Review of Dirac field (aside)

$$\psi = \sum_{s,p} \left[b^s_p(p)\, u^s(p)\, e^{-ip\cdot x} + d^{s\dagger}(p)\, v^{s}\, e^{ip\cdot x} \right]$$

where $s = \pm\frac{1}{2}$ and $\sum_p \equiv \int \frac{d^3p}{(2\pi)^3 (2E_p)}$

$b^\dagger$, $d^\dagger$ create +ve, −ve freq particles. Use relativistic normalisation $|p\rangle = b^\dagger(p)|0\rangle$,

$\langle p|q\rangle = (2\pi)^3\, 2E_p\, \delta^{(3)}(\underline{p}-\underline{q})$

In chiral rep $u^s = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \xi^s \\ \sqrt{p\cdot\bar\sigma}\ \xi^s \end{pmatrix}$, $v^s = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \eta^s \\ -\sqrt{p\cdot\bar\sigma}\ \eta^s \end{pmatrix}$ where $\sigma^\mu = (I, \sigma^i)$, $\bar\sigma^\mu = (I, -\sigma^i)$

Helicity is the projection of any momentum onto direction of lin mom

$$h = \underline{J}\cdot\hat{\underline{p}} = \underline{S}\cdot\hat{\underline{p}} \qquad \text{where } \underline{J} = -i\underline{r}\times\nabla + \underline{S}$$

spin $S_i = \frac{i}{4}\varepsilon_{ijk}\gamma^j\gamma^k = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix}$ [chiral rep]

massless spinor satisfying $\not{p}u = 0$ and can show $h u^s(p) = \frac{\gamma^5}{2} u^s(p)$

$\ulcorner$ $\frac{\gamma^0}{p^0}(\gamma^0 p^0 - \underline{\gamma}\cdot\underline{p})\, u^s(p) = 0$

$\Rightarrow\ (1 - \gamma^0\underline{\gamma}\cdot\hat{\underline{p}})\, u^s(p) = 0$ use $\gamma^5\gamma^0\gamma^i = \frac{i}{2}\varepsilon^{ijk}\gamma^j\gamma^k = 2S^i$

$(\gamma^5 - \gamma^5\gamma^0\gamma^i\hat{p}_i)\, u^s(p) = 0$

$(\gamma^5 - 2\underline{S}\cdot\hat{\underline{p}})\, u^s(p) = 0$ $\square$ $\lrcorner$

$h u_{L,R} = \frac{\gamma^5}{2} u_{L,R} = \mp\frac{1}{2} u_{L,R}$ where $u_{L,R} = P_{L,R}\, u$ with $P_{L,R} = \frac{1}{2}(1 \mp \gamma^5)$

$u_{L,R}$ has helicity $-\frac{1}{2}, +\frac{1}{2}$.

Note:
- Chiral states are only eigenstates of Dirac equation when $m=0$
- Helicity can be defined for $m=0$ and $m\neq 0$ but it's not a L.I. when $m\neq 0$

  Also only a one-to-one correspondence between helicity and chirality when $m=0$

2.2 Gauge sym (Local sym)

Promoting α to $\alpha(x)$ a f^n of x, the kinetic term in the Dirac $\mathcal{L}$ is no longer invariant,

$$\bar\psi i\not\partial\psi \longrightarrow \bar\psi i\not\partial\psi - \bar\psi\gamma^\mu\psi\,\partial_\mu\alpha(x)$$

We introduce a gauge deriv, D_μ, s.t.

$$D_\mu\psi(x) \longrightarrow \exp[i\alpha(x)]\, D_\mu\psi(x)$$

To do we introduce a gauge field $A_\mu(x)$

$$D_\mu\psi(x) = (\partial_\mu + igA_\mu)\,\psi(x) \qquad \text{where transf}^n \text{ for } A_\mu \text{ is}$$

$$A_\mu \longrightarrow A_\mu - \frac{1}{g}\partial_\mu\alpha(x) \qquad \text{so that } \bar\psi i\not{D}\psi \longrightarrow \bar\psi i\not{D}\psi .$$

We can introduce a kinetic term for A_μ, $\mathcal{L}_k = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ or $ig F_{\mu\nu} = [D_\mu, D_\nu]$.

2.3 Types of symmetry

Symmetries can manifest themselves in a no. of ways:

1) Symmetry is intact (exact), e.g. $U(1)_{EM}$ and $SU(3)_c$ in SM
2) " is broken by an anomaly (not a true sym), e.g. global axial symmetry for massless fields in SM
3) " is explicitly broken by some terms in $\mathcal{L}$. May be still useful to consider if the sym breaking terms are small, e.g. (global) isospin symmetry relating u, d quarks
4) " is respected by $\mathcal{L}$ but not by the vacuum "hidden symmetry"
   a) spontaneously broken symmetry: vacuum expectation value (VEV) for one or more scalar fields, e.g. $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$
   b) even without scalar fields, dynamical symmetry breaking from quantum symmetry, e.g. $SU(2)_L \times SU(2)_R$ global sym in strong interaction

3 Discrete sym

Parity, P: $(t, \underline{x}) \rightarrow (t, -\underline{x})$

Time-reversal, T: $(t, \underline{x}) \rightarrow (-t, \underline{x})$

Charge conj.: particle $\longleftrightarrow$ anti-particle