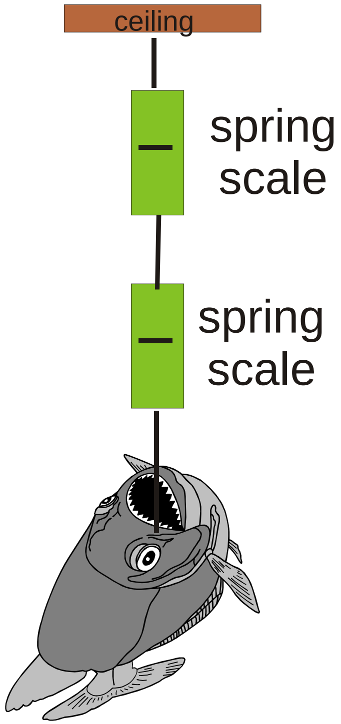

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B. The bottom scale will read 0; the top scale will read 5 lb.

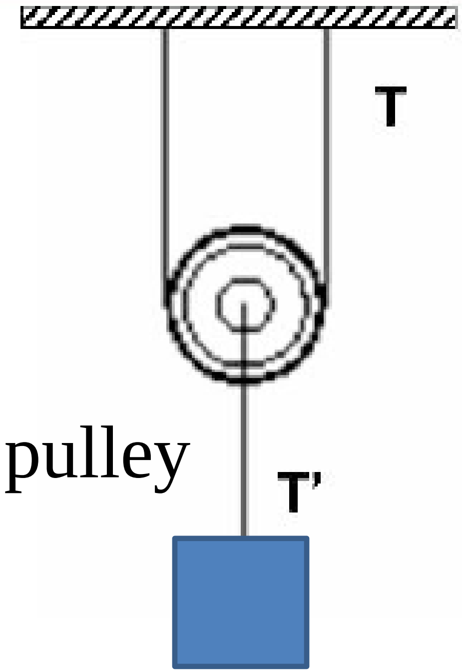
C. The sum of the two readings will be 5 lb. but the individual readings are uncertain.

D. Both scales will read 5 lb; their sum is 10 lb.

The tension in the rope is 5 lb **everywhere in the rope** and both scales read the tension.

Their sum will be 10 lb - but who cares. The sum of **T** at every point in a rope would be infinite!

4 options



An ideal pulley is attached by an ideal rope to two points of the ceiling, as shown in the figure.

A single ideal rope hanging from the center of the pulley supports a mass *m* at rest

$\Rightarrow$ the tension in the lower rope is $T' = mg$.

What is the tension T in the rope attached to the ceiling?

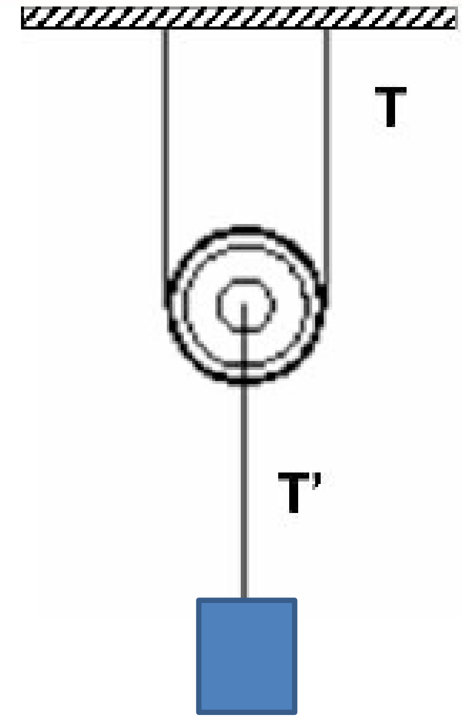
A. $T = T'$

B. $T = 2T'$

C. $T = \frac{1}{2} T'$

D. None of the above.

4 options



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A single ideal rope hanging from the center of the pulley supports a mass *m* at rest

$\Rightarrow$ the tension in the lower rope is $T' = mg$.

What is the tension *T* in the rope attached to the ceiling?

A. $T = T'$

B. $T = 2T'$

C. $T = \frac{1}{2} T'$

D. None of the above.

No acceleration of the pulley means

$2T - T' = 0$

ENGINEERING DISASTERS

http://en.wikipedia.org/wiki/Hyatt_Regency_walkway_collapse

A Hyatt Regency hotel in Kansas City featured walkways hanging from steel rods. The walkways failed in 1981; **114 people died** because of a mistake that any PHYS 121 student could catch.

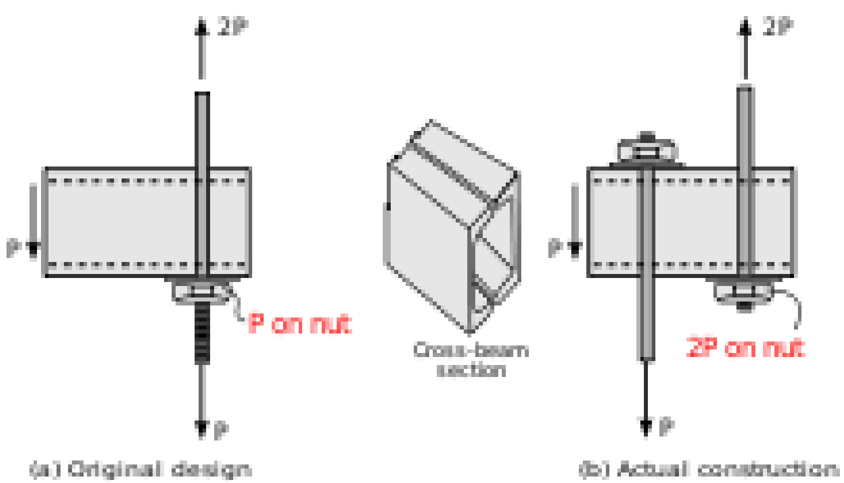
- deadliest structural collapse in US history until the 2001 World Trade Center collapse.
- ~ hanging multiple blocks from a rope.

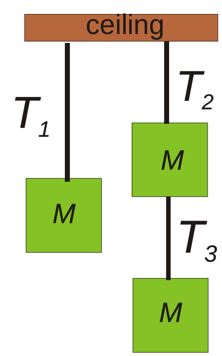
As designed, each *nut* on the steel rods supported its share of one walkway.

But it's hard to get those nuts on unless the entire rod is threaded!

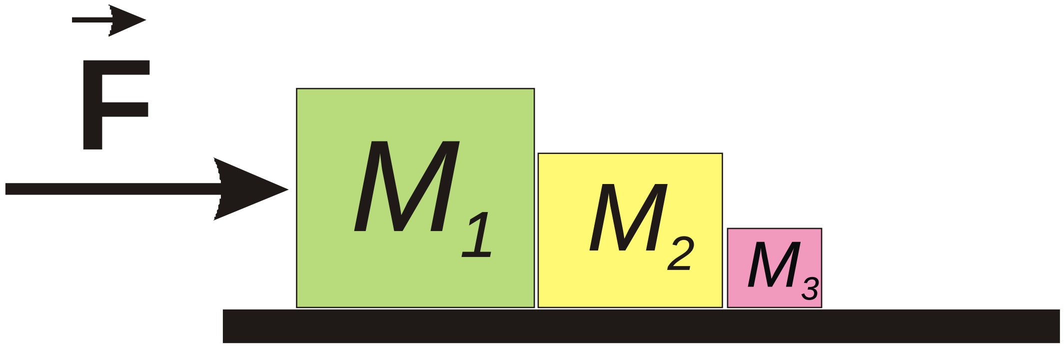
As *built*, some nuts had to support 2 or 3 walkways.

