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Total Valuation Adjustment with Collateralisation for Interest Rate Derivatives

Master's Thesis
Espoo, August 9, 2016

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ABSTRACT OF MASTER'S THESIS

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Title: Total Valuation Adjustment with Collateralisation for Interest Rate Derivatives			
Date:	August 9, 2016	**Pages:**	58
Major:	Systems and Operations Research	**Code:**	F3008
Supervisor:	Professor Ahti Salo		
Instructor:	M.Sc. (Tech.) Kimmo Lehikoinen		

Failures in financial risk management that led, e.g., to the credit crisis of 2008 have casted doubts on the practice. New efforts have been made, especially regarding counterparty credit risk (CCR), which is prevalent in the over the counter (OTC) derivatives market. After the credit crisis, the pricing of derivatives has changed considerably due to the CCR.

To obtain the truer market price of a derivative, the old pricing framework needs to be amended with credit valuation adjustment (CVA) – i.e. the price of CCR. However, this is not enough because – in addition to its counterparty – the bank itself can also go bankrupt. Hence, debt valuation adjustment (DVA), which refers to a party's own default risk, must also be priced in. These two together form a simplified total valuation adjustment (TVA).

In this thesis, this kind of TVA framework including collateralisation is presented for interested rate derivatives. Concerning this, firstly, literature is reviewed on part of the valuation adjustments. Secondly, the framework is introduced. Finally, the effects of collateralisation agreements on valuation adjustments are studied in an example case.

Keywords:	derivatives, counterparty credit risk, valuation adjustments
Language:	English

Aalto-yliopisto
Perustieteiden korkeakoulu
Teknillisen fysiikan ja matematiikan koulutusohjelma

DIPLOMITYÖN TIIVISTELMÄ

Tekijä:	Jaakko Reinvall		
Työn nimi: Vakuussopimusten käyttö korkojohdannaisten kokonaisarvonoikaisussa			
Päiväys:	9. elokuuta 2016	**Sivumäärä:**	58
Pääaine:	Systeemi- ja operaatiotutkimus	**Koodi:**	F3008
Valvoja:	Professori Ahti Salo		
Ohjaaja:	Diplomi-insinööri Kimmo Lehikoinen		

Rahoitusriskien hallinnan epäonnistumiset, jotka johtivat esimerkiksi vuoden 2008 luottokriisin syntymiseen, ovat saaneet epäilyt heräämään tätä käytännön harjoittamista kohtaan. Uusia ponnistuksia tehdään, erityisesti kohdistuen vastapuoliriskiin, joka on yleinen yksityisesti vaihdettujen johdannaisten markkinoilla. Johdannaisten hinnoittelu on muuttunut paljon luottokriisin jälkeen johtuen vastapuoliriskistä.

Oikeamman markkinahinnan saamiseksi johdannaiselle, vanhaa hinnoittelukehystä on parannettava ottaen huomioon vastapuolen luottoriskin arvonoikaisu. Tämä ei kuitenkaan riitä sillä vastapuolen lisäksi pankki itse voi myös mennä konkurssiin. Siten oman luottoriskin arvonoikaisu, joka viittaa pankin omaan luottotappioon, pitää myös hinnoitella. Nämä kaksi yhdessä muodostavat yksinkertaistetun kokonaisarvonoikaisun.

Tässä diplomityössä esitellään tämänkaltainen viitekehys vakuussopimukset sisältäen korkojohdannaisille. Tähän liittyen ensimmäiseksi tehdään kirjallisuuskatsaus arvonoikaisuista. Toiseksi perehdytään itse kehykseen. Lopuksi vakuussopimusten vaikutuksia arvonoikaisuihin tutkitaan esimerkkitapauksessa.

Asiasanat:	johdannaiset, vastapuoliriski, arvonoikaisut
Kieli:	Englanti

Acknowledgements

I would like to express my gratitude to my instructor, Kimmo Lehikoinen, and to my supervisor, Ahti Salo, for feedback, guidance and support through this thesis process.

Also, I am grateful to my family for their support during my studies.

Espoo, August 9, 2016

Jaakko Reinvall

Abbreviations and Acronyms

AVA	Additional Valuation Adjustment
BCVA	Bilateral Credit Valuation Adjustment
BSDE	Backward Stochastic Differential Equation
CCP	Central Clearing Party
CCR	Counterparty Credit Risk
CDS	Credit-Default Swap
CE	Current Exposure
CRA	Collateral Rate Adjustment
CSA	Credit Support Annex
CVA	Credit Valuation Adjustment
DVA	Debt Valuation Adjustment
EE	Expected Exposure
EPE	Expected Positive Exposure
FVA	Funding Valuation Adjustment
IRD	Interest Rate Derivative
ISDA	International Swaps and Derivatives Association, Inc.
KVA	Capital Valuation Adjustment
LIBOR	London Interbank Offered Rates
LMM	Libor Market Model
LVA	Liquidity Valuation Adjustment
MTA	Minimum Transfer Amount
OIS	Overnight Index Swap
OTC	Over The Counter
PFE	Potential Future Exposure
RC	Replacement Cost
RVA	Rating Valuation Adjustment
SABR	Stochastic, Alpha, Beta, Rho model

TBTF	Too Big To Fail
TVA	Total Valuation Adjustment
XVA	General acronym for different valuation adjustments

Contents

Chapter 1

Introduction

1.1 Derivatives Pricing

A financial derivative, or simply a *derivative* (as referred in this thesis) is a financial contract whose value is dependent on, or *derived* from, market factors, such as stock prices, interest rates and currency exchange rates. Derivatives can cause global economic crises if the risks related to them are badly managed. The credit crisis of 2008 and the ongoing eurozone crisis are examples of the far-reaching effects that failures in *financial risk management* – i.e. in the practice of controlling exposure to different kinds of financial risk – may produce. The credit crisis began from the US and spread to become a worldwide economic crisis affecting both the financial markets and the real economy. During that crisis, several financial institutions – including, among others, Bear Stearns, Fannie Mae and Freddie Mac – were bailed out by the U.S. Government, because they were considered systematically too important to be allowed to go bankrupt. Often, the term *too big to fail* (TBTF) is used to refer to these kinds of companies. Taxpayers' money has also been needed to support failing banks and now also countries in the eurozone crisis. Because of these crises, not even the institutions with the highest credit ratings can be regarded as risk-free any more and, moreover, derivatives market cannot credibly have financial institutions relying on TBTF argument as its basis. Thus, the importance of financial risk management is at its all-time high.

Derivatives have been significantly involved in these crises. For example, Lehman Brothers, a bank whose bankruptcy came to symbolize the credit crisis, had a notional value of 800 billion USD in *over the counter* (OTC) derivatives – bilateral privately-negotiated contracts – when it went bankrupt. At a high level, the issues of derivatives have to do with leverage, complexity related both to these products and to the financial network and the constantly changing financial markets. Furthermore, due to their bilateral nature, OTC derivatives are subject to *counterparty credit risk* (CCR), which is the risk of a default of one of the parties in a financial contract between two parties, called the party and the counterparty throughout this thesis. Although in most cases only the party's point of view is considered, it is good to remember that from the counterparty's viewpoint the party is its counterparty equivalently. Significant CCRs in the financial markets have the potential to cause systemic crisis because the default of a large institution can be the first domino to fall in this highly interconnected system.

The limelight was put on CCR in the most recent financial crises. In other words, the possibility of a counterparty default became very much a reality in the aftermath of the crises. This affected the pricing of derivatives so that it became necessary to take into account counterparty risk to obtain their true market value. Subtracting the price of counterparty risk – called *credit valuation adjustment* (CVA) – from the value of a derivative as calculated before the crises should have then given the "correct" price. Yet, a party itself also has a default risk – this is a counterparty risk viewed from the counterparty's point of view – which had to be priced in, too. The price of financial institution's own default risk is referred to as *debt valuation adjustment* (DVA). Hence, together these two adjustments cover the default risk of both parties and the combined adjustment for this is called *bilateral credit valuation adjustment* (BCVA).

Regardless of the improvement to pricing derivatives by BCVA, the valuation framework was incomplete, because other issues needed to be addressed as well. Next to the "list" of adjustments was added *funding valuation adjustment* (FVA), which takes into account funding costs of derivative transactions, it was followed by many other aspects affecting the valuation of derivatives including *liquidity valuation adjustment* (LVA), *replacement cost* (RC), *overnight index swap* (OIS) rates versus *London interbank offered rates* (LIBOR), *capital valuation adjustment* (KVA), *additional valuation adjustment* (AVA), *rating valuation adjustment* (RVA), *collateral rate adjustment* (CRA), etc. (Crépey et al., 2014). The term XVA is generally used to refer to these different pricing adjustments (Carver, 2013). Furthermore, Crépey et al. (2014, p. 63) refer to the aggregate value of XVAs with *total valuation adjustment* (TVA). Crépey et al. (2014, p. xvi) stress that XVAs generate substantial profit and loss figures for financial institutions. For example, Hull and White (2014, p. 2) note that J.P. Morgan Chase benefited 1.4 billion USD from DVA gain which meant 15 % increase in the bank's net income in 2011. The effect could be negative as well, as was the case when the investment bank implemented a framework for FVA which resulted in a 1.5 billion USD loss (Whittall, 2014). The full breadth of all the XVAs is out of scope of this thesis and therefore the focus of the thesis is on pricing consistently only the most studied XVAs – CVA, DVA and FVA.

CCR can be mitigated by several ways of which the most common are *netting* and *collateralisation* – also known as margining. These are contractual arrangements that may be used to reduce credit exposure. Netting allows offsetting positive and negative derivative contract values with a specific counterparty in a case that the counterparty defaults. It is a relevant arrangement in OTC derivatives markets due to significant amounts of bi-directional transactions between two parties. Credit exposure can be further reduced by collateralisation – that is, an arrangement in which parties agree to post assets to support big exposures. Hull and White (2014) point out that collateralisation has become increasingly important in OTC derivatives markets because of the changes to globally agreed standards. They mention two requirements both in which initial margin and variation margin are required to be posted by the two parties in a derivatives contract. Firstly, the requirement of clearing most OTC derivatives through central clearing parties (CCPs) and, secondly, the requirement that non-centrally cleared derivatives be properly collateralized. Margin requirements are more important in bilaterally cleared derivatives; and the research focus of the thesis is on these type of contracts. The official framework for margin requirements for non-centrally cleared derivatives was published by (Basel Committee on Banking Supervision, 2013).

