CS-E4500 Problem Set 3

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In this report I have used Gathen and Gerhard (2013), chapter *Division with Remainder using Newton Iteration* as a resource.

Problem 1

Arithmetic with rational numbers in radix-point representation.

(a)

Let us work in base $B = 3$. Multiply 22122.21201 and 22121.22001. Present the result in radix-point representation.

$$22122.21201 \cdot 22121.22001 = 2202201011.1020220201$$

(b)

Let us work in base $B = 7$. Add 145.2332632 and 1345053.103. Present the result in radix-point representation.

$$145.2332632 + 1345053.103 = 1345231.3362632$$

Problem 2

The relation between a polynomial

$$f(x) = a_0 + a_1 x + ... + a_n x^n$$

and its *n-reversal*

$$
\begin{aligned}
x^n f(x^{-1}) &= x^n(a_0 + a_1 x^{-1} + ... + a_n x^{-n}) \\
&= x_0 x^n + a_1 x^{n-1} + ... + a_n \\
&= \operatorname{rev}_n f(x)
\end{aligned}
$$

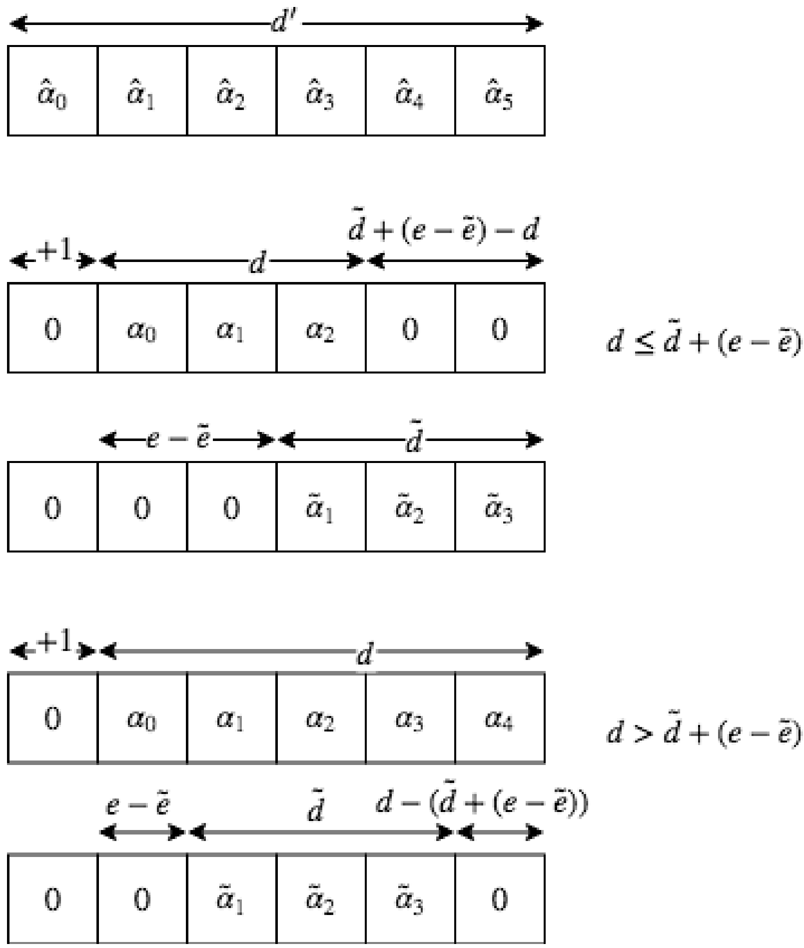
Then the reversal of the polynomial

$$
a(x) = q(x)b(x) + r(x), \quad a, b, q, r \in R[x]
$$

where $n = \deg a \geq \deg b = m$ and $\deg r \leq m - 1$ is

$$
\begin{aligned}
\operatorname{rev}_n a(x) &= x^n a(x^{-1}) \\
&= x^n(q(x^{-1})b(x^{-1}) + r(x^{-1})) \\
&= x^n q(x^{-1})b(x^{-1}) + x^n r(x^{-1}) \\
&= (x^{n-m}q(x^{-1}))(x^m b(x^{-1})) + x^{n-m+1}(x^{m-1}r(x^{-1})) \\
&= \operatorname{rev}_{n-m} q(x) \operatorname{rev}_m b + x^{n-m+1} \operatorname{rev}_{m-1} r.
\end{aligned}
$$

Problem 3



Two numbers in radix-point representation

$$\alpha = sB^e \sum_{i=0}^{d-1} \alpha_i B^{-i}$$

and

$$\tilde{\alpha} = \tilde{s}B^{\tilde{e}} \sum_{i=0}^{\tilde{d}-1} \tilde{\alpha}_i B^{-i}$$

can be summed by matching the radix points of the numbers, padding the *missing* digits with zeros and then using integer addition. The resulting number will also be in the radix-point representation potentially shifting the radix point and increasing the number of digits.

Let choose the operands α and $\tilde{\alpha}$ such that $e \geq \tilde{e}$. Then the addition can also be represented in the radix point representation

$$\begin{aligned}\hat{\alpha} &= \alpha + \tilde{\alpha} \\ &= \hat{s} B^{\hat{e}} \sum_{i=0}^{\hat{d}-1} \hat{\alpha}_i B^{-i}\end{aligned}$$

where the exponent $\hat{e} = \max\{e, \tilde{e}\} = e$ or $\hat{e} = \max\{e, \tilde{e}\} + 1 = e + 1$ if there is a carry and

$$\hat{d} = \max\{d, \tilde{d} + (e - \tilde{e})\} + 1$$

is the number of digits in sum where $+1$ accounts for the carry.

Problem 4

References

Gathen, J. von zur and Gerhard, J., 2013. *Modern Computer Algebra.* 3rd ed. New York, NY, USA: Cambridge University Press.