PHYS 121 – SPRING 2015

Chapter 11: Collisions

version 03/27/2015, ~ 100 slides

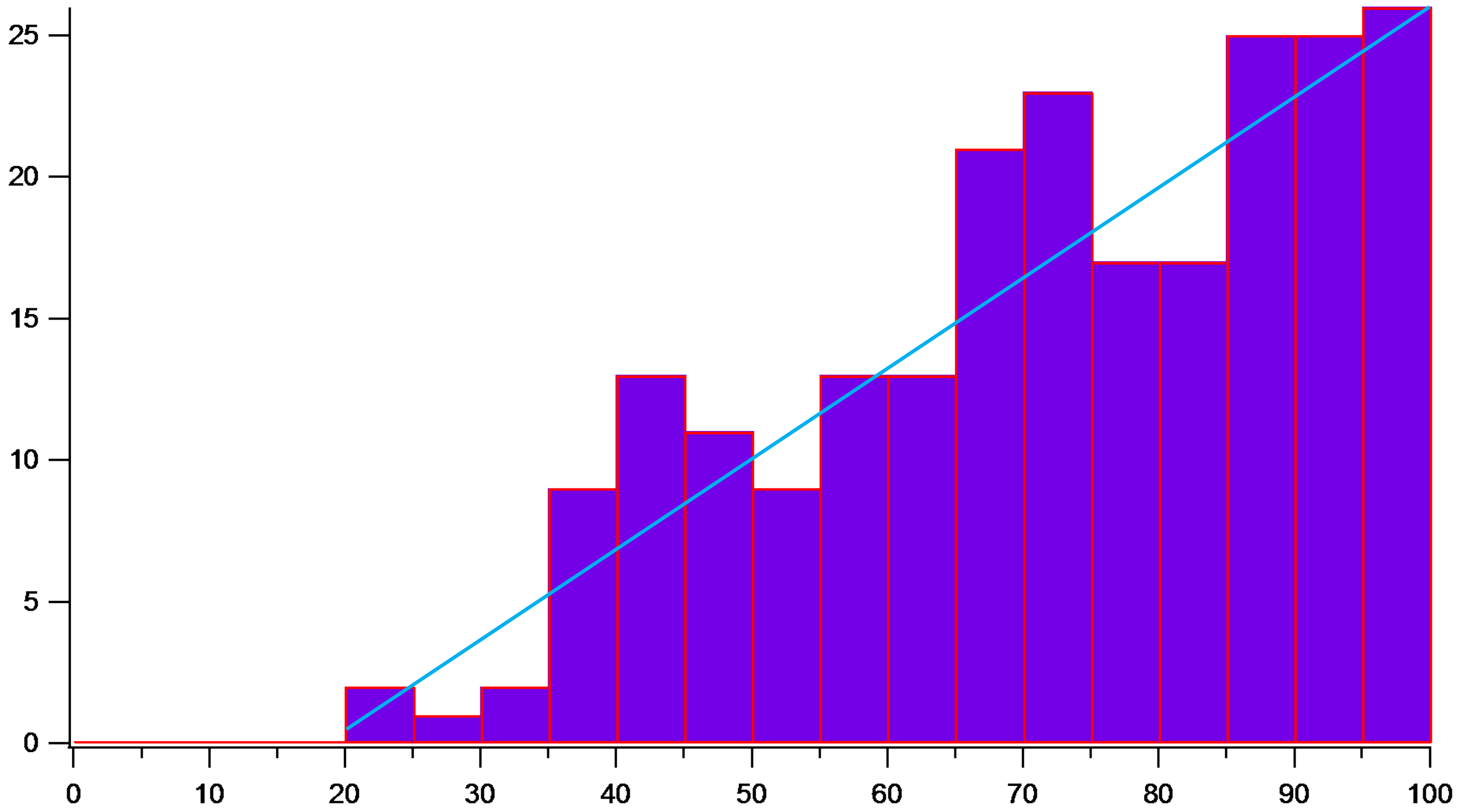
We finished this material on Friday, March 27.

http://www.texaspacificrailway.org/wrecks/texas-and-pacific-railway-wrecks-collision-of-two-texas-and-pacific--cannon-balls--in-jefferson--texas-image-1453.html

ANNOUNCEMENTS

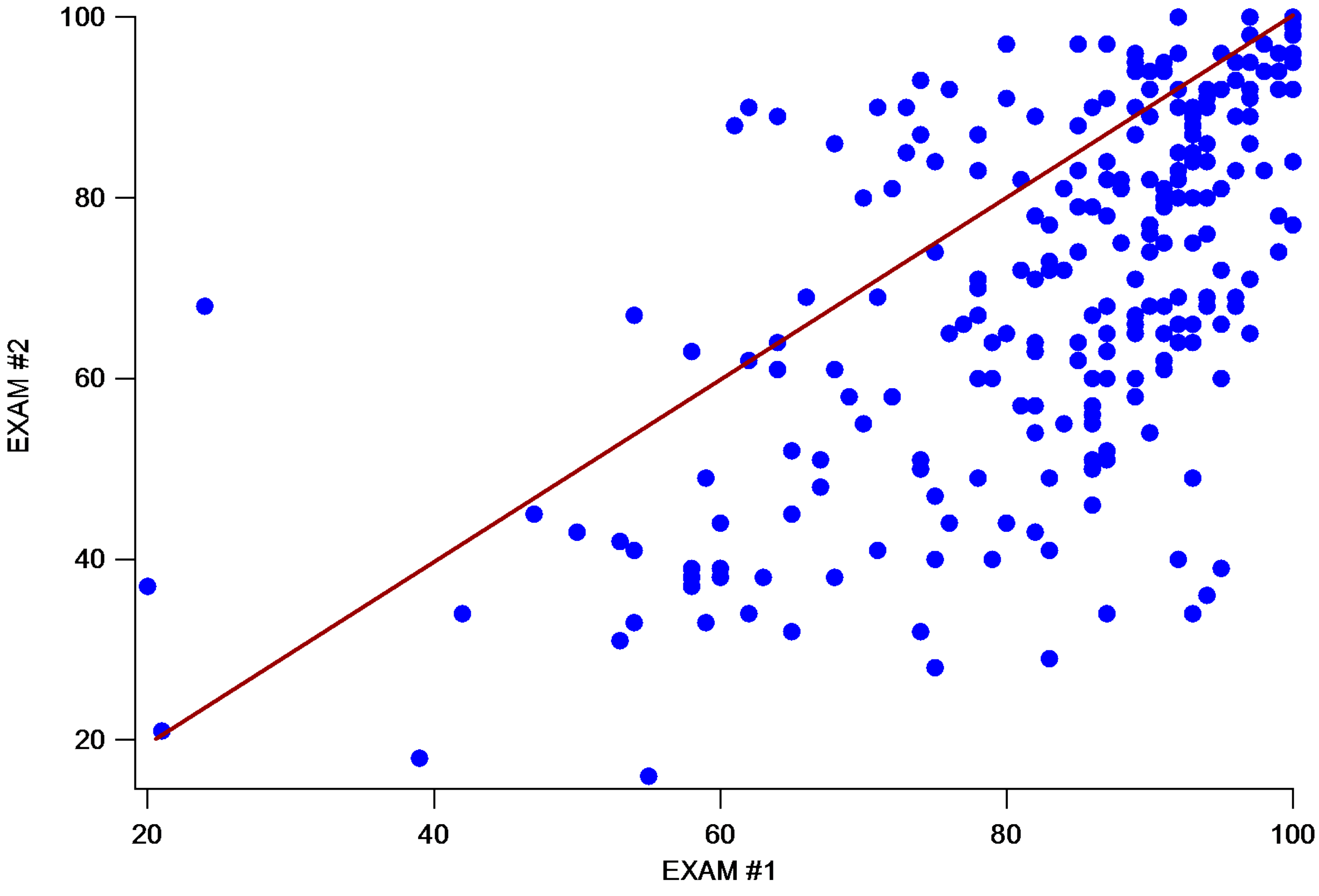
- **The average on Exam #2 is 70.2 ± 20.0%.**
- Many students have exam grades ≥ 20% lower than Exam #1.
- The solutions were posted this morning.
- The SI's will conduct reviews of this exam.
- Final exam will include a problem from one of the hour exams.
- My office hours tomorrow are shifted ≥ 30 minutes ~ 3:30 PM.
- Prof. Christenson has office hours 3 – 5 PM today.

EXAM #2, 2015 GRADE DISTRIBUTION



Five scores of 100%!

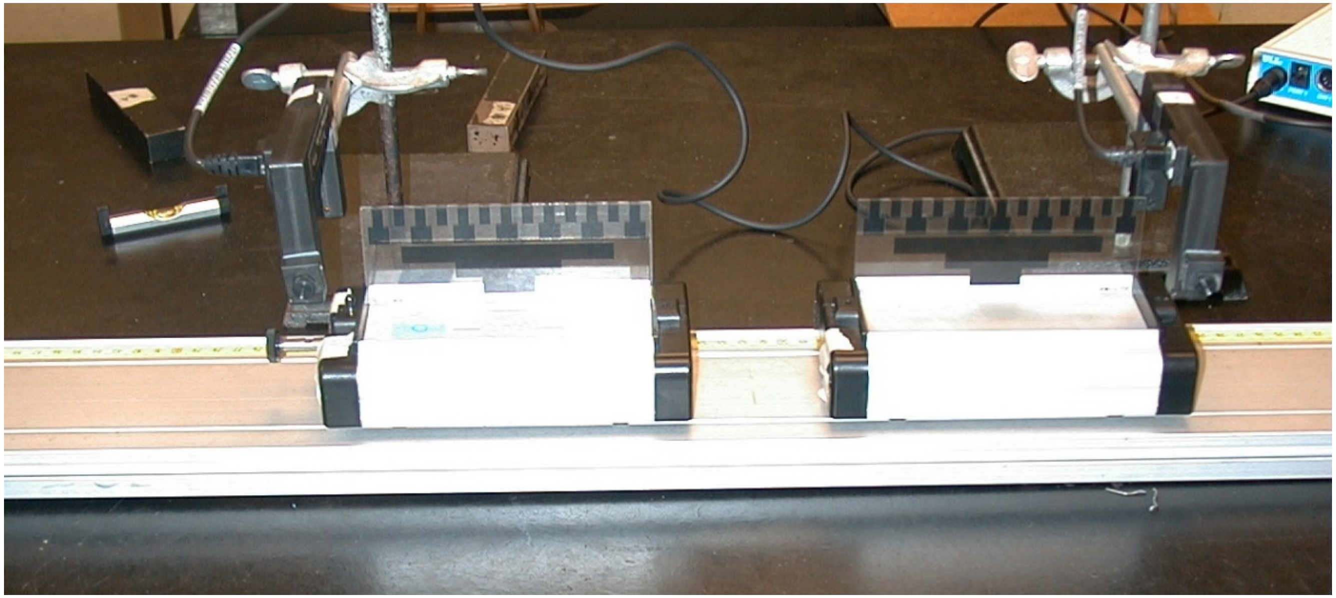
EXAM #2 *vs.* EXAM #1



Lab #4: Collisions - Conservation of Momentum

February 25 – March 5

You will consider
collisions between two carts
on a low-friction track. Under ideal circumstances, i.e., when
no external forces
are exerted on either cart, the
net sum of the momenta of the two carts must be unchanged,
so that the momentum gained by one cart is equal and opposite to the momentum lost by the other.



Center of Mass DEMO

– bonus points are available

This is a good time to test our theory that,
in the absence of external forces, the CM doesn't accelerate
by putting some students on skateboards.

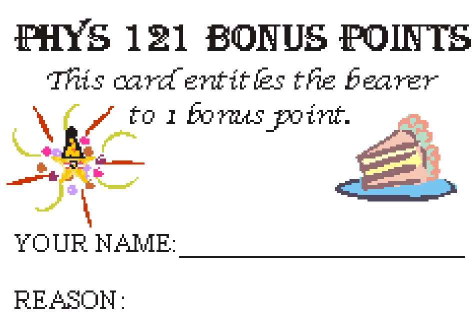
I need 3 volunteers;

- 2 students who have about the same mass as each other, &
- 1 other student who has a very different mass.

Volunteers should:

- be coordinated enough to stand (*or sit*) on a moving skateboard
- have good health insurance
- have no lawyers in the family or as friends.

https://www.youtube.com/watch?v=OQhRFdn1ijI&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr&index=4



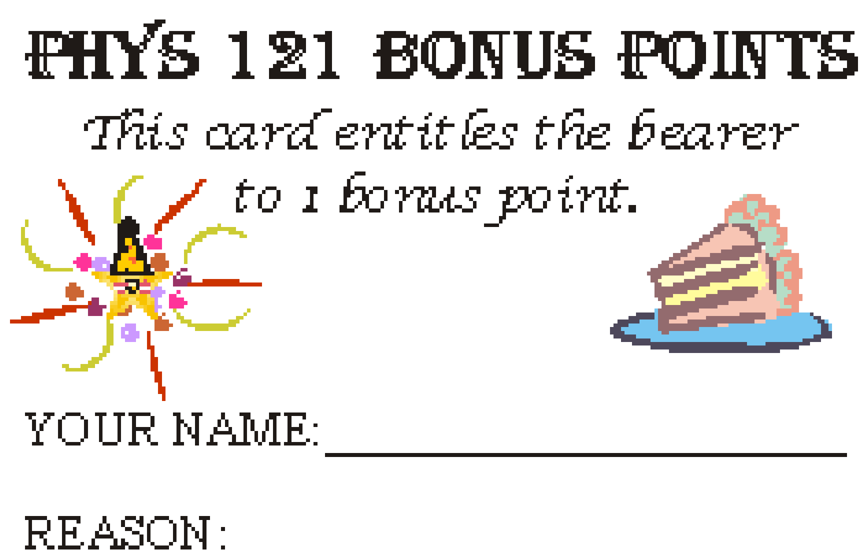
DEMOS

bonus points are available

External forces essentially act at the CM.

CM of tossed bat – 2 (*coordinated*) volunteers

https://www.youtube.com/watch?v=IyIpno9AwjU&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr



COLLISIONS

Understanding collisions is EXTREMELY important

to understanding our universe, *i.e.* physics!

- Collisions are not just carts hitting other carts or bats hitting a ball.
- Every step you take is a collision of your foot with the ground.
- Blood stays in your body due to collisions between blood cells & artery walls.
- The air molecules you breath collide with each other 10^{10} times/second and make it to your lungs only because of collisions in your throat.
- Every solid, including you, is held together in part due to collisions.

If you don’t believe this, try eliminating collisions by stepping into a vacuum.

DON’T TRY THAT!

http://brokensecrets.com/2011/02/11/people-dont-explode-in-space/

http://www.geoffreylandis.com/vacuum.html

http://westernsloth.blogspot.com/2012/10/space-freefall-and-boiling-blood.html

COLLISIONS & FORCES

- The forces between objects during a collision are:
  - **complicated** functions of time and position and
  - **very** hard to handle with **F** = *m***a**.
- **Conservation of Momentum** is crucial in understanding collisions.
- The key concept (*the magic words you need to invoke*)

During the relatively **short time of a typical collision** (~*msec*), the **effect of most external forces is negligible compared to the effect of large "*internal*" forces between the colliding objects**.

- Gravity isn't critical during the time a ball is in contact with a bat.
- Friction from the ground isn't your biggest concern when a large truck traveling at 70 mph hits your Prius, Kia or Porsche.
- Note that we are NOT relying on the *absence* of external forces to justify the use of conservation of momentum in collisions.

