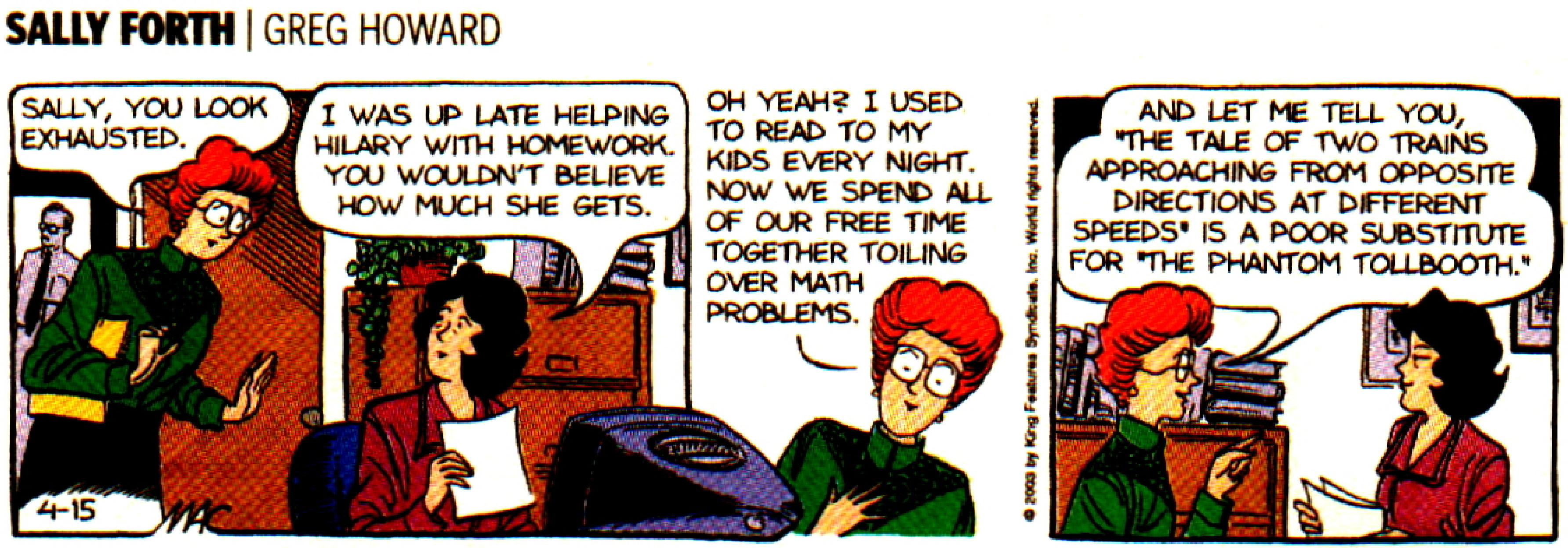
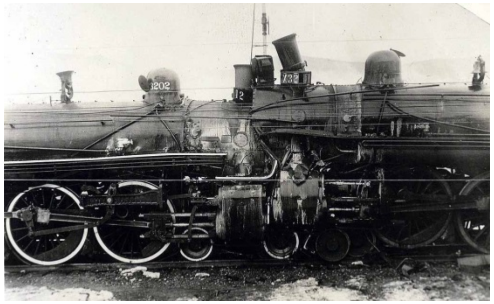
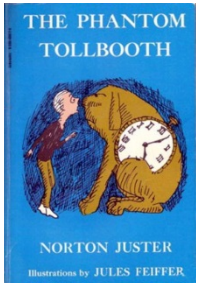
PHYS 121 – SPRING 2015







Chapter 2: Motion Along a Straight Line

version 01/14/2015, ~97 slides

http://justacarguy.blogspot.com/2011/08/train-crashes.html

Completed on Wednesday, January 21.

ANNOUNCEMENTS

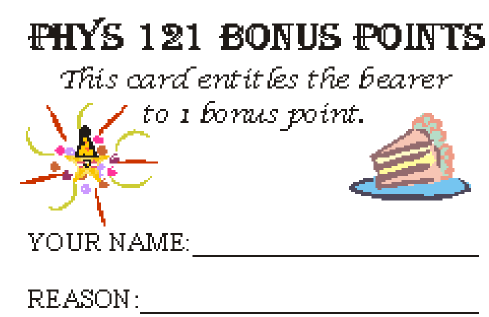
Fridays are Case Spirit Days!

??Anyone who comes to class this Friday wearing clothing with a

CWRU or Case logo

can pick up a bonus point coupon after class??

but the item must be visible to the general public.



http://case.bncollege.com/webapp/wcs/stores/servlet/CategoryDisplay?categoryId=40366&catalogId=10001&langId=-1&storeId=16053#&topCatId=40000

KINEMATICS

We need to define several quantities.

A big part of learning physics is learning the jargon; understanding the precise definitions assigned to various words.

KINEMATICS ≡ study of motion

without regard for the forces that cause that motion

Forces are included when we get to **DYNAMICS** in Chapter 5 of Ohanian.

TRANSLATIONAL MOTION

A ‘*normal*’ object like a person or a ball can travel:

- in 3 dimensions
- along curved paths
- while spinning on an axis
- or rolling on a surface
- and changing shape as it moves.

TRANSLATIONAL MOTION

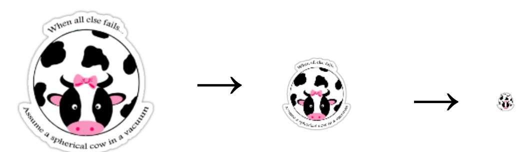
SIMPLIFY!

We'll temporarily focus on objects that can be represented by points in space.

$\Rightarrow$ no spinning

$\Rightarrow$ no rolling

$\Rightarrow$ no shape shifting



$\Rightarrow$ no internal structure

$\Rightarrow$ all objects have 0 dimensions ≡ (ideal) ***particles***.

Our first spherical cow of the semester is very small!

Ideal particles are fully characterized by their position in space at some given time and by their mass.

TRANSLATIONAL MOTION ≡

only the particle's position changes with time.

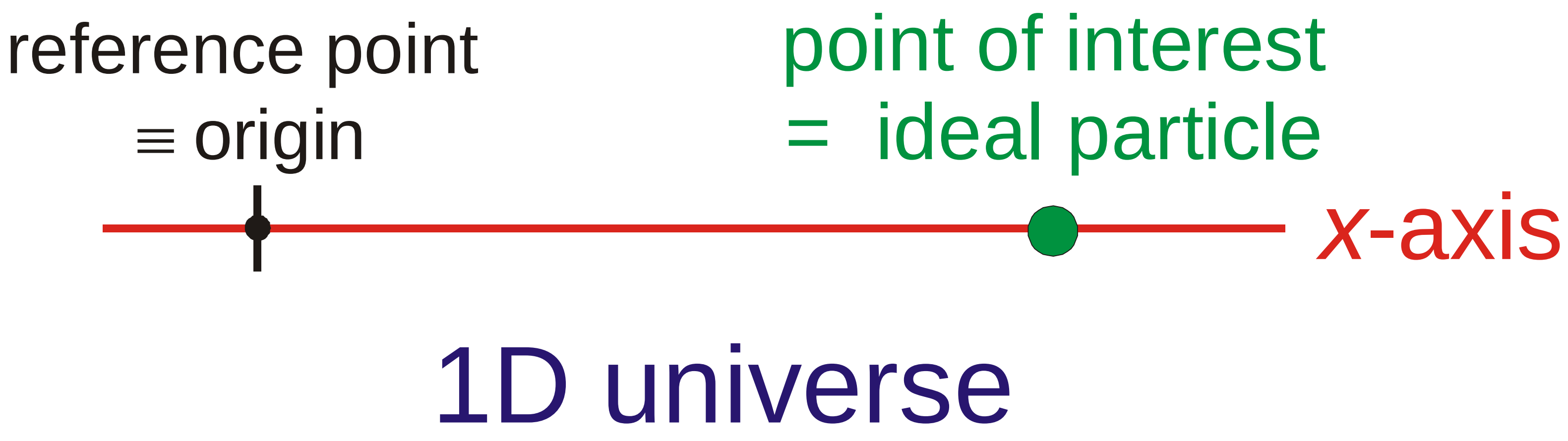
(they can't spin or rotate, yet)

SIMPLIFY SOME MORE!

Our particles can only travel in one dimension, 1D.

$\Rightarrow$ our universe is an infinitely long line.

- 1D $\Rightarrow$ it takes only 1 value to specify the **position** of a point with respect to some reference point $\equiv$ **the origin**.
- Horizontal systems are often labeled as an *x*-axis while vertical systems are often labeled as a *y*-axis (*but this is only a convention & not a requirement*).



DISPLACEMENT or POSITION

Displacement ≡ distance between two **positions** in space, a ***'final'*** position compared to an ***'initial'*** or ***'original'*** position or reference point.

Ohanian waits until section 3.1 to introduce displacement.

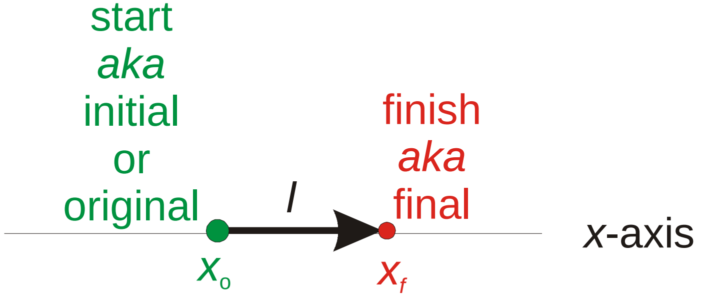
- It's the ***change in position*** that matters, not the positions themselves.

  The displacement in the figure = the distance between the start and finish.

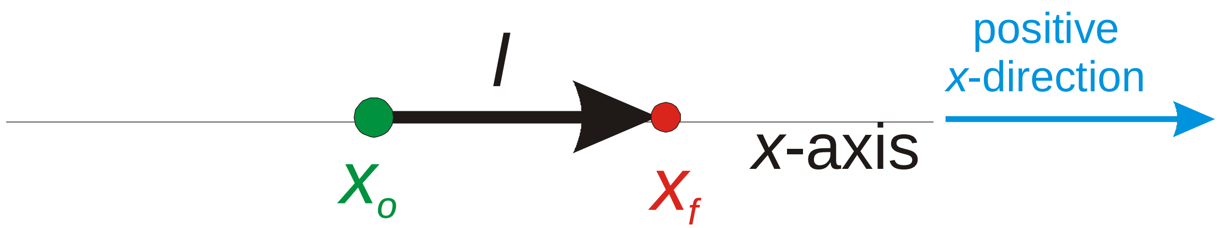
- This displacement, including the direction of travel, can be represented by the length, l (*in meters*), of the arrow in the figure.

$$l = \Delta x = x_{\text{final}} - x_{\text{initial or original}}$$

Δ = Greek symbol for capital *delta* ≡ *"change in"*

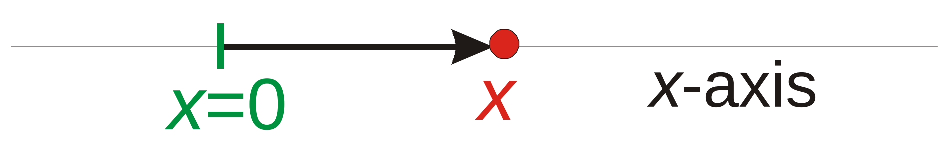


DISPLACEMENT or POSITION



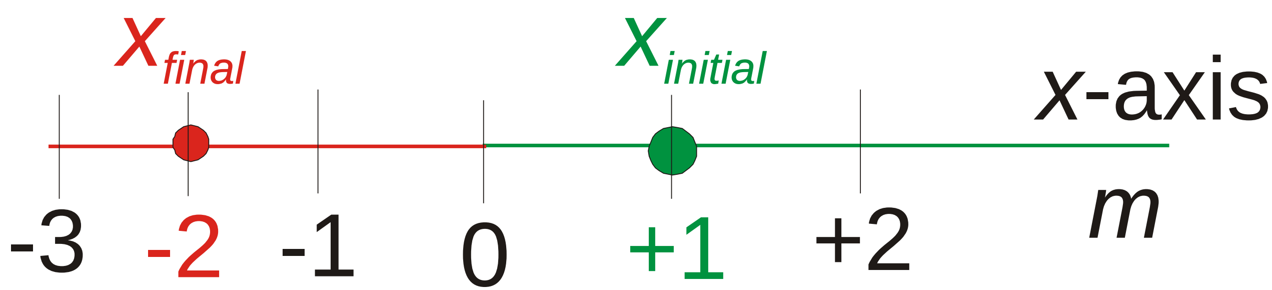
- The displacement is positive above, assuming we adopt the convention that the positive direction is towards the right.

- The *position* of a point is its displacement from the *origin* we've defined.

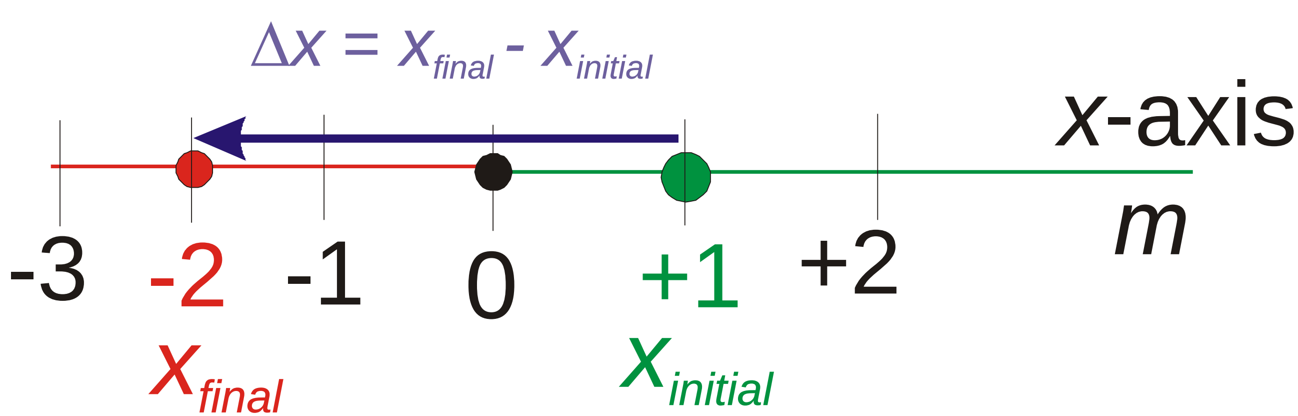


DISPLACEMENT

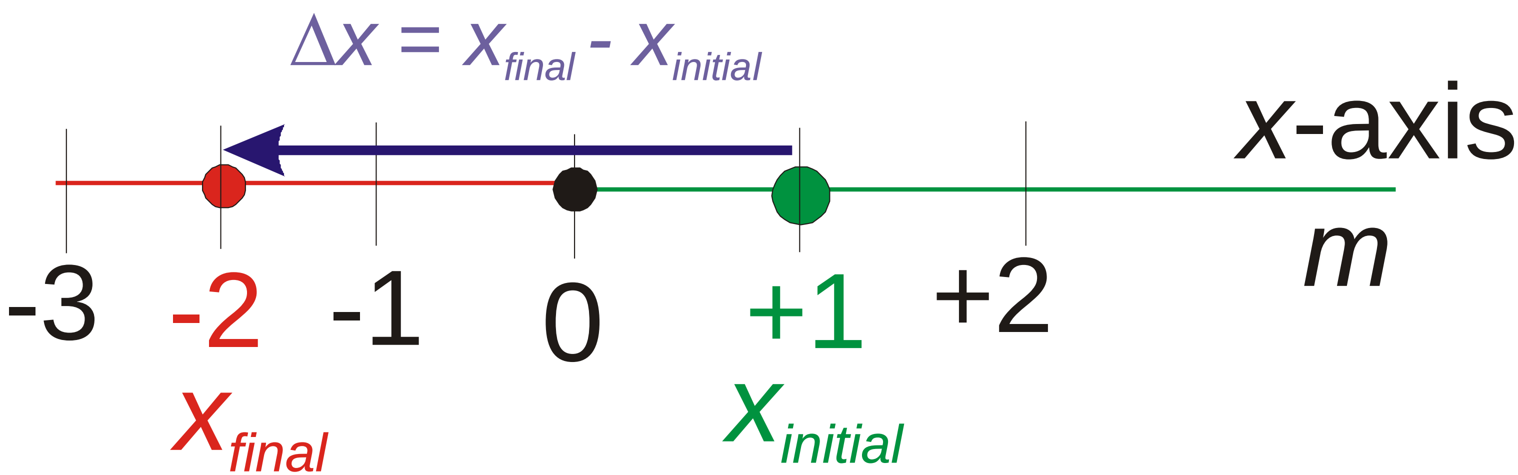
- Positions and displacements can be negative or positive.
- Position x_{final} = -2 m



- Displacement $\equiv \Delta x = x_{final} - x_{initial} = [(-2) - (+1)] = -3$ m



DISPLACEMENT



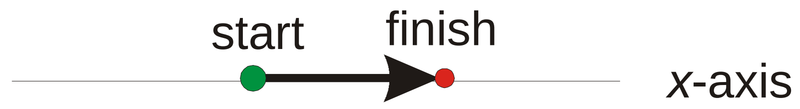
Ohanian uses x_2 and x_1 instead of x_{final} and $x_{initial}$.

but it's easier to keep track of *final* and *initial* than *1* & *2*.

I might use: x_i (for *initial*) *or* x_o (for *original*) *or* x_1

x_f (for *final*) *or* x_2

ORIGINS



- Displacement is independent of the origin.