1.2 Research Objectives

This thesis has two main objectives. Firstly, to build simplified, but consistent TVA framework that can be used to price interest rate derivatives. This requires reviewing literature of counterparty risk and other related subjects such as CVA, DVA, FVA, LVA, RC, RVA, CRA, wrong-way risk, multiple funding curves and collateral. Due to the significant amount of complexity involved in XVAs and their aggregation, the framework must be simplified so that it can be applied to market data. Thus, only the most relevant aspects are retained meaning CVA, DVA, FVA and collateralisation. Secondly, to increase understanding of the effects of collateralisation, another objective is to show how XVAs and TVA change according to different collateral agreements. Numerical simulations for interest rate derivatives are carried out to study this.

Calculating TVA is complex and requires methods at multiple levels. Firstly, the future exposures of derivatives, collateral assets, default probabilities, etc. need to be modelled. In this thesis, Monte Carlo simulation is chosen for computing these exposures, because, as Gregory (2012) points out, the method is suitable for dealing with many complexities involved in the calculation, for example collateralisation. Furthermore, Monte Carlo simulation is a state-of-the-art method for exposure modelling. As interest rate derivatives are of main interest in the thesis, Hull-White 1-factor model (Hull and White, 1990) is considered for future interest rates simulation. Regarding collateral assets, *credit support annex* (CSA) functions as basis for collateralization scheme. Bloomberg market data – e.g. implied volatilities, credit-default swap (CDS) spreads, discount rates – is used to obtain numerical results. Secondly, each XVA is calculated from the exposures and, lastly, those XVAs are aggregated. The difficulty lies in the latter rather than the former, because pricing XVAs can be done using quite simple formulas after exposures have been obtained, but the consistent aggregation of XVAs to a single TVA has proved problematic (Crépey et al., 2014). Because of the significant complexities, in the numerical section of this thesis, TVA is limited to the most studied XVAs of CVA, DVA, FVA and collateralisation.

While CCR has been studied in the literature extensively post the credit crisis, consistent calculation of TVA that takes the most important XVAs into consideration has been discussed in published works only in recent years. Duffie and Huang (1996) were the first to consider BCVA. They presented a model for valuing claims subject to default by both contracting parties, such as swap and forward contracts, but did not include collateralisation in their model. Other notable contributions to the study of BCVA are Bielecki and Rutkowski (2004) and Brigo and Capponi (2009). Only recently, BCVA has been studied in combination with funding costs. One of the earliest works on the subject is due to Morini and Prampolini (2010) followed by Castagna (2011). However, both of these works concern only simple financial contracts such as loans, which means the results obtained in the papers do not apply generally. There are, though, few works in which the authors try to build a comprehensive and consistent TVA framework. Crépey et al. (2014) give an advanced overview on a number of XVAs and other related aspects. The view on funding has been built in Crépey (2011, 2012a,b), which are based on a *backward stochastic differential equation* (BSDE) approach for pricing and hedging bilateral counterparty risk under funding constraints. Other related works that take funding costs into account with a general approach include Pallavicini et al. (2011, 2012) and Bielecki and Rutkowski (2013).

Chapter 2

Counterparty Credit Risk

Corporations are complex entities and face many risks. One of these risks is a *financial risk* which is a blanket term for several risk types concerning financial transactions. Counterparty credit risk is a financial risk that has become particularly important due to the financial crises in the past six years. This chapter introduces CCR along with the relevant concepts surrounding it.

2.1 Intersection of Financial Risks

Financial risk includes, among others, market risk, credit risk, liquidity risk and operational risk. These are some of the main financial risk types and they all have a connection to CCR. Hence, they are described and the connection with CCR is explained below.

Market risk is probably the most studied financial risk. It refers to the risk that arises from changes in the values of *underliers* (or underlying variables), which are market factors – such as stock prices, interest rates, currency exchange rates, foreign exchange rates, commodity prices and credit spreads. The risk can be either linear or non-linear. In the linear case, the risk is manifested by a direct exposure to the movements of the underlying variables. In the other case of non-linear risk, the risk arises from the market volatility. (Gregory, 2012, pp. 9-10)

Bielecki and Rutkowski (2004, p. 3) suggest that *credit risk* can be divided into risk subcategories such as: counterparty credit risk (also called default risk), *spread risk*, and *downgrade risk*. For a reminder, counterparty credit risk is defined as the risk of loss that may result when the counterparty cannot completely fulfill its contractual terms (e.g. fixed interval payments). Spread risk describes the risk of change in a contract's value due to a variation in credit spreads. Downgrade risk is the risk that arises from a possibility of a downgrade by a rating agency. As for the modelling of credit risk, there are, at least, three key quantities which need to be considered in credit risk quantification: the probability of the counterparty defaulting, the potential exposure at default and the associated recovery value.

Gregory (2012, p. 10) states that *liquidity risk* can be divided into two different forms, namely, asset liquidity risk and funding liquidity risk. The former refers to the risk of not being able to execute a transaction at quoted market prices due to the size of the position being unfit to other market participants. The latter refers to the risk of lacking cash or cash equivalents to routinely settle payments with possible consequences of assets' early selling for cash and the realization of losses.

Gregory (2012, p. 10) describes *operational risk* as the risk which relates to people, systems, internal processes and external events. It consists, amongst other things, of issues such as human errors (e.g. mistakes in processing information), disruptions in systems (e.g. breakdown of hardware, telecommunications failure), failed processes (e.g. failure of modelling new products) and external risks (e.g. changes in government policies). The definition includes also legal risk – the risk of firm's interpretation of contracts, laws or regulations being different from that of the regulatory authority – but excludes strategic and reputational risk.

The concept of CCR is further clarified here as it is separated from both reference credit risk and traditional lending risk. Firstly, it is important to distinguish between *reference credit risk* and CCR. Reference credit risk is present in a derivative where the credit risk arises only because of a third-party, which functions as a reference entity in the derivative – both the party and the counterparty are assumed to be default-free. CCR refers instead to the credit risk that exists because of actual defaultable parties in the derivative (Bielecki and Rutkowski, 2004, p. 3). Secondly, the distinction between *lending risk* and CCR is made. Gregory (2012, p. 22) suggests that the primary differences between lending risk and CCR lie in two characteristics, namely laterality and future value uncertainty. Lending risk, which is a traditional type of credit risk, is unilateral (one-sided) and there is no significant uncertainty related to the future values of financial contracts including such risk. Whereas CCR is usually bilateral (two-sided) and the financial contracts containing that kind of risk mostly have substantially uncertain future values. An example of the financial contract including lending risk is a repayment mortgage. Only the lender in the contract takes lending risk as the borrower may fail to repay completely the borrowed amount, but the default of the lender would not cause a loss to the borrower because no cash flows from the lender to the borrower exists in this contract. Due to the repayments in the contract, the borrowed amount decreases over time, however, the remaining sum of money is predictable with good accuracy. Respectively, an over the counter derivative – such as interest rate swap – is an example of the financial contract that contains CCR. In general, the value of the derivative can change between positive and negative values over time and, hence, the party is subject to the default of the counterparty and the counterparty is subject to the default of the party. The derivative's value is determined by the future cash flows, which are most likely highly uncertain.

CCR is the intersection of many different financial risks – including all of the ones introduced above. This is one of the reasons why significant complexity is involved with the risk. Let us now go through how the financial risks above are connected to CCR. Firstly, market risk is under consideration. As Gregory (2012, p. 10) points out, the market risk of a derivative can be removed by entering into another derivative that offsets the original derivative. For example, looking this from the party's point of view, it can have a derivative with a counterparty A and a similar but offsetting derivative with another counterparty B. Hence, the party is not exposed to market risk, but because the two counterparties differ, the party faces CCR. Secondly, as for funding liquidity risk, it may increase as a result of mitigating CCR by collateralisation. Furthermore, this kind of risk mitigation increase operational complexity and thus create new operational risks (Gregory, 2012, p. 11).

2.2 Derivatives Market

The worldwide derivatives market reached a notional higher than 700 trillion (i.e. $7 \cdot 10^{14}$) USD in 2011. That is about ten times the size of the world GDP in the same year. Even considering that there are some issues with the derivatives market size figure including e.g. double counting, the market is still huge (Crépey et al., 2014, p. 6). The market may be classified into two: over the counter (OTC) derivatives and exchange-traded derivatives. It is reported in Basel Committee on Banking Supervision (2014a, p. A141) that the size of the OTC market – measured by total notional amount outstanding – was 691 trillion USD in June 2014. By contrast, the size of the exchange-traded market was 73 trillion USD, hence also the OTC market is almost ten times the size of the exchange-traded market (Basel Committee on Banking Supervision, 2014a, p. A146). In this thesis, we will focus on the OTC derivatives, because exchange-traded products are not subject to CCR. Also, the size of the OTC market gives importance to the selection. Nonetheless, exchange-traded derivatives will be briefly discussed next to highlight differences between them and OTC derivatives.

Several of the simplest financial derivatives are traded through exchanges. These exchanges provide liquidity to the market and they make it convenient for the market participants to trade and unwind positions. Exchange-traded derivatives are not exposed to CCR – not at least in theory – because an exchange acts as a central counterparty between the trading parties and it is extremely unlikely to not being able to fulfill its contractual payment obligations. To increase liquidity and trading transparency in the market, exchange-traded derivatives are standardized. In contrast to exchange-traded derivatives, OTC derivatives are usually bilaterally traded and allow customisation. They are privately-negoatiated contracts, in which the parties involved are exposed to the CCR of each other. (Gregory, 2012, p. 15)

Interest rate derivatives (IRD) comprise the largest product segment in the OTC markets as shown by Figure 2.1 below. In terms of both notional amount and gross market value, the market for IRD stands out clearly as the largest from the beginning of the given timeline that starts from year 2007. For example, the notional amounts of outstanding IRD totalled 563 trillion USD – 81.5 % of the total for OTC derivatives – at mid-2014. At the same time, the second largest product segment, foreign exchange (FX) derivatives, summed to only 75 trillion USD – 10.8 % of the total for OTC derivatives. The significance of the IRD segment can also be seen in the gross market values, which measure the cost of replacing outstanding derivatives at market prices. The market values of IRD totalled 13 trillion USD – 77.3 % of the total for OTC derivatives – at mid-2014, whereas, at the same time, the market values of FX derivatives summed to merely 1.7 trillion USD – 9.87 % of the total for OTC derivatives.

