# LABORATORY # 05

## Basic Filters and Frequency Response

### Purpose

The goal of this laboratory is to study the frequency characteristics of Low and High Pass Filters. Specifically, signal amplitude, signal phase, and how they are related to input frequency. In addition, the concept of cutoff frequency will be introduced.

### Theoretical Background

A filter is a device or configuration that allows a specific range of input frequencies to pass and rejects signals whose frequency is outside of the specified range. Among the many types of filters, in this laboratory you will be introduced to basic Low and High Pass Filters. These filters consist of nothing more than a configuration made up of a single resistor and capacitor.

### Low Pass Filter

Low Pass Filters allow low frequencies to pass and block high frequencies. The ideal Low Pass Filter should allow all frequencies below some specified frequency, call it $f_c$ (cutoff), to pass and allow no frequencies above $f_c$ to pass. It is not possible to design an ideal Low Pass Filter. Therefore the design characteristics of the filters used in this laboratory will be non-ideal. It is important to be aware there are more sophisticated filtering devices on the market that approach the ideal case.

Figure 5.1, on the next page, shows a typical configuration for a Low Pass Filter. The mathematical development to follow will show that the circuit is acting as a Low Pass Filter at the output terminal A-B. The input frequency and amplitude, E, will directly influence the magnitude and phase of $V_C$. In other words, at certain input frequencies the Low Pass Filter will operate on the input **E** to produce $\mathbf{V_c}$. The frequency where this action occurs is determined by values chosen for the resistor (R) and the capacitor (C).

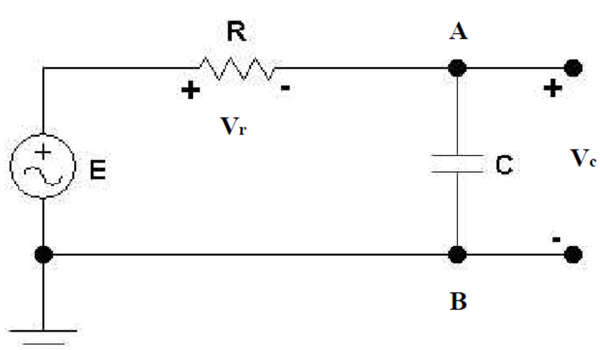


Figure 5.1 LPF

The voltage across the capacitor in Figure 5.1 can be written as

$$V_c = I(-jX_c)$$

Where

$$I = \frac{E}{Z}$$

And

$$Z = R - jX_c$$

After substitution $V_c$ can be rewritten as

$$V_c = \frac{-jEX_c}{R - jX_c}$$

Recalling that

$$X_c = \frac{1}{2\pi fC}$$

And substitution leads to

$$\frac{V_c}{E} = \frac{1}{1 + j2\pi fRC}$$

If we use the definition of gain, and take the magnitude, the final form can be written as

$$\left|\frac{V_c}{E}\right| = \frac{1}{\sqrt{(1 + (2\pi fRC)^2)}}$$

From inspection of the above equation, it is apparent that as the frequency (f) increases, the magnitude of the gain decreases. In short, the circuit in Figure 5.1 works as a Low Pass Filter. Also take note that the product RC has an effect on the overall gain. The phase angle for the Low Pass Filter can be expressed as

$$\tan(\phi) = -2\pi fRC$$

Note that the phase is initially at zero radians, where ideal gain is at its maximum. This shows that the phase of the output will approach -π/2 radians as the frequency increases.

### High Pass Filter

High Pass Filters allow high frequencies to pass and block low frequencies. The ideal High Pass Filter should allow all frequencies above some specified frequency, call it $f_{c\ (cutoff)}$, to pass and allow no frequencies below $f_c$ to pass. It will be shown mathematically that the circuit in Figure 5.2 is acting as a High Pass Filter at the output terminal A-B,.

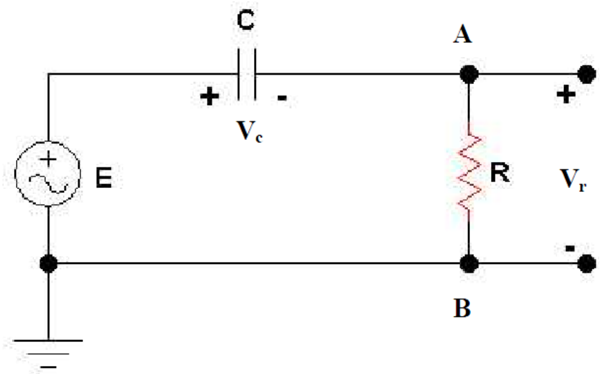


Figure 5.2 HPF

The voltage across the resistor, in Figure 5.2, can be written as

$$V_r = IR$$

$$I = \frac{E}{Z}$$

$$Z = R - jX_c$$

Using the definition of Xc, and making substitutions, allows the equation to be rewritten as