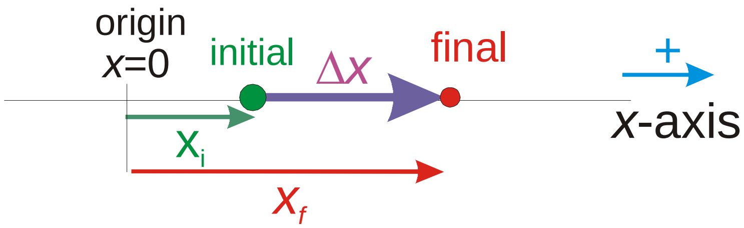
  Δx is the same if the origin is at your nose or in a galaxy far, far away.

- *Einstein's Theory of Relativity* tells us that there is no special, correct place for an origin.

  You can place the origin any place you like to make your calculation as easy as possible.

CHOOSE WISELY!

DISPLACEMENT RELATIVE TO AN ORIGIN



- In the figure above, it might have been wise to place the origin where the object starts – but this is not necessary.
- **The important thing is to tell your audience what your choices are, particularly if that audience is your grader!**

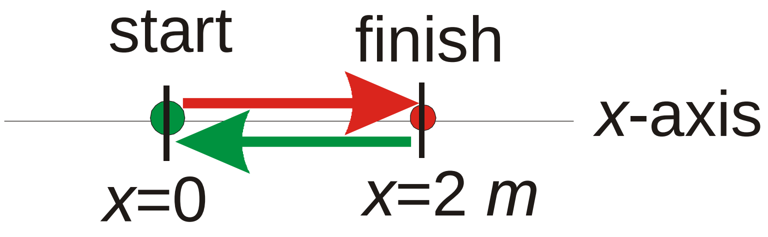
A figure, with labels, helps!

DISPLACEMENT *vs* DISTANCE

- *Displacement* is the **net change** in position from the start to the end of the motion.
- *Distance traveled* = how far the object travels during its motion.
- Displacement need not equal distance.

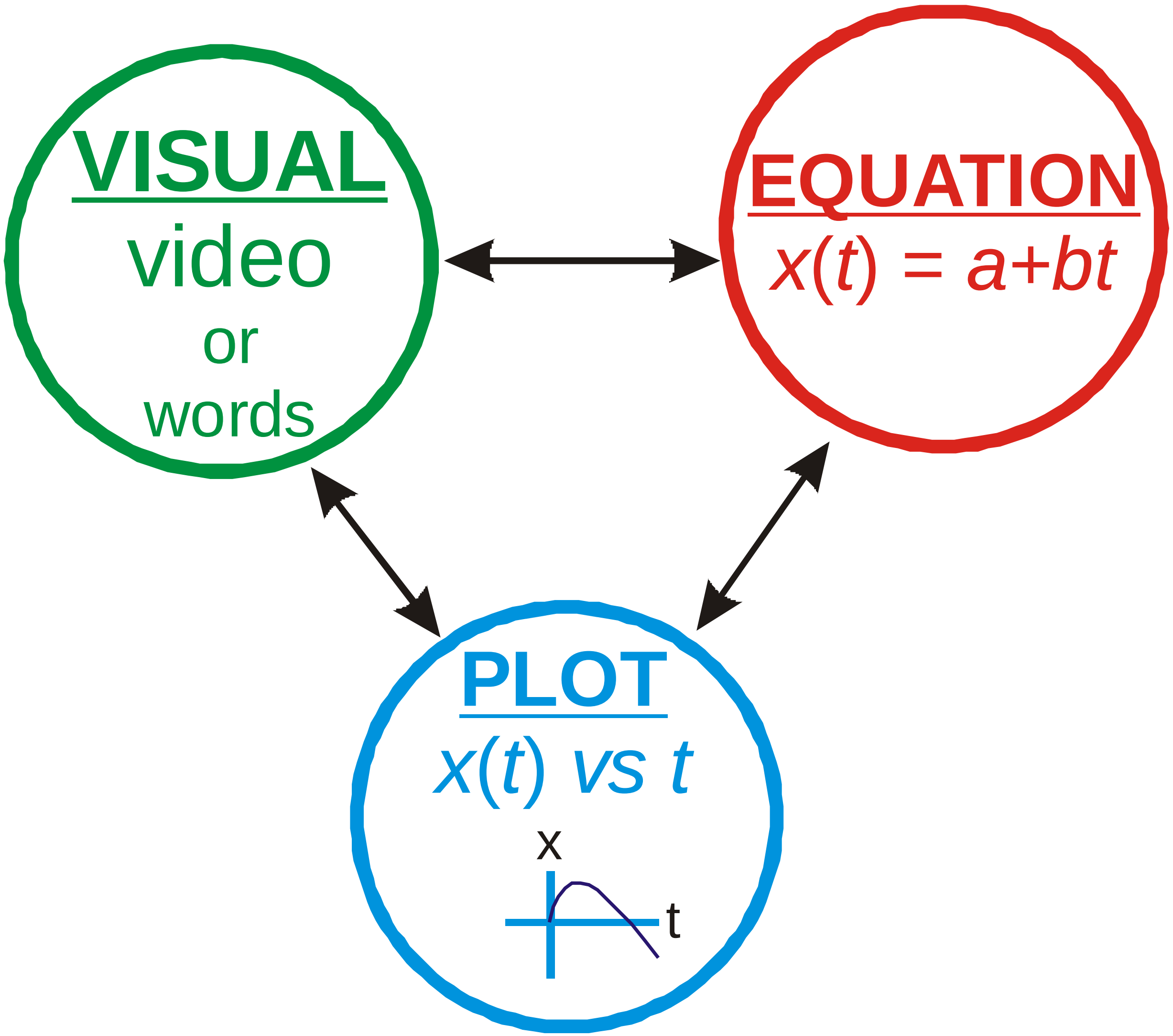
For example, if an object moves 2 *m* to the right and then 2 *m* to the left,

- Displacement $\Delta x = x_{final} - x_{initial} = 0\ m$
- Distance traveled = 4 *m*

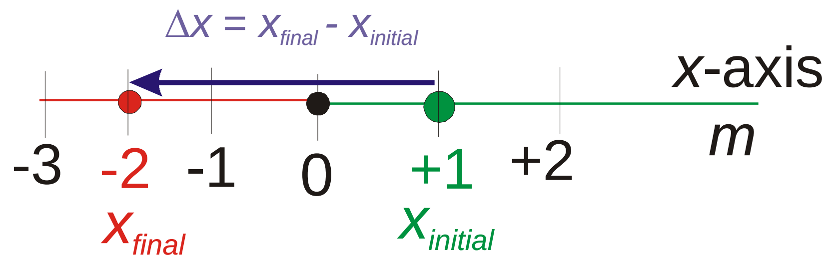


DESCRIBING MOTION

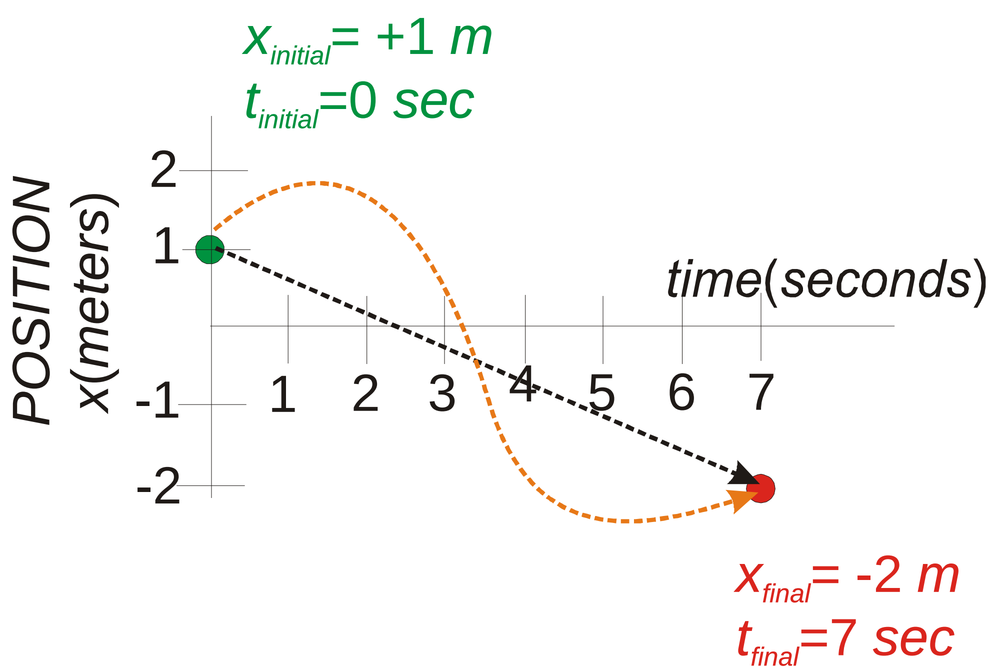
PHYS 121 students must be able to describe motion using each method illustrated below - and translate between them.



GRAPHS/ PLOTS

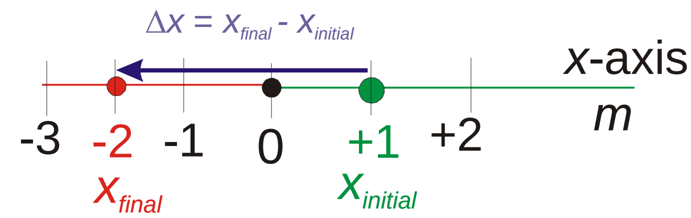


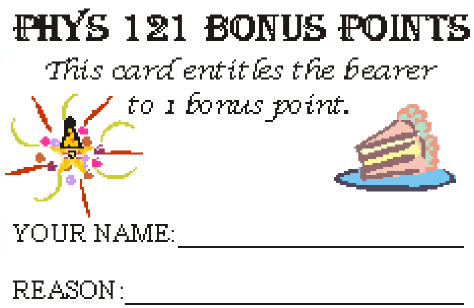
- The figure above is an incomplete description of *motion*; it doesn't include *time*.
- A plot of *x(t) vs. t* shows *position* (*vertical axis*) as a function of *time* (*horizontal axis*) assuming the motion takes place over 7 seconds.
- Depending on <u>how</u> the particle in the figure above moves, the displacement might be plotted as shown below.



DEMO

2 volunteers needed





PHYS 121 BONUS POINTS

This card entitles the bearer to 1 bonus point.

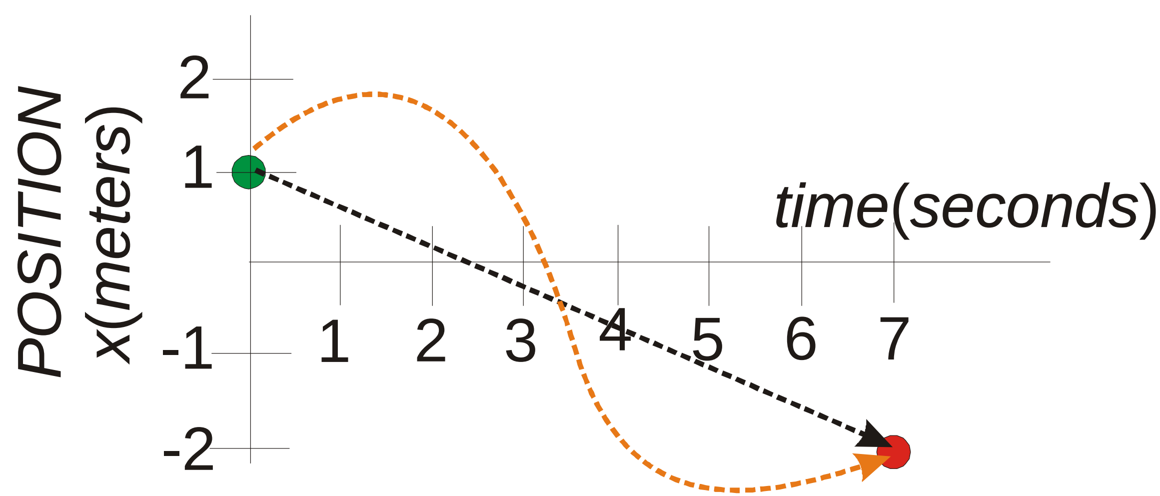
YOUR NAME:____________________

REASON:____________________

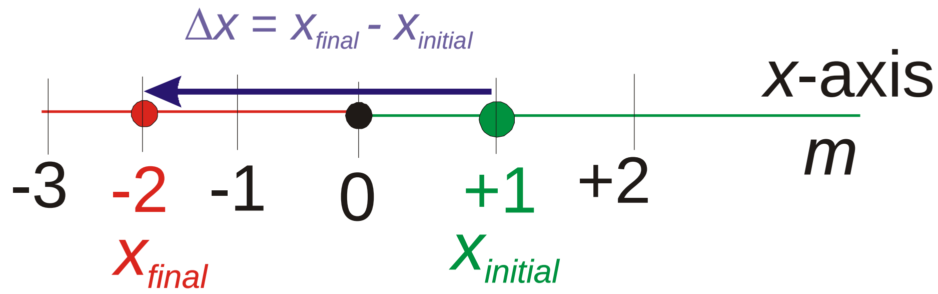
Duplicate the two types of motion in the plot of x(t).

$x = 0 \equiv$ Dr. C.

$+x \equiv$ towards audience's right



EQUATIONS



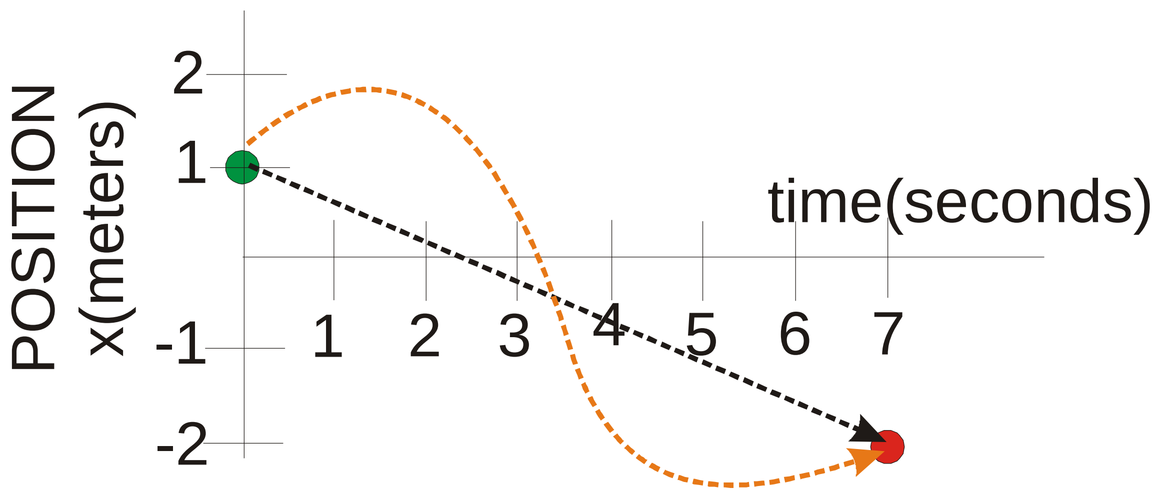
- An equation $x(t)$ describes x *as a function of time.*
- The equation for the 'direct' path in the plot is a straight line.

$$x(t) = b + mt : \; b = \text{intercept} \quad \& \quad m = \text{slope}$$

$$x(t) = 1 - (3/7)t \;\; \textit{with } x \textit{ in } m, \, t \textit{ in } s.$$

$$1 = \text{position at t} = 0$$

$$-3/7 = \text{slope in } m/s$$



VELOCITY & SPEED

The terms *velocity* and *speed* have special, distinct meanings in physics.

- *Speed* is the rate at which position changes with time

$$s = \left| \frac{dx}{dt} \right|$$

- *Velocity* tells you not just the *speed* at which an object is moving, but also the *direction* in which it is traveling.
  - In a 1D horizontal universe, the velocity can be positive or negative, for motion right or left.
  - Speed is never negative and doesn't have a direction.

$$s = |v|$$

- This distinction is critical when we expand our studies to 2D & 3D
  - but even in 1D we're frequently more interested in the velocity of an object, since it might matter whether it's moving to the left or right.

FRAMES of REFERENCE

Frame of Reference ≡ choice of axes & origin for some *observer*.

Everyone is entitled to their own personal Frame of Reference , FoR.

unless Dr. C. or the text makes the choice for you.

Often there are WISE choices & POOR choices

$\Rightarrow$ give your choice some THOUGHT.

Consider 2 points on the chalkboard from your point of view & from mine.

Positions of each point are different for each observer's personal FoR,

with your origin at ~ your nose.

Displacements between the two points are the same for everyone.

Velocities & speed can vary in different Frames of Reference.

What is the velocity of those points?

As you run out the side door of Strosacker when class ends?

We'll handle kinematics with *relative motion* in chapter 4, after introducing vectors plus 2D & 3D universes.

CATEGORIES OF SPEED

Speed & velocity can be categorized as either *instantaneous* or *average*.

- The *average speed* at which an object moves is given by

$$s_{average} = \overline{s} = \frac{\text{overall distance traveled}}{\text{time required}}$$

A flat bar over some quantity like *s* in the equation above indicates that we're considering its average over time.

- The *average velocity* over some time interval Δt

  = ***net*** change in position Δx divided by the time interval Δt over which this change took place.

$$v_{average} = \overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

where x_f is the final position of the object at time t_f
while x_i is the initial position of the object at time t_i.

~~AVERAGE VELOCITY~~

The *average velocity* is **NOT**

$$v_{average} = \bar{v} \neq \frac{v_f + v_i}{2} \neq \frac{v_f - v_i}{t_f - t_i}$$

$$v_{average} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

AVERAGE SPEED

The average speed can be greater that the average velocity if an object changes direction one or more times during its travels.

