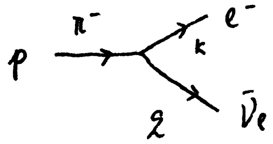
6.4 – π decay

$\pi^-(\bar{u}\, d) \rightarrow e^- \bar{\nu}_e$ (assume $m_\nu = 0$)



The d & $\bar{u}$ don't propagate freely, they are bound inside π^-. The relevant currents are

- $J^\alpha_{lept} = \bar{\nu}_e \gamma^\alpha (1-\gamma^5)\, e$
- $J^\alpha_{had} = \bar{u}\gamma^\alpha(1-\gamma^5)\,(V_{ud}\, d + \dots)$

$$= \underset{\gamma^\alpha}{\underset{\uparrow}{V^\alpha_{had}}} - \underset{\gamma^\alpha\gamma^5}{\underset{\uparrow}{A^\alpha_{had}}}$$

The amplitude is

$$M = \langle e^-(k)\, \bar{\nu}_e(q) \,|\, \mathcal{L}^{eff}_W \,|\, \pi^-(p)\rangle$$

$$= -\frac{G_F}{\sqrt{2}} \langle e^-(k)\, \bar{\nu}_e(q) | \bar{e}\,\gamma_\alpha(1-\gamma^5)\,\nu_e |0\rangle \langle 0 | J^\alpha_{had} | \pi^-(p)\rangle$$

[QCD is P-invariant and π has spin 0
parity = $-$ve $\quad \langle 0|\bar{u}\gamma^\alpha d|\pi^-(p)\rangle = 0$ $\quad$ can't write down Lorentz covariant structure]

Thus get

$$M = \frac{1}{\sqrt{2}} G_F\, \bar{u}_e(k)\,\gamma_\alpha(1-\gamma^5)\, V_{\nu_e}(q)\, \langle 0 | \cancelto{0}{V^\alpha_{had}} - A^\alpha_{had} | \pi^-(p)\rangle$$

Parametrise the unknown non-perturbative QCD part in the pion decay const F_π

$$\langle 0|\bar{u}\gamma^\alpha\gamma^5 d|\pi^-(p)\rangle = i\sqrt{2}\, p^\alpha F_\pi$$

$$\therefore\ M = i G_F V_{ud}\, \bar{u}_e(k)\, \not{p}\,(1-\gamma^5)\, V_{\nu_e}(q)\, F_\pi$$

$$= i G_F F_\pi V_{ud}\, m_e\, \bar{U}_e(k)\,(1-\gamma^5)\, V_{\nu_e}(q)$$

$$\sum_{\substack{spins\\ e,\nu_e}} |M|^2 = \sum_{spins} |G_F F_\pi V_{ud} m_e|^2 \left[\bar{u}_e(k)(1-\gamma^5) V_{\nu_e}(q)\, \overset{\bar{V}_{\nu_e}(q)}{\vee}(1+\gamma^5)\, u_e(k)\right]$$

$$= 8\,|G_F F_\pi V_{ud} m_e|\,(k\cdot q)$$

This again shows helicity suppression:

Spin-0 π^- decays to +ve helicity $\bar{\nu}_e$ and +ve helicity e^-. But if $m_e = 0$ this is RH chirality and is forbidden

weak interaction couples to LH chiral particles (RH antiparticles)

$$\underset{\Leftarrow \frac{1}{2}}{\overset{\bar{\nu}_e}{\longleftarrow}} \quad \overset{\cdot}{\pi^-} \quad \underset{\Rightarrow \frac{1}{2}}{\overset{e^-}{\longrightarrow}}$$

$$\Gamma_{\pi\to e\bar{\nu}_e} = \frac{1}{2m_\pi}\int\frac{d^3k}{(2\pi)^3 2k^0}\int\frac{d^3q}{(2\pi)^3 2q^0}(2\pi)^4\delta^{(4)}(p-k-q)\,8\,|G_F F_\pi m_e V_{ud}|^2\,(k\cdot q)$$