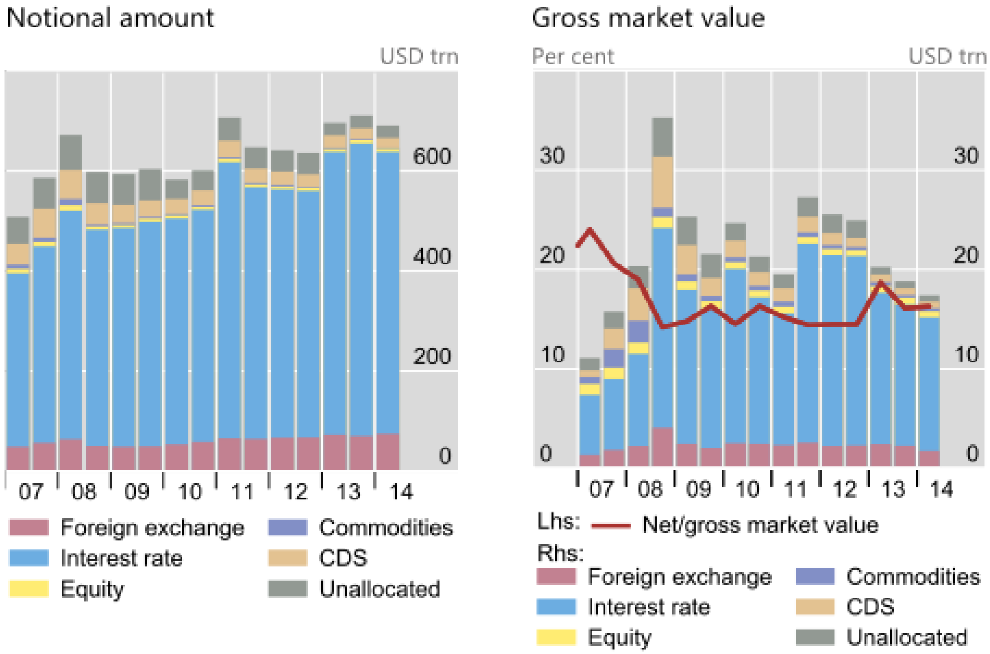

Figure 2.1: Outstanding positions in OTC derivatives market by product segment. Figure reproduced from Basel Committee on Banking Supervision (2014b, p. 19).

2.3 Exposure Measures

Let us consider the counterparty's default in a single derivative. After the default, the party must close out the derivative with the counterparty. The party can then decide whether or not to enter into a similar derivative – essentially replacing the closed out derivative. However, regardless of the party's decision, assuming that the party wants its market position to remain unchanged and therefore replaces the derivative is convenient for determining the loss due to the defaulting counterparty. The loss equals the cost of replacing the derivative, because the party's market position stays the same. The replacement cost depends on the sign of the derivative's value at the time of the default, and, hence, the two possible cases are examined next. Let us call the derivative in which the counterparty defaults *original derivative*, and the respective derivative that replaces the original derivative *replacement derivative*. Firstly, if the original derivative has a negative value for the party, it has to pay this value to the counterparty and enter into the replacement derivative with another counterparty receiving the value of the original derivative. This results in the net loss of zero. Secondly, if the original derivative has a positive value for the party, it receives nothing from the counterparty and enters into the replacement derivative, which costs the value of the original derivative, with another counterparty. This results in the net loss equal to the value of the original derivative. In summary, the party loses if the derivative value is positive and neither loses nor profits if the derivative value is negative. Hence, in case of a single derivative, the party's exposure is the maximum of the derivative's value and zero as expressed by the following equation:

$$E_i(T) = \max\{0, V_i(T)\} = V_i(T)^+, \tag{2.1}$$

where $V_i(T)$ is the value of the derivative i at time T. (Zhu and Pykhtin, 2007, pp. 16-17)

Canabarro and Duffie (2003) provide basic definitions for counterparty exposure measures. Counterparty exposure is defined similarly to the exposure above, but for the portfolio of derivatives – instead of a single derivative – with a counterparty. However, netting affects counterparty exposure calculation and, thus, the definition is elaborated in subsection 2.4.1. The other related counterparty exposure measures are introduced below.

- **Current exposure (CE)** means the current value of the exposure to a counterparty.

- **Potential future exposure (PFE)** is defined as the maximum expected exposure on a future date with a specific statistical confidence.

$$PFE_{\alpha}(T) = \max\{x \in \mathbb{R} : P(E(T) > x) = 1 - \alpha\} \tag{2.2}$$

  where α is a given statistical confidence level and $E(T)$ is the exposure at a future time T. For example, the $PFE_{99\,\%}(T)$ is the level of potential exposure that is exceeded with the probability of only a 1 %.

- **Expected exposure (EE)**, $EE(T)$, is the average exposure on a future time T. When T is let to vary over a specific time period, the curve of $EE(T)$ is called the expected exposure profile.

- **Expected positive exposure (EPE)** is defined as the average $EE(T)$ over a specific time period.

2.4 Risk Mitigation

Reducing credit exposure by contractual arrangements such as netting and collateralisation is the primary way of mitigating CCR. Concerning, amongst other things, netting and collateralisation, Gregory (2012, p.45) notes that the ISDA Master Agreement, which is a document published by the International Swaps and Derivatives Association, Inc. (ISDA), outlines the general terms for international OTC derivatives transactions. The purpose of the document is to provide a legally more certain bilateral framework for the market participants.

2.4.1 Netting

The counterparty's default requires arrangements. As one of the arrangements, the party has to close out the derivatives that it had entered into with the counterparty. It is preferable for the party to be allowed to offset positive and negative derivative values with the counterparty in closing out of the derivatives. This is possible with a contractual arrangement called netting. However, it depends on a jurisdiction whether netting is allowable or not (Gregory, 2012, p. 46). Gregory (2012, p. 41) remarks that netting is relevant to the OTC market in comparison to most business relations, because often the transactions between the market participants are bi-directional. Without netting, each derivative is valued separately and the total loss with the counterparty is the sum of the derivatives having positive values. Thus, the maximum netted loss following the bankruptcy of the counterparty is always less than or equal to the value of the maximum loss which is not netted. This can be expressed for the portfolio consisting of n derivatives with the inequality

$$E_{N_+}(T) = \max\{0, \sum_{i=1}^{n} V_i(T)\} \leq \sum_{i=1}^{n} \max\{0, V_i(T)\} = E_{N_-}(T), \tag{2.3}$$

where $E_{N_+}(T)$ is netted exposure and $E_{N_-}(T)$ is non-netted exposure.

As in Gregory (2012, pp. 48-49), let us consider the following example where the party has multiple derivatives with the counterparty. From the party's perspective, the values of the derivatives which have a positive value sum up to +100 million USD and the values of the derivatives with a negative value total −95 million USD. Without netting, the party's exposure is +100 million USD, whereas, with netting, it is only +5 million USD. From the counterparty's viewpoint things change a little, thus let us consider also this viewpoint. Now, the sum of the contracts which have a positive value is +95 million USD and the sum of the contracts having a negative value is −100 million USD. Without netting, the counterparty's exposure is +95 million USD, whereas, with netting the total value goes negative and hence there is zero exposure – though the counterparty would have to pay the negative amount of −5 million USD in the event of the party's default. Thus, netting between the institutions in this example would reduce the risk.

2.4.2 Collateralisation

Collateralisation is another contractual arrangement for reducing credit exposure. This way of risk mitigation can go beyond netting so that if a netted exposure is still large, it may be further limited by collateralisation.

In lending agreements (e.g. in case of a mortgage loan), *collateral* refers to those assets that a borrower guarantees to give to a lender to secure the payment of a loan. The assets may refer to, amongst other things, equipment, real estate, buildings, cash, jewellery, art and financial securities. If the borrower defaults, the lender is entitled to the collateral to cover its losses. However, the realisation of the collateral may be delayed during the bankruptcy process (Gregory, 2012, p.60).

Collateral is also often utilized in derivatives transactions. Consider a party's exposure with netting which is large and positive. Thus, if the counterparty of the party defaulted, the effect would be substantial. However, to limit those kind of exposures, a collateral arrangement in which the parties involved specify the conditions for posting collateral to support big exposures may be used. In contrast to lending agreements, Gregory (2012, p.60) points out, collateral in derivatives is easily liquidable due to derivatives legislation and the type of the collateral – cash or liquid securities – used in these transactions. Cash is the main type of collateral received and delivered against non-cleared OTC derivative transactions. As an example, it accounted for 74.9 % and 78.3 % of total collateral for received and delivered collateral respectively in 2014. Government securities is the second most common asset category. Together with cash, their share of the total collateral has been approximately 90 % in prior years as stated in (International Swaps and Derivatives Association, 2014, pp. 7-8). In this thesis, for the sake of simplification, collateral postings are assumed to be cash.

A Credit Support Annex (CSA) is a legal document which sets the rules for credit support (collateral) concerning OTC derivatives. It is part of the ISDA Master Agreement. The most of all non-cleared OTC transactions are performed under a CSA across different derivative products, e.g. 97 % of credit derivatives and 86 % of fixed income derivatives are collateralised (International Swaps and Derivatives Association, 2014, pp. 10-11).