For example, if you flew at 600 mph to Hawaii and immediately returned to Cleveland at 600 mph,

your average speed will be 600 mph

but your average velocity would be 0

since $\Delta x = 0$.

(*Personally, this time of year, I'd stay in Hawaii.*)

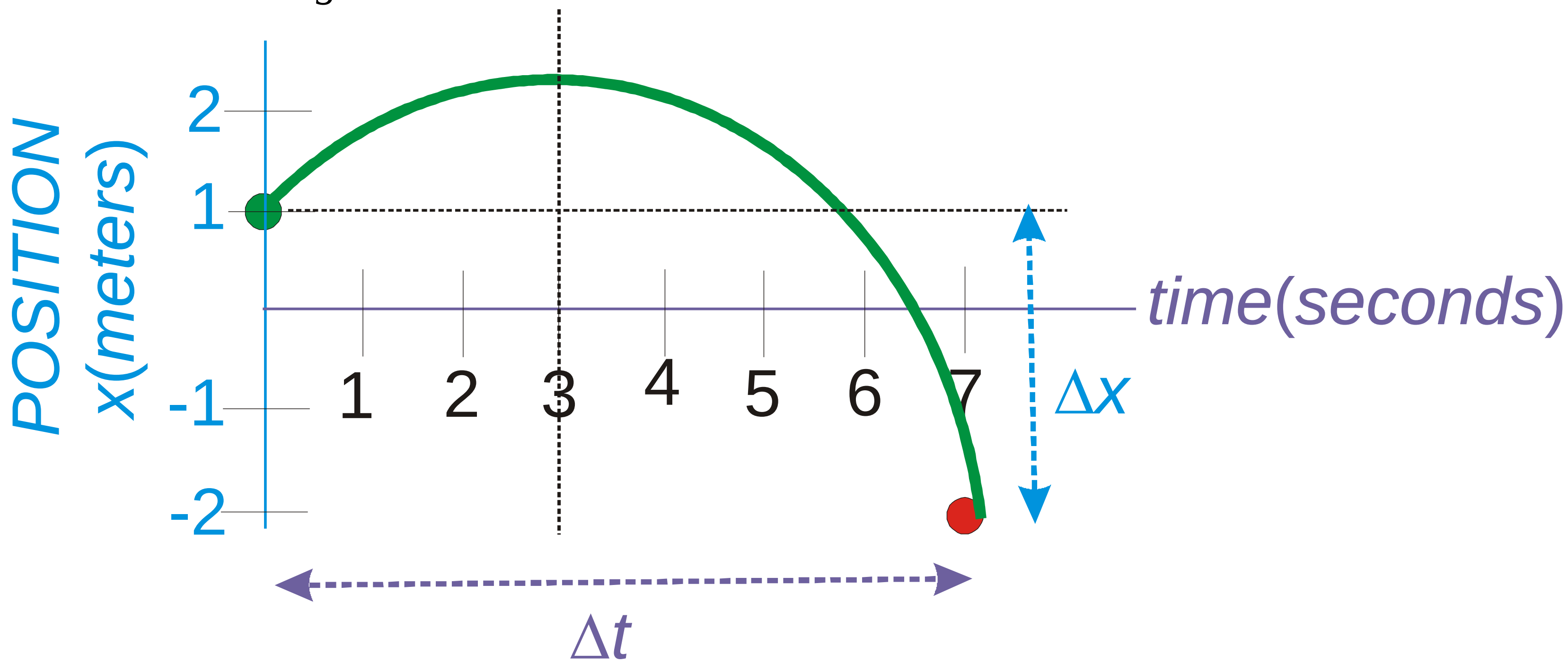
http://sbhawaii.com/images/Kaanapali_beach_Maui.jpg

AVERAGE VELOCITY in a GRAPH

In a plot of $x(t)$, $v_{average}$ between two points is Δx divided by Δt.

Focus on the initial & final positions & times.

$$v_{avg} = \Delta x/\Delta t = (-2 - 1)/7 = -3/7\ m/s$$

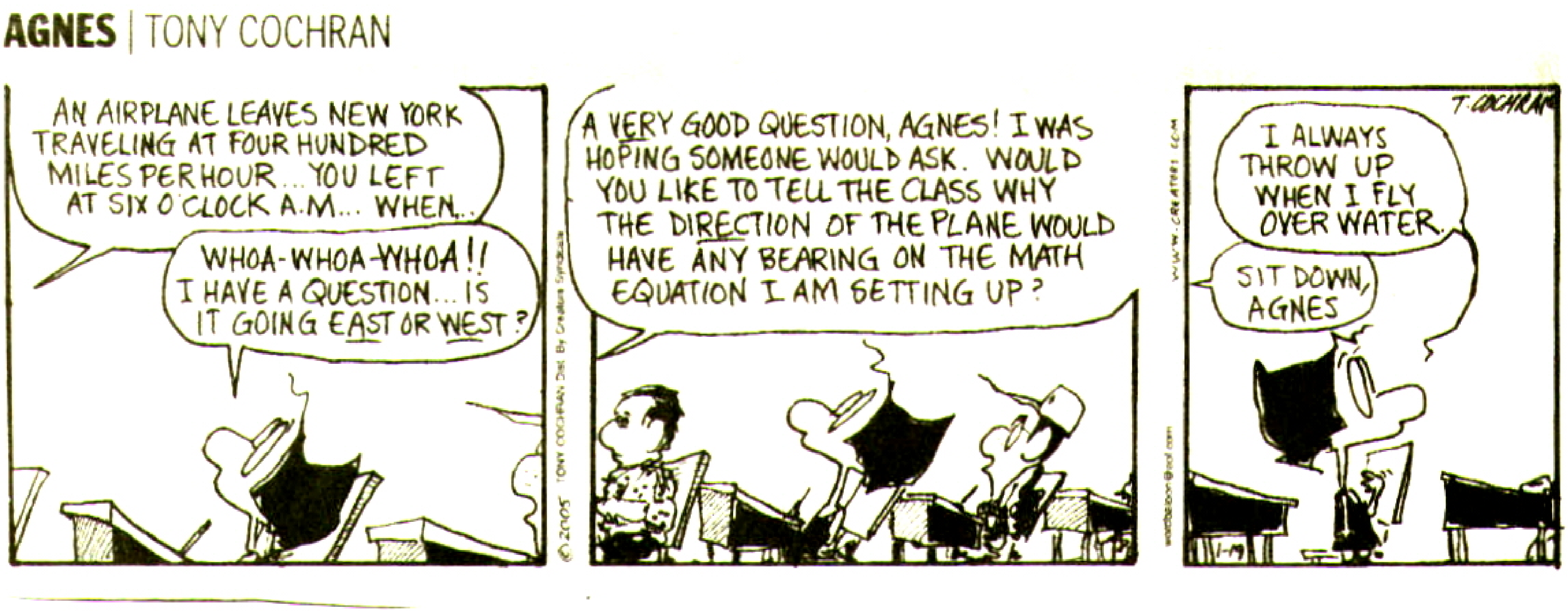


$$s_{avg} = \text{overall distance}/\Delta t$$

$\Rightarrow$ consider '*Turning Point*" at $t = 3\ sec$

$$s_{avg} = (1\ m + 4\ m)/7 = 5/7\ m/s$$

PHYS 121 – SPRING 2015



Chapter 2: Motion Along a Straight Line, *continued*

version 01/16/2015, ~97 slides

We made it to slide #24 on Wednesday, January 14.

Get your clickers/apps ready.

ANNOUNCEMENTS

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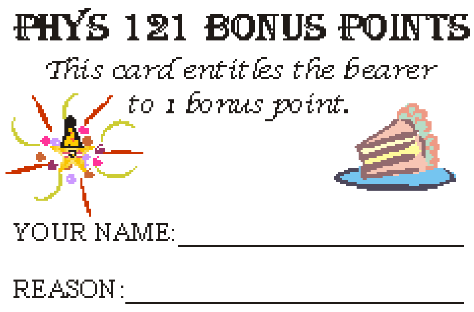
can pick up a bonus point coupon after class

but the item must be visible to the general public.

Remember to fill out & return your bonus point coupons.



http://case.bncollege.com/webapp/wcs/stores/servlet/CategoryDisplay?categoryId=40366&catalogId=10001&langId=-1&storeId=16053#&topCatId=40000

INSTANTANEOUS SPEED

The *instantaneous* velocity (*or speed*) is the velocity (*or speed*) at some given *moment* of time.

"moment" = as fast as you can measure, in the limit as $\Delta t \rightarrow 0$

- *Instantaneous* is normally more interesting than *average v*.

  If you see *v* without a subscript, you should assume it stands for an instantaneous velocity.
- Calculus provides an easy way to express this concept.

$$v_{\text{instantaneous}} = v = \lim_{\Delta t \rightarrow 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

$$v_{\text{instantaneous}} = v = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt} \qquad v(t) = \frac{dx(t)}{dt}$$

The (*instantaneous*) velocity (*as a function of time*) is the time derivative of the position (*as a function of time*).

We'll often leave out the (t).

For example, if x as a function of time is given by

$$x = a + bt + ct^2$$

where a, b & c are constants, then

$$v_{\text{instantanteous}} = v = b + 2ct$$

Note that a is the position at time $t = 0$, $a = x_o$

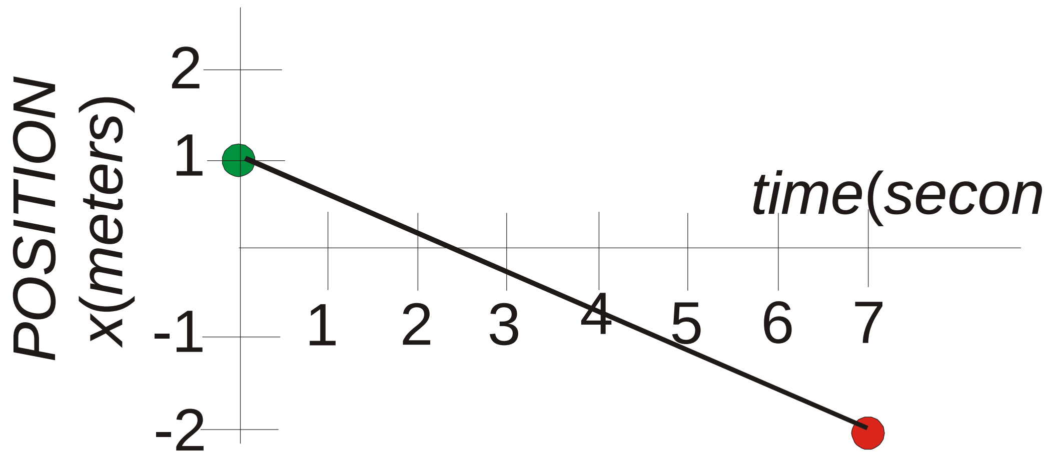
b is the velocity at $t = 0$, $b = v_o$

$\rightarrow$ $\mathbf{x = x_o + v_o t + ct^2}$ *in this example*

$$v_{\text{instantaneous}} = v = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

d*x*/d*t* at any given time is the **slope** of *x*(*t*) at that time

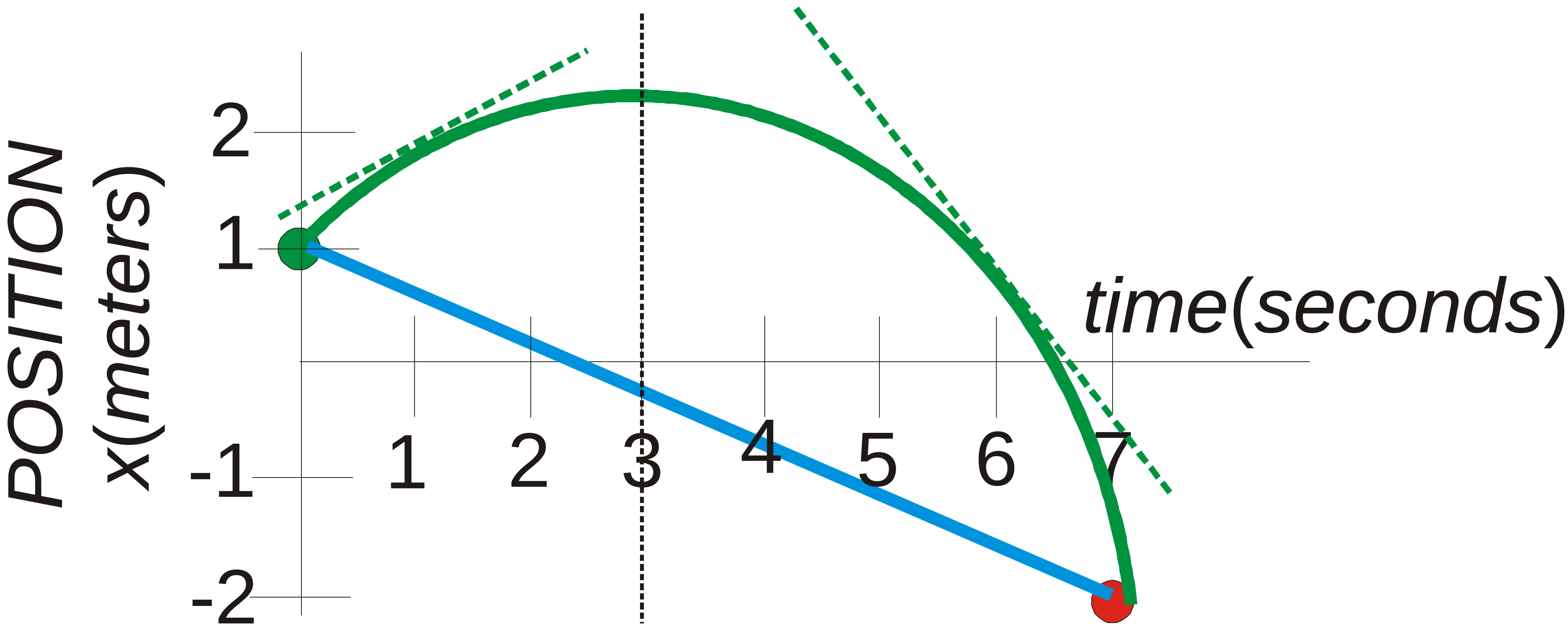
The velocity at any given time can be found from a plot of *x*(*t*) from the slope (*the tangent to the line being plotted*) at that time.



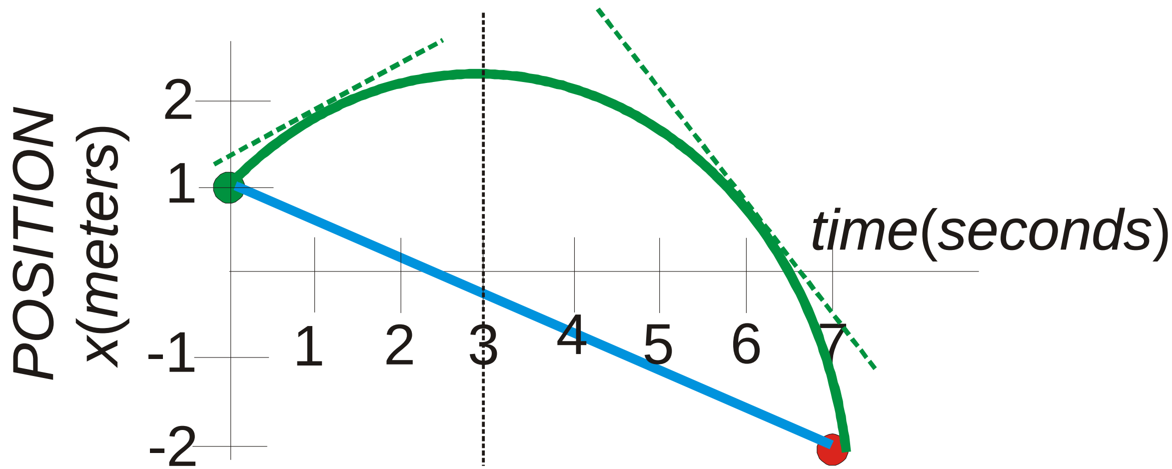
If *x*(*t*) is a straight line, then $v_{average} = \Delta x/\Delta t = dx/dt = v_{instantaneous}$.

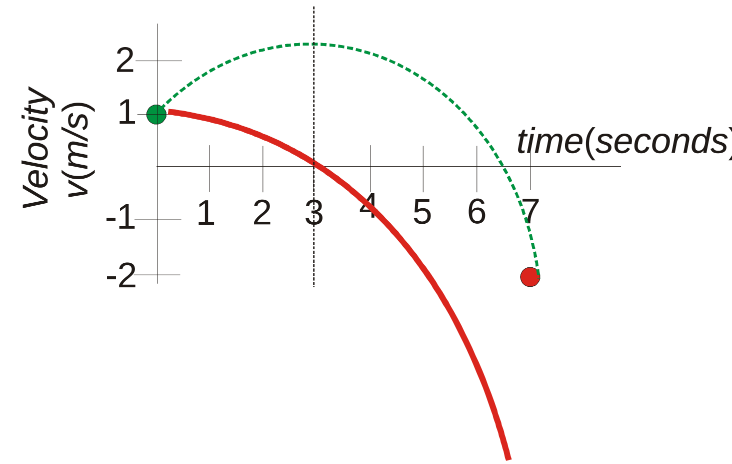
In the plot of *x*(*t*) above,

$v_{instantaneous} = v_{average} = [(-2 - 1)m \,/\, (7 - 0)s] = -3/7\ m/s = -0.429\ m/s$



What can you say about v if $x(t)$ is <u>not</u> a straight line ~ green curve?





What can you say about v if $x(t)$ is not a straight line ~ green curve.