↑ in rest frame of π

$$= \frac{|G_F F_\pi m_e V_{ud}|^2}{4\pi^2 m_\pi}\int\frac{d^3k}{E|\vec{k}|}\,\delta(m_\pi - E - |\vec{k}|)\,(E|\vec{k}| + |\vec{k}|^2)$$

$$\left(E = k^0 = \text{energy of } e^-,\quad q^0 = |\vec{q}| = |\vec{k}|\right)$$

$$= \frac{|G_F F_\pi m_e V_{ud}|^2}{4\pi^2 m_\pi}\int\frac{4\pi k^2\,dk}{E}\,\frac{(E+k)}{\left(1+\frac{k_0}{E}\right)}\,\delta(k-k_0) \quad \text{where } k_0 = \frac{m_\pi^2 - m_e^2}{2m_\pi}$$

$$\Gamma_{\pi\to e\bar{\nu}_e} = \frac{|G_F F_\pi V_{ud}|^2}{4\pi}\,m_e^2\, m_\pi\left(1 - \frac{m_e^2}{m_\pi^2}\right)^2$$

The expression for $\Gamma_{\pi\to\mu\bar{\nu}_\mu}$ is the same with $m_e \to m_\mu$. Ratio is

$$\frac{\Gamma_{\pi\to e\bar{\nu}_e}}{\Gamma_{\pi\to\mu\bar{\nu}_e}} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right) \simeq 1.28\times 10^{-4}$$

c.f. exp $= 1.230(4)\times 10^{-4}$. Reasonable agreement, need quantum loop effects to bring into better agreement. Note: ratio $\ll 1$ ∵ $m_\mu \gg m_e$ so less helicity suppressed.

6.5 – $K^0 - \bar{K}^0$ Mixing

Kaons contain a s quark/antiquark. Flavour eigenstates:

$K^0\ (\bar{s}d)$, $\bar{K}^0\ (\bar{d}s)$, $K^+\ (\bar{s}u)$, $K^-\ (\bar{u}s)$

Are the lightest kaons, they have: $J^P = 0^-$ (i.e. pseudoscalars)
↑ spin ↑ parity

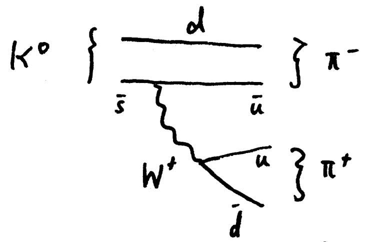
For kaons at rest we can take relative

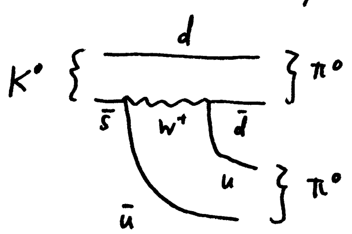
$\hat{C}\hat{P}|K^0\rangle = -|\bar{K}^0\rangle$, $\hat{C}\hat{P}|\bar{K}^0\rangle = -|K^0\rangle$

The CP eigenstates are

$|K^0_\pm\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \mp |\bar{K}^0\rangle)$, $\hat{C}\hat{P}|K^0_\pm\rangle = \pm|K^0_\pm\rangle$

Let's consider: $K^0 \longrightarrow \pi^0\pi^0$ and $\pi^+\pi^-$ the relevent Feynman diagrams are





recall π^0 has $J^P = 0^-$

From conservation of angular momentum: total angular momentum of $\pi\pi = 0 \Rightarrow$ orbital $L = 0$

c.f. for K^0: $\hat{C}\hat{P}|\pi^+\pi^-\rangle = (-1)^L|\pi^+\pi^-\rangle = |\pi^+\pi^-\rangle$
$\hat{C}\hat{P}|\pi^0\pi^0\rangle = \ldots\ldots\ |\pi^0\pi^0\rangle$ } CP +1 eigenstates

If CP conserved for weak $K^0_+ \to \pi\pi$ but $K^0_- \not\to \pi\pi$
"short" lived "long" lived