A5.1 - Noise Filtering using 1€ Filter

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The Algorithm

The *One Euro Filter*[1] (Casiez, Roussel and Vogel, 2012) implements a low pass filter

$$\hat{X}_i = \alpha X_i + (1 - \alpha)\hat{X}_{i-1} \qquad \text{(low-pass-filter)}$$

where the smoothing factor $\alpha \in [0, 1]$, instead of being a constant, is adaptive, i.e. dynamically computed using information about the rate of change (speed) of the signal. This aims to balance the jitter versus lag trade-off since people are more sensitive to jitter at low speeds and more sensitive to lag at high speeds. The smoothing factor is defined as

$$\alpha = \frac{1}{1 + \frac{\tau}{T_e}}, \qquad \text{(smoothing-factor)}$$

where

1) The τ is time constant computed using the cutoff frequency.

$$\tau = \frac{1}{2\pi f_C} \qquad \text{(time-constant)}$$

2) The cutoff frequency f_C which is designed to increase linearly as the speed increases.

$$f_C = f_{C_{min}} + \beta|\dot{\hat{X}}_i| \qquad \text{(cutoff-frequency)}$$

3) The sampling period T_e which is the time difference between the samples

$$T_e = T_i - T_{i-1}. \qquad \text{(sampling-period)}$$

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[1] http://cristal.univ-lille.fr/~casiez/1euro/

4) The speed $\hat{\dot{X}}_i$ which is calculated using (low-pass-filter) with a constant cutoff frequency f_{C_d}, and therefore constant (smoothing-factor), from the discrete derivative of the samples

$$\dot{X}_i = \frac{X_i - X_{i-1}}{T_e}. \qquad \text{(derivative)}$$

There are two configurable parameters in the model.

1) The minimum cutoff frequency. Decreasing this value will decrease slow speed jitter.

$$f_{C_{min}} > 0 \qquad \text{(minimum-cutoff-frequency)}$$

2) The speed coefficient. Increasing this value will decrease speed lag.

$$\beta > 0 \qquad \text{(speed-coefficient)}$$

Having only two parameters makes the model simple and easier to understand and tune.

Results

By empirically testing values for the parameters, some good ranges found for the values were $f_{C_{min}} \in [0.01, 0.005]$ and $\beta = [0.01, 0.001.]$. The attached code and set of figures will display the relationship between these parameters and the filtered signal.

References

Casiez, G., Roussel, N. and Vogel, D., 2012. 1€ filter: A simple speed-based low-pass filter for noisy input in interactive systems. In: *Proceedings of the sigchi conference on human factors in computing systems.* ACM, pp.2527–2530.