This was beyond their design limit. **OOPS!**





MULTIPLE BLOCKS ~ Ohanian 5.47

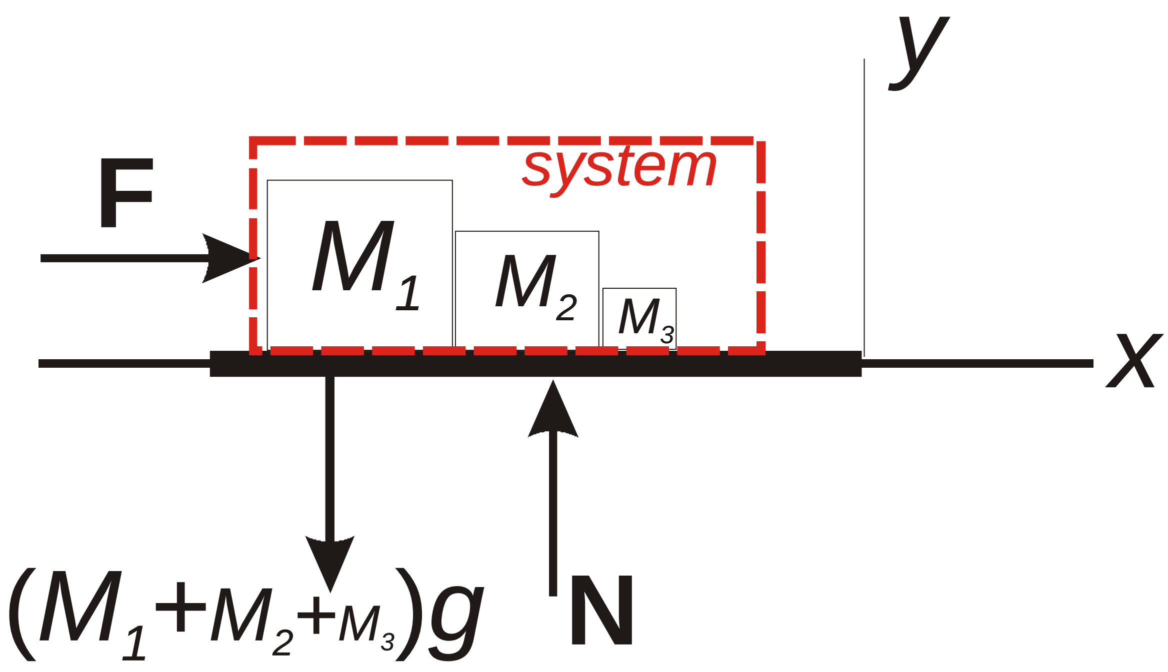


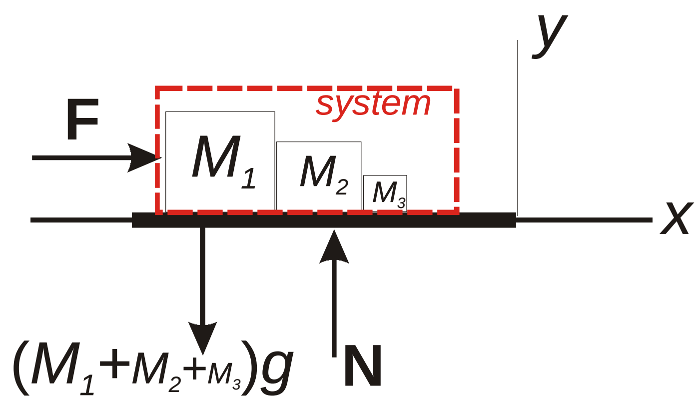
- Consider three blocks of masses $M_1 = 2M_2 = 4M_3$ that are initially at rest (*on a frictionless, horizontal surface*) and touching each other.
- If a force **F** is applied to the left face of M_1, what is the acceleration of this system and what are the contact (*normal*) forces between each pair of blocks?
- This problem requires wise choices of 'system'.

1. Define your system **wisely** & determine which forces are internal or external to your system.
   - I'll temporarily choose my system to be all three masses, which means **F** is an external force.
   - If you make any other choice, you have chosen ***poorly*** → the problem is more difficult to solve since you'll need to worry about contact forces between the blocks before you're ready to calculate them directly.

2. Draw a **<u>free body diagram</u>**, including coordinate system.

- This is a 1D problem and I'll choose an *x*-axis along the table.
- The position of the origin isn't important.
- The 3 forces acting on my system are shown below.



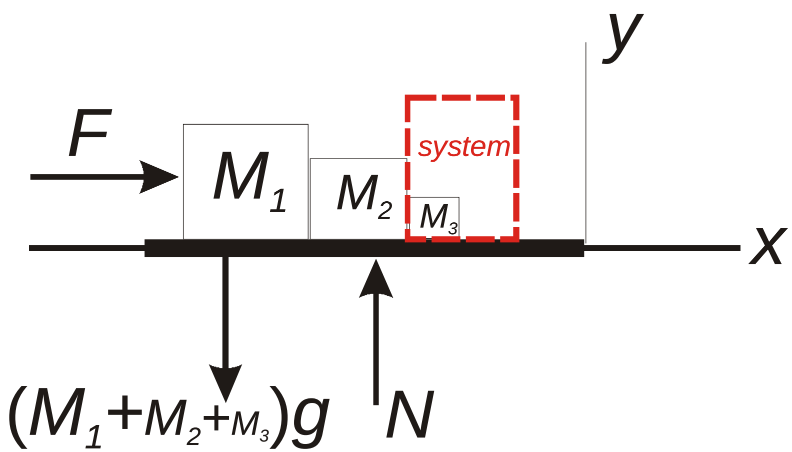


3. Write the concepts/equations you will use.

- *a* must be the same for all three blocks (*otherwise they won't remain in contact*) and we can use this to find the contact forces between the blocks.
- With my choice of system, there's one external force in the *x*-direction and $\Sigma F_x = ma_x$ gives

$$a = F/(M_1 + M_2 + M_3)$$

$$a = F/(4M_3 + 2M_3 + M_3) = F/(7M_3)$$



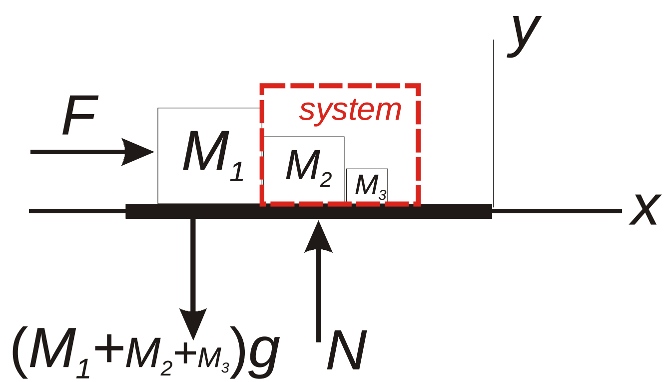
4. Solve for the contact forces between the blocks.

- There are NO contact forces in my previous system.

⇒ Redefine your system (*to be just the third block*).

- The contact force between M_2 and $M_3 \equiv F_3$ is the only force acting on the third block (*in the x-direction*) and must be the source of its acceleration.

$$F_3 = M_3 a = M_3[F/(7M_3)] = F/7$$



- Redefine your system to be just the second and third blocks, as a pair.
- The contact force acting between M_1 and $M_2 \equiv F_2$

$$F_2 = (M_2+M_3)\,a = (M_2+M_3)[F/(7M_3)] = (3M_3)[F/7M_3] = (3/7)F$$

[ASIDE: You could have instead defined your system as just the second block with contact forces acting on both sides but this makes the math slightly harder.]

CONSISTENCY CHECK

Try to think of some way to check your results.

- Note that M_2 feels a contact force acting on both faces,
- (3/7)F pushing it towards the right and (1/7)F pushing it towards the left, for a net force of (2/7)F pushing it towards the right.
- It's mass is $2M_3$ so it's acceleration is $a = F/M = (1/7)(F/M_3)$.
- This is the same acceleration we found above, which is reassuring – and necessary!

GOOD!

My answer is "self-consistent".

