Homework2

April 8, 2025

1 Advanced Macroeconomics 1: Homework 2

1.1 Settings

[1]:
```python
##################################################
#   Package Imports
##################################################

### Data manipulation ###
import pandas as pd  # DataFrame handling
import numpy as np  # Numerical operations
import random  # For reproducibility

### Visualization ###
import matplotlib.pyplot as plt  # General plotting
import seaborn as sns  # Statistical data visualization

### Time Series & Statistical Models ###
import statsmodels.api as sm  # Statistical models
from statsmodels.tsa.stattools import (
    adfuller,  # Augmented Dickey-Fuller Test
    kpss,      # KPSS Test
    acf, pacf  # Autocorrelation functions
)
from statsmodels.tsa.arima.model import ARIMA  # ARIMA model
from statsmodels.graphics.tsaplots import (
    plot_acf,  # ACF plot
    plot_pacf  # PACF plot
)
from statsmodels.stats.diagnostic import acorr_ljungbox  # Ljung-Box test

from arch.unitroot import PhillipsPerron, ZivotAndrews  # PP and Zivot-Andrews␣
 ↪tests

### Machine Learning Metrics ###
from sklearn.metrics import mean_squared_error  # Evaluation metric

### Extra ###
```

```
import warnings
warnings.filterwarnings("ignore")

### External Data Sources (optional) ###
import ipeadatapy as ipea  # For accessing IPEA data (if needed)

# Set seed for reproducibility
seed = 42
np.random.seed(seed)
random.seed(seed)
```

[2]:
```
rgb_color = (162 / 255, 37 / 255, 56 / 255)  # Red
```

[3]:
```
warnings.filterwarnings("ignore")
```

1.2 Questions

1.2.1 Question 1

Using the time series provided in the course drive (in `.txt`, `.csv`, and `.xls` formats), perform unit root tests and verify the stationarity of each series. Indicate whether each series is stationary or not.

Apply the following tests: - Augmented Dickey-Fuller (ADF) - Phillips-Perron (PP) - Kwiatkowski–Phillips–Schmidt–Shin (KPSS) - Zivot-Andrews test for structural breaks - Any other relevant tests you consider necessary

Include your code in the appendix. Be prepared to explain your results and code if asked.

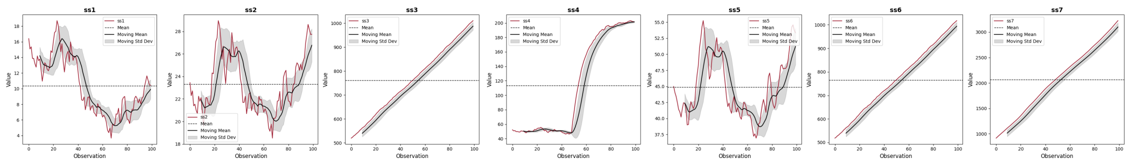
Data

[4]:
```
# Load the data
file_path = r"data_exercicio.csv"
df = pd.read_csv(file_path)

# Set index to a RangeIndex
df.index = pd.RangeIndex(start=0, stop=len(df), step=1)
```