- v is **positive** between 0 – 3 *seconds*, where the slope is positive.
- v is **negative** between 3 – 7 *seconds.*
- The instantaneous velocity, v, is **zero** at about 3 *sec*, where the slope is 0.
- The average velocity from 0 to 7 *seconds* **is the same** as for the blue path,

$$\Delta x/\Delta t = -0.429\ m/s.$$

- v_{avg} of the two lines is **NOT the same** in most other time *intervals*.
- v for the green curve **=** v_{avg} = v for the blue curve **at ~4 *s***

when the slope of the green curve = slope of the blue curve.

Can you sketch a plot of $v(t)$ as a function of t?

The red line is the derivative of the green curve.

$$v(t) \to x(t) \quad ???$$

$$v = \frac{dx}{dt}$$

You can rearrange the equation for *v* to get

$$dx = vdt$$

$$\int_o^f dx = \int_o^f vdt \underset{\textit{iff}\ \text{v is constant}}{=} v\int_o^f dt$$

$$x_f - x_o = v\left(t_f - t_o\right)$$

$$\Delta x = v\Delta t$$

$$\text{if } t_o \equiv 0 \quad \& \quad t_f \equiv t \quad \& \quad x_f \equiv x$$

$$x = x_o + vt$$

IF *v* is CONSTANT *wrt time*!

If *v* changes with *time*, use

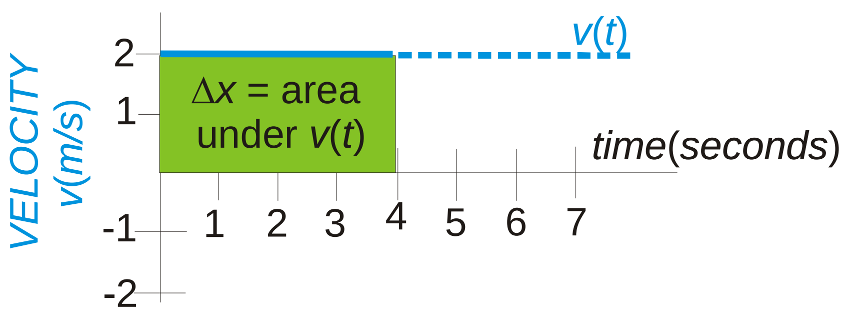
$$\Delta x = \int_o^f dx = \int_o^f v(t)\,dt \qquad x_f = x_o + \int_o^f v(t)\,dt$$

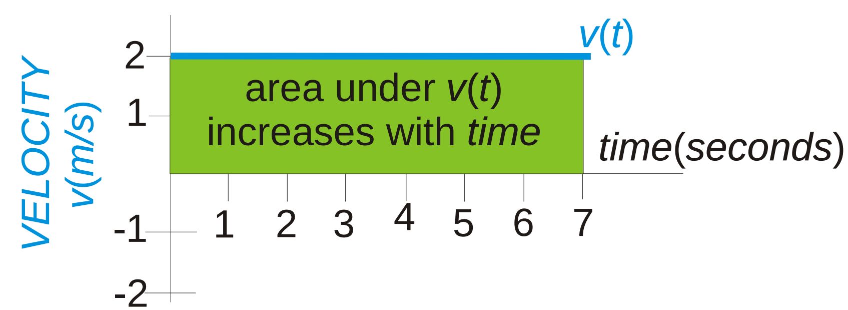
POSITION as an INTEGRAL

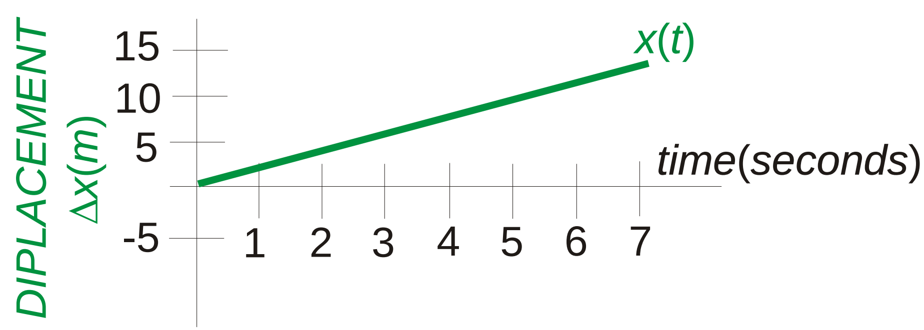
$$\Delta x = x_f - x_o = \int_o^f v\,dt \qquad\qquad x_f = x_o + \int_o^f v\,dt$$

Δx = area under the curve in a plot of $v(t)$ *vs.* t.

A plot for $v(t)$ when v is constant in time looks like:





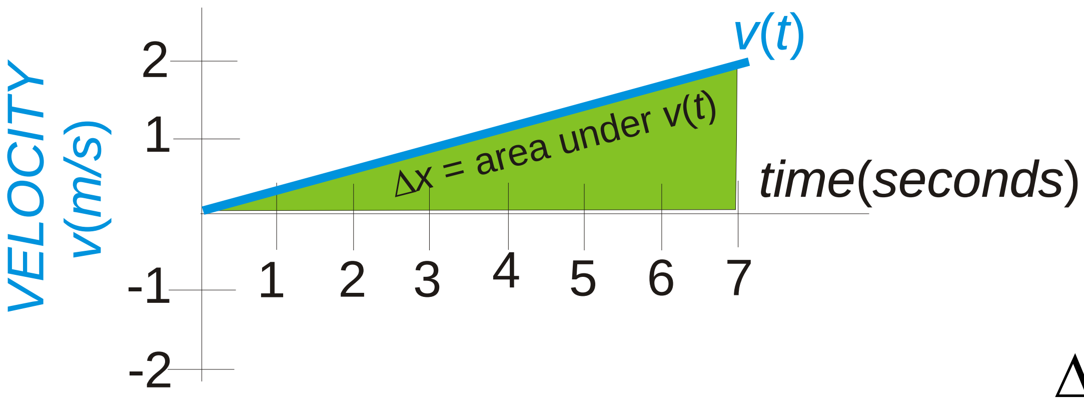


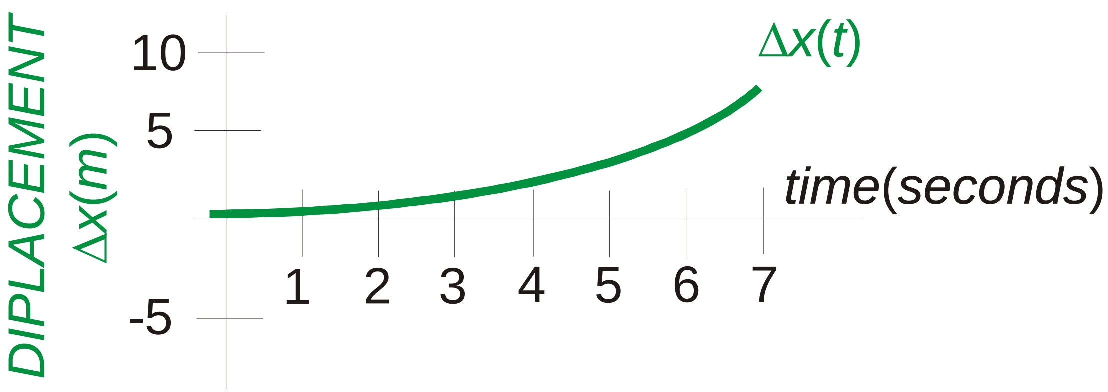
POSITION as an INTEGRAL

$$\Delta x = x_f - x_o = \int_o^f v\,dt \qquad x_f = x_o + \Delta x = x_o + \int_o^f v\,dt$$

applies even if v changes with time.

A plot for $v(t)$ when v increases linearly with time, $v = \alpha t$, starting at $v = 0$ when $t = 0$, time looks like:





$$\Delta x = x_f - x_o = \int_o^f v\,dt = \int_o^f (\alpha t)\,dt = \frac{\alpha t^2}{2}$$

MOTION in STAGES

Don't be confused by motion in stages, like

- A woman drives due east at 40 *mph* for 30 *minutes* and at 20 *mph* for an additional 15 *minutes*.

  How far does she travel & what is her average speed?

- Calculate Δx for *each stage* & add to find Δx_{total}

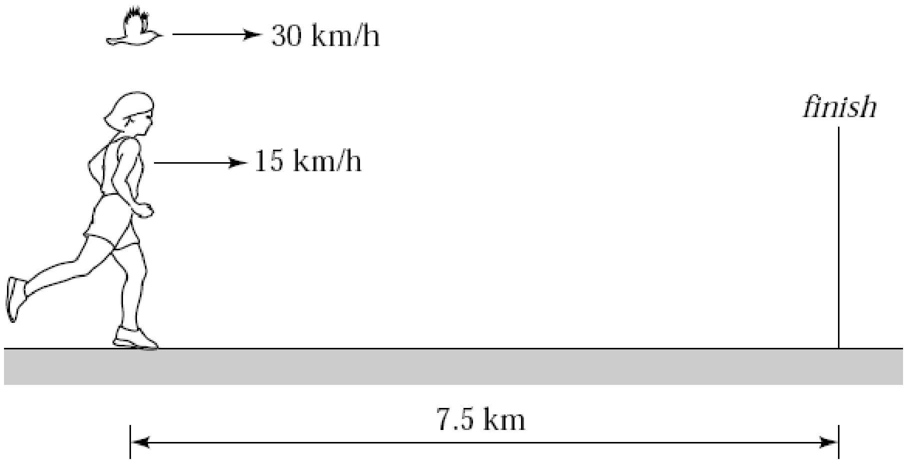
  $\Delta x_{\text{total}} = \Delta x_{\text{1st stage}} + \Delta x_{\text{2nd stage}} = 20\ miles + 5\ miles = \underline{25\ miles}$

- Use the net Δx to find $v_{avg} = \Delta x/\Delta t$.

$v_{avg} = 25\ miles/45\ minutes = 25\ miles/(3/4\ hr) = \underline{33\ mph}$

Note that $v_{\text{avg}} \neq \frac{1}{2}(v_o + v_f) = 30$ mph

- 5 options



A woman is running at a steady 15 *km/hr*. When she is 7.5 *km* from her destination, a bird over her head begins flying to the finish at 30 *km/hr*. When the bird reaches the finish line, it turns around and flies back to the runner and then turns around again, repeating the back-and-forth trips until the woman reaches the finish line. How many kilometers does the bird travel?

A. 10 *km*

B. 15 *km*

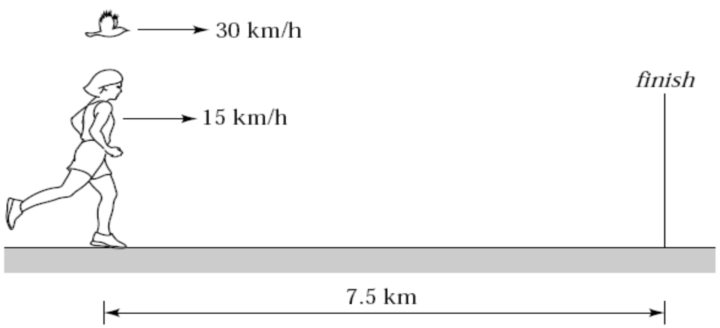
C. 20 *km*

D. 30 *km*

E. none of the above.

HINT: How long does it take the runner to reach the finish line?

- 5 options



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B. 15 *km*

C. 20 *km*

D. 30 *km*

E. none of the above.

The trick to solving this problem is to THINK CLEARLY!

- The runner travels 7.5 *km* at 15 *km/hr*; this takes 30 minutes.
- The bird flies 30 *km/hr* for 30 *minutes*, traveling 15 *km* total.

6 options

An object travels from one point in space to another point. After it arrives at its destination, its displacement is:

A. either greater than or equal to
B. always greater than
C. always equal to
D. either smaller than or equal to
E. always smaller than
F. either smaller or larger

than the distance it traveled.

6 options

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B. always greater than

C. always equal to

D. either smaller than or equal to

E. always smaller than

F. either smaller or larger

than the distance it traveled.

The displacement is the distance between the final and the initial positions (*and can be negative*).

The distance traveled is the length of the path traversed; it is always positive and can be larger than the displacement if a turning point is included.

- 4 options

POSSIBLE EXAM #2 & #3 DATES

A. Wednesday, March 18 & April 15

B. Friday, March 20 & April 17

C. Wednesday, March 25 & April 22

D. Friday, March 27 & April 24

- ENGR 131 EXAMS: February 5, March 17 & April 16
- ENGR 145 EXAMS: Thursdays, February 5, March 5, April 2 & 23.
- MATH 122 EXAMS: Tuesdays, February 3, March 3, April 14.

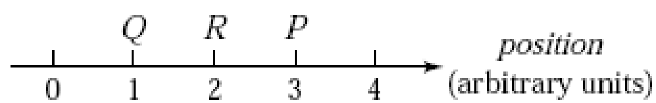
PLOTS & VISUALIZATION

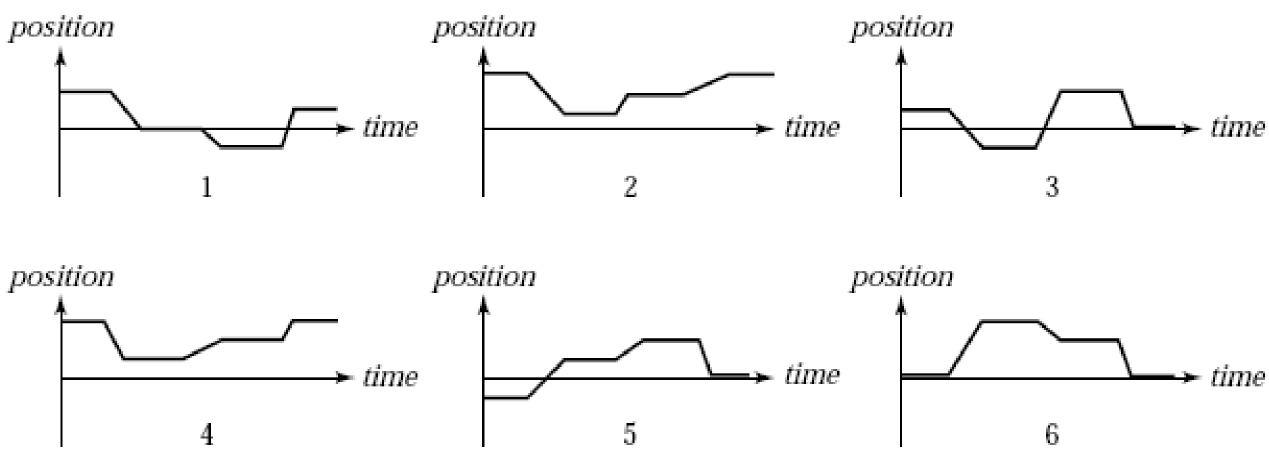
6 options 

A PHYS 121 student moves as follows along the horizontal line shown below.

- She is initially standing at point P, staying there for a few seconds after $t = 0$.
- She then strolls slowly to point Q and stays there for a moment.
- She sees a chocolate chip cookie at R, runs to grab it and pauses.
- She then walks slowly back to P where she stops and eats the cookie.

Which of the position vs. time graphs below correctly represents this motion.





PLOTS & VISUALIZATION

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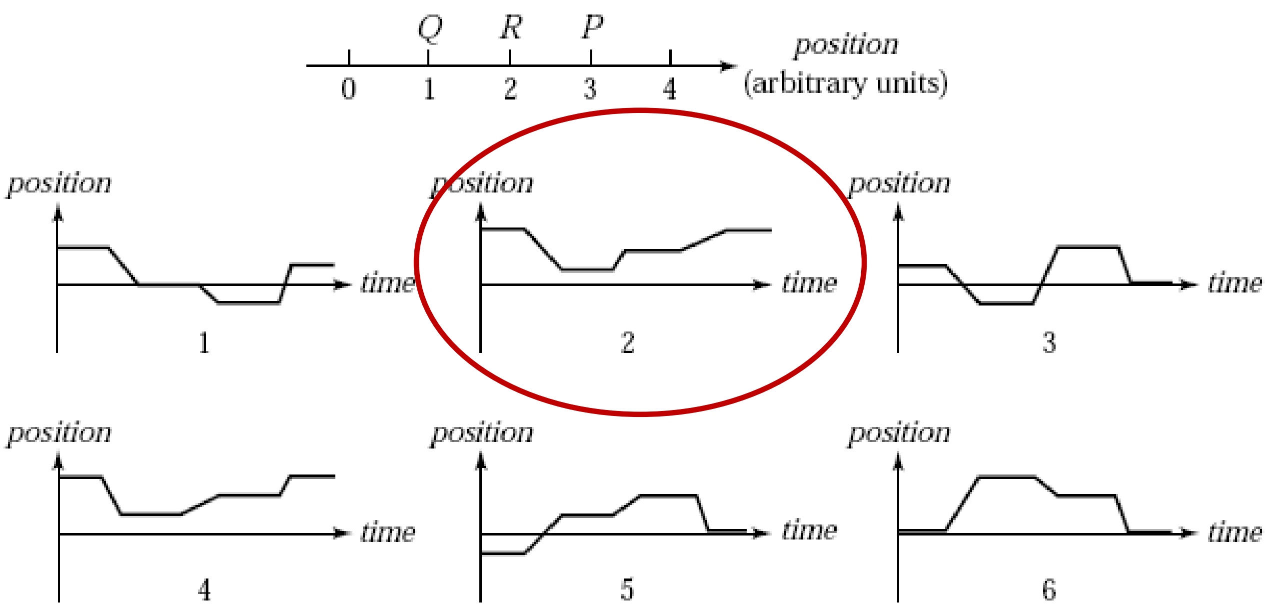


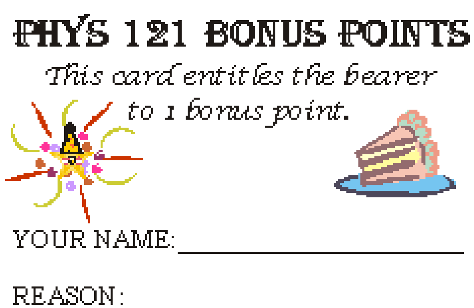
Is it easier to visualize the motion with the text or with the plot?