UNIFORM BARS

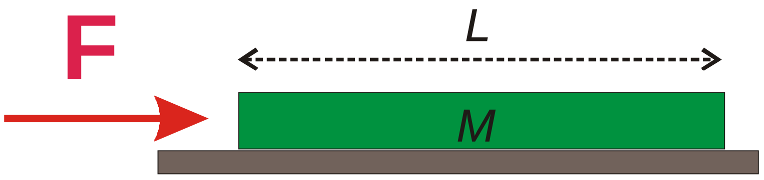
extrapolating from 3 blocks to ∞

A long, uniform bar lies on a flat, frictionless table.

The bar has mass M and length L.

We can define a *linear mass density* $\lambda = M/L$ (*Greek letter lambda*).

You apply a horizontal force to one end, pushing on the bar.



QUESTION

How do the internal *compressive* forces vary inside the bar?

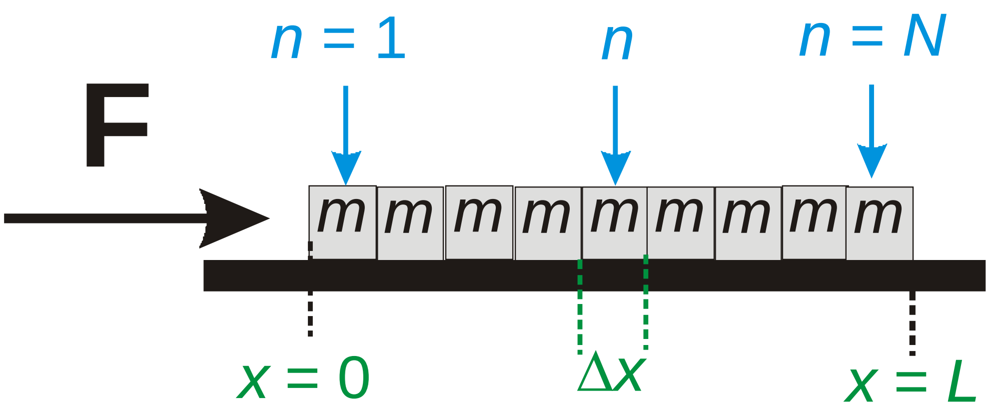
Pushing on the bar leads to *compressive forces* trying to squeeze sections of the bar together.

Pulling on the bar would lead to *tensile forces* trying to pull sections of the bar apart.

APPROACH

Divide the bar up into a finite number of small blocks & find the contact forces between each pair of blocks.

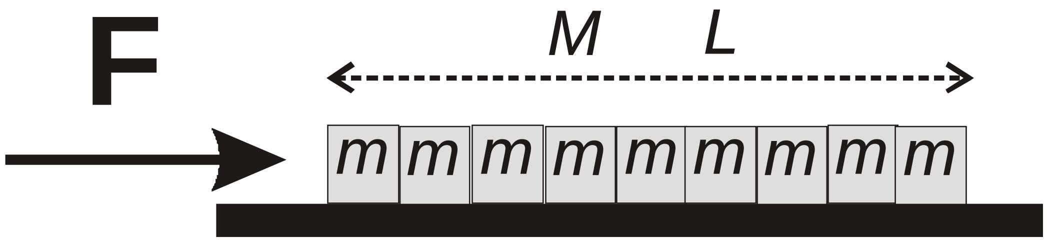
Then take the limit as the number of blocks $\rightarrow \infty$.

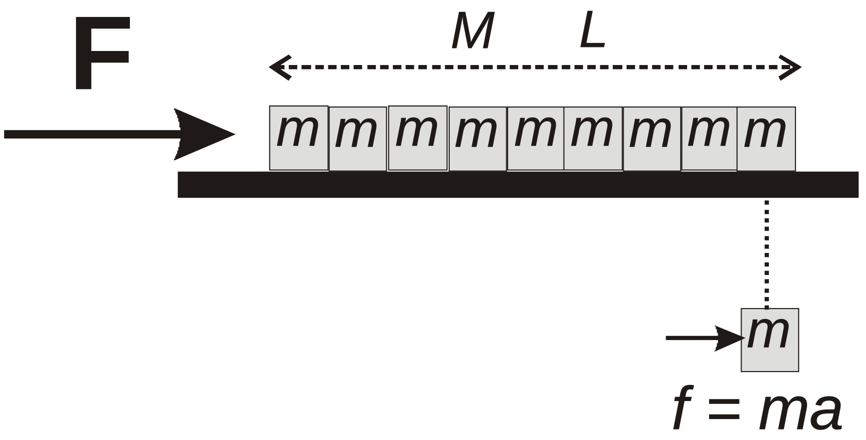


N = number of smaller blocks

n = index referring to the n'th bock from F

m = mass of a small block = $M/N = (M/L)(L/N) = \lambda(\Delta x)$

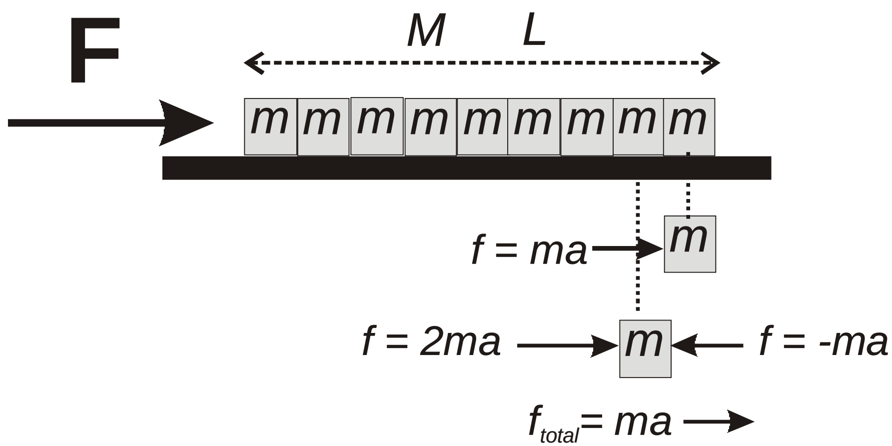




- Taking the system as the entire bar, we know it accelerates to the right at $a = F/M$.
- Taking the system as the last *m* on the right, the force on <u>IT</u> must be

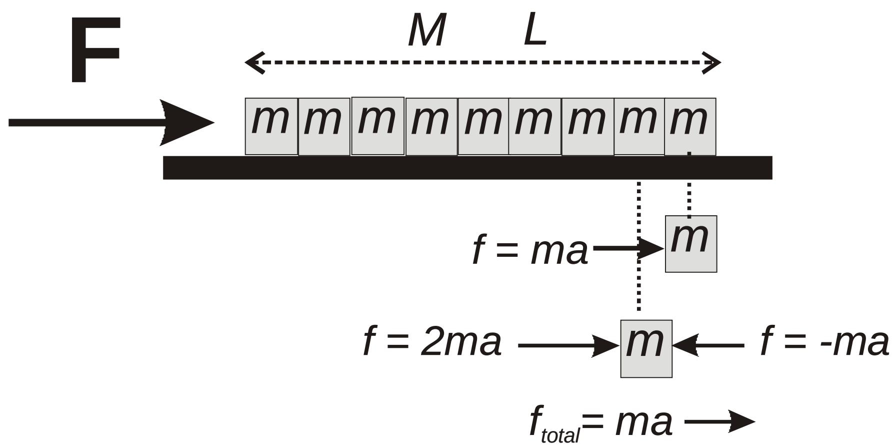
$$f_{\text{last mass on right}} = ma.$$

- The <u>net</u> force acting on <u>EVERY</u> little *m* must be *ma*.
- But **F** acts only on the block on the far left.
- The net force acting on every other block consists of a contact force on each of its sides.



- There is only one contact force acting the last block.
- The contact force on its left face must be $f = ma$.
- This leads to a reaction force f =-*ma* that pushes to the left on block N-1.
- There must be a net force of $f = ma$ to the right on each small block

$\Rightarrow$ there must be a force $f = 2ma$ on the left side of block N-1 pushing it towards the right.