Plot the series

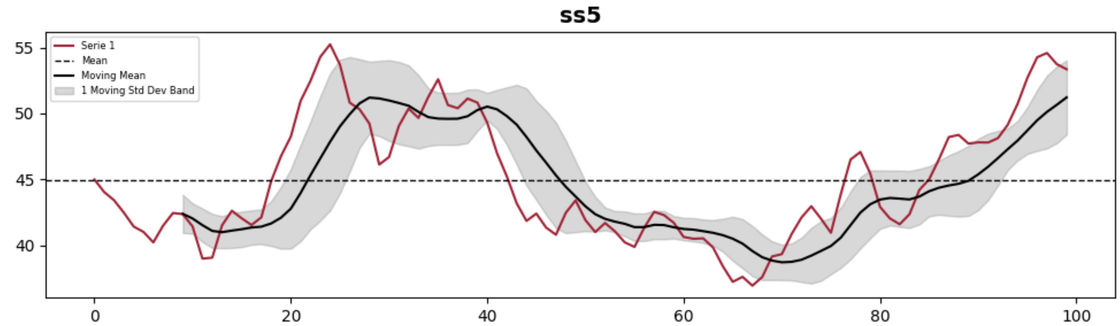
[5]:
```
# Define the columns of the original series (assuming they are the first 4)
series_columns = 'ss1', 'ss2', 'ss3', 'ss4', 'ss5', 'ss6', 'ss7'

# Set the window size for the moving average and moving standard deviation
window_size = 10



# Create side-by-side subplots (1 row, one column for each series)
fig, axes = plt.subplots(1, len(series_columns), figsize=(5 *␣
↪len(series_columns), 5), sharex=True)
```

```python
# If there is only one series, wrap the axis in a list to maintain the loop
if len(series_columns) == 1:
    axes = [axes]

# Loop to generate the plot for each series
for ax, column in zip(axes, series_columns):
    # Calculate the moving average and moving standard deviation locally
    moving_mean = df[column].rolling(window=window_size).mean()
    moving_std = df[column].rolling(window=window_size).std()

    # Calculate the overall mean of the original series
    mean_value = df[column].mean()

    # Plot the original series
    ax.plot(df.index, df[column], label=column, color = rgb_color ,linewidth=1.
 ↪5)

    # Draw a horizontal line indicating the mean
    ax.axhline(mean_value, color='black', linestyle='--', linewidth=1,␣
 ↪label='Mean')

    # Plot the moving average
    ax.plot(df.index, moving_mean, color='black', linewidth=1.5, linestyle='-',␣
 ↪label='Moving Mean')

    # Plot the moving standard deviation band
    ax.fill_between(df.index, moving_mean - moving_std, moving_mean +␣
 ↪moving_std,
                    color='grey', alpha=0.3, label='Moving Std Dev')

    # Remove background and extra borders using the current subplot (ax)
    ax.set_facecolor('white')         # Remove background color

    # Set title, labels, and grid
    ax.set_title(column, fontsize=14, fontweight='bold')
    ax.set_xlabel("Observation", fontsize=12)
    ax.set_ylabel("Value", fontsize=12)
    ax.legend()
    ax.grid(False)

plt.tight_layout()
plt.show()
```



[6]:
```
# Graph for specific serie
column = "ss5"

# Define the moving window size
window_size = 10

# Calculate moving average and moving standard deviation
df['Moving_Mean'] = df[column].rolling(window=window_size).mean() # Not using
df['Moving_Std'] = df[column].rolling(window=window_size).std()

# Calculate mean and standard deviation
mean_value = df[column].mean()
std_dev = df[column].std()

# Create the plot
plt.figure(figsize=(12, 3))
plt.plot(df.index, df[column], color=rgb_color, linewidth=1.5, label="Serie 1")
plt.axhline(mean_value, color='black', linestyle='--', linewidth=1,␣
 ↪label='Mean')  # Dotted line for mean

# Plot standard deviation bands
#plt.fill_between(df.index, mean_value - std_dev, mean_value + std_dev,␣
 ↪color='gray', alpha=0.3, label='1 Std Dev Band')

# Plot moving average
plt.plot(df.index, df['Moving_Mean'], color='black', linewidth=1.5,␣
 ↪linestyle='-', label="Moving Mean")

# Plot moving standard deviation bands
plt.fill_between(df.index,
                 df['Moving_Mean'] - df['Moving_Std'],
                 df['Moving_Mean'] + df['Moving_Std'],
                 color='grey', alpha=0.3, label='1 Moving Std Dev Band')

# Add titles and labels
plt.title(column, fontsize=14, fontweight='bold')
plt.xlabel('', fontsize=12)
plt.ylabel('', fontsize=14)
plt.legend(fontsize = 6)
```

```
plt.show()
```