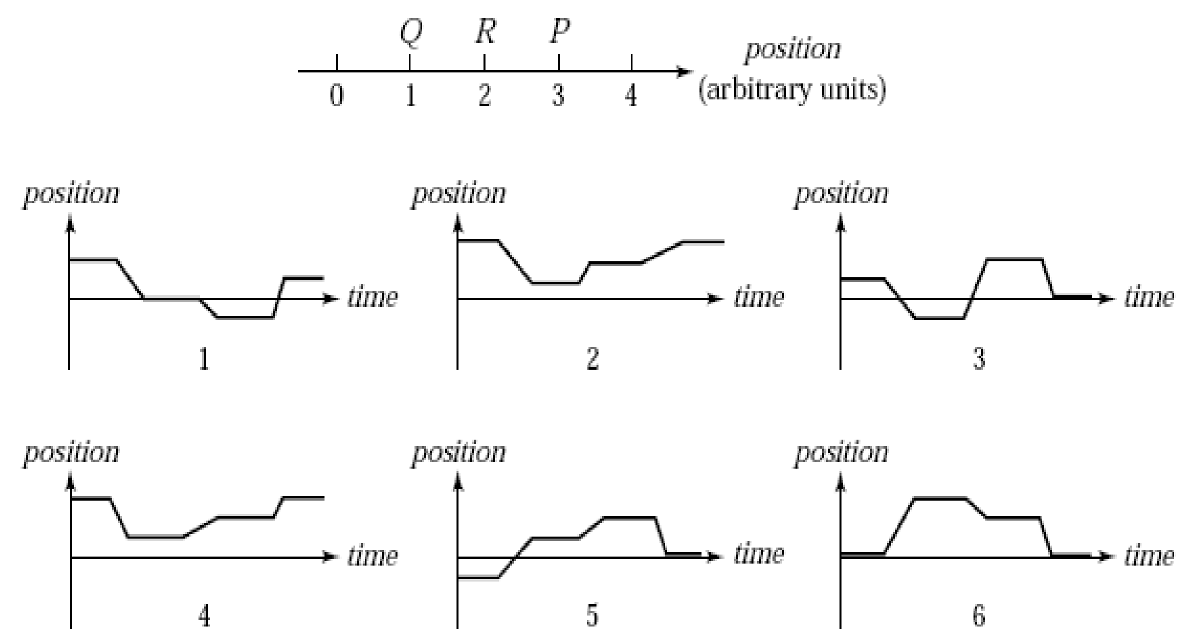
DEMO - PLOTS & VISUALIZATION

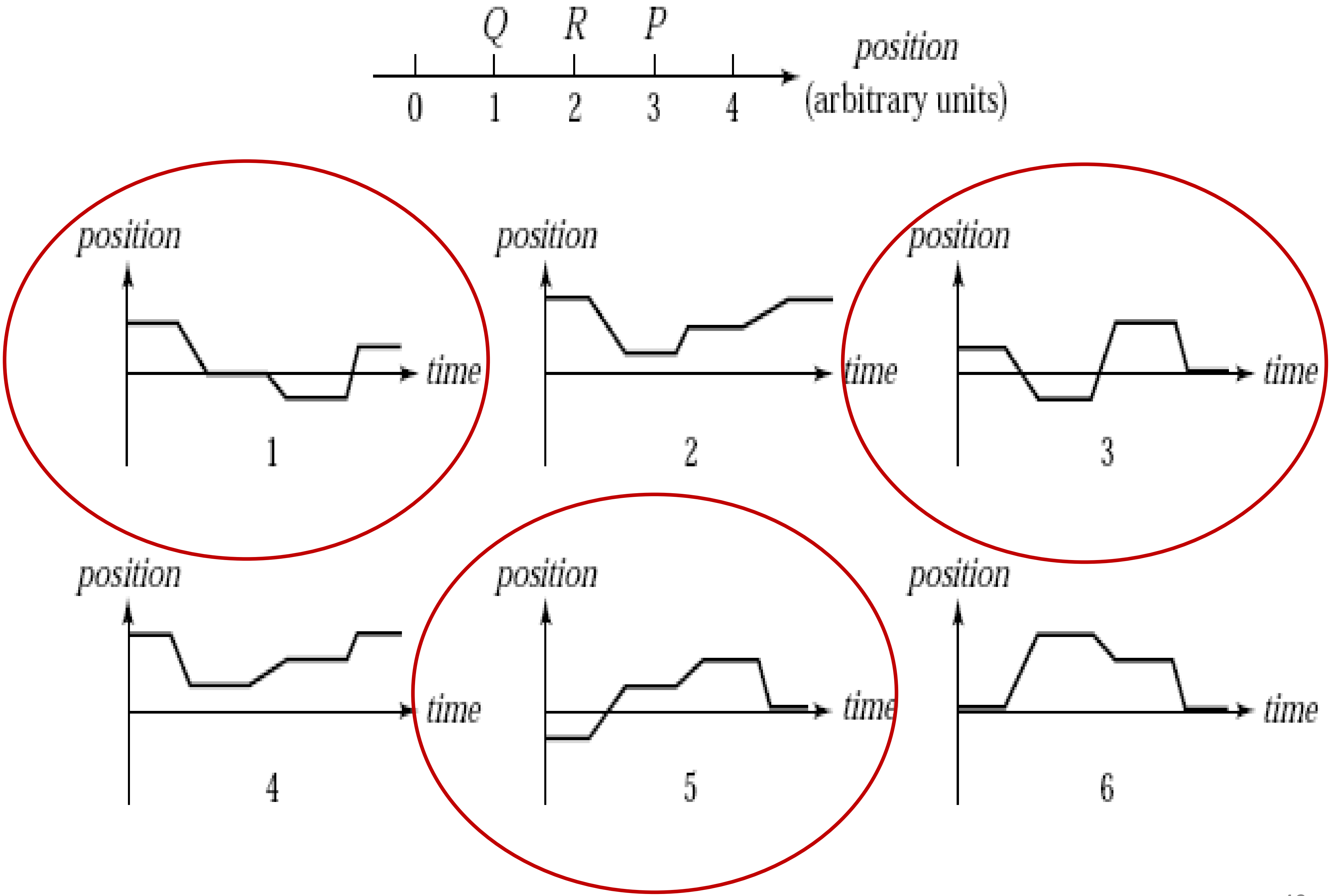
THREE VOLUNTEERS ARE NEEDED; A BONUS POINT IS AVAILABLE.

- Your charge is to duplicate the motion in plot #1, #3 or #5 below.
- Position is in meters, positive is towards the exit.
- The origin of the universe = Dr. C.
- The total time for each plot is 10 seconds.
- The best effort, as judged by the class, wins a bonus point.

See the next slide for an enlarged copy of this figure.






Q R P
0 1 2 3 4
position
(arbitrary units)
position
time
1
position
time
2
position
time
3
position
time
4
position
time
5
position
time
6

ACCELERATION

The relationship of acceleration to velocity mirrors the relationship of velocity to position.

The *average* acceleration ≡ *overall* change in velocity Δv divided by the time interval Δt for the change.

$$a_{average} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad v_{average} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The *instantaneous* acceleration is the acceleration at some moment of time.

$$a_{\text{instantaneous}} = a = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$v_{\text{instantaneous}} = v = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

ACCELERATION

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

If, for example, *v* as a function of time is given by

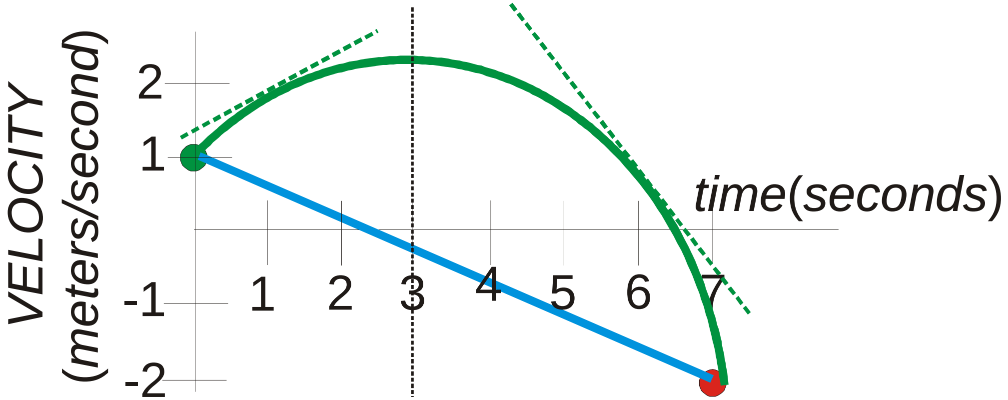
$$v = v_o + \alpha t + \beta t^2$$

where v_o is the velocity at time $t = 0$

and α and β (*Greek letters alpha & beta*) are constants, then

$$a = \alpha + 2\beta t$$

The acceleration at any given time = slope (*tangent*) in a plot of $v(t)$ *vs.* t.



ACCELERATION

Note that you can invert the derivative in $a = \dfrac{dv}{dt}$

to get $dv = adt$

and integrate to get

$$\Delta v = \int_o^f dv = \int_o^f adt \underset{iff \;\; \text{a is constant}}{=} a\int_o^f dt$$

$$v_f - v_o = a\left(t_f - t_o\right)$$

$$\Delta v = a\Delta t$$

$$\text{if } t_o \equiv 0 \quad \& \quad t_f \equiv t \qquad \& \quad v_f \equiv v$$

$$v = v_o + at$$

$$\int_o^f dx = \int_o^f vdt \underset{iff \;\; \text{v is constant}}{=} v\int_o^f dt$$

$$x_f - x_o = v\left(t_f - t_o\right)$$

$$\Delta x = v\Delta t$$

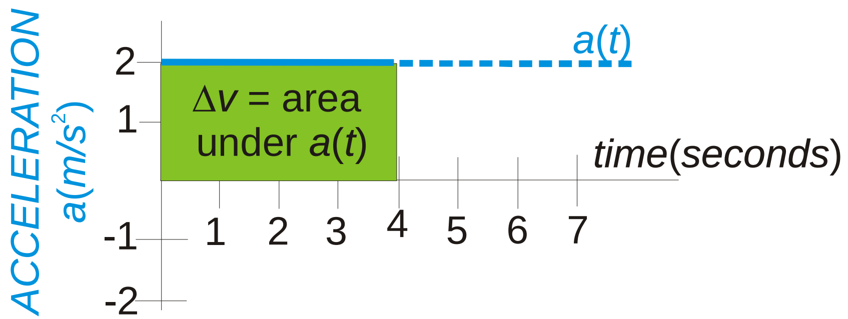
$$\text{if } t_o \equiv 0 \quad \& \quad t_f \equiv t \quad \& \quad x_f \equiv x$$

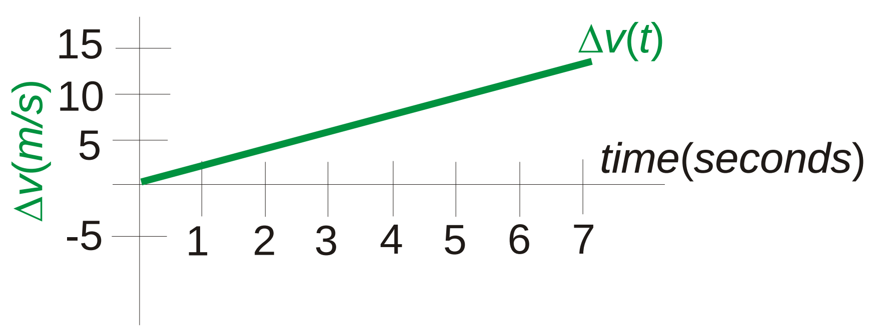
$$x = x_o + vt$$

VELOCITY as an INTEGRAL

$$\Delta v = \int_o^f dv = \int_o^f a dt \qquad v = v_o + \Delta v = v_o + \int dv = v_o + \int a dt$$

Whether or not a is constant, the change in v can also be interpreted as the area under the curve in a plot of $a(t)$ *vs*. t.





MOTION WITH CONSTANT ACCELERATION

Most 1-D motion problems based on **constant** acceleration can be solved using one of 4 equations derived from the definitions of instantaneous velocity and acceleration

Be careful and look for some hint that the acceleration is constant before you apply the methods described on the next slide!

If the acceleration is not constant, you cannot solve the problem with those equations but must use the derivatives & integrals shown below.

$$v = \frac{dx}{dt} \qquad \& \qquad a = \frac{dv}{dt}$$

$$x = x_o + \int_0^f v\,dt \qquad \& \qquad v = v_o + \int_0^f a\,dt$$

MOTION WITH CONSTANT ACCELERATION

If – and only if – the acceleration a is constant

and we define $t_o = 0$, then

$$v = v_o + \int_0^f a\,dt = v_o + a\int_0^f dt \rightarrow v = v_o + at$$

$$x = x_o + \int_0^f v\,dt \rightarrow x = x_o + \int_0^f \left(v_o + at\right)dt \rightarrow x = x_o + v_o t + \tfrac{1}{2}at^2$$

With a little bit of algebra, shown in Ohanian, you can manipulate these equations to obtain two particularly useful relationships.

$$x = x_0 + \tfrac{1}{2}\left(v_o + v\right)t$$

$$v^2 - v_o^2 = 2a\left(x - x_o\right)$$

4 EQUATIONS FOR CONSTANT ACCELERATION

$$v = v_o + at \qquad x = x_o + v_o t + \tfrac{1}{2} a t^2$$

$$x = x_0 + \tfrac{1}{2}\left(v_o + v\right)t \qquad v^2 - v_o^2 = 2a\left(x - x_o\right)$$

- GOOD NEWS: You **do not** have to memorize these equations for exams.
- All 4 of these equations are based on the basic definitions of displacement, velocity & acceleration + some calculus & algebra
  - $\Rightarrow$ they aren't fundamentally important in their own right
  - $\Rightarrow$ all you need is $v = \mathrm{d}x/\mathrm{d}t$ & $a = \mathrm{d}v/\mathrm{d}t$
- But they ARE handy in solving PHYS 121 kinematics problems.
  - Students tend to approach constant acceleration problems by seeing what terms you are given & which you need to solve for out of (x, x_o, v, v_o, a, t) and then identifying which of these 4 equations works.
  - You need an equation that contains ONLY ONE unknown term.
- Make certain, before you use these equations, that the acceleration is constant; otherwise these equations don't apply!

TRICKS & TIPS

for solving constant acceleration problems

- **TURNING POINTS**: $\boldsymbol{v} = \mathbf{0}$ at the instant an object changes direction, such as the top of an upward throw.

  You have to go through v = 0 to get from +v to =-v!

- **CONVENIENCE**: It's often convenient to choose a starting or ending point where $v = 0$, and you are free to incorporate an existing turning point into your analysis even if it isn't required or specified.

  Use the top of a trajectory if you like.

TRICKS & TIPS

for solving constant acceleration problems

- **SYMMETRY**: $v_{up} = v_{down}$ at the same height in vertical motion with acceleration due to gravity.

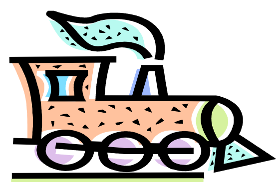
  (*TIME REVERSAL SYMMETRY & CONSERVATION of ENERGY*)

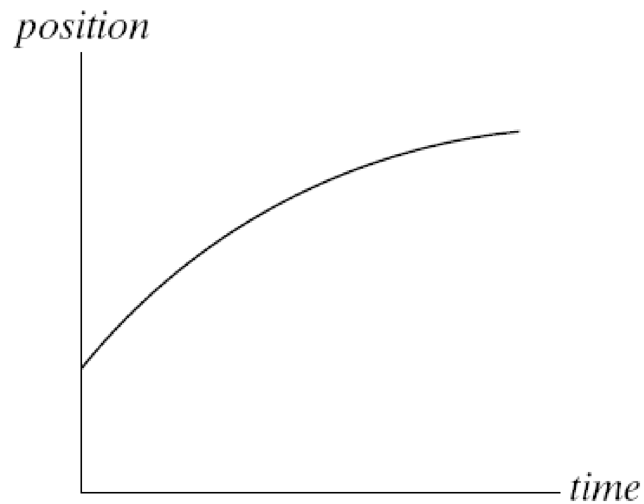
- **SIGNS**: It's possible to have a positive velocity and a negative acceleration & *vice versa*.

  Don't be confused by negative positions, velocities & accelerations.

- 4 options **Using a plot of x(t) to find v and a.**

A train moves along a track with a position as a function of time plotted below.





During the range of times shown, the train (*choo-choo*):

A. Speeds up continuously (*i.e.*, *accelerates*)

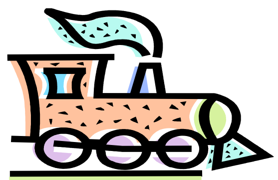
B. Slows continuously

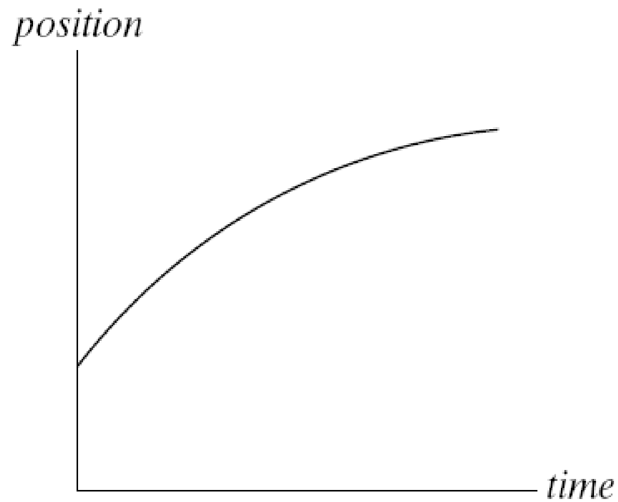
C. Speeds up at first and then starts slowing.