We next consider some of the most important features covered in a CSA. Brigo et al. (2013) point out that these include, amongst other things, *threshold*, *minimum transfer amount* (MTA), *eligible collateral*, *frequency of margin calls*, *variation margin* and *independent amount.*

- **Threshold** is defined as a limit for the portfolio of derivatives that, if exceeded, means the beginning of collateralisation. Hence, the threshold is the level of uncollateralised exposure. Only the part of the exposure which exceeds the threshold will be collateralised.

- **Minimum transfer amount** is the smallest amount of collateral that is agreed to be transferable. This means that the exposure has to be above the sum of a threshold and a minimum transfer amount before any collateral calls can be made.

- **Eligible collateral** refers to assets that are permissible to be used as collateral.

- **Margin period or risk** refers to the time period from a collateral call to receiving the collateral. This period is longer than the period merely between collateral calls, i.e., the period that defines the frequency of monitoring and calling of collateral. Essentially, margin period of risk defines the length of time without receiving collateral where any increase in exposure will be uncollateralised.

- **Variation margin** is an adjustment to collateral amount due to changes in derivatives' market values.

- **Independent amount** (also sometimes called initial margin) is a fixed amount of collateral that must be posted regardless of the variation margin.

Chapter 3

Valuation Adjustments

This chapter gives an introduction to valuation adjustments (or XVAs), whereas in the modelling chapter 4, more details are presented for selected XVAs: CVA, DVA and BCVA. The chapter follows for the most part the book of Crépey et al. (2014).

3.1 Overnight Index Swap versus London Interbank Offered Rates

Along with valuation adjustments, the multi-curve world post the crisis of 2007 need to be addressed. This concerns differences between LIBORs and OIS rates.

A *risk-free interest rate* is a theoretical rate of return of an asset that is not subject to risk of financial loss (see, e.g., Luenberger, 2009, p. 165). The risk-free term structure of interest rates is an important input in the pricing of derivatives. Hull and White (2013, p. 2) mention that "It is used for defining the expected growth rates of asset prices in a risk-neutral world and for determining the discount rate for expected payoffs in this world." A *treasury rate* is an interest rate that is paid on *treasury bills* and *treasury bonds*, which are financial products sold by a government to borrow money. According to the old derivatives pricing theory and practice (see, e.g., Hull, 2009, pp. 73-74), derivatives are priced with a single risk-free interest rate – meaning a proxy such as a treasury rate. Hull (2009, p. 74) points out that in practice, though, LIBORs are more applied than treasury rates in the pricing of derivatives.

London interbank offered rates (LIBORs), respectively *Euro interbank offered rates* (EURIBORs), are daily published reference rates that are calculated as an average of the interest rates at which a particular panel of banks are confident of obtaining unsecured – i.e. uncollateralised – loans for certain periods (see, e.g., Crépey et al., 2014, p. 48). Because of the low default probabilities of banks, the term structure of LIBORs was considered to be approximately risk-free. Similarly, the treasury rates of the governments that had a low probability of default and due to this were rated highly (e.g. with a AAA or Aaa credit rating) were then considered approximately risk-free. LIBORs as risk-free rates were preferred to treasury rates due to the following advantages:"better liquidity, the lack of problems with technical factors (such as repo specialness and tax issues) and the close links between LIBORs and funding costs" (Gregory, 2012). Furthermore, Hull and White (2013, p. 2) point out that by applying LIBOR as the risk-free rate, it was easy to price the most widely traded derivative – an interest rate swap that exchanges LIBOR for a fixed rate.

The credit crisis stressed the fact that LIBORs are not risk-free, but include CCR. During the crisis, the trust between banks decreased due to default concerns. LIBORs increased, and they diverged with regard to other rates (Hull and White, 2013, p. 2; Crépey et al., 2014, pp. 48-49). Crépey et al. (2014, p. 48) point out that at the same time *overnight interest rate swaps* (OIS) rates gained popularity. These rates are calculated by compounding an overnight interbank rate (O/N) i.e. a rate at which overnight unsecured funding can be obtained in the interbank market. The divergence that occured between the OIS rate in the euro market (EONIA rate) and LIBOR is shown in the Figure 3.1 below.

OIS rates seem to be better approximate for a risk-free rate than LIBORs. Hull and White (2013, pp. 7-8) argue that the OIS rate is a good approximation for a longer term risk-free rate, because the credit risk in an OIS is small. Regarding collateralised derivatives, Gregory (2012, p. 287) provides another reason: OIS is the underlying rate in these derivatives. Hull and White (2013, p. 3) point out that LIBORs are still used in majority of the non-collateralised derivatives, using the rationale that gives a better estimate of the party's cost of funding than the OIS rate. They continue by stating that due to a traditional principle in finance, which conveys the idea of the evaluation of an investment is determined by the risk of the investment, not how it is funded, this

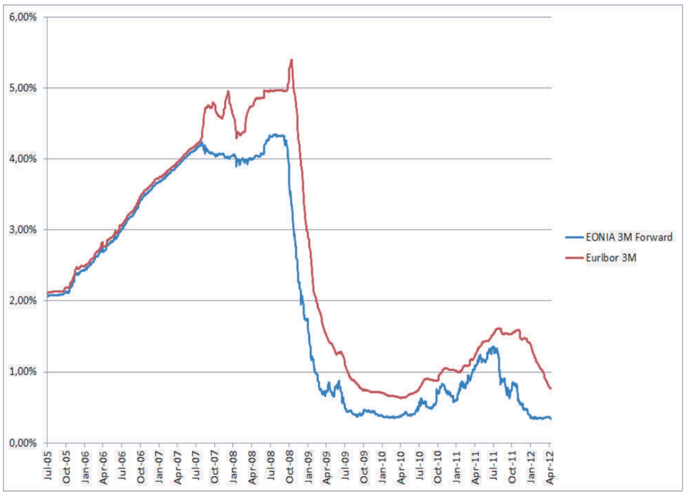

Figure 3.1: The 3-month Euribor and the 3-month EONIA-swap rate. Significant difference between the rates occurred on 6.8.2007. Figure reproduced from Crépey et al. (2014, p. 49).

rationale is questionable. Also, because LIBORs are fixed by key derivatives market makers, insider issues and market manipulation are of concern (Wheatley, 2012).

Nowadays, hence, OIS seems to be the most relevant discounting rate. LIBORs used to be the underlying interest rate for both projecting and discounting future cash flows. LIBORs can still be used to calculate future cash flows if they underlie some derivative. But discounting rate should be OIS rate. Although OIS may not be the "true" risk-free rate, at least it should be more correct one. This makes pricing of a single currency IRS an exotic problem involving multiple curves.

3.2 Bilateral Credit Valuation Adjustment

Crépey et al. (2014, pp. 15-28) provide a useful introduction to CVA and DVA. Consider the party having a possibility to trade a derivative with either a defaultable counterparty or a default-free counterparty. Other things being equal, minimizing counterparty risk is beneficial and, hence, it is strictly better for the party to choose the default-free counterparty. This means that the defaultable counterparty is chosen only if the party is compensated on the extra risk it will take with this choice. In other words, the defaultable counterparty must reduce the price of the derivative and that reduction is called Credit Valuation Adjustment (CVA). As a matter of fact, *credit valuation adjustment* is the value difference between the price of the derivative without default risk and the price of the derivative with default risk of the counterparty included. Hence, CVA is a price itself and, as such, it is risk neutral or pricing measure expectation of future discounted cash flows. This is an expected value at valuation time of the discounted future cash flows including the counterparty risk on the specific portfolio. Since it is a price, it is computed entirely under the pricing measure, $\mathbb{Q}$. Thus, measuring risk statistics or the probability measure, $\mathbb{P}$, are not used here. Although CVA became more and more important after the credit crisis of 2008, the concept had existed for a while before that, see, for example, (Duffie and Huang, 1996), (Bielecki and Rutkowski, 2004), (Brigo and Masetti, 2005) and (Zhu and Pykhtin, 2007).

Brigo and Masetti (2005, pp. 6-8) consider a generic claim with default risk of the counterparty included. They show that the price of this claim equals the price of an equivalent claim, where the default risk is not included minus an option term. Hence, CVA is actually this option term. Now, this makes the valuation of derivatives model dependent even in derivatives that are model independent without counterparty risk. As an example, consider the valuation of swaps. Without counterparty risk the valuation is model independent: it requires no dynamical model for the term structure of interest rates, discounting and projecting curves at initial times suffice; with counterparty risk the valuation is model dependent: it requires an interest rate model. This implies difficulties in advancing the existing pricing models to include counterparty risk.

Consider the same situation as above, but now from the counterparty's point of view. Because the party is risk-free and only the counterparty itself has a positive default probability, the counterparty marks a positive adjustment to the risk-free price of the derivative. Hence, the price increased to the counterparty by the same amount (i.e. CVA) as it decreased for the party. This allows both parties to agree on the price. Increasing of the price for the counterparty is explained by the compensation of its own default risk. This positive adjustment that is added to the risk-free price by the counterparty is called *debt valuation adjustment* (DVA). The adjustment is positive because the early default of the counterparty would imply a discount on its payments concerning the derivative, which means a gain for the counterparty. As only the default risk of the counterparty is included, the adjustments are also called Unilateral DVA (UDVA) and Unilateral CVA (UCVA). Denoting the party by P and the counterparty by CP, in this case, $UCVA(P) = UDVA(CP)$. This means that the adjustment made by both parties to the risk-free price is the same, but the party subtracts it of the price and the counterparty adds it to the price. Also, because the party is default-free, it holds true that $UDVA(P) = UCVA(CP) = 0$.

However, instead of these unilateral cases the interesting case is the bilateral one where the both parties have default probability greater than zero. This is because of the the credit crisis of 2008, which showed that not even the institutions with the highest credit ratings can be regarded as risk-free any more. Thus, consider the bilateral case next. The situation now requires that both the possibility of default by the party and the possibility of default by the counterparty are consistently taken into account in the valuation. Crépey et al. (2014, p. 23) state it clearly: "Now both parties will mark a positive (bilateral) CVA to be subtracted and a positive (bilateral) DVA to be added to the default risk-free price (MtM) of the deal."

