Joshua H Nieve 2021-16253-MN-1
BSCoE 3-2L Discrete Mathematics 06/15/2022

CMPE 30043: DISCRETE MATHEMATICS
PROBLEM SET #1: SET THEORY

1. Let $A = \{x \in \boldsymbol{U} \mid x \text{ is a multiple of } 2\}$ $C = \{x \in \boldsymbol{U} \mid x \text{ is a multiple of } 3\}$
$B = \{x \in \boldsymbol{U} \mid x \text{ is a perfect square}\}$ $\boldsymbol{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 4, 6, 8, 10\}$ $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$B = \{1, 4, 9\}$ $\bar{B} = \{2, 3, 5, 6, 7, 8, 10\}$
$C = \{3, 6, 9\}$ $\bar{C} = \{1, 2, 4, 5, 7, 8, 10\}$

Find each set.

a.) $A \cup B$

a. $A \cup B = \{1\ 2\ 4\ 6\ 8\ 9\ 10\}$

b.) $A \cup C$

b. $A \cup C = \{2\ 3\ 4\ 6\ 8\ 9\ 10\}$

c.) $B - \bar{C}$

c. $B - \bar{C} = \{9\}$

d.) $\bar{B} \cap (A \cup C)$

d. $\bar{B} \cap (A \cup C) = \{2, 3, 6, 8, 10\}$

e.) $(C - B) \cup (B - A)$

e. $(C - B) \cup (B - A)$

$C - B = \{3, 6\}$

$B - A = \{1, 9\}$

$(C - B) \cup (B - A) = \{1, 3, 6, 9\}$

f.) $\overline{C - \overline{(B - A)}}$

f. $\overline{C - \overline{(B - A)}}$

$\overline{(B - A)} = \{2, 3, 4, 5, 6, 7, 8, 10\}$

$C - \overline{B - A} = \{9\}$

$\overline{C - \overline{(B - A)}} = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$

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2. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $X = \{1, 3, 5, 6, 7\}$
$Y = \{y \mid y = 2x, x \in Z\}$ $Z = \{z \mid z \text{ is a prime number}\}$

$X = \{1, 3, 5, 6, 7\}$ $\bar{X} = \{2, 4, 8, 9, 10\}$
$Y = \{4, 6, 10\}$ $\bar{Y} = \{1, 2, 3, 5, 7, 8, 9\}$
$Z = \{2, 3, 5, 7\}$ $\bar{Z} = \{1, 4, 6, 8, 9, 10\}$

Enumerate:

a.) $\overline{X - \bar{Z}}$

a. $\overline{X - \bar{Z}}$
$X - \bar{Z} = \{3, 5, 7\}$
$\overline{X - \bar{Z}} = \{1, 2, 4, 6, 8, 9, 10\}$

b.) $\overline{\bar{X} \cup (Y \cap \bar{X})}$

b. $\overline{\bar{X} \cup (Y \cap \bar{Z})}$
$Y \cap \bar{Z} = \{4, 6, 10\}$
$\bar{X} \cup (Y \cap \bar{Z}) =$ ~~[illegible]~~ $\{2\ 4\ 6\ 8\ 9\ 10\}$
$\overline{\bar{X} \cup (Y \cap \bar{Z})} = \{2, 8, 9\}$

c.) $\bar{Z} - \overline{Y - X}$

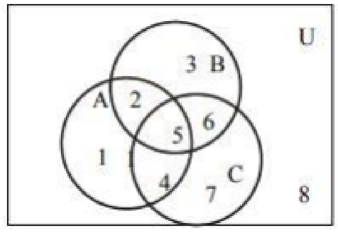
c. $\bar{Z} - \overline{Y - X}$
$Y - X = \{4, 10\}$
$\overline{Y - X} = \{1, 2, 3, 5, 6, 7, 8, 9\}$
$\bar{Z} - \overline{Y - X} = \{4, 10\}$