D. Moves at a constant velocity.

 - 4 options **Using a plot of x(t) to find v and a.**

A train moves along a track with a position as a function of time plotted below.





During the range of times shown, the train (*choo-choo*):

A. Speeds up continuously (*i.e.*, *accelerates*)

B. Slows continuously

C. Speeds up at first and then starts slowing.

D. Moves at a constant velocity.

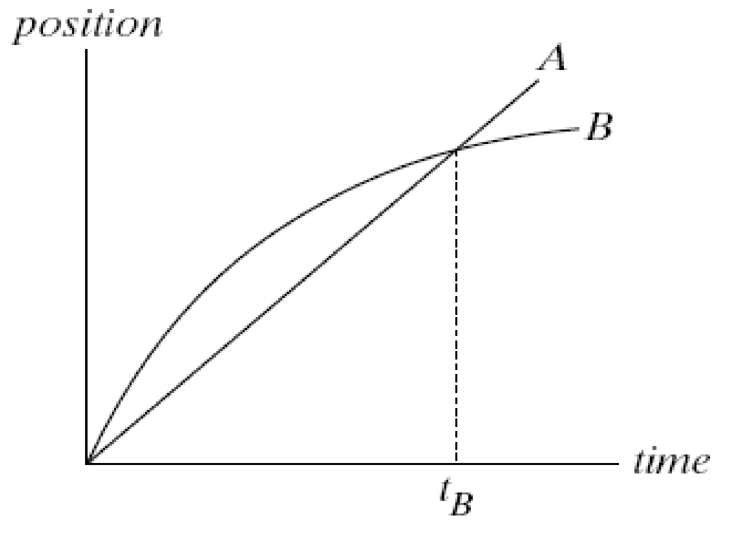
The position is always positive and increasing,

the velocity (slope) is always positive, but

the acceleration is negative since the slope decreases as time increases.

- 5 options

The graph below shows position as a function of time for two trains traveling on parallel tracks. Which of the following statements is true?



A. At time t_B, both trains have the same velocity.

B. Both trains speed up (*accelerate*) all the time.

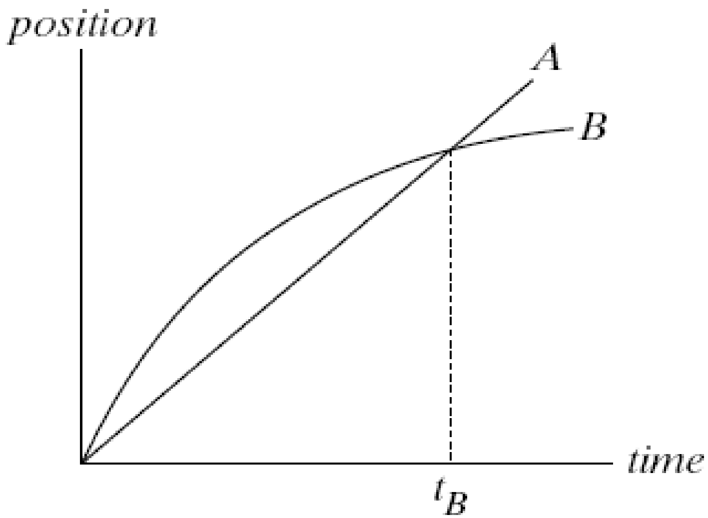
C. Both trains have the same velocity at some time before t_B.

D. Somewhere on the graph, both trains have the same acceleration.

E. None of the above.

 - 5 options

The graph below shows position as a function of time for two trains traveling on parallel tracks. Which of the following statements is true?



A. At time t_B, both trains have the same velocity.

B. Both trains speed up (*accelerate*) all the time.

C. Both trains have the same velocity at some time before t_B.

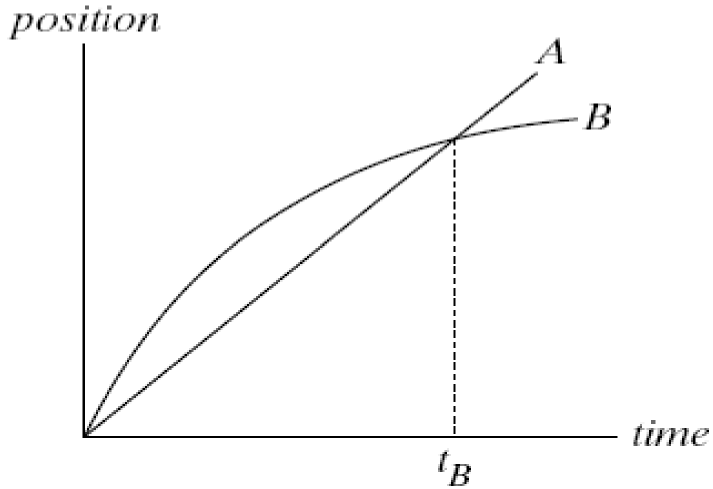
D. Somewhere on the graph, both trains have the same acceleration.

E. None of the above.

The velocity is the slope or tangent of the curves and both plots have the same tangent somewhere near ½ t_B.

 - 5 options

Consider the average and instantaneous velocities for train *B* following the curved path. (*Ignore train A.*) Which of the following is true?



A. Its average and instantaneous velocities are the same between $t = 0$ and $t = t_B$.

B. Its average and instantaneous velocities are the same at t_B.

C. Its average and instantaneous velocities are the same at $t = 0$.

D. The average and instantaneous velocities are the same for only an instant of time.

E. Its average and instantaneous velocities are never the same.

 - 5 options

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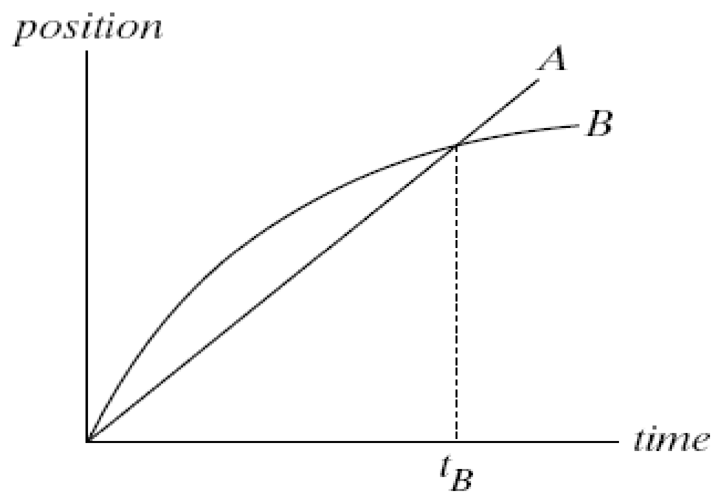


A. Its average and instantaneous velocities are the same between $t = 0$ and $t = t_B$.

B. Its average and instantaneous velocities are the same at t_B.

C. Its average and instantaneous velocities are the same at $t = 0$.

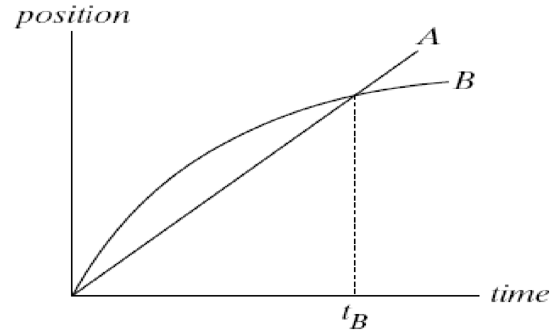
D. The average and instantaneous velocities are the same for only an instant of time. ***That instant is the same time as in the previous question, when the slopes of the two curves are parallel, since line A shows the average speed of train B between $t = 0$ and $t = t_B$***

E. Its average and instantaneous velocities are never the same.

- 5 options

WHY ARE THE OTHER ANSWERS WRONG?

Consider the average and instantaneous velocities for train *B* following the curved path. (*Ignore train A.*) Which of the following is true?



A. Its average and instantaneous velocities are the same between $t = 0$ and $t = t_B$.

The slope of curve *B* changes and ≠ slope of line *A* everywhere in this interval.

The instantaneous velocity only equals the average velocity if the velocity is constant.

Otherwise the instantaneous velocity changes with time.

B. Its average and instantaneous velocities are the same at t_B.

C. Its average and instantaneous velocities are the same at $t = 0$.

There is no such thing as an average velocity AT SOME TIME, an average is over a time interval.

E. Its average and instantaneous velocities are never the same.

We identified a point where they are the same – and this must be true in general since you can't always be traveling faster or slower than your average velocity.

PHYS 121: We made it slide #63 on Friday, 1/16/2015.

PLACE YOUR HOMEWORK IN THE APPROPRIATE FOLDER ON THE STAGE or the BACK of the BALCONY.

There's a stapler on stage.

The folders are arranged from left to right.

GROUP #1: A – C: Aguilar - Choi

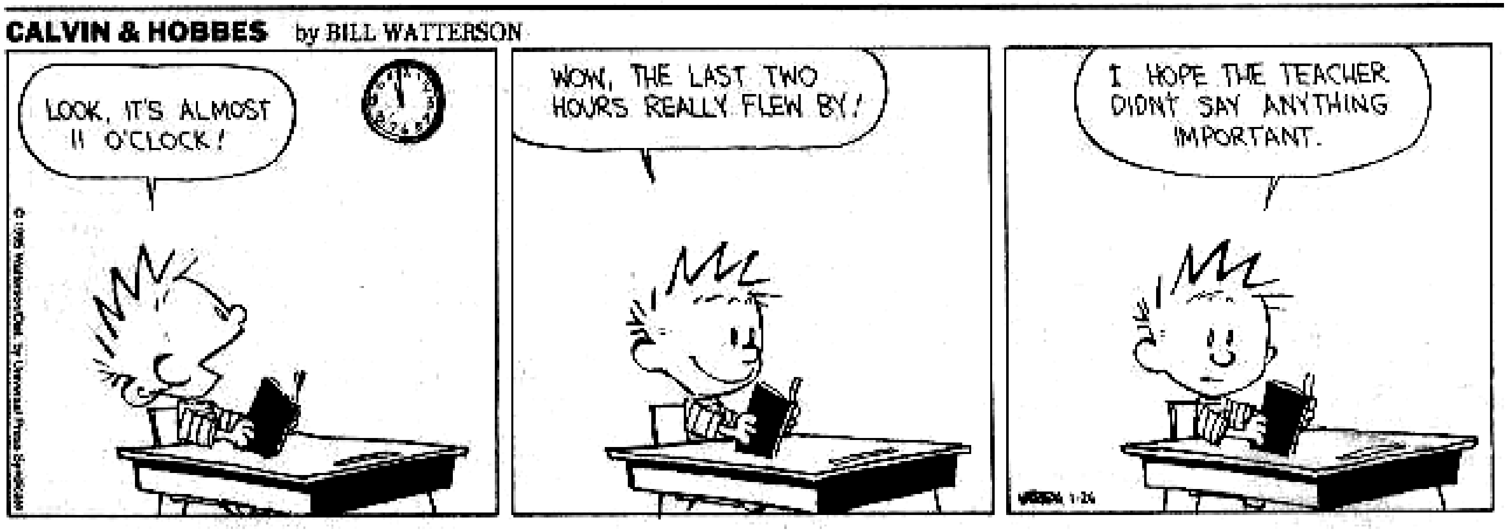
GROUP #2: Co – Gr: Conover - Guo

GROUP #3: H – K: Hall - Krynicki

GROUP #4: L – Pa: LaLonde – Owens

GROUP #5: P – S: Padilla - Sun

GROUP #6: T – Z: Teitelbaum – Zuccala



Get your clickers/apps ready.

ANNOUNCEMENTS

➢ HOMEWORK #1 INQUIRIES FROM STUDENTS

Significant Figures: Ohanian 2.64, t = 30 sec, d = 700 m

One significant figure?

- See http://en.wikipedia.org/wiki/Significant_figures for a good treatment of significant figures → 30 & 700 are ***'ambiguous'*.**
- If in doubt, use 2 or 3 significant figures.
- You can carry additional significant figures along in your calculations, until the end.
- Answers may vary slightly, depending on when you reduce to significant figures. Graders are instructed to '*be generous*'.
- Error analysis, as taught in the lab, is the correct way to handle uncertainty,

  - but you'd need the estimated error in the input data

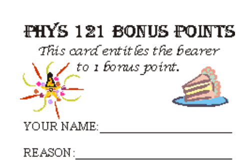
  - and a **LOT** more time for each homework problem.

ANNOUNCEMENTS

- **HOMEWORK #1 INQUIRIES FROM STUDENTS**
  - ***'Checking your answer'*** (*magnitude*, *units*, *etc.*) on homework or exams is not a requirement that will be graded, unless the problem specifies some specific check. It's just common sense.
  - ***'Practice problems'*** are optional. They are not to be handed in and you don't need to do them if you don't need the practice.
  - ***Dr. C.*** problems after the practice problems ARE required.

ANNOUNCEMENTS

- **CLICKERS**
  - Everyone whose clicker is registered by midnight Friday earns 1 Bonus Point.
  - Emails have been sent to people whose clickers are registered properly – or not.
  - If you have problems registering your clicker, contact Turning Technologies, not Dr. C. But we would like to hear about common problems and solutions to clicker issues so that we can communicate them to the class.
- **EXAM DATES**
  - Keep an eye out for a survey, distributed *via* email, on dates for Exam #2 & #3. These dates should be finalized by next Monday.



POSSIBLE EXAM #2 & #3 DATES

Survey closes 9 AM Friday.

A. Wednesday, March 18 & April 15
B. Friday, March 20 & April 17
C. Wednesday, March 25 & April 22
D. Friday, March 27 & April 24

- ENGR 131 EXAMS: February 5, March 17 & April 16
- ENGR 145 EXAMS: Thursdays, February 5, March 5, April 2 & 23.
- MATH 122 EXAMS: Tuesdays, February 3, March 3, April 14.

As of Today, 8:00 AM		
Answer	**Response**	**%**
Wednesday, March 18 & April 15	13	13%
Friday, March 20 & April 17	99	80%
Wednesday, March 25 & April 22	7	6%
Friday, March 27 & April 24	4	3%
Total	123	100%

ANNOUNCEMENTS

Your graded homework will be returned in class next Monday.

- You can retrieve it before class starts or after it ends

  (*if you act quickly, before the next class arrive*s.)
- Your work can be retrieved at other times from the Rockefeller 104D corridor.

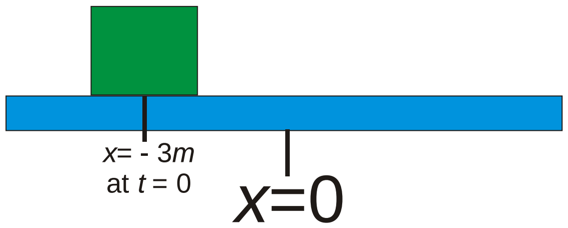
 - 6 options

How many hours did you spend on today's PHYS 121 homework assignment?

1. 1 hour or less
2. 1 – 2 hours
3. 2 – 3 hours
4. 3 – 4 hours
5. 5 – 6 hours
6. > 6 hours

The lecture is 'worth' 3 credits ⇒ you should expect to spend ~ 6 hours on reading & homework each week.

- 4 options



A wood block sliding along a horizontal table has a position that changes with time according to the equation

$$x = -3 -5t$$

with time in seconds and distance in meters.

Which of the following is the best answer for times $t > 1$?

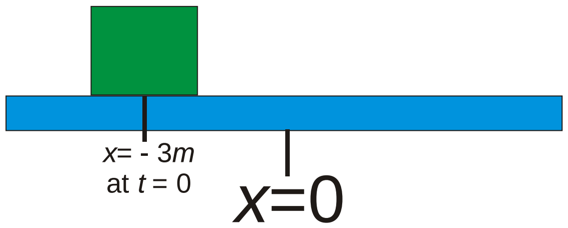
A. the acceleration of the block is zero

B. the velocity along the line is -5 m/s

C. it will never get to positive values of x

D. all of the above

- 4 options



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B. the velocity along the line is -5 *m/s*

C. it will never get to positive values of *x*

D. all of the above

***x* for t = 1 is -2 *m* and grows increasingly negative as time increases**

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(-3-5t\right) = -5 \text{ m/s} \quad \text{so B is true}$$

$$\text{a} = \frac{dv}{dt} = \frac{d}{dt}\left(-5\right) = 0 \quad \text{so A is true}$$

VARYING ACCELERATION

How do you solve a problem when a isn't constant?

- You can't use $x = x_o + v_o t + \frac{1}{2}at^2$ or any of the other handy-dandy formulae for constant acceleration.
- Given $a(t)$, solve for $x(t)$ using

$$x(t) = \int v(t)\,dt = \int \left[\int a(t)\,dt \right] dt$$

- Given $a($**x**$)$ instead of $a(t)$, it's tricky to solve for $v(t)$ & $x(t)$.
- We will consider this type of problem later this spring.

