6.3 μ decay (cont)

$$\frac{1}{2}\sum_{spins}|M|^2 = S_1^{\alpha\beta}\, S_{2\alpha\beta}\,\frac{G_F^2}{4}$$

$$S_1^{\alpha\beta} = \mathrm{Tr}\left[(\not{k}+m_e)\gamma^\alpha(1-\gamma^5)\not{q}\,\gamma^\beta(1-\gamma^5)\right]$$

$$S_{2\alpha\beta} = \mathrm{Tr}\left[\not{q}'\gamma_\alpha(1-\gamma^5)(\not{p}+m_\mu)\gamma_\beta(1-\gamma^5)\right]$$

$$S_1^{\alpha\beta} = 8\left[k^\alpha q^\beta + k^\beta q^\alpha - k\cdot q\, g^{\alpha\beta} - i\,\varepsilon^{\alpha\beta\mu\rho}k_\mu q_\rho\right]$$

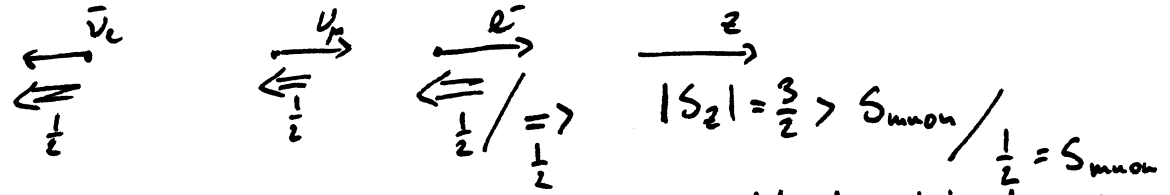
$$S_{2\alpha\beta} = 8\left[q'_\alpha p_\beta + q'_\beta p_\alpha - q'\cdot p\, g_{\alpha\beta} - i\,\varepsilon_{\alpha\beta\mu\rho}\, q'^\mu p^\rho\right]$$

This leads to, $\frac{1}{2}\sum_{spins}|M|^2 = 64\, G_F^2\,(p\cdot q)(k\cdot q')$

Consider the case when e^- and ν_μ go out along $+z$ and ν_e along $-z$:

$$k\cdot q' = \sqrt{m_e^2+k_z^2}\; q_z' - k_z q_z' \longrightarrow 0 \quad \text{if } m_e = 0$$

- weak interaction only couples to LH chiral particles (RH antiparticles)



If $m_e \neq 0$, LH and RH components are coupled (-ve helicity not same as LH chirality) $\Rightarrow$ may occur 'helicity suppressed'

Decay rate
$$\Gamma = \frac{1}{2m_\mu}\int\frac{d^3k}{(2\pi)^3 2k^0}\int\frac{d^3q}{(2\pi)^3 2q^0}\int\frac{d^3q'}{(2\pi)^3 2q'^0}(2\pi)^4\delta^{(4)}(p-k-q-q')\frac{1}{2}\sum_{spins}|M|^2$$

$$= \frac{G_F^2}{8\pi^5 m_\mu}\int\frac{d^3k\, d^3q\, d^3q'}{k^0 q^0 q'^0}\,\delta^{(4)}(p-k-q-q')\,(p\cdot q)(k\cdot q')$$

Consider,
$$I_{\mu\nu}(p-k) = \int\frac{d^3q\, d^3q'}{q^0 q'^0}\,\delta^{(4)}(p-k-q-q')\, q_\mu q'_\nu$$

From sym, $I_{\mu\nu}(p-k) = \overset{(f^n)}{a}((p-k)^2)\,(p-k)_\mu(p-k)_\nu + \overset{(f^n)}{b}((p-k)^2)\, g_{\mu\nu}(p-k)^2$