d.) $Y \cup (\bar{X} - Z)$

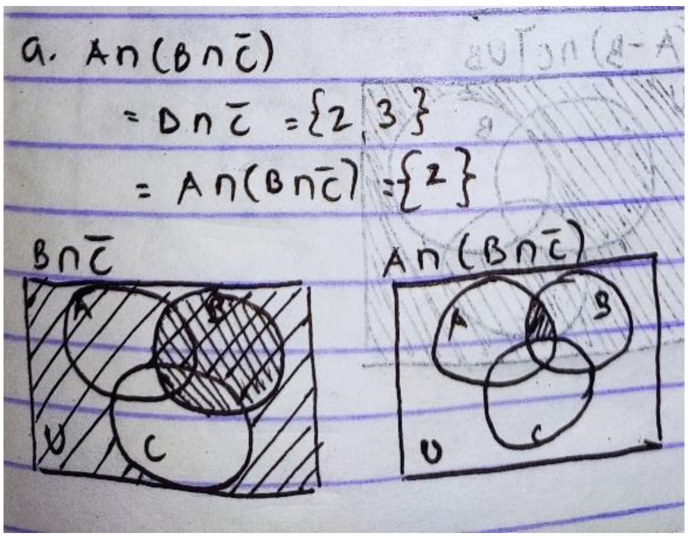
d. $Y \cup (\bar{X} - Z)$
$\bar{X} - Z = \{4, 8, 9, 10\}$
$Y \cup (\bar{X} - Z) = \{4, 6, 8, 9, 10\}$

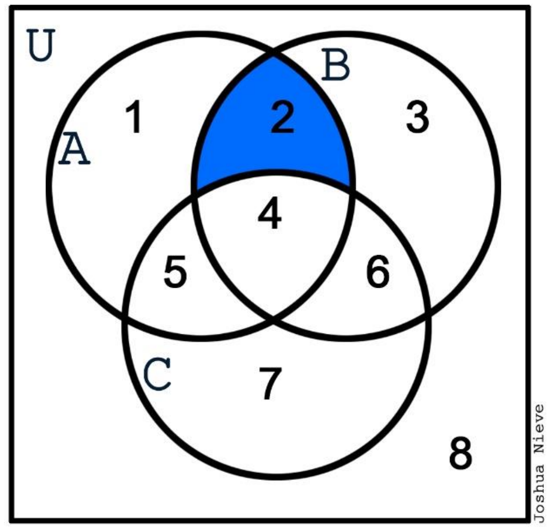
3. Let U be the universal set and let A, B and C be subsets of U. Sketch a Venn diagram for each set.

a.) $A \cap (B \cap \bar{C})$
b.) $(A \cap \bar{B}) \cup \bar{C}$
c.) $\overline{[(A - B) \cap C] \cup B}$