The CVA of the party equals the DVA of the counterparty; The DVA of the party equals the CVA of the counterparty. Let us denote the risk-free price from the point of view of a party, p, by $P_{rf}(p)$. The final price, $P(p)$, to a party, p, is obtained as follows: $P(p) = P_{rf}(p) - CVA(p) + DVA(p)$. Because $P_{rf}(P) = -P_{rf}(CP)$, $DVA(P) = CVA(CP)$ and $DVA(CP) = CVA(P)$, it holds true that $P(P) = -P_{rf}(CP) + CVA(CP) - DVA(CP) = -(P_{rf}(CP) - CVA(CP) + DVA(CP)) = -P(CP)$ and so the price agreeable for both parties. The industry usually refers to the difference of $CVA - DVA$ by Bilateral CVA (BCVA) – the term is used similarly in the thesis.

Duffie and Huang (1996) were one of the first ones to model BCVA. They considered swaps and forwards in their model. Brigo and Capponi (2009) consider BCVA in detail. They introduce a general arbitrage-free framework for calculating BCVA and then consider Credit Defaults Swaps (CDS) specifically as underlying portfolio in their analysis. Other aspects of BCVA are studied in (Brigo et al., 2011b, 2014, 2011a). Brigo et al. (2014) develop an arbitrage-free BCVA framework that includes collateralisation with possible rehypothecation. Considering CDS contracts specifically, they show that even under high frequency collateralisation the effectiveness of collateralisation is limited if the party's and the counterparty's default times are dependent. In this thesis, counterparty risks are limited via the BCVA framework.

Chapter 4

Total Valuation Adjustment for Interest Rate Derivatives

Pricing XVAs requires calculating collateralised exposures and default probabilities. Regarding the exposures, market scenarios need to be simulated and derivatives valued at future dates. As for the probabilities, they may be obtained from credit-default swap (CDS) spreads. Therefore, all these factors, which are present in the formulas of XVAs are considered in this chapter.

4.1 Simulation Framework

Zhu and Pykhtin (2007) present a simulation framework for computing counterparty credit exposure, which is also used in this thesis. As an output of the framework, one may calculate different statistics of the exposure distribution. The extended form of this framework includes scenario simulation, derivatives valuation, and portfolio aggregation.

Firstly, *market scenarios*, which are realizations of market factors that affect the values of the derivatives in a portfolio, need to be generated for a set of future dates. Market factors are usually modelled by a stochastic differential equation (SDE). To simulate future values of these factors, Zhu and Pykhtin (2007) state that there are two methods, which they call "Path-Dependent Simulation (PDS)" and "Direct Jump to Simulation Date (DJS)". In a PDS method, market factors are generated through time for each future date up to and including the furthest simulation date. Whereas, in a DJS method, each generation of market factors for a given future date is a simulation of its own. Zhu and Pykhtin (2007) point out that at a particular simulation date, the distribution of a market factor is identical irrespective of the method used. They note that despite this a PDS method may be a better fit for path-dependent, American/Bermudan and asset-settled derivatives.

Secondly, derivatives are valued at future dates using the simulated scenarios. However, there are a few problems related to the valuation. Zhu and Pykhtin (2007) point out that one of the problems concerns path-dependent, American/Bermudan and asset-settled derivatives. They state that the value of these derivatives may depend either on earlier happened event (e.g., the exercise of an option) or on the full continuous path before the valuation date (e.g., barrier or Asian options). Because the derivatives are valued at a discrete set of future simulation dates, there is, for example, uncertainty whether or not an option has been exercised or if a barrier has been hit. Furthermore, the problem is even worse for the DJS method, where there is no relation between scenarios at a given simulation date and simulation dates previous to that. For this reason, the PDS approach is chosen to be used in this thesis.

4.2 Interest Rate Models

Interest rates are a significant factor in the pricing of *interest rate swaptions* considered in the thesis. All the most important aspects of interest rates are covered in this section.

4.2.1 Pricing Fundamentals

Interest rates are usually quoted on an annual basis. However, there are different approaches to calculating interest accrued over a specific time period. This means mainly that the compounding frequency, m, can be varied. The most commonly used compounding frequencies are: annual ($m = 1$), semi-annual ($m = 2$), quarterly ($m = 4$), monthly ($m = 12$), weekly ($m = 52$), daily ($m = 360$) and *continuous compounding* ($m \to \infty$). The compounding frequency can be viewed as a unit of measurement. Thus, conversion from one compounding frequency to another changes accrued interest. (Hull, 2009, pp. 76)

The *term structure* of interest rates – also called *yield curve* – is a graph of interest rates as a function of *maturity*, which itself is defined as the period of time under which a financial contract is in effect (Glasserman, 2004, p. 559). Selecting an appropriate term structure is important in the valuation of many derivatives. Hence, in the thesis, interest rates are presented with the notation: an annual interest rate $R(T)$ for maturity T. As mentioned in section 3.1, LIBOR is the main reference rate for swaptions and thus LIBOR is used as the term structure of interest rates for computing and discounting cash flows.

Consider an investment period of T years and an annual interest rate of $R(T)$ with the compounding frequency of m times a year for the whole period. Then the interest that accrues over this period is (see e.g. Glasserman, 2004, p. 559)

$$\left(1+\frac{R(T)}{m}\right)^{mT}-1. \tag{4.1}$$

From the expression 4.1, assuming now continuous compounding, accrued interest is obtained as follows,

$$\left[\lim_{m\to\infty}\left(1+\frac{R(T)}{m}\right)^{mT}\right]-1=e^{R(T)\cdot T}-1. \tag{4.2}$$

In practice, expressions 4.1 and 4.2 must be slightly amended. A *day count convention*, which is a method for counting the accrual of interest over time affects the calculation of these expressions. It determines the expression for a *day count factor*, δ_k, which describes the ratio of a given time period, $\Delta T_k = T_{k+1} - T_k$, to a year. Hence, for example, in the expression 4.1 the fraction of $1/m$ is replaced by δ_k and, because of this, the expression changes to

$$\prod_{k=0}^{mT-1}(1+\delta_k R(T))-1, \tag{4.3}$$

where $T=\sum_{k=0}^{mT-1}\Delta T_k$. The related *discount factor* from time, T_{mT}, to time, T_0, of the expression 4.3 is

$$D(T_0, T_{mT}) = \frac{1}{\prod_{k=0}^{mT-1}(1+\delta_k R(T))}, \tag{4.4}$$

(Glasserman, 2004, p. 559). One of the day count conventions is *Actual/*360 (ACT/360) which is defined as the actual number of days in a time period divided by 360. It is commonly used in the pricing of interest rate swaps. However, to avoid unnecessary complexity, 30/360 day count convention – according to which each month has 30 days and a year has 360 days – is used in this thesis (Neftci, 2008, pp. 33-34).

A *present value* refers to the current summed monetary worth of the future cash flows. Let us consider m future cash flows. A future cash flow, $CF(T_i)$, occuring at an arbitrary time, T_i, $i \in \{n+1, n+2, ..., n+m\}$, is discounted to the present moment, T_n, with a discount factor, $D(T_n, T_i)$. Now, the present value of all the future cash flows can be obtained from

$$PV(T_n) = \sum_{i=n+1}^{n+m} D(T_n, T_i) CF(T_i). \tag{4.5}$$

An *interest rate swap* (IRS) is a contract agreed between two parties to exchange interest rate cash flows, which are based on a specific notional amount. A plain vanilla type – which refers to swapping fixed-level payments to variable-level payments – is a commonly traded IRS (see, e.g., Neftci, 2008, p. 124). Consider a derivative which either is an IRS, or in which the underlying is an IRS. From the viewpoint of a party that is paying a fixed rate, the derivative is described as a *payer*; from the viewpoint of a party that is receiving a fixed rate, the derivative is described as a *receiver*.

Now, let us consider a payer plain vanilla IRS from the party's point of view. The payment dates are $\{T_{n+1}, T_{n+2}, ..., T_{n+m}\}$, where $T_i = i\delta$ and furthermore δ is a period (e.g., half a year). On these dates, the party makes and receives payments, which are calculated from fixed rate, R, floating rate, $r(T_i)$, which is fixed a period before at T_{i-1}, notional amount, N, and the period, δ. The value of the IRS at the beginning of it, at time T_n, is the difference of the present value of the floating payments, $PV_{FLT}(T_n)$, and the present value of the fixed payments, $PV_{FIX}(T_n)$, that is

$$PV_{SWAP}(T_n) = PV_{FLT}(T_n) - PV_{FIX}(T_n) = N\delta \sum_{i=n+1}^{n+m} D(T_n, T_i)(r(T_i) - R), \tag{4.6}$$

where $D(T_n, T_i)$ is the discount factor over the period $[T_n, T_i]$. Glasserman (2004, p. 563) shows that this can be expressed as

$$PV_{SWAP}(T_n) = N - NR\delta \sum_{i=n+1}^{n+m} D(T_n, T_i) - ND(T_n, T_{n+m}). \tag{4.7}$$

The IRS is priced at zero so that both the party and the counterparty are willing to agree to the IRS without paying any additional costs. The *swap rate* is defined as the fixed rate, R, that makes the value of the IRS equal to zero. The swap rate, given by the value of the IRS (4.7), is

$$R_S(T_n) = \frac{1 - D(T_n, T_{n+m})}{\delta \sum_{i=n+1}^{n+m} D(T_n, T_i)}. \tag{4.8}$$

Furthermore, Glasserman (2004, p. 564) presents the *forward* swap rate at time $t < T_n$ for the IRS, which is

$$R_S(t) = \frac{D(t, T_n) - D(t, T_{n+m})}{\delta \sum_{i=n+1}^{n+m} D(t, T_i)}. \tag{4.9}$$

An *option* is a contract which gives its holder the right, but not the obligation, to buy or sell an underlying asset or instrument (e.g. a swap, a stock, a commodity, a bond) at a certain price – called the *strike price* – on a specified date or dates. If the holder of an option indeed buys or sells an underlier, the holder *exercises* the option. However, there is a time limit on when an option may be exercised and this last day is named as an *expiration date*. A buying or selling price of an option itself is called a *premium*.