IMPULSE

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \Rightarrow \quad d\vec{p} = \vec{F}dt$$

- Integrate over the time of interest (*the collision*)

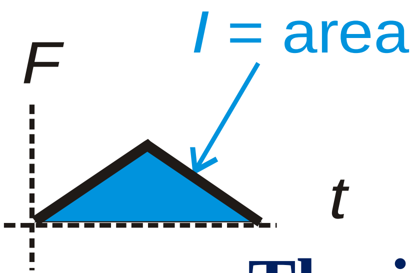
$$\int_{t_o}^{t_f} d\vec{p} = \Delta\vec{p} = \vec{p}_{final} - \vec{p}_{initial} = \int_{t_o}^{t_f} \vec{F}(t)\,dt$$

- The integral on the right, and therefore Δ**p**, is useful enough to have its own name and symbol, the

IMPULSE, I (*some texts use* **J**)

$$\vec{I} \equiv \int_{t_o}^{t_f} \vec{F}(t)\,dt = \Delta\vec{p}$$

***F** & **p** are vectors and therefore **I** is a vector, too,*
but we'll only worry about 1D systems for now.



IMPULSE

$$\vec{I} \equiv \int_{t_o}^{t_f} \vec{F}(t)\,dt = \Delta\vec{p}$$

The *impulse* delivered to an object by some force acting on that object is the time integral of the force = area under the curve of a plot of F(*t*) *vs.* *t*.

- The impulse tells you the change in the object's momentum.
- Impulse is most useful when the force is a complicated function of time, as in a collision.
- Impulse lets you connect **forces**, **momentum** and **time**.
- We'll often think in terms of ***average forces***,

$$\mathbf{I} = \Delta\mathbf{p} = m\Delta\mathbf{v} = m(\mathbf{a}_{\mathbf{avg}}\Delta t) = (m\mathbf{a}_{\mathbf{avg}})\Delta t = \mathbf{F}_{\mathbf{AVG}}\Delta t$$

$$\mathbf{I = F_{average}\Delta t}$$

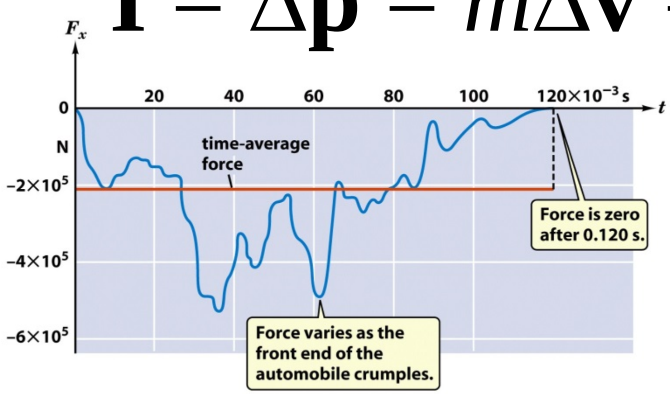


Figure 11-2 Physics for Engineers and Scientists 3/e

BOUNCING BALL

A 0.62-kg ball falls vertically & hits the floor with a speed of 20 m/s.
It rebounds vertically upwards with an initial speed of 10 m/s.

What is the impulse on the ball (*due to the floor*)
during the ball's 1st collision with the floor?

http://www.lhsboosterclub.com/wp-content/uploads/2013/09/BasketballStockImage.jpg

BOUNCING BALL

$$\vec{I} = \int_{t_o \,=\, \text{first contact}}^{t_f \,=\, \text{ball loses contact}} \vec{F}(t)\,dt$$

where $F(t)$ is the force exerted by the floor on the ball as a function of time while the ball is in contact with the floor.

It's almost hopeless to estimate $F(t)$ and do the integral on the right (*although you can measure $F(t)$ with modern sensors*).

HOWEVER we can use the fact that

$$I \equiv \int F dt = p_{final} - p_{initial}$$

Assuming that up is positive, $v_f = +10$ m/s while $v_i = -20$ m/s

$I = \Delta p = M\Delta v = 0.62$ kg [10 m/s - (-20 m/s)]

$I = 19$ kg-m/s or 19 N-s

This is the impulse the floor delivers to the ball, since we've used the momentum of the ball in our calculation; the impulse the ball delivers to the floor is -30 N-s.

BOUNCING BALL

If the ball is in contact with the floor for 25 milliseconds = 0.025 s, what is the average force exerted on the ball during this collision?

$$I = F_{AVG}\Delta t$$

$$\Rightarrow F_{AVG} = I/\Delta t$$

$$F_{AVG} = 19\text{ N-s}/0.025\text{ s} = 760\text{ N}$$

A 0.62 kg ball weighing ~ 6 N experiences a 760 N force!

TRANSLATED TO AMERICAN:

A 22 ounce = 1.4 pound ball experiences a 171 lb. force!

COLLISIONS HURT!

BOUNCING BALL

How would your answer for the impulse change if the ball just went 'splat' and didn't bounce at all in a 25 msec collision?

The impulse would be *smaller* because the change in momentum would be smaller and the average force would be smaller.

$$I \equiv \int F dt = p_{final} - p_{initial}$$

A SPLAT hurts *less* than an elastic rebound,

although this is probably counter-intuitive!

If you don't believe this, try hitting yourself on the head with a snowball and then with a iceball of the same mass.

Actually, don't try it – trust the physics!

http://transueland.blogspot.com/2011/10/megans-goose-egg.html

http://www.killinofirm.com/media/brain-injury.jpg

DEMO

A ball rolls down an incline & hits a piece of wood balanced on one end.

Which type of ball is more likely to knock the wood over,

i.e. which delivers the larger impulse to the wood?

A. a bouncy, elastic ball?

B. a ‘dead’ ball?

https://www.youtube.com/watch?v=kjhFlhYrsow&list=PL_g3d400fWB9i9NxFcTYvUqNeeuL0xoyr

USING IMPULSE

HOW CAN YOU REDUCE THE AVERAGE FORCE?

$$I = F_{average}\Delta t = m\Delta v \;\rightarrow\; F_{average} = m\frac{\Delta v}{\Delta t}$$

The concept of impulse gives us another way to understand how to ***reduce*** the average force experienced during a collision, such as

- A landing after falling from some height
- A car crashing into another car or into a stationary object

➢ Increase Δt to maximize the time spent ‘colliding’.

➢ Decrease Δv by setting $v_{final} = 0$ instead of $v_{final} = -v_{initial}$

  - Land with your legs ready to bend or be prepared to roll as you land.
  - Use padding or build a crush zone into your structure.

https://www.google.com/search?q=1963+plymouth+belvedere&espv=2&biw=1416&bih=988&source=lnms&tbm=isch&sa=X&ei=EBEQVcufLpL3ggTo1IGYAQ&ved=0CAgQ_AUoAw#tbm=isch&q=1963+plymouth+belvedere&imgdii=orJXkzXpWAqZwM%3A%3B-elqI-8ycksPsM%3BorJXkzXpWAqZwM%3A&imgrc=orJXkzXpWAqZwM%253A%3BV18rZSoNp3PwRM%3Bhttp%253A%252F%252Ffarm5.staticflickr.com%252F4042%252F4495093670_2bf7cc85ca_o.jpg%3Bhttp%253A%252F%252Fwww.boldride.com%252Fride%252F1963%252Fplymouth-belvedere%3B2560%3B1707

USING IMPULSE

HOW CAN YOU REDUCE THE AVERAGE FORCE?

HISTORY / SERMON / ENGINEERING / LAWSUITS

$$I = F_{average}\Delta t = m\Delta v \quad \rightarrow \quad F_{average} = m\frac{\Delta v}{\Delta t}$$

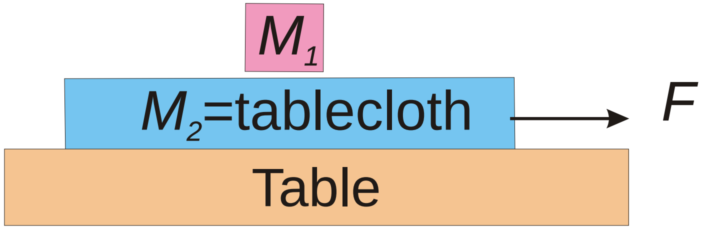
Before the 1970's, cars were built as rigidly and resistant to deforming as possible (*body on frame construction*).

- Dashboards were metal & heads would bounce off.
- No seatbelts or airbags: passengers bounced around the car during collisions.
- Cars survived collisions when their occupants did not.
- This began to change when Ralph Nader published *Unsafe at Any Speed in 1965* *http://en.wikipedia.org/wiki/Unsafe_at_Any_Speed* .

Modern cars incorporate crush zones.

- Parts of the car's structure crumple in a controlled fashion in a collision.
- This makes cars more expensive to repair, but still less expensive than repairing the people inside the car.

IMPULSIVE DEMO



Explain the trick of pulling a tablecloth out from under a set of dishes without dragging the dishes onto the floor.

- Our system is M_1. We don't care what happens to M_2.
- The only HORIZONTAL force on M_1 is $F_{kinetic\ friction} = \mu_k M_1 g$.

unless you pull too slowly so that static friction applies & $\mu_k \rightarrow \mu_s$

$$F_{pull} > (M_1 + M_2)a = (M_1 + M_2)\frac{F_s}{M_1} = (M_1 + M_2)\left(\frac{\mu_s \cancel{M_1} g}{\cancel{M_1}}\right) \quad \Rightarrow \quad a > \mu_s g$$

- $F_{kinetic\ friction} = \mu_k M_1 g$ is independent of the speed of the tablecloth.

$\Rightarrow$ the IMPULSE $\quad I = F_{friction}\Delta t = \Delta p \quad \Rightarrow \quad \Delta p = (\mu_k M_1 g)\Delta t$.

$\Rightarrow$ If Δt is small, I is small $\Rightarrow M_1 v_{1\text{-}final}$ is small & M_1 remains on the table.

Δt is small if you pull the tablecloth very fast.

DO YOU BELIEVE?

COLLISIONS

TWO WAYS TO CLASSIFY COLLISIONS

1. **DIMENSIONS**
   - 1D collisions are relatively easy to solve.
   - 2D collisions are harder to solve & require more information.
   - 3D collisions are too painful for PHYS 121 students
     – and not needed for examining collisions between two particles, which can only be 1D or 2D.
2. **ENERGY LOSS**
   - ELASTIC COLLISION ≡ no energy is lost.
   - INELASTIC COLLISION ≡ some energy is lost.
   - COMPLETELY INELASTIC COLLISION ≡ some energy is lost & the colliding objects stick together.

<u>COLLISIONS & MOMENTUM</u>

Collisions are by definition **<u>short in time</u>**.

$$\Rightarrow \Delta p = F_{external}\Delta t \to 0$$

$\Rightarrow$<u>MOMENTUM</u> IS CONSERVED DURING A COLLISION

It's $\Delta t \to 0$ that matters, not $F_{external}$.

COLLISIONS & ENERGY

ENERGY IS CONSERVED

IN ELASTIC COLLISIONS

BUT NOT IN INELASTIC COLLISIONS

Energy lost during inelastic collisions is converted into heat and deformation of the objects.

The latter types of energy (*heat & deformation*) cannot be easily converted back into macroscopic motion.

Macroscopic + Microscopic Energy is conserved even in inelastic collisions but that fact isn't useful in analyzing macroscopic motion.

COMPLETELY INELASTIC 1D COLLISIONS

Two blocks collide & stick together.

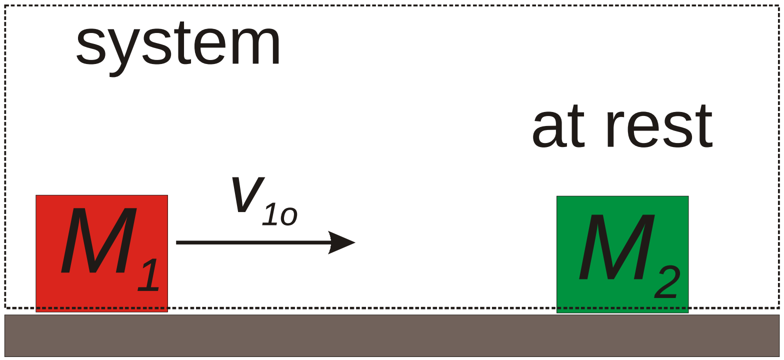
- Assume M_2 is initially at rest. (*simplifies math & changes nothing important*)
- Given v_{1o} of M_1, what is the final velocity v_f of the joined blocks?

Ohanian uses *v* and *v'* for before & after.

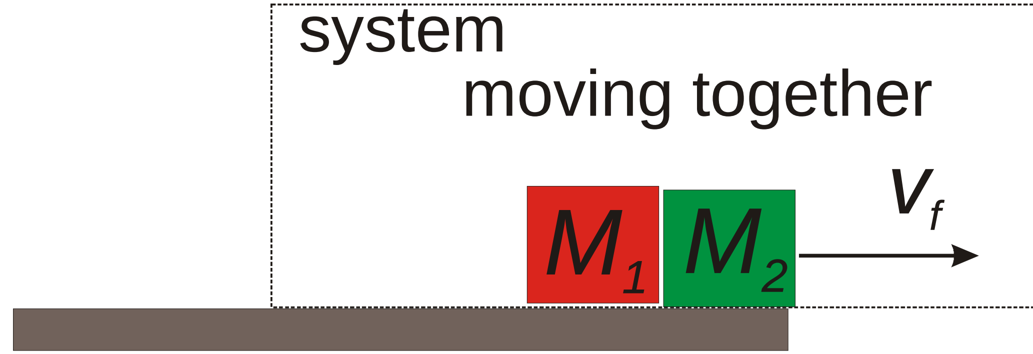
- You can't solve this problem directly with $F = ma$ since we don't know the forces acting on each mass during the collision.

(*We could solve this problem with center of mass concepts.*)

- The 4 equations for constant acceleration don't apply since the acceleration of the blocks is probably not constant during the collision.



Before Collision



After Collision

- Define the system as both blocks.
- There are *no external forces* in the *x*-direction & it's a collision.

Either one of these conditions is sufficient to say P_x is conserved.

$$P_f = P_o$$

$$P_f = (M_1 + M_2)v_f$$

$$P_o = M_1 v_{1o}$$

$$P_f = P_o \quad \rightarrow \quad (M_1 + M_2)v_f = M_1 v_{1o}$$

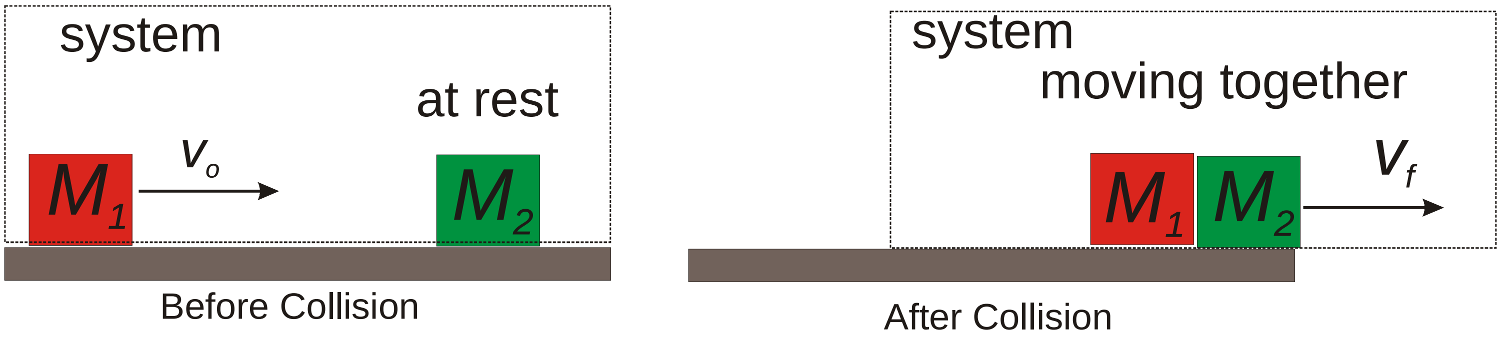
$$v_f = v_{1o}\left(\frac{M_1}{M_1 + M_2}\right)$$

REALITY CHECK

$$v_f = v_{1o}\left(\frac{M_1}{M_1 + M_2}\right)$$

- If $M_1 << M_2$, *(you throw a snow flake M_1 at a parked truck M_2)* they both end up at rest, $v_f \sim 0$.
- If $M_1 >> M_2$ *(you drive a truck M_1 into a snow flake M_2)* then M_1 barely slows, $v_f \sim v_{1o}$; the truck M_1 just picks up the snowflake M_2 and carries it along for the ride.
- If $M_2 = M_1$ then the pair of blocks continues at ½ the speed of M_1.
- Note that we can also obtain this answer by analyzing the CM.