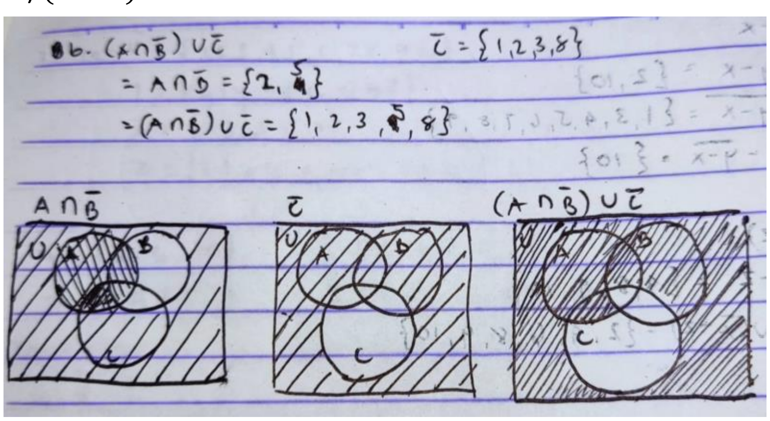


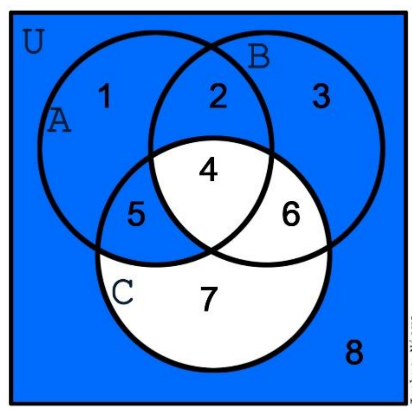
a.) $A \cap (B \cap \bar{C})$



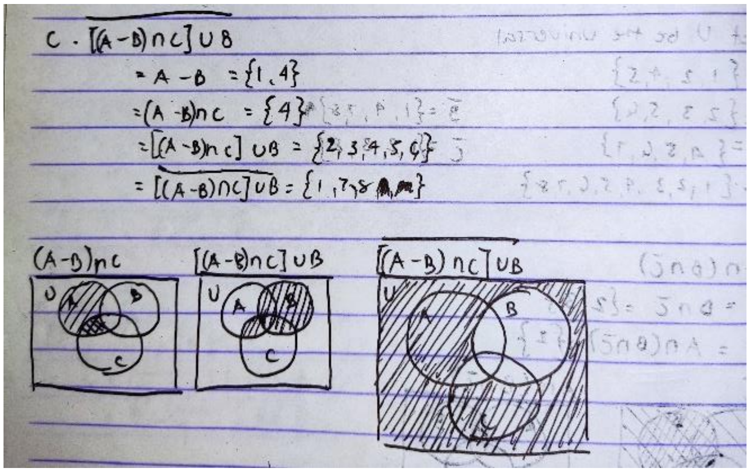


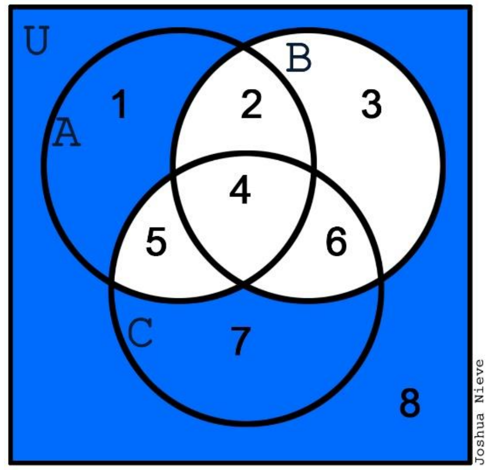
b.) $(A \cap \bar{X}) \cup \bar{C}$





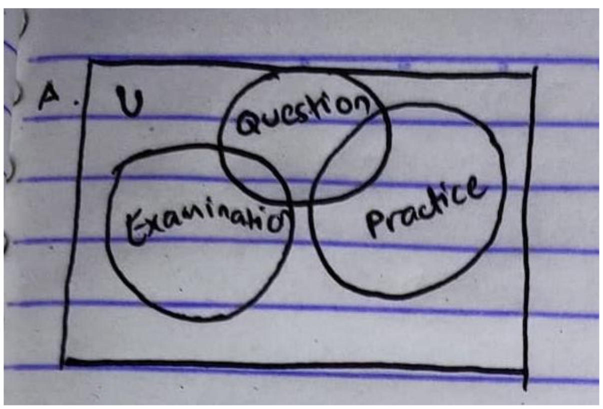
c.) $\overline{[(A-B)\cap C]\cup B}$



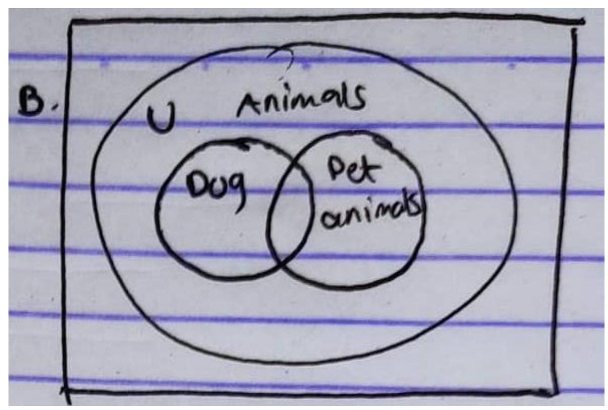


4. Represent the Venn diagrams that indicate the following:

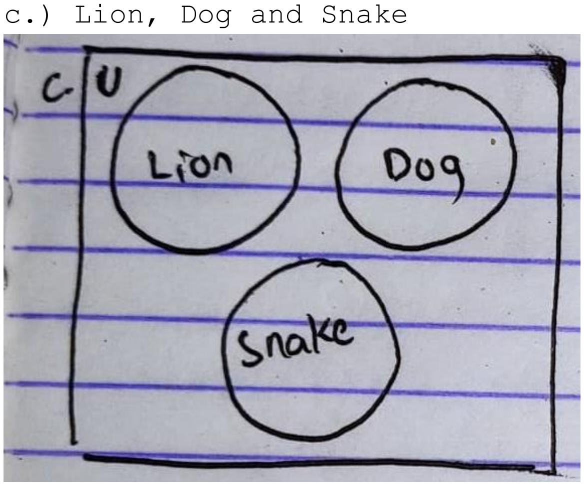
a.) Examination, Question and Practice



b.) Dogs, pet animals and animals

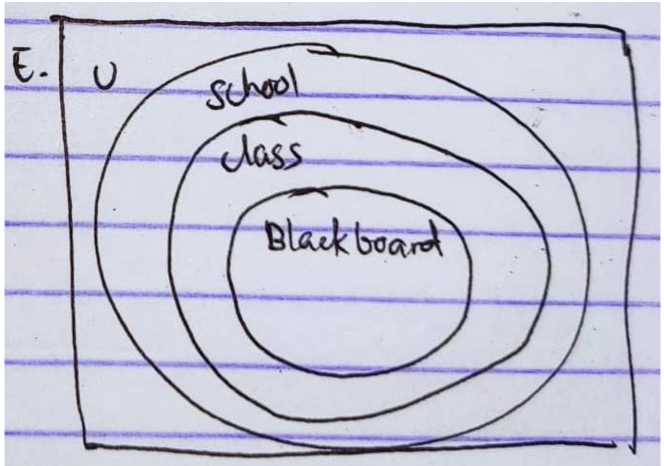


c.) Lion, Dog and Snake



d.) Judge, Thieves and Criminals

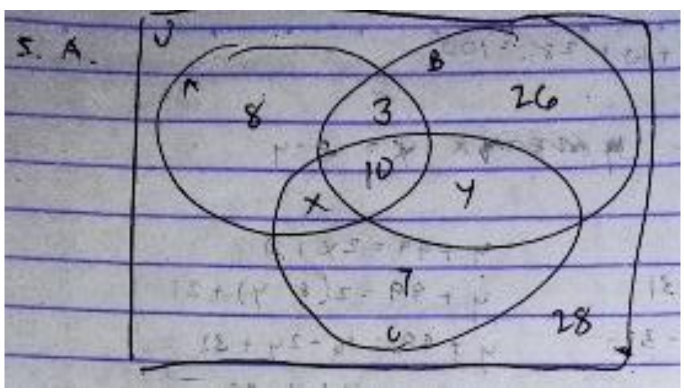
e.) Class, Blackboard and School



5. In a school, 100 students have access to three software packages A, B and C

28 did not use any software
8 used only packages A
26 used only packages B
7 used only packages C
10 used all three packages 13 used both A and B

a.) Draw a Venn diagram with all sets enumerated as for as possible. Label the two subsets which cannot be enumerated as x and y, in any order



b.) If twice as many students used package B as package A, write down a pair of simultaneous equations in x and y.

B. $8 + 3 + 10 + 26 + 7 + 28 + x + y = 100$

$82 + x + y = 100$

$x + y = 100 - 82$

$x + y = 18$

$26 + y + 3 + 10 = 2(8 + 3 + x + 10)$

$26 + y + 3 + 10 = 16 + 6 + 2x + 20$

$y + 39 = 2x + 42$

c.) Solve these equations to find x and y.

c. $x + y = 18$ $\quad y = 18 - x$

$y + 39 = 2x + 42$

$18 - x + 39 = 2x + 42$

$2x + x = 39 + 18 - 42$

$3x = 15$

$x = 5$

then

$82 + x + y = 100$

$82 + 5 + y = 100$

$87 + y = 100$

$y = 100 - 87$

$y = 13$ $\quad x = 5$

d.) How many students used package C?