Options can be classified in many ways. The option is called a *call option* (*put option*) if the right is to buy (sell) an underlier. Another way to separate options from each other is according to exercise dates. The valid exercise dates for an *American option* are all the dates from the start date up to and including the expiration date while an *European option* may be exercised only at the expiration date. Moreover, there exist other options with different kinds of exercise styles, but the majority of the options is comprised of American and European types. A third way to categorize options is on the basis of their *moneyness* which can be defined as the relative position of an underlying's price, $S(t)$, at time, t, with respect to the strike price, K. Considering a call option (put option) at time t, the three categories of moneyness are the following. First, if $S(t) > K (K > S(t))$, the option is *in-the-money* (ITM); second, if $S(t) = K$, the option is *at-the-money* (ATM); third, if $S(t) < K (K < S(t))$, the option is *out-of-the-money* (OTM).

The *pay-off* of an option is the value of the option at the expiration date. Let us determine a European call option with the following terms: strike price is denoted by K, underlying is a stock price denoted by $S(t)$ at time t and, lastly, expiration time is marked with T. Now, the pay-off of the option is

$$(S(T) - K)^+ = \max\{0, S(T) - K\}. \tag{4.10}$$

The expected present value of the option can be obtained by calculating $E[D(t,T)(S(T)-K)^+]$ where $D(t,T)$ is discount factor from T to t and $E[]$ is expectation with a distribution for the random variable $S(T)$. This is the *price* of a European call option.

4.2.2 Black-Scholes Model

Black and Scholes (1973) developed a formula for options pricing. This Black-Scholes model is the centerpiece of derivatives pricing and it will be shortly presented here. Firstly, the assumptions of the formula are listed here:

1. The option contract is the same as in the previous section. Thus, a European call option with a stock as an underlying is under consideration.

2. The risk-free interest rate, r, is constant

3. The following stochastic differential equation (SDE) characterizes the dynamics

of the stock price:

$$dS(t) = \mu(S(t))S(t)dt + \sigma S(t)dW(t), \quad t \in [0,\infty), \tag{4.11}$$

where $\mu(S(t))$ is the mean rate of return (or the drift), the parameter σ is the stock price volatility and $W(t)$ is a Wiener process (or a Brownian motion) under the real world probability measure, $\mathbb{P}$.

4. Corporate actions such as dividends and stock splits are excluded.

5. Transaction costs and bid-ask spreads are not taken into account.

If both sides of the SDE (4.11) are divided by $S(t)$, then $dS(t)/S(t)$ may be interpreted as the percentage change in the stock price that depends on the drift, volatility and movements according to Brownian motion. Substituting the mean rate of return, $\mu(S(t))$, with the risk-free rate r results in risk-neutral view.

From the assumptions above, the Black-Scholes partial differential equation (PDE) can be derived. The PDE is

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0, \tag{4.12}$$

where C is the price of a European call option. In some cases partial differential equations can be solved analytically if the boundary conditions are suitable. Choosing the pay-off of the option (4.10) as a boundary condition, gives – in this case of a European call option – a closed-form solution called the Black-Scholes formula:

$$C(t) = S(t)\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2), \tag{4.13}$$

where

$$d_1 = \frac{log(S(t)/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \tag{4.14}$$

$$d_2 = \frac{log(S(t)/K) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}, \tag{4.15}$$

and the $\Phi(x)$ denotes the cumulative standard normal probability function:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du. \tag{4.16}$$

A modification of the original Black-Scholes option pricing model is the Black model – also known as the Black-76 model – introduced by Black (1976). The Black formula is frequently used in pricing interest rate derivatives and thus it is interesting in regard to this work. The difference between the Black-Scholes and the Black-76 is that in the Black-Scholes, the underlying of an option is a stock, whereas in the Black-76, the underlying is a forward contract. Hence, let us replace a stock price, $S(t)$, with a corresponding forward price, $F(t)$, here. In this case, the corresponding PDE will be

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC = 0, \tag{4.17}$$

with the following boundary condition:

$$C(F(T), T) = (F(T) - K)^+. \tag{4.18}$$

The Black's formula solves the PDE when European options are in question. The option price is then

$$C(F(t), t) = e^{-r(T-t)}[F(t)\Phi(d_1) - K\Phi(d_2)], \tag{4.19}$$

where

$$d_1 = \frac{log(F(t)/K) + \sigma^2(T-t)/2}{\sigma\sqrt{T-t}}, \tag{4.20}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \tag{4.21}$$

4.2.3 Interest Rate Swaption

A *swaption* is an option in which the underlying asset is a swap. Specifically interest rate swaptions are of interest here. A "n x m" swaption (or "n-into-m") swaption is a n-year option to enter into a m-year swap (see, e.g., Glasserman, 2004, p. 566). An interest rate swaption can be either physically-settled or cash-settled concerning exercising its underlying. In a *physically-settled* case, the actual interest rate cash flows are swaped until the expiration date; in a *cash-settled* case, only the net present value of the interest rate cash flows is exchanged at the exercise date and then the swaption terminates (see, e.g., Cesari et al., 2009, p. 81).

Let us now consider a European payer interest rate swaption with a expiration date T_n. Thus, the owner of the swaption has the right to enter into a payer swap at time T_n in accordance with predetermined terms including fixed rate, K. Let there be m payment dates: $T_{n+1}, ..., T_{n+m}$. Denote by N the notional principal. Using the value of the interest rate swap (4.7) and the swap rate (4.8), the price of the underlying interest rate swap at the expiration date T_n can be simplified as follows

$$\begin{aligned} V(T_n) &= N - NK\delta \sum_{i=n+1}^{n+m} D(T_n, T_i) - ND(T_n, T_{n+m}) \\ &= NR_S(T_n)\delta \sum_{i=n+1}^{n+m} D(T_n, T_i) - NK\delta \sum_{i=n+1}^{n+m} D(T_n, T_i) \\ &= N\delta \sum_{i=n+1}^{n+m} D(T_n, T_i)(R_S(T_n) - K). \end{aligned} \tag{4.22}$$

Furthermore, the pay-off of the swaption is:

$$[V(T_n)]^+ = N\delta \sum_{i=n+1}^{n+m} D(T_n, T_i)(R_S(T_n) - K)^+. \tag{4.23}$$

By observing the maximization part of the equation (4.23), similarity to a call option on a forward swap rate can be seen. Hence, modeling the dynamics of a forward swap rate is required. Now the swaption price is given by computing the expected present value of the swaption pay-off (4.23). Conventionally swaptions are priced with a version of Black's formula. The formula gives the swaption price at time t:

$$N\delta \sum_{i=n+1}^{n+m} D(t, T_i)(R_S(t)\Phi(d_1) - K\Phi(d_2)), \tag{4.24}$$

where

$$d_1 = \frac{log(R_S(t)/K) + \sigma^2(T_n - t)/2}{\sigma\sqrt{T_n - t}}, \tag{4.25}$$

$$d_2 = d_1 - \sigma\sqrt{T_n - t}. \tag{4.26}$$

4.2.4 Hull-White 1-Factor

An interest rate is the only market factor that is necessary for pricing interest rate swaptions. Pricing the swaptions at multiple future times requires simulating interest rates with term structure models.

Short rate (also known as *instantaneous short rate*), $r(t)$, is the interest rate which holds only an infinitesimally short time interval at time t. Many short rate models as stochastic processes have been developed and presented as good approximations of actual interest rate processes (Luenberger, 2009, p. 406). Pricing with a short rate is relevant only in risk-neutral setting, not with a real world assumption (Hull, 2009, p. 673).

A typical characteristic of interest rate processes is their tendency to move back towards the long-run average through time. It is called *mean reversion*. For example, if $r(t)$ has a markedly higher value than its average, then it should have a negative trend because of mean reversion, while if $r(t)$ is significantly smaller than its average, its trend should be positive because of mean reversion.(Hull, 2009, p. 675)

Hull and White (1990) developed a model which was an extension to a one of the existing short rate models. In one version of the model, the short rate is assumed to satisfy the SDE,

$$dr = [\theta(t) - ar]dt + \sigma(t)dW \iff dr = a[\theta(t)/a - r]dt + \sigma(t)dW, \tag{4.27}$$

where a is the rate of mean reversion, $\theta(t)/a$ is the average to which the short rate reverts to, σ is the annual standard deviation for the short-term interest rate and W is the Brownian motion.

The important advantage of short rate models is that their calibration and simulation is simpler compared to another typical interest rate model class of LIBOR market model (LMM) (Gregory, 2012, p. 167). Although the calibration and simulation of swaption prices require appropriate methods, they are not the core of the methods section. Thus, simplicity at this point is fitting. In the context of the level of Hull-White one-factor model, it becomes apparent that this model has three noteworthy advantages that make it a realistic model. First, the model is a *no-arbitrage* model i.e. a model that is fitted to today's term structure of interest rates (see, e.g., Hull, 2009, p. 124). Second, negative interest rates have recently become reality – e.g. Euro LIBOR rates ranging from overnight to 3 months were negative on August 10, 2015 (Intercontinental Exchange, Inc., 2015). Hull-White one-factor model allows negative interest rates and, thus, corresponds to reality also from this aspect. Third, as noted above, the model allows for mean reversion. Based on these advantages, interest rates are simulated with the Hull-White one-factor model in this thesis.

4.2.5 Calibration

Practical aspects of calibration of the Hull-White one-factor model are briefly described here. As shown above the short rate reverts to the average determined by $\theta(t)$ and the parameter is chosen so that it is consistent with today's term structure of interest rates. In the calibration itself the difference between the given market prices of derivatives and the prices of derivatives computed by the Hull-White model are minimized. The *implied volatility* is a formula-dependent variable that gives the market price of an option when the known value of it is substituted to an option pricing formula. In the case of interest rate derivatives it is called *Black volatility*, since the option pricing formula is the Black formula (Neftci, 2008, p. 433). The Financial Instruments Toolbox$^{\text{TM}}$ of MATLAB is used in the calibration.