Consider $g^{\mu\nu}I_{\mu\nu}(p-k) = \int\cdots q\cdot q' = a(p-k)^2 + 4b(p-k)^2$

$$(q+q')^2 = q^2 + q'^2 + 2q\cdot q' = 2q\cdot q'$$

But from mom. conservation $= (p-k)^2$

$a + 4b = \frac{I}{2}$ ①, $\quad I = \int\frac{d^3q}{q^0}\int\frac{d^3q'}{q'^0}\,\delta^4(p-k-q-q')$

and $(p-k)^\mu (p-k)^\nu I_{\mu\nu} = a(p-k)^4 + b(p-k)^4 = \int \cdots\, q\cdot(p-k)\; q'\cdot(p-k)$

$(q\cdot q')\quad (q\cdot q')$

$$a+b = \frac{I}{4} \qquad (2)$$

Evaluate in frame with $\underline{p}-\underline{k}=0$, $\underline{q}=-\underline{q}'$.

These lead to:

$$\Gamma = \frac{G_F^2}{(2\pi)^4 3 m_\mu}\int \frac{d^3k}{k^0}\left[2p\cdot(p-k)\; k\cdot(p-k) + (p\cdot k)(p-k)^2\right]$$

Recall that we ~~consider~~ quote Γ in rest frame of decaying particle

$p\cdot k = m_\mu E$ where $E = k^0$

$p\cdot p = m_\mu^2$, $k\cdot k = m_e^2$

$\frac{m_e}{m_\mu} \approx 0.0048 \ll 1$ approx $m_e \approx 0$ is reasonable

$$\Gamma = \frac{G_F^2}{(2\pi)^4 3 m_\mu}\int \frac{d^3k}{E}\left[2m_\mu^2\, m_\mu E - 2(m_\mu E)^2 - 2(m_\mu E)^2 + m_\mu E\, m_\mu^2\right]$$

$$= \frac{4\pi G_F^2\, m_\mu}{(2\pi)^4\, 3}\int_0^{m_\mu/2} dE\, E^2(3m_\mu - 4E)$$

$E_{min} = 0$ (e is at rest)

E_{max} is when $\nu_\mu, \bar\nu_e$ in same direction and opposite to e^-

$$E + (E_{\bar\nu_e} + E_{\nu_\mu}) = m_\mu, \quad E - (E_{\bar\nu_e} + E_\mu) = 0$$

$$E_{max} = \frac{m_\mu}{2}$$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- $\mu \to e\bar\nu_e\nu_\mu$ is only possible decay of μ

From exp measurement $\tau_\mu = 2.1970\times10^{-6}$ s

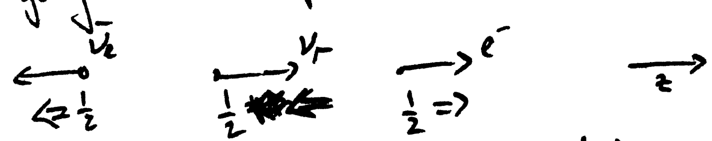
$\Rightarrow G_F = 1.164\times10^{-5}\ \text{GeV}^2$

- one loop corrections are suppressed by $\sim 10^{-6}$

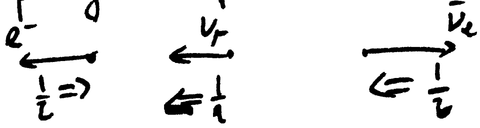
- Experimentally, G_F consistent from $\tau \to e\bar\nu_e\nu_\tau$ and $\tau\to\mu\bar\nu_\mu\nu_\tau$

$\Rightarrow$ lepton universality

- Note, going back to setup where $e, \bar\nu_e, \nu_\mu$ are along +ve/−ve z axis ($m_e \neq 0$)



Under parity transform, momenta reverse but spins don't



But (at least in limit of massless neutrinos) this isn't allowed (weak int doesn't couple to RH ν_μ).

Weak decays violate P.

6.4 π decay $\pi^-(\bar{u}d) \to e^- \bar{\nu}_e$ (assume $m_{\nu_e} = 0$)