A7.1 - Rearrangement of a Linear Menu

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The cost c_{ij} for assigning an item i to a position j is defined by the expected time to select the item: $c_{ij} = p_i \cdot d_j \cdot r$. Thus the problem can be formulated as:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} p_i \cdot d_j \cdot r \cdot x_{ij}$$

subject to

$$\sum_{i=1}^{N} x_{ij} = 1 \qquad \forall j = 1..N$$

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$$x_{ij} \in \{0, 1\} \qquad \forall i, j = 1..N$$

- x_{ij} denotes if an item i is assigned to position j.
- p_i is the frequency distribution of the menu items. There are two conditions that must hold for a the distribution: $\sum_{i=1}^{N} p_i = 1$ and $p_i \geq 0$.
- $d_j \geq 0$ is the distance from the start of the menu.
- $r > 0$ is the constant reading cost.

If there are two frequency distributions p^{novice} and they p^{expert} they can be merged into one by taking a weighted sum of the elements

$$p_i = w_1 p_i^{novice} + w_2 p_i^{expert}$$

where $w_1 + w_2 = 1$ and $w_i > 0$ for all elements $i \in \{i, ..., N\}$. It can be verified that the result is also a frequency ditribution:

$$
\begin{aligned}
\sum_{i=1}^{N} p_i &= \sum_{i=1}^{N} (w_1 p_i^{novice} + w_2 p_i^{expert}) \\
&= w_1 \sum_{i=1}^{N} p_i^{novice} + w_2 \sum_{i=1}^{N} p_i^{expert} \\
&= w_1 + w_2 \\
&= 1
\end{aligned}
$$

Also, $p_i > 0$ for all i because the weights are positive and the elements in the two distributions are positive by definition.

The linear assignment problem can be solved in the same way as usual by using the resulting frequency from merging the two distributions. Changing the weight effects how much each frequency distribution contributes to the optimal solution, i.e. if the weight for novice distribution is higher than for experts, the results will favor novices over experts.