Standard Model 14, 15

Summary of EW theory

$\mathcal{L}_{gauge,\phi}$ - masses for $W^\pm$, Z and Higgs. $W^\pm$, Z - H interactions and H-H interactions

$\mathcal{L}_{lept,\phi}$ - lepton masses, lepton - H interactions

$\mathcal{L}^{EW}_{lept}$ - lepton - $W^\pm$, Z, γ int (PMNS matrix : ν oscillations / mixing and probably $\cancel{CP}$)

$\mathcal{L}_{quark,\phi}$ - quark masses, quarks - H int.

$\mathcal{L}^{EW}_{quark}$ - quark int. with $W^\pm$, Z, γ
(CKM matrix : quark flavour mixing & CP violation)

6. Weak Decays

6.1. Effective Lagrangian

We'll consider some processes where energies, momenta $\ll M_W, M_Z$, so we can use an effective field theory (Fermi weak Lagrangian)

The weak interaction part of the Lagrangian is

$$\mathcal{L}_W = \frac{g}{2\sqrt{2}} \left(J^\mu W_\mu^+ + J^{\mu +} W_\mu^- \right) \leftarrow \text{charged current}$$

$$+ \frac{g}{2\cos\theta_W} J_n^\mu Z_\mu \leftarrow \text{neutral current}$$

and the S-matrix is

$$S = \mathcal{T} \exp\left[i \int d^4x \, \mathcal{L}_W(x) \right]$$

For small g, can Taylor expand, and assuming no $W^\pm$, Z in initial / final states.

$$\langle f | S | i \rangle = \langle f | \mathcal{T} \Big\{ 1 - \frac{g^2}{8} \int d^4x \, d^4x' \Big[J^{\mu +}(x) \, D^W_{\mu\nu}(x-x') \, J^\nu(x')$$

$$+ \frac{1}{\cos^2\theta_W} J_n^{+\mu}(x) \, D^Z_{\mu\nu}(x-x') \, J_n^\nu(x') \Big] + O(g^4) \Big\} | i \rangle$$

where we've used Wick's theorem and $D^W_{\mu\nu}(x-x') = \langle \mathcal{T} W_\mu^-(x) W_\nu^+(x') \rangle$ (Feynman propagator) & same for Z

$$D_{\mu\nu}^{Z,W}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \tilde{D}_{\mu\nu}^{Z,W}(p)$$

$$\tilde{D}_{\mu\nu}^{Z,W}(p) = \frac{i}{p^2 - M^2 + i\varepsilon}\left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{M_{Z,W}^2}\right) \qquad \text{[show later]}$$

At low energies (e.g. decays of quarks & leptons except top quark) $m_{Z,W} \gg p^2$ where p is any combination of initial & final state momenta. We can approximate the propagators by,

$$\tilde{D}_{\mu\nu}^{W}(p) \simeq \frac{i g_{\mu\nu}}{M_W^2} \qquad \text{(and same for } Z\text{)}$$

$$\tilde{D}_{\mu\nu}^{W}(x-y) = \frac{i}{M_W^2} g_{\mu\nu}\, \delta^4(x-y)$$

can describe this interaction by a "contact interaction", 4-fermion interaction

We find

$$-\frac{g^2}{8}\{\cdots\} \longrightarrow -\frac{ig^2}{8M_W^2} J^{\mu T}(x) J^{\nu}(x')\, g_{\mu\nu}\, \delta^4(x-x')$$

and similar for neutral current. The effective lagrangian is

$$i\mathcal{L}_W^{\text{eff}}(x) = -\frac{iG_F}{\sqrt{2}}\left[J^{\mu\dagger}(x) J_\mu(x) + \rho J_n^{\mu\dagger}(x) J_{n\mu}(x)\right]$$

where $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$, $\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W}$

We can write $\rho = 1 + \Delta\rho$ where 1 is the tree level vertex and $\Delta\rho$ is from quantum loops (and sensitive to BSM physics)

($\simeq 0.008$ in SM + process dependent parts)

$$\langle f|S|i\rangle \simeq \langle f|\mathcal{T}[1 + i\int d^4x\, \mathcal{L}_W^{\text{eff}} + \cdots]|i\rangle$$

$$\approx \langle f|\mathcal{T}\exp\left(i\int d^4x\, \mathcal{L}_W^{\text{eff}}\right)|i\rangle$$

Note that $[G_F] = -2$ (since $[J] = 3$)

$\Rightarrow$ non-renormalisable. This OK for energy scale $\ll M_W, M_Z$. The $\frac{1}{m_W^2}$ in G_F indicates that Fermi theory breaks down at energy scale $\sim M_W$

Aside. Z propagator (similar for $W^\pm$)

We'll gloss over subtleties here (gauge, ghosts) [see P&S §21]

Work here in unitary gauge (?)

$$\mathcal{L}_Z^{free} = -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

To find propagators introduce external current j^μ coupled to Z^μ

So $\mathcal{L} \to \mathcal{L}_Z^{free} + j^\mu(x) Z_\mu(x)$

EL eqns $\Rightarrow$

$$-\partial^2 Z_\rho + \partial_\rho \partial^\mu Z_\mu - m_Z^2 Z_\rho = j_\rho$$

$$\Rightarrow \partial^2 Z_\rho - \partial_\rho \partial^\mu Z_\mu + m_Z^2 Z_\rho = -j_\rho \qquad (\star)$$

Take ∂^ρ of this

$$\overset{\to 0}{\cancel{\partial^2 \partial^\rho Z_\rho - \partial^2 \partial^\mu Z_\mu}} + m_Z^2 \partial^\rho Z_\rho = -\partial^\rho j_\rho$$

Sub back into $(\star)$

$$\Rightarrow (\partial^2 + m_Z^2) Z_\mu = -\left(g_{\mu\nu} + \frac{\partial_\mu \partial_\nu}{m_Z^2}\right) j^\nu$$

Therefore, $Z_\mu(x) = i\int d^4y \, D^Z_{\mu\nu}(x-y)\, j^\nu(y)$

$$D^Z_{\mu\nu}(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \tilde{D}^Z_{\mu\nu}(p) \qquad \text{Fourier trans}$$

$$\Rightarrow \tilde{D}^Z_{\mu\nu}(p) = \frac{-i}{p^2 - m_Z^2 + i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_Z^2}\right) \qquad \text{similar for W boson}$$

6.2. Decay rates & Cross-sections

Questions we can ask of particle physics experiments boil down to

① How frequent does X decay to $A_1 + A_2 + \ldots$?

② Given N collisions between A & B how many times do we produce X ?

<u>Decay rate</u> Γ_X is no. of decays of X in its rest frame divided by no. of X present per unit time.

The lifetime $\tau_X = \frac{1}{\Gamma_X}$

$\Gamma_X = \sum_i \Gamma_{X \to F_i}$ where $\Gamma_{X \to F_i}$ is the particle decay rate to the final state F_i.

We need $\langle f | S | i \rangle$ with $i = X$. Write $S = 1 + iT$ where 1 corresponds to nothing happening