Stationarity Analysis of Time Series

[7]:
```
def stationarity_tests_verbose(series, name):
    results = {}

    # ADF (Augmented Dickey-Fuller)
    try:
        adf_p = adfuller(series.dropna(), autolag='AIC')[1]
        results['ADF'] = (
            "stationary (unit root rejected)" if adf_p < 0.05
            else "non-stationary (unit root not rejected)"
        )
    except:
        results['ADF'] = "error"

    # DF (Dickey-Fuller classic)
    try:
        df_p = adfuller(series.dropna(), maxlag=0, regression='c')[1]
        results['DF'] = (
            "stationary (unit root rejected)" if df_p < 0.05
            else "non-stationary(unit root not rejected)"
        )
    except:
        results['DF'] = "error"

    # PP
    try:
        pp_p = PhillipsPerron(series.dropna()).pvalue
        results['PP'] = (
            "stationary (unit root rejected)" if pp_p < 0.05
            else "non-stationary (unit root not rejected)"
        )
    except:
```

```
        results['PP'] = "error"

    # KPSS
    try:
        kpss_p = kpss(series.dropna(), regression='c', nlags='auto')[1]
        results['KPSS'] = (
            "stationary (null of stationarity not rejected)" if kpss_p > 0.05
            else "non-stationary (null of stationarity rejected)"
        )
    except:
        results['KPSS'] = "error"

    # Zivot-Andrews
    try:
        za_p = ZivotAndrews(series.dropna()).pvalue
        results['Zivot-Andrews'] = (
            "stationary with structural break (unit root rejected)" if za_p < 0.
↪05
            else "non-stationary (unit root not rejected)"
        )
    except:
        results['Zivot-Andrews'] = "error"

    return pd.Series(results, name=name)
```

[30]:
```
series_names = ['ss1', 'ss2', 'ss3', 'ss4', 'ss5', 'ss6', 'ss7']

results_df = pd.DataFrame()

for col in series_names:
    results_df = pd.concat([results_df, stationarity_tests_verbose(df[col],␣
↪col)], axis=1)

for col in series_names:
    print(f"Results for {col}")
    print(stationarity_tests_verbose(df[col], col))
    print("\n" + "-"*50 + "\n")
```

```
Results for ss1
ADF                        non-stationary (unit root not rejected)
DF                         non-stationary (unit root not rejected)
PP                         non-stationary (unit root not rejected)
KPSS                non-stationary (null of stationarity rejected)
Zivot-Andrews              non-stationary (unit root not rejected)
Name: ss1, dtype: object

--------------------------------------------------
```

```
Results for ss2
ADF                       non-stationary (unit root not rejected)
DF                        non-stationary (unit root not rejected)
PP                        non-stationary (unit root not rejected)
KPSS               stationary (null of stationarity not rejected)
Zivot-Andrews             non-stationary (unit root not rejected)
Name: ss2, dtype: object

--------------------------------------------------

Results for ss3
ADF                       non-stationary (unit root not rejected)
DF                        non-stationary (unit root not rejected)
PP                        non-stationary (unit root not rejected)
KPSS               non-stationary (null of stationarity rejected)
Zivot-Andrews             non-stationary (unit root not rejected)
Name: ss3, dtype: object

--------------------------------------------------

Results for ss4
ADF                           non-stationary (unit root not rejected)
DF                            non-stationary (unit root not rejected)
PP                            non-stationary (unit root not rejected)
KPSS                   non-stationary (null of stationarity rejected)
Zivot-Andrews    stationary with structural break (unit root re…
Name: ss4, dtype: object

--------------------------------------------------

Results for ss5
ADF                       non-stationary (unit root not rejected)
DF                        non-stationary (unit root not rejected)
PP                        non-stationary (unit root not rejected)
KPSS               stationary (null of stationarity not rejected)
Zivot-Andrews             non-stationary (unit root not rejected)
Name: ss5, dtype: object

--------------------------------------------------

Results for ss6
ADF                       non-stationary (unit root not rejected)
DF                        non-stationary (unit root not rejected)
PP                        non-stationary (unit root not rejected)
KPSS               non-stationary (null of stationarity rejected)
Zivot-Andrews             non-stationary (unit root not rejected)
Name: ss6, dtype: object
```

```
--------------------------------------------------

Results for ss7
ADF                        non-stationary (unit root not rejected)
DF                         non-stationary (unit root not rejected)
PP                         non-stationary (unit root not rejected)
KPSS                non-stationary (null of stationarity rejected)
Zivot-Andrews              non-stationary (unit root not rejected)
Name: ss7, dtype: object

--------------------------------------------------
```