- Continue this pattern for all the blocks.
- For the *n*'th block from the left, there are

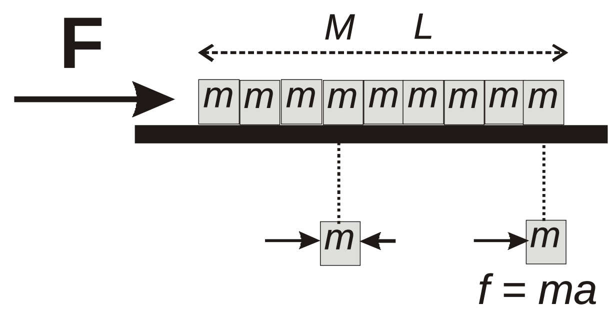
$(N - n)$ blocks to its right

$\Rightarrow$ a contact force $(N - n)ma$ on its right face, pushing left,

$+$ a contact force $(N - n + 1)ma$ on its left face pushing right.

- Each block (*except those on the ends*) experiences contact forces on both its left and right; these act as compressive forces.

A rod made of Jell-O would compress noticeably.



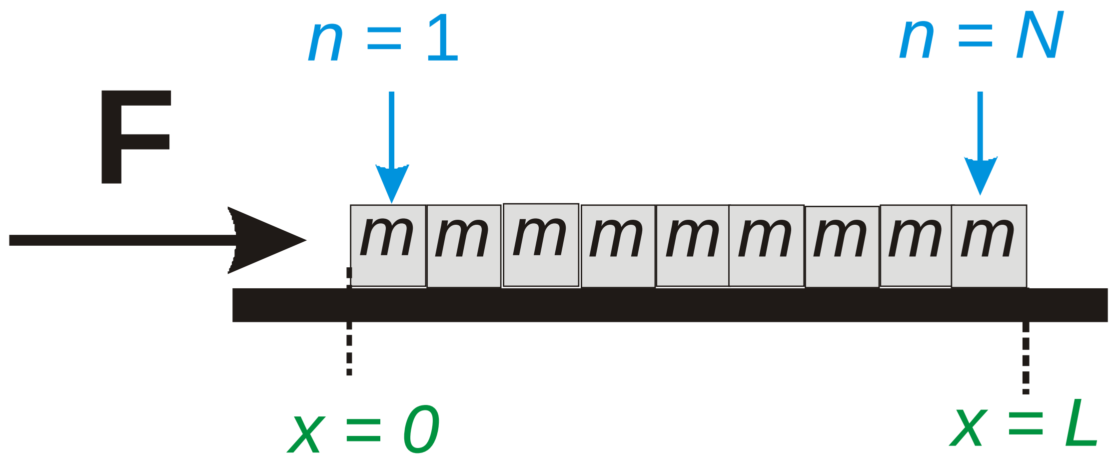
- Plugging in $a = F/M = F/(Nm)$ for the overall system.
- For the n'th block the contact forces on its left and right faces change from $(N - n + 1)ma$ & $(N - n)ma$

LEFT FACE: $F_{left} = (N - n+1)(F/N) = (1 - n/N)F + F/N$

RIGHT FACE: $F_{right} = (N - n)(F/N) = (1 - n/N)F$

- If N is large, then F/N is small and the compressive forces on both faces are almost equal to each other

internal compression $= (1 - n/N)F$



Let the number of blocks tend towards infinity.

Define distance along the rod as a continuous variable x.

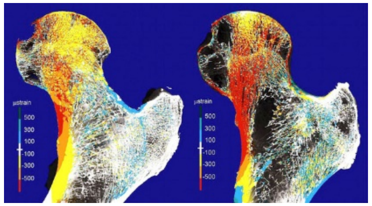
$$\Rightarrow \quad (1 - n/N)F \text{ becomes } (1 - x/L)F$$

Notice that this result is a ***linear*** function of x, changing smoothly from F to 0 as x changes from 0 to L.

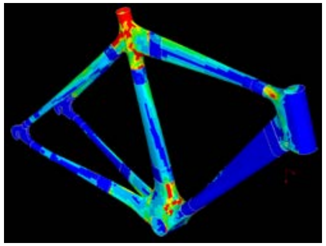
As long as a bar (*or rope*) is uniform in cross-section, the internal compressive forces must vary linearly along its length.

The left end of the bar experiences stronger forces than the right end.

If we were to reverse the direction of F and pull on the bar, this compression would change into a tension. ONLY the SIGNS change.



CULTURAL INTERLUDE
Finite Element Analysis, FEA



You've just encountered a simple 1-D version of one of the most powerful tools available to an engineer.

Finite Element Analysis uses a computer program to break an object down into small bits called *finite elements* and solve the differential equations that describe how forces and torques are transferred between neighboring elements (*incorporating the mechanical properties of those elements*).

FEA is used and taught in materials science, civil engineering, mechanical/aerospace engineering, biomedical engineering, *etc*.

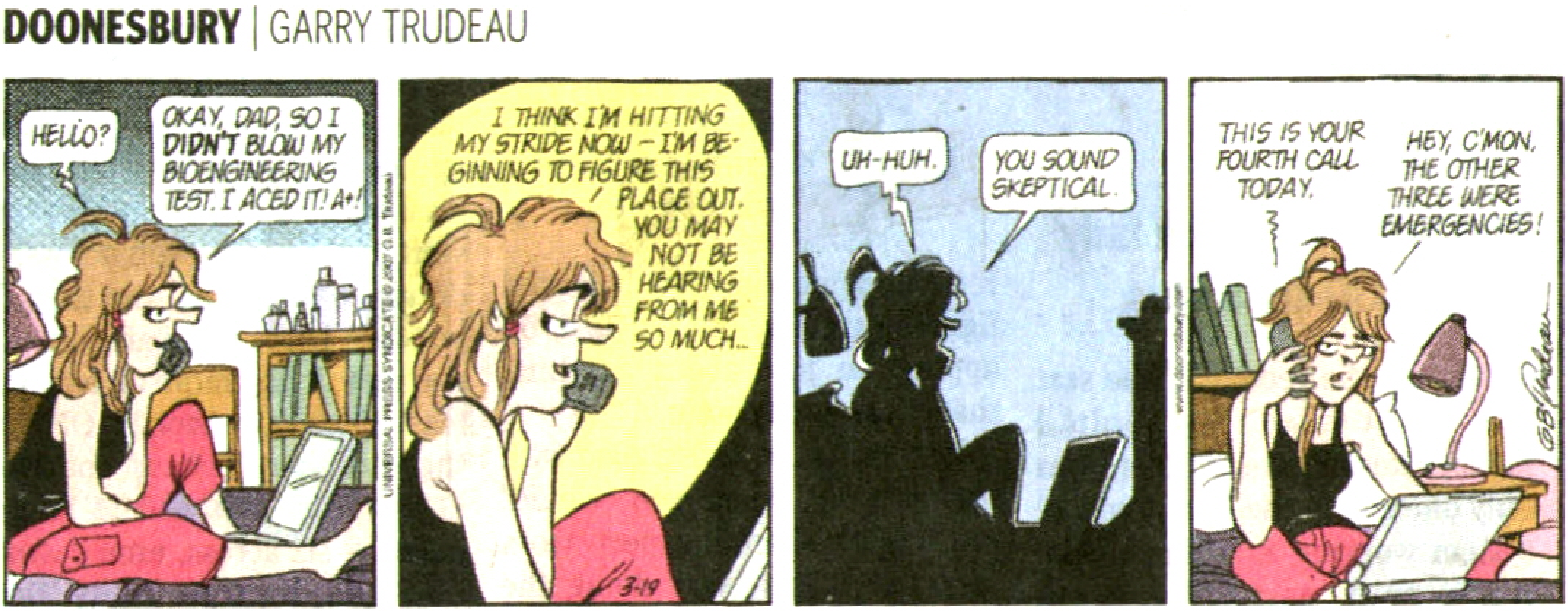
See images at:
https://www.google.com/search?q=finite+element+analysis&hl=en&source=lnms&tbm=isch&sa=X&ei=Y4UzUfqhBofv0QHj2oHQDw&ved=0CAoQ_AUoAQ&biw=1273&bih=875

Particularly http://www.mechanicalengineeringblog.com/2234-types-of-finite-element-analysis-finite-element-analysis-capabilities-finite-element-analysis-engineering-services/

We made it to slide #131 on

Friday, February 6.