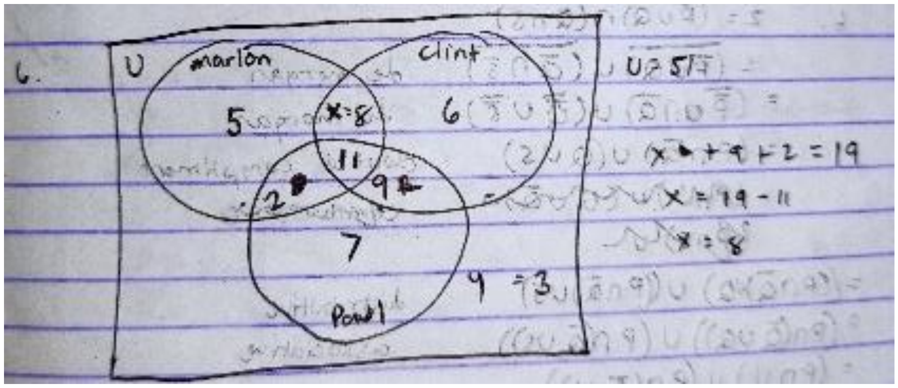
d. $7 + x + y + 10$

$7 + 5 + 13 + 10$

$= 35$ students used package C

6. A survey was carried out to find if students in the theater department liked the following three actors: Marlon Brando, Clint Eastwood and Paul Newman. Exactly 51 students participated in the survey.

- 26 students liked Marlon Brando.
- 34 students liked Clint Eastwood.
- 29 students liked Paul Newman.
- 2 students liked Paul Newman and Marlon Brando, but not Clint Eastwood.
- 9 students liked Paul Newman and Clint Eastwood, but not Marlon Brando.
- 19 students liked exactly two of the following three actors: Marlon Brando, Clint Eastwood and Paul Newman.
- 11 students liked all of the following three actors: Marlon Brando, Clint Eastwood and Paul Newman.



a.) How many students liked exactly one of the following three actors: Marlon Brando, Clint Eastwood and Paul Newman?

a. $5 + 6 + 7 = 18$ students liked exactly one

b.) How many students liked none the following three actors: Marlon Brando, Clint Eastwood and Paul Newman?

b. $5 + 8 + 6 + 11 + 9 + 2 + 7 + y = 51$

$48 + y = 51$

$y = 51 - 48$

$= 3$ students liked none

c.) How many students liked at most two of the following three actors: Marlon Brando, Clint Eastwood and Paul Newman?

c = ~~substract~~

$5 + 8 + 6 + 2 + 9 + 7 + 3 = 40$ students

or

$51 - 11 = 40$ students

7. Simplify each of the following sets using Set Identities

a.) $Z = (A - B) \cup (A \cap B)$

7. a. $Z = (A - B) \cup (A \cap B)$

$= (A \cap \bar{B}) \cup (A \cap B)$ set difference

$A \cap (B \cup \bar{B})$ distributive law

$A \cap (U)$ complement law

$A \cap U$ Identity Law

$= A$

b.) $Z = \overline{(\bar{P} \cup Q) \cap (\bar{Q} \cap \bar{S})}$

b. $Z = \overline{(\bar{P} \cup Q) \cap (\bar{Q} \cap \bar{S})}$

$= \overline{(\bar{P} \cup Q)} \cup \overline{(\bar{Q} \cap \bar{S})}$ de morgan

$= (\bar{\bar{P}} \cap \bar{Q}) \cup (\bar{\bar{Q}} \cup \bar{\bar{S}})$ de morgan

$= (P \cap \bar{Q}) \cup (Q \cup S)$ double compliment

commutative

$= ((P \cap \bar{Q}) \cup Q) \cup ((P \cap \bar{Q}) \cup S)$ distributive

$= (P \cap (\bar{Q} \cup Q)) \cup (P \cap (\bar{Q} \cup S))$ associative

$= (P \cap U) \cup (P \cap (\bar{Q} \cup S))$ complement / identity

$= P \cup (P \cap (\bar{Q} \cup S))$

$= A \cup (A \cap B)$ ABSORPTION LAW

$= P$

c.) $Z = S \cap (P \cup \bar{S}) \cap (Q \cup \bar{S}) \cap \overline{P \cap Q \cap S}$

c. $Z = S \cap (P \cup \bar{S}) \cap (Q \cup \bar{S}) \cap \overline{P \cap Q \cap S}$

$= (S \cap P) \cup (S \cap \bar{S}) \cap (Q \cup \bar{S}) \cap \overline{P \cap Q \cap S}$ distributive

$= (S \cap P) \cup \phi \cap (Q \cup \bar{S}) \cap \overline{P \cap Q \cap S}$ compliment law

