Spacetime symmetry

- **Lorentz** $SO(3,1)$ $\mathbb{R}^{3,1}$

$$SO(3,1) \xrightarrow{\text{Wick}} SO(4) \qquad \mathbb{R}^4$$

$$\mathcal{L}_{\mathbb{C}}(SO(3,1)) = \mathcal{L}_{\mathbb{C}}(SO(4)) = D_2$$
$$= A_1 \oplus A_1$$

globally, $SO(4) = \dfrac{SU(2) \times SU(2)}{\mathbb{Z}_2}$

Field	$SO(3,1)$	$A_1 \oplus A_1$	
Scalar	$\underline{1}$	(R_0, R_0)	$\varphi(x)$
Dirac fermion	LH$\oplus$RH Weyl fermion	$(R_1, R_0) \oplus (R_0, R_1)$	$\lambda_\alpha(x), \bar{\lambda}_{\dot\alpha}(x)$ $\alpha = 1,2$ $\dot\alpha = 1,2$
Vector	$\underline{4}$	(R_1, R_1)	$A_{\alpha\dot\alpha} = \sigma^\mu_{\alpha\dot\alpha} A_\mu$

σ^μ: γ matrices, $\sigma = (1, \underline{\sigma})$

- **Poincaré**

Lorentz rotations $M_{\mu\nu}$
+ Translations P_μ — ideal

not simple

Extensions

- conformal invariance — massless free fields, $\mathcal{L} = \frac{1}{2}(\partial\phi)(\partial\phi)$

scale invariant

$$D: \; x_\mu \to x_\mu' = \lambda^{-1} x_\mu$$
$$\phi(x) \to \lambda^{\Delta} \phi(x')$$

Δ: dimension of ϕ

quasi-theorem for unitary QFT:
scale invariance $\Rightarrow$ conformal invariance

conformal group in $D=4$ is $SO(4,2)$ — dim 15

$$\left(\begin{array}{c|c} \begin{matrix} 0 & D \\ -D & 0 \end{matrix} & P_\mu + K_\mu \\ \hline \begin{matrix} P_\nu \\ + \\ K_\nu \end{matrix} & M_{\mu\nu} \end{array}\right)$$

CFT

No-go theorem (Coleman - Mandula)

... not with Lie algebras ...

Supersymmetry

"graded" Lie algebra $\quad \mathfrak{g}_{SUSY} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ ← Boson ← Fermi

Lie superalgebra

$\forall\, X \in \mathfrak{g}_{SUSY}$ define grade $|X| = 0, 1$

$$[X, Y] = -(-1)^{|X||Y|}[Y, X]$$

Internal symmetry $\quad$ - $\mathcal{H}_{1,\text{particle}}$ in QFT

unitary repn of $\mathcal{L}(G)$

- Symmetry => degeneracy

\# particles $\quad$ dim(R)

quantum numbers $=$ weights of R

		Q	I	M	
nucleon (fermions)	p	+1	+1/2	938	MeV
	n	0	-1/2	940	MeV

		Q	I	M	
π-mesons (bosons)	π^+	+1	+1	139	MeV
	π^0	0	0	135	MeV
	π^-	-1	-1	139	MeV

⤳ approximate $SU(2)_I$ symmetry

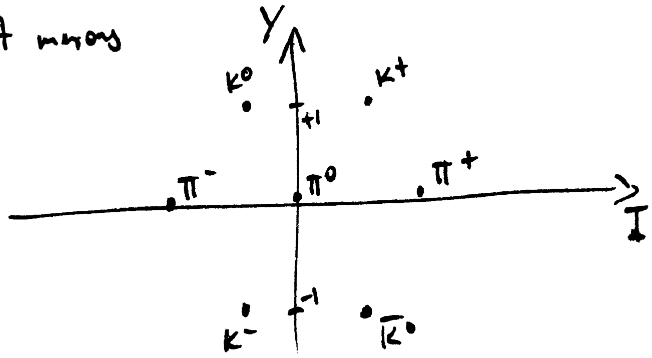
$H = 2I$

$\begin{pmatrix} |p\rangle \\ |n\rangle \end{pmatrix} \sim R_1$

$\begin{pmatrix} \pi^+ \\ \pi^- \\ \pi^0 \end{pmatrix} \sim R_2$

- new particles
  - mesons (= bosons)
  - baryons (= fermions)
- new conserved q number
  - hypercharge (Y)

8 light mesons



	$SU(3)_{\text{flavour}}$
quark	$\underline{3}$
$\bar{q}$	$\bar{\underline{3}}$

mesons $\quad q\bar{q}$ $\quad$ baryons $\quad qqq$

$\underline{3} \otimes \bar{\underline{3}} = \underline{1} \oplus \underline{8}$ $\qquad$ $\underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{1} \oplus \underline{8} \oplus \underline{8} \oplus \underline{10}$

QCD $\quad SU(3)_{\text{colour}} \quad q = \underline{3}$

$$\mathcal{L} = i\bar{\psi}_{\dot\alpha}\, \sigma_\mu^{\dot\alpha\alpha} D^\mu \psi_\alpha + \frac{1}{4g^2} K(F_{\mu\nu}, F^{\mu\nu})$$

$$D^\mu = \partial_\mu + R(A_\mu)$$