1.2.2 Question 2

Using the time series obtained from the IPEADATA website, apply unit root tests to the following variables:

- Gross Domestic Product (GDP) at market prices: chained volume index (base year 1995 = 100)
- IPCA - General Price Index (base December 1993 = 100)

Determine whether each series is stationary or not.

Apply the following tests: - Augmented Dickey-Fuller (ADF) - Phillips-Perron (PP) - Kwiatkowski–Phillips–Schmidt–Shin (KPSS) - Zivot-Andrews test for structural breaks - Any other relevant tests you consider necessary

Include your code in the appendix. Be prepared to explain your results and interpretations if asked.

Data

[33]:
```python
# Here we use a keyword to search for available time series or browse them␣
↪manually on the IPEA website
ipea.metadata()

# GDP series
ipea.describe('SCN104_PIBPM104')
```

[33]:
```
               PIB - preços de mercado - índice real encadeado (média 1995 =
100)
Name            PIB - preços de mercado - índice real encadead…
Code                                             SCN104_PIBPM104
Big Theme                                          Macroeconômico
Theme code                                                      8
Source          Instituto Brasileiro de Geografia e Estatístic…
Source acronym                                    IBGE/SCN Trim.
Comment         O produto interno bruto (PIB) é o total dos be…
Last update                         2025-03-07T13:32:00.727-03:00
Frequency                                              Trimestral
Measure                                                         -
```

```
Unit                                                              None
Status                                                               A
```

[34]:
```python
# IPCA series
ipea.describe('PRECOS12_IPCA12')
```

[34]:
```
                          IPCA - geral - índice (dez. 1993 = 100)
Name                      IPCA - geral - índice (dez. 1993 = 100)
Code                                              PRECOS12_IPCA12
Big Theme                                          Macroeconômico
Theme code                                                      9
Source          Instituto Brasileiro de Geografia e Estatístic…
Source acronym                                        IBGE/SNIPC
Comment         O Índice Nacional de Preços ao Consumidor Ampl…
Last update                         2025-03-12T16:26:01.02-03:00
Frequency                                                  Mensal
Measure                                                         -
Unit                                                         None
Status                                                          A
```

