String Theory

Signature $(+,-,-,-)$

Curvature $(\nabla_a \nabla_b - \nabla_b \nabla_a)\nabla_c = R_{abc}{}^{d}\nabla_d$

Ricci Tensor $R_{ad} = R_{abcd}\,\eta^{bc}$

Ricci Scalar $R = \eta^{ab} R_{ab}$

Gamma matrices

$\{\gamma^a, \gamma^b\} = 2\eta^{ab}$

γ^0 antihermitian

γ^i hermitian

Dirac conjugate $\bar{\Psi}_D = \Psi^\dagger \gamma^0$

Charge conjugation $\gamma^{a\,T} = -C\gamma^a C^{+}$

$C^{+} = -C$

Majorana ~~spinor~~ conjugate

$$\bar{\Psi}_M = \Psi^T C$$

Majorana spinors are such that the Dirac conjugate is equal to the Majorana ~~s~~ conjugate

[Morally equivalent to saying spinors are real rather than complex]

Indices

$a, b, c \ldots$	spacetime
$i, j, k, \ldots$	spatial
greek	world-sheet indices

Natural units: defined in terms of G $(\hbar, c = 1)$

Mass-scale - Planck mass

$$m_p = \left(\frac{m\hbar}{8\pi}\right)^{1/2} = 2.177 \times 10^{-5}\,g = 10^{19}\,GeV \Rightarrow G = \frac{1}{m_p^2}$$

Length scale - Planck length

$$l = \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.6 \times 10^{-33}\ cm$$

planck time - ~~t~~

$$t = \left(\frac{\hbar G}{c^5}\right)^{1/2} = 5.391 \times 10^{-34}\,s$$

This means that Mass $\sim$ (Length)$^{-1}$

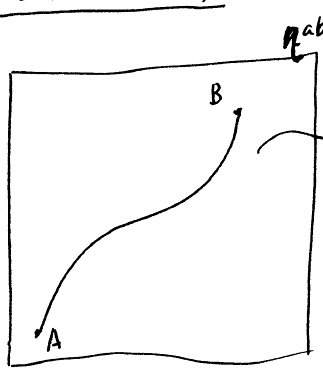
Perturbative string theory - fairly well understood

perturbative description of gravity

True nature is a mystery

Not a theory of spacetime

Particle Theory



Worldline

$X^a(\tau)$

Some parameter along the worldline

Point particle action

$$I = -m\int_A^B ds$$

s proper distance along the worldline

dimensionless

Not reparameterisation invariant

A reparameterisation invariant action

$$I = -m\int d\tau \sqrt{-\eta_{ab}\frac{dx^a}{d\tau}\frac{dx^b}{d\tau}}$$

$\tau \to \bar{\tau}(\tau)$

could choose τ to be proper time

$\tau \to \bar{\tau}(\tau)$ is an example of gauge invariance

$\frac{\delta I}{\delta x^a} = 0 \Rightarrow$ equations with square roots

Proper distance

$e\, d\tau \quad\Rightarrow\quad ds^2 = -e^2 d\tau^2$

Einbein

looks like a 1-D metric tensor

Introduce an auxilary field e into the action

$$I = \frac{1}{2}\int d\tau \left(\frac{1}{e}\eta_{ab}\frac{dx^a}{d\tau}\frac{dx^a}{d\tau} - em^2\right) \quad \otimes$$

e not dynamical, E.O.M $\eta_{ab}\frac{dx^a}{d\tau}\frac{dx^b}{d\tau} = -e^2m^2$

Substitute back into action ⊗ to give back $-m\int d\tau \sqrt{\quad}$,

Gauge choice $e = 1/m$ will give back τ = proper time

Introducing e is termed - introduction of an auxilary field

no momentum conjugate to it

"Hubbard -Stratomovich transform"

Gauge invariance of ⊗

$$e \to e + \delta e$$

$$\left.\begin{aligned} \delta \tau &= \frac{d}{d\tau}(\xi \tau) \\ \delta x^a &= \xi \frac{dx^a}{d\tau} \end{aligned}\right\} \quad \delta I = 0$$

(integrate by parts)

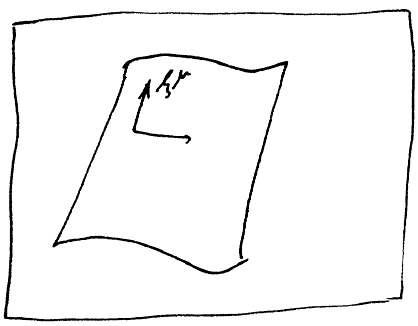
Momentum conjugate to x

$$p_a = \frac{\delta I}{\delta \dot{x}^a} = \frac{1}{e}\frac{dx^b}{d\tau}\eta_{ab}$$

The choice $e = \frac{1}{m}$ gives $p^a = m\dot{x}^a$

Replace η_{ab} by g_{ab} result in the e.o.m being the geodesic equation

String Theory



String is a 2-D object.

Metric induced by string embedded in spacetime

$$X^a(\xi^\mu)$$

coordinates in the world sheet

Metric induced in the surface

$$\gamma_{\mu\nu} = \frac{\partial X^a}{\partial \xi^\mu}\frac{\partial X^b}{\partial \xi^\nu}\eta_{ab}$$

must have Lorentz signature

[like saying a particle trajectory must be time like]

Action for the String

$$I = -T\int dA$$

surface area element

T- string tension dimension $\text{Mass}/\text{Length} = 1/(\text{Length})^2$

Introduce a scale into the theory.

This is the only scale in the theory

$T = \frac{1}{2\pi\alpha'}$ α' – Inverse string tension

- Regge slope parameter

Nambo - Goto action really invented by Dirac