Representation R_Λ of g

Highest weight $\Lambda \in \mathcal{L}_W[g]$

$\Lambda = \sum_{i=1}^{r} \Lambda^i w_{(i)}$ ← fundamental weights

↖ Dynkin labels R_Λ

for a highest weight $\Lambda^i \geq 0$ ← Λ is a dominant integral weight $\in \bar{\mathcal{L}}_W[g]$

for g f.d., simple, complex

- R_Λ is irreducible

remaining weights are of form $\lambda = \Lambda - \mu$ $\quad \mu = \sum_{i=1}^{r} \mu^i \alpha_{(i)} \quad \mu^i \geq 0$

Irreps of $A_2 \simeq \mathcal{L}_C(SU(3))$

Each dominant weight $\Lambda = \Lambda^1 w_{(1)} + \Lambda^2 w_{(2)} \quad \Lambda^1, \Lambda^2 \in \mathbb{Z} \quad \Lambda^1, \Lambda^2 \geq 0$

$\in \bar{\mathcal{L}}_W[A_2]$

get an irrep, $R_{(\Lambda^1,\Lambda^2)}$ of A_2

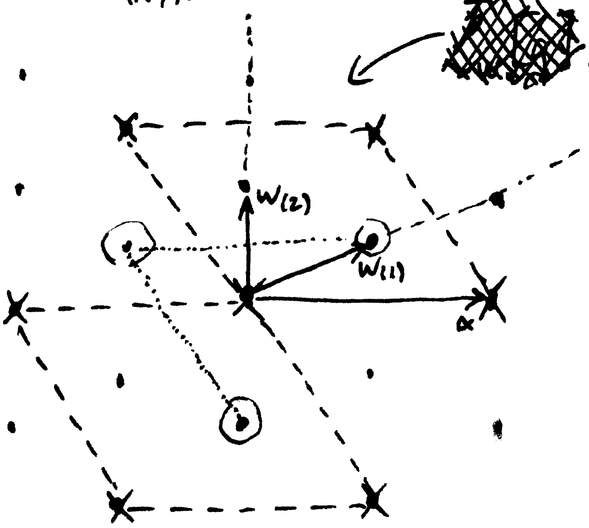
- Can show $\dim R_{(\Lambda^1,\Lambda^2)} = \frac{1}{2}(\Lambda^1+1)(\Lambda^2+1)(\Lambda^1+\Lambda^2+2)$

- If $\Lambda_1 \neq \Lambda_2$ get a pair of repns

$R_{(\Lambda^1,\Lambda^2)} = \bar{R}_{(\Lambda^2,\Lambda^1)}$

weights of conjugate repns are related by

$\lambda \in S_{(\Lambda^1,\Lambda^2)} \Leftrightarrow -\lambda \in S_{(\Lambda^2,\Lambda^1)}$



$R_{(0,0)}$	$\underline{1}$	trivial
$R'_{(1,0)}$	$\underline{3}$	fundamental
$R_{(0,1)}$	$\underline{\bar{3}}$	anti-fundamental
$R_{(2,0)}$	$\underline{6}$	
$R_{(0,2)}$	$\underline{6}$	
$R_{(1,1)}$	$\underline{8}$	adjoint

$\Lambda = (1,1)$

$\alpha_{(1)} = \alpha = (2,-1)$

$\alpha_{(2)} = \beta = (-1,2)$

$\Lambda \in S_R^{(1,1)}$ [X on figure]

$\Lambda - \alpha_{(1)} = (-1,2) \in S_R \quad \Lambda - \alpha_{(2)} = (2,-1) \in S_R$

$\Rightarrow \Lambda - \alpha_{(1)} - \alpha_{(2)}$, $\Lambda - \alpha_{(1)} - 2\alpha_{(2)} \in S_R$, $\quad \Lambda - \alpha_{(1)} - \alpha_{(2)}$, $\Lambda - 2\alpha_{(1)} - \alpha_{(2)} \in S_R \quad \Rightarrow \Lambda - 2\alpha_{(1)} - 2\alpha_{(2)} = (-1,-1) \in S_R$

