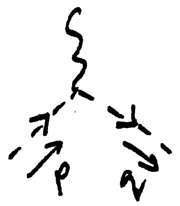
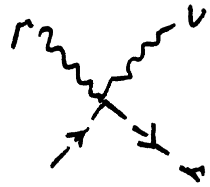
Charged scalars $\mathbb{C}$ scalar ψ

$$\mathcal{L} = \underbrace{(D_\mu \psi)^\dagger D^\mu \psi}_{\partial_\mu \psi^\dagger \partial^\mu \psi \, - \, ie A_\mu (\psi^\dagger \partial^\mu \psi - \psi \partial^\mu \psi^\dagger) \, + \, e^2 A_\mu A^\mu \psi^\dagger \psi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

2 vertices



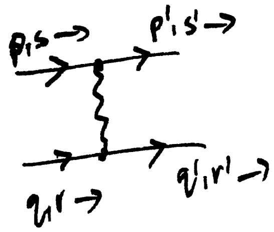
$-ie(p+q)_\mu$



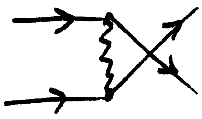
$+2ie^2 \eta_{\mu\nu}$

QED processes

1. $e^-e^- \to e^-e^-$

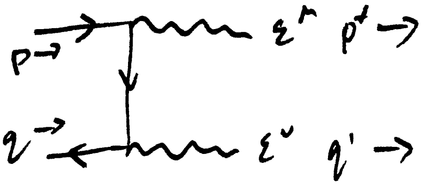


$+$

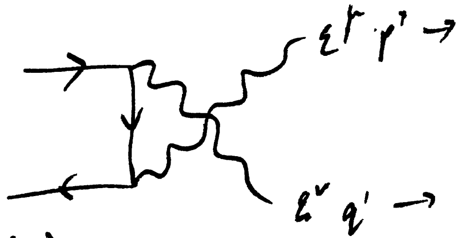


$$= -i(ie)^2 \left\{ \frac{[\bar{u}^{s'}(p')\gamma^\mu u^s(p)][\bar{u}^{r'}(q')\gamma_\mu u^r(q)]}{(p'-p)^2} - \frac{[\bar{u}^s(p')\gamma^\mu u^r(q)][\bar{u}^r(q')\gamma_\mu u^s(p)]}{(p-q)^2} \right\}$$

2. $e^+e^- \to \gamma\gamma$

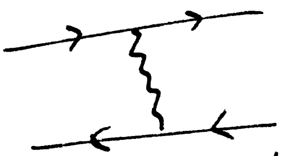


$+$



$$= i(ie)^2 \left\{ \frac{[\bar{v}^r(q)\gamma^\nu(\not{p}-\not{p}'+m)\gamma^\mu u^s(p)}{(p-p')^2-m^2} + \frac{[\bar{v}^r(q)\gamma^\mu(\not{p}-\not{q}'+m)\gamma^\nu u^s(p)}{(p-q')^2-m^2} \right\} \varepsilon^\mu(p')\, \varepsilon^\nu(q')$$

3. $e^+e^- \to e^+e^-$ "Bhabha scattering"



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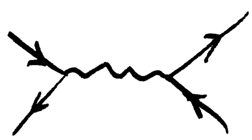
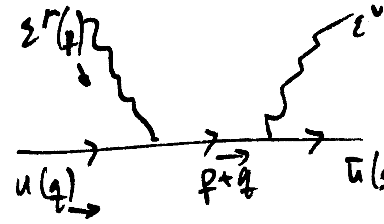


$$\mathcal{A} = -i(ie)^2 \left\{ \frac{[\bar{u}^{s'}(p')\gamma^\mu u^s(p)][\bar{v}^r(q)\gamma_\mu v^{r'}(q')]}{(p-p')^2} + \frac{[\bar{v}^r(q)\gamma^\mu u^s(p)][\bar{u}^{s'}(p')\gamma_\mu v^{r'}(q')]}{(p+q)^2} \right\}$$

