3.2 Parity $\quad \hat{P}: x^\mu \to x_P^\mu = (x^0, -x^i)$

Scalar fields (cont.)

$\hat{P}: |p\rangle \to \eta_a^* |p_P\rangle$ where η_a^* is a complex phase

so $\hat{P}\, a^\dagger(p)|0\rangle = \eta_a^* a^\dagger(p_P)|0\rangle$ and since $\hat{P}P = I$ and $\hat{P}|0\rangle = |0\rangle$,

$\hat{P}\, a^\dagger(p)\, \hat{P}^{-1} = \eta_a^* a^\dagger(p_P)$

and to conserve normalisations, $\hat{P} a(p) \hat{P}^{-1} = \eta_a a(p_P)$.

Similarly, $\hat{P} c^\dagger(p) \hat{P}^{-1} = \eta_c^* c^\dagger(p_P)$ etc.

Then,
$$\hat{P}\phi(x)\hat{P}^{-1} = \sum_p \left[\hat{P} a(p) \hat{P}^{-1} e^{-ip\cdot x} + \hat{P} c^\dagger(p) \hat{P}^{-1} e^{+ip\cdot x} \right]$$
$$= \sum_p \left[\eta_a a(p_P) e^{-ip\cdot x} + \eta_c^* c^\dagger(p_P) e^{+ip\cdot x} \right]$$
$$\begin{bmatrix}\text{relabel}\\ p_P \leftrightarrow p\end{bmatrix} = \sum_{p_P} \left[\eta_a a(p) e^{-ip_P\cdot x} + \eta_c^* c^\dagger(p) e^{+ip_P\cdot x} \right]$$
$$\begin{bmatrix}\text{use}\\ p_P\cdot x = p\cdot x_P\end{bmatrix} = \sum_{p_P} \left[\eta_a a(p) e^{-ip\cdot x_P} + \eta_c^* c^\dagger(p) e^{+ip\cdot x_P} \right]$$
$$\begin{bmatrix}\text{note}\\ \sum_{p_P} = \sum_p\end{bmatrix} = \sum_p \left[\dots \right]$$

This doesn't 'look like' $\phi(x_P)$ unless $\eta_a = \eta_c^* \equiv \eta_P$.
Also otherwise $[\phi(x), \hat{P}\phi^\dagger(y)\hat{P}^{-1}]$ would not vanish for spacelike $x-y_P$.

Therefore, $\hat{P}\phi(x)\hat{P}^{-1} = \eta_P \phi(x_P)$ where η_P is the intrinsic parity of $\phi(x)$.

For a real scalar field, $a = c$ and so $\eta_a = \eta_c$ and so $\eta_a = \eta_P = \eta_P^*$ i.e. $\eta_P = \pm 1$. (scalar, pseudoscalar fields).

For a complex scalar, η_P may be complex but if there is an associated conserved charge Q, η_P can be related to Q [Weinberg, sec. 2.2 and 2.3].

Vector field

polarisation vectors $(\lambda = -1, 0, 1)$
$$V^\mu(x) = \sum_{p,\lambda} \left[\varepsilon^\mu(\lambda,p) a^\lambda(p) e^{-ip\cdot x} + \varepsilon^{\mu *}(\lambda,p) c^{\dagger\lambda}(p) e^{+ip\cdot x} \right]$$

note that λ doesn't change under $\hat{P}$

Using similar steps to above
$$\hat{P} V^\mu(x) \hat{P}^{-1} = \sum_p \left[\varepsilon^\mu(\lambda, p_P) a^\lambda(p) e^{-ip\cdot x_P} \eta_a + \varepsilon^{\mu *}(\lambda, p_P) c^{\dagger\lambda}(p) e^{+ip\cdot x_P} \eta_c^* \right]$$

Use $\varepsilon^\mu(\lambda, p_P) = -\mathbb{P}^\mu{}_\nu \varepsilon^\nu(\lambda, p)$ (show this using explicit form for ε^μ and Lorentz transform).

$$\hat{P} V^\mu(x) \hat{P}^{-1} = -\mathbb{P}^\mu{}_\nu \eta_P V^\nu(x_P)$$ where for same reasons as above $\eta_P = \eta_a = \eta_c^*$

- vectors have $\eta_P = -1$
- axial vectors have $\eta_P = 1$

Dirac field

Creation/annihilation operators behave like for scalar fields, and the spin component s is unchanged.

$$\hat P\, b^s(p)\, \hat P^{-1} = \eta_b\, b^s(p_P)\,, \quad \hat P\, d^{s\dagger}(p)\, \hat P^{-1} = \eta_d^*\, d^{s\dagger}(p_P)$$

Then,

$$\hat P\, \psi(x)\, \hat P^{-1} = \sum_{p,s} \left[\eta_b\, b^s(p_P)\, u^s(p)\, e^{-ip\cdot x} + \eta_d^*\, d^{s\dagger}(p_P)\, v^s(p)\, e^{+ip\cdot x}\right]$$

$$[\text{as above}] = \sum_{p,s} \left[\eta_b\, b^s(p)\, u^s(p_P)\, e^{-ip\cdot x_P} + \eta_d^*\, d^{s\dagger}(p)\, v^s(p_P)\, e^{+ip\cdot x_P}\right]$$

Use $u^s(p_P) = \gamma^0 u^s(p)$ and $v^s(p_P) = -\gamma^0 v^s(p)$, verify using Lorentz boosts in a particular rep.

$$= \gamma^0 \sum_{p,s} \left[\eta_b\, b^s(p)\, u^s(p)\, e^{-ip\cdot x_P} - \eta_d^*\, d^{s\dagger}(p)\, v^s(p)\, e^{+ip\cdot x_P}\right]$$

Again, require $\eta_b = -\eta_d^* = \eta_P$ so that the anti-com. $\{\bar\psi(x), \hat P \psi(y) \hat P^{-1}\}$ vanishes for spacel. $x - y_P$.

Therefore,

$$\psi_P(x) \equiv \hat P\, \psi(x)\, \hat P^{-1} = \eta_P\, \gamma^0\, \psi(x_P)$$

$$\bar\psi_P(x) \equiv \hat P\, \bar\psi(x)\, \hat P^{-1} = \eta_P^*\, \bar\psi(x_P)\, \gamma^0$$

Note, • $\hat P\, \psi_R\, \hat P^{-1} = \gamma^0\, \psi_R\, \eta_P$

• if $\psi(x)$ satisfies Dirac eq, so does $\psi_P(x)$.

We can now determine how various fermion bilinears transform.

$$\bar\psi(x)\,\psi(x) \longrightarrow \bar\psi(x_P)\,\psi(x_P) \qquad \text{scalar}.$$