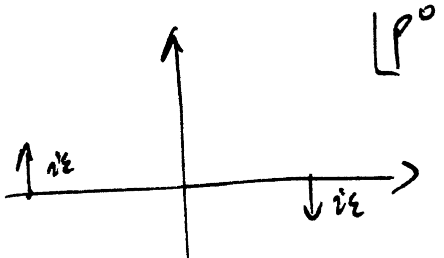


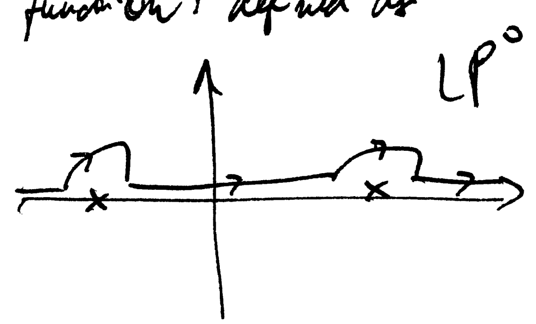
$$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip\cdot(x-y)}}{p^2 - m^2 + i\varepsilon}, \quad \varepsilon > 0$$

The propagator is the Green's function of the KG op.
Avoiding singularities

$$\left(\partial_t^2 - \nabla^2 + m^2 \right) \Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} (-p^2+m^2)\, e^{-ip\cdot(x-y)}$$

$$= -i \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} = -i\delta^4(x-y) \qquad \text{(didn't use the contour)}$$

For some purposes, it's useful to pick other contours, e.g. the <u>retarded</u> Green's function: defined as



which in terms of ops is

$$\Delta_R(x-y) = \begin{cases} [\phi(x), \phi(y)] & : x^0 > y^0 \\ 0 & : y^0 > x^0 \end{cases}$$

This is useful if we start with some initial field configuration and work out how it evolves in the presence of a source. This solves the "inhomogeneous KG eq":

$$\partial_\mu \partial^\mu \phi + m^2 \phi = J(x) \leftarrow \text{background } f^n$$

One also defines the "advanced Green's f^n" $\Delta_A(x-y)$ which vanishes when $y^0 < x^0$. (Useful if we know the end point of a field ~~c~~ configuration and want to know where it came from). Feynman propagators are the most useful in QFT.

3 Interacting Fields

Free theories are special – can determine the exact spectrum but nothing interacts.
These facts are related: quadratic $\mathcal{L} \Rightarrow$ linear EoM $\Rightarrow$ exact quantisation $\Rightarrow$
$\Rightarrow$ multi-part^c states with no interaction

Let's add interaction terms to $\mathcal{L}$ – higher powers of the fields
We start by ~~adding~~ asking what type of small perturbations we can add to $\mathcal{L}$, e.g. for a $\mathbb{R}$ scalar ϕ.

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 - \sum_{n=3} \frac{\lambda_n}{n!}\phi^n$$ coupling constants

Recall $[S] = 0$, $S = \int d^4x \, \mathcal{L}$ and $[d^4x] = -4$ =>

$[\mathcal{L}] = 4$, $[\partial_\mu] = 1$ => $[\phi] = 1$, $[m] = 1$, $[\lambda_n] = 4 - n$

Which terms are "small"? Might say '$\lambda_n \ll 1$' but that only works for dimless quantities, $\exists$ 3 classes of terms:

① $[\lambda_3] = 1$ dimless parameter is $\left(\frac{\lambda_3}{E}\right)$ where E is the energy of the configuration of interest

So $\frac{\lambda_3 \phi^3}{3!}$ is a small perturbation at high energies.

$E \gg \lambda_3$ but a large perturbation at low energies ($E \ll \lambda_3$). Called relevant (at low energies). In a relativistic theory, $E > m$, so we can always make this perturbation small by taking $\lambda_3 \ll m$

② $\frac{\lambda_4 \phi^4}{4!}$ is small if $\lambda_4 \ll 1$, called marginal perturbation.

③ $\frac{\lambda_n \phi^n}{n!}$ $\forall n > 4$ has dimensionless parameter $\lambda_n E^{n-4}$ small at low energies, large at high energies. Called irrelevant (at low energies).

It's typically impossible to avoid high E in QFT (cf vacuum energy discussion). We might then expect problems with irrelevant ops. These are technically non-renormalisable theories - see AQFT.

In this course, we study only weakly coupled field theories - ones that can truly be considered as small perturbations of the free theory at all energies.

e.g.s ① ϕ^4 theory: $\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$ $(\lambda \ll 1)$

We can already guess the effects of the final term by noting that here $[H, N] \neq 0$.

=> part^c # not conserved

Expanding the last term in $\mathcal{L}$ in terms of $a_p, a_p^\dagger$:

we get $a_p^\dagger a_p^\dagger a_p^\dagger a_p^\dagger$, $a_p^\dagger a_p^\dagger a_p^\dagger a_p$, ... which will create/destroy part^c.

② Scalar Yukawa theory

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - M^2 \psi^* \psi - \frac{1}{2} m^2 \phi^2 - g \psi^* \psi \phi$$

$g \ll m, M$. Here, still have $[Q, H] = 0 \Rightarrow$ # ψ part^c − ~~#~~ψ antipart^c is cons.

But no part^c # conservation for ϕ.

A worry: the potential $V = M^2 \psi^* \psi + \frac{1}{2} m^2 \phi^2 + g \psi^* \psi \phi$ has a stable local minimum at $\phi = \psi = 0$ but is unbounded below for large $(-g\phi)$, so we can't push the theory too far.

Strongly coupled theories are a major research topic.