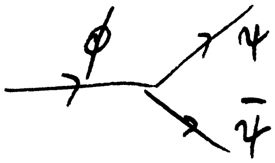
Meson decay



$$|i\rangle = \sqrt{2E_p}\, a_p^\dagger |0\rangle$$

$$|f\rangle = \sqrt{4E_{q_1}E_{q_2}}\; b_{q_1}^\dagger c_{q_2}^\dagger |0\rangle$$

$$\langle f|S|i\rangle = -ig\langle f|\int d^4x\, \psi^\dagger(x)\psi(x)\phi(x)|i\rangle$$

$$= -ig\langle f|\int d^4x\, \psi^\dagger(x)\psi(x)\int\frac{d^3k}{(2\pi)^3}\sqrt{\frac{2E_p}{2E_k}}\, a_k a_p^\dagger \, e^{-ik\cdot x}|0\rangle$$

$$= -ig\langle f|\int d^4x\, \psi^\dagger(x)\psi(x)\, e^{-ip\cdot x}|0\rangle$$

$$= -ig\langle 0|\int d^4x\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{\sqrt{4E_{q_1}E_{q_2}}}{\sqrt{4E_{k_1}E_{k_2}}}\, c_{q_2}b_{q_1}c_{k_1}^\dagger b_{k_2}^\dagger\, e^{i(k_1+k_2-p)\cdot x}|0\rangle$$

$$= -ig\langle 0|\int d^4x\, e^{i(q_1+q_2-p)\cdot x}|0\rangle$$

$$= -ig\,\delta^4(q_1+q_2-p)\,(2\pi)^4$$

Wick's Theorem

We want to compute $\langle f|T\{H_I(x_1),\ldots,H(x_n)\}|i\rangle$ where $|i\rangle$ and $|f\rangle$ are e'states of the free theory. The H_I contain creation/annihilation ops so life will be made easier if we arrange the annihilation ones on RHS. So we need to go from t-ordered products to normal ordered products. (Normal ordering).

Consider a $\mathbb{R}$ scalar field $\phi = \phi^+(x) + \phi^-(x)$ with

$$\phi^+ = \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\, a_p e^{-ip\cdot x} \qquad \phi^- = \int\frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\, a_p^\dagger e^{ip\cdot x}$$

Choose $x^0 > y^0$, then

$$T\phi(x)\phi(y) = \phi(x)\phi(y) = \left(\phi^+(x)+\phi^-(x)\right)\left(\phi^+(y)+\phi^-(y)\right)$$

$$= \phi^+(x)\phi^+(y) + \phi^-(x)\phi^+(y) + [\phi^+(x),\phi^-(y)] + \phi^-(y)\phi^+(x) + \phi^-(x)\phi^-(y)$$

$$\Rightarrow T\phi(x)\phi(y) = :\phi(x)\phi(y): + D(x-y) \qquad \int\frac{d^3p}{(2\pi)^3}e^{-ip(x-y)}$$

Meanwhile, for $y^0 > x^0$,

$$T\phi(x)\phi(y) = :\phi(x)\phi(y): + D(y-x)$$

$$\rightarrow T\phi(x)\phi(y) = :\phi(x)\phi(y): + \Delta_F(x-y)$$

where Δ_F, the Feynman propagator, is $\Delta_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i e^{ik\cdot(x-y)}}{k^2-m^2+i\varepsilon}$

NB: $T\phi\phi$ and $:\phi\phi:$ are both ops. The difference between them, Δ_F is a c-valued f^n.

<u>Definition</u> Lets define a contraction of a pair of fields in a string of ops $\ldots\phi(x_1)\ldots\phi(x_2)\ldots$ to mean we replace these ops with the appropriate c-f^n

$\overbrace{\phi(x_1)\,\phi}(x_2) = \Delta_F(x_1-x_2)$ leaving other ops untouched

For C scalar field $T\psi(x)\psi^\dagger(y) = :\psi(x)\psi^\dagger(y): + \Delta_F(x-y)$ so we define $\overbrace{\psi(x)\,\psi^\dagger}(y) = \Delta_F(x-y)$, whereas $\overbrace{\psi(x)\,\psi}(y) = 0 = \overbrace{\psi^\dagger(x)\,\psi^\dagger}(y)$.

