

# Problem Set 7

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March 10, 2020

## H7.1

Let  $L \in \mathbf{ZPP} = \mathbf{RP} \cap \mathbf{coRP}$ . Then we have two Monte Carlo Turing machines,  $N_1$  which decides  $L$  and  $N_2$  decides which  $\bar{L}$ . We compose PTM  $M$  using  $N_1$  and  $N_2$  such that:

- 1) If  $x \in L$ , then  $x \notin \bar{L}$  and all computations for  $N_2$  halt with “yes”, thus  $Pr[M(x) = 0] = 0$ .
- 2) If  $x \notin L$ , then all computations for  $N_1$  halt with “no”, thus  $Pr[M(x) = 1] = 0$
- 3) Otherwise  $Pr[M(x) = ?] \leq (1/2)^2 \leq 1/3$ .

The probability of getting a definite answers in  $k$  iterations is atleast  $1 - 2^{-k}$  (polynomial). Consequence from condition (1) is that  $M(x) = 1$  if and only if  $x \in L$ .

## H7.2

## H7.3