$= (S \cap P) \cap (Q \cup \bar{S}) \cap \overline{P \cap Q \cap S}$ identity

$= ((S \cap P) \cap Q) \cup ((P \cap S) \cap \bar{S}) \cap \phi$ distributive

$= (P \cap (S \cap Q)) \cup (P \cap (S \cap \bar{S})) \cap \phi$ associative

$= (P \cap (S \cap Q)) \cup \phi \cap \overline{P \cap Q \cap S}$ complement / identity

$= P \cap (S \cap Q) \cap \bar{P} \cup \bar{Q} \cup \bar{S}$ de morgan

$= (S \cap Q) \cap P \cap \bar{P} \cup \bar{Q} \cup \bar{S}$ commutative

$= (S \cap Q) \cap \phi \cup \bar{Q} \cup \bar{S}$ identity

$= (S \cap Q) \cap \phi \cup \bar{Q} \cup \bar{S} = \phi \cup \bar{Q} \cup \bar{S} = \bar{Q} \cup \bar{S}$ identity

$S \cap \bar{S} \cap (P \cup Q) \cap \overline{P \cap Q \cap S}$ distributive

$\phi \cap (P \cup Q) \cap \overline{P \cap Q \cap S}$ complement / identity

$= A \cap Q = \phi$

$\phi \cap \overline{P \cap Q \cap S}$ de morgan

$\phi \cap \bar{P} \cup \bar{Q} \cup \bar{S}$ identity

$\phi \cup \bar{Q} \cup \bar{S}$

$= \bar{Q} \cup \bar{S}$

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8. Construct the truth table of the following set equations.

a.) $Z = (A - B) \cup (\mathrm{A} \cap \mathrm{B})$

A. $Z = (A - \bar{B}) - \overline{\bar{A} \cap (A - \bar{B})}$

A, B = 2 = $2^2 = 4$

A	B	A $\bar{B}$	A − $\bar{B}$	$\bar{A}$	$\bar{A} \cap (A - \bar{B})$	$\overline{\bar{A} \cap (A - \bar{B})}$	$(A - \bar{B}) - \overline{\bar{A} \cap (A - \bar{B})}$
0	0	1	0	1	0	1	0
0	1	0	0	1	0	1	0
1	0	1	0	0	0	1	0
1	1	0	1	0	0	1	0

b.) $Z = \overline{[(A - B) \cap \mathrm{C}] \cup \mathrm{B}}$

B. $Z = \overline{[(A - B) \cap C] \cup B}$

A, B, C = 3 = $2^3 = 8$

A	B	C	A − B	(A − B) ∩ C	[(A − B) ∩ C] ∪ B	$\overline{[(A - B) \cap C] \cup B}$
0	0	0	0	0	0	1
0	0	1	0	0	0	1
0	1	0	0	0	1	0
0	1	1	0	0	1	0
1	0	0	1	0	0	1
1	0	1	1	1	1	0
1	1	0	0	0	1	0
1	1	1	0	0	1	0

c.) $Z = S \cap (\mathrm{P} \cup \bar{S}) \cap (\mathrm{Q} \cup \bar{S}) \cap \overline{P \cap \mathrm{Q} \cap \mathrm{S}}$

C. $Z = S \cap (P \cup \bar{S}) \cap (Q \cup \bar{S}) \cap \overline{P \cap Q \cap S} = 0$

S, P, Q = 3 = $2^3 = 8$

P	Q	S	$\bar{S}$	P ∪ $\bar{S}$	Q ∪ $\bar{S}$	P ∩ Q	P ∩ Q ∩ S	$\overline{P \cap Q \cap S}$	S ∩ (P ∪ $\bar{S}$)	S ∩ (P ∪ $\bar{S}$) ∩ (Q ∪ $\bar{S}$)	S ∩ (P ∪ $\bar{S}$) ∩ (Q ∪ $\bar{S}$) ∩ $\overline{P \cap Q \cap S}$
0	0	0	1	1	1	0	0	1	0	0	0
0	0	1	0	0	0	0	0	1	0	0	0
0	1	0	1	1	1	0	0	1	0	0	0
0	1	1	0	0	1	0	0	1	0	0	0
1	0	0	1	1	1	0	0	1	0	0	0
1	0	1	0	1	0	0	0	1	1	0	0
1	1	0	1	1	1	1	0	1	0	0	0
1	1	1	0	1	1	1	1	0	1	1	0