VARYING ACCELERATION EXAMPLE

$$a = kt$$

where k is some constant

What is $x(t)$, assuming $t_o \equiv 0$?

$$v(t) = v_o + \int_o^f a(t)\,dt = v_o + \int_o^f (kt)\,dt = v_o + \tfrac{1}{2}kt^2$$

$$x(t) = x_o + \int_o^f v(t)\,dt = x_o + \int_o^f \left(v_o + \tfrac{1}{2}kt^2\right)dt$$

$$x(t) = x_o + v_o t + \tfrac{1}{6}kt^3$$

Check your answer by taking the derivative of $x(t)$ twice $\rightarrow$

$$v(t) = \frac{dx(t)}{dt} = v_o + \tfrac{1}{2}kt^2 \qquad a(t) = \frac{dv(t)}{dt} = kt$$

FREE FALL

The force of gravity *near the earth's surface* causes a *downward* acceleration

$$a_{\text{gravity}} \equiv g \approx 9.81\ \text{m/s}^2 \approx 32.2\ \text{ft/s}^2$$

- Although this acceleration varies slightly with location on the earth's surface and with altitude above it, we can treat it as a constant acceleration as long as you include or assume the *magic words*

  "near the earth's surface"

- Free Fall problems can be solved using formulae for constant acceleration, with $a = -g$

  or $a = +g$ if you define down as positive.

- Unless you are told otherwise, assume that we're ignoring air resistance
  - even though air resistance is significant for many free fall problems.
  - We'll deal with air resistance later this semester.
- It's common to use a *y-axis* for vertical motion.

$+y$

 = 3 options

If you release an object from rest, it accelerates downward at 9.8 m/s^2.

If you instead **throw** that same object downward, its downward acceleration after release is:

A. less than 9.8 m/s^2

B. 9.8 m/s^2

C. greater than 9.8 m/s^2

= 3 options

If you release an object from rest, it accelerates downward at 9.8 m/s^2.

If you instead **throw** that same object downward, its downward acceleration after release is:

A. less than 9.8 m/s^2

B. 9.8 m/s^2

C. greater than 9.8 m/s^2

acceleration does not depend on v_o

- 6 options

You throw a ball straight up into the air (*near the earth's surface*).

At the highest point of the ball's path:

A. The velocity and acceleration of the ball are zero.

B. The velocity is non-zero but the acceleration is zero.

C. The velocity is zero but the acceleration is non-zero.

D. Neither the velocity nor the acceleration are zero.

E. The ball hangs motionless for a second or so before falling.

F. Your older brother grabs the ball and runs away; you start crying.

- 5 options

You throw a ball straight up into the air (*near the earth's surface*).

At the highest point of the ball's path:

A. The velocity and acceleration of the ball are zero.

B. The velocity is non-zero but the acceleration is zero.

C. The velocity is zero but the acceleration is non-zero.

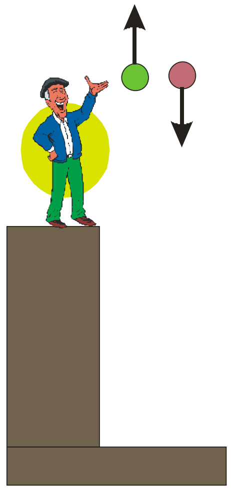
D. Neither the velocity nor the acceleration are zero.

E. The ball stops hangs motionless for a second or so before falling.

F. Your older brother grabs the ball and runs away; you start crying.

- The acceleration is $-g$ the entire time but the velocity is 0 if the ball stops even momentarily.
- $v = 0$ at the top of the path but only for an infinitesimal time, not for a "second or so".
- You can think of this as zero slope in a plot of $y(t)$ = parabolic function of t.
- If the velocity wasn't zero at the top of the path, it wouldn't be the top!
- **When you hear the phrase "maximum height" in the context of projectile motion, you can translate this into "the point where $v_y = 0$".**

- 3 options



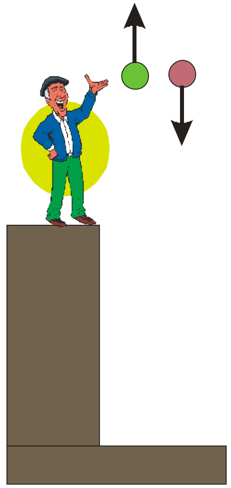
A balding, but **handsome**, physics professor standing at the edge of a cliff throws one ball straight up and another ball straight down, both with the same initial speed. Neglecting air resistance, **which ball hits the ground with the greater speed**; the one that is thrown up or the one that is thrown down?

A. The one that is thrown up.

B. The one that is thrown down.

C. They both hit the ground with the same speed.

- 3 options



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$$v^2 = v_o^2 + 2a\left(x - x_o\right)$$

A. The one that is thrown up.

B. The one that is thrown down.

C. They both hit the ground with the same speed.

Only the NET displacement matters in determining the final velocity and this net displacement is the same in both cases.

You can also use *symmetry* concepts. The ball that is thrown up will return to its initial height with the same speed it had initially, but in the opposite direction, so the rest of its travel is identical to the ball that is thrown down.

4 options

You throw a ball upward with an initial velocity v_o.

Which of the equations below is the best choice for calculating how far up the ball travels before momentarily coming to a halt?

A. $v = v_0 + at$

B. $v^2 = v_o^2 + 2a\Delta x$

C. $x = x_0 + v_o t + ½\, at^2$

D. $x = x_o + ½(v_o + v)t$

4 options

You throw a ball upward with an initial velocity v_o.

Which of the equations below is the best choice for calculating how far up the ball travels before momentarily coming to a halt?

A. $v = v_0 + at$

B. $v^2 = v_o^2 + 2a\Delta x$

C. $x = x_o + v_o t + ½\, at^2$

D. $x = x_o + ½(v_o + v)t$

You know:

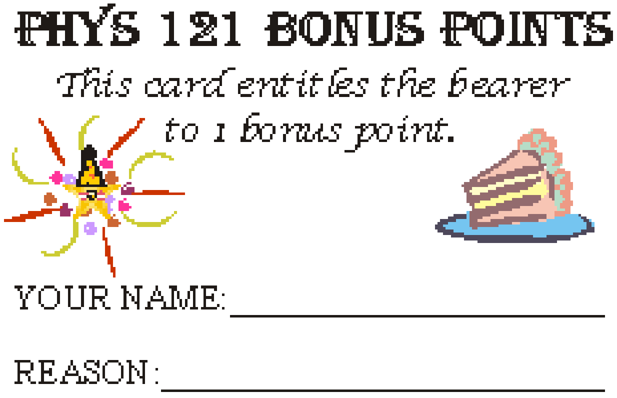
- $v = 0$ at the top of the path
- v_o
- $a = -g$

so $\Delta x = (v^2 - v_o^2)/(2a) = -v_o^2/(-2g)$.

Note that you aren't given the time, t, in the air and don't want to solve for it, so ignore any equation with t in it.

Note, too, that some prefer converting x to y in these equations.

Anyone who can juggle three balls for a few minutes during the next clicker question can earn a bonus point.



http://www.juggling.org/pics/cartoons.html

4 options

You want to juggle three balls. You can hold one in each hand but need to keep the third ball in the air for t = 1.2 seconds at a time since this is how long it takes you to shuffle the other two balls between your hands and throw one of them upwards.

Which of the equations below is the best choice for calculating <u>how far up</u> you have to throw each ball in turn?

A. $v = v_o + at$

B. $v^2 = v_o^2 + 2a\Delta x$

C. $x = x_o + v_o t + ½ at^2$

D. $x = x_o + ½(v_o + v)t$

4 options

You want to juggle three balls. You can hold one in each hand but need to keep the third ball in the air for t = 1.2 seconds at a time since this is how long it takes you to shuffle the other two balls between your hands and throw one of them upwards. Which of the equations below is the best choice for calculating how far up you have to throw each ball in turn?

A. $v = v_0 + at$

C. $x = x_0 + v_o t + ½ at^2$

B. $v^2 = v_o^2 + 2a\Delta x$

D. $x = x_o + ½(v_o + v)t$

This one's a bit tricky. You want Δx and you know a & t but not v or v_0.

It might seem that NONE of these equations will work. This is a good time to use a trick mentioned early. Consider ½ the motion, starting at the top of the path with $v_0 = 0$ until you catch the ball. This means t= 0.6 sec instead of 1.2 sec.

$$(x_0 – x) = - v_o t - ½ at^2 = 0 – ½(-9.8 \text{ m/s}^2)(0.6 \text{ s})^2 = 1.76 \text{ m}$$

- 4 options

If you drop a large pumpkin and a small pumpkin from the roof of Strosacker on Halloween, which should hit the ground first, ignoring air resistance?

A. The large pumpkin

B. The small pumpkin

C. Both should hit the ground at the same time

D. They'll both hover in mid-air, because it's Halloween and the laws of physics don't apply.

- 4 options

If you drop a large pumpkin and a small pumpkin from the roof of Strosacker on Halloween, which should hit the ground first, ignoring air resistance?

A. The large pumpkin

B. The small pumpkin

C. Both should hit the ground at the same time

D. They'll both hover in mid-air, because it's Halloween and the laws of physics don't apply.

Mass doesn't appear in our simple (*simplistic*?) description of free fall motion.

None of the equations include *M*.

 - 4 options

A LECTURE DEMO

Coffee Filters

Which ‘*falls*’ faster, a piece of paper or a rubber ball?

A. a rubber ball

B. a piece of paper (*coffee filter*)

C. they have the same downward acceleration

D. none of the above, this sounds like a trick question

- 4 options

A LECTURE DEMO

Coffee Filters

Which '*falls*' faster, a piece of paper or a rubber ball?

A. a rubber ball

B. a piece of paper (*coffee filter*)

C. they have the same downward acceleration

D. none of the above, this sounds like a trick question

You do, however, have to eliminate air resistance as a factor. If you don't then, A is the correct answer.

MORE CULTURE

What changes about kinematics at very high speeds, so that classical mechanics fails and

Einstein's Theory of Special Relativity

must be used?

You'll find out if you take
PHYS 221, Introduction to Modern Physics,
but most of you won't,
so here are the key points.

MORE CULTURE

- The speed of light $\equiv c$ is the maximum possible velocity

  $\beta \equiv v/c$ can vary from 0 to 1 : $\gamma \equiv (1-\beta^2)^{-1/2}$ varies from 1 to ∞
- The *Lorentz Transformations* describe motion in a frame of reference moving at speed *v* in the *x* direction with respect to some observer.

$$y' = y \qquad z' = z \qquad x' = \gamma\left(x - vt\right) \qquad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

- SIMULTANEOUS events for one person are not simultaneous for another.
- TIME DILATION: moving clocks run slower (*TWIN PARADOX*)
- LENGTH CONTRACTION: objects shrink along the direction of travel

INTERSECTIONS

(15 slides for review on your own)

Ohanian includes homework problems like 2.19 with the cheetah but doesn't cover the concepts separately.

Two objects labeled #1 & #2, moving at constant velocities

$v_1 = 2\ m/s$ & $v_2 = 1\ m/s$

start at positions

$x_{1o} = -4\ m$ & $x_{2o} = 0\ m$

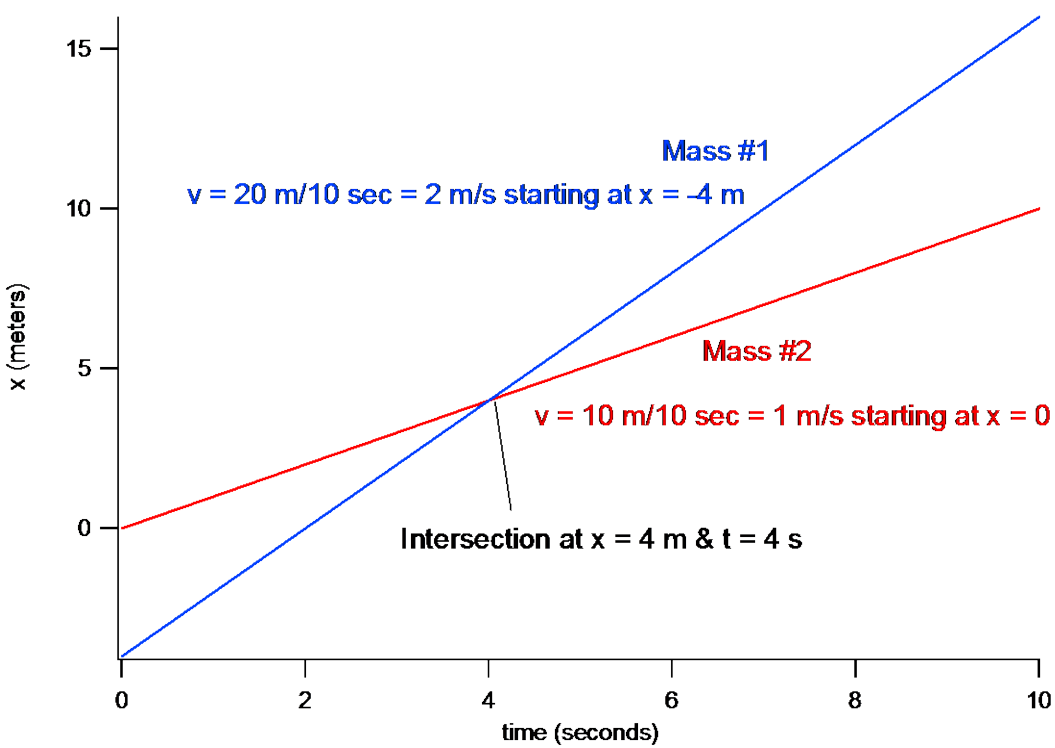
respectively.

WHEN and WHERE will their paths intersect?

INTERSECTIONS

Thinking graphically, in terms of a plot of x(t) vs. t:

- Different (*constant*) velocities are represented by lines of different slopes.
- Different initial points → different intercepts with the vertical x axis at t = 0
  - *Different starting TIMES would be represented by different intercepts with the horizontal time axis at x = 0.*
  - *In an equation, different starting times could use t and t ± T.*
- Paths intersect when they have the same value for x at the same time *t*.



INTERSECTIONS *continued*

Analytically, in terms of equations:

$x_1(t)=x_{1o}+v_1t \qquad x_2(t)=x_{2o}+ v_2t$

CROSSING at some time *t* requires

(*some people would use t' instead of t to mark this as a special time*)

$$x_1(t) = x_2(t)$$

$$x_{1o}+v_1t \;=\; x_{2o}+v_2t$$

$$-4\ m + (2\ m/s)t = 0\ m + (1\ m/s)t \qquad t=\frac{x_{1o}-x_{2o}}{v_2-v_1}$$

<u>WHEN: t = 4 s</u>

To find <u>WHERE</u>, plug *t* = 4 *s* into either equation of motion

$$x_1(t) = x_{10} + v_1t = -4\ m + (2\ m/s)(4\ s) = \underline{4\ m}$$

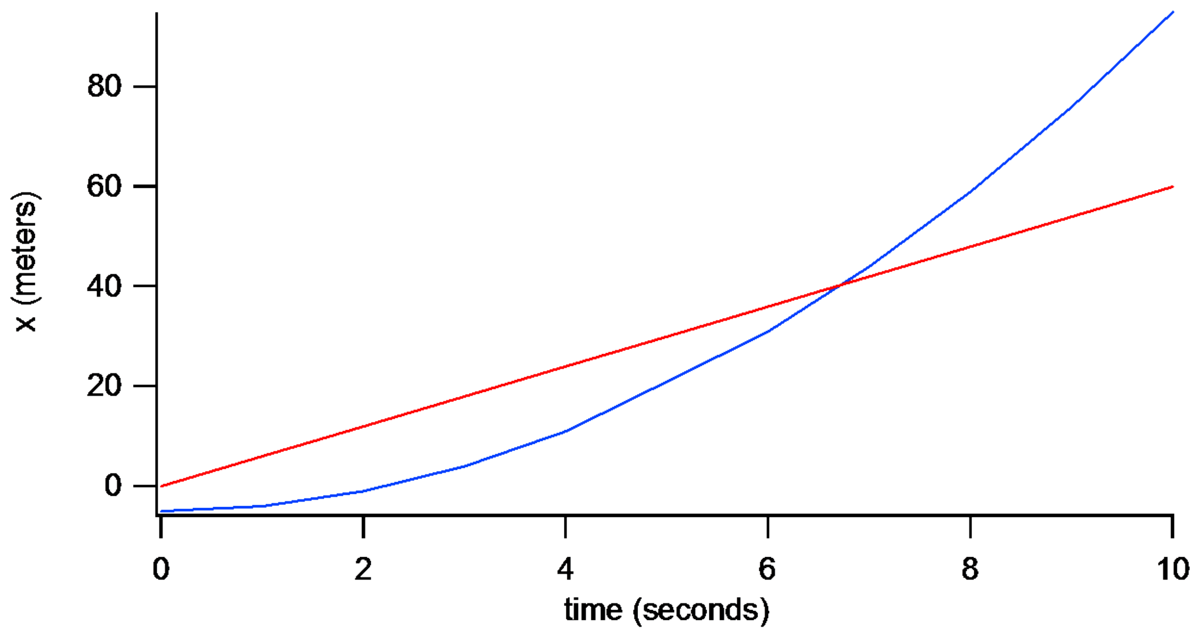
INTERSECTIONS when 1 object is accelerating

Graphically, with a plot of $x(t)$:

- The accelerating object is represented by a plot of $x(t) = x_0 + v_o t + \frac{1}{2} a t^2$ which is a second order polynomial in t.
- Their paths intersect when they have the same x at the same time t.
- The (*Origin*) plot below is of

$$x_1(t) = 0\ m + (6\ m/s)t \qquad \& \qquad x_2(t) = -5\ m + (1\ m/s^2)t^2$$

The intersection occurs at about $x = 40\ m$, $t = 6.7$ seconds



ANALYTICALLY, with EQUATIONS:

$x_1(t) = 0\ m + (6\ m/s)t \qquad x_2(t) = -5\ m + (1\ m/s^2)t^2$

$$x_1(t) = x_2(t)$$

$$0 + 6t = -5 + 1t^2$$

$$t^2 - 6t - 5 = 0$$

$$t = [6 \pm (36 + 20)^{½}]/2 = [6 \pm 7.48]/2$$

= 6.74 *seconds* or -0.74 *seconds*

QUADRATIC EQUATIONS

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What is the meaning of

t = 6.74 *seconds* <u>or</u> -0.74 *seconds*

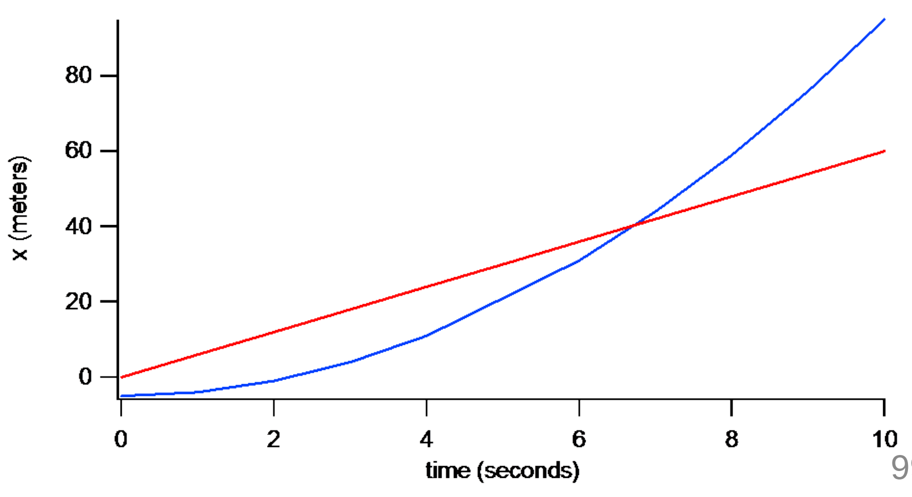
A quadratic equation always has two solutions

but it's possible that only one of them is meaningful in some given situation

We want t = 6.74 *seconds*.

t = -0.74 *sec* corresponds to a crossing before t = 0.

