5.1 (continued)

- Scalar acquiring a VEV, w.l.o.g. $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$

$$D_\mu \phi = (\partial_\mu + \underset{SU(2)}{i g W_\mu^a \tau^a} + \tfrac{i}{2} g' B_\mu)\phi$$

coupling of $U(1)$ to Higgs

Find $\tau^a = \frac{1}{2}\sigma^a$, $f^{abc} = \varepsilon^{abc}$

- SSB $\mu^2 = -\lambda v^2 < 0$. Some of the gauge fields get mass.

$(D_\mu\phi)^\dagger(D_\mu\phi)$ contains

$$\frac{1}{2}\frac{v^2}{4}\left[g^2(W^1)^2 + g^2(W^2)^2 + (-gW^3 + g'B)^2\right]$$

- Define: $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm i W_\mu^2)$, $\begin{pmatrix}Z_\mu\\A_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}W_\mu^3\\B_\mu\end{pmatrix}$

We see that

$m_W = \frac{vg}{2}$, $m_Z = \frac{v}{2}\sqrt{g^2+g'^2}$, θ_W is the Weinberg angle, satisfying

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

photon A_μ with $m_\gamma = 0$

so $SU(2) \times U(1) \to U(1)_{EM}$

Tree level prediction: $m_W = m_Z \cos\theta_W$

Experimentally: $m_W \approx 80\,\text{GeV}$, $m_Z \approx 91\,\text{GeV}$, $m_\gamma \lesssim 10^{-8}\,\text{eV}$

- Also Higgs boson gets mass $m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$ λ and so m_H are not predicted by SM
- Also get $W^\pm$, Z – Higgs & Higgs – Higgs interactions

5.2 Coupling to Leptons (quarks later)

$$D_\mu = \partial_\mu + i g W_\mu^a \underset{\text{generators}}{T^a} + i g' Y B_\mu$$

hypercharge – coupling of leptons to $U(1)$ gauge

$T^3(1-\sin^2\theta_W) - \sin^2\theta_W Y$

$= T^3 - \sin^2\theta_W Q$

$$D_\mu = \partial_\mu + \frac{ig}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) + \frac{ig Z_\mu}{\cos\theta_W}(\cos^2\theta_W T^3 - \sin^2\theta_W Y)$$

$$+ \underbrace{ig\sin\theta_W}_{\text{magnitude of electric charge}} A_\mu \underbrace{(T^3 + Y)}_{Q = U(1)_{EM}\text{ charge}}$$

where $T^\pm = T^1 \pm i T^2$

Experimentally $W^{\pm}$ only couple to LH quarks & leptons

$\therefore$ put RH fermions in the trivial rep of $SU(2)$ (where $T^a = 0$)

e.g. $R(x) = e_R(x)$. By contrast LH fermions are in the fundamental rep. $T^a = \tau^a$

$$L(x) = \begin{pmatrix} \nu_{e_L}(x) \\ e_L(x) \end{pmatrix} \leftarrow \text{electron neutrino} \qquad e_L(x) := \tfrac{1}{2}(1-\gamma^5)\, e(x)$$

* N.B. For now we are assuming that neutrinos are massless, and LH only

For $R(x)$: $Q = Y = -1$

For $L(x)$: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ & since $Q = \tau^3 + Y$, have $Y = (-\tfrac{1}{2}) \otimes \mathbb{1}_2$; $\tau^3 = \tfrac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

~ in fact need to $\otimes$ this with 4x4 spinor space (which is why we can slash the D operator)

Putting this together : $\mathcal{L}^{EW}_{lept} = \bar{L}\, i \not{D} L + \bar{R}\, i \not{D} R$

<u>What about fermion masses?</u>

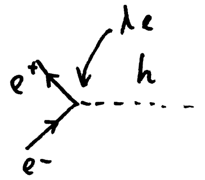
Fermion mass term break gauge invariance (explicitly). But we can consider fermion - Higgs interactions. $(\bar{e}_L(x), \bar{\nu}_{e_L}(x))$

$$\mathcal{L}_{lept.\phi} = -\sqrt{2}\,\lambda_e\,(\bar{L}\phi R + \bar{R}\phi^{+} L) \qquad \lambda_e = \text{Yukawa coupling}$$

[check gauge invariance. Note $\sum Y = 0$ in each term]

Working in unitary gauge and expanding : $\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

$$\Rightarrow \mathcal{L}_{lept,\phi} = -\lambda_e\,(v+h)\,(\bar{e}_L e_R + \bar{e}_R e_L) = -\underbrace{m_e}_{\text{fermion mass } = \lambda_e v}\,\bar{e}e - \lambda_e h \bar{e}e$$



N.B. The Higgs - fermion coupling $\propto m_e$

Now go back to fermion - gauge boson interaction (from $\mathcal{L}^{EW}_{lept}$)

$$\mathcal{L}^{EW,int}_{lept} = \frac{g}{2\sqrt{2}}\,(J^\mu W_\mu^{+} + J^{\mu +} W_\mu^{-}) + e J^\mu_{EM} A_\mu + \frac{g}{2\cos\theta_W} J^\mu Z_\mu$$