  CM & momentum conservation are equivalent for this problem but momentum is more generally useful.



ENERGY CHECK

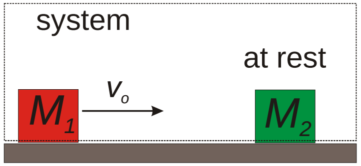
$$v_f = v_{1o}\left(\frac{M_1}{M_1+M_2}\right)$$

$$K_o = \tfrac{1}{2}M_1 v_{1o}^2$$

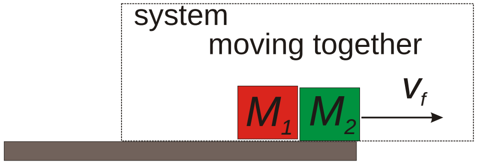
$$K_f = \tfrac{1}{2}(M_1+M_2)v_f^2 = \tfrac{1}{2}(M_1+M_2)v_{1o}^2\left(\frac{M_1}{M_1+M_2}\right)^2$$

$$= \tfrac{1}{2}\frac{M_1^2}{M_1+M_2}v_{1o}^2 = \left(\tfrac{1}{2}M_1 v_{1o}^2\right)\left(\frac{M_1}{M_1+M_2}\right)$$

$$\text{Since } \left(\frac{M_1}{M_1+M_2}\right) < 1$$



Before Collision



After Collision

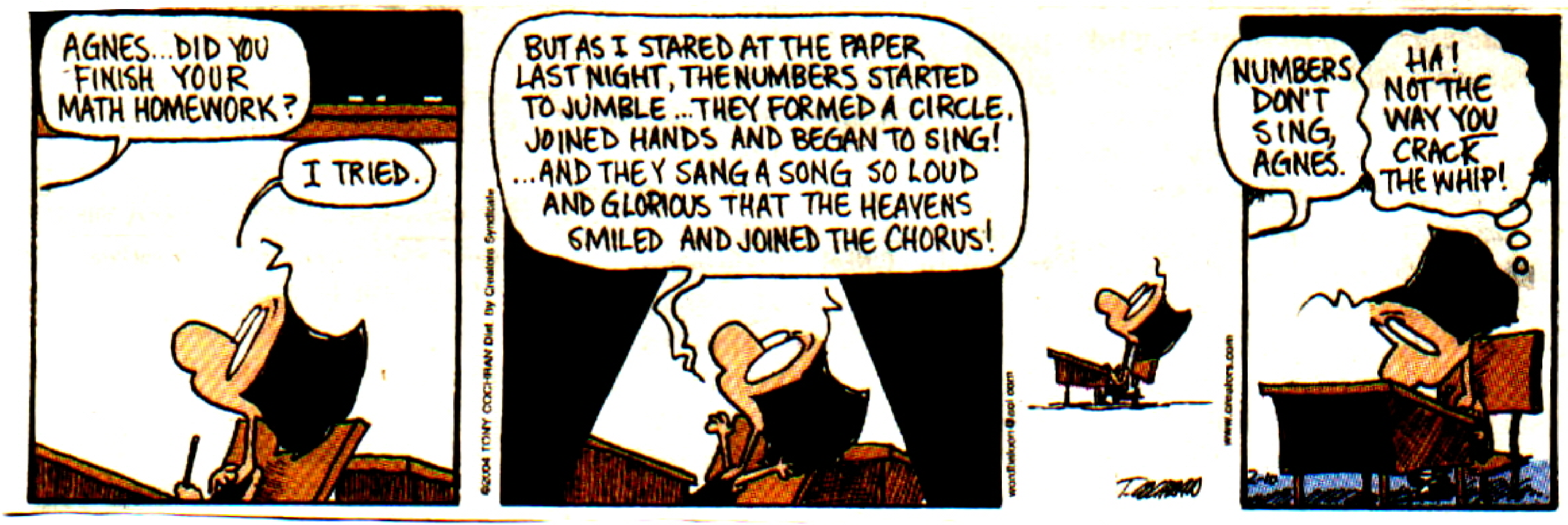
$$\tfrac{1}{2}M_1 v_{1o}^2 > \tfrac{1}{2}M_1 v_{1o}^2\left(\frac{M_1}{M_1+M_2}\right)$$

$$K_o > K_f$$

A completely inelastic collision MUST result in a loss of kinetic energy.
Making M_2 relatively bigger makes the energy loss bigger.

We made it to slide #29 on Monday, March 23

PHYS 121 – SPRING 2015



Chapter 11: Collisions

version 03/25/2015, ~ 100 slides

We made it to slide #29 on Monday, March 23.

Get your clickers ready.

ANNOUNCEMENTS

- Late and re-graded exams are available in the Rock. 104 hallway.
- To improve your performance on the next exam and your course grade, attend the SI sessions regularly.
- Make certain you understand how to do all the homework problems.
- Trust the posted solutions, not the grader's evaluation of your homework, particularly since they grade only a fraction of the problems on each assignment.
- Review the lecture notes, particularly clicker questions, and make sure you UNDERSTAND the answers to those questions.
- Check out last year's exams.
- Extra help is available during (& *outside*) office hours and through the ESS tutor program.

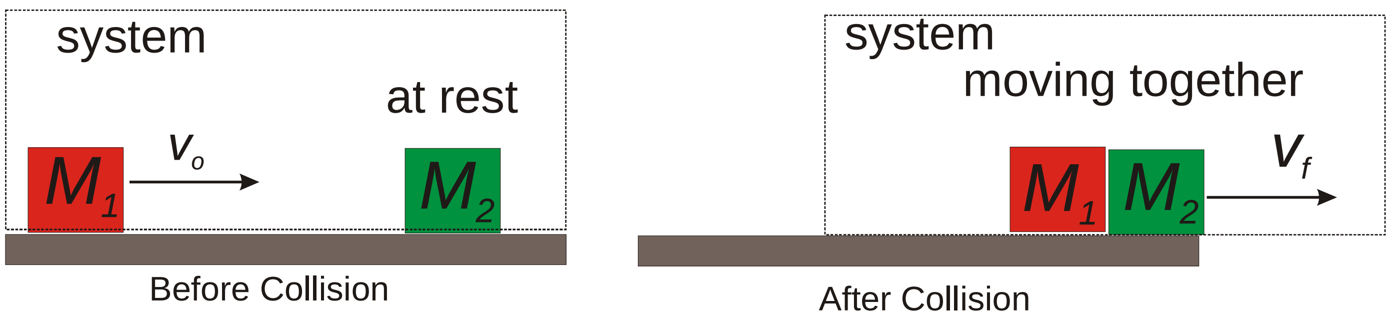
EXPLOSIONS

Classical physics looks the same if time runs backwards

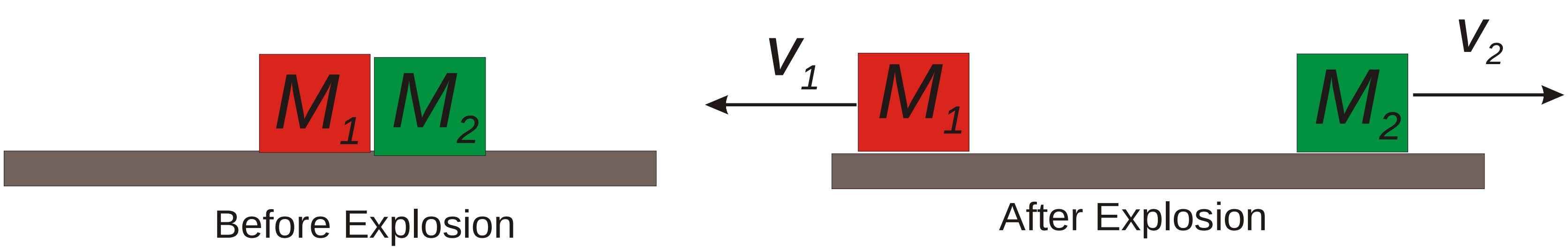
$\equiv$ *time reversal symmetry*.

A completely inelastic collision reversed in time

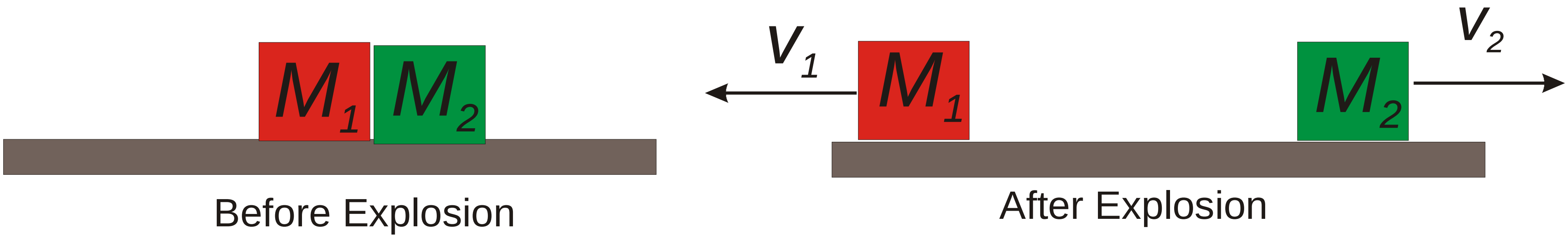
looks like an explosion.



$\Rightarrow$ Explosion problems can be solved with the same techniques used for completely inelastic collision.



EXPLOSIONS



A block at rest suddenly explodes into pieces M_1 & M_2.

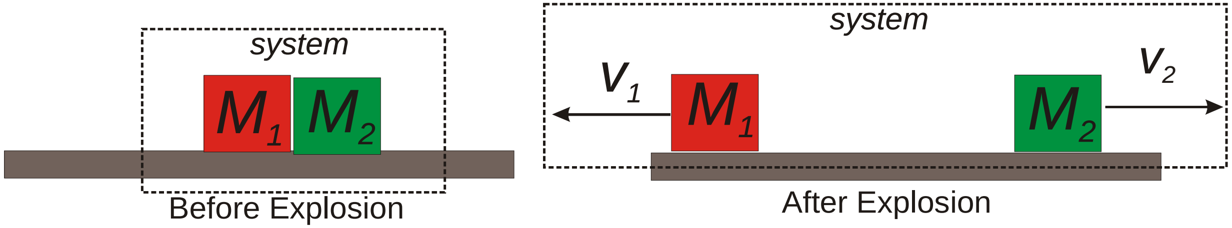
What are the velocities of each piece?

You can't use $F = ma$ since the forces during an explosion are very complicated and the acceleration is NOT constant.

You CAN solve it using the Center of Mass

but we'll use Conservation of Momentum.

EXPLOSIONS



A block at rest suddenly explodes into two pieces of mass M_1 and M_2.

What are the velocities of each piece?

Defining the system as $M_1 + M_2$

There are no external forces (*in the x-direction*) involved in the explosion & the explosion is fast!

$\Rightarrow P_x$ is conserved $\Rightarrow P_f = P_o = 0$

$$P_f = M_1 v_{1f} + M_2 v_{2f}$$

$$M_1 v_{1f} + M_2 v_{2f} = 0$$

$$v_{2f} = -v_{1f}\,(M_1/M_2)$$

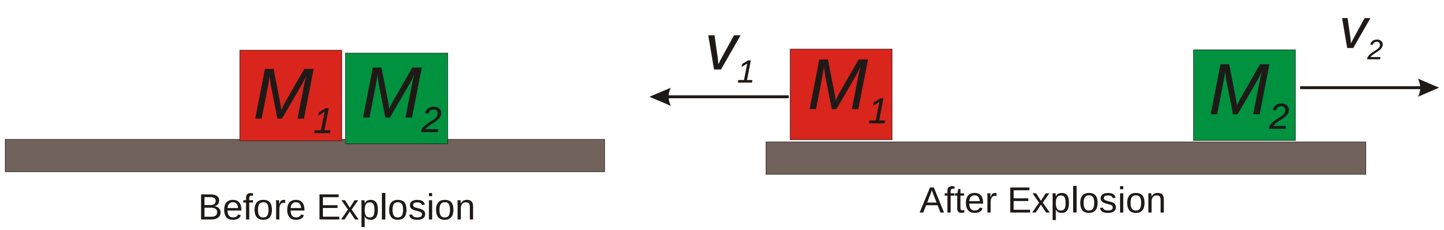
This is all you can say without more information

REALITY CHECK

$v_{2f} = -v_{1f}\,(M_1/M_2)$

- ✓ If $M_2 = M_1$ each block ends up with the same speed in opposite directions.
- ✓ If $M_2 >> M_1$ then v_2 is much smaller than v_1.
- ✓ Conservation of momentum provides only 1 relationship for 2 unknowns (*the two final velocities*) $\Rightarrow$ can't solve for v_1 & v_2.
- If you know how much energy the explosion releases, you have 2 equations for 2 unknowns & can determine v_1 & v_2 .

$$E_{final} = \tfrac{1}{2} M_1 v_{1f}^2 + \tfrac{1}{2} M_2 v_{2f}^2$$



COLLISIONS as **PART** of a PROCESS

- Objects normally move <u>BEFORE & AFTER</u> a collision.
- Analyze the collision **<u>separately</u>** from motion before & after.
- Energy might be conserved before and/or after the collision even if it isn't conserved during the collision – and *vice versa*.

COLLISIONS as PART of a PROCESS

- M_1 slides down a frictionless ramp where it collides and sticks to M_2.

  How fast are the blocks moving after the collision?

  Use $mgh = ½mv^2$ (*Conservation of Energy*) to find v_o of M_1. Then analyze the collision.

- A block moving at v_o strikes & sticks to another block attached to a spring.

  How far is the spring compressed?

  Analyze the collision first, then use $½M_{total}v^2{}_{after\text{-}collision} = ½kx^2$. (*Conservation of Energy*)

  A mass hits and sticks to a second hanging mass. How high do they swing?

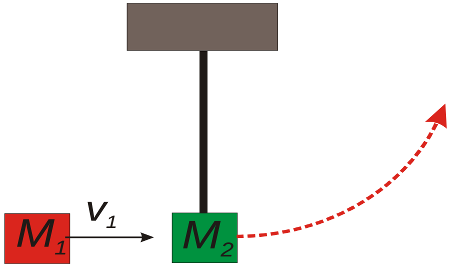
  Analyze the collision, then use conservation of energy $½M_{total}v^2 = mgh$

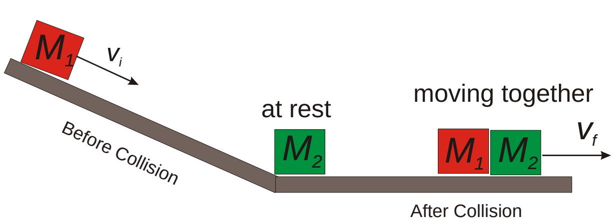
- A pair of blocks collide on a surface with friction μ_k and stick together.

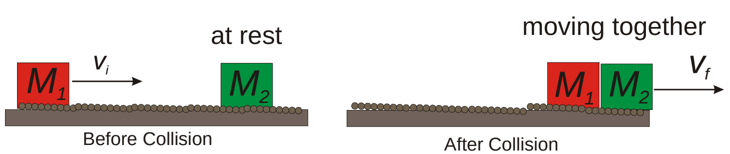
  Where do the blocks end up?

  Analyze the collision, then

  use $½M_{total}v^2{}_{after\text{-}collision} = \mu_k M_{total}gx$ for x.

