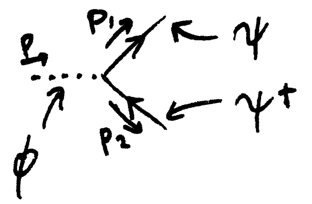
Feynman diagrams

Draw Feynman diagrams to represent the expansion of $\langle f | S | i \rangle$ and associate # (or $\int$) to them.

Terms in the expansion:

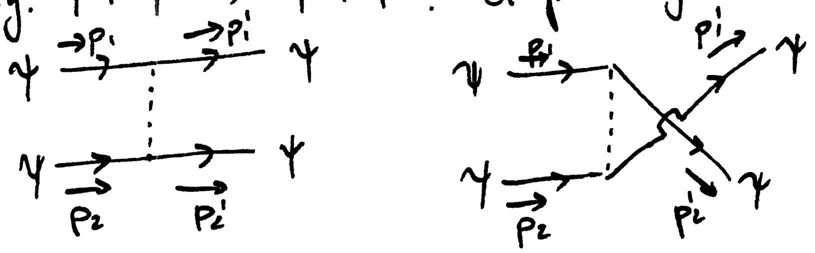
- external line $\forall$ part^c in $|i\rangle$ and $|f\rangle$
  Assign a directed momentum p, an arrow for ψ-part^c to denote charge
  Incoming (outgoing) arrow in the initial state for ψ ($\bar{\psi}$) and reverse for final state.
- join the lines w/ vertices

$\psi^+ \psi \phi$

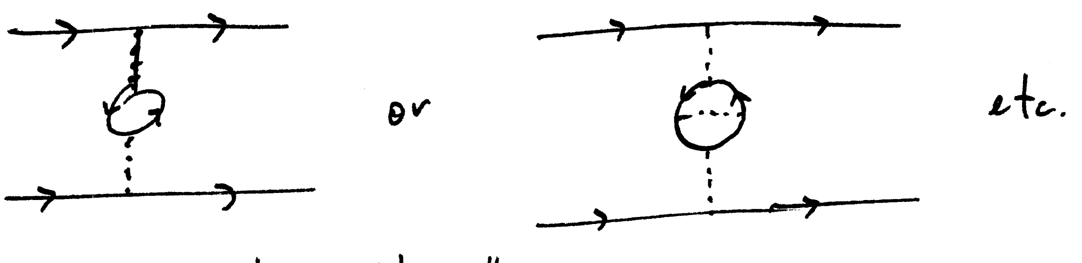


Each diagram 1:1 term in $\langle f | (S-1) | i \rangle$

e.g. $\psi + \psi \to \psi + \psi$. Simplest diagrams:



$\exists$ more complicated diagrams:



Use Feynman rules to associate a #.

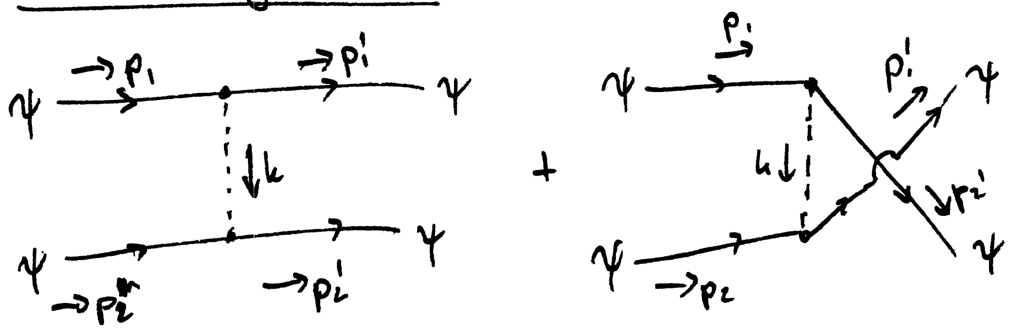
(a) momentum k_i to each internal line i

(b) factor $(-ig)(2\pi)^4 \delta^4(\sum_i k_i)$ to each vertex ← sum of all momenta flowing into the vertex

(c) factor $\int \frac{d^4k}{(2\pi)^4} D(k^2)$ for each internal line momentum k

$$D = \frac{i}{k^2 - m^2 + i\varepsilon} \text{ for } \phi, \quad D = \frac{i}{k^2 - \mu^2 + i\varepsilon} \text{ for } \psi$$