Plot the series

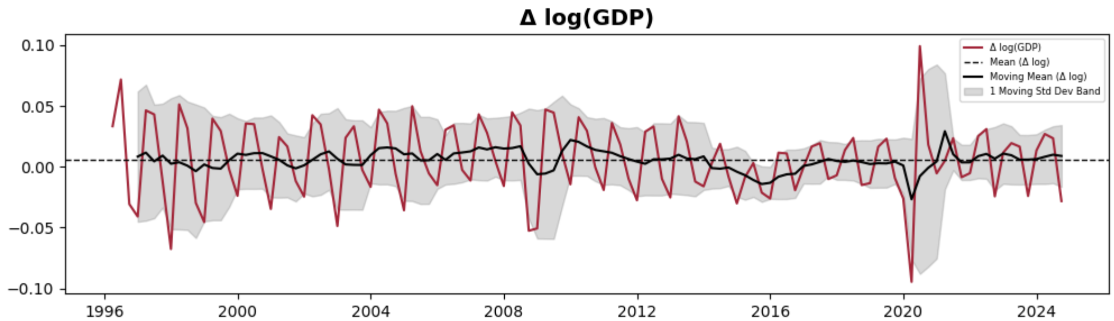
[35]:
```python
gdp = ipea.timeseries('SCN104_PIBPM104')

gdp = gdp.rename(columns={
    'RAW DATE': 'date',
    'VALUE (-)': 'GDP',
    'CODE': 'code',
    'DAY': 'day',
    'MONTH': 'month',
    'YEAR': 'year'
})
################ Plot ################

column = "GDP"

# Define the moving window size
window_size = 4

# Calculate moving average and moving standard deviation
gdp['Moving_Mean'] = gdp[column].rolling(window=window_size).mean()
gdp['Moving_Std'] = gdp[column].rolling(window=window_size).std()

# Calculate mean and standard deviation
mean_value = gdp[column].mean()
std_dev = gdp[column].std()

# Create the plot
```

```
plt.figure(figsize=(12, 3))
plt.plot(gdp.index, gdp[column], color=rgb_color, linewidth=1.5, label="GDP")
plt.axhline(mean_value, color='black', linestyle='--', linewidth=1,␣
 ↪label='Mean')  # Dotted line for mean

# Plot standard deviation bands
#plt.fill_between(gdp.index, mean_value - std_dev, mean_value + std_dev,␣
 ↪color='gray', alpha=0.3, label='1 Std Dev Band')

# Plot moving average
plt.plot(gdp.index, gdp['Moving_Mean'], color='black', linewidth=1.5,␣
 ↪linestyle='-', label="Moving Mean")

# Plot moving standard deviation bands
plt.fill_between(gdp.index,
                 gdp['Moving_Mean'] - gdp['Moving_Std'],
                 gdp['Moving_Mean'] + gdp['Moving_Std'],
                 color='grey', alpha=0.3, label='1 Moving Std Dev Band')

# Add titles and labels
plt.title(column, fontsize=14, fontweight='bold')
plt.xlabel('', fontsize=12)
plt.ylabel('', fontsize=14)
plt.legend(fontsize = 6)
plt.show()
```

[36]:
```
gdp = ipea.timeseries('SCN104_PIBPM104')

gdp = gdp.rename(columns={
    'RAW DATE': 'date',
    'VALUE (-)': 'GDP',
    'CODE': 'code',
    'DAY': 'day',
    'MONTH': 'month',
```

```
    'YEAR': 'year'
})

################ Plot Growth Rate ################
# ------ Difference of logs over time represents the growth rate ------

column = "GDP"

# Apply log transformation to the GDP column
gdp['Log_GDP'] = np.log(gdp[column])

# Calculate the first difference of the log
gdp['Log_Diff'] = gdp['Log_GDP'].diff()

# Define the moving window size
window_size = 4

# Calculate moving average and moving standard deviation of the differenced log␣
↪series
gdp['Moving_Mean'] = gdp['Log_Diff'].rolling(window=window_size).mean()
gdp['Moving_Std'] = gdp['Log_Diff'].rolling(window=window_size).std()

# Calculate overall mean and standard deviation of the differenced log series
mean_value = gdp['Log_Diff'].mean()
std_dev = gdp['Log_Diff'].std()

# Create the plot
plt.figure(figsize=(12, 3))

# Plot the differenced log series
plt.plot(gdp.index, gdp['Log_Diff'], color=rgb_color, linewidth=1.5, label="Δ␣
↪log(GDP)")

# Horizontal line for the overall mean
plt.axhline(mean_value, color='black', linestyle='--', linewidth=1, label='Mean␣
↪(Δ log)')

# Plot the moving average
plt.plot(gdp.index, gdp['Moving_Mean'], color='black', linewidth=1.5,␣
↪linestyle='-', label="Moving Mean (Δ log)")

# Plot the moving standard deviation band
plt.fill_between(gdp.index,
                 gdp['Moving_Mean'] - gdp['Moving_Std'],
                 gdp['Moving_Mean'] + gdp['Moving_Std'],
                 color='grey', alpha=0.3, label='1 Moving Std Dev Band')
```

