Standard Model

I Introduction

In standard model:

Forces are mediated by gauge bosons (spin 1)

EM – photon γ (QED)

Weak interaction – $W^\pm$, Z bosons

Strong interaction – gluons g (QCD)

Matter (spin $\frac{1}{2}$ fermions)

Neutrinos ν_e, ν_μ, ν_τ – only interact via weak int (neutral)

Charged leptons e, μ, τ – EM and weak int

Quarks $\binom{u}{d}, \binom{c}{s}, \binom{t}{b}$, charge $\binom{+\frac{2}{3}}{-\frac{1}{3}}$ – all 3 interactions

Higgs boson (spin 0)

gives mass to $W^\pm$, Z and fermions

Gauge bosons are manifestations of local symmetries

Gauge group in SM:

$$\underbrace{SU(3)_c}_{\substack{\uparrow \\ SU(3)\text{ colour} \\ QCD}} \times \underbrace{\underbrace{SU(2)_L}_{\substack{\uparrow \\ \text{chiral } SU(2) \\ \text{left}}} \times \underbrace{U(1)_Y}_{\substack{\uparrow \\ \text{hypercharge}}}}_{\text{Electroweak}}$$

spontaneous symmetry breaking $\to U(1)_{EM}$ and gives weak and EM int

References

Conventions: dictionary ArXiV 1209.6213. Will use $\eta = \eta_e = \eta' = \eta_Z = \eta_\theta = \eta_Y = \eta_e = +1$

2 Chiral and Gauge symmetries

We review some concepts from last term and set up notation. Throughout we'll use natural units $\hbar = c = 1$.

2.1 Chiral sym

Consider a spin $\frac{1}{2}$ Dirac fermion ψ which satisfies the Dirac equation

$$(i\not{\partial} - m)\psi = 0 \quad , \quad \not{\partial} = \gamma^\mu \partial_\mu$$

Dirac matrices γ^μ satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ where $g^{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

Define $\gamma^5 = +i\gamma^0\gamma^1\gamma^2\gamma^3$, $(\gamma^5)^2 = I$, $\{\gamma^5, \gamma^\mu\} = 0$

We'll generally use the Chiral (or Weyl) basis

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Consider the massless limit of the Dirac equation

$$\not{\partial}\psi = 0 \quad \Rightarrow \quad \not{\partial}\gamma^5\psi = 0$$

Define $P_{R,L} = \frac{1}{2}(1 \pm \gamma^5)$ projection operators $(P_{R,L})^2 = P_{R,L}$

$$P_R + P_L = I \qquad P_L P_R = P_R P_L = 0$$

Define $\psi_{R,L} = P_{R,L}\psi$ then $\gamma^5 \psi_{R,L} = \pm \psi_{R,L}$ definite chirality (right and left-handed)

In chiral rep

$$P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} , \quad P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$\psi_{R,L}$ only contains lower (upper) components. $\psi_{R,L}$ annihilate RH (LH) chiral particles

Exercise: consider $\bar{\psi} = \psi^+ \gamma^0$

A massless Dirac fermion has a $U(1)_L \times U(1)_R$ global symmetry

$$U(1)_{L,R} : \psi_{L,R}(x) \longrightarrow e^{i\alpha_{L,R}} \psi_{L,R}(x)$$

$$\bar{\psi}_{L,R}(x) \longrightarrow e^{-i\alpha_{L,R}} \bar{\psi}_{L,R}(x)$$

as can be seen from the Dirac Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

But the mass term explicitly breaks this symmetry to a vector sym $U(1)_V$ where $\alpha_L = \alpha_R \equiv \alpha$