$$\frac{V_r}{E} = \frac{j2\pi f RC}{1 + j2\pi f RC}$$

Using the definition of gain and taking the magnitude gives the final form of

$$\left|\frac{V_r}{E}\right| = \frac{2\pi f RC}{\sqrt{(1+(2\pi f RC)^2)}}$$

Noticing that 2RC in above Equation is a constant for any given circuit leads to

$$\left|\frac{V_r}{E}\right| \propto \frac{f}{\sqrt{1+f^2}}$$

This equation is recognizable as an increasing function of frequency that approaches unity as the frequency increases. Therefore, the circuit in Figure 5.2 works as a High Pass Filter. The phase angle for the High Pass Filter can be expressed as

$$\tan(\phi) = \frac{1}{2\pi f RC}$$

Note the phase is initially at π/2 radians, where ideal gain is zero. It shows that the phase of the output will approach zero radians as the frequency increases.

### *Cutoff Frequency*

The cutoff frequency, denoted by $f_C$, is defined as the frequency at which the magnitude of gain is equal to

$$\frac{1}{\sqrt{2}}$$

Considering the Low Pass Filter configuration

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(1+(2\pi f RC)^2)}}$$

Replacing f with $f_c$, and then solving for the cutoff frequency yields

$$f_c = \frac{1}{2\pi RC}$$

This is true for both Low and High Pass Filters. You determine the cutoff frequency by setting the product value of (RC). Keeping in mind that a low pass configuration will pass frequencies below $f_c$ and the high pass configuration will pass frequencies above $f_c$.

### Theoretical Analysis

Let the input signal be a 4V peak-to-peak sinusoid with R = 1 kΩ, and C = 1μF for both the Low and High Pass Filter sections.

### *Low Pass Filter*

Referring to circuit in Figure 5.1:

- Fill up a table similar to the one given below. (Table 5.1).
- Plot gain in dB vs. frequency. Use data from Table 5.1.
- Plot phase vs. frequency. Use data from Table 5.1.
- Determine the cutoff frequency from your tabulated data you made in step 1. Compare it with the theoretical value. Mark the cutoff frequency on your magnitude plot.

Table 5.1

| | Theoretical | |
|---|---|---|
| f (HZ) | Gain (dB) | Phase Angle (Degree) |
| 25 | | |
| 50 | | |
| 75 | | |
| 100 | | |
| 150 | | |
| 200 | | |
| 300 | | |
| 500 | | |
| 600 | | |
| 700 | | |
| 800 | | |
| 900 | | |
| 1000 | | |

### *High Pass Filter*

Referring to circuit in Figure 5.2:

- Fill up a table similar to the one given below. (Table 5.2).
- Plot gain in dB vs. frequency. Use data from Table 5.2.
- Plot phase vs. frequency. Use data from Table 5.2.
- Determine the cutoff frequency from your tabulated data you made in step 1. Compare it with the theoretical value. Mark the cutoff frequency on your magnitude plot.

Table 5.2

| | Theoretical | |
|---|---|---|
| f (HZ) | Gain (dB) | Phase Angle (Degree) |
| 25 | | |
| 50 | | |
| 75 | | |
| 100 | | |
| 150 | | |
| 200 | | |
| 300 | | |
| 500 | | |
| 600 | | |
| 700 | | |
| 800 | | |
| 900 | | |
| 1000 | | |

## Simulation

***Low Pass Filter***

Draw the circuit as shown in Figure 5.3. Note the connection of a new instrument. This instrument is called a Bode Plotter. The Bode Plotter produces a graph of a circuit's frequency response. It is capable of measuring a signal's voltage gain and phase shift. Both gain and phase are plotted against frequency on the screen. The vertical and horizontal scales, F and I, should be preset to display the area of interest on the Bode screen.

For the simulation section let your input signal be 4 V peak-to-peak sinusoid, f = 25-1000 Hz, R = 1 kΩ, and C = 1μF.