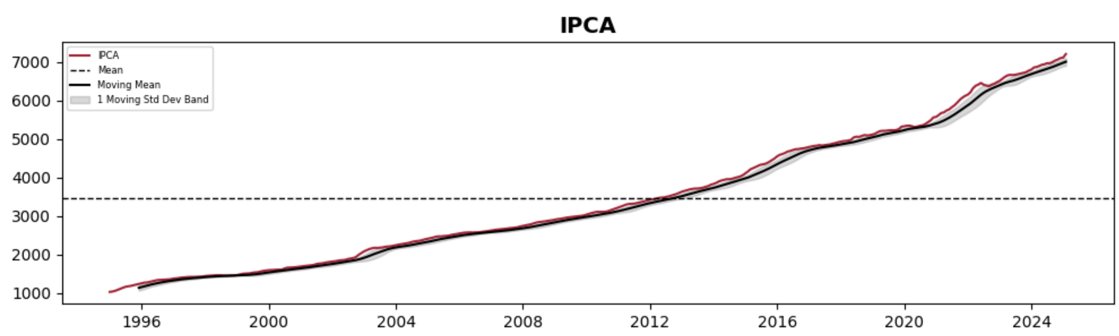
```python
# Add title and legend
plt.title('Δ log(' + column + ')', fontsize=14, fontweight='bold')
plt.xlabel('', fontsize=12)
plt.ylabel('', fontsize=14)
plt.legend(fontsize=6)
plt.show()
```



[37]:

```python
ipca = ipea.timeseries('PRECOS12_IPCA12')

ipca = ipca.rename(columns={
    'RAW DATE': 'date',
    'VALUE (-)': 'IPCA',
    'CODE': 'code',
    'DAY': 'day',
    'MONTH': 'month',
    'YEAR': 'year'
})
################ Plot ################


column = "IPCA"

ipca = ipca[ipca.index >= '1995-01-01']

# Define the moving window size
window_size = 12

# Calculate moving average and moving standard deviation
ipca['Moving_Mean'] = ipca[column].rolling(window=window_size).mean()
ipca['Moving_Std'] = ipca[column].rolling(window=window_size).std()

# Calculate mean and standard deviation
mean_value = ipca[column].mean()
std_dev = ipca[column].std()
```

