Gauge theories with vector-like couplings to fermions, e.g. QCD and QED, are symmetric (invariant) under P, C, T. However, theories that only involve LH particles are not, e.g. weak int. In fact, the combination CP is violated by the weak int. From the CPT theorem, T is violated. This ~~CP~~ has important consequences and it's one of the Sakharov conditions required for a matter-antimatter asym.

To understand these statements, we first consider theories that are sym under C, P and T.

3.1 Symmetry operators

General Poincaré transformation can be written

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$$

Proper Lorentz transform has $\det\Lambda = +1$.

Parity $\Lambda^\mu{}_\nu = \mathbb{P}^\mu{}_\nu = \mathrm{diag}(1,-1,-1,-1)$, Time reversal, $\mathbb{T}^\mu{}_\nu = \mathrm{diag}(-1,1,1,1)$ are "improper" Lorentz transfm.

Wigner: if physics invariant under $\Psi \to \Psi'$, where Ψ, Ψ' are some vectors in a Hilbert space, then there's an operator W s.t. $\Psi' = W\Psi$ where W is <u>either</u>

- unitary and linear

$$(W\Phi, W\Psi) = (\Phi, \Psi) \quad \text{and} \quad W(\alpha\Phi + \beta\Psi) = \alpha W\Phi + \beta W\Psi, \quad \alpha,\beta \in \mathbb{C}, \ \Phi,\Psi \text{ vectors}$$

<u>OR</u>

- anti-unitary and anti-linear

$$(W\Phi, W\Psi) = (\Phi, \Psi)^* \quad \text{and} \quad W(\alpha\Phi + \beta\Psi) = \alpha^* W\Phi + \beta^* W\Psi$$

Consider an infinitesimal Poincaré transfm

$$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \omega^\mu{}_\nu, \quad a^\mu = \varepsilon^\mu \quad \text{where } \omega \text{ and } \varepsilon \text{ are small params.}$$

The corresponding operator W can be expanded as

$$W(\Lambda, a) = W(I+\omega, \varepsilon) = 1 + \frac{i}{2}\omega_{\mu\nu} J^{\mu\nu} - i\varepsilon_\mu P^\mu$$

$J^{\mu\nu}$: generate rot + boosts; $P^0 = H$, the Hamiltonian; P^i = linear mom ops

From the general composition rule:

$$\hat{P} W \hat{P}^{-1} = W(\mathbb{P}\Lambda\mathbb{P}^{-1}, \mathbb{P}a) \qquad \hat{P} = W(\mathbb{P}, 0)$$

$$\hat{T} W \hat{T}^{-1} = W(\mathbb{T}\Lambda\mathbb{T}^{-1}, \mathbb{T}a) \qquad \hat{T} = W(\mathbb{T}, 0)$$

Inserting expansion for W in terms of ω and ε on both sides and compare coeffs of $-\varepsilon_0$,

$$+\hat{P}\, iH\, \hat{P}^{-1} = +iH$$

$$+\hat{T}\, iH\, \hat{T}^{-1} = -iH$$

Now consider an energy eigenstate Ψ with energy E,

$$(\Psi, iH\Psi) = iE$$

Assuming that $\hat{P}$ and $\hat{T}$ are symmetries of the theory, $\hat{P}\Psi$ and $\hat{T}\Psi$ should be eigenstates with energy E.

- Treat $\hat{P}$ as linear:

$$(\hat{P}\Psi, iH\hat{P}\Psi) = (\hat{P}\Psi, \hat{P}\, iH\Psi) = (\hat{P}\Psi, \hat{P}\, iE\Psi) \overset{\downarrow}{=} iE(\hat{P}\Psi, \hat{P}\Psi) = iE \ \checkmark$$

[also check unitary property]

- Treat $\hat{T}$ as linear:

$$(\hat{T}\Psi, iH\hat{T}\Psi) = (\hat{T}\Psi, -\hat{T}iH\Psi) = -(\hat{T}\Psi, \hat{T}iE\Psi)$$

$$\underset{\text{linear}}{=} -iE(\hat{T}\Psi, \hat{T}\Psi) = -iE \quad \times$$

- Treat $\hat{T}$ as antilinear:

$$\underset{\text{antilinear}}{=} +iE(\hat{T}\Psi, \hat{T}\Psi) = +iE \quad \checkmark$$

[also check antiunitary property]

3.2 Parity

$$x^\mu \to x_P^\mu = (x^0, -\underline{x})$$

$$p^\mu \to p_P^\mu = (p^0, -\underline{p})$$

Scalar fields

Consider a complex scalar field.

$$\phi(x) = \sum_p \Big[\underset{\substack{\uparrow \\ \text{annih op} \\ \text{for particle}}}{a(p)} e^{-ip\cdot x} + \underset{\substack{\nwarrow \\ \text{creation op} \\ \text{for antiparticle}}}{c^\dagger(p)} e^{+ip\cdot x} \Big]$$

$\hat{P}$ maps $\underset{\text{(particle)}}{|p\rangle} \to \eta^{a*} |p_P\rangle$ where η^{a*} is a complex phase