QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$

$$D_\mu = \partial_\mu + ieA_\mu$$

Work in Coulomb gauge: $\nabla \cdot A = 0$

E.o.M for A^0 is

$$-\nabla^2 A_0 = e\psi^\dagger\psi = ej^0 \qquad j^\mu = \bar{\psi}\gamma^0\psi$$

This has solution $\quad A_0(x) = e\int d^3x' \dfrac{j^0(\underline{x}', t)}{4\pi|\underline{x} - \underline{x}'|}$

In Coulomb gauge, we can rewrite the Maxwell part of the Lagrangian as

$$L_M = \int d^3x \, \frac{1}{2}(\underline{E}^2 - \underline{B}^2)$$

$$= \int d^3x \, \frac{1}{2}(\dot{\underline{A}} - \underline{\nabla}A_0)^2 - \frac{1}{2}B^2$$

$$= \int d^3x \, \frac{1}{2}\dot{\underline{A}}^2 + \frac{1}{2}(\underline{\nabla}A_0)^2 - \frac{1}{2}\underline{B}^2$$

$\nabla \cdot \underline{A} = 0$ means the cross term vanish from integration by parts

$$L_M = \int d^3x \left\{ \frac{1}{2}\dot{\underline{A}}^2 + \underbrace{\frac{e^2}{2}\int d^3x' \frac{j_0(x)\, j(x')}{4\pi|x - x'|}} - \frac{1}{2}B^2 \right\}$$

n.l. term! Arises as an artifact of working in Coulomb gauge (doesn't appear in Lorentz gauge)

Compute H:

$$\underline{\Pi} = \frac{\partial\mathcal{L}}{\partial\dot{\underline{A}}} = \dot{\underline{A}} \qquad \Pi_\psi = \frac{\partial\mathcal{L}}{\partial\dot{\psi}} = i\dot{\psi}^\dagger$$

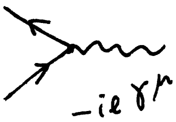
$$\Rightarrow H = \int d^3x \left\{ \frac{1}{2}\dot{\underline{A}}^2 + \frac{1}{2}\underline{B}^2 + \bar{\psi}(-i\gamma^i\partial_i + m)\psi - e\underline{j}\cdot\underline{A} \right.$$

$$\left. + \frac{e^2}{2}\int d^3x' \frac{j_0(x)\, j_0(x')}{4\pi|x - x'|} \right\}$$

$\bar{\psi}\gamma_0\psi$

$\underline{j} = \bar{\psi}\underline{\gamma}\psi$

Feynman Rules



$-ie\gamma^\mu$

transverse photon propagator $\mu \sim\sim \nu$

$$D^{tr}_{ij} = \frac{i}{p^2 + i\varepsilon}\left(\delta_{ij} - \frac{p_i p_j}{|\underline{p}|^2}\right)$$

instantaneous n.l. int^n $\underset{x}{\bullet} - - - \underset{y}{\bullet}$

$$\frac{(e\gamma_0)^2\, \delta(x^0 - y^0)}{4\pi|\underline{x} - \underline{y}|}$$ what do we do with this

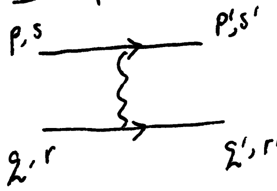
Since it comes from A_0, we could try to massage it into a D_{00} piece of the γ propagator.

In momentum space $\dfrac{\delta(x^0 - y^0)}{4\pi|\underline{x} - \underline{y}|} = \displaystyle\int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{|\underline{p}|^2}$

we now define the γ propagator

$$D_{\mu\nu}(p) = \begin{cases} \dfrac{i}{p + i\varepsilon}\left(\delta_{ij} - \dfrac{p_i p_j}{|\underline{p}|^2}\right) & \mu = i \neq 0 \\ & \nu = j \neq 0 \\ \dfrac{i}{|\underline{p}|^2} & \mu = \nu = 0 \\ 0 & \text{otherwise} \end{cases}$$

Examples $e^-e^- \to e^-e^-$



$$\sim e^2 [\bar{u}(p')\gamma^\mu u(p)]\, D_{\mu\nu}(k)\, [\bar{u}(q')\gamma^\nu u(q)]$$

$$k \equiv p - p' = q' - q$$

claim: can replace $D_{\mu\nu}(k)$ by $\dfrac{-i\eta_{\mu\nu}}{k^2}$

proof: $(\not{p} - m)u(p) = 0$

Define the spinor combinations

$$\alpha^\mu = \bar{u}(p')\gamma^\mu u(p)$$

$$\beta^\mu = \bar{u}(q')\gamma^\mu u(q)$$

Then $k_\mu \alpha^\mu = \bar{u}(p')(\not{p}' - \not{p})u(p) = \bar{u}(p')(m - m)u(p) = 0$

$k_\mu \beta^\mu = 0$ similarly

By the same token $k_\mu \beta^\mu = 0$

So, our Feynman diagram is given by $\alpha^\mu D_{\mu\nu}\beta^\nu$

$$= i\left(\frac{\underline{\alpha}\cdot\underline{\beta}}{k^2} - \frac{(\underline{\alpha}\cdot\underline{k})(\underline{\beta}\cdot\underline{k})}{|\underline{k}|^2 k^2} + \frac{\alpha^0\beta^0}{|\underline{k}|^2}\right)$$

$$= i\left(\frac{\underline{\alpha}\cdot\underline{\beta}}{k^2} - \frac{k_0^2\alpha^0\beta^0}{|\underline{k}|^2 k^2} + \frac{\alpha^0\beta^0}{|\underline{k}|^2}\right)$$

$$= i\left(\frac{\underline{\alpha}\cdot\underline{\beta}}{k^2} - \frac{1}{|\underline{k}|^2 k^2}\underbrace{(k_0^2-k^2)}_{|\underline{k}|^2}\alpha^0\beta^0\right)$$

$$= -i\frac{\alpha\cdot\beta}{k^2} = \alpha^\mu\left(-\frac{i\eta_{\mu\nu}}{k^2}\right)\beta^\nu$$

from current conservation

Can make same substitution for

 $\sim e^2[\bar{v}(q)\gamma^\mu u(p)]\,D_{\mu\nu}(p+q)\,[\bar{u}(p')\gamma^\nu v(q')]$

It's a fact that we can always use the nice L.I. version

$D_{\mu\nu} = -\frac{i\eta_{\mu\nu}}{k^2}$ – in fact it's also the propagater in Lorenz gauge (Feynman gauge)

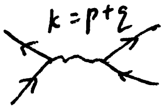
In general

$$D_{\mu\nu} = -\frac{i}{p^2}\left(\eta_{\mu\nu} + (\alpha-1)\frac{p_\mu p_\nu}{p^2}\right)$$

this cancels in all physical diagrams

example:

 $k=p-p'$ $\quad \bar{u}(p')\gamma^\mu u(p)\,k_\mu = 0$

$k=p+q$
 $\bar{v}(p)\gamma^\mu u(q)\,k_\mu = \bar{v}(p)(\not{p}+\not{q})u(q) = 0$