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CMPE 30043: DISCRETE MATHEMATICS
PROBLEM SET #2: LOGIC

1. Let p = "It is raining"
q = "I am wearing my rubbers",
s = "I am carrying my umbrella".

Express each of the following statements in symbols:

a) "It is raining and I am not wearing my rubbers."

a. $\equiv p \land \neg q$

b) "It is not true that it is raining and I am wearing my rubbers."

b. $\equiv \neg(p \land q)$

c) "Either it is raining, or it is not true that it is raining or I am wearing my rubbers."

c. $\equiv p \lor \neg p \lor q$

d) "Either it is not raining, or I am wearing my rubbers and I am carrying my umbrella."

d. $\equiv \neg p \lor (q \land s)$

e) "Either it is raining and I am wearing my rubbers, or it is not raining and I am not carrying my umbrella."

e. $\equiv (p \land q) \lor \neg(p \land s)$

2. Let's consider a compound propositions where
A ="Angelo comes to the party", B ="Bruno comes to the party",
C ="Carlo comes to the party", D ="David comes to the party".

Represent the following statements into a Boolean expression:

a) "If David comes to the party then Bruno and Carlo come too"

a. $\equiv D \rightarrow B \land C$

b) "David comes to the party if and only if Carlo comes and Angelo doesn't come"

b. $\equiv D \leftrightarrow \neg(C \land \neg A)$

c) "If David comes to the party, then, if Carlo doesn't come then Angelo comes"

c. $\equiv D \rightarrow (\neg C \rightarrow A)$

d) "Carlo comes to the party provided that David doesn't come, but, if David comes, then Bruno doesn't come"

$d \equiv ((C \rightarrow \neg D) \wedge (D \rightarrow \neg B))$

e) "Angelo, Bruno and Carlo come to the party if and only if David doesn't come, but, if neither Angelo nor Bruno come, then David comes only if Carlo comes"

$e. \equiv (A \wedge B \wedge C \leftrightarrow \neg D) \wedge$
$(\neg A \wedge \neg B \rightarrow (D \rightarrow C))$

3. Rephrase the following statements in the form "If P, then Q" or "P iff Q".

a.) If it is hot outside, you buy an ice cream cone, and if you buy an ice cream cone, it is hot outside.

a ≡ you buy an ice cream cone if and only if it is hot outside

b.) It rains if it is a weekend day, and it is a weekend day if it rains.

b ≡ it rains if and only if weekend day.

c.) If you read the news paper every day, you will be informed and conversely.

c. ≡ if you read the newspaper everyday, then you will be informed and conversely

d.) To get tenure as a professor, it is sufficient to be world-famous.

d ≡ if you are world-famous, then you will get tenure as a professor

e.) It is necessary to walk 8 miles to get to the top of the Peak.

e ≡ if you walk 8 miles, then you will get ~~to~~ the top of the peak

4. Write negations of each of the following conditional statements.

a.) If Master lives in the Philippines, then he lives in Pasig.

4. a ≡ if master does not lives in the Philippines, then he does not lives in Pasig.

b.) If my car is in the repair shop, then I cannot get to class.

b ≡ If my car is not in the repair shop, then I can get to class

c.) If x is prime then x is odd or x is 2.

c ≡ if x is not price, then x is neither odd or 2.

d.) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

d ≡ if n is not divisible by 6, then n is not divisible by 2 and 3.

5. Given the following conditional statements:

a.) Write the inverse of

i.) If it snows today, I will ski tomorrow

i ≡ $P \to Q$
≡ $\neg P \to \neg Q$
≡ if it didn't snow today, then I will not ski tomorrow

ii.) If my car is in the repair shop, then I cannot get to class.

ii ≡ $P \to \neg Q$
≡ $\neg P \to Q$
≡ if my car wasn't in the repair shop, then I can get to class.

b.) Write the converse of

i.) If today is Friday, then 2 + 3 = 5.

6. i ≡ $P \to Q$
≡ $Q \to P$
≡ If 2+3=5, then today is Friday

ii.) If P is a square, then P is a rectangle.

ii ≡ $P \to Q$
≡ $Q \to P$
≡ if P is a rectangle, then P is a square

c.) Write the contrapositive of

i.) If a shape is not a rectangle, then the shape does not have four sides.

i. 1 ≡ ¬P → ¬Q
≡ Q → P
≡ if the shape have four sides, then the shape is a rectangle

ii.) If master owns a car, then he is rich.

ii ≡ P → Q
≡ ¬Q → ¬P
≡ If he is not rich, then master doesn't own a car

6. Construct the Truth Table and Determine whether each of the following compound proposition is a Tautology, Contradiction or Contingency.

a) $Z \equiv (\neg P \rightarrow Q) \lor [(P \land \neg R) \leftrightarrow Q]$

$Z \equiv (\neg P \rightarrow Q) \lor [(P \land \neg R) \leftrightarrow Q]$ $2^n = 8$

