• Assume $\sum_{x \in \text{hadrons}} |x\rangle\langle x| = \sum_{x \in g, q, \bar{q} \text{ states}} |x\rangle\langle x|$

"quark-hadron duality" (Assume that $e^+e^- \to \gamma^* \to q\bar{q}$ can be separated from hadronisation.)

$$\rho_h^{\mu\nu}(q^2) = N_c \sum_f Q_f^2 \int \frac{d^3k_1}{(2\pi)^3 2k_1^0} \int \frac{d^3k_2}{(2\pi)^3 2k_2^0} (2\pi)^3 \delta^{(4)}(q - k_1 - k_2) \times \mathrm{Tr}\left[(\not{k}_1 + m_f)\gamma^\mu(\not{k}_2 - m_f)\gamma^\nu\right]\Big|_{k_1^2 = k_2^2 = m_f^2}$$

$N_c$ ← no of colours $= 3$

To do this integral, use similar steps to $\mu$ decay calc. One thing to consider

$$I^{\mu\nu} = \int \frac{d^3k_1}{k_1^0} \int \frac{d^3k_2}{k_2^0} \delta^{(4)}(q - k_1 - k_2)\, k_1^\mu k_2^\nu \Big|_{k_1^2 = k_2^2 = m_f^2}$$

Argue we can write, $I^{\mu\nu} = A(q^2) q^\mu q^\nu + B(q^2) g^{\mu\nu}$

Contract with $g_{\mu\nu}$ and $q_\mu q_\nu$ to obtain eqs for A, B

Use $q^2 = (k_1 + k_2)^2 = 2m_f^2 + 2k_1 \cdot k_2$

$$\Rightarrow \rho_h(q^2) = \frac{N_c}{12\pi^2} \sum_f Q_f^2 \, \Theta(q^2 - 4m_f^2) \left(1 - \frac{4m_f^2}{q^2}\right)^{1/2} \frac{q^2 + 2m_f^2}{q^2}$$

If all $m_f \to 0$; $\rho_h(q^2) = \frac{N_c}{12\pi^2} \sum_f Q_f^2$

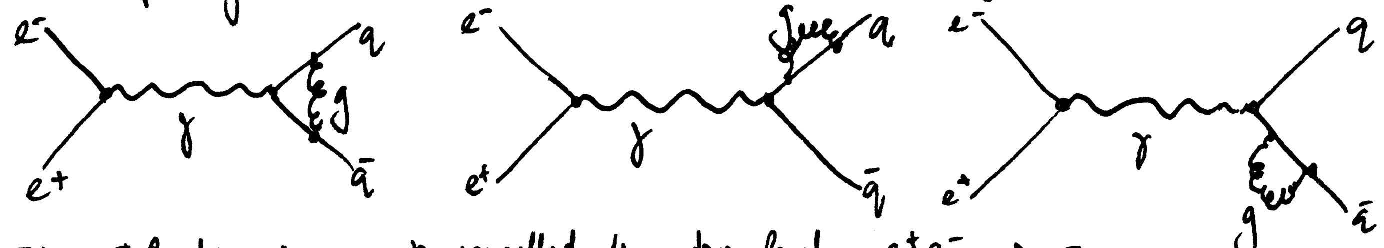
$$\sigma_{LO}(e^+e^- \to \text{hadrons}) = N_c \frac{4\pi\alpha^2}{3q^2} \sum_f Q_f^2$$

LO ← leading order

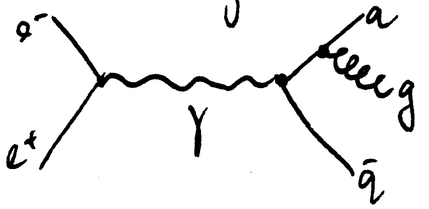
Consider the ratio, $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

$$R_{LO} = N_c \sum_f Q_f^2 = \begin{cases} \frac{2}{3}N_c & \text{when } u,d,s \text{ "active"} \\ \frac{10}{9}N_c & \quad " \quad u,d,s,c \quad " \\ \frac{11}{9}N_c & \quad " \quad u,d,s,c,b \quad " \end{cases}$$

One loop diagrams are UV finite but have IR divergent




The IR divergence is cancelled by tree level $e^+e^- \to q\bar{q}g$




etc.

## 7.4 Deep inelastic scattering

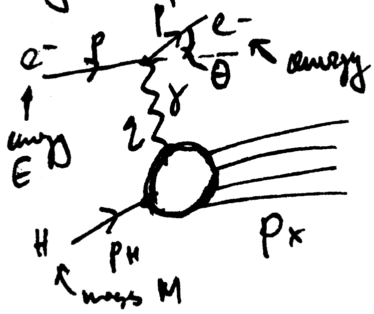
Scattering electrons off protons:

- low energies – proton appears point-like (Rutherford and Mott scattering)
- higher energies (smaller wavelengths) – the charge dist must be taken into account (form factors)

→ elastic scattering

- higher energies – elastic cross section falls off and dominant process is inelastic scattering off constituents of proton

Early exps led to the idea of "partons" inside hadrons that are weakly interacting – can now understand in QCD (asymptotic freedom)




$p\cdot p' = |p||p'|\cos\theta$ $\quad q = p - p'$

$Q^2 \equiv -q^2 = 2p\cdot p' = 2EE'(1-\cos\theta) \geq 0$

$X$ ← Hadron breaks up into final state $X$ about which we have no interest.

Treat $e^-$ as massless

Define $\nu \equiv p_H\cdot q$ $\quad p_X^2 = (p_H+q)^2 \overset{\text{show this}}{\geq} M^2 \Rightarrow Q^2 \leq 2\nu$

It's useful to define dimless params

$x \equiv \dfrac{Q^2}{2\nu}$ "Bjorken x" $0 \leq x \leq 1$, $\quad y \equiv \dfrac{\nu}{p_H\cdot p}$ "inelasticity" $0 \leq y \leq 1$

Calc the cross section in rest frame of hadron H $\quad y = \dfrac{\nu}{ME} = \dfrac{E-E'}{E}$, $\nu = M(E-E')$

$$\mathcal{M} = (ie)^2\, \bar{u}_e(p')\gamma^\mu u_e(p) \left(\frac{-ig_{\mu\nu}}{q^2}\right) \langle X|\, J_h^\nu \,|H(p_H)\rangle$$

$$d\sigma = \frac{1}{4ME\underbrace{|v_e - v_H|}_{=1}} \frac{d^3p'}{(2\pi)^3\, 2p'^0} \sum_{X,p_H} (2\pi)^4 \delta^{(4)}(q+p_H-p_X)\, \frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2$$