PHYS 121 – SPRING 2015



Chapter 5: Newton's Laws of Motion

version 02/09/2015, ~ 155 slides

We made it to slide #88 on Wednesday, February 4, 2015.

ANNOUNCEMENTS

EXAM #1

Pick up a ***BLUE BOOK*** when you enter Strosacker.

Sit in 1st, 3rd, 5th, *etc*. seats from an aisle.

HOMEWORK #3 Class average = $(43.5 \pm 6.7)/50 = (87.1 \pm 13.5)\%$

Lab #3: Conservation of Mechanical Energy

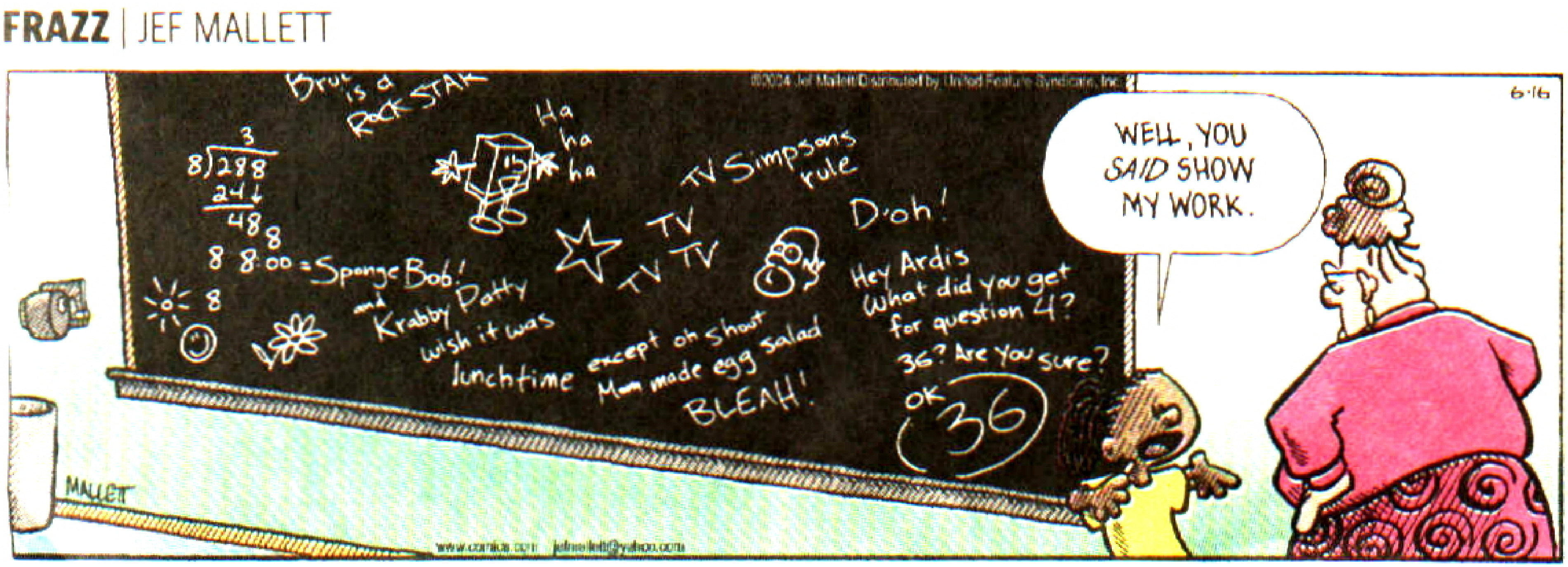
February 11 – 19, Chapter 8 of Ohanian

In this experiment, you will test the theory of conservation of mechanical energy. To do that, you will make two types of measurements:

a. conversion of gravitational potential energy of a falling body into the kinetic energy of the body and a cart to which it is tied,

b. conversion of gravitational potential energy into the potential energy stored in a stretched spring.

PHYS 121 – SPRING 2015



Chapter 5: Newton's Laws of Motion

version 02/13/2015, ~ 155 slides

We made it to slide #135 on Monday, February 9, 2015.

Get your clickers ready.

ANNOUNCEMENTS

Are there any romantics in PHYS 121 this semester?

"Singing Valentines", http://caseglee.webs.com/singing-valentines.

Physicists must be romantics; they see beauty in F = mA!

Engineers Week & WISER, Feb. 14 – 27

http://engineering.case.edu/delpp/eweek

"Testing your engineering knowledge by building a contraption to keep light bulbs intact after falling in the Nord atrium."

Start with your physics knowledge!

$$\mathbf{F = ma}$$

$$\mathbf{a = (v^2 - v_o{}^2)/(2\Delta x)}$$

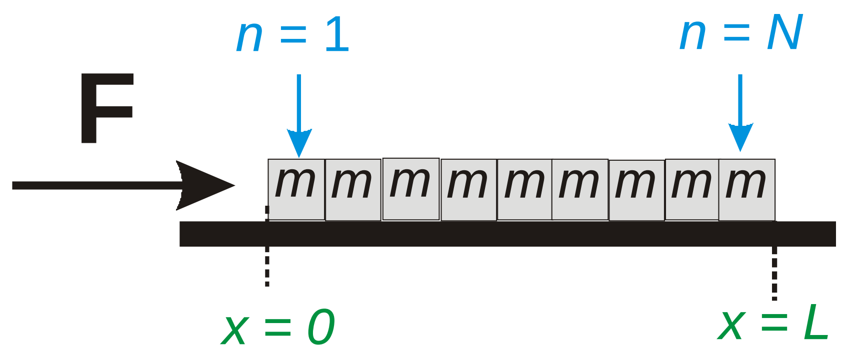
smaller F ↔ larger Δx

ANNOUNCEMENTS

- **Exam #1 will be graded over the weekend & returned Monday.**
  - **We will not review it in lecture but the solutions will be posted on Blackboard by Monday afternoon.**
  - **Make certain you understand any mistakes you made; the final exam will include a problem from one of the hour exams.**
- **Blackboard Postings**
  - **Blackboard will be updated with grades only *occasionally* this semester.**
  - **The first occasion will be after the 1st exam grades are settled and before Spring Break (*March 9-13*) when midterm grades are due.**
  - **You'll be able to check your exam, homework, lab and bonus point records.**
- **Midterm grades will be based on Exam #1 plus homework & lab.**
  - **No bonus points will be included and no homework grades will be dropped.**
  - **The grade rubric will be 50% exam, 25% homework & 25% lab**
  - **Breakpoints for letter grades will be 90% =A, 80% =B, 70% =C & 60% =D.**

REMINDER:
UNIFORM BARS

extrapolating from 3 blocks to ∞



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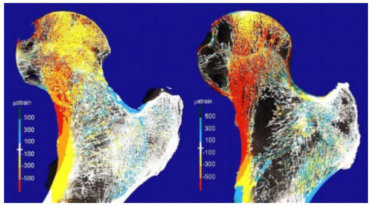
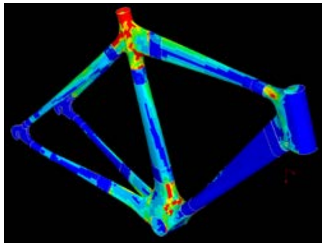


CULTURAL INTERLUDE
Finite Element Analysis, FEA



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https://www.google.com/search?q=finite+element+analysis&hl=en&source=lnms&tbm=isch&sa=X&ei=Y4UzUfqhBofv0QHj2oHQDw&ved=0CAoQ_AUoAQ&biw=1273&bih=875

Particularly http://www.mechanicalengineeringblog.com/2234-types-of-finite-element-analysis-finite-element-analysis-capabilities-finite-element-analysis-engineering-services/

