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$$V_{CKM} = U_u^\dagger U_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Not every CKM matrix element is independent.

Constraints: unitary, global phase invariance of the quark fields

2 generation case

unitary $\rightarrow$

$$V = \begin{pmatrix} \cos\theta_c \, e^{i\alpha} & \sin\theta_c \, e^{i\beta} \\ -\sin\theta_c \, e^{i(\alpha+\gamma)} & \cos\theta_c \, e^{i(\beta+\gamma)} \end{pmatrix}$$

Terms in the Lagrangian are invariant under $U(1)$ global transformation of any quark field, $q_L^i \longmapsto e^{i\alpha^i} q_L^i$

4 fields $\Rightarrow$ can perform 3 such transforms to eliminate α, β, and γ.

$$V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$ where θ_c is the Cabibbo angle ≈ 0.22

Then weak charged current is

$$J^{\mu,+} = \cos\theta_c \bar{u}_L \gamma^\mu d_L + \sin\theta_c \bar{u}_L \gamma^\mu s_L - \sin\theta_c \bar{c}_L \gamma^\mu d_L + \cos\theta_c \bar{c}_L \gamma^\mu s_L$$

3 generation case

9 independent parameters $\xrightarrow{\text{unitary}}$ 3 angles and 6 phases

$\downarrow$ 6 quark fields, eliminate 5 phase

3 angles and 1 phase

(4 free parameters in total)

$\lambda \equiv V_{us} \approx \sin\theta_c \approx 0.22 \ll 1$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$ not exact

The fact that V_{CKM} has a phase means that the Yukawa matrices λ^{ij} cannot be real. Therefore the Standard Model Lagrangian violates CP.

5.4 Neutrino oscillations and mass

We now know that mass eigenstates & weak eigenstates for neutrinos are not equivalent. The analogous mixing matrix in U_{PMNS}. If neutrinos are Dirac fermions (like quarks), there are 3 angles and 1 phase (CP violation in general).

If neutrinos are Majorana fermions, 3 angles + 3 phases
bc no longer independent anti-particle field (CP violation in general)

<u>Dirac fermions</u>: $N^i = \nu_R^i = (\nu_{e_R}, \nu_{\mu_R}, \nu_{\tau_R})$

Must modify $\mathcal{L}_{lept,\phi} = -\sqrt{2}\,(\lambda^{ij}\bar{L}^i\phi R^j + \lambda_\nu^{ij}\bar{L}^i\phi^c N^j + h.c.)$

Exactly like for quarks, Neutrinos get mass because

$$-\sum_i M_\nu^i(\bar{\nu}_R\nu_L^i + \bar{\nu}_L^i\nu_R^i)$$

<u>Majorana fermions</u>.

Neutrinos are electrically neutral, could be its own antiparticle

$$d^s(p) = b^s(p)$$

Take the C-parity = 1 (w.l.o.g)

$$\nu_R(x) = \nu_L^c(x) = C\bar{\nu}_L^T(x)$$

$$\left(\text{Here } \nu^c(x) = \underset{C\bar{\nu}^T=}{\gamma_C}\,(C^{-1}\nu(x)) = \nu(x)\right)$$

⇒ RH neutrino field not independent, it's charge conj. of LH field.

Mass term: $-\frac{1}{2}\sum_i M_\nu^i(\bar{\nu}_L^{ic}\nu_L^i + \bar{\nu}_L^i\nu_L^{ic})$

To arise from a Higgs VEV, we need a term

$$\mathcal{L}_{\nu\phi M} = -\frac{Y^{ij}}{M}(L^{iT}\phi^c)\,C\,(\phi^{cT}L^j) + h.c.$$

This dim 5 op. is non-renormalisable. This is OK as long as we think of SM as effective field theory at energy scale << scale of "new physics"

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Summary of EW theory

$\mathcal{L}_{\text{gauge},\phi}$ - masses for $W^\pm$, Z and Higgs. $W^\pm$, Z - H interactions and H-H interactions

$\mathcal{L}_{\text{lept},\phi}$ - lepton masses, lepton - H interactions

$\mathcal{L}^{EW}_{\text{lept}}$ - lepton - $W^\pm$, Z, γ int (PMNS matrix : ν oscillations / mixing and probably $\cancel{CP}$)

$\mathcal{L}_{\text{quark},\phi}$ - quark masses, quarks - H int.

$\mathcal{L}^{EW}_{\text{quark}}$ - quark int. with $W^\pm$, Z, γ
(CKM matrix : quark flavour mixing & CP violation)

6. Weak Decays

6.1. Effective lagrangian

We'll consider some processes where energies, momenta $\ll M_W, M_Z$, so we can use an effective field theory (Fermi weak lagrangian)

The weak interaction part of the lagrangian is

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}}\left(J^\mu W_\mu^+ + J^{\mu +} W_\mu^-\right) \leftarrow \text{charged current}$$

$$+ \frac{g}{2\cos\theta_W} J_n^\mu Z_\mu \leftarrow \text{neutral current}$$

and the S-matrix is

$$S = T \exp\left[i\int d^4x\, \mathcal{L}_W(x)\right]$$

For small g, can Taylor expand, and assuming no $W^\pm$, Z in initial / final states.

$$\langle f|S|i\rangle = \langle f|T\Big\{1 - \frac{g^2}{8}\int d^4x\, d^4x' \Big[J^{\mu +}(x)\, D^W_{\mu\nu}(x-x')\, J^\nu(x')$$

$$+ \frac{1}{\cos^2\theta_W} J_n^{+\mu}(x)\, D^Z_{\mu\nu}(x-x')\, J_n^\nu(x')\Big] + O(g^4)\Big\}|i\rangle$$

where we've used Wick's theorem and $D^W_{\mu\nu}(x-x') = \langle T W_\mu^-(x) W_\nu^+(x')\rangle$ (Feynman propagator) & same for Z