## CMPE 30043: DISCRETE MATHEMATICS

PROBLEM SET #2: LOGIC

p = "It is raining" 1. Let

q = "I am wearing my rubbers",

s = "I am carrying my umbrella".

Express each of the following statements in symbols:

a) "It is raining and I am not wearing my rubbers."

b) "It is not true that it is raining and I am wearing my rubbers."

c) "Either it is raining, or it is not true that it is raining or I am wearing my rubbers."

d) "Either it is not raining, or I am wearing my rubbers and I am carrying my umbrella."

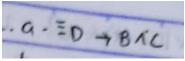
e) "Either it is raining and I am wearing my rubbers, or it is not raining and I am not carrying my umbrella."

2. Let's consider a compound propositions where

A ="Angelo comes to the party", B = "Bruno comes to the party", C = "Carlo comes to the party", D = "David comes to the party".

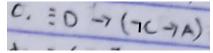
Represent the following statements into a Boolean expression:

a) "If David comes to the party then Bruno and Carlo come too"



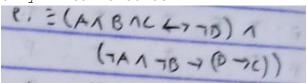
b) "David comes to the party if and only if Carlo comes and Angelo doesn't come"

c) "If David comes to the party, then, if Carlo doesn't come then Angelo comes"



d) "Carlo comes to the party provided that David doesn't come, but, if David comes, then Bruno doesn't come"

e) "Angelo, Bruno and Carlo come to the party if and only if David doesn't come, but, if neither Angelo nor Bruno come, then David comes only if Carlo comes"



- 3. Rephrase the following statements in the form "If P, then Q" or "P
- a.) If it is hot outside, you buy an ice cream cone, and if you buy an ice cream cone, it is hot outside.

b.) It rains if it is a weekend day, and it is a weekend day if it rains.

## b = it rains if and only if weekend day.

c.) If you read the news paper every day, you will be informed and conversely.

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_	,	be	informed	and	convers	240	10	5-1)	- 9-	

d.) To get tenure as a professor, it is sufficient to be world-famous.

d =	if you	va	ve world-f	amous	Henry	you	will	get	HUNCH	٤
	1		professor		7		*	T	7 7	

e.) It is necessary to walk 8 miles to get to the top of the Peak.

4. Write negations of each of the following conditional statements.

a.) If Master lives in the Philippines, then he lives in Pasig.

b.) If my car is in the repair shop, then I cannot get to class.

to class

c.) If x is prime then x is odd or x is 2.

di if x is not price, then x is neither odd or 2.

d.) If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

d = if n is not divisible by by then n is not divisible by 2 and 3.

- 5. Given the following conditional statements:
  - a.) Write the inverse of
    - i.) If it snows today, I will ski tomorrow

i = P-7Q = nP-7Q = if it didn't snow today, then I will not ski tomorrow

ii.) If my car is in the repair shop, then I cannot get to class.

ii = P-7 7Q = -P-> Q = if my car wash t in the repair stop, then I can get to dass.

b.) Write the converse of

i.) If today is Friday, then 2 + 3 = 5.

6. I = P-7Q = Q > P = 1f 2+3=5 | Hen today is Friday

ii.) If P is a square, then P is a rectangle.

ii = P -> Q = Q -> P = if P is a rectargle, then P is a square

c.) Write the contrapositive of

i.) If a shape is not a rectangle, then the shape does not have four sides.

= Q -> P	6, t- 0 = 1
if the shape have four sides, t	ten the shape is a
rectorgle	redtagle

ii.) If master owns a car, then he is rich.

i) =	P-7Q	1 9	13
=	72-77		
4 %	If he is not nich, then master doesn't own	9 0	ar

6. Construct the Truth Table and Determine whether each of the following compound proposition is a Tautology, Contradiction or Contingency.

a)  $Z \equiv (\neg P \rightarrow Q) \ V \ [(P \land \neg R) \leftrightarrow Q]$ 

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7	T	T FA	s do to do	SuiF 41	FA	longs	F) 2-	in to	19	0

$$b) \quad Z \equiv [\neg P \rightarrow (P \rightarrow Q)] \rightarrow [Q \rightarrow (P \rightarrow P)]$$