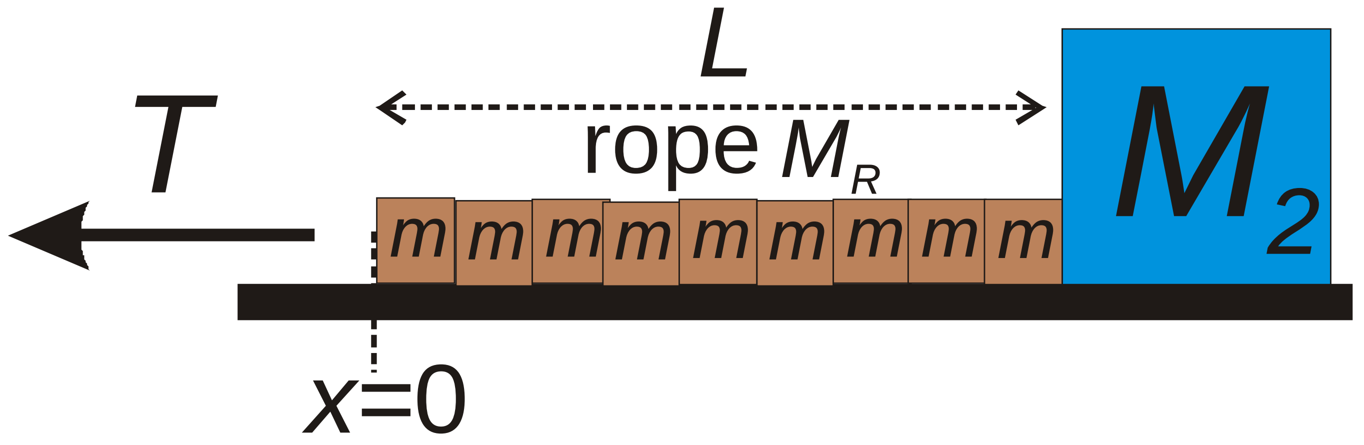
ROPES

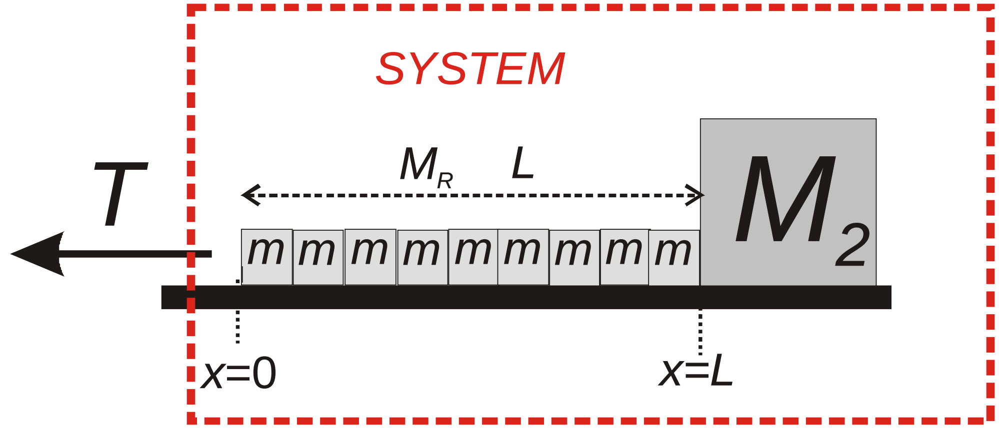
We can now let our ropes have mass, but calling them a rope **still** implies some other properties.

- Ropes can only exert forces along their length.
- Ropes can't push on an object, only pull.
- Ropes can only experience tension, not compression.

We can modify our analysis of a rod to handle a rope with mass M_R pulling another mass M_2.

What is the tension within the rope, as a function of x?





Focusing temporarily on the entire rope + M_2 system:

$$T = (M_R + M_2)a \qquad a = T/(M_R + M_2)$$

The tension **at** $M_2 = M_2 a = TM_2/(M_R + M_2)$

We could analyze this system like we did earlier but let's instead use the *concept* that the tension must vary linearly along a uniform rope.

From $x = 0$ to $x = L$, the tension must decrease linearly from

$$T \quad to \quad TM_2/(M_R + M_2)$$

From $x = 0$ to $x = L$, T decreases <u>linearly</u> from

$$T \quad to \quad TM_2/(M_R + M_2)$$

The general formula for a straight line is $y = mx + b$

- b is the intercept with the y-axis for $x = 0$
- m is the slope = $\Delta y/\Delta x$

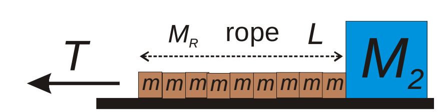
For our rope system, y is the tension T, x is x,

$$\Delta y_{total} = \Delta T = [TM_2/(MR + M_2) - T] \quad \& \quad \Delta x_{total} = L$$

So the tension in the rope as a function of x, $T(x)$, is given by

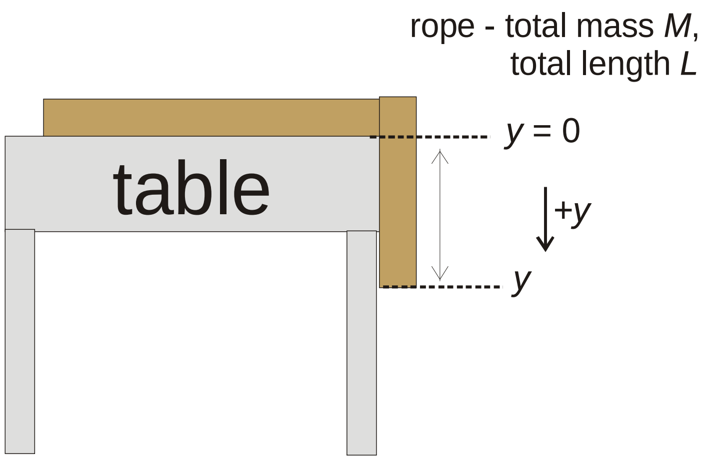
$$T(x) = \frac{\Delta T}{\Delta x}x + T_{\text{at x=0}} = \frac{\dfrac{TM_2}{M_R+M_2} - T}{L}x + T = \frac{\dfrac{TM_2}{M_R+M_2} - \dfrac{T(M_R+M_2)}{M_R+M_2}}{L}x + T = \frac{\dfrac{-TM_R}{M_R+M_2}}{L}x + T$$

$$T(x) = T\left[1 - \left(\frac{x}{L}\right)\left(\frac{M_R}{M_R+M_2}\right)\right]$$



Check the limit as $M_R \to 0$, $x \to 0$, $x \to L$.

~ OHANIAN 5.77**



A uniform rope of length L and mass M is held at rest stretched out on a frictionless table with a fraction s_o of rope hanging off the edge of the table.

Once the rope is released, it will start sliding off the table.

Describe its motion, $y(t)$.

THIS IS A 1D PROBLEM,

with our 1D universe bent 90° at the edge of the table.

- 1D $\Rightarrow$ one value tells you the position of a particle.
- The leading end of the rope is at a position y as shown in the figure.
- The rope hanging off the table has length y and mass $(y/L)M$.
- The part of the rope on the table has length $(L - y)$ and mass $(1 - y/L)M$.
- The rope starts with $v = 0$ and $y_o = s_oL$ where s_o is a fraction $0 \leq s_o \leq 1$

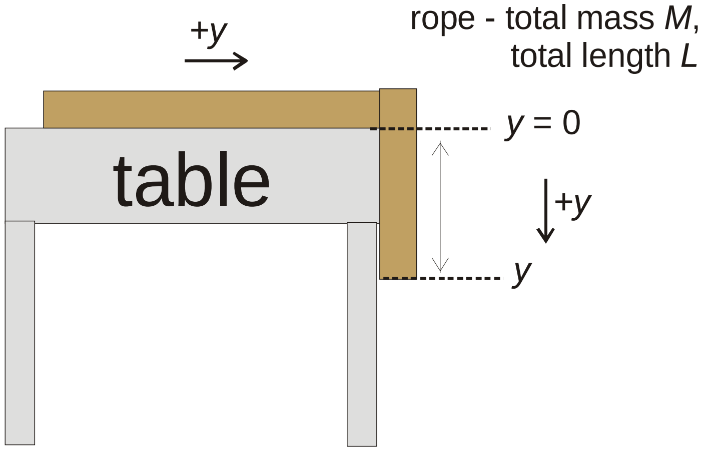
$$F_{total} = F_{gravity\ on\ (y/L)M} = Ma$$

$$\left(\frac{y}{L}\right)Mg = Ma$$

so

$$a = \left(\frac{y}{L}\right)g$$

$$\frac{d^2y}{dt^2} = \left(\frac{g}{L}\right)y$$



What is the solution $y(t)$ to this differential equation? $\frac{d^2y}{dt^2} = \left(\frac{g}{L}\right)y$

$$y(t) = Ae^{\sqrt{\frac{g}{L}}\,t}$$

with $A \rightarrow$ constant determined by initial conditions.