Nuclear Scattering (revisited)



$$(-ig)^2\int\frac{d^4k}{(2\pi)^4}(2\pi)^8\,\frac{i}{k^2-m^2+i\varepsilon}\Big[\delta^4(p_1-p_1'-k)\,\delta^4(p_2-p_2'+k)$$
$$+\delta^4(p_1-p_2'-k)\,\delta^4(p_2-p_1'+k)\Big]$$

$$= i(-ig)^2\left[\frac{1}{(p_1-p_1')^2-m^2}+\frac{1}{(p_1-p_2')^2-m^2}\right](2\pi)^4\,\delta^4(p_1+p_2-p_1'-p_2') \qquad (*)$$

Physical interpretation:
Nucleons exchange a meson of momentum $k=p_1-p_1'=p_2-p_2'$ (in 1st diagram).
The meson does not necessarily satisfy $k^2=m^2$, then called "off-shell" or "virtual meson".
External are "on-shell", satisfying $p_i^2=m^2$.

Amplitudes

$\int d^4x$ in $\psi+\psi\to\psi+\psi$ scattering at order g^2:
def^n amplitude $\mathcal{A}_{fi}$ by eliminating 4-momentum conserving δ-s. (translational invariance and common to all S-matrix elements)

$$\langle f|(S-1)|i\rangle = i\ \mathcal{A}_{fi}\ (2\pi)^4\ \delta^4(p_f-p_i)$$

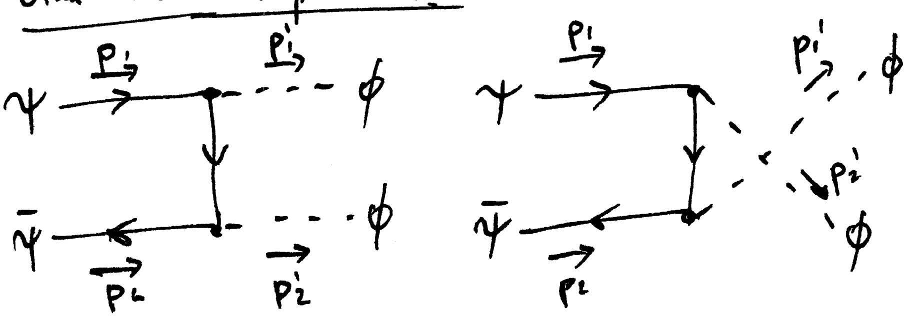
i: by convention to match non-rel QM

p_f: Σ of final state 4-momenta

p_i: Σ of initial state 4-momenta

Compute $\mathcal{A}_{fi}$ (Feynman rules):
- All possible diagrams with 4-momentum conservation at each vertex
- Factor of $(-ig)$ for each vertex
- Factor im propagator for each internal line
- Integrate over 4-momentum k in each loop $\int\frac{d^4k}{(2\pi)^4}$

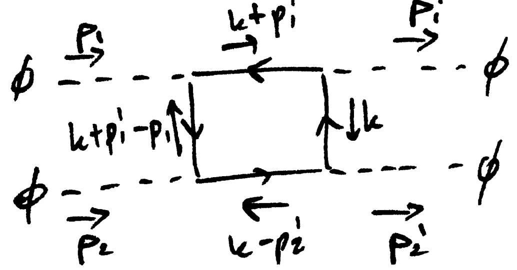
Other tree-level processes



$$\mathcal{A}=(-ig)^2\left[\frac{1}{(p_1-p_1')^2-\mu^2}+\frac{1}{(p_1-p_2')^2-\mu^2}\right]$$

(drop $i\varepsilon$ in denominator as it never vanishes)

$\phi\phi \to \phi\phi$



Momentum conservation in lower left corner ::

$$\sum_{i \in \text{final}} p_i - \sum_{i \in \text{initial}} p_i = 0$$

Our graph is

$$i\mathcal{A} = (-ig)^4 \int \frac{d^4k}{(2\pi)^4} \frac{i^4}{(k^2-\mu^2+i\varepsilon)((k+p_1')^2-\mu^2+i\varepsilon)((k+p_1'-p_1)^2-\mu^2+i\varepsilon)((k-p_2')^2-\mu^2+i\varepsilon)}$$

At large k, $\int \frac{d^4k}{k^8}$ is convergent, <u>but</u> not always the case.