Tensor Products

Let R_Λ and $R_{\Lambda'}$ be irreps of $\underset{\sim}{g}$ with repn spaces $V_\Lambda, V_{\Lambda'}$

$$V_\Lambda = \bigoplus_{\lambda \in S_\Lambda} V_\lambda \quad , \quad V_{\Lambda'} = \bigoplus_{\lambda' \in S_{\Lambda'}} V_{\lambda'}$$

- If $\lambda \in S_\Lambda$ and $\lambda' \in S_{\Lambda'}$

$\lambda + \lambda' \in \mathcal{L}_W[\underset{\sim}{g}]$ is a weight of tensor product repn $R_\Lambda \otimes R_{\Lambda'}$

proof:

$$v_\lambda \in V_\lambda \implies R_\Lambda(H^i)\, v_\lambda = \lambda^i v_\lambda$$

$$v_{\lambda'} \in V_{\lambda'} \implies R_{\Lambda'}(H^i)\, v_{\lambda'} = \lambda'^i v_{\lambda'}$$

$$\implies (R_\Lambda \otimes R_{\Lambda'})(H^i)(v_\lambda \otimes v_{\lambda'}) = \left(R_\Lambda(H^i)\, v_\lambda\right) \otimes v_{\lambda'} + v_\lambda \otimes \left(R_{\Lambda'}(H^i)\, v_{\lambda'}\right)$$

$$= (\lambda + \lambda')^i (v_\lambda \otimes v_{\lambda'}) \quad \square$$

Hence, weight set of $R_\Lambda \otimes R_{\Lambda'}$ is

$$S_{\Lambda \otimes \Lambda'} = \{ \lambda + \lambda' : \lambda \in S_\Lambda , \lambda' \in S_{\Lambda'} \}$$

- Finite, simple, complex $\underset{\sim}{g}$

$$R_\Lambda \otimes R_{\Lambda'} = \bigoplus_{\Lambda'' \in \bar{\mathcal{L}}_W} \mathcal{N}^{\Lambda''}_{\Lambda, \Lambda'} R_{\Lambda''}$$

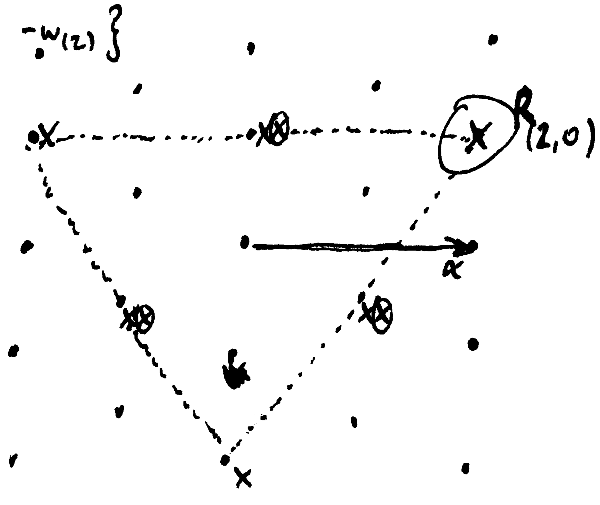
multiplicities $\mathcal{N} \in \mathbb{Z}_{\geq 0}$

<u>Example</u> $\underset{\sim}{g} = A_2$

$S_{(1,0)} = \{ w_{(1)}, -w_{(1)} + w_{(2)}, -w_{(2)} \}$

$R_{(1,0)} \otimes R_{(1,0)} = R_{(2,0)} \oplus R_{(0,1)}$

$\underline{3} \otimes \underline{3} = \underline{6} \oplus \underline{\bar{3}}$



	SU(3) colour	SU(3) flavour
q	$\underline{3}$	$\underline{3}$
$\bar{q}$	$\underline{\bar{3}}$	$\underline{\bar{3}}$

only singlets appear

$\underline{3} \otimes \underline{\bar{3}} = \underline{1} \oplus \underline{8}$ → mesons $q\bar{q}$

$q\bar{q}$

$\underline{3} \otimes \underline{\bar{3}} = \underline{1} \oplus \underline{8}$

$\underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \dots$

qqq