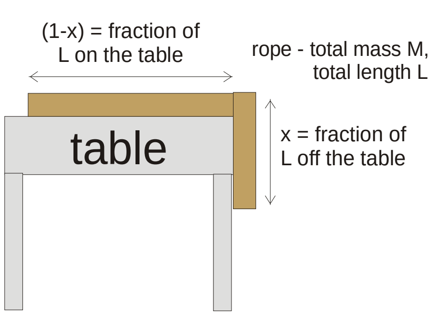
$$y(t=0) = s_o L = Ae^{\sqrt{\frac{g}{L}}\,0} = A$$

$$\Rightarrow \quad y(t) = s_o L e^{\sqrt{\frac{g}{L}}\,t}$$

$$v(t) = s_o L\sqrt{\frac{g}{L}}e^{\sqrt{\frac{g}{L}}\,t} = s_o\sqrt{gL}\;e^{\sqrt{\frac{g}{L}}\,t}$$

$$a(t) = s_o L\frac{g}{L}e^{\sqrt{\frac{g}{L}}\,t} = s_o g e^{\sqrt{\frac{g}{L}}\,t}$$

$\Rightarrow$ The rope falls off the table with a speed that increases exponentially with time, until it's entirely off the table.



REALITY CHECK

If $s_o = 1$ at $t = 0$, $a = g$ as we expect.

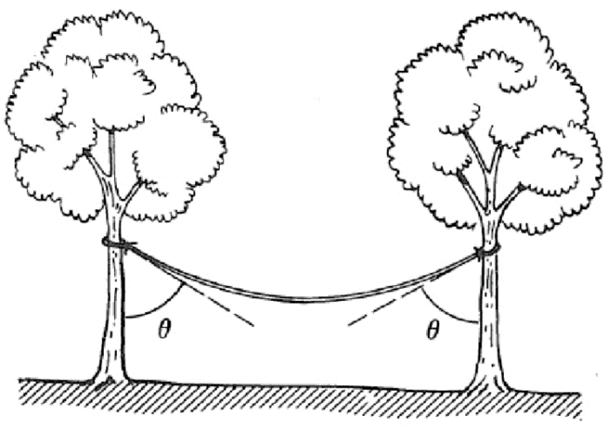
If $s_o = 0$ at $t = 0$, $a = 0$ as we expect.

What changes if there is friction with the table?

You'll find out soon!

HANGING ROPES

The rope in the figure has mass M and hangs from each tree at the same height at angle θ. What is the tension at the ends of the rope? (*from C. Covault*)



ANSWER

Let the system be the entire rope.

The sum of forces acting on it must be 0.

$$\Sigma F_y = 2T\cos\theta - Mg = 0.$$

So T at the ends is $Mg/(2\cos\theta)$

NOTICE: As $\theta \rightarrow 90°$, $T \rightarrow \infty$!

CULTURAL INTERLUDE

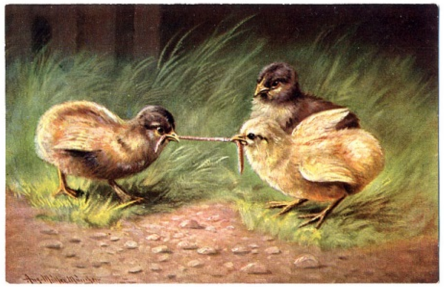
A complete solution to the problem of hanging ropes would show that the rope takes on a shape defined as a *catenary*.

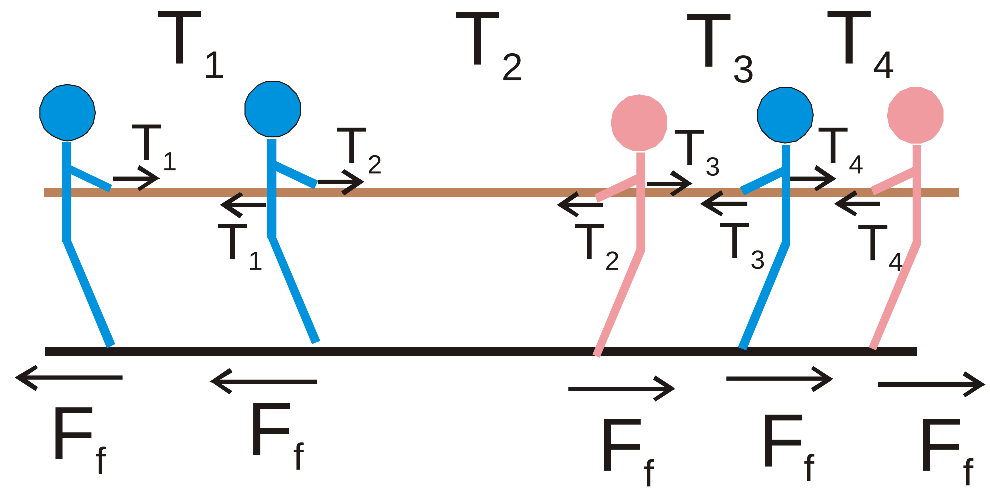
Sophomore physics majors study this problem.

See http://en.wikipedia.org/wiki/Catenary

http://eyedealpostcards.com/images/postcards/SchonianChicksTugOfWarWWorm11675.jpg

TUG OF WAR





Consider 5 students involved in a tug of war as shown above (*on the right*).

What is the tension in each section of the (*ideal*) rope?

- The tension is uniform between each student because the rope is massless.
- There will be 4 different values for this tension as shown in the figure.
- a = acceleration of the rope - and the students holding it.

  Assuming equal strength for each student, a will be to the right.

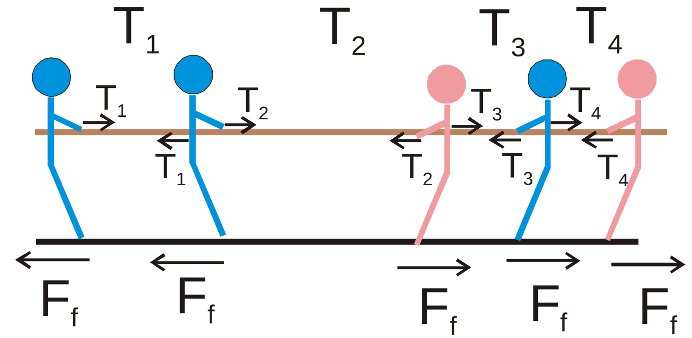
The forces acting on each student are:

- F_f = friction on floor, a reaction force to pushing on the floor.

  We'll assume for convenience that this is the same for each person.

- T_i = tension from the rope in front of and behind your hands.

TUG OF WAR



The acceleration a can be found by defining all 5 students as your system.

$a = \Sigma F/\Sigma M$ but the tension is an INTERNAL force so

$$a = (3F_f - 2F_f)/5M = F_f/5M$$

Now work your way through the individual students starting from the left.

$T_1 - F_f = Ma = F_f/5$ solving for T_1: $T_1 = (6/5)F_f$

Note that $T_1 > F_f$ as required for the system to accelerate to the right.