4.3 Risk Neutral versus Real World

There is an issue regarding two probability measures used to do finance stuff. Informal, but useful introductions to the these probability measures are provided in (e.g., Crépey et al., 2014, p. 17; Brigo et al., 2013, p. 7-8). The statistics of random variables such as future time losses depend on the probability measure being used. The historical or physical probability measure (also called real world measure), $\mathbb{P}$, is the probability measure used for e.g. historical estimation of financial variables, historical volatility calculations and maximum likelihood estimation. Simulating the financial variables, for example in computing VaR, up to the risk horizon is done under the probability measure, $\mathbb{P}$. Also, predicting future market variables through, e.g., economic forecast or technical analysis is done implicitly under $\mathbb{P}$. Thus, the statistics of the observed world are useful for risk measurement and prediction. However, another probability measure is needed for pricing a financial product such as an option or a structured product, when it is priced in a no-arbitrage framework. According to the no-arbitrage theory, the expected values of future discounted cash flows are to be taken under a different probability measure, namely $\mathbb{Q}$. The two measures are connected by a mathematical relationship that depends on risk aversion, or market price of risk. In simple models, the expected rate or return under $\mathbb{P}$ is given by the risk-free rate plus the market price of risk times the volatility. Estimating the expected value of the rate of return of an asset under $\mathbb{P}$ is hard, whereas under $\mathbb{Q}$, one knows that the rate of return will be the risk-free rate, since dependence on the $\mathbb{P}$ rate of return can be hedged away through replication techniques. Hence, the price of a derivative does not depend on the actual expected future return – i.e. the expected future return under $\mathbb{P}$.

4.4 Exposure Calculations

After the simulation of derivative values, exposures can be computed. Netting and collateralisation must be taken into account as described in section 2.4. Firstly, let us consider exposure at counterparty-level. As Zhu and Pykhtin (2007, p. 17) point out, there may be several netting agreements with a single counterparty. Also, it is possible that some of the derivatives are not covered by any netting agreement. Netted and non-netted exposures of the inequality 2.3 can now be applied to derivatives that are under a netting agreement and to derivatives that are not under any netting agreement respectively. Denote the k th netting agreement with a counterparty by NA_k. The counterparty-level exposure is now

$$E(T) = \sum_k \max\{0, \sum_{i \in NA_k} V_i(T)\} + \sum_{i \notin \{NA\}} \max\{0, V_i(T)\}. \tag{4.28}$$

Then the other risk mitigation technique is applied to the counterparty-level exposure. Collateralisation is typically modelled as follows.Zhu and Pykhtin (2007, p. 20) state that firstly, the collateral amount, $C(T)$, at a given simulation time, T, is determined by the difference $\Delta(T-S) = E(T-S) - H$, where $E(T-S)$ is the uncollateralised exposure at simulation time, $T-S$, with S being the margin period of risk and H is the threshold value. Now, the collateral value is

$$C(T) = \begin{cases} \Delta(T-S), & \text{if } \max\{0, \Delta(T-S) \geq M \\ 0, & \text{if } \max\{0, \Delta(T-S)\} < M, \end{cases}$$

where M is the MTA. Hence, the collateral is zeroed if it is less than the MTA. Then the collateral $C(T)$ is subtracted from the uncollateralised exposure, $E(T)$, which gives the collateralised exposure at the simulation time, T,

$$E_C(T) = \max\{0, E(T) - C(T)\}. \tag{4.29}$$

According to (Gregory, 2012, pp. 146-147), a margin period of risk of 10 or 20 days is a standard assumption. The case studies of the thesis are carried out for three collateralisation schemes: no-CSA, one-way CSA and two-way CSA. The one-way CSA scheme is tested both from the perspective of the party and from the perspective of the counterparty. In the two-way CSA scheme, the value of a threshold is varied.

One of the key factors affecting the quantification of counterparty credit risk is the exposure expected to be lost in default. This notwithstanding, it is not clear what exposure measure should be used. Crépey et al. (2014, p. 76) point out that the terms used in counterparty risk research are quite fluid. As an example, they mention their use of expected positive exposure, $EPE(t)$, which usually corresponds to expected (conditional) exposure denoted by $EE(t)$ in the literature. Also, these exposures are often calculated assuming the statistical probability $\mathbb{P}$, rather than the pricing measure $\mathbb{Q}$. However, as discussed in section 4.3, the pricing measure, $\mathbb{Q}$, should be used for the calculation of the price of a derivative. For this reason, expected exposure is calculated under $\mathbb{Q}$ in the thesis.

Modelling the expected exposure follows the approach presented in (Zhu and Pykhtin, 2007, pp. 21-22). It is assumed that the exposure is independent of the credit quality of the counterparty. Now,

$$CVA = (1-R)\int_0^T EE^*(t)\,\mathrm{d}PD(0,t), \tag{4.30}$$

where $EE^*(t)$ is the $\mathbb{Q}$-measure (or risk-neutral) discounted expected exposure (EE) given by

$$EE^*(t) = E^Q[\frac{D_0}{D_t}E(t)]. \tag{4.31}$$

Since expectation in equation 4.31 is taken under $\mathbb{Q}$-measure, evolution models for market factors should be arbitrage free. This makes it necessary to properly calibrate the parameters of the evolution model, such as drifts and volatilities, properly. As mentioned in the subsection 4.2.4, the Hull-White one-factor model is used only under risk-neutral setting. Furthermore, the parameters of the model are calibrated properly to market options prices as described in the subsection 4.2.5.

4.5 Default Probabilities

A *credit default swap* (CDS) is the most popular credit derivative and it is relevant to this thesis via default probabilities. This contract insures against the default risk of a particular party. This party is called the *reference entity*. The buyer of CDS has the right to sell bonds issued by the reference entity for their face value if the reference entity defaults and the seller of the CDS has the obligation to buy the bonds for their face value if the reference entity defaults. Hence, the seller insures the buyer against the reference name defaulting. The face value of the bonds is called the *notional principal* of the CDS. The *CDS spread* is the annual amount paid, as a percentage of the notional principal, by the buyer for the contract. Estimated risk-neutral default probabilities can be implied from CDS spreads. (Hull, 2009, pp. 518-519, 523)

In order to estimate risk-neutral default probabilities, the associated recovery rate must also be known. *A recovery rate*, R, refers to the amount that would be recovered in the event of a counterparty defaulting. Usually, recovery rates are assumed to be a percentage of the notional amount (the exposure). They can also be expressed through *loss given default* (LGD), L, where $L = 1 - R$. Recovery rates depend on the seniority of the claim. Normally, OTC derivatives would rank *pari passu* – i.e. have the same seniority and therefore should expect to receive the same recovery value – with senior unsecured debt, which is the reference in most CDS contracts. If the recovery claim for counterparty risk is different, this must be quantified. (Gregory, 2012, pp. 32, 209-210)

4.6 Simulation Specifications

The specifications of the simulations carried out in the thesis are presented here. Firstly, the time increment used in simulations is determined. Based on the discussion in section 4.4, and for the convenience of the simulations, margin period of risk is assumed to be 10 days – or 1/36 years. This factor also effectively determines the time increment, δ, because no other factor requires shorter interval. Furthermore, shorter time increments than this become quickly computationally too intensive for the purposes of this thesis. Hence, simulation times are separated by the constant time increment $\delta = 10$ days. Let the time grid for simulation be $\{T_1, T_2, ..., T_n\}$, where $T_i = (i-1)\delta$.

Secondly, the number of Monte Carlo trials is decided. It has been observed that around 10,000 trials provide satisfactory results in reasonable time (Cesari et al., 2009, p. 131). However, this proves to be too high for the case studies of the thesis computed on a basic classroom desktop computer (64-bit, 16.0 GB RAM). Thus, it is settled for 5,000 trials.

4.7 Bilateral Credit Valuation Adjustment

Now that all the necessary factors to calculate BCVA have been introduced, we begin modelling BCVA from Unilateral CVA and then proceed to BCVA by including DVA.

Modelling CVA in this thesis follows the approach presented in (Zhu and Pykhtin, 2007, pp. 21-22). It is assumed that the party's exposure to the counterparty is independent of the credit quality of the counterparty. This gives

$$CVA = (1-R)\int_0^T EE^*(t)\,\mathrm{d}PD(0,t). \tag{4.32}$$

However, counterparty level calculations for discounted EE require simulations. The simulations are carried out according to the simulation framework in section 4.6. Thus, the integral of equation 4.32 is approximated

$$CVA = (1-R)\sum_{i=1}^{T} EE^*(t_i)PD(t_{i-1},t_i). \tag{4.33}$$

Gregory (2012, p. 267) presents the definition of BCVA, which follows directly from that of unilateral CVA, with the assumption that the party is also subject to default. The BCVA

$$\begin{aligned}BCVA = (1-R_C)\sum_{j=1}^{n} EE(t_j)[1-PD_P(0,t_{j-1})]PD_C(t_{j-1},t_j)\\+(1-R_P)\sum_{j=1}^{n} NEE(t_j)[1-PD_C(0,t_{j-1})]PD_P(t_{j-1},t_j),\end{aligned} \tag{4.34}$$

where the subscript C refers to the counterparty and subscript, P, to the party. This definition is used also in the thesis. It is assumed that the default of the party and the counterparty are independent and that simultaneous default cannot occur.

Chapter 5

Case Studies

5.1 Data

A single derivative is considered in this case study. The derivative is 5x5 USD ATM European payer interest rate swaption. The swaption is assumed to be physically-settled. Furthermore, the underlying swap of the swaption is assumed to have quarterly payment frequency and 30/360 day count convention. The data for calibration consists of the OIS yield curve and of the Black volatilities of USD ATM European swaptions. The interest rates of the yield curve for different maturities on the date of 30.9.2014 are presented in the Table 5.1. In Figure 5.1 the rates are plotted out. The Black volatilities obtained on the date of 30.9.2014 are presented in the Table 5.2. The swaptions differ by expiration date and tenor (i.e. the maturity of the underlying swap). In Figure 5.2 the Black volatilities are plotted out. CDS spreads are shown in the Table 5.3 and Table 5.4. Bank A is the party and Bank B is the counterparty in the derivative. Default probabilities are calculated based on the market spreads.

