4.4 Higgs mechanism (cont.)

$$V(\phi^*\phi) = \mu|\phi|^2 + \lambda|\phi|^4 \qquad , \lambda > 0$$

- If $\mu^2 < 0$, minima $|\phi_0|^2 = -\frac{\mu^2}{2\lambda} \equiv \frac{V^2}{2}$

w.l.o.g. expand around real ϕ_0,

$$\phi(x) = e^{i\theta(x)/V}(V+\eta)/\sqrt{2} \qquad \text{where } \theta, \eta \in \mathbb{R} \; , \; V > 0$$

Substitute into $\mathcal{L}$, for small fluctuations $\phi(x) \approx \frac{1}{\sqrt{2}}(V+\eta+i\theta)$,

$$V(\phi^*\phi) = \lambda\left(|\phi|^2 - \frac{V^2}{2}\right)^2 = \frac{\lambda}{4}\left(V^2 + \eta^2 + \theta^2 + 2V\eta - V^2\right)^2 \quad \text{(up to const)}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\eta\,\partial^\mu\eta + 2\mu^2\eta^2) + \frac{1}{2}\partial_\mu\theta\,\partial^\mu\theta - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + qVA_\mu\partial^\mu\theta$$
$$+ \frac{q^2V^2}{2}A_\mu A^\mu + \mathcal{L}_{int} \leftarrow \begin{bmatrix}\text{terms with} \\ >2 \text{ fields}\end{bmatrix}$$

Appear to have mass for η and A_μ but not θ. Strange $A_\mu\partial^\mu\theta$ term. To see what's going on, transform to unitary gauge,

$$A_\mu \longrightarrow A_\mu + \frac{1}{qV}\partial_\mu\theta(x) \qquad \text{where} \quad \alpha(x) = -\frac{1}{V}\theta(x)$$

$$\phi \longrightarrow e^{-i\theta/V}\phi = \frac{1}{\sqrt{2}}(V+\eta)$$

$$\mathcal{L} = \frac{1}{2}(\partial^\mu\eta\,\partial_\mu\eta + 2\mu^2\eta^2) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2V^2}{2}A_\mu A^\mu + \mathcal{L}_{int}$$

- massive η field $m_\eta^2 = -2\mu^2 = 2\lambda V^2$ "Higgs boson"
- photon with mass $m_A^2 = q^2V^2$
- Goldstone mode θ has been "eaten" to become the longitudinal polarisation of A_μ.

$$\mathcal{L}_{int} = \frac{q^2}{2}A_\mu A^\mu\eta^2 + q\,m_A A_\mu A^\mu\eta - \frac{\lambda}{4}\eta^4 - m_\eta\sqrt{\frac{\lambda}{2}}\,\eta^3$$

4.5 Nonabelian theories

<u>Review</u>

$\psi_i(x) \to \cancel{\ } U_{ij}(x)\,\psi_j(x) = \exp(i t^a \theta^a(x))_{ij}\,\psi_j(x)$

real

$i,j = 1, \dots, n$

unitary matrices for n-dim rep of unitary group

Hermitian generators of that rep forming a Lie algebra

$\bar\psi_i(x) \to \bar\psi_j(x)\,\exp(-i t^a \theta^a(x))_{ji}$

Lie algebra: $[t^a, t^b] = i f^{abc} t^c$ structure constants

$\mathrm{Tr}(t^a t^b) = \mathrm{Tr}(R)\,\delta^{ab}$ (normalisation)

Dynkin index for R ($=\frac{1}{2}$ for fund. rep)

In the SM, fermions live in fundamental or trivial ($n=1$) reps.

Covariant derivative $(D_\mu)_{ij} = \partial_\mu \delta_{ij} + ig(t^a A^a_\mu)_{ij}$ (dropping i,j ind)

$[D_\mu, D_\nu] = ig\, t^a F^a_{\mu\nu}$, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$

The gauge part of $\mathcal{L}$ is $\mathcal{L}_g = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} = -\frac{1}{2}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$

Next chapter, will discuss EW theory $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

On example sheet 2, you'll consider other patterns of SSB, e.g. $SU(2) \to U(1)$ and cases where the scalar transforms in different reps.

5 Electroweak theory

We will make choices to construct a theory that is capable of describing experimental data.

5.1 EW gauge theory (Gauge + Higgs part)

- Gauge symmetry is $SU(2)_L \times U(1)_Y$
- Complex scalar (Higgs) field: doublet (fund) rep of $SU(2)$ and hypercharge, $Y=\frac{1}{2}$.

Under a gauge transformation

$\phi(x) \to e^{i\alpha^a(x)\tau^a} e^{i\beta(x)\frac{1}{2}}\phi(x)$ where $\tau^a = \frac{\sigma^a}{2}$ $(a=1,2,3)$

- Scalar acquires a VEV, w.l.o.g. $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$.