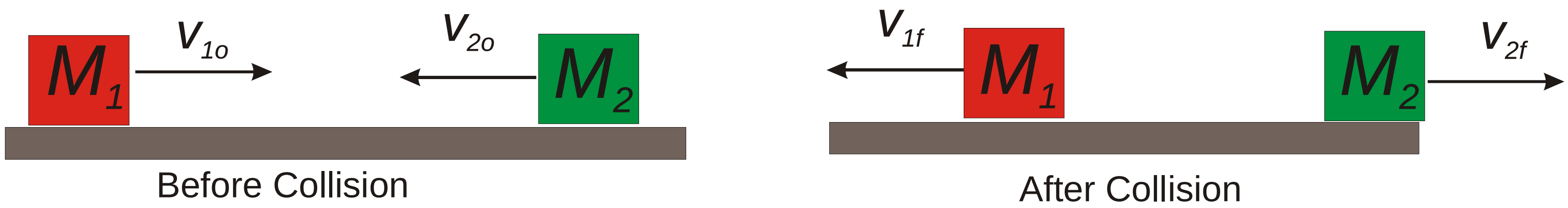






ELASTIC COLLISIONS IN 1D

more general than Ohanian's analysis

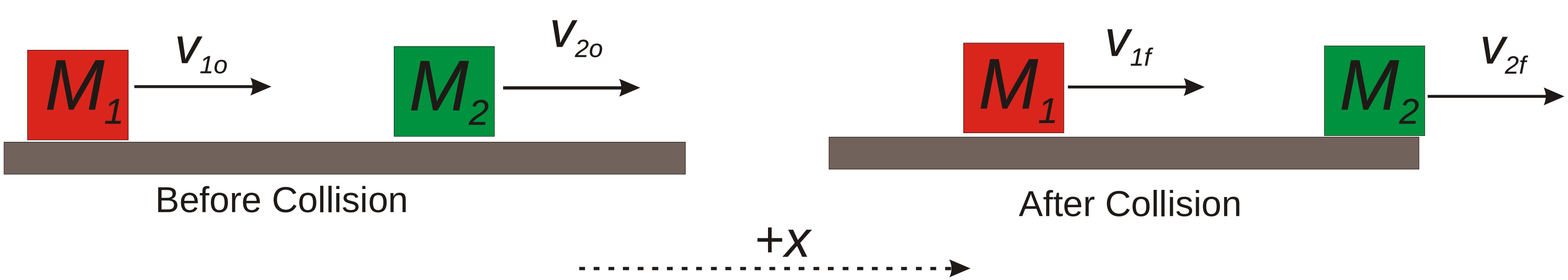


Two masses moving towards each other collide **elastically**.

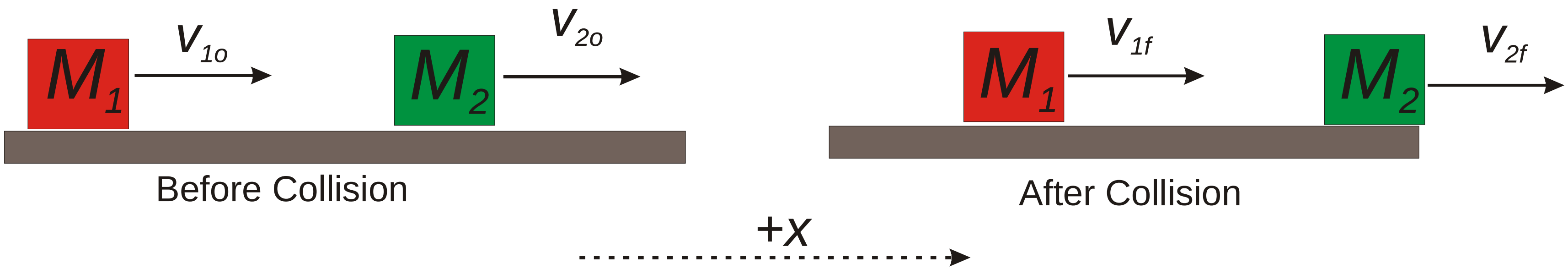
"ELASTICALLY" ⇒ kinetic energy is conserved.

Let all four velocity vectors point to the right (*positive x*)

(*but some of these velocity vectors may have negative values, signifying movement to the left*)



ELASTIC COLLISIONS IN 1-D



TWO PRINCIPLES of PHYSICS APPLY

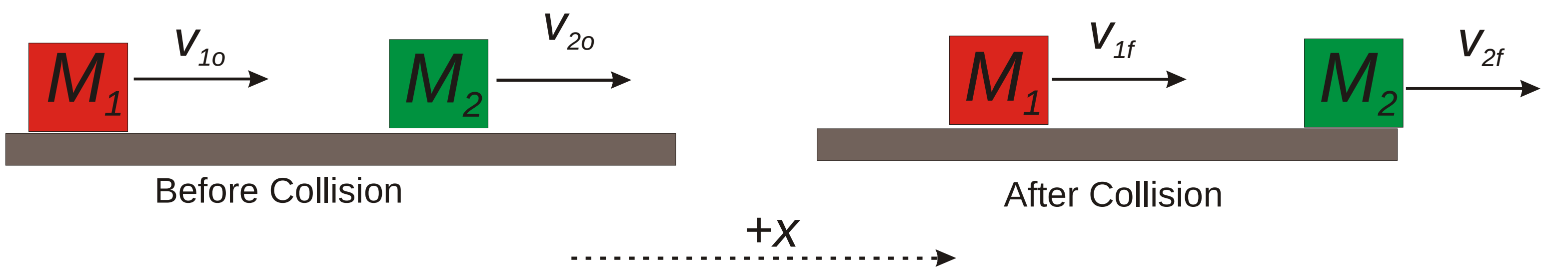
1. **MOMENTUM IS CONSERVED**, $\Delta t \sim 0$

$$P_{total} = \sum_i m_i v_i = \text{constant}$$

2. **ENERGY IS CONSERVED** because it's *elastic*

$$K_{total} = \sum_i \frac{1}{2} m_i v_i^2 = \text{constant}$$

ELASTIC COLLISIONS IN 1-D



There are 6 parameters/variables.

$$M_1 \quad M_2 \quad v_{1o} \quad v_{2o} \quad v_{1f} \quad v_{2f}$$

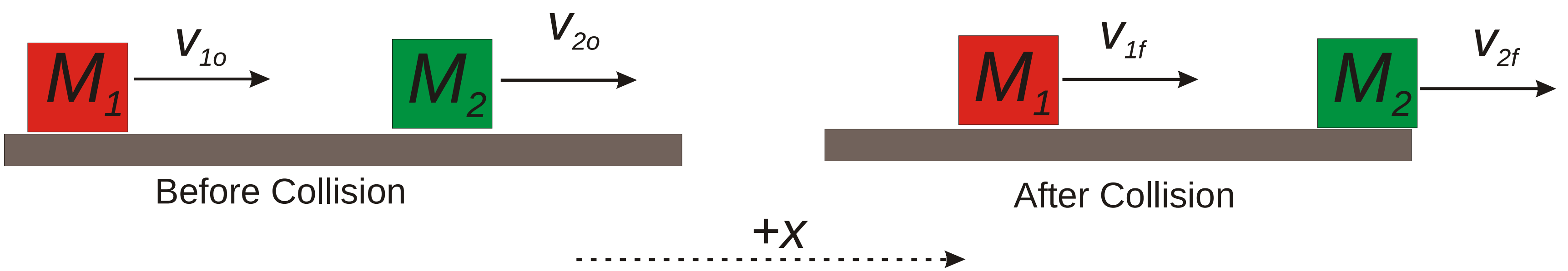
Conservation of energy & momentum provide

2 independent relationships between them

$$K_{total} = \sum_i \frac{1}{2} m_i v_i^2 = \text{ constant} \qquad P_{total} = \sum_i m_i v_i = \text{constant}$$

$\Rightarrow$ You need 4 independent bits of information about the 6 parameters in order to solve for the other 2 parameters.

ELASTIC COLLISIONS IN 1-D



- Typically you are given the two masses and two initial velocities

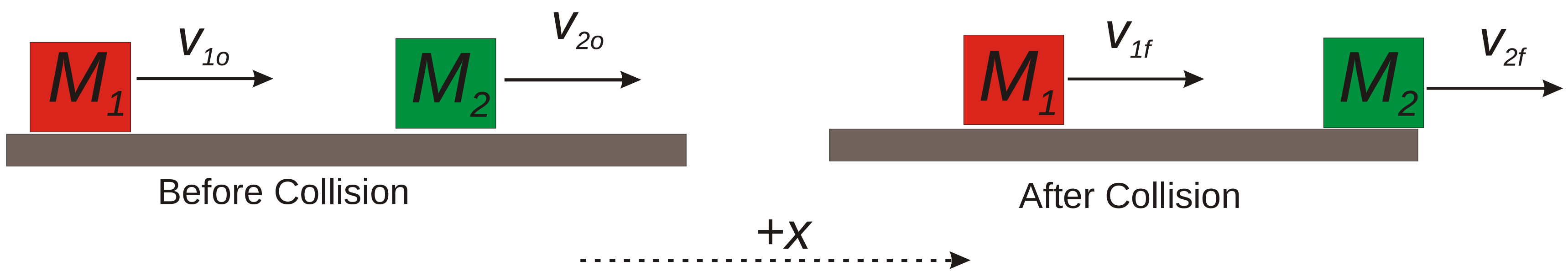
  & asked to solve for the two final velocities

  but we can mix this up any way we like

- **NOTE THAT** a **2D** elastic collision includes 4 more parameters since you need *x* & *y* components for each velocity term.
- Conservation of momentum provides one eq. for each dimension
- But conservation of energy does not!

⇒You need more information to solve 2D & 3D elastic collisions, such as the final speed or direction of one of the particles.

ELASTIC COLLISIONS IN 1-D



First, we'll write out what the physics tells us.

$$K_{total} = \sum_i \frac{1}{2} m_i v_i^2 \underset{initial}{=} \frac{1}{2} M_1 v_{1o}^2 + \frac{1}{2} M_2 v_{2o}^2 \underset{final}{=} \frac{1}{2} M_1 v_{1f}^2 + \frac{1}{2} M_2 v_{2f}^2$$

$$P_{total} = \sum_i m_i v_i \underset{initial}{=} M_1 v_{1o} + M_2 v_{2o} \underset{final}{=} M_1 v_{1f} + M_2 v_{2f}$$

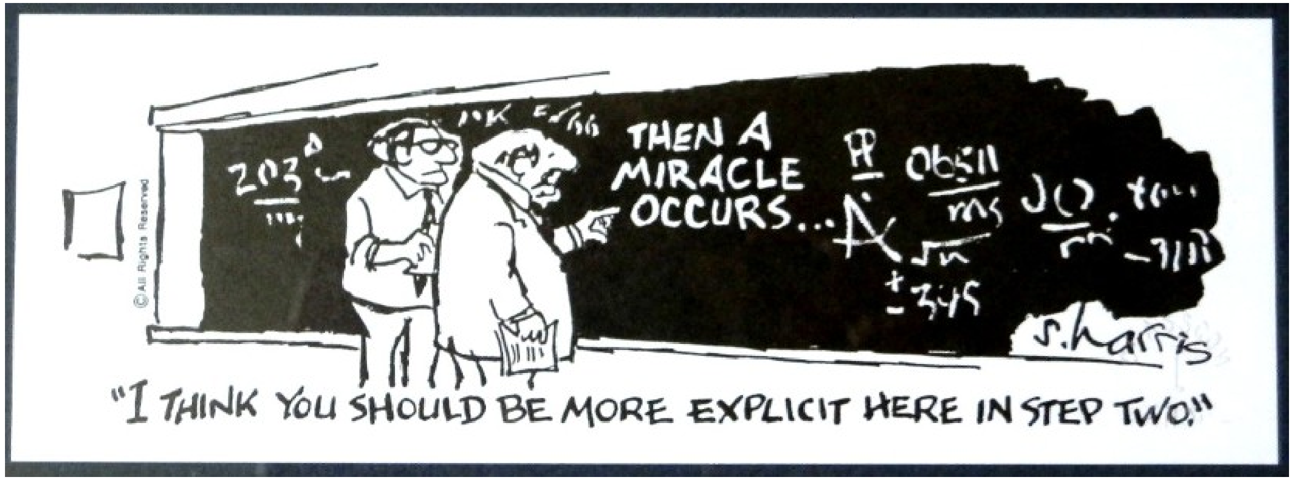
Now do the math

$\rightarrow$ solve the pair of equations shown above for v_{1f} & v_{2f}

in terms of the other parameters.

GOOD NEWS: We know it's possible; 2 eq. for 2 unknowns!

BAD NEWS: THE MATH *(Are you ready? Take off your shoes like Prof. Butler?)*



ATAMO = "and then a miracle occurs"

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right)$$

$$v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

OBVIOUSLY!

Actually the math is not all that hard. Here's most of it, to **read on your own**!

$$K_{total} = \sum_i \frac{1}{2} m_i v_i^2 \underset{initial}{=} \frac{1}{2} M_1 v_{1o}^2 + \frac{1}{2} M_2 v_{2o}^2 \underset{final}{=} \frac{1}{2} M_1 v_{1f}^2 + \frac{1}{2} M_2 v_{2f}^2$$

$$\rightarrow M_1 \left(v_{1o}^2 - v_{1f}^2 \right) = M_2 \left(v_{2f}^2 - v_{2o}^2 \right)$$

EQ #1, from factoring: $M_1 \left(v_{1o} - v_{1f} \right)\left(v_{1o} + v_{1f} \right) = M_2 \left(v_{2f} - v_{2o} \right)\left(v_{2f} + v_{2o} \right)$

now rearrange the conservation of momentum equation

$$P_{total} = \sum_i m_i v_i \underset{initial}{=} M_1 v_{1o} + M_2 v_{2o} = M_1 v_{1f} + M_2 v_{2f}$$

EQ #2: $M_1 \left(v_{1o} - v_{1f} \right) = M_2 \left(v_{2f} - v_{2o} \right)$

divide Eq #1 by Eq #2 to get

$$\left(v_{1o} + v_{1f} \right) = \left(v_{2f} + v_{2o} \right)$$

$$v_{2f} = v_{1o} + v_{1f} - v_{2o}$$

plug this back into Eq. #2

$$M_1 \left(v_{1o} - v_{1f} \right) = M_2 \left(\left(v_{1o} + v_{1f} - v_{2o} \right) - v_{2o} \right)$$

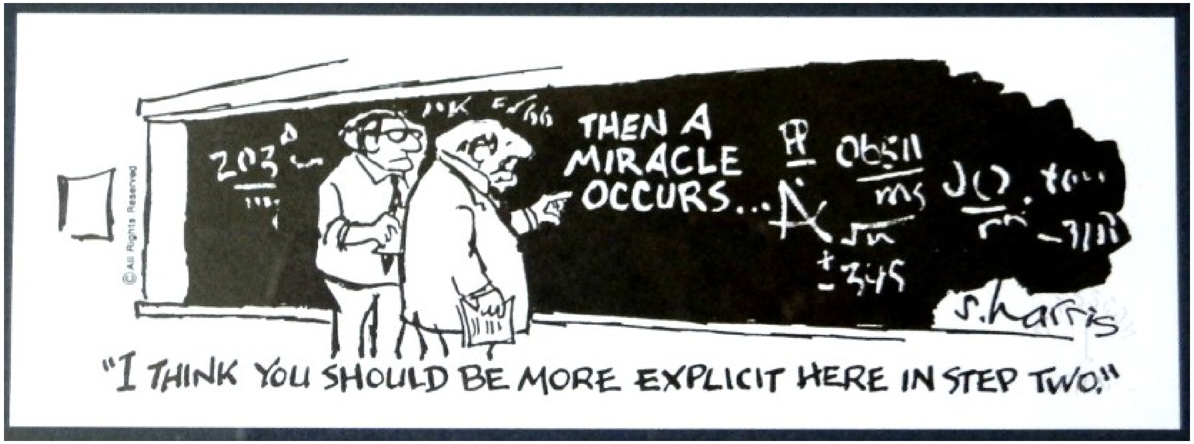
and rearrange to get v_{1f}

$$\boxed{v_{1f} =} \frac{\left(M_1 - M_2 \right) v_{1o} + \left(2M_2 \right) v_{2o}}{M_1 + M_2} = \boxed{v_{1o} \left(\frac{M_1 - M_2}{M_1 + M_2} \right) + v_{2o} \left(\frac{2M_2}{M_1 + M_2} \right)}$$

You should be able to solve for $v_{2f} = v_{1o} + v_{1f} - v_{2o}$ now.

