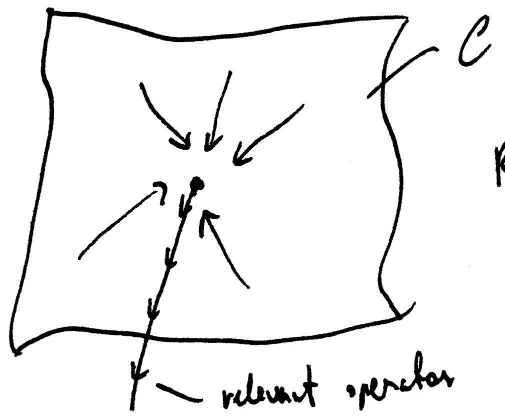


∞-dim^{n} surface of irrelevant couplings

Remaining consist of marginal operators: i.e. those with $\Delta_i = d$. To lowest order, these are unchanged by RG flow, but we have to examine higher order corrections to decide whether these operators are marginally relevant, marginally irrelevant or exactly marginal.

Marginally (ir)relevant operators may stay roughly constant for long periods of RG evolution. Because of this, these operators are often important phenomenologically. In d dimensions, $[\phi] = \frac{d-2}{2}$ for a scalar field, $[\partial]=1$, so we have the following operators: (even in ϕ)

d	Relevant	Marginal
2	ϕ^{2k} $\forall k \geq 0$	$(\partial\phi)^2, \phi^{2k}(\partial\phi)^2$ $k>0$
3	ϕ^{2k} $k=1,2$	$(\partial\phi)^2, \phi^6$
4	ϕ^2	$(\partial\phi)^2, \phi^4$
$d>4$	ϕ^2	$(\partial\phi)^2$

Taking the continuum limit

We've started with some effective theory at a high scale Λ_0, and studied its couplings as we flow down to low energies. In high energy physics, we'd instead like to take the continuum limit $\Lambda_0 \to \infty$.

To do this, consider following the RG flow backwards. If we start from g_i^* (critical pt), then the couplings are independent of scale, so we can take the limit $\Lambda_0 \to \infty$ after computing the path integral, and will obtain finite results.
If we start with a theory anywhere on the critical surface, then since its couplings $\sim \left(\frac{\Lambda}{\Lambda_0}\right)^{\Delta_i - d}$ with $\Delta_i > d$, then again sending $\Lambda_0 \to \infty$ drives the theory to the critical point. Notice, since $\mathcal{C}$ has finite codim, we only need to tune finitely many couplings to ensure we start on $\mathcal{C}$. The continuum theory is again a CFT (perhaps Gaussian).

However, suppose we begin at an experimental scale $\Lambda \ll \Lambda_0$ with a theory that has non-zero values for relevant couplings. We appear to lose control of this theory as $\Lambda_0 \to \infty$. The way to obtain a finite limit is to <u>tune</u> the initial values $g_i(\Lambda_0)$. Suppose the RG trajectory passes closest to the critical pt at some scale μ. On dimensional grounds, we must have

$$\mu = \Lambda_0 f(g_i(\Lambda_0))$$

We want to tune the values of $g_i(\Lambda_0)$ so $\lim_{\Lambda_0 \to \infty} \Lambda_0 f(g_i(\Lambda_0))$ is finite. Note that $f(g_i(\Lambda_0)) = 0$ defines the critical surface.

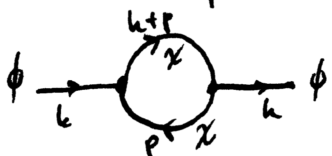
To achieve this, we modify the action by introducing <u>counterterms</u>.

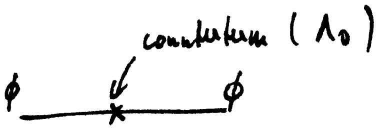
i.e. $$S^{eff}_{\Lambda_0}[\phi] = S_{\Lambda_0}[\phi, g_i(\Lambda_0)] + \hbar S^{CT}[\phi, g_i(\Lambda_0), \Lambda_0]$$ ← explicit dependence on Λ_0

The counterterm action modifies the couplings in the original action <u>by hand.</u>
These should be chosen s.t.

$$\lim_{\Lambda_0 \to \infty} \left[\int_{C^\infty(M)_{(\Lambda,\Lambda_0]}} \mathcal{D}\chi \, \exp\left(-\frac{S^{eff}_{\Lambda_0}[\phi+\chi, g_i(\Lambda)]}{\hbar} - S^{CT}[\phi+\chi, \Lambda_0] \right) \right] \text{ is finite.}$$

So in practice, we compute the path integral perturbatively. If we evaluate a 1-loop diagram ($\mathcal{O}(\hbar^0)$) using vertices + props from the original action, we'll get an answer depending on Λ_0. This result typically diverges as $\Lambda_0 \to \infty$. The vertices in S^{CT} provide further contributions to the loop process, and we tune these <u>by hand</u> so that the sum is finite as $\Lambda_0 \to \infty$.



as a divergent loop integral,


<u>Calculating RG evolution</u>

We note that (in $d > 2$), the only marginal / relevant operator that involves derivatives of ~~the~~ ϕ is the kinetic term $(\partial\phi)^2$. This suggests we can find a simple truncation of the RG flow by restricting to actions of the form.

$$S[\varphi] = \int d^dx \, \tfrac{1}{2}(\partial\varphi)^2 + V(\varphi)$$

It's still hard to compute the path integral. We split $\varphi = \overset{\text{low}}{\phi} + \overset{\text{high}}{\chi}$ and want to compute $S^{eff}_{\Lambda}[\phi] = -\hbar \log\left[\int_{C^\infty(M)_{(\Lambda,\Lambda_0]}} \mathcal{D}\chi \, e^{-S[\phi+\chi]} \right]$. To make progress, we assume we lower the cutoff infinitesimally $\Lambda = \Lambda_0 - \delta\Lambda$.