4.4 Higgs Mechanism

Gauge theories do not satisfy all of the axioms supposed in Goldstone's theorem; depending on the choice of gauge, one of the axioms must be violated. Take QED as an example, quantise in Lorenz gauge → negative norm states; quantise in radiation gauge → no L.I. but no negative norm states.

Scalar electrodynamics

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi) - V(\phi^* \phi)$$

Gauge invariance

$$\phi(x) \longmapsto e^{i\alpha(x)} \phi(x) \qquad D_\mu = \partial_\mu + i g A_\mu$$

$$A_\mu(x) \longmapsto A_\mu(x) - \frac{1}{g} \partial_\mu \alpha(x)$$

Take the scalar potential

$$V(\phi^*\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4, \text{ with } \lambda > 0$$

Consider $\mu^2 < 0$, minima $|\phi_0|^2 = -\frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$

w.l.o.g. expand around real ϕ_0

$$\phi(x) = e^{i\theta(x)/v} (v + \eta)/\sqrt{2} \qquad \text{where } \theta, \eta \in \mathbb{R},\ v > 0$$

Substitute into $\mathcal{L}$, for small fluctuation $\phi(x) \simeq \frac{1}{\sqrt{2}}(v + \eta + i\theta)$

$$V(\phi^*\phi) = \lambda \left(|\phi|^2 - \frac{v^2}{2} \right)^2 = \frac{\lambda}{4} (v^2 + \eta^2 + \theta^2 + 2v\eta - v^2)^2 \quad \text{up to const}$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \eta \partial^\mu \eta + 2\mu^2 \eta^2) + \frac{1}{2} \partial_\mu \theta \partial^\mu \theta - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + g v A_\mu \partial^\mu \theta$$

$$+ \frac{g^2 v^2}{2} A_\mu A^\mu + \mathcal{L}_{int} \left[\begin{matrix} \text{terms with} \\ >2 \text{ fields} \end{matrix} \right]$$

Appear to have mass for η and A_μ but not θ. Strange $A_\mu \partial^\mu \theta$ term. To see what's going on, transform to unitary gauge.

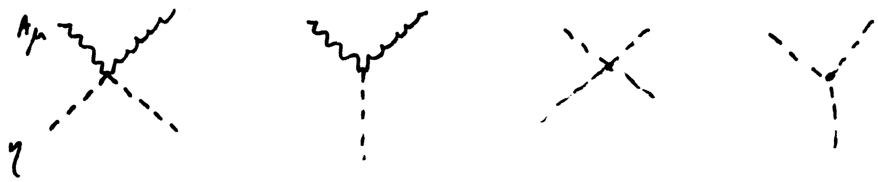
$$A_\mu \longrightarrow A_\mu + \frac{1}{gv} \partial_\mu \theta(x) \qquad \text{where } \alpha(x) = -\frac{1}{v}\theta(x)$$

$$\phi \longrightarrow e^{-i\theta/v} \phi = \frac{1}{\sqrt{2}} (v + \eta)$$

$$\mathcal{L} = \underbrace{\frac{1}{2} (\partial^\mu \eta \partial_\mu \eta + 2\mu^2 \eta^2)}_{\substack{\text{massive } \eta \text{ field} \\ \text{"higgs boson"}}} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \underbrace{\frac{g^2 v^2}{2} A_\mu A^\mu}_{\substack{\text{photon mass} \\ m_A^2 = g^2 v^2}} + \mathcal{L}_{int}$$

Goldstone mode θ has been "eaten" to become the longitudinal polarisation of A_μ

$$\mathcal{L}_{int} = \frac{g^2}{2} A_\mu A^\mu \eta^2 + g m_A A_\mu A^\mu \eta - \frac{\lambda}{4}\eta^4 - m_\eta \sqrt{\frac{\lambda}{2}}\,\eta^3$$



4.5 Non-abelian theories

Review

$$\psi_i(x) \to U_{ij}(x)\,\psi_j(x) = \exp\left(i t^a \theta^a(x)\right)_{ij} \psi_j(x)$$

U_{ij}: unitary matrices form n-dim rep of unitary group

t^a: Hermitian generators of that rep forming a Lie algebra

$$\bar\psi_i(x) \to \bar\psi_j(x)\exp\left(-i t^a \theta^a(x)\right)_{ji}$$

Lie algebra: $[t^a, t^b] = i f^{abc} t^c$

f^{abc}: structure const

$$\mathrm{Tr}(t^a t^b) = T(R)\,\delta^{ab}$$

(Dynkin index for R ($=\frac{1}{2}$ for fundamental rep))

In the SM, fermions live in fundamental or trivial reps.

Covariant derivatives $(D_\mu)_{ij} = \partial_\mu \delta_{ij} + ig(t^a A_\mu^a)_{ij}$ (A_μ^a: gauge field)

$$[D_\mu, D_\nu] = ig\, t^a F^a_{\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

The gauge part of $\mathcal{L}$ is $\mathcal{L}_g = -\frac{1}{4} F^a_{\mu\nu}F^{a\mu\nu} = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$

Next chapter, will discuss EW theory $SU(2)_L \times U(1)_Y \to U(1)_{EM}$

5. Electroweak theory

We will make choices to construct a theory that is capable of describing experimental data

5.1. EW gauge theory

- Gauge symmetry is $SU(2)_L \times U(1)_Y$
- Complex scalar (Higgs) field: doublet (fundamental) rep of $SU(2)$ and hypercharge $Y = \frac{1}{2}$ under a gauge transformation

$$\phi(x) \to e^{i\alpha^a(x)\tau^a} e^{i\beta(x)\frac{1}{2}}\phi(x) \quad \text{where } \tau^a = \frac{\sigma^a}{2} \quad (a = 1, 2, 3)$$