You might prefer

OBVIOUSLY!



$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

- GOOD NEWS: You're not expected to memorize equations like this.
- BAD NEWS: You SHOULD understand a few *special cases*.
- If you START with specific, simple initial conditions like Ohanian does, with $v_{2o} = 0$, the algebra is MUCH simpler & you can do it (*on an exam*)!
- You can always pretend you are in a reference frame where $v_{2o} = 0$ and translate the result into the lab frame. $v_{2\text{-}lab} = v_{2\text{-}0} + v_{0\text{-}lab}$
- But if you're going to translate between frames of reference to analyze collisions, use the CM frame!

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

SPECIAL CASE #1: If $M_1 = M_2 = M$ then

$$v_{1f} = v_{2o} \quad \& \quad v_{2f} = v_{1o}$$

- The two masses *exchange velocities* when they collide!
- If $v_{2o} = 0$, then $v_{1f} = 0$ & $v_{2f} = v_{1o}$
- You can use this concept in billiards; the cue ball stops.

although the spin you can put on the cue ball let's you vary the result.

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

SPECIAL CASE #2: $M_1 \ll M_2 \quad \& \quad v_{2o} = 0$

You throw a light mass at a heavy mass which is initially at rest.

$$v_{1f} = -v_{1o} \quad \& \quad v_{2f} = 0$$

- M_1 bounces back with its same initial speed.
- M_2 doesn't move.
- This should make perfect sense.

If you drop an elastic ball onto the ground, the ball bounces back up and the Earth stays put - approximately.

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

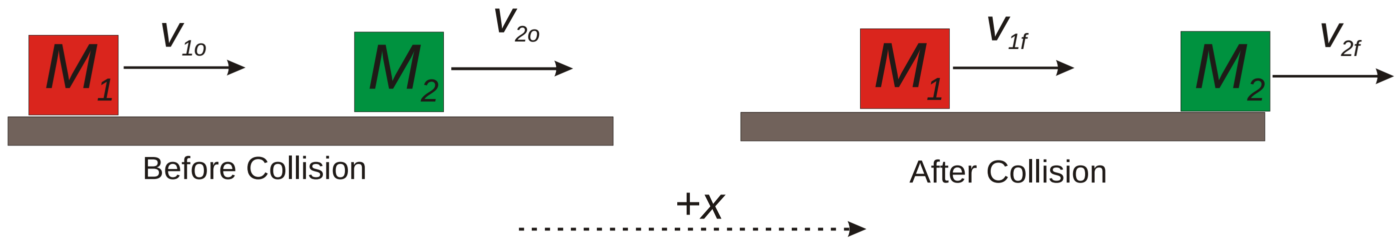
SPECIAL CASE #3: $M_1 >> M_2$ & $v_{20} = 0$

You throw a heavy object at a light object which is initially at rest.

$$v_{1f} = v_{1o} \qquad \& \qquad v_{2f} = 2v_{1o}$$

- M_1 continues on its way unaffected by the collision.
- M_2 takes off at **twice** the velocity as M_1.
- The behavior of M_1 should make sense.
- But the behavior of M_2 may be hard to accept without more evidence – *i.e. an experiment or demonstration*.

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$



<u>**SPECIAL CASE #3:**</u> $M_1 >> M_2$ & $v_{20} = 0$

You throw a heavy object at a light object which is initially at rest.

<u>**ALTERNATE ANALYSIS**</u> $v_{1f} = v_{1o}$ & $v_{2f} = 2v_{1o}$

- Imagine looking at the collision from the reference frame* of M_1

  (* *add* $-v_{1o}$ *to all velocities to shift to this frame*)

  - **<u>You</u>** see M_1 as a massive object at rest, $v^*_{1o} = 0$ & $v^*_{1f} = 0$
  - M_2 approaches with velocity $v^*_{2o} = -v_{1o}$ (*instead of 0*) and bounces back with the same velocity $v^*_{2f} = +v_{1o}$.
  - To return to the *lab* frame of reference, add $+v_{1o}$ to both final velocities, resulting in speeds of v_{1o} and $2v_{1o}$ in the lab frame, as above.

$$\cancel{v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right)} \quad \& \quad \cancel{v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)}$$

<u>YOU</u>

should not use general purpose collisions solutions to solve most homework or exam problems.

Start with conservation of momentum and write an equation for your specific situation.

$$P_f = P_o \;\rightarrow\; M_1 v_{1f} + M_2 v_{2f} = M_1 v_{1o} + M_2 v_{2o}$$

- **This might include the masses plus initial & final velocities in each direction, some of which are known.**
- **Inelastic collisions (*or explosions*) will have the same terms for the final (*or initial*) velocities.**
- **2D collisions have 2 equations for conservation of momentum.**

$$\cancel{v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)}$$

<u>YOU</u>

should not use general purpose collisions solutions to solve homework or exam problems.

Write an equation for energy, if applicable.

? $K_f = K_o$? $\rightarrow$ $\frac{1}{2}M_1 v^2_{1f} + \frac{1}{2}M_2 v^2_{2f} = \frac{1}{2} M_1 v^2_{1o} + \frac{1}{2} M_2 v^2_{2o} \pm E_{lost\ or\ gained}$

- **Energy is conserved for elastic collisions.**
- **Energy is lost for inelastic collisions.**

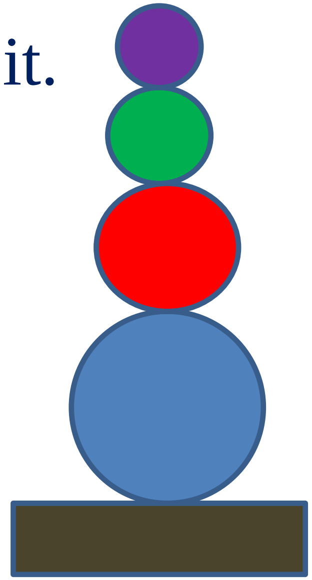
- **Check whether you have enough information to solve for the unknowns, 1 independent equation for each unknown.**
- **Solve the problem.**

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

BALL DROP

A set of falling balls of progressively decreasing mass collide elastically after falling a height *h*.

- Each has a speed $v = (2gh)^{1/2}$ just before the collision.
- The bottom ball has mass $M_1 >> M_2$
- M_1 collides with the floor & bounces back up before M_2 hits it.
- $v_1 = +v$ just after M_1 collides with the floor (*positive is up*)
- $v_2 = -v$ just before M_2 collides with M_1
- Plug this into the formulae above: $v_{1o} = +v, \quad v_{2o} = -v$



$$v_{2f} = 2v + v = 3v$$

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

BALL DROP

$$v_{2f} = 2v + v = 3v$$

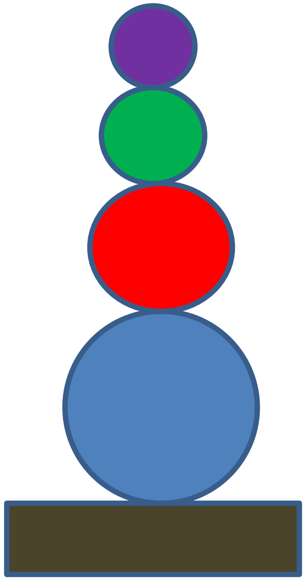
- Add a third even smaller ball on top of the second ball

$$v_{1o} = +3v \quad \& \; v_{2o} = -v$$

$$v_{2f} = 3v(2) + v = 7v$$

- A 4th ball will rebound at

$$v_{2f} = 7v(2) + v = 15v$$



- But $mgh = \frac{1}{2} mv^2$ so $15v \rightarrow 225\ h$

$\rightarrow$ the 4th ball bounces a LOT higher!

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

4-ball DROP demo - the actual masses in the demo are:

Ball #1 =66.3 grams, #2 = 29.5 grams, #3 = 9.8 grams, #4 = 3.7 grams

- If you approximate this as a doubling of mass for each successive ball, say they have mass 8*M*, 4*M*, 2*M*, & *M* respectively,
  then instead of 3*v* after the first collision we have, instead of 3*v*,

$$v_{2f} = v\left(\frac{2 \bullet 8M}{8M + 4M}\right) - v\left(\frac{4M - 8M}{8M + 4M}\right) = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}v$$

The second collision results in (5/3)(4/3)*v*+(1/3)*v* = (23/9) *v* = 2.56 *v*

The third collision results in (23/9)(4/3)+(1/3)*v* = 3.75 *v*

which will translate to the lightest ball bouncing 14 times its original height when dropped

because it's the square of the velocity that matters: $mgh = \frac{1}{2}mv^2$

LET'S TRY IT!

PHYS 121 – SPRING 2015

Chapter 11: Collisions

version 03/27/2015, ~ 100 slides

We made it to slide #57 on Wednesday, March 25.

Get your clickers ready.

5 options

ANNOUNCEMENTS

- Welcome high school visitors!
- Blackboard grade postings will be updated next week,
  after Exam #2 issues are settled.
- To make up for lost lectures (*2 school closings*), would you prefer:
  A. Schedule two extra lectures ~ Saturdays at 8 AM?
  B. Dr. C. records 2 MediaVision lectures with required viewing?
  C. Post lecture files with required reading?
  D. Post lecture files with optional reading (*no homework/exam problems*)?
  E. Forget about it? (*IT = details of gravitational motion?*)
- Some of the following clicker questions have appeared on exams.

4 options

$P = mv$ $K = \frac{1}{2} mv^2 = P^2/2m$

$W = F\Delta x$ $W = \Delta K$

A ping-pong ball and a bowling ball are rolling towards you. Both balls have the same momentum and you exert the same force to stop each.

How do the distances needed to stop them compare?

A. It obviously takes a shorter distance to stop a ping-pong ball.

B. Momentum & force the same $\Rightarrow$ takes the same distance.

C. It takes a longer distance to stop a fast ping-pong ball!

D. It's not possible to answer without more information.

4 options

A ping-pong ball and a bowling ball are rolling towards you. Both balls have the <u>same momentum</u> and you exert the <u>same force</u> to stop each.

How do the <u>distances</u> needed to stop them compare?

A. It obviously takes a shorter distance to stop a ping-pong ball.

B. Force & momentum the same $\Rightarrow$ takes the same distance.

C. It takes a longer distance to stop a fast ping-pong ball!

D. It's not possible to answer without more information.

- **Since the ping-pong ball has less mass, it must have a larger velocity in order to match the momentum of the bowling ball.**
- **The ping-pong ball must have MUCH more kinetic energy,**

  since $K = \frac{1}{2} mv^2 = P^2/2m$.
- **$P = mv$ is the same $\Rightarrow$ $K \sim 1/m$ is more for the ping-pong ball.**
- **$\Delta K = W \Rightarrow$ you need to do more work to stop the ping-pong ball!**
- **$W = F\Delta x$ but F is the same for both balls**

  $\Rightarrow \Delta x$ is larger for the ping-pong ball.

<u>BEWARE OF FAST MOVING PING-PONG BALLS!</u>

4 options

A mass m with speed v hits another mass M head-on in a 1D ***elastic*** collision. M was originally at rest.

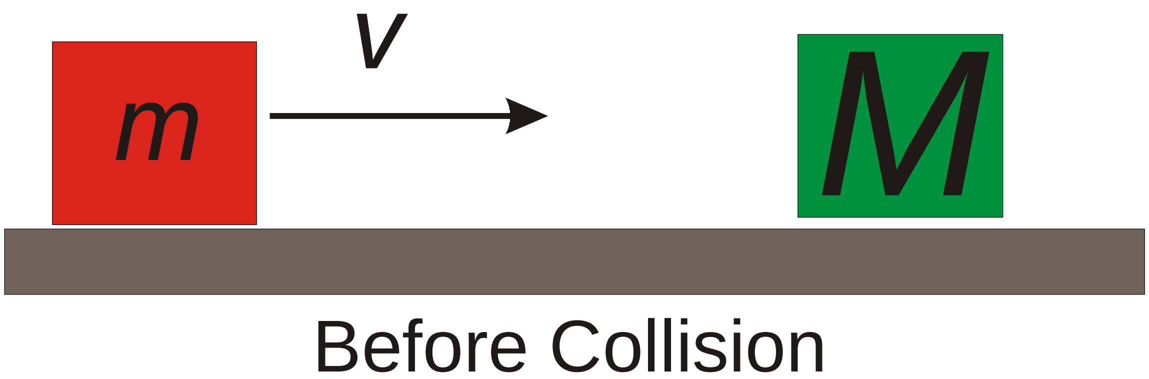
What should the size of M be relative to m in order that M ends up with the maximum possible KINETIC ENERGY for a given m & v?

A. $M << m$

B. $M = m$

C. $M = 2m$

D. $M >> m$



$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

4 options

A mass m with speed v hits another mass M head-on in a 1D elastic collision. M was originally at rest.

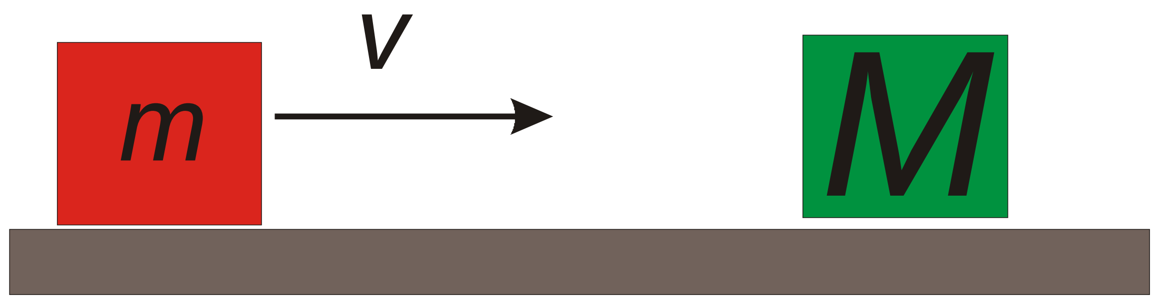
What should the size of M be relative to m in order that M ends up with the maximum possible <u>KINETIC ENERGY</u> for a given m & v?

A. $M << m$

B. $M = m$

C. $M = 2m$

D. $M >> m$



Before Collision

M can get all of m's original KE ($\frac{1}{2}mv^2$) if m stops after the collision.

You can't do better than that in an elastic collision.

This requires $M = m$.

- 4 options

A mass *m* with speed *v* hits another mass *M* head-on in a 1D elastic collision. *M* was originally at rest.

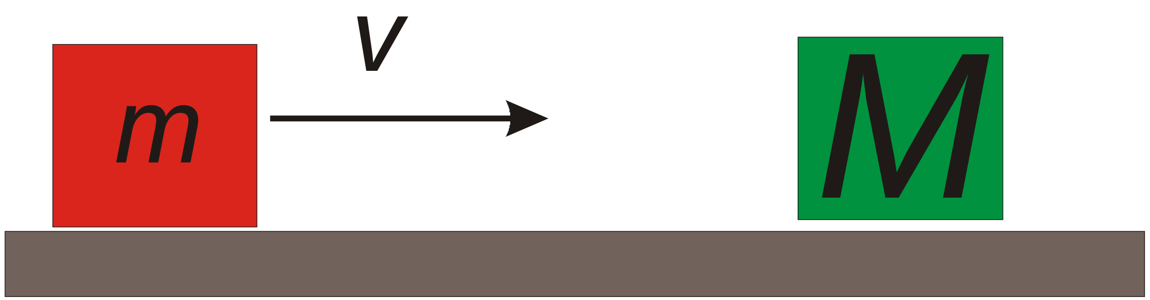
What should be the size of *M* relative to *m* in order that *M* gets the most <u>MOMENTUM</u> possible?