4. Compton scattering
$\gamma e^- \to \gamma e^-$



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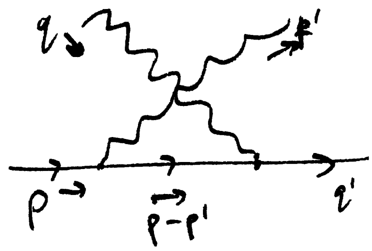
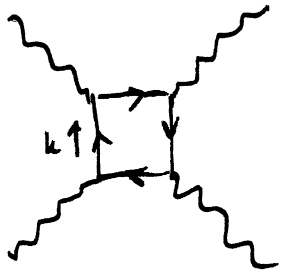


5. $\gamma\gamma \to \gamma\gamma$

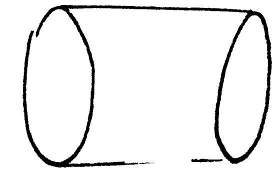
$$\frac{i(\not{k}+m)}{k^2-m^2+i\varepsilon} \leftarrow \frac{i}{\not{k}-m+i\varepsilon}$$



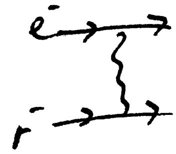
$$\sim \frac{1}{16\pi^2}\int_0^\Lambda \frac{d^4k}{k^4} \overset{\text{naively}}{\sim} \frac{1}{16\pi^2}\ln\Lambda$$

"log divergence"

but gauge symmetry renders the diagram finite



could add muons
$e^-\mu^- \to e^-\mu^-$



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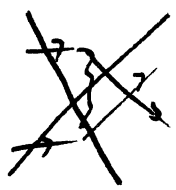
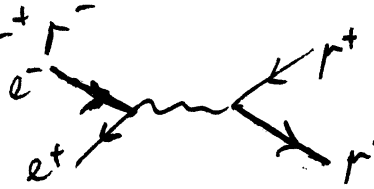


$$\sim \frac{1}{(p-p')^2}$$

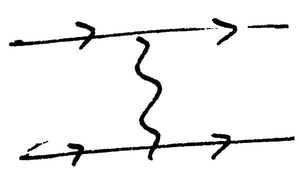
$e^+e^- \to \mu^+\mu^-$



$$\sim \frac{1}{(p+q)^2}$$

<u>The Coulomb Potential</u>

$e^-e^- \to e^-e^-$



$$(-i)(ie)^2 \frac{[\bar{u}(p')\gamma^\mu u(p)][\bar{u}(q')\gamma_\mu u(q)]}{(p-p')^2}$$

In non-relativistic limit, $u(p) \to \sqrt{m}\begin{pmatrix}\xi\\ \xi\end{pmatrix}$

$\bar{u}^s(p')\gamma^0 u^r(p) \to 2m\,\delta^{rs}$

$\bar{u}(p)\gamma^i u(q) \to 2m\,(\xi^\dagger\ \xi^\dagger)\begin{pmatrix}-\sigma^i & 0\\ 0 & \sigma^i\end{pmatrix}\begin{pmatrix}\xi\\ \xi\end{pmatrix} = 0$

<u>Average</u> over initial spins + sum over final spins

The $\mathcal{M} \sim \dfrac{+ie^2(2m)^2}{|\mathbf{p}-\mathbf{p}'|^2} \Rightarrow U(r) = e^2\displaystyle\int \frac{d^3p}{(2\pi)^3}\frac{e^{i\mathbf{p}\cdot\mathbf{r}}}{|\mathbf{p}|^2} = \frac{e^2}{4\pi r}$

$e^+e^- \to e^+e^-$

$+i(ie)^2\ [\bar{u}(p')\gamma^\mu u(p)][\bar{v}(q)\gamma_\mu v(q')] \xrightarrow{NR} \bar{v}^r(q)\gamma^0 v^s(q') = 2m\,\delta^{rs}$

like e^+e^- but with minus sign $\Rightarrow$ <u>attractive potential</u>