7 QCD (cont)

• SU(2) isospin : p, n in doublet, π^+, π^0, π^- in triplet
(global) $m_p \neq m_n$ so broken sym (though reasonable approx)

• Strange hadrons observed – extend to $SU(3)_F$ sym.
Not a good sym but useful for classifying hadrons
* DO NOT CONFUSE WITH GAUGE $SU(3)_c$ *

– Lightest mesons are in $\underline{8}$ and $\underline{1}$ multiplets.
– Lightest baryons $\underline{8}$ and $\underline{10}$.

• Quark model : baryons are bound states of 3 'constituent' quarks (spin-$\frac{1}{2}$ fermions) and mesons are quark and antiquark. u, d, s form $\underline{3}$ of $SU(3)_F$; u, d have isospin $I = \frac{1}{2}$, $I_z = \pm\frac{1}{2}$, $m_u \simeq m_d < m_s$

• Problem : Δ^{++} is uuu, spin $= \frac{3}{2}$ so wavefunction appears symmetric in violation of Fermi stats => need extra quantum number "colour" and postulate all observable states are colour singlets (no net colour) => confinement

$q\bar{q}$, qqq are allowed, as are $qq\bar{q}\bar{q}$, $qqqq\bar{q}$, but qq, $q\bar{q}q$ etc. are not allowed.

• Predict Ω^- baryon (sss) => subsequently observed.

7.1 QCD Lagrangian

Modern description of strong int of quarks is QCD. Gauge theory with $SU(3)_c$ sym. Strong force is mediated by gauge bosons called gluons. Symmetry is exact and gluons are massless.

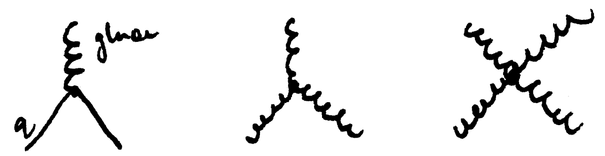
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu} + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

$$D_\mu = \partial_\mu + ig A^a_\mu T^a \quad , \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A_\mu^b A_\nu^c$$

T^a are generators of SU(3) fundamental rep.

$$[T^a, T^b] = i f^{abc} T^c$$

$T^a = \frac{1}{2}\lambda^a$ where λ^a are Gell-Mann matrices.
$a = 1, 2, \dots, 8$



7.2 Renormalisation

A theory has an $\mathcal{L}$ that contains a set of couplings g_i (include masses in this set). For each of these, we need a physical / observable / derived quantity g_i^0 and an expression (renorm condition)

$$g_i^0 = G_i^0(\{g_j(\mu)\}, \mu)$$

where $\{g_j(\mu)\} \equiv g(\mu)$ are renormalised couplings and μ is the renorm point. We'll consider pert. expressions for G_i^0.

How do renorm couplings change as we vary μ?

$$\beta(g(\mu), \mu) = \mu \frac{d}{d\mu} g_i(\mu)$$

g_i^0 do not depend on μ.

$$\mu \frac{d}{d\mu} G_i^0(g(\mu), \mu) = \left(\mu \frac{\partial}{\partial\mu} + \beta_j \frac{\partial}{\partial g_j}\right) G_i^0(g(\mu), \mu) = 0$$

Ignoring quark masses, general expr for $SU(N)$ gauge theory to one loop

$$\beta(g) = -\frac{\beta_0 g^3}{16\pi^2} + O(g^5) \leftarrow \text{higher order}$$

$$\beta_0 = \frac{11}{3} N - \frac{4}{3} \sum_f T_f$$ where T_f is Dynkin index of fermion f rep.

($\mathrm{Tr}\, t_f^a t_f^b = T_f \delta^{ab}$, where t_f^a are generators)

- Assumed that RH + LH couple equally
- For fund rep, $T_f = \frac{1}{2}$.

One-loop expr for QCD, $\beta_0 = 11 - \frac{2}{3} n_f$. n_f = no of quark flavours

There are 6 flavours of quarks but the no. of 'active' quarks depends on energy scale.

energies $\ll m_{top} \approx 173$ GeV — $n_f = 5$

energies ~ 100 MeV — $n_f = 3$.

The 'strong coupling', $\alpha_s = \frac{g^2}{4\pi}$

$$\mu \frac{d\alpha_s}{d\mu} = -\frac{\beta_0}{2\pi} \alpha_s^2 \quad \text{(neglecting higher orders)}$$

$$\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha_s}{\alpha_s^2} = -\frac{\beta_0}{2\pi} \int_{\mu_0}^{\mu} \frac{d\mu}{\mu}, \quad \alpha_s(\mu) = \frac{2\pi}{\beta_0} \frac{1}{\ln(\mu/\mu_0) + \frac{2\pi}{\beta_0 \alpha_s(\mu_0)}}$$