Table 5.1: OIS yield curve as of 30.9.2014.

Maturity (year)	**Rate (%)**
1	0.2000
2	0.6374
3	1.1058
4	1.4708
5	1.7341
7	2.1078
10	2.4576
12	2.6227
15	2.7912
20	2.9490

Table 5.2: The (mid) Black volatilities of USD ATM European swaptions as of 30.9.2014. The volatilities are expressed in basis points (bps). The data was retrieved from Bloomberg.

Tenor (year) / Expiry (year)	1	2	3	4	5	6	7	8	9	10
1	52.87	44.12	38.82	35.52	33.38	31.40	30.01	27.85	26.79	26.77
2	41.25	36.58	34.11	32.23	30.74	29.59	28.56	26.76	26.06	26.14
3	35.84	33.90	31.72	30.19	28.92	28.04	27.24	25.83	25.26	25.42
4	32.88	31.06	29.77	28.71	27.66	26.91	26.29	25.02	24.51	24.72
5	30.45	28.87	28.01	27.22	26.44	25.80	25.27	24.15	23.78	24.06
6	28.93	27.62	26.78	26.04	25.35	24.82	24.37	23.37	23.06	23.37
7	27.54	26.45	25.60	24.90	24.33	23.88	23.51	22.63	22.39	22.71
8	26.34	25.35	24.59	23.98	23.47	23.07	22.73	21.92	21.69	21.99
9	25.17	24.27	23.62	23.10	22.65	22.30	21.99	21.24	21.01	21.29
10	23.97	23.24	22.70	22.25	21.87	21.57	21.27	20.58	20.35	20.61

Table 5.3: The senior CDS spreads with the recovery value of 40% of Bank B on 29/9/2014. The mid spreads are calculated as average of ask and bid spreads.

Term	**Bid (bps)**	**Ask (bps)**	**Mid (bps)**
6 m	22.420	33.040	27.730
1 yr	25.700	36.890	31.295
2 yr	41.200	51.820	46.510
3 yr	57.010	67.010	62.010
4 yr	76.820	83.720	80.270
5 yr	89.000	94.000	91.500
7 yr	103.010	113.020	108.015
10 yr	120.020	130.020	125.020

Table 5.4: The senior CDS spreads with the recovery value of 40% of Bank A on 29/9/2014. The mid spreads are calculated as average of ask and bid spreads.

Term	**Bid (bps)**	**Ask (bps)**	**Mid (bps)**
6 m	3.510	8.010	5.760
1 yr	6.974	14.067	10.521
2 yr	13.920	26.213	20.067
3 yr	20.180	38.260	29.220
4 yr	36.995	50.715	43.855
5 yr	53.180	63.170	58.175
7 yr	56.590	70.572	63.581
10 yr	61.680	81.620	71.650

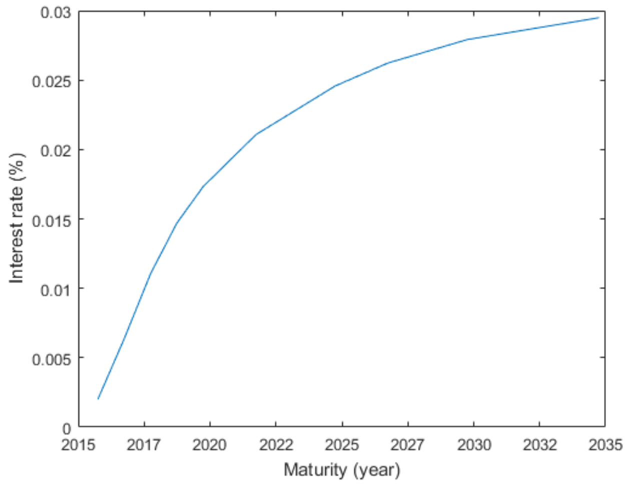

Figure 5.1: OIS yield curve as of 30.9.2014

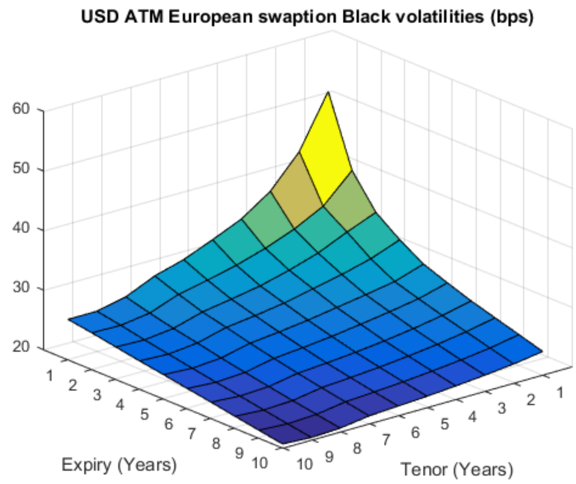

Figure 5.2: The (mid) Black volatilities of USD ATM European swaptions as of 30.9.2014. The volatilities are expressed in basis points (bps).

5.2 Results

Different collateralization arrangements are tested here. CVA, DVA and BCVA calculated with the varying collateralisation arrangements in effect are compared among themselves and to non-collateralised CVA, DVA and BCVA. The results are shown in the Figure 5.3 below. They show the effectiveness of two-way collateralisation with thresholds. Especially, using a zero-threshold descreases CVA and BCVA close to zero, although DVA hardly changes. It is also interesting to observe ineffectiveness of one-way collateralisation agreements, which lead to absolute increases in BCVA compared to case, where there is no collateralisation agreements. At least, differences between CVA and DVA in these two one-way cases are explained by the differences in the default probability curves of the parties.

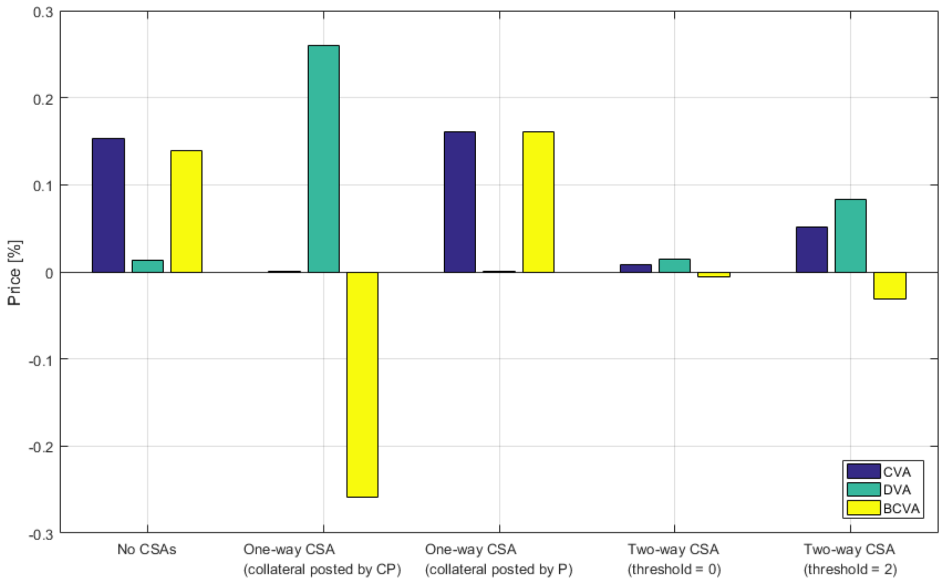

Figure 5.3: CVA, DVA and BCVA expressed as percentage of the notional.

Chapter 6

Conclusions

In this thesis, the problems related to XVAs have been studied. Firstly, the issue of building a consistent, but simplified TVA framework was studied. Secondly, as an example case of applying this TVA framework, numerical simulations were carried out for interest rate options.

TVA can be modelled so that it is consistent, but this usually requires simplification at many stages of the modelling process, if the objective is to make it suitable for practical simulations. In the case example, it was shown that collateralisation is very effective with a two-sided zero-threshold agreement. Overall two-way collateralisation is key to reducing exposure and therefore XVAs. However, XVAs are filled with different issues and a number of assumptions, such as, pricing with a single-curve must be made in theses of this scope.

The potential for future research in relation to the TVA topic is vast. In search of true TVA, it is not sufficient to be limited to only the most prominent of XVAs – i.e. CVA and DVA – as was the case in this thesis. Hence, incorporating more valuation adjustments would be one direction worth taking. The most notable one of these XVAs is FVA. This valuation adjustment is not very well understood yet, but theoretical framework for it has been proposed in a few papers (see, e.g., Burgard and Kjaer, 2011). FVA has not been welcomed without criticism, quite the opposite, because the subject is surrounded by controversy (see, e.g., Hull and White, 2012). Nevertheless, the valuation adjustment needs sound principles behind it or should be entirely disregarded, because the reality is that major banks report their FVAs and there is variation in their calculation methods.

Collateralization appears in the counterparty risk articles relatively rarely, but has recently gained popularity (see, e.g., Brigo et al., 2014). On the surface level, it is uncomplicated to include it in simulation models. However, it can be explored in depth due to many parameters that are connected to it, such as, rehypothecation, threshold, minimum transfer amount, margin period of risk, variation margin and independent amount.

In this thesis the scope was limited to interest rate derivatives and specifically to interest rate swaptions. Generally, counterparty risk aspects are considered with interest rate swaps and credit default swaps, although, at least in theory, these aspects are present in other asset categories as well. Therefore, it would be interesting to understand, for example, the effects of collateralisation on XVAs, when FX, commodity or equity derivatives are studied.

Compared to the future directions listed above, there are also some minor research paths worth covering. Banks deal with many counterparties and may enter numerous derivatives with a single counterparty. Thus, studying a case where a bank has multiple counterparties and a portfolio of derivatives with each one of them would a realistic one. However, this may require computational resources that are not available to most of the researchers. Even in this thesis, the number of simulations had to be limited. Therefore, considering analytical approximations for XVAs could be a useful research direction. Additionally, using multiple interest rate curves in the models would bring valuation adjustments closer to the real world. Furthermore, experimenting with different interest rate models, for example, the stochastic, alpha, beta, rho (SABR) models (Hagan et al., 2002) could produce interesting results.

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