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7	F	F	F	T	T	Jeck	J. + +J	
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c) $Z \equiv (P \land R) \leftrightarrow [\neg S \lor (Q \rightarrow P)]$	$S V (Q \rightarrow P) \mid$	V	¬5	$\leftrightarrow$	K)	Λ	(P	≡	Z	C)
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F	Т	T	7	F	F	F	F	T	
7	F	F	F	F	T	T	7	F	
T	F	F	T	F	F	T	Т	F	
T	F	T	F	T	Т	7	7	T	
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7. Simplify the following using the algebra of propositions. Show your solutions.

a)  $A \equiv \neg \{ [Q \ V \ (P \ \land \ Q)] \ \land \ [P \ \land \ (P \ \land \ Q)] \}$ 

	TOUR OF THE PROPERTY OF THE
=1{[av(Pna)] 1 [pn(Pna)]}	absorption
=7{QATPA(PAQ)]}	associative/idempotence
= 7 { Q x (P x Q) }	associative -
= 7 {(a/a)/P}	idenpotence
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~ 70 X 7P	[(1-9/-9/-1/-1/9-9)-9-9-] =
	Company of the Compan

b)  $B = \{ [\neg (P \land S) \lor \neg Q] \land (P \lor \neg Q) \land \neg (P \land S) \}$ 

$D = \{ [\neg (I \land S) \lor \neg Q] \land (F \lor \neg Q) \}$	Λ¬(r Λ 3) }
0 = (F(PAS) V-Q] A(PV-Q) A-(PAS)}	(am) value
= {(7PX75V7Q) 1 (PV7Q) 1 (7PV75)}	de norgans
= {(7PV7SV7Q) 1 (7PV7S) 1 (PV7Q)}	Commutativity
= {-(PASAQ) A (-PV-15) A (PV-1Q)}	Commente demo your
= {7(PASA(ISVIP)AQA(IQVP)}	commutativity
= [7(P1(S175)V7P) 1 (Q17Q)VP}	associative
={7(PA7PV&t3) A tVP}	idempotence
= (7(PAL)A (7)	identity
= {7(PAt) 1 t}	de mergan Paleatary
={m(rpvf)/t}	floridentity of -=
E -PAt Sideson	identify ?
= -P ) santameti	[ ] [ (QAQ) ] F ;

 $c) \quad C \equiv [\neg P \rightarrow (P \rightarrow Q)] \rightarrow [Q \rightarrow (P \rightarrow P)]$ 

C = [-1P-7 (P-4Q)] -> [Q-7(P-7P)]	implication of
= 7[pv(7pva)] v [7a vp]	associative
= - [(PV-P) VQ] V[-QVP] - 1-10-19	A Torridentity = a
= 7(tvQ) v [-QVP]	demorgan
	(2 + 19) ridentity (19)-13
= fv(7QUP)	o-19) identity (2-19)
= TOVP OF Q7P	( E-PV-SVAC) A (PV-C)

d)  $X \equiv [(P \lor \neg Q) \land (Q \lor \neg S) \land S \land \neg (P \land Q \land S)]$ 

[(2NDNA) - N 2 N(2+VB) N(D+VA)] = X . O	,
= [(PVTQ)/Q] V[(PVTQ)/AS]]/ S AT[PAQAS)]	distributive
= PV[(PV7Q)AS]AS A7(PAQAS)]	associative identity
= PV SA(PV7Q) A7 (PAQAS)	idempotence
= SV PA(PV7A) A (PAQAS)	commutativity
= SV PA(PV7Q) A7 (PAQAS)	absorption
= SV PAT (PAQAS)	de morgans
= SVPA (7PV7QV7S)	associative
= SV(PA7P)V7QV7S	identity
= SV t V 7 Q V 7 S	identify
(= t)	

- $8. \ \mbox{Supply}$  the reasons for each step needed to show that the following argument is valid.
- a) The Chairman on the board of  ${\tt XYZ}$  Automobile Company was strongly urging that the

company purchase Ace Rubber Company. He based his recommendation on the following argument.