A. $M << m$

B. $M = m$

C. $M = 2m$

D. $M >> m$



Before Collision

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

- 4 options

A mass *m* with speed *v* hits another mass *M* head-on in a 1D elastic collision. *M* was originally at rest.

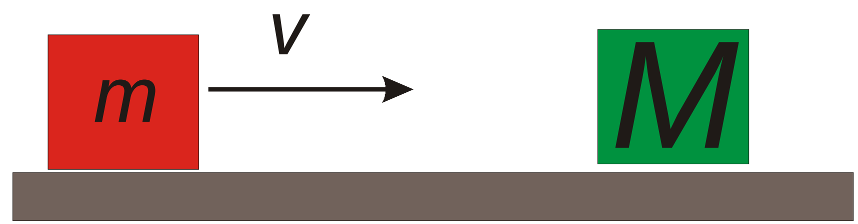
What should be the size of *M* relative to *m* in order that *M* gets the most MOMENTUM possible?

A. $M << m$

B. $M = m$

C. $M = 2m$

D. $M >> m$



Before Collision

- Momentum is a vector!
- The change in *M*'s momentum (*the impulse*) is equal (*but opposite*) to the impulse *m* receives.
- *m*'s impulse is biggest if it bounces straight back with velocity = $-v$ so that its impulse is $2mv$.
- This happens if *M* is relatively heavy.
- The final KE of *M* is small because $\frac{1}{2}MV^2 = \frac{1}{2}MV\,V = \frac{1}{2}(2mv)\,V = 0$ since V goes to 0 even though the momentum term $2mv$ is as large as possible, thanks to the large *M*.
- The large *M* compensates for the small *v* but can't compensate for v^2.

- 4 options

A mass *m* with speed *v* hits another mass *M* head-on in a 1D elastic collision. *M* was originally at rest.

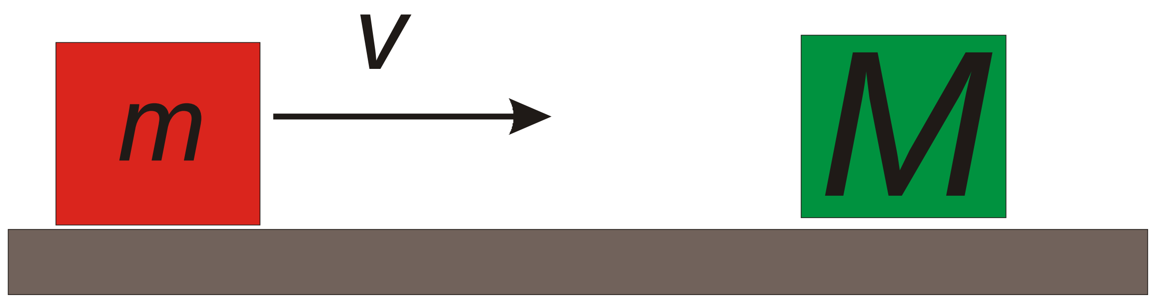
What should be the size of *M* relative to m in order that *M* gets the largest <u>VELOCITY</u> possible?

A. $M << m$

B. $M = m$

C. $M = 2m$

D. $M >> m$



Before Collision

$$v_{1f} = v_{1o}\left(\frac{M_1 - M_2}{M_1 + M_2}\right) + v_{2o}\left(\frac{2M_2}{M_1 + M_2}\right) \quad \& \quad v_{2f} = v_{1o}\left(\frac{2M_1}{M_1 + M_2}\right) + v_{2o}\left(\frac{M_2 - M_1}{M_1 + M_2}\right)$$

- 4 options

A mass m with speed *v* hits another mass *M* head-on in a 1D elastic collision. *M* was originally at rest.

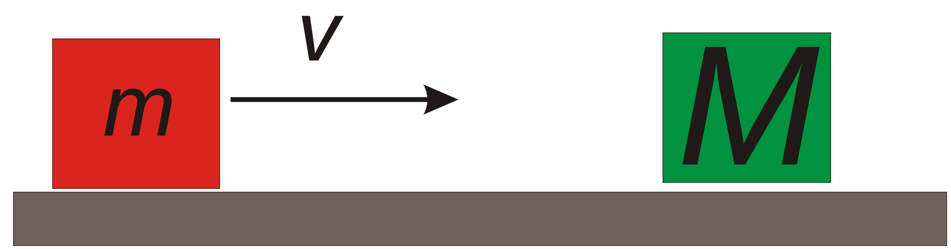
What should be the size of *M* relative to m in order that *M* gets the largest <u>VELOCITY</u> possible?

A. $M << m$

B. $M = m$

C. $M = 2m$

D. $M >> m$



Before Collision

As we decrease *M* from $M >> m$ (*where it has zero velocity after the collision*) to $M = m$ (*where it has speed v afterwards*) we end up with a speed $2v$ for $M << m$ as shown earlier in these notes.

Notice the trend!

- 5 options

A golf ball is fired at a bowling ball initially at rest and bounces back elastically.

Compared to the bowling ball, the golf ball after the collision has

A. more momentum but less kinetic energy.
B. more momentum and more kinetic energy.
C. less momentum and less kinetic energy.
D. less momentum but more kinetic energy.
E. none of the above

- 5 options

A golf ball is fired at a bowling ball initially at rest and bounces back elastically.

Compared to the bowling ball, the golf ball after the collision has

A. more momentum but less kinetic energy.

B. more momentum and more kinetic energy.

C. momentum and less kinetic energy.

D. less momentum but more kinetic energy.

E. none of the above

- The golf ball bounces back at nearly its incident speed, whereas the bowling ball hardly budges.
- Since the golf ball bounces back with nearly its original speed, it must also have nearly its original kinetic energy, losing very little to the bowling ball since the total kinetic energy is conserved.
- The golf ball ends up with momentum $-mv$ but the bowling ball must gain momentum $+2mv$ to conserve momentum.
- Also, $K = p^2/(2m)$. p is 4x bigger for the bowling ball but its mass is MUCH larger.

- 4 options

You are on a cart, initially at rest on a track with no friction.

You throw balls at a partition that is rigidly mounted on the cart.

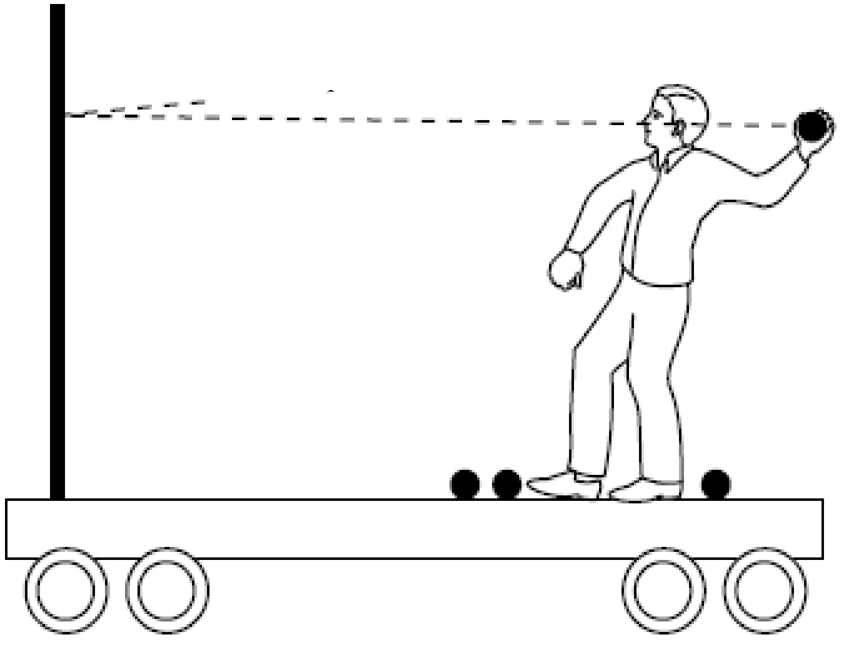
If you catch the balls as they bounce back, does the cart move?

A. Yes, it moves to the right.

B. Yes, it moves to the left.

C. No, it remains in place.

D. All of the above.



- 4 options

You are on a cart, initially at rest on a track with no friction.

You throw balls at a partition that is rigidly mounted on the cart.

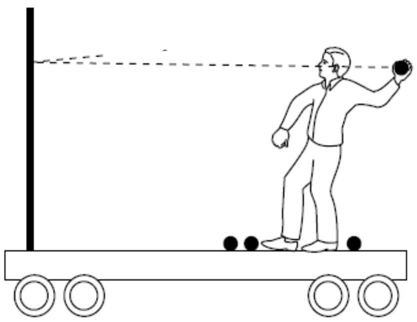
If you catch the balls as they bounce back, does the cart move?

A. Yes, it moves to the right.

B. Yes, it moves to the left.

C. No, it remains in place.

D. All of the above.



From conservation of momentum, the cart shifts

- *Backwards, to the right, when you throw the ball,*
- *then forward when a ball hits the partition and*
- *returns to rest when you catch* the ball.

The momentum change during the collision with the partition is twice that associated individually with the throw and the catch.

- 4 options

Suppose you are on a cart, initially at rest on a track with no friction. You throw balls at a partition that is rigidly mounted on the cart.

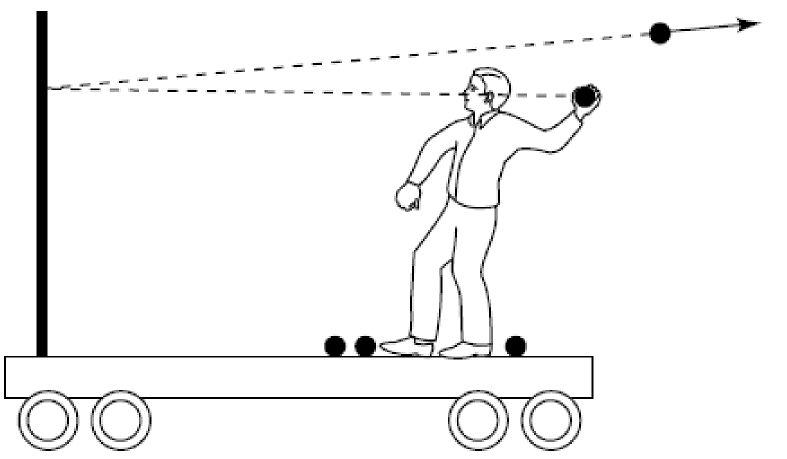
If the balls bounce straight back as shown in the figure but *you don't catch them*, is the cart in motion when you're done?

A. Yes, it's moving to the right.

B. Yes, it's moving to the left.

C. No, it's stopped.

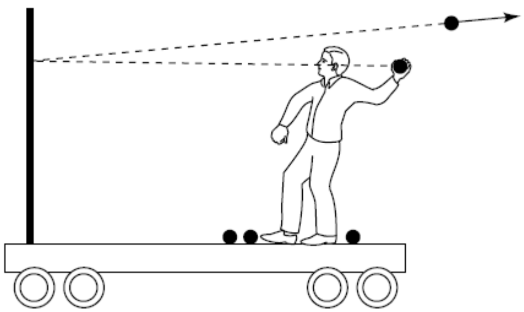
D. All of the above.



- 4 options

Suppose you are on a cart, initially at rest on a track with no friction. You throw balls at a partition that is rigidly mounted on the cart.

If the balls bounce straight back as shown in the figure but *you don't catch them*, is the cart in motion when you're done?



A. Yes, it moves to the right.

B. Yes, it moves to the left.

C. No, it remains in place.

D. All of the above.

- Because all the balls bounce back to the right (*eventually*), then to conserve momentum, the cart must move to the left.
- This is equivalent to throwing the balls directly out the back, like a rocket engine – which we will discuss shortly.

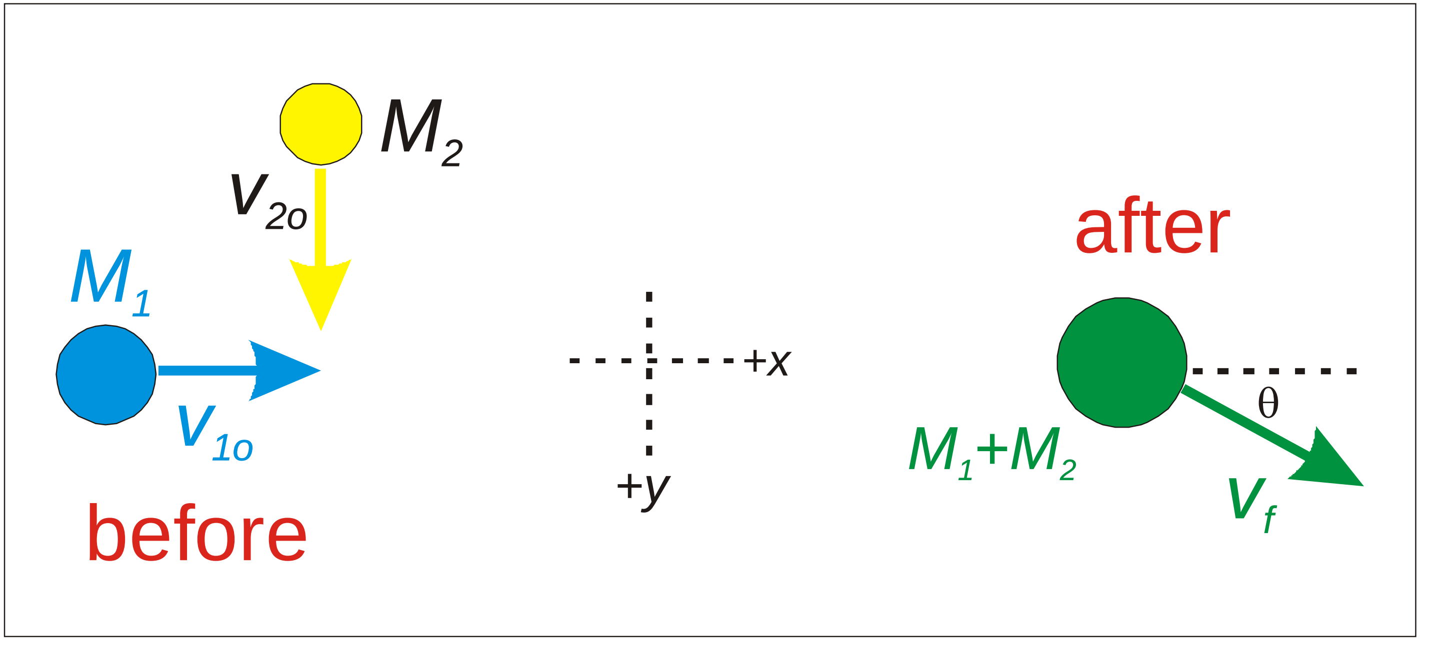
COMPLETELY INELASTIC COLLISIONS IN 2-D

FUSION & FISSION

Consider a 2D completely inelastic collision

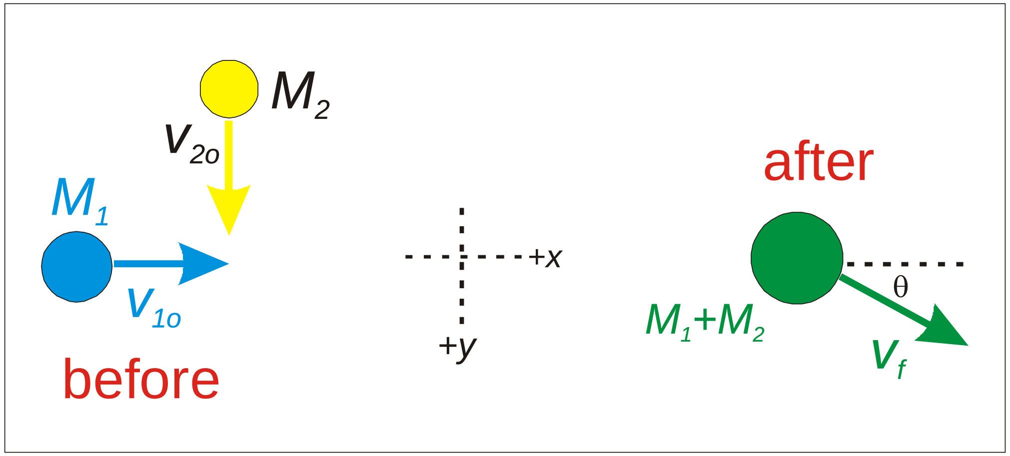
(*Two masses merge into one = fusion.*)

Assume, for simplicity, that the masses approach each other at right angles.