P	Q	R	¬P	¬P→Q	¬R	P∧¬R	(P∧¬R)↔Q	Z	
F	F	F	T	F	T	F	T	T	
F	F	T	T	F	F	F	T	T	
F	T	F	T	T	T	F	F	T	
F	T	T	T	T	F	F	F	T	
T	F	F	F	T	T	T	F	T	TAUTOLOGY
T	F	T	F	T	F	F	T	T	
T	T	F	F	T	T	T	T	T	
T	T	T	F	T	F	F	F	T	

b) $Z \equiv [\neg P \rightarrow (P \rightarrow Q)] \rightarrow [Q \rightarrow (P \rightarrow P)]$

$Z \equiv [\neg P \rightarrow (P \rightarrow Q)] \rightarrow [Q \rightarrow (P \rightarrow P)]$

P	Q	¬P	P→Q	¬P→(P→Q)	P→P	Q→(P→P)	Z	
F	F	T	T	T	T	T	T	
F	T	T	T	T	T	T	T	TAUTOLOGY
T	F	F	F	T	T	T	T	
T	T	F	T	T	T	T	T	

$c)$ $Z \equiv (P \land R) \leftrightarrow [\neg S \lor (Q \to P)]$

$Z \equiv$
$(P \land R) \leftrightarrow [\neg S \lor (Q \to P)] = 2^n = 2^4 = 16$

P	Q	R	S	P∧R	¬S	Q→P	¬S∨(Q→P)	Z
F	F	F	F	F	T	T	T	F
F	F	F	T	F	F	T	T	F
F	F	T	F	F	T	T	T	F
F	F	T	T	F	F	T	T	F
F	T	F	F	F	T	F	T	F
F	T	F	T	F	F	F	F	T
F	T	T	F	F	T	F	T	F
F	T	T	T	F	F	F	F	T
T	F	F	F	F	T	T	T	F
T	F	F	T	F	F	T	T	F
T	F	T	F	T	T	T	T	T
T	F	T	T	T	F	T	T	T
T	T	F	F	F	T	T	T	F
T	T	F	T	F	F	T	T	F
T	T	T	F	T	T	T	T	T
T	T	T	T	T	F	T	T	T

contingency

7. Simplify the following using the algebra of propositions. Show your solutions.

$a)$ $A \equiv \neg\{[Q \lor (P \land Q)] \land [P \land (P \land Q)]\}$

$\equiv \neg\{[Q \lor (P \land Q)] \land [P \land (P \land Q)]\}$ absorption
$\equiv \neg\{Q \land [P \land (P \land Q)]\}$ associative/idempotence
$\equiv \neg\{Q \land (P \land Q)\}$ associative
$\equiv \neg\{(Q \land Q) \land P\}$ idempotence
$\equiv \neg\{Q \land P\}$ de morgan's
$\equiv \neg Q \lor \neg P$

$b)$ $B \equiv \{[\neg(P \land S) \lor \neg Q] \land (P \lor \neg Q) \land \neg(P \land S)\}$

$B \equiv \{[\neg(P \land S) \lor \neg Q] \land (P \lor \neg Q) \land \neg(P \land S)\}$
$\equiv \{(\neg P \lor \neg S \lor \neg Q) \land (P \lor \neg Q) \land (\neg P \lor \neg S)\}$ de morgans
$\equiv \{(\neg P \lor \neg S \lor \neg Q) \land (\neg P \lor \neg S) \land (P \lor \neg Q)\}$ Commutativity
$\equiv \{\neg(P \land S \land Q) \land (\neg P \lor \neg S) \land (P \lor \neg Q)\}$ ~~Commuta~~ demorgans
$\equiv \{\neg(P \land S \land (\neg S \lor \neg P) \land Q \land (\neg Q \lor P)\}$ commutativity
$\equiv \{\neg(P \land (S \land \neg S) \lor \neg P) \land (Q \land \neg Q) \lor P\}$ associative
$\equiv \{\neg(P \land \neg P \lor t) \land t \lor P\}$ idempotence
$\equiv \{\neg(P \land t) \land t\}$ identity
$\equiv \{(\neg P \lor f) \land t\}$ demorgan ~~identity~~
$\equiv -P \land t$ identity
$\equiv -P$ identity

c) $C \equiv [\neg P \rightarrow (P \rightarrow Q)] \rightarrow [Q \rightarrow (P \rightarrow P)]$

$C \equiv [\neg P \rightarrow (P \rightarrow Q)] \rightarrow [Q \rightarrow (P \rightarrow P)]$ implication

$\equiv \neg[P \vee (\neg P \vee Q)] \vee [\neg Q \vee P]$ associative

$\equiv \neg[(P \vee \neg P) \vee Q] \vee [\neg Q \vee P]$ identity

$\equiv \neg(t \vee Q) \vee [\neg Q \vee P]$ demorgan

$\equiv (f \wedge \neg Q) \vee [\neg Q \vee P]$ identity

$\equiv f \vee (\neg Q \vee P)$ identity

$\equiv \neg Q \vee P$ or $Q \rightarrow P$

d) $X \equiv [(P \vee \neg Q) \wedge (Q \vee \neg S) \wedge S \wedge \neg(P \wedge Q \wedge S)]$