<u>Wick's theorem</u> For any collection of fields $\phi_1 = \phi_1(x_1)$, $\phi_2 = \phi_2(x_2)$, ...

$$T(\phi_1\phi_2\ldots\phi_N) = :\phi_1\phi_2\ldots\phi_N: + :\text{all possible contractions}:$$

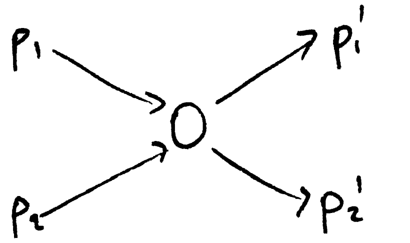
e.g. $T(\phi_1\phi_2\phi_3\phi_4) = :\phi_1\phi_2\phi_3\phi_4: + \overbrace{\phi_1\phi_2}:\phi_3\phi_4: + \overbrace{\phi_1\phi_3}:\phi_2\phi_4: + 4 \text{ similar}$
$+ \overbrace{\phi_1\phi_2}\overbrace{\phi_3\phi_4} + \overbrace{\phi_1\phi_3}\overbrace{\phi_2\phi_4} + \overbrace{\phi_1\phi_4}\overbrace{\phi_2\phi_3}$

Proof: Already shown for $N=2$.
Suppose it's true for $\phi_2 \ldots \phi_n$ and now add ϕ_1 with
$x_1^0 > x_k^0 \quad \forall k \in \{2, \ldots, n\}$

$T\phi_1\phi_2\ldots\phi_n = (\phi_1^+ + \phi_1^-)(:\phi_2\ldots\phi_n: + :\text{other contractions}:)$

The ϕ_1^- term stays where it is - it's already normal ordered.
The ϕ_1^+ term has to make its way past the ϕ_n^- ops, so we can write the RHS as a normal ordered product. Each time it moves past ϕ_n^-, we pick up a factor of $\overbrace{\phi_1\phi_n}$.

<u>Nuclear Scattering</u> $\psi + \psi \to \psi + \psi$



$|i\rangle = \sqrt{4E_{p_1}E_{p_2}}\; b^\dagger_{p_1} b^\dagger_{p_2}|0\rangle$

$|f\rangle = \sqrt{4E_{p_1'}E_{p_2'}}\; b^\dagger_{p_1'} b^\dagger_{p_2'}|0\rangle$

Look at order g^2 in $\langle f|(S-1)|i\rangle$ $\quad$ -1 as not interested in the case with no scattering

We have the expansion of $S-1$

$$\frac{(-ig)^2}{2}\int d^4x_1\, d^4x_2\; T\left[\psi^\dagger(x_1)\psi(x_1)\phi(x_1)\psi^\dagger(x_2)\psi(x_2)\phi(x_2)\right]$$

Using Wick's theorem, ∃ a term in this string which is

$$:\psi^\dagger(x_1)\psi(x_1)\psi^\dagger(x_2)\psi(x_2):\ \phi(x_1)\phi(x_2)$$

contributes to scattering because the 2 ψ fields annihilate the initial ψ's and ψ† create the final ψs.

$$\langle p_1', p_2'|:\psi^\dagger(x_1)\psi(x_1)\psi^\dagger(x_2)\psi(x_2):|p_1, p_2\rangle$$

$$=\langle p_1', p_2'|:\psi^\dagger(x_1)\psi^\dagger(x_2):|0\rangle\langle 0|:\psi(x_1)\psi(x_2):|p_1, p_2\rangle$$

$$=\left(e^{ip_1'\cdot x_1+ip_2'\cdot x_2}+e^{ip_1'\cdot x_2}+e^{ip_1'\cdot x_2+ip_2'\cdot x_1}\right)$$

$$\times\left(e^{-ip_1 x_1-ip_2 x_2}+e^{-ip_1x_2}\ \cancel{\pm}\ {}^{-ip_2x_1}\right)$$? WTF

$$=e^{ix_1(p_1'-p_1)+ix_2(p_2'-p_2)}+e^{ix_1(p_2'-p_1)+ix_2(p_1'-p_2)}+x_1\leftrightarrow x_2$$

Insert into ∫ to get, at order g^2

$$\frac{(-ig)^2}{\cancel{2}}\int d^4x_1\, d^4x_2\left[e^{\cdots}+e^{\cdots}+\cancel{(x_1\leftrightarrow x_2)}\right]2\int\frac{e^{ik(x_1-x_2)}}{k^2-m^2+i\varepsilon}\frac{d^4k}{(2\pi)^4}$$

The x_1x_2 ∫s give us δ-f^n.

$$=(-ig)^2\int\frac{d^4k}{(2\pi)^4}\frac{i(2\pi)^8}{k^2-m^2+i\varepsilon}\left\{\delta^4(p_1'-p_1+k)\,\delta^4(p_2'-p_2-k)\right.$$

$$\left.+\delta^4(p_2'-p_1+k)\,\delta^4(p_1'-p_2-k)\right\}$$

$$=(-ig)^2\left[\frac{1}{(p_1-p_1')^2-m^2}+\frac{1}{(p_2-p_2')^2-m^2}\right](2\pi)^4\delta^4(p_1+p_2-p_1'-p_2')$$