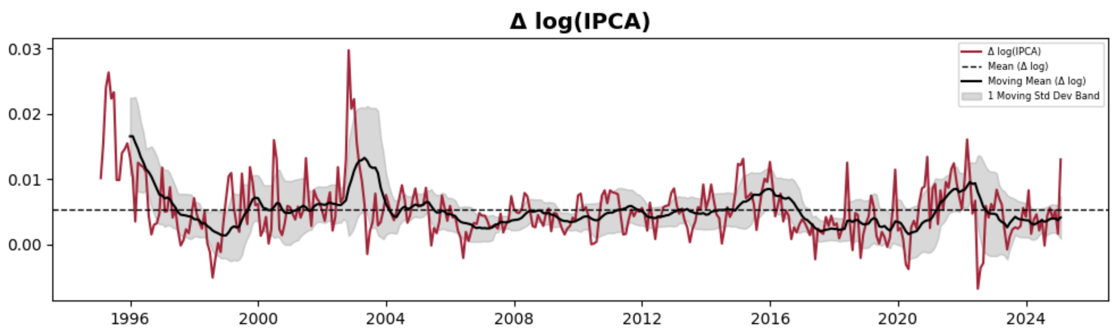
```
# Create the plot
plt.figure(figsize=(12, 3))
plt.plot(ipca.index, ipca[column], color=rgb_color, linewidth=1.5, label="IPCA")
plt.axhline(mean_value, color='black', linestyle='--', linewidth=1,␣
↪label='Mean')  # Dotted line for mean

# Plot standard deviation bands
#plt.fill_between(ipca.index, mean_value - std_dev, mean_value + std_dev,␣
↪color='gray', alpha=0.3, label='1 Std Dev Band')

# Plot moving average
plt.plot(ipca.index, ipca['Moving_Mean'], color='black', linewidth=1.5,␣
↪linestyle='-', label="Moving Mean")

# Plot moving standard deviation bands
plt.fill_between(ipca.index,
                 ipca['Moving_Mean'] - ipca['Moving_Std'],
                 ipca['Moving_Mean'] + ipca['Moving_Std'],
                 color='grey', alpha=0.3, label='1 Moving Std Dev Band')

# Add titles and labels
plt.title(column, fontsize=14, fontweight='bold')
plt.xlabel('', fontsize=12)
plt.ylabel('', fontsize=14)
plt.legend(fontsize = 6)
plt.show()
```



[38]:
```
ipca = ipea.timeseries('PRECOS12_IPCA12')

ipca = ipca.rename(columns={
    'RAW DATE': 'date',
    'VALUE (-)': 'IPCA',
    'CODE': 'code',
```

```
    'DAY': 'day',
    'MONTH': 'month',
    'YEAR': 'year'
})

################ Plot Growth Rate ################
# ------ Difference of logs over time represents the growth rate ------

column = "IPCA"

# Filter the data from 1995 onwards
ipca = ipca[ipca.index >= '1995-01-01'].copy()

# Apply log transformation to the IPCA column
ipca['Log_IPCA'] = np.log(ipca[column])

# Calculate the first difference of the log
ipca['Log_Diff'] = ipca['Log_IPCA'].diff()

# Define the moving window size
window_size = 12

# Calculate moving average and moving standard deviation of the differenced log␣
↪series
ipca['Moving_Mean'] = ipca['Log_Diff'].rolling(window=window_size).mean()
ipca['Moving_Std'] = ipca['Log_Diff'].rolling(window=window_size).std()

# Calculate overall mean and standard deviation of the differenced log series
mean_value = ipca['Log_Diff'].mean()
std_dev = ipca['Log_Diff'].std()

# Create the plot
plt.figure(figsize=(12, 3))

# Plot the differenced log series
plt.plot(ipca.index, ipca['Log_Diff'], color=rgb_color, linewidth=1.5, label="Δ␣
↪log(IPCA)")

# Horizontal line for the overall mean
plt.axhline(mean_value, color='black', linestyle='--', linewidth=1, label='Mean␣
↪(Δ log)')

# Plot the moving average
plt.plot(ipca.index, ipca['Moving_Mean'], color='black', linewidth=1.5,␣
↪linestyle='-', label="Moving Mean (Δ log)")

# Plot the moving standard deviation band
```

```
plt.fill_between(ipca.index,
                 ipca['Moving_Mean'] - ipca['Moving_Std'],
                 ipca['Moving_Mean'] + ipca['Moving_Std'],
                 color='grey', alpha=0.3, label='1 Moving Std Dev Band')

# Add title and legend
plt.title('Δ log(' + column + ')', fontsize=14, fontweight='bold')
plt.xlabel('', fontsize=12)
plt.ylabel('', fontsize=14)
plt.legend(fontsize=6)
plt.show()
```



