Standard Model 19

Define $\eta_{+-} = \dfrac{|\langle \pi^+\pi^- | H | K_L^0 \rangle|}{|\langle \pi^+\pi^- | H | K_S^0 \rangle|}$

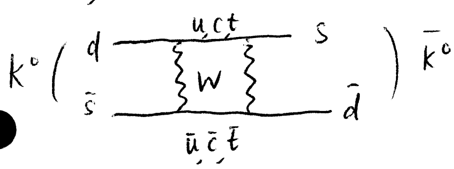
Exp $\eta_{+-} \approx \eta_{00} \approx 2.2 \times 10^{-3} \neq 0 \Rightarrow$ CP violation

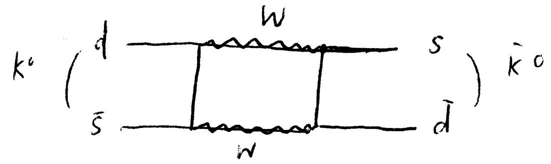
Two possible ways for $\cancel{CP}$

- Direct $\cancel{CP}$ of $s \to u$ due to phase in V_{CKM}
- Indirect $\cancel{CP}$ due to $K^0 \to \bar{K}^0$ or vice-versa, then decaying (ultimately due to V_{CKM})

Turns out indirect $\cancel{CP}$ is mainly responsible and dominant contribs are:

"box diagram" $\Delta S = 2$





Next to leading order in perturbation theory

$$\therefore \; |K_S^0\rangle = \frac{1}{\sqrt{1+|\varepsilon_1|^2}}\left(|K_+^0\rangle + \varepsilon_1 |K_-^0\rangle\right) \approx |K_0^+\rangle$$

$$|K_L^0\rangle = \frac{1}{\sqrt{1+|\varepsilon_2|^2}}\left(|K_-^0\rangle + \varepsilon_2 |K_+^0\rangle\right) \approx |K_0^-\rangle \qquad \varepsilon_1, \varepsilon_2 \in \mathbb{C}$$

Assume two state mixing and ignore details of strong interaction.

$$|K_S(t)\rangle = a_S(t)\,|K^0\rangle + b_S(t)\,|\bar{K}^0\rangle$$

$$|K_L(t)\rangle = a_L(t)\,|K^0\rangle + b_L(t)\,|\bar{K}^0\rangle$$

$$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle \Rightarrow$$

next to leading order weak Hamiltonian

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = R\begin{pmatrix} a \\ b \end{pmatrix} \quad \text{where} \quad R = \begin{pmatrix} \langle K^0|H|K^0\rangle & \langle K^0|H'|\bar{K}^0\rangle \\ \langle \bar{K}^0|H|K^0\rangle & \langle \bar{K}^0|H|\bar{K}^0\rangle \end{pmatrix}$$

Because Kaons decay, R is not Hermitian. We can write it as $R = M - \frac{i}{2}\Gamma$ where M is Hermitian (mass matrix) and Γ is the decay matrix (also Hermitian)

If $\hat{\Theta} = \hat{C}\hat{P}\hat{T}$, $\Theta H' \Theta^{-1} = H'^{\dagger}$ (see earlier notes), $\Theta|\bar{K}^0\rangle = -|K^0\rangle$ (consider rest frame)

and $\Theta|K^0\rangle = -|\bar{K}^0\rangle$, since $\hat{T}|K^0\rangle = |K^0\rangle$

↑ due to $\hat{C}\hat{P}$

$\therefore R_{11} = (K^0, H' K^0) = (\Theta^{-1}\Theta K^0, H'\Theta^{-1}\Theta K^0) = (\bar{K}^0, H'^{\dagger}\bar{K}^0)^{*}$ ← because $\hat{T}$ antilinear

$= (\bar{K}^0, H'\bar{K}^0) = R_{22}$

If $\hat{T}$ was a good symmetry (i.e. $\hat{C}\hat{P}$ is a good symmetry)

$$R_{12} = (K^0, H'\bar{K}^0) = (\bar{K}^0, H' K^0) = R_{21}$$

We can show

$$\varepsilon_1 = \varepsilon_2 = \varepsilon = \frac{\sqrt{R_{12}} - \sqrt{R_{21}}}{\sqrt{R_{12}} + \sqrt{R_{21}}}$$

If $\hat{C}\hat{P}$ is conserved, then $R_{12} = R_{21}$, $\varepsilon_1 = \varepsilon_2 = 0$.

the direct source of $\cancel{CP}$

One can also show that $\eta_{+-} = \varepsilon + \varepsilon'$ and $\eta_{00} = \varepsilon - 2\varepsilon'$

Exp. $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$, $|\varepsilon'|/|\varepsilon| = (1.66 \pm 0.23) \times 10^{-3}$

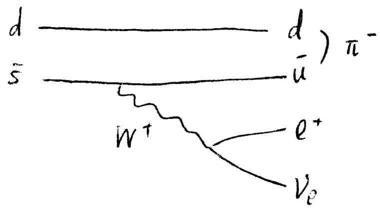
Other decays can be used to probe $K^0_{L,S}$ e.g.

Semileptonic decays:

$K^0 \to \pi^- e^+ \nu_e$ $\qquad \bar{K}^0 \not\to \pi^- e^+ \nu_e$

$K^0 \not\to \pi^+ e^- \bar{\nu}_e$ $\qquad \bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$

(cannot draw a diagram)



If CP conserved, then we'd expect $\Gamma(K^0_{L,S} \to \pi^- e^+ \nu_e) = \Gamma(K^0_{L,S} \to \pi^+ e^- \bar{\nu}_e)$

Define. $$A_L = \frac{\Gamma(K^0_L \to \pi^- e^+ \nu_e) - \Gamma(K^0_L \to \pi^+ e^- \bar{\nu}_e)}{\Gamma(K^0_L \to \pi^- e^+ \nu_e) + \Gamma(K^0_L \to \pi^+ e^- \bar{\nu}_e)}$$

Exp $A_L = (3.32 \pm 0.06) \times 10^{-3}$ $(\approx 2\,\mathrm{Re}\,\varepsilon)$

7. QCD

Find proton and neutrons have similar masses, and so do the pions (π^+, π^-, π^0). This lead to the ideal of $SU(2)_I$

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