"If we buy Ace Rubber Co., then we can make our own tires. Our earnings will be higher

if we sell our cars cheaper. People will invest in our company provided that our earnings are

higher. Now, it is impossible to make our own tires and not sell our cars cheaper. Therefore, if we

buy Ace Rubber Co. then the people will invest in our company."

Justification for acquiring ALC Rubber Company. The arquement for this is that if xyz company. The ACC Rubber Company it will be able to produce it's own tires from other sources. Since xyz corporation now sells its car at high rates to pay the tires! expenditures.

However, if they manufacture their own tires, the business will be able to provide its automobiles for a fair price. Sales will eventually rise, and customers will as well, People would also be notivated to increase their mainvestments in the firm a fter viewing its profitability. Therefore, we can confidently state that purchasing Acc Rubber Company will be a wise more and in the long term, improve the parabable profits of xyz company.

b) At the end of a long and heated trial, the defense attorney sums up his case as follows:

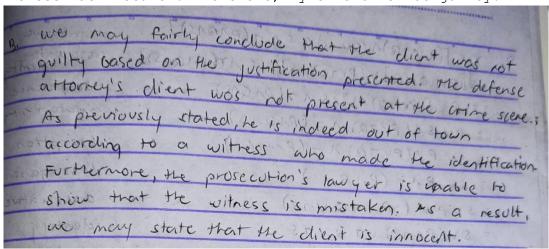
"If my client were guilty (G), then he must have been at the scene of the crime (A). It is

certainly not true that he was at the scene of the crime and at the same time was out of town (0).

Now, if the witness who identified my client as being out of town was not mistaken  $(\sim M)$ , then

my client must have been out of town. But, the Prosecution Attorney was not able to prove that

the witness was mistaken. Therefore, my client is not guilty."



On the basis of this summation, should the defendant be found guilty or not guilty?

c) Hypothesis: (Conditional Proof)

 $P \rightarrow R$ 

 $Q \rightarrow R$ 

 $\therefore (P \ V \ Q) \rightarrow R$ 

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Proof reason.
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Q78 premise de principa
Pray NACP = 12 9 TON 2021
(A-TR) 1 (Q-TR) 1,2 conjuntion
RVR 3,4 constructive dilemma
Read 5 Toutology day to
(PVB) 7R 3-4 constructive dilemna
the argument is valid, the
defendant is not guilty
will eventually one out the

d) Hypothesis:

 $\neg (P \land Q)$ 

 $\neg R \rightarrow Q$ 

 $\neg P \rightarrow R$ 

 $\therefore R$ 

·· 1	
D. Proff	Reason
1 7(PAQ)	Premise
2 7R7 Q	Premise
3 7P->R	Premise
9 7 97778	2. Frons
5 7Q7R	4 DN
G (7P-7R) 1 (1Q	-7R) 3,5 conj
7 76 1 20	1 DM
& RVR	6 100
9 R	8 Tauto
A STATE OF THE PARTY OF THE PAR	

## e) Hypothesis:

P

 $P \rightarrow Q$ 

S V R

 $R \rightarrow \neg Q$ 

 $\therefore S V T$ 

E proof	Reacon
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P-7Q	premise
SVR	
R-77Q	. 7
Q	1,2 MP
TR	4 5 MT
2	3 6 DS
SVT	7 Add

f) Hypothesis: (Indirect Proof)

 $\neg Q \ V \ R$ 

 $P \rightarrow \neg R$ 

Q

 $\therefore \neg P$ 

F	Proof	Reason
1	TOUR	, ,
2	PATR	Premise
3	Q	
4	P/f.	. ) Add Premisc
5	QTR	1 Impl
6	R	3 5 mp
7	70	2 4 MT
8	PATP	47 Con1
9	F	8 negation