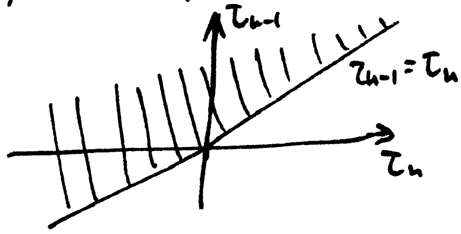
S-matrix $\quad S = T \exp\left(-i\int_{-\infty}^{\infty} dt\; V(t)\right)$

$$S_T \equiv \hat{T} S \hat{T}^{-1} = \sum_n (+i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_{n-1}} dt_n\, V(t_1)V(t_2)\cdots V(t_n)$$

$$\left[\begin{matrix}\tau_i = -t_{n+1-i} \\ \int_{-\infty}^{\infty} d\tau_n \int_{\tau_n}^{\infty} d\tau_{n-1} \\ = \int_{-\infty}^{\infty} d\tau_{n-1}\int_{-\infty}^{\tau_{n-1}} d\tau_n\end{matrix}\right] = \sum_n (+i)^n \int_{-\infty}^{\infty} d\tau_n \int_{\tau_n}^{\infty} d\tau_{n-1} \cdots \int_{\tau_2}^{\infty} d\tau_1\, V(\tau_n)V(\tau_{n-1})\cdots V(\tau_1)$$



$$S_T = \sum_n (+i)^n \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\tau_1} d\tau_2 \cdots \int_{-\infty}^{\tau_{n-1}} d\tau_n\, V(\tau_n)V(\tau_{n-1})\cdots V(\tau_1)$$

$$S_T = S^\dagger$$

Consider $|\eta\rangle$ and $|\xi\rangle$ with $|\eta_T\rangle = \hat{T}|\eta\rangle$, $|\xi_T\rangle = \hat{T}|\xi\rangle$. Then,

$$\begin{aligned}\langle \eta_T | S | \xi_T\rangle &= (\hat{T}\eta, S\hat{T}\xi) = (\hat{T}\eta, S_T^\dagger \hat{T}\xi) = (\hat{T}\eta, \hat{T}S^\dagger \xi) \\ &= (\eta, S^\dagger \xi)^* \qquad [\text{anti-unitary } \hat{T}] \\ &= (\xi, S\eta) = \langle \xi | S | \eta\rangle .\end{aligned}$$

If $\hat{T}\mathcal{L}_I(x)\hat{T}^{-1} = \mathcal{L}_I(x_T)$, S-matrix elements are equal for time-reversed processes where the initial and final states are swapped.

3.5 CPT theorem

Theorem: Any Lorentz inv $\mathcal{L}$ with a Hermitian Hamiltonian should be invariant under the product of C, P and T.

- All observations suggest that CPT is respected in nature.

3.6 Baryogenesis – generation of matter-antimatter asym in Universe

Sakharov – 3 conditions that are necessary:

1. Baryon # violation: $X \to Y + B$ ← excess baryons
   (or leptogenesis: lepton # asymmetry → baryon excess)
2. Non-equilibrium: otherwise $\Gamma(Y+B \to X) = \Gamma(X \to Y + B)$
3. C and CP violation: otherwise, $\frac{dB}{dt} \propto \Gamma(X \to Y + B) - \Gamma(\bar{X} \to \bar{Y} + \bar{B})$ if C sym,
   $\Gamma(X \to nq_L) + \Gamma(X \to nq_R) = \Gamma(\bar{X} \to n\bar{q}_L) + \Gamma(\bar{X} \to n\bar{q}_R)$ if CP sym.

4 Spontaneous symmetry breaking

• Hidden symmetries which are present in $\mathcal{L}$ but not observables

4.1 Spontaneous sym breaking of a discrete sym

Consider a real scalar field $\phi(x)$ with a sym $V(\phi)$ [discrete sym $\phi \to -\phi$]

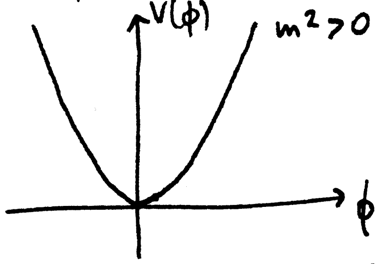
$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$

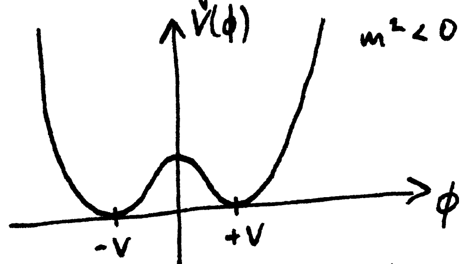
E.g. ϕ^4 - theory: $V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4$, $\lambda > 0$

• Usual case is $m^2 > 0$ (massive scalar field) and $V(\phi)$ has a minimum at $\phi = 0$
Can consider perturbations about $\phi = 0$ (for small λ).

• If $m^2 < 0$, we can write $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 + \text{const}$, where $v = \sqrt{\frac{-m^2}{\lambda}}$.

Now $\phi = 0$ is a maximum and there are two degenerate minima at $\phi = \pm v$ (vacua).





We see that ϕ has acquired a non-zero vacuum expectation value (VEV). W.l.o.g. consider small excitations around $\phi = v$, $\phi(x) = v + f(x)$. Then we can write,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \lambda\left(v^2 f^2 + v f^3 + \frac{1}{4} f^4\right) + \text{const}$$

Therefore, f is a scalar field with mass $m_f^2 = 2v^2\lambda$. This $\mathcal{L}$ is not inv under $f \to -f$.
The symmetry of the original $\mathcal{L}$ has been broken by the VEV of ϕ.