Standard Model 13

$$\mathcal{L}_{lept}^{int} = \frac{g}{2\sqrt{2}}\left(J^\mu W_\mu^+ + J^{\mu +} W_\mu^-\right) + e J^\mu_{EM} A_\mu + \frac{g}{2\cos\theta_W} J^\mu_n Z_\mu$$

where $J^\mu_{EM} = -\bar{e}\gamma^\mu e$ (EM current)

$J^\mu = \bar{\nu}_{e_L}\gamma^\mu(1-\gamma^5) e$ (charged weak current)

$J^\mu_n = \frac{1}{2}\left[\bar{\nu}_{e_L}\gamma^\mu(1-\gamma^5)\nu_{e_L} - \bar{e}\gamma^\mu(1-\gamma^5 - 4\sin^2\theta_W)e\right]$ neutral weak current

SM has 3 generations. e, μ, τ

$$L^1 = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix} \quad L^2 = \begin{pmatrix} \nu_{\mu_L} \\ \mu_L \end{pmatrix} \quad L^3 = \begin{pmatrix} \nu_{\tau_L} \\ \tau_L \end{pmatrix} \qquad R^1 = e_R \quad R^2 = \mu_R \quad R^3 = \tau_R$$

SU(2) doublet

and

$$\mathcal{L}_{lept,\phi} = -\sqrt{2}\left(\lambda^{ij}\bar{L}^i\phi R^j + \lambda^{+ij}\bar{R}^i\phi^+ L^j\right)$$

N.B. λ^{ij} are not predicted by the SM. Diagonalise by writing $\lambda\lambda^+ = U\Lambda^2U^+$ and $\lambda^+\lambda = S\Lambda^2S^+$ where U and S are unitary matrices, and Λ^2 is diagonal and +ve.
non-negative eigenvalues

$\lambda\lambda^+ = U\Lambda S^+ S\Lambda U^+$ and $\lambda = U\Lambda S^+$, $\lambda^+ = S\Lambda U^+$

$\lambda^+\lambda = S\Lambda U^+ U\Lambda S^+$ $\qquad \Lambda = U^+\lambda S$

Then change basis

$L^i \to U^{ij}L^j \qquad R^i \to S^{ij}R^j$

This diagonalizes $\mathcal{L}_{lept,\phi}$ while leaving $\mathcal{L}_{lept}^{EW}$ invariant because $\bar{L}$ only couples to L

Simultaneous diagonalisation $\Rightarrow$ mass eigenstates are also eigenstates of the EW of the interaction.

5.3 Quark flavour

6 flavoured quarks. (as far as we know)

- RH fields are SU(2) singlets

$U_R^i = (u_R, c_R, t_R)$ $Y = Q = \frac{2}{3}$

$d_R^i = (d_R, s_R, b_R)$ $Y = Q = -\frac{1}{3}$

- RH fields are SU(2) doublets

$$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L\right) \quad Y = \frac{1}{6}$$

$i = 1, 2, 3$ label generations

$$\mathcal{L}^{EW}_{quark} = \bar{Q}_L i \not{D} Q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R$$

$$\mathcal{L}_{quark,\phi} = -\sqrt{2}\left[\lambda_d^{ij} \bar{Q}_L^i \phi d_R^j + \lambda_u^{ij} \bar{Q}_L^i \phi^c u_R^j + h.c. \right]$$

$(\phi^c)^\alpha = \varepsilon^{\alpha\beta} \phi^{+\beta}$ transforms in fundamental rep of $SU(2)$

Needed to ensure $\mathcal{L}_{quark,\phi}$ is gauge invariant

[Note $\sum Y = 0$ in each term]

Diagonalising λ_u and λ_d as for leptons.

$\lambda_u = U_u \Lambda_u S_u^\dagger$, $\lambda_d = U_d \Lambda_d S_d^\dagger$ (U, S unitary, Λ diagonal)

Transform fields,

$u_L \to U_u u_L$, $d_L \to U_d d_L$

$u_R \to S_u u_R$, $d_R \to S_d d_R$

Recall that $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$ then $\lambda_d^{ij} \bar{Q}_L^i \phi d_R^j \to \bar{Q}_L \phi \Lambda_d d_R$ etc

and the $\phi = \phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$ gives

$$-\sum_i m_d^i (\bar{d}_L^i d_R^i) + m_u^i \bar{u}_L^i u_R^i + h.c. \qquad m_q^i = v \Lambda_q^{ii}$$

[In this basis $\mathcal{L}_{quark,\phi}$ is inv. under P, C, T.
Note $\mathcal{L}_{gauge,\phi}$ is also inv. under P, C, T.]

However, this basis transformation has an effect on $\mathcal{L}^{EW}_{quark}$

$\bar{u}_R i \not{D} u_R$ and $\bar{d}_R i \not{D} d_R$ are unchanged but the $W_\mu^\pm$ piece in $\bar{Q}_L i \not{D} Q_L$ is transformed.

$$\left(\frac{g}{2\sqrt{2}} J^{\pm\mu} W_\mu^\pm \right)$$

$$J^{\mu+} = \bar{u}_L^i \gamma^\mu d_L^i \to \bar{u}_L^i \gamma^\mu \underbrace{(U_u^\dagger U_d)^{ij}}_{\text{not diagonal}} d_L^j$$

Interactions with $W^\pm$ lead to intergenerational quark couplings. Weak eigenstates are linear combinations of the mass eigenstates. We can't simultaneously diagonalise.

The Cabibbo-Kobyashi-Maskawa (CKM) matrix is $V_{CKM} = U_u^\dagger U_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \cdot & \ddots & \cdot \\ \cdot & \cdot & V_{tb} \end{pmatrix}$