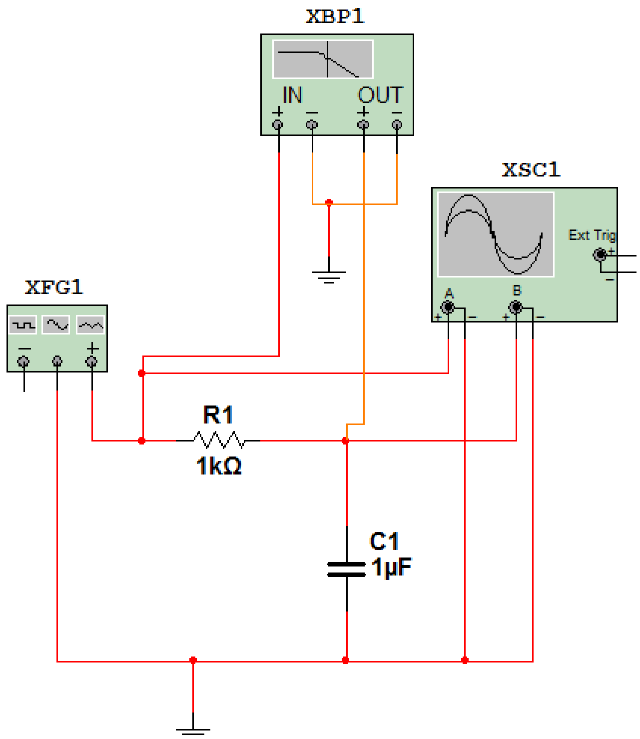

Figure 5.3

- Open the Bode Plotter. Set the plotter for a magnitude reading by depressing the Magnitude button. Then depress Log buttons under both the Vertical and Horizontal headings. Set the I and F parameters as needed. See Figure 5.4 below.
- Set up your function generator with the defined inputs and simulate the circuit. Make a printout of the Bode Plotter output. Interpret the output of the plot. Does it show Low Pass or High Pass Filter characteristics?
- Determine the cutoff frequency by moving the cursor line. Position the cursor line by clicking on the left and right arrows. The cursor position is displayed in the windows next to the arrows. Recall the cutoff frequency occurs when the output magnitude is 0.707 (or -3dB), the magnitude of the input. Make a print of the output with the cursor line positioned at the cutoff frequency on the screen.
- Make the necessary changes in settings on the Bode Plotter in order to get Phase characteristic. Depress the Phase button. Then simulate the circuit and make a printout of the output.

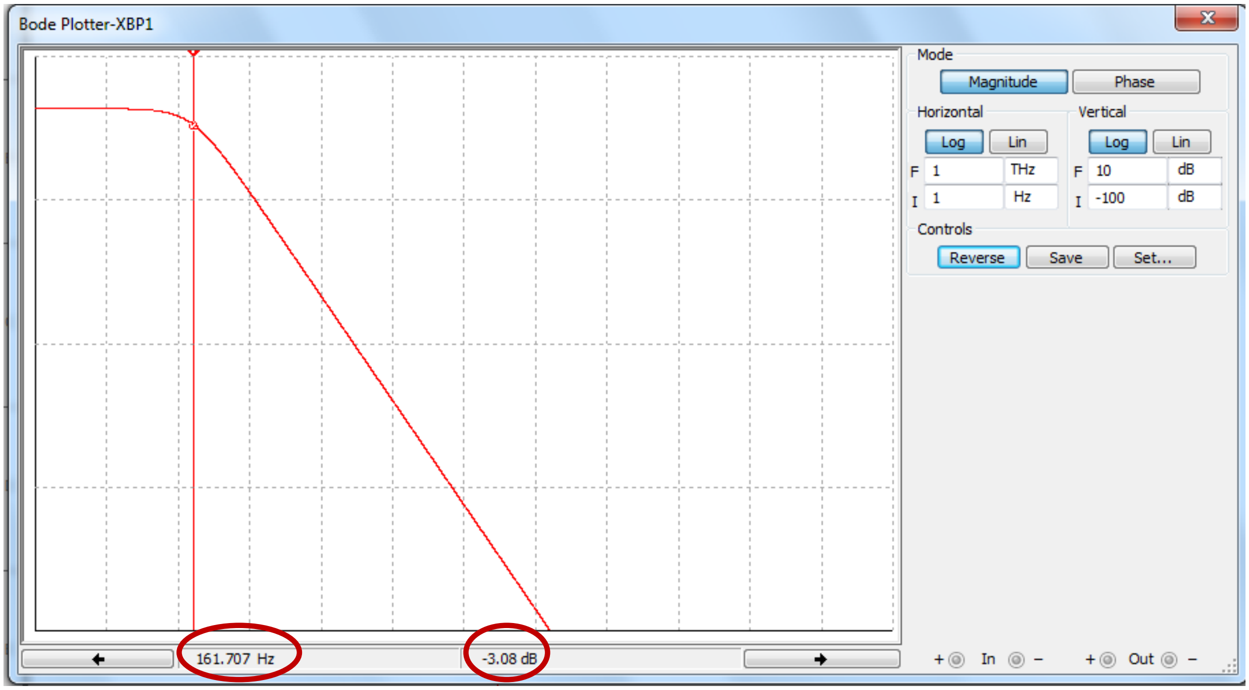

Figure 5.4 Example Bode Plotter output

### *High Pass Filter*

Use the same circuit as in Figure 5.3 but switch the positions of R and C to convert the circuit into a High Pass Filter configuration. Use the same values from the Low Pass Filter simulation for your input and circuit parameters. Repeat steps 1 through 3 from Low Pass Filter simulation. However, you will need to reset the Bode Plotter settings to get the magnitude and phase characteristics for the High Pass Filter. Turn the circuit on and adjust F and I for both the Vertical and Horizontal settings. Once you have a viewable output, stop the simulation. <u>Now restart the simulation and take your data</u>.

## Laboratory Procedure

Build both a Low Pass Filter (Figure 5.1) and a High Pass Filter (Figure 5.2) in the laboratory. Follow all the steps from the Theoretical Analysis.