Next you can either isolate the second student from the left as a single object or consider the two leftmost students together. The algebra is a bit easier with the latter choice.

$T_2 - 2F_f = 2Ma = (2/5)F_f$ solving for T_2: $T_2 = (12/5)F_f$

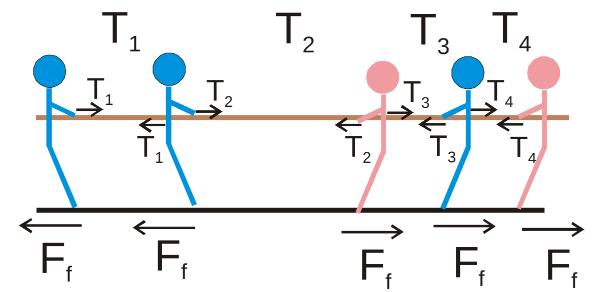
Consider the 3 leftmost students as your system.

$T_3 - 2F_f + F_f = 3Ma = (3/5)F_f$ solving for T_3: $T_3 = (8/5)F_f$

Consider the 4 leftmost students as your system.

$T_4 - 2F_f + 2F_f = 4Ma = (4/5)F_f$ solving for T_4: $T_4 = (4/5)F_f$

TUG OF WAR



The tension is largest between the teams – which should come as no surprise.

That tension is 12/5 = 2.4 times the force provided by any individual.

How large might the tension be with over 2000 participants as in the '***Harrisburg incident***'?

http://news.google.com/newspapers?nid=1368&dat=19780614&id=PwQkAAAAIBAJ&sjid=6BEEAAAAIBAJ&pg=3734,2630177

Should we try this with PHYS 121 students?

DEMO

I need 8 volunteers!

[Tug of war](https://www.youtube.com/watch?v=MgRvs3_G0N8&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr)

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See http://what-if.xkcd.com/127/ for extreme tug-of-war situations, like pulling apart an iron bar. You'd need 25 people per team for a ½" diameter bar.

TUG OF WAR = ROPE PULL

Greek Week ~ end of spring semester

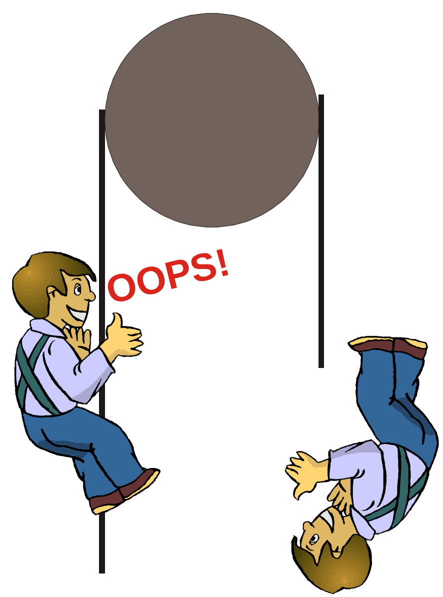
Each fraternity and sorority chapter enters a team.

10 per team, last person 'tied in'.

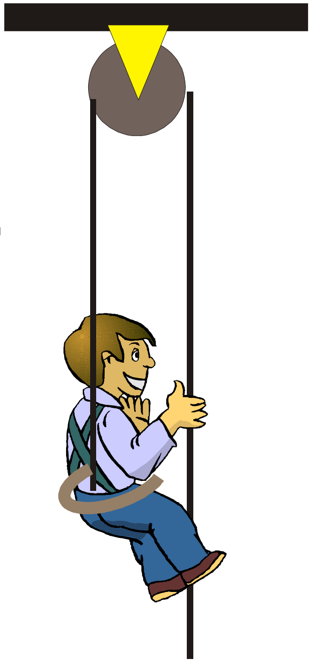
Can you spot a PHYS 121 SI in two of these photos?

ROPES & *ACTIVE* MASSES

ACTIVE = moving *wrt* the rope



BOOTSTRAPPING



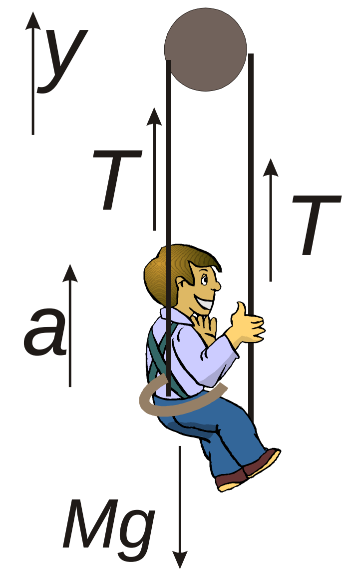
One end of a rope is tied around your waist. The rope is strung over a pulley and you try to pull yourself up using the loose end.

Analyze this situation.

The system is YOU!

There are TWO factors of T acting upwards on you – the rope pulling on your waist and the rope pulling (*back*) on your hands.

$$2T - Mg = Ma \quad \rightarrow \quad a = 2T/M - g$$



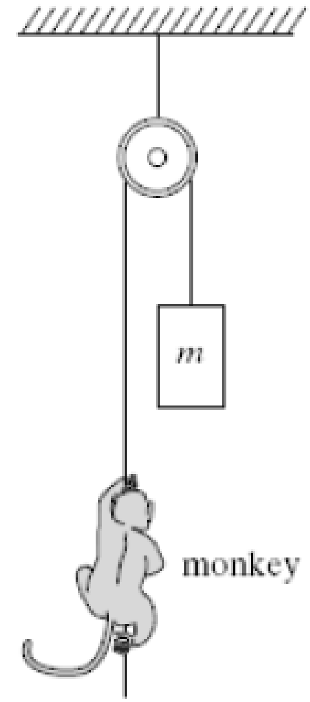
You can hang motionless or move with constant velocity, $a = 0$, using only $T = Mg/2$ instead of $T = Mg$.

$\Rightarrow$ this system conveys a mechanical advantage.

BUT this is only in terms of the force, not the work you do.

You need to pull twice as much rope through your hands. We'll see in chapter 8 that the work you do is the same in both cases.

4 options



A monkey of mass *m* clings motionless to a rope that passes over a pulley. Attached to the rope on the other side of the pulley, but somewhat higher, there is a box of bananas also of mass *m* that exactly balances the monkey's weight.

The monkey starts climbing the rope to reach the box of bananas, moving up a distance *L* measured *wrt* the pulley

What happens to the box of bananas?

A. It does not move.

B. It also moves up a distance *L* towards the pulley.

C. It moves down a distance *L* down, away from the pulley.

D. It moves up a distance 2*L* towards the pulley.

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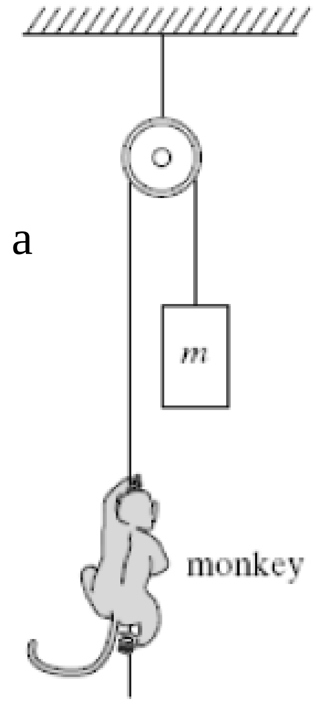

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The tension acting on both the monkey and the box are the same; both point upwards and are <u>greater</u> than *mg* down, so both accelerate up at the same rate and move the same distance in the same time interval.

So the box keeps moving away from the monkey!

How sad!

Although the box will eventually be stopped by the pulley.

DEMO

Monkey & Banana

- If a monkey climbs or descends, the other side with an equal weight moves with it!
  https://www.youtube.com/watch?v=JTyU49isrp0&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr

THE END!

http://us.123rf.com/400wm/400/400/feverpitched/feverpitched0803/feverpitched080300154/2685279-the-end-road-sign-with-dramatic-blue-sky-and-clouds.jpg