COMPLETELY INELASTIC COLLISIONS IN 2-D

FUSION & FISSION



The parameters/variables of interest are

$$M_1 \quad M_2 \quad \vec{v}_{1o} \quad \vec{v}_{2o} \quad \vec{v}_f$$

where the three velocities are vectors, each described by a magnitude & direction or by two vector components.

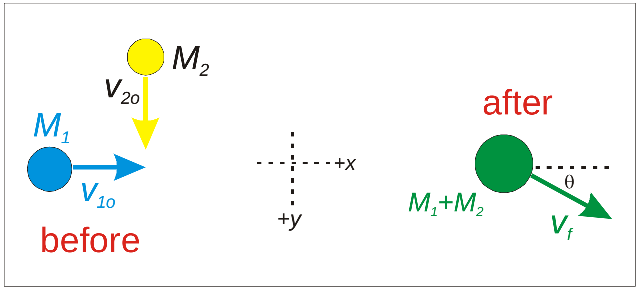
So there are **8** parameters/variables altogether.

although 2 of the directions are specified for this example.

$\Rightarrow$ Given $M_1, M_2, \vec{v}_{1o}$ & $\vec{v}_{2o}$ you can solve for $\vec{v}_f$ with 2 equations, which conservation of momentum gives you.

CONSERVATION OF MOMENTUM provides **2** relationships in 2D.

$$\vec{P}_{total} = \sum_i m_i \vec{v}_i \underset{initial}{=} M_1 \vec{v}_{1o} + M_2 \vec{v}_{2o} \underset{final}{=} \left(M_1 + M_2\right) \vec{v}_f$$



x-direction: $\mathrm{M}_1 v_{1o} = \left(M_1 + M_2\right) v_{xf}$

y-direction: $\mathrm{M}_2 v_{2o} = \left(M_1 + M_2\right) v_{yf}$

$$v_{xf} = \frac{\mathrm{M}_1}{\left(M_1 + M_2\right)} v_{1o} \quad \& \quad v_{yf} = \frac{\mathrm{M}_2}{\left(M_1 + M_2\right)} v_{2o}$$

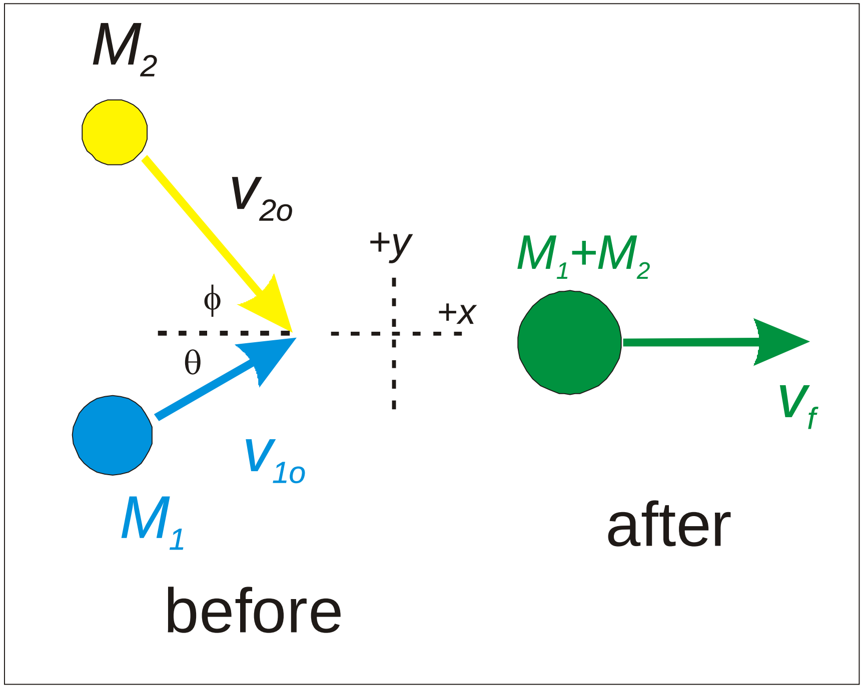
$$\left|\vec{v}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \frac{1}{\left(M_1 + M_2\right)} \sqrt{M_1^2 v_{1o}^2 + M_2^2 v_{2o}^2}$$

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{M_2 v_{2o}}{M_1 v_{10}}\right)$$

DONE!

2D COMPLETELY INELASTIC COLLISION WITH ARBITRARY ANGLES

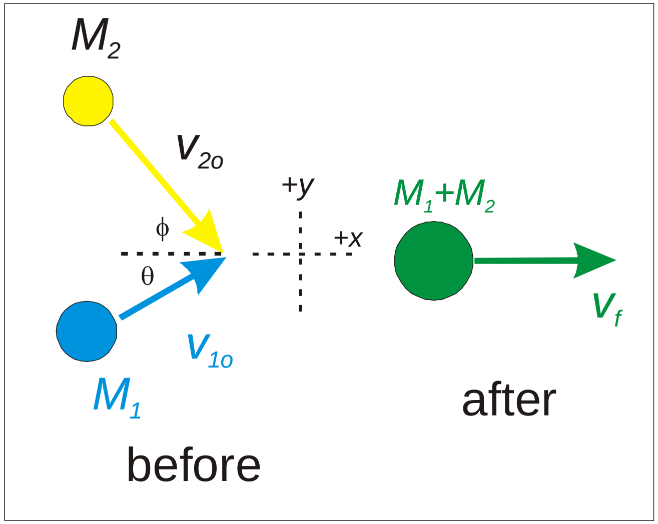
($\theta + \phi$ in the figure below)



CHOOSE WISELY

Align the *x*-axis with the outgoing combined mass.

aka the Center of Mass!



Conservation of momentum provides TWO equations.

x-direction: $\mathrm{M}_1 v_{1o}\cos\theta + \mathrm{M}_2 v_{2o}\cos\phi = (M_1 + M_2) v_{xf}$

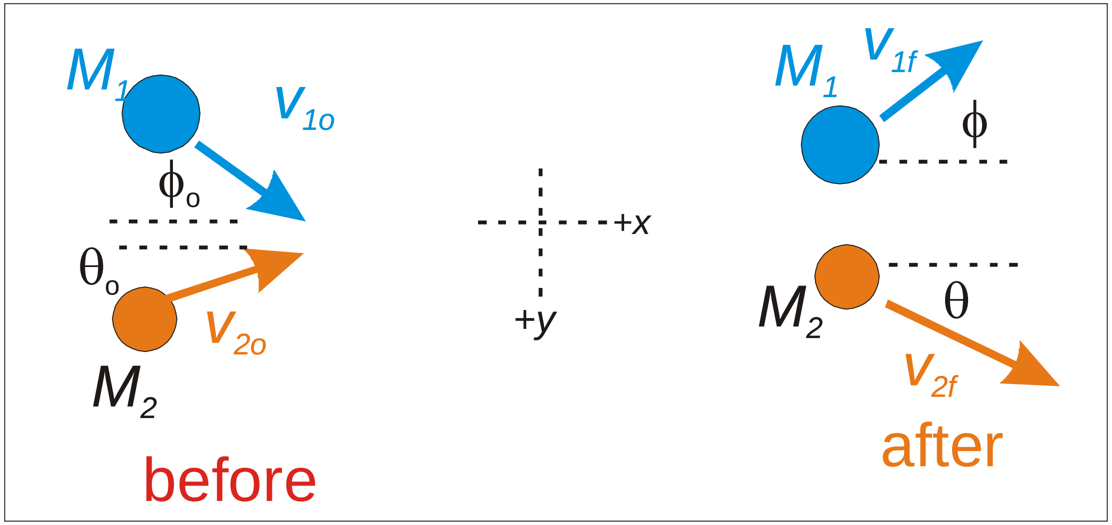
y-direction: $\mathrm{M}_1 v_{1o}\sin\theta - \mathrm{M}_2 v_{2o}\sin\phi = (M_1 + M_2) v_{yf} = 0$

$$v_{xf} = \frac{\mathrm{M}_1 v_{1o}\cos\theta + \mathrm{M}_2 v_{2o}\cos\phi}{(M_1 + M_2)} \quad \& \quad v_{yf} = 0$$

You can solve this problem without aligning the x-axis as shown.

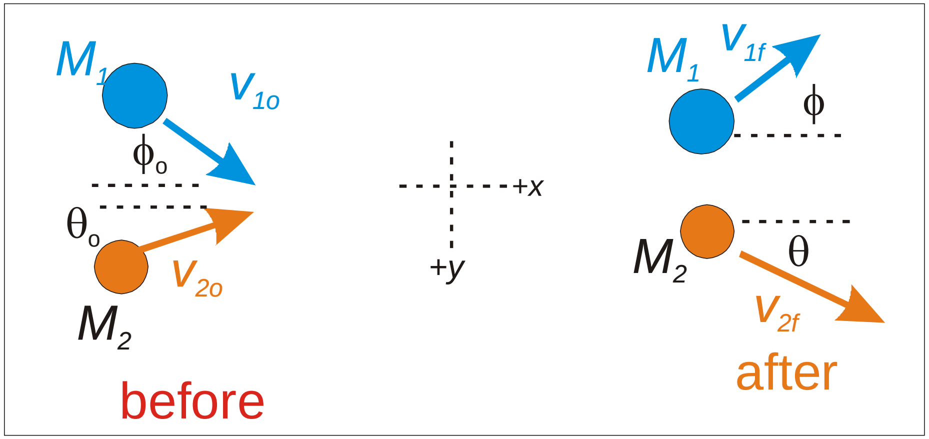
The physics is the same but the algebra is messier.

ELASTIC COLLISIONS IN 2D



- Momentum is conserved (*in both directions, independently*) = 2 eq.
- Kinetic energy is conserved = 1 equation (*because we said it is elastic*)
- 10 parameters: M_1, M_2, v_{1ox}, v_{1oy}, v_{2ox}, v_{2oy}, v_{1fx}, v_{1fy}, v_{2fx}, v_{2fy}
- Even if you are given the masses & both initial velocities, you are 1 equation short of being able to solve for the 4 final velocity terms.
- Let's see what you CAN say about the collision and then we'll discuss the missing information/equation.

ELASTIC COLLISIONS IN 2D



- <u>Momentum is conserved</u>

  x-direction: $M_1 v_{1ox} + M_2 v_{2ox} = M_1 v_{1fx} + M_2 v_{2fx}$

  y-direction: $M_1 v_{1oy} + M_2 v_{2oy} = M_1 v_{1fy} + M_2 v_{2fy}$

- <u>Kinetic energy is conserved</u>

  $\frac{1}{2} M_1(v^2_{1ox} + v^2_{1oy}) + \frac{1}{2} M_2(v^2_{2ox} + v^2_{2oy}) =$

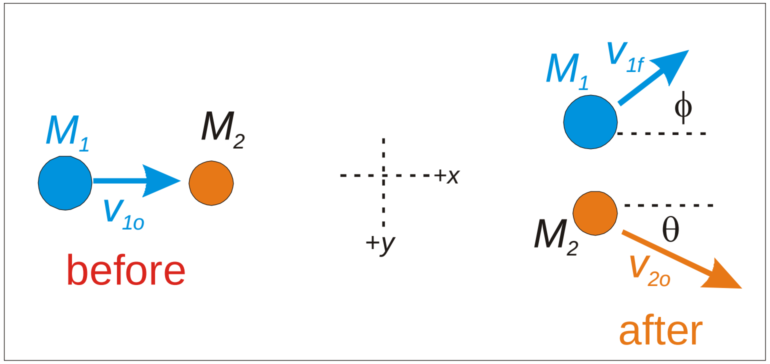
  $\frac{1}{2} M_1(v^2_{1fx} + v^2_{1fy}) + \frac{1}{2} M_2(v^2_{2fx} + v^2_{2fy})$

- Attempts to "solve" these equations "will be futile"

  although you can simplify them for specific parameter values.

- It will be more useful to simplify the problem itself.

ELASTIC COLLISIONS IN 2D



SIMPLIFY, SIMPLIFY Thoreau in *Walden*

http://en.wikiquote.org/wiki/Henry_David_Thoreau

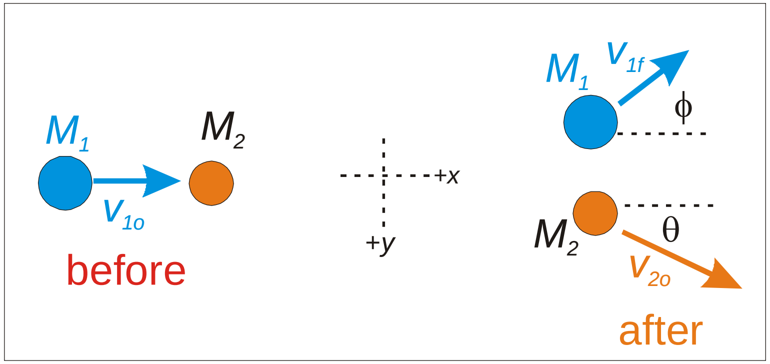
- Let $M_1 = M_2 = M \Rightarrow M$ can be divided out of our equations.
- Let $\mathbf{v}_{2o} = 0$; we can always transfer our results between the initial rest frame of M_2 and the lab frame by adding $\mathbf{v}_{2o}$.
- Align our *x*-axis with $\mathbf{v}_{10}$ - it's a free universe & we can align our *x*-axis any way we like; just don't choose poorly.
- Momentum is conserved *(M divides out of the equation.)*

  Eq. #1: $\vec{v}_{1o} = \vec{v}_{1f} + \vec{v}_{2f}$
- Kinetic energy is conserved: *(½M divides out of the equation.)*

  Eq. #2: $\vec{v}_{1o}^{\,2} = \vec{v}_{1f}^{\,2} + \vec{v}_{2f}^{\,2}$

ELASTIC COLLISIONS IN 2D

DO THE MATH



Eq. 1:

$$\vec{v}_{1o} = \vec{v}_{1f} + \vec{v}_{2f}$$

Eq. 2:

$$\vec{v}_{1o}^{\,2} = \vec{v}_{1f}^{\,2} + \vec{v}_{2f}^{\,2}$$

- Square Eq. #1

$$\vec{v}_{1o}^{\,2} = \left(\vec{v}_{1f} + \vec{v}_{2f}\right)\bullet\left(\vec{v}_{1f} + \vec{v}_{2f}\right)$$

$$\vec{v}_{1o}^{\,2} = \vec{v}_{1f}^{\,2} + 2\vec{v}_{1f}\bullet\vec{v}_{2f} + \vec{v}_{2f}^{\,2} \qquad \text{Eq. \#3}$$

- Compare Eq. #2 & Eq. 3 $\Rightarrow$

$$\vec{v}_{1f}\bullet\vec{v}_{2f} = v_{1f}v_{2f}\cos\left(\theta+\phi\right) = 0$$

$\Rightarrow$ the masses move off at right angles to each other;

$$\theta + \phi = 90°$$

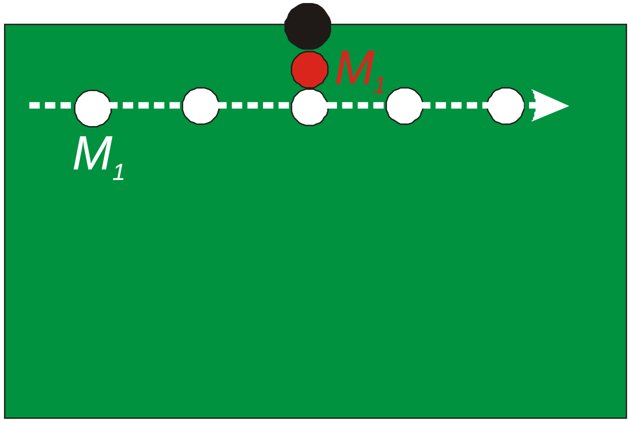
http://www.cuesportgroup.com/tournaments/page/2

http://games.brothersoft.com/backspin-billiards.html

POOL or BILLIARDS

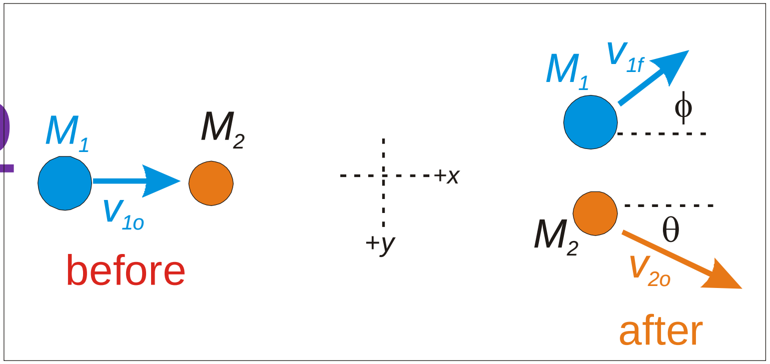
Billiard balls travel at right angles after a collision (*ignoring spin*)

Dr. C.'s favorite shot: If M_1 barely touches (“*kisses*”) M_2, M_1 will continue ~ straight ahead with almost all of its initial velocity while M_2 moves off slowly at a right angle.



