Question 1 - Solving Recurrence Relation

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Solve the recurrence

$$T(n) = T(3n/4) + 1, \quad T(1) = 1$$

by using iterative method. You may assume that $n = 4^k$ for some integer k.

Using iteration on the recurrence relation we obtain

$$\begin{aligned} T(1) &= 1 \\ T(4/3) &= T(1) + 1 = 2 \\ T((4/3)^2) &= T(4/3) + 1 = 3 \\ &\vdots \\ T((4/3)^k) &= k + 1. \end{aligned}$$

By substituting

$$x = (4/3)^k \Leftrightarrow k = \log_{4/3} x$$

we obtain a solution to the recurrence relation

$$T(x) = \log_{4/3} x + 1.$$