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



5 options

A speeder driving at a constant v passes a hidden police car. He does not see the police car and continues on at constant speed. After a delay of time T to check her radar gun and start her engine, the police officer begins to accelerate at constant acceleration a to catch the speeder. $t = 0$ is defined as the time the police car starts to accelerate.

Which of the following equations should you use to solve for the time $\underline{t}$ that it takes the police car to catch the speeder?

A. $\frac{1}{2}at^2 = vt$

B. $\frac{1}{2}at^2 = v(t - T)$

C. $\frac{1}{2}a(t - T)^2 = v(t + T)$

D. $\frac{1}{2}at^2 = v(t + T)$

E. None of the above.

A speeder driving at a constant v passes a hidden police car. She does not see the police car nor does she notice it as the police car begins a time T later to accelerate at constant acceleration a from rest to catch her. Which of the following equations is the correct one to solve for the time t that it takes the police car to catch the speeder, where $t=0$ is the time the police car begins to accelerate?

A. $\frac{1}{2}at^2 = vt$

B. $\frac{1}{2}at^2 = v(t - T)$

C. $\frac{1}{2}a(t - T)^2 = v(t + T)$

D. $\frac{1}{2}at^2 = v(t + T)$

E. None of the above.

$x(t)$ for the police officer is $x = x_o + v_o t + \frac{1}{2}at^2 = \frac{1}{2}at^2$

with $x_o = 0$ & $v_o = 0$

$x(t)$ for the speeder is $x = x_o + vt = vT + vt = v(t + T)$

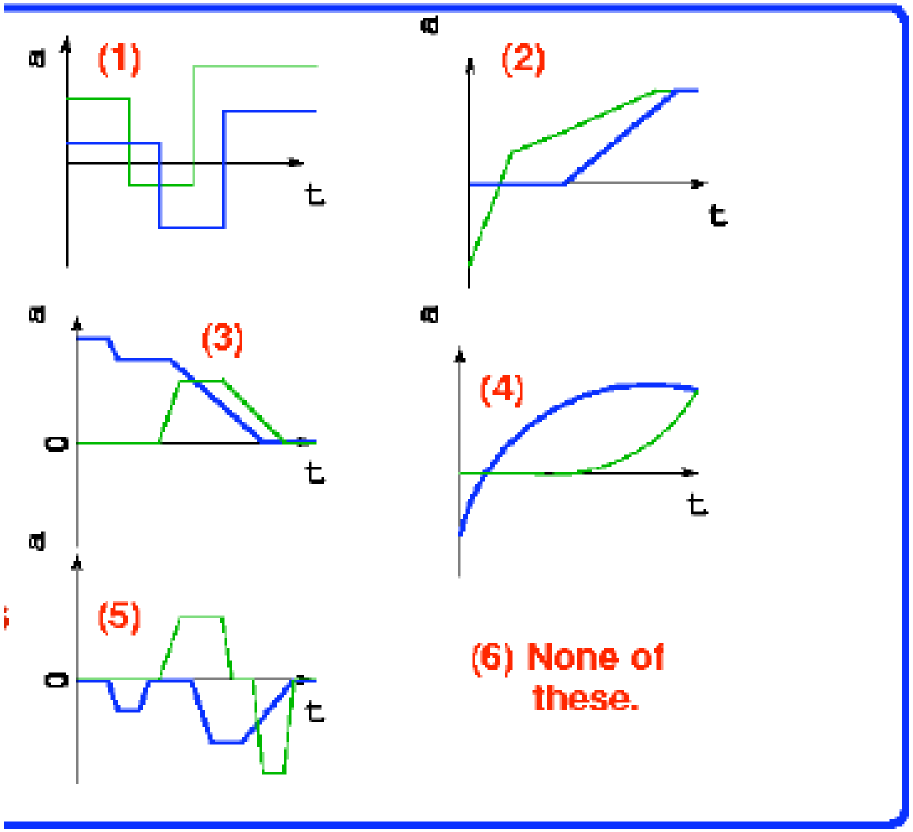
You can think of $x(t)$ for the speeder either as:

$x_{o\text{-}speeder} = vT$ *or*

$x_{o\text{-}speeder} = 0$ but $t_{speeder} = (t + T)$

6 options

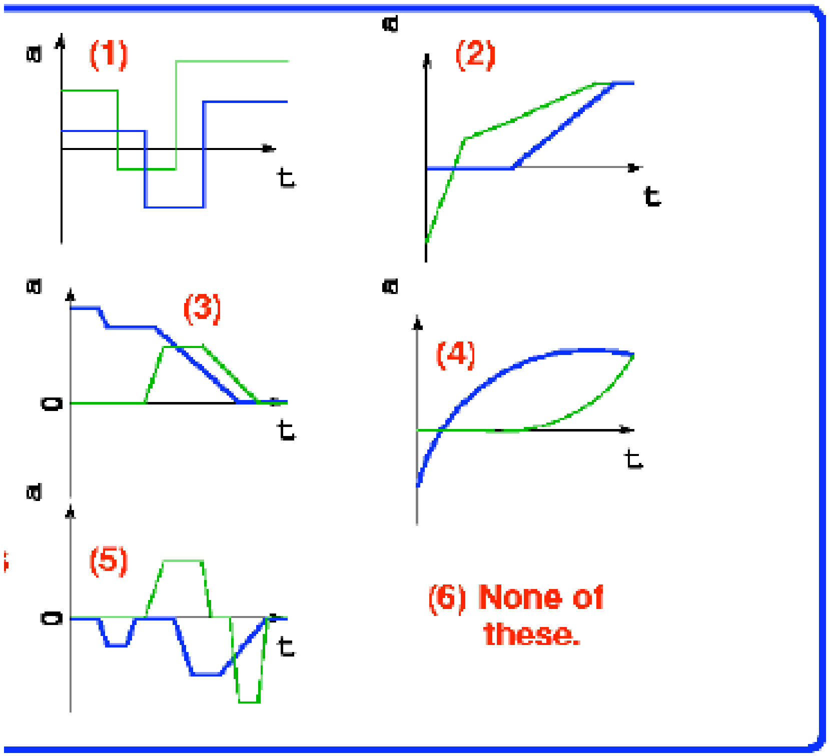
Sam is speeding down the road when he notices a police car with a radar gun stopped by the side of the road, waiting to ambush him. Sam slows down to the speed limit, but too late, the police car pulls out, accelerates to catch him and forces Sam to a stop. Which of the plots on the right might describe the **<u>acceleration as a function of time</u>** for the two cars during this sequence?



-You are not told which curve belongs to which car; figure it out.

6 options

Sam is speeding down the road when he notices a police car with a radar gun, stopped by the side of the road, waiting to ambush him. Sam slows down to the speed limit, but too late, the police car pulls out, accelerates to catch him and forces Sam to a stop. Which of the plots on the right might describe the acceleration as a function of time for the two cars during this sequence?



The answer is #5, with Police = green Sam = blue

Considering the police car alone eliminates #1 - #4. The police car has zero acceleration at first, then positive acceleration for awhile to catch the speeder and then negative acceleration down to zero speed as both cars stop. The only curve that satisfies this is the green one in (5).

Sam is initially moving at constant (*too high*) speed which corresponds to 0 acceleration, notices the police car and decelerates with a negative a to the speed limit and then has to decelerate to a stop when this doesn't work.

GENERAL INTERSECTIONS in 1D

Two objects are accelerating at rates a_1 & a_2
starting at times t_1 & t_2
from positions x_{1o} & x_{2o}
with velocities v_{1o} & v_{2o}

<u>WHEN</u> and <u>WHERE</u> will their paths intersect?
or how far apart will they be at time t_3?
or when or where will they have the same velocity?
etc. etc.

GENERALINTERSECTIONS in 1D

cultural interlude

One option is to consider the motion from the '*rest frame*' of reference of one of the objects instead of from the '*lab frame*'.

We'll introduce this method in chapter #4, with

$$\vec{R}_{AB} = \vec{R}_{A-lab} + \vec{R}_{lab-B} \qquad \vec{v}_{AB} = \vec{v}_{A-lab} + \vec{v}_{lab-B} \qquad \vec{a}_{AB} = \vec{a}_{A-lab} + \vec{a}_{lab-B}$$

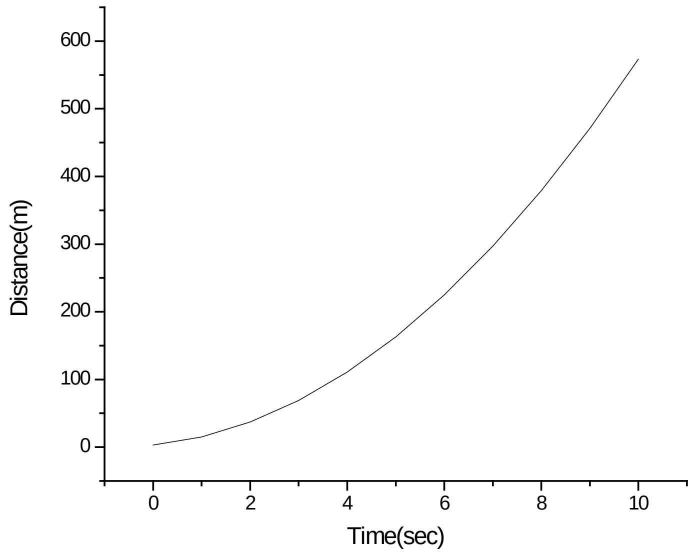
- You then have only one moving object to consider and can transfer back to the lab frame when you are done.
- Most people find this more challenging than solving directly from the lab frame of reference – particularly since your rest frame is itself accelerating!
- So we'll stick with the lab frame analysis until we have a chance to review concepts of relative motion.

GENERAL INTERSECTIONS in 1D
Graphical Solution

The general formula for $x(t)$ for motion in 1-D with constant acceleration is

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

The plot on this page was made with Origin using parameter values of x_o = 3 *m*, v_o = 7 *m/s*, a = 10 *m/s*^{2,} t ranging from 0 to 10 sec.



<u>VARYING THE PARAMETERS WOULD</u>:

$x_o \rightarrow$ shift the curve up or down

$v_o \rightarrow$ shift the minimum of the parabola by

$t = -v_o/a$ (*from dx/dt = 0 for a max or min*)

$a \rightarrow$ vary the curvature of the parabola, making it wider for smaller a or narrower for larger a

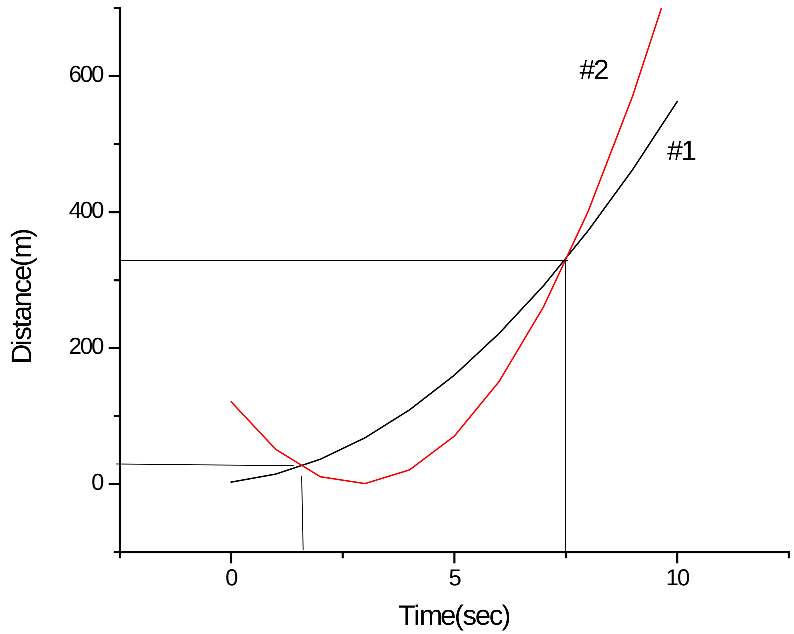
$t \rightarrow$ shift the curve left or right.

GENERAL INTERSECTIONS in 1D
Graphical Solution

If a second object with constant acceleration is added to this plot.

- Intersections on the plot → intersections of the objects.
  - *There may be 0, 1 or 2 intersections depending on the parameters & the range of times you plot*
- If the slopes are parallel at some t, the velocities are equal at that t.

#1	#2
$x_o = 3\ m$	$1\ m$
$v_o = 7\ m/s$	$5\ m/s$
$a = 10\ m/s^2$	$30\ m/s^2$
t from 0 to 10 *sec*	starts 3 *s* later



There are intersections at approximately (*using Origin's data point reader*)
1.5 *sec*, 25 *m* & 7.6 *sec*, 350 *m*

INTERSECTIONS in 1D
Analytic Solution

$$x = x_o + v_o t + \tfrac{1}{2} a t^2$$

The equations of motion for these objects are (*minus the units*):

$$x_1 = 3 + 7t + 5t^2 \qquad x_2 = 1 + 5(t-3) + 15(t-3)^2$$

You can solve for intersections simply by setting these equal, calculating t and then plugging t into either equation to find x.

$$3 + 7t + 5t^2 = 1 + 5(t-3) + 15(t-3)^2 = 1 + (5t - 15) + (15t^2 - 90t + 135)$$

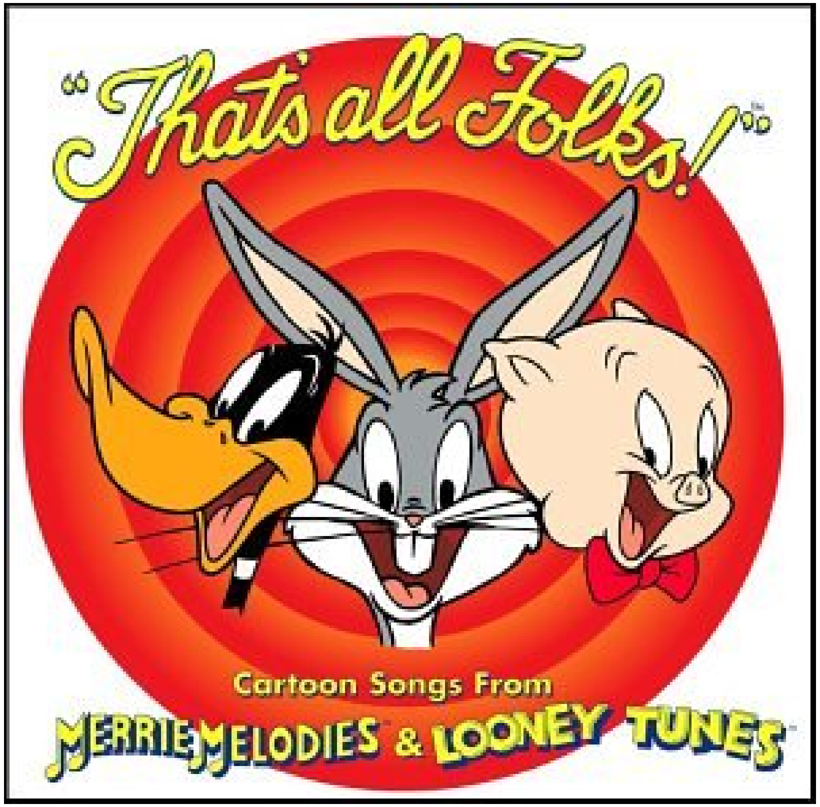
$$3 + 7t + 5t^2 = 121 + -85t + 15t^2$$

$$10t^2 - 92t + 118 = 0$$

$$t = \frac{92 \pm \sqrt{92^2 - 4 \cdot 10 \cdot 118}}{2 \cdot 10} = \frac{92 \pm 61.2}{20} = 7.66 \text{ sec or } 1.54 \text{ sec}$$

Plug these times into either of the $x(t)$ formulae to find x.
1.54 *sec* $\rightarrow$ 25.6 *m* & 7.66 *sec* $\rightarrow$ 350.0 *m*

THE END!



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