ELASTIC COLLISIONS IN 2D

cultural interlude



$$\theta + \phi = 90°$$

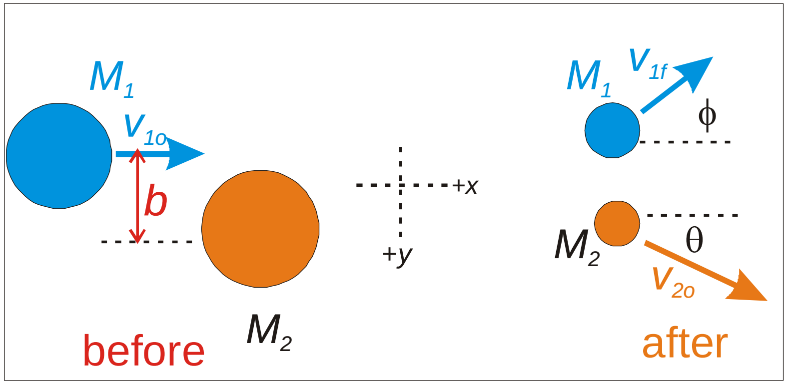
Is this all we can say about a 2D elastic collision? (*What is θ ?*)

YES, without more information.

- Perhaps an outgoing velocity or the "angular momentum".
- The angular momentum (*not introduced yet in PHYS 121*) can be described as the *impact parameter* or how far off-center the objects are when they collide. *See Ohanian *6.63.*
- For example, we WERE able to solve a head-on collision = our earlier 1D elastic collision where the first ball stops and the second moves off with $v_{2f} = v_{10}$ because the impact parameter and angular momentum = 0.

ELASTIC COLLISIONS IN 2D

cultural interlude



- The impact parameter = symbol b as shown in the figure above.
- The initial angular momentum of M_2 is $L = M_1 v_{1o} b$
- We could write out the final angular momentum in terms of M_1, M_2, $\mathbf{v}_{1f}$ & $\mathbf{v}_{2f}$ but not for another couple of weeks.
- The point is that this is the 4th equation we need in order to completely analyze an arbitrary 2D elastic collision.

And it's very annoying to carry out in practice!

And you don't need to know this for homework or exams in PHYS 121!

Ohanian 6.63 has not been assigned for homework.

ROCKET PROPULSION

(not covered in Ohanian but included in most intro textbooks)

Earlier we showed that

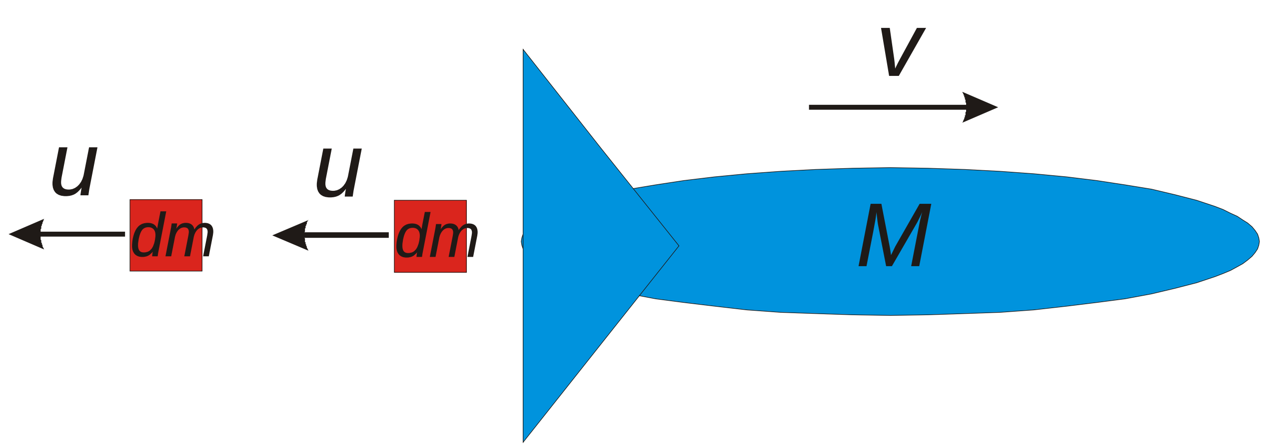
$$F = dp/dt = d(mv)/dt = m(dv/dt) + v(dm/dt)$$

and we assumed that m is constant so that

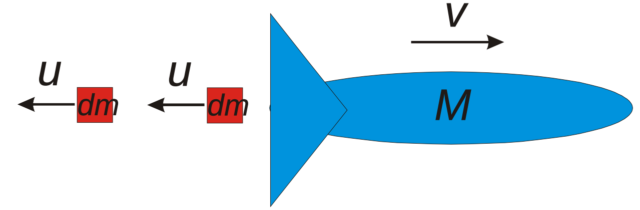
$$F = dp/dt = m(dv/dt) = ma$$

Now consider the case where m is not constant.

An object ejects bits of mass dm as it travels.



ROCKET PROPULSION



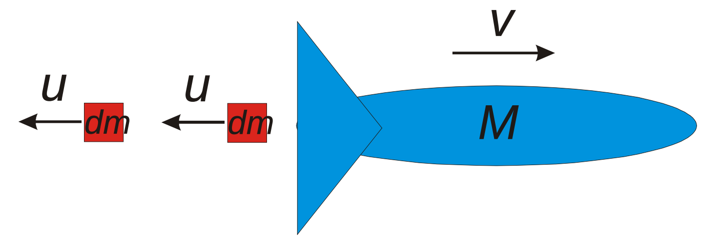
- $v(t)$ = forward velocity of the rocket at any given time
- u = speed of mass (*gas*) ejected from the tail
  - measured with respect to the rest frame of the rocket
  - u points in our negative direction.
- Speed of the gas in our lab frame is:

$$v_{gas\text{-}lab} = v_{gas\text{-}rocket} + v_{rocket\text{-}lab} = -u + v$$

- M = mass of the rocket at any given time
- dM is the change in the mass of the rocket **($dM < 0$)**

= mass of the gas ejected in a small interval of time.

ROCKET PROPULSION



Momentum Conservation $\rightarrow \; P_o = P_f$

$$Mv = (M + dM)(v + dv) - dM(v - u)$$

$$Mv = (Mv + Mdv + vdM + dMdv) - (vdM - udM)$$

$$\cancel{Mv} = \cancel{Mv} + Mdv + \cancel{vdM} + \cancel{dMdv} - \cancel{vdM} + udM$$

where $dMdv = 0$ because it's the product of TWO infinitesimal quantities $\Rightarrow$ infinitely smaller than other terms.

$$Mdv = -udM \qquad or \qquad dv = -u(dM/M)$$

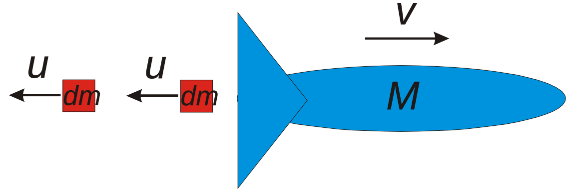
Integrating both sides

$$\int_o^f dv = -u\int_o^f \frac{dM}{M} \quad \rightarrow \quad v_f - v_o = -u\ln M_o^f = -u\left(\ln M_f - \ln M_0\right) = -u\ln\frac{M_f}{M_o} = +u\ln\frac{M_o}{M_f}$$

$$v - v_o = u\ln(M_o/M)$$

where v_o is the velocity when $M = M_o$

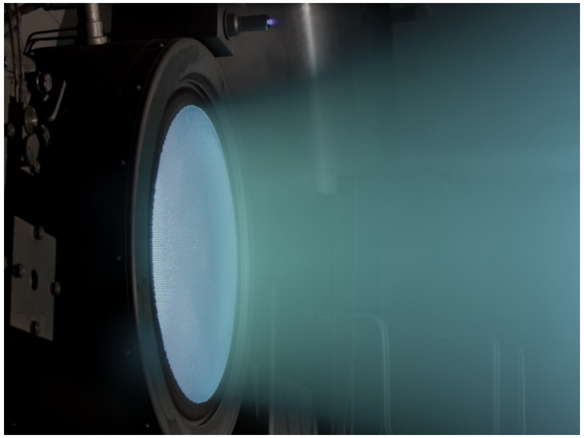
ROCKET PROPULSION



$$v = v_o + u \ln(M_o/M)$$

- Logarithmic behavior is a losing proposition – it takes forever to reach the final point.
- For a single stage chemical rocket, most of the payload may be fuel.
- NASA-Glenn is a world-leader in ion propulsion systems which have much bigger *u* terms.

http://www.nasa.gov/multimedia/imagegallery/image_feature_2416.html



4 options

A nucleus of mass M and speed V decays into two smaller nuclei of masses m_1 and m_2 with speeds v_1 and v_2 respectively . The angles with which the daughter nuclei emerge relative to the original parent direction are shown in the figure.

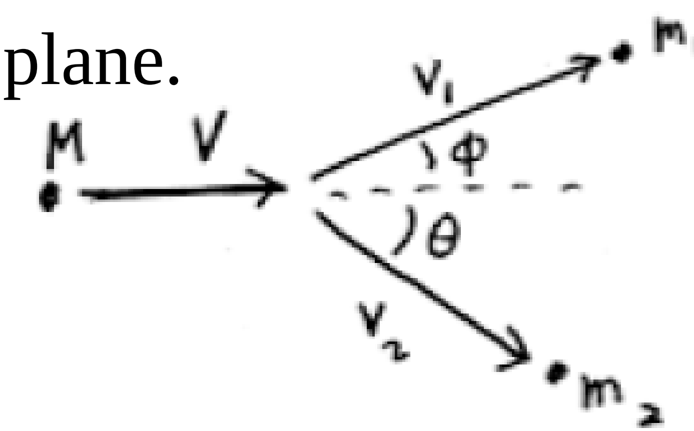
Which of the following statement is in general NOT true?

A. $MV = m_1v_1\cos\phi + m_2v_2\cos\theta$

B. $M_1v_1\sin\phi = m_2v_2\sin\theta$

C. $\frac{1}{2} MV^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$

D. The reaction always takes place in a plane.



4 options

A nucleus of mass M and speed V decays into two smaller nuclei of masses m_1 and m_2 with speeds v_1 and v_2 respectively . The angles with which the daughter nuclei emerge relative to the original parent direction are shown in the figure.

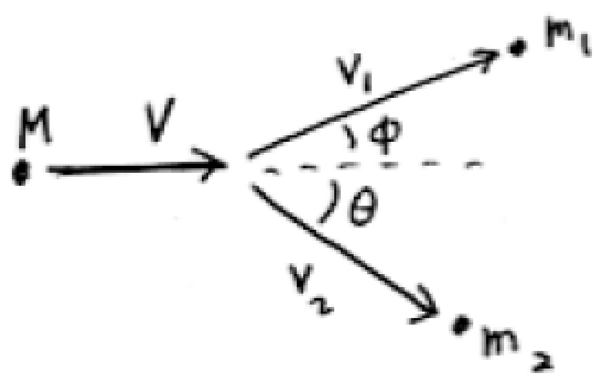
Which of the following statement is in general NOT true?

A. $MV = m_1 v_1 \cos\phi + m_2 v_2 \cos\theta$

B. $M_1 v_1 \sin\phi = m_2 v_2 \sin\theta$

C. $\frac{1}{2} MV^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

D. The reaction always takes place in a plane.



Fission is inelastic, kinetic energy is <u>NOT</u> conserved.

The other options are consequences of momentum conservation and are always true.

- 3 options

Suppose rain falls vertically and accumulates in an open cart rolling along a straight horizontal track with negligible friction. The <u>momentum</u> of the cart

A. increases.

B. does not change.

C. decreases.

- 3 options

Suppose rain falls vertically and accumulates in an open cart rolling along a straight horizontal track with negligible friction.

The momentum of the cart

A. increases.

B. does not change.

C. decreases.

The rain has no momentum in the horizontal direction & so does not change the momentum of the cart.

Note, however, that the **mass of the cart increases** as rain is added, so the **speed of the cart is reduced**!

The cart has to accelerate the rain up to its horizontal speed.

$M_f v_f = M_i v_i$ with $M_f > M_i$ & $v_f < v_i$ by the same factor.

$\Rightarrow$ There is a $F_{rain\text{-}cart}$ $\Rightarrow$ there is a $F_{cart\text{-}rain}$

- 3 options

Suppose rain falls vertically and accumulates in an open cart rolling along a straight horizontal track with negligible friction.

The kinetic energy of the cart

A. increases.

B. does not change.

C. decreases.

- 3 options

Suppose rain falls vertically and accumulates in an open cart rolling along a straight horizontal track with negligible friction.

The kinetic energy of the cart

A. increases.

B. does not change.

C. decreases.

$M_f v_f = M_i v_i$ with $M_f > M_i \Rightarrow v_f < v_i$ by the same factor.

$\frac{1}{2} M_f v_f^2 < \frac{1}{2} M_i v_i^2$ since $v_f < v_i$ while $\frac{1}{2} M_f v_f = \frac{1}{2} M_i v_i$

It's simpler to use $K = P^2/2M$. If P is constant but M increases, then K decreases.

- 3 options

Is it possible for a stationary object that is struck by a moving object to have a larger final momentum than the initial momentum of the incoming object?

A. Yes.
B. No, this would violate conservation of momentum.
C. It depends on the values of v_{10} and the masses.
D. I don’t know, plus I’m sick & tired of clicker questions.

- 3 options

Is it possible for a stationary object that is struck by a moving object to have a larger final momentum than the initial momentum of the incoming object?

A. Yes.

B. No, this would violate conservation of momentum.

C. It depends on the values of v_{10} and the masses

D. I don't know, plus I'm sick & tired of clicker questions.

If the moving object bounces back at all, then the stationary object gains momentum $M_{moving}(v_o + v_f)$ since Δp of the moving object is - $M_{moving}(v_o + v_f)$

http://healthy-lifestyle.most-effective-solution.com/wp-content/uploads/2011/10/the-end-is-here.jpg