D. $X \equiv [(P \vee \neg Q) \wedge (Q \vee \neg S) \wedge S \wedge \neg(P \wedge Q \wedge S)]$

$\equiv \{[(P \vee \neg Q) \wedge Q] \vee [(P \vee \neg Q) \wedge S]\} \wedge S \wedge \neg(P \wedge Q \wedge S)]$ distributive

$\equiv P \vee [(P \vee \neg Q) \wedge S] \wedge S \wedge \neg(P \wedge Q \wedge S)$ associative identity

$\equiv P \vee S \wedge (P \vee \neg Q) \wedge \neg(P \wedge Q \wedge S)$ idempotence

$\equiv S \vee P \wedge (P \vee \neg Q) \wedge \neg(P \wedge Q \wedge S)$ commutativity

$\equiv S \vee P \wedge \neg(P \wedge Q \wedge S)$ absorption

$\equiv S \vee P \wedge (\neg P \vee \neg Q \vee \neg S)$ de morgans

$\equiv S \vee (P \wedge \neg P) \vee \neg Q \vee \neg S$ associative

$\equiv S \vee t \vee \neg Q \vee \neg S$ identity

$\equiv t$ identity

8. Supply the reasons for each step needed to show that the following argument is valid.

a) The Chairman on the board of XYZ Automobile Company was strongly urging that the company purchase Ace Rubber Company. He based his recommendation on the following argument.

"If we buy Ace Rubber Co., then we can make our own tires. Our earnings will be higher if we sell our cars cheaper. People will invest in our company provided that our earnings are higher. Now, it is impossible to make our own tires and not sell our cars cheaper. Therefore, if we buy Ace Rubber Co. then the people will invest in our company."

A. I agree with the chairman of XYZ automobile's justification for acquiring Ace Rubber Company. The arguement for this is that if XYZ company puchases Ace Rubber Company, it will be able to produce it's own tires from othe sources. Since XYZ corporation now sells its car at high rates to pay the tires' expenditures.
However, if they manufacture their own tires, the business will be able to provide its automobiles for a fair price. Sales will eventually rise, and customers will as well, People would also be motivated to increase their ~~m~~ investments in the firm after viewing its profitability. Therefore, we can confidently state that purchasing Ace Rubber Company will be a wise move and in the long term, improve the ~~profitables~~ profits of XYZ company.

b) At the end of a long and heated trial, the defense attorney sums up his case as follows:

"If my client were guilty (G), then he must have been at the scene of the crime (A). It is
certainly not true that he was at the scene of the crime and at the same time was out of town (O).
Now, if the witness who identified my client as being out of town was not mistaken (~M), then
my client must have been out of town. But, the Prosecution Attorney was not able to prove that
the witness was mistaken. Therefore, my client is not guilty."

B. We may fairly conclude that the client was not guilty based on the justification presented. The defense attorney's client was not present at the crime scene. As previously stated, he is indeed out of town according to a witness who made the identification. Furthermore, the prosecution's lawyer is unable to show that the witness is mistaken. As a result, we may state that the client is innocent.

On the basis of this summation, should the defendant be found guilty or not guilty?

c) Hypothesis: (Conditional Proof)

$P \rightarrow R$

$Q \rightarrow R$

$\therefore (P \lor Q) \rightarrow R$

c

Proof	reason
$P \rightarrow R$	premise
$Q \rightarrow R$	premise
$P \lor Q$	ACP
$(P \rightarrow R) \land (Q \rightarrow R)$	1,2 conjunction
$R \lor R$	3,4 constructive dilemma
R	5 Tautology
$(P \lor Q) \rightarrow R$	3-6 constructive dilemma

the argument is valid, the defendant is not guilty

d) Hypothesis:

$\neg(P \land Q)$

$\neg R \rightarrow Q$

$\neg P \rightarrow R$

$\therefore R$

D.	Proof	Reason
1	$\neg(P \land Q)$	Premise
2	$\neg R \rightarrow Q$	Premise
3	$\neg P \rightarrow R$	Premise
4	$\neg Q \rightarrow \neg\neg R$	2. Trans
5	$\neg Q \rightarrow R$	4 DN
6	$(\neg P \rightarrow R) \land (\neg Q \rightarrow R)$	3,5 Conj
7	$\neg P \lor \neg Q$	1 DM
8	$R \lor R$	6 7CD
9	R	8 Tauto

e) Hypothesis:

P

$P \to Q$

$S \vee R$

$R \to \neg Q$

$\therefore S \vee T$

E Proof	Reason
P	
$P \to Q$	premise
$S \vee R$	
$R \to \neg Q$	
Q	1,2 MP
$\neg R$	4 5 MT
S	3 6 DS
$S \vee T$	7 Add

f) Hypothesis: (Indirect Proof)

$\neg Q \vee R$

$P \to \neg R$

Q

$\therefore \neg P$

F	Proof	Reason
1	$\neg Q \vee R$	
2	$P \to \neg R$	Premise
3	Q	
4	$P / \therefore f$	Add Premise
5	$Q \to R$	1 Impl
6	R	3 5 MP
7	$\neg P$	2 6 MT
8	$P \wedge \neg P$	4 7 Conj
9	f	8 negation