Stationarity Analysis of Time Series

[39]:
```
def stationarity_tests_verbose(series, name):
    results = {}

    # ADF (Augmented Dickey-Fuller)
    try:
        adf_p = adfuller(series.dropna(), autolag='AIC')[1]
        results['ADF'] = (
            "stationary (unit root rejected)" if adf_p < 0.05
            else "non-stationary (unit root not rejected)"
        )
    except:
        results['ADF'] = "error"

    # DF (Dickey-Fuller classic)
    try:
        df_p = adfuller(series.dropna(), maxlag=0, regression='c')[1]
        results['DF'] = (
            "stationary (unit root rejected)" if df_p < 0.05
            else "non-stationary (unit root not rejected)"
        )
    except:
```

```
        results['DF'] = "error"

    # PP
    try:
        pp_p = PhillipsPerron(series.dropna()).pvalue
        results['PP'] = (
            "stationary (unit root rejected)" if pp_p < 0.05
            else "non-stationary (unit root not rejected)"
        )
    except:
        results['PP'] = "error"

    # KPSS
    try:
        kpss_p = kpss(series.dropna(), regression='c', nlags='auto')[1]
        results['KPSS'] = (
            "stationary (null of stationarity not rejected)" if kpss_p > 0.05
            else "non-stationary (null of stationarity rejected)"
        )
    except:
        results['KPSS'] = "error"

    # Zivot-Andrews
    try:
        za_p = ZivotAndrews(series.dropna()).pvalue
        results['Zivot-Andrews'] = (
            "stationary with structural break (unit root rejected)" if za_p < 0.
↪05
            else "non-stationary (unit root not rejected)"
        )
    except:
        results['Zivot-Andrews'] = "error"

    return pd.Series(results, name=name)
```

[42]:

```
# Run tests on GDP and Log_GDP
gdp_result = stationarity_tests_verbose(gdp['GDP'], name='GDP')
log_gdp_result = stationarity_tests_verbose(gdp['Log_GDP'], name='Log_GDP')

# Run tests on IPCA and Log_IPCA
ipca_result = stationarity_tests_verbose(ipca['IPCA'], name='IPCA')
log_ipca_result = stationarity_tests_verbose(ipca['Log_IPCA'], name='Log_IPCA')

# Exibir como texto, limpo para PDF
for name, result in [('GDP', gdp_result),
                     ('Log_GDP', log_gdp_result),
                     ('IPCA', ipca_result),
```

```python
                       ('Log_IPCA', log_ipca_result)]:
    print(f"Stationarity Test Results for {name}")
    print(result)
    print("\n" + "-"*60 + "\n")
```

```
Stationarity Test Results for GDP
ADF                        non-stationary (unit root not rejected)
DF                         non-stationary (unit root not rejected)
PP                         non-stationary (unit root not rejected)
KPSS                non-stationary (null of stationarity rejected)
Zivot-Andrews              non-stationary (unit root not rejected)
Name: GDP, dtype: object

------------------------------------------------------------


Stationarity Test Results for Log_GDP
ADF                        non-stationary (unit root not rejected)
DF                         non-stationary (unit root not rejected)
PP                         non-stationary (unit root not rejected)
KPSS                non-stationary (null of stationarity rejected)
Zivot-Andrews              non-stationary (unit root not rejected)
Name: Log_GDP, dtype: object

------------------------------------------------------------


Stationarity Test Results for IPCA
ADF                        non-stationary (unit root not rejected)
DF                         non-stationary (unit root not rejected)
PP                         non-stationary (unit root not rejected)
KPSS                non-stationary (null of stationarity rejected)
Zivot-Andrews              non-stationary (unit root not rejected)
Name: IPCA, dtype: object

------------------------------------------------------------


Stationarity Test Results for Log_IPCA
ADF                           non-stationary (unit root not rejected)
DF                                    stationary (unit root rejected)
PP                            non-stationary (unit root not rejected)
KPSS                   non-stationary (null of stationarity rejected)
Zivot-Andrews    stationary with structural break (unit root re…
Name: Log_IPCA, dtype: object

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```