6.2 (cont) Decay Rate

When $S = 1 + iT$ Removing the "Boring" part, define invariant amplitude M by

$$\langle f|S-1|i\rangle = (2\pi)^4 \delta^{(4)}(p_f - p_i)\, iM_{fi}$$

$$\text{Prob } i \to f = P(i\to f) = \frac{|\langle f|S-1|i\rangle|^2}{\langle f|f\rangle \langle i|i\rangle}$$

Working in a finite spatial volume V (to avoid subtleties from non-renormalisable state) and temporal extent T

$$(2\pi)^4 \delta^4(0) = VT$$

Our normalisation is then $\langle i|i\rangle = (2\pi)^3\, 2p_i^0\, \delta^{(3)}(0) = 2p_i^0 V$ ($= m_i$ since it's taken to be stationary)

and $\langle f|f\rangle = \prod_r 2p_r^{(0)} V$ where r labels final state particles

$$p(i\to f) = \frac{|M_{fi}|^2}{2m_i \cancel{V}} (2\pi)^4 \delta^4(p_i - \sum_r p_r) \cancel{V} T \prod_r \left(\frac{1}{2p_r^0 V}\right)$$

We never measure final state momenta with infinite precision

$$\Gamma(i\to f) = \frac{1}{T}\int p(i\to f) \prod_r \left[\frac{V}{(2\pi)^3} d^3p_r\right]$$

Lorentz invariant measure is

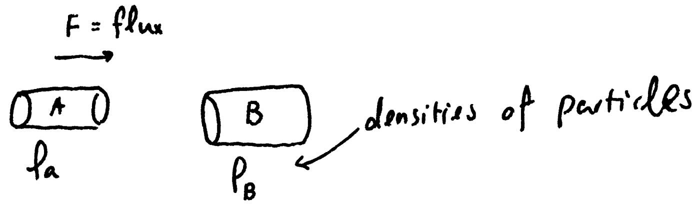
$$d\rho_f = (2\pi)^4 \delta^4(p_i - \sum_r p_r) \prod_r \frac{d^3p_r}{(2\pi)^3 2p_r^0}$$

$$\Gamma(i\to f) = \frac{1}{T}\int \frac{|M_{if}|^2}{2m_i} T \prod_r \left(\frac{(2\pi)^3}{V}\right) d\rho_f \prod_r \left(\frac{V}{(2\pi)^3}\right)$$

$$\Gamma(i\to f) = \frac{1}{2m_i}\int |M_{fi}|^2 d\rho_f$$

Cross. Sections

Consider 2 colliding beams



$n = F\sigma$

n: # of scattering events per unit time per target particle

F: incident flux $\underbrace{|\vec{v}_A - \vec{v}_B|}_{\text{relative velocity}}\rho_A$ = # of incoming particles per unit area per unit time

σ: cross-section

Total # of scattering events per unit time $= N = n\rho_B V = F\sigma\rho_B V$

$$= |\vec{v}_A - \vec{v}_B|\,\rho_A\rho_B V\sigma$$

Our normalisation is such that $\rho_A = \rho_B = \frac{1}{V}$

Consider $dN \propto$ (finite state momenta $d\rho_f$)

$$dN = d\sigma\,\frac{|\vec{v}_A - \vec{v}_B|}{V} = \frac{1}{2E_A 2E_B V}\,|M_{fi}|^2\,d\rho_f$$

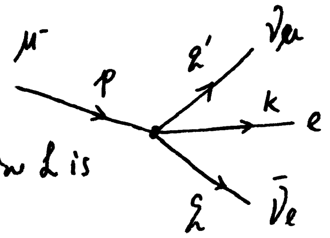
↑ generalising our earlier expression for $P(i \to f)$

$$\boxed{d\sigma = \frac{1}{|\vec{v}_A - \vec{v}_B|\,4E_A E_B}\,|M_{fi}|^2\,d\rho_f}$$

N.B. often cross-sections are measured in barns, $1\text{ barn} = 10^{-28}\,m^2$

6.3 Muon Decay

$\mu^- \longrightarrow e\,\bar{\nu}_e\,\nu_\mu$



(Assume $m_\nu = 0$)

The relevant bit of effective $\mathcal{L}$ is

$$-\frac{G_F}{\sqrt{2}}\,J^{\alpha\dagger}J_\alpha$$

where $J^\alpha = \underbrace{\bar{\nu}_e\gamma^\alpha(1-\gamma^5)e + \bar{\nu}_\mu\gamma^\alpha(1-\gamma^5)\mu}_{\text{relevant}} + \bar{\nu}_\tau\gamma^\alpha(1-\gamma^5)\tau$

$m_\mu \approx 106\,MeV$, $m_W \approx 80\,GeV$, so $m_\mu \ll m_W \rightarrow$ Fermi theory good approximation

$$M = \langle e(k)\,\bar{\nu}_e(q)\,\nu_\mu(q')\,|\,\mathcal{L}_W^{eff}\,|\,\mu(p)\rangle$$

$$= -\frac{G_F}{\sqrt{2}}\langle e(k)\,\bar{\nu}_e(q)\,|\,\bar{e}\gamma^\alpha(1-\gamma^5)\nu_e\,|\,0\rangle\langle\nu_\mu(q')\,|\,\bar{\nu}_\mu\gamma^\alpha(1-\gamma^5)\mu\,|\,\mu(p)\rangle$$

$$= \frac{G_F}{\sqrt{2}}\,\bar{u}_e(k)\gamma^\alpha(1-\gamma^5)v_{\nu_e}(q)\,\bar{u}_{\nu_\mu}(q')\gamma_\alpha(1-\gamma^5)u_\mu(p)$$ ↲ Wick (only one possible contraction)

- We are not interested in final state spins → sum over final state spin
- We don't know spin of μ^- → average over initial state spin

$$\frac{1}{2}\sum_{spins}|M|^2 = \frac{1}{2}\frac{G_F^2}{2}\sum_{spins}\left[\bar{u}_e(k)\gamma^\alpha(1-\gamma^5)v_{\nu_e}(q)\,\bar{v}_{\nu_e}(q)\gamma^\beta(1-\gamma^5)u_e(k)\right]$$
$$\times\left[\bar{u}_{\nu_\mu}(q')\gamma_\alpha(1-\gamma^5)u_\mu(p)\,\bar{u}_\mu(p)\gamma_\beta(1-\gamma^5)u_{\nu_\mu}(q')\right]$$

$$= \frac{G_F^2}{4}S_1^{\alpha\beta}S_{2\alpha\beta}$$

$$S_1^{\alpha\beta} = Tr\left[(\not{k}+m_e)\gamma^\alpha(1-\gamma^5)(\not{q})\gamma^\beta(1-\gamma^5)\right]$$

$$S_{2\alpha\beta} = Tr\left[(\not{q}')\gamma_\alpha(1-\gamma^5)(\not{p}+m_\mu)\gamma_\beta(1-\gamma^5)\right]$$

(use $\sum_s u_s(